Instantons in Deformed Supersymmetric Gauge Theories

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Based on the work
[hep-th/0705.3532, JHEP 07 (2007) 068]
(K.Ito, H.Nakajima and S.S)

Closed string background and deformed SYM theories

- Constant NS-NS 2-form B_{mn}

→Noncommutative SYM theory in D-brane world-volume

[Seiberg-Witten (1999)]

- Constant R-R $\mathcal{F}_{mnabc}$

→SYM theory in deformed geometry (non(anti)commutative superspace)

[Ooguri-Vafa, Seiberg (2003)]

We focus on deformed SYM theories defined on D-branes w.v. in the presence of R-R backgrounds

R-R backgrounds in IIB string theory

- Self-dual R-R 5-form field strength →deformation of superspace geometries

N=1 superspace deformation
[Ooguri-Vafa, Seiberg (2003)]

N=2 non-singletly deformed harmonic superspace from open string amplitudes
[Ito-S.S (2006)]

N=4 superspace deformation from open string amplitudes
[Ito-Kobayashi-S.S (2006)]

- R-R 1-form → deformation of superspace geometry with specific structure

N=2 (N=4) singletly deformed harmonic superspaces (v.o. charge conservation)

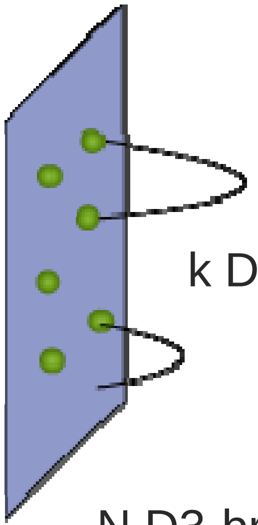
- R-R 3-form deformation?→ *[Ito-Nakajima-S.S. (2007)]*

D3/D(-1) system and instantons

ADHM construction = Systematic construction of instantons in gauge theories

[Atiyah-Hitchin-Drinfeld-Manin (1978)]

ADHM construction of instantons in SYM
→ D3/D-instanton brane configuration



<u>U(N), k-instanton = k D(-1), N D3</u>

D3/D(-1), D(-1)/D(-1) open string → ADHM moduli

<u>instanton effective action of 4D SYM theory</u>
→ D(-1)-brane effective action S_{eff}

$$\mathcal{F}^{(k)}(a) = \int d\widehat{\mathcal{M}}(k) e^{-\frac{8\pi k}{g_{\text{YM}}^2} - S_{\text{eff}}}$$

D-instanton effective action with SD R-R 3-form background

D3/D(-1) with self-dual R-R 3-form bkg. (N=2 graviphoton bkg. ε)

D-instanton effective action = Deformed instaton effective action

→ Ω -background deformation of N=2 instanton effective action

[Billo-Frau-Fucito-Lerda (2006)]

→BRST exact after topological twisting

→localization technique for moduli space integration of instanton partition function

Explicit calculation of instanton p.f.

[Nekrasov-Okounkov (2003)]

$$Z(a;\varepsilon) = \exp\left[\frac{\mathcal{F}_{n.p}(a;\varepsilon)}{\varepsilon^2}\right] = \exp\left[\sum_{k=1}^{\infty}\frac{1}{\varepsilon^2}\mathcal{F}^{(k)}(a;\varepsilon)\right]$$

$$\lim_{\varepsilon\to 0}\mathcal{F}_{n.p.}(a;\varepsilon) = \mathcal{F}_{SW}(a)$$

R-R backgrounds naturally regularize the moduli space integration

Deformed SYM in the presence of SD 3-form R-R bkg.

[Ito-Nakajima-S.S (2007)]

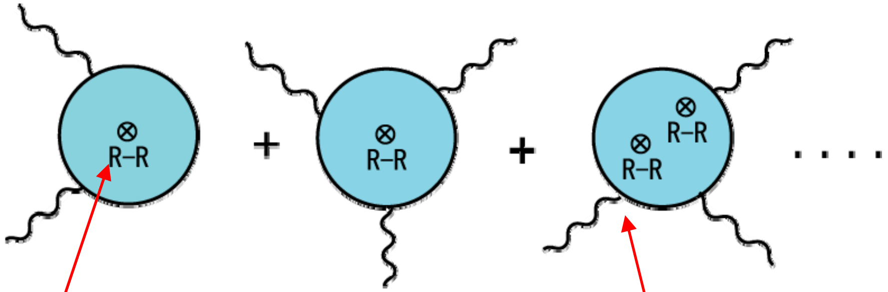
Filed theory calculations of deformed instantons

(consistency check)

Deformed SYM theory in the presence of self-dual R-R 3-form background

→ Open string disk amplitudes with closed string vertex insertion (NSR-formalism)

$$\mathcal{L} \sim A_{\mathrm{SYM}} +$$



R-R vertex operator

open string vertex operator

deformed action

$$\xrightarrow{\alpha' \to 0} \quad \mathcal{L} = \mathcal{L}_{\mathrm{SYM}} + \delta\mathcal{L}$$

RR correction

N=4 SYM in the presence or self-dual R-R 3-form bkg.

N Euclidean D3-branes in $\mathbf{R}^{10}$ → N=4 U(N) SYM

Self-dual R-R 3-form field strength background $\mathcal{F}^{\mu\nu a}$

- N=2 graviphoton and vector-like backgrounds are unified into N=4 graviphoton background
- All information of N=2 is included in the N=4
- # of non-zero disk amplitudes to calculate is less than N=2 case

N=4 open/closed string vertex operators

Field	Vertex Operator
Vector	$V_A^{(-1)}(y;p) = (2\pi\alpha')^{\frac{1}{2}} \frac{A_\mu(p)}{\sqrt{2}} \psi^\mu(y) e^{-\phi(y)} e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}$ $V_A^{(0)}(y;p) = 2i(2\pi\alpha')^{\frac{1}{2}} A_\mu(p) \left(\partial X^\mu(y) + i(2\pi\alpha')^{\frac{1}{2}} p\cdot\psi\psi^\mu(y)\right) e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}$
6 real scalars	$V_\varphi^{(-1)}(y;p) = (2\pi\alpha')^{\frac{1}{2}} \frac{\varphi_a(p)}{\sqrt{2}} \psi^a(y) e^{-\phi(y)} e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}$ $V_\varphi^{(0)}(y;p) = 2i(2\pi\alpha')^{\frac{1}{2}} \varphi_a(p) \left(\partial X^a(y) + i(2\pi\alpha')^{\frac{1}{2}} p\cdot\psi\psi^a(y)\right) e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}$
Gluino	$V_\Lambda^{(-1/2)}(y;p) = (2\pi\alpha')^{\frac{3}{4}} \Lambda^{\alpha A}(p) S_\alpha(y) S_A(y) e^{-\frac{1}{2}\phi(y)} e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}$ $V_{\bar\Lambda}^{(-1/2)}(y;p) = (2\pi\alpha')^{\frac{3}{4}} \bar\Lambda_{\dot\alpha A}(p) S^{\dot\alpha}(y) S^A(y) e^{-\frac{1}{2}\phi(y)} e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}$
Auxiliary field	$V_H^{(0)}(y;p) = a(2\pi\alpha') H_{\mu\nu}(p) \psi^\mu\psi^\nu(y) e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}$ $V_{H_{A\varphi}}^{(0)}(y;p) = b(2\pi\alpha') H_{\mu a}(p) \psi^\mu\psi^a(y) e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}$ $V_{H_{\varphi\varphi}}^{(0)}(y;p) = c(2\pi\alpha') H_{ab}(p) \psi^a\psi^b(y) e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}$
R-R field	$V_{\mathcal{F}}^{(-1/2,-1/2)}(z,\bar z) = (2\pi\alpha')\mathcal{F}^{(\alpha\beta)[AB]} \left[S_\alpha(z)S_A(z)e^{-\frac{1}{2}\phi(z)} S_\beta(\bar z)S_B(\bar z)e^{-\frac{1}{2}\phi(\bar z)}\right]$

note: closed string left and right mover are identified by boundary condition *[Billo-Frau-Pesando-Lerda (2004)]*

Non-zero amplitudes— selection rules

$\langle \cdots \mathcal{F} \rangle$

(I) $\mathcal{F} \sim S_\alpha S_A S_\beta S_B$ Charge cancellation combination

(II) α' power counting $D = \dfrac{1}{2\pi^2 \alpha'^2}$ Disk normalization + vertex operators

Zero-slope scaling

$$(2\pi\alpha')^{1/2}\mathcal{F} = C = \text{fixed}$$

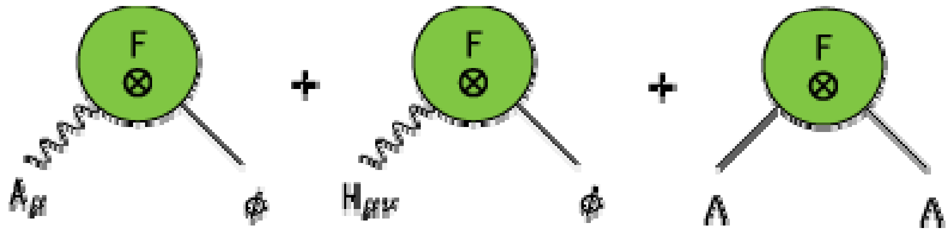
(III) Lorentz and internal index structure Ex. $\mathcal{F}^{(\alpha\beta)} \langle S_\alpha S_\beta \rangle \sim \mathcal{F}^{(\alpha\beta)} \varepsilon_{\alpha\beta} = 0$

There are three non-vanishing contributions $\Rightarrow$

$$\boxed{\langle A_\mu \varphi \mathcal{F} \rangle + \langle H_{\mu\nu} \varphi \mathcal{F} \rangle + \langle \Lambda\Lambda\mathcal{F} \rangle}$$

Deformed N=4 SYM action

$$\mathcal{L} = \mathcal{L}_{\mathrm{SYM}}^{\mathcal{N}=4} +$$



$$+\cdots$$

$$\langle\!\langle V_{X_1}^{(q_1)} \cdots V_{\mathcal{F}}^{(-\frac{1}{2},-\frac{1}{2})} \cdots \rangle\!\rangle$$

$$= C_{D_2} \int \frac{\prod_{i=1}^{n} dy_i \prod_{j=1}^{n_{\mathcal{F}}} dz_j d\bar{z}_j}{dV_{CKG}} \langle V_{X_1}^{(q_1)}(y_1) \cdots V_{\mathcal{F}}^{(-\frac{1}{2},-\frac{1}{2})}(z_1, \bar{z}_1) \cdots \rangle$$

<u>Zero-slope limit → N=4 deformed action</u>

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_{\mathrm{SYM}}^{\mathcal{N}=4} + \delta\mathcal{L} \\
&= \frac{1}{k g_{\mathrm{YM}}^2} \mathrm{Tr}\Big[-\frac{1}{2}\left(F_{\mu\nu}^{(+)} - i\varphi_a C_{\mu\nu}^{\ a}\right)^2 - i\Lambda^{\alpha A}(\sigma^\mu)_{\alpha\dot{\beta}} D_\mu \overline{\Lambda}^{\dot{\beta}}{}_A - \frac{1}{2}(D_\mu\varphi_a)^2 + \frac{1}{4}[\varphi_a, \varphi_b]^2 \\
&\quad + \frac{1}{2}(\Sigma^a)^{AB}\overline{\Lambda}_{\dot{\alpha}A}[\varphi_a, \overline{\Lambda}^{\dot{\alpha}}{}_B] + \frac{1}{2}(\overline{\Sigma}^a)_{AB}\Lambda^{\alpha A}[\varphi_a, \Lambda^B{}_\alpha] + \varepsilon_{ABCD}\Lambda_\alpha{}^A\Lambda_\beta{}^B C^{(\alpha\beta)[CD]}\Big]
\end{aligned}$$

Supersymmetry and consistecy with the Myers action

Remaining *deformed* SUSY

		rank of $C^{(a3)[12]}$		
		0	1	2
rank of $C^{(a3)[34]}$	0	$\mathcal{N}=(2.2)$	$\mathcal{N}=(3/2.1)$	$\mathcal{N}=(1.1)$
	1	$\mathcal{N}=(3/2.1)$	$\mathcal{N}=(1.0)$	$\mathcal{N}=(1/2.0)$
	2	$\mathcal{N}=(1.1)$	$\mathcal{N}=(1/2.0)$	$\mathcal{N}=(0.0)$

Myers action
$$S_{CS} = \frac{\mu_3}{k}\mathrm{STr}\int_{\mathcal{M}_4}\sum_n P[e^{i\lambda \mathrm{i}_\varphi \mathrm{i}_\varphi}\lambda^{\frac{1}{2}}\mathcal{A}^{(n)}]e^{\lambda F}$$

Self-dual 3-form (and its dual 7-form)
$$\frac{\mu_3}{k}\mathrm{STr}\int_{\mathcal{M}_4} P[e^{i\lambda \mathrm{i}_\varphi^2}\lambda^{\frac{1}{2}}\mathcal{A}^{(2)}]e^{\lambda F}\Big|_{SD} = \frac{1}{2kg_{\mathrm{YM}}^2}\int_{\mathcal{M}_4} d^4x\ \mathrm{Tr}\,[\varphi_a F_{\mu\nu}]\,(2\pi\alpha')^{\frac{1}{2}}\mathcal{F}^{\mu\nu a}$$
$$\frac{\mu_3}{k}\mathrm{STr}\int_{\mathcal{M}_4} P[e^{i\lambda \mathrm{i}_\varphi^2}\lambda^{\frac{1}{2}}\mathcal{A}^{(6)}]e^{\lambda F}\Big|_{SD} = 0$$

Bosonic part is consistent with the deformed action derived from amplitudes calculation

N=4 ADHM solution

[K.Ito, H.Nakajima, S.S, work in progress]

From the deformed action $\mathcal{L} = \mathcal{L}^{\mathcal{N}=4}_{\mathrm{SYM}} + \delta\mathcal{L}$, instanton equation and its solution are obtained

N=4 ADHM solution $a'_{ij}, w_{uj\dot\alpha}, \bar{w}^{\dot\alpha}_{uj}, \mu^A_{uj}, \mathcal{M}'^A_\alpha$: ADHM moduli

$$A^{(0)}_\mu = -i\bar{U}\partial_\mu U.$$

$$\Lambda^{(0)A}_\alpha = \Lambda_\alpha(\mathcal{M}^A) = \bar{U}(\mathcal{M}^A f \bar{b}_\alpha - b_\alpha f \bar{\mathcal{M}}^A)U.$$

$$\varphi^{(0)}_a = -\frac{1}{4}(\bar{\Sigma}^a)_{AB}\bar{U}\mathcal{M}^A f \bar{\mathcal{M}}^B U + \bar{U}\begin{pmatrix} \phi_a & 0 \\ 0 & \chi_a \mathbf{1}_{2\times 2} + \mathbf{1}_{k\times k} C_a \end{pmatrix} U$$

$$U_{\lambda v} = \begin{pmatrix} V_{uv} \\ (U'_\alpha)_{iv} \end{pmatrix} \qquad \mathcal{M}^A{}_{\lambda i} = \mathcal{M}^A_{(u+l\beta)i} = \begin{pmatrix} \mu^A{}_{ui} \\ (\mathcal{M}'^A_\beta)_{li} \end{pmatrix}$$

$$V = (1 - w_{\dot\alpha} f \bar{w}^{\dot\alpha})^{\frac{1}{2}}. \quad U'_\alpha = -(a' + x)_{\alpha\dot\alpha} f \bar{w}^{\dot\alpha} \bar{V}^{-1}. \quad f \to |x|^{-2} \ (|x| \to \infty)$$

$$\phi_a = \langle \varphi_a \rangle$$

ADHM constraint

ADHM constraint

$$(\bar{\Delta}\Delta) = f^{-1}1_{2\times 2}$$

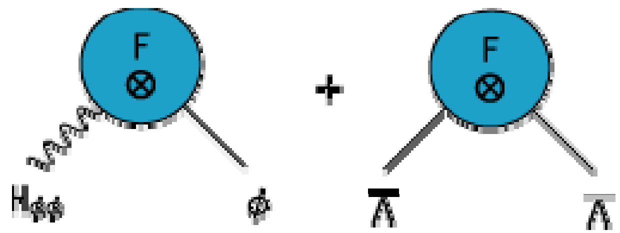
$$\bar{\mathcal{M}}^A\Delta + \bar{\Delta}\mathcal{M}^A = 0$$

$$\Delta_{\lambda j\dot{\alpha}} = a_{\lambda j\dot{\alpha}} + b^3_{\lambda j}x_{3\dot{\alpha}} = \begin{pmatrix} w_{uj\dot{\alpha}} \\ (a'_{ij} + \delta_{ij}x)_{\alpha\dot{\alpha}} \end{pmatrix}. \quad (\lambda = u + i\alpha)$$

Constraint on χ_a

$$\frac{1}{2}\{\bar{w}^{\dot{\alpha}}w_{\dot{\alpha}}, \chi_a\} + [a'_\mu, [a'^\mu, \chi_a]] = \frac{1}{4}(\bar{\Sigma}^a)_{AB}\bar{\mathcal{M}}^A\mathcal{M}^B + \bar{w}^{\dot{\alpha}}\phi_a w_{\dot{\alpha}} + C^{\mu\nu a}[a'_\mu, a'_\nu]$$

Comment on mass deformation
(R-R 3-form with internal SD condition)

$$\mathcal{L} = \mathcal{L}_{\mathrm{SYM}}^{\mathcal{N}=4} +$$ 

Mass deformation of the instanton effective action can be achieved by
Another type of R-R 3-form backgorund (self-dual in internal space)

$$\begin{aligned}
\delta\mathcal{L} &= \frac{1}{kg_{\mathrm{YM}}^2}\mathrm{Tr}\left[(\bar{\Sigma}^a\Sigma^b\bar{\Sigma}^c)_{AB}\varphi_a\varphi_b\varphi_c\right]C^{(AB)} + \frac{2}{kg_{\mathrm{YM}}^2}\mathrm{Tr}\left[\bar{\Lambda}_{\dot{\alpha}A}\bar{\Lambda}^{\dot{\alpha}}_{B}\right]C^{(AB)} \\
&= \frac{1}{4}\frac{1}{kg_{\mathrm{YM}}^2}\mathrm{Tr}\left[(\bar{\Sigma}^a\Sigma^b\bar{\Sigma}^c)_{AB}(\bar{\Sigma}^a\Sigma^b\bar{\Sigma}^d)_{CD}\varphi_c\varphi_d\right]C^{(AB)}C^{(CD)} \\
&= \frac{1}{2kg_{\mathrm{YM}}^2}\int d^2\theta\ \mathrm{Tr}\left[(m_i\Phi_i^2)\right] + \frac{1}{2kg_{\mathrm{YM}}^2}\int d^2\bar{\theta}\ \mathrm{Tr}\left[(\bar{m}_i\bar{\Phi}_i^2)\right]
\end{aligned}$$

for appropriate mass matrix background

N=4 → N=2 reduction

Set-up

Orbifolding $\mathbf{R}^4 \times \mathbf{C} \times \mathbf{C}^2/\mathbf{Z}_2$

N (fractional) D3-branes at orbifold fixed point

D3/D3 Vertex operator projection
→U(N) N=2 SYM on the branes

$$V_{\mathcal{F}}^{(-1/2,-1/2)}(z,\bar{z})=(2\pi\alpha')\mathcal{F}^{(\alpha\beta)[ij]}\left[S_\alpha(z)S^{(-)}(z)S_i(z)e^{-\frac{1}{2}\phi(z)}S_\beta(\bar{z})S^{(-)}(\bar{z})S_j(\bar{z})e^{-\frac{1}{2}\phi(\bar{z})}\right]$$

Constant graviphoton background

$$V_{\mathcal{F}}^{(-1/2,-1/2)}(z,\bar{z})=(2\pi\alpha')\mathcal{F}^{(\alpha\beta)[\hat{i}\hat{j}]}\left[S_\alpha(z)S^{(+)}(z)S_{\hat{i}}(z)e^{-\frac{1}{2}\phi(z)}S_\beta(\bar{z})S^{(+)}(\bar{z})S_{\hat{j}}(\bar{z})e^{-\frac{1}{2}\phi(\bar{z})}\right]$$

Vector-like background (after orbifold projection)

N=4 → N=2 background decomposition

graviphoton

$$C^{(AB)} = \frac{1}{2}\begin{pmatrix} C^{(ij)} & 0 \\ 0 & \bar{C}^{(\hat{i}\hat{j})} \end{pmatrix}$$

Vector bkg.

N=4 → N=2 reduction

N=4 → N=2 orbifold projection for open string v.o.

$$\Lambda_\alpha^A = \overline{\Lambda}_A^{\dot\alpha} = 0 \ (A = 3,4)$$

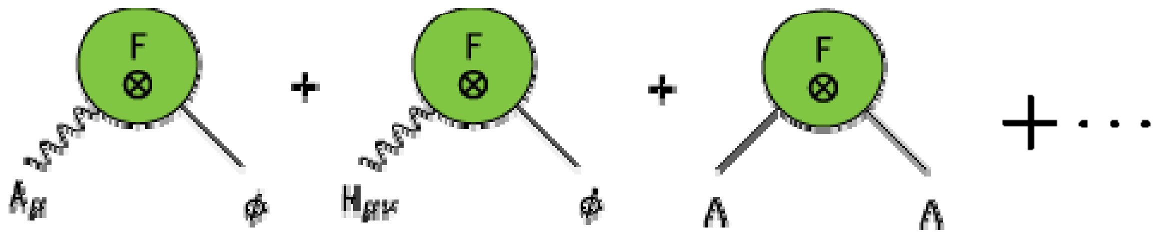
$$\varphi_a = 0 \ (a = 7,8,9,10)$$

$$\varphi = \frac{1}{\sqrt{2}}(\varphi_5 - i\varphi_6), \bar\varphi = \frac{1}{\sqrt{2}}(\varphi_5 + i\varphi_6)$$

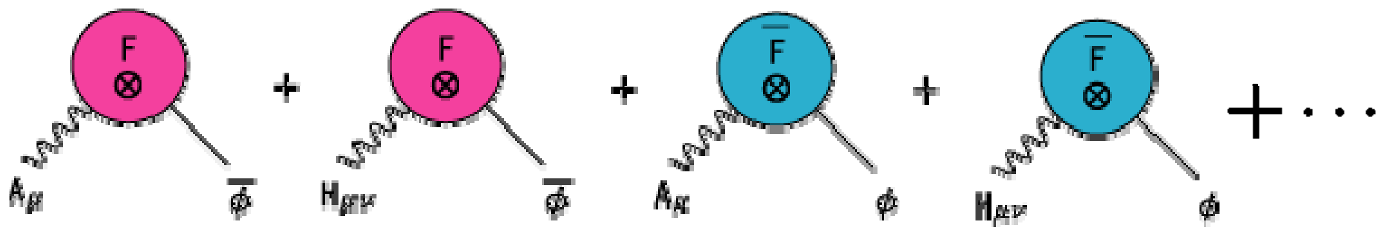
$$\begin{aligned}
\mathcal{L}^{\mathcal{N}=4\to\mathcal{N}=2} &= \frac{1}{kg_{\mathrm{YM}}^2}\mathrm{Tr}\Big[iF_{\mu\nu}\bar\varphi C^{\mu\nu} - i\varphi F_{\mu\nu}\bar C^{\mu\nu} + \frac{\sqrt{2}}{2}\Lambda_\alpha{}^i\Lambda_{3i}\bar C^{(\alpha 3)} \\
&\qquad + \frac{1}{2}(\bar\varphi C^{\mu\nu})^2 - (\bar\varphi C_{\mu\nu})(\varphi\bar C^{\mu\nu}) + \frac{1}{2}(\varphi\bar C^{\mu\nu})^2\Big] \\
&= \frac{1}{kg_{\mathrm{YM}}^2}\mathrm{Tr}\left[-\frac{1}{2}\left(F_{\mu\nu}^{(+)} - (i\bar\varphi C_{\mu\nu} - i\varphi\bar C_{\mu\nu})\right)^2 + \frac{\sqrt{2}}{2}\Lambda_\alpha{}^i\Lambda_{3i}\bar C^{(\alpha 3)}\right]
\end{aligned}$$

Amplitudes reduction

$$\mathcal{L} = \mathcal{L}_{\text{SYM}}^{\mathcal{N}=4} + $$



$$\mathcal{L} = \mathcal{L}_{\text{SYM}}^{\mathcal{N}=2} + $$



→graviphoton + vector-like background

Deformed N=2 SYM : SUSY

$$\mathcal{L} = \mathcal{L}_{\mathrm{SYM}}^{\mathcal{N}=2} + \delta\mathcal{L} + \mathcal{O}(C^3)$$

$$\begin{aligned}\delta\mathcal{L} \;=\; \frac{1}{k g_{\mathrm{YM}}^2}\mathrm{Tr}\Big[& i F_{\mu\nu}\bar{\varphi}C^{\mu\nu} - i\varphi F_{\mu\nu}\bar{C}^{\mu\nu} + \frac{\sqrt{2}}{2}\Lambda_\alpha{}^i\Lambda_{\beta i}\bar{C}^{(\alpha\beta)} \\ & + \frac{1}{2}(\bar{\varphi}C^{\mu\nu})^2 - (\bar{\varphi}C_{\mu\nu})(\varphi\bar{C}^{\mu\nu}) + \frac{1}{2}(\varphi\bar{C}^{\mu\nu})^2\Big]\end{aligned}$$

Supersymmetry is *deformed* and the number of SUSY is dependent on the rank of the backgorund

		rank of $C^{(\alpha\beta)}$		
		0	1	2
rank of $\bar{C}^{(\alpha\beta)}$	0	$\mathcal{N}=(1,1)$	$\mathcal{N}=(1,0)$	$\mathcal{N}=(1,0)$
	1	$\mathcal{N}=(1/2,1)$	$\mathcal{N}=(1/2,0)$	$\mathcal{N}=(1/2,0)$
	2	$\mathcal{N}=(0,1)$	$\mathcal{N}=(0,0)$	$\mathcal{N}=(0,0)$

Deformed ADHM from deformed SYM *[K.Ito, H.Nakajima, S.S, work in progress]*

ADHM solution

$$A_\mu^{(0)} = -i\overline{U}\partial_\mu U.$$

$$\Lambda_\alpha^{(0)i} = \Lambda_\alpha(\mathcal{M}^i) = \overline{U}(\mathcal{M}^i f \overline{b}_\alpha - b_\alpha f \overline{\mathcal{M}}^i)U.$$

$$\varphi^{(0)} = i\frac{\sqrt{2}}{4}\epsilon_{ij}\overline{U}\mathcal{M}^i f \overline{\mathcal{M}}^j U + \overline{U}\begin{pmatrix} \phi & 0 \\ 0 & \chi \mathbf{1}_{2\times 2} - \mathbf{1}_{k\times k} C \end{pmatrix} U$$

$$\bar{\varphi}^{(0)} = \bar{U}\begin{pmatrix} \bar{\phi} & 0 \\ 0 & \bar{\chi}\mathbf{1}_{2\times 2} + \mathbf{1}_{k\times k}\bar{C} \end{pmatrix} U$$

Deformed constraint

$$\mathbf{L}\chi = -i\frac{\sqrt{2}}{4}\epsilon_{ij}\bar{\mathcal{M}}^i\mathcal{M}^j + \bar{w}^{\dot{\alpha}}\phi w_{\dot{\alpha}} + C^{\mu\nu}[a'_\mu, a'_\nu]$$

$$\mathbf{L}\bar{\chi} = \bar{w}^{\dot{\alpha}}\bar{\phi}w_{\dot{\alpha}} + \bar{C}^{\mu\nu}[a'_\mu, a'_\nu]$$

$$\mathbf{L} = \frac{1}{2}\{\bar{w}^{\dot{\alpha}}w_{\dot{\alpha}}, *\} + [a'_\mu, [a'^\mu, *]]$$

Instanton effective action

Instanton effective action $f_{mn} = 2C_{mn}, \ \bar{f}_{mn} = 2\bar{C}_{mn}$

$$
\begin{aligned}
S_{\text{eff}}^{(0)} \ = \ & 2\pi^2 \text{tr}_k[-2([\chi^\dagger, a'_m] - 2i\bar{f}^{mn}a'_n)([\chi, a'^m] - 2if^{mn}a'_n) \\
& + (\chi^\dagger \bar{w}_{\dot{\alpha}} - \bar{w}_{\dot{\alpha}}\bar{a})(w^{\dot{\alpha}}\chi - aw^{\dot{\alpha}}) + (\chi\bar{w}_{\dot{\alpha}} - \bar{w}_{\dot{\alpha}}a)(w^{\dot{\alpha}}\chi^\dagger - \bar{a}w^{\dot{\alpha}}) \\
& + i\frac{\sqrt{2}}{2}\bar{\mu}^i\epsilon_{ij}(\mu^j\chi^\dagger + \bar{a}\mu^j) - i\frac{\sqrt{2}}{4}M^{\alpha i}\epsilon_{ij}([\chi^\dagger, M^j_\alpha] - \frac{i}{2}\bar{f}_{mn}(\sigma^{mn})_{\alpha 3}M^{3j})] \\
& + S_{ADHM}
\end{aligned}
$$

$$
S_{ADHM} = \text{tr}_k[-i\vec{D}\cdot\vec{\tau}^{\dot{\alpha}}{}_{\dot{3}}(\bar{w}^{\dot{3}}w_{\dot{\alpha}} + \bar{a}'^{\dot{3}\alpha}a'_{\alpha\dot{\alpha}}) - i\lambda^{\dot{\alpha}}_i(\bar{\mu}^i w_{\dot{\alpha}} + \bar{w}_{\dot{\alpha}}\mu^i + [a'_{\alpha\dot{\alpha}}, M^{\alpha i}])].
$$

[Billo-Frau-Fucito-Lerda (2006)]

→Precisely reproduces D-instanton effective action in the presence of Omega-background (up to $\mathcal{O}(C\bar{C})$ term)

R-R deformed N=2 instanton ⇔ N=2 Omega-background effective action

R-R deformed N=1,4 instanton $\overset{?}{\Leftrightarrow}$ N=1,4 Omega-background effective action

[K.Ito, H.Nakajima and S.S, work in progress]

Summary and future work

- Derived deformed N=4 SYM action that correctly reproduces N=2 D3/D(-1) system with self-dual R-R background and also omega-background deformation of instanton effective action
- ADHM solution in the deformed N=2,4 SYM is found
- Gravitational correction to instanton solutions

[future work]

- Perform complete calculation of N=4 instanton effective action and find the similarity with N=2 deformed instanton calculus
- N=1 generalization

Omega-background deformation

6D Omega-background

$$ds_6^2 = dzd\bar{z} + (dx^\mu + \Omega^\mu dz + \bar{\Omega}^\mu d\bar{z})^2$$
$$\Omega^\mu \equiv \Omega^{\mu\nu} x_\nu, \ \ \overline{\Omega}^\mu = \overline{\Omega}^{\mu\nu} x_\nu$$

Omega-deformed N=2 SYM $\mathcal{L}(\Omega) = \mathcal{L}_{\mathsf{SYM}}^{\mathcal{N}=2} + \delta\mathcal{L}$

$$\begin{aligned}\delta\mathcal{L} \ = \ +\frac{1}{k}\mathrm{Tr}\Big[&\sqrt{2}gF_{\mu\nu}D^\mu\bar{\varphi}\Omega^\nu - \sqrt{2}gF_{\mu\nu}D^\mu\varphi\overline{\Omega}^\nu + ig^2D_\mu\bar{\varphi}D_\nu\bar{\varphi}\Omega^\mu\Omega^\nu \\ &-i\sqrt{2}g^2D_\mu\bar{\varphi}[\varphi.\bar{\varphi}]\Omega^\mu - i\sqrt{2}g^2D_\mu\varphi[\varphi.\bar{\varphi}]\overline{\Omega}^\mu - 2g^2F_{\mu\rho}F_\nu{}^\rho\Omega^\mu\overline{\Omega}^\nu \\ &-2ig^2D_\mu\bar{\varphi}D_\nu\varphi\Omega^\mu\overline{\Omega}^\nu + ig^2D_\mu\varphi D_\nu\varphi\overline{\Omega}^\mu\overline{\Omega}^\nu - 2ig^3[\varphi.\bar{\varphi}]F_{\mu\nu}\Omega^\mu\overline{\Omega}^\nu\Big]\end{aligned}$$

→ Deformed instanton effective action

$$S_{\mathsf{eff}}^{(0)}(\Omega, \overline{\Omega} = 0) = S_{RR}^{(0)}$$

BRST-exact after topological twisting

→Applicable localization technique in moduli space integration

→Instanton partition function can be calculated *[Nekrasov-Okounkov (2004)]*