Flavour Violating Interactions of Supersymmetric Particles

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Work in Progress

Layout

- Part 1:

  Deriving RGEs with thresholds.

- Part 2:

  Illustrative Calculation: $\widetilde{t}_1 \to c\widetilde{Z}_1$.

Introduction of Thresholds

- Non-degenerate SUSY spectrum.
- Necessary for true two loop accuracy.
- Introduce step functions which alter RG running as particles are removed from effective theory.

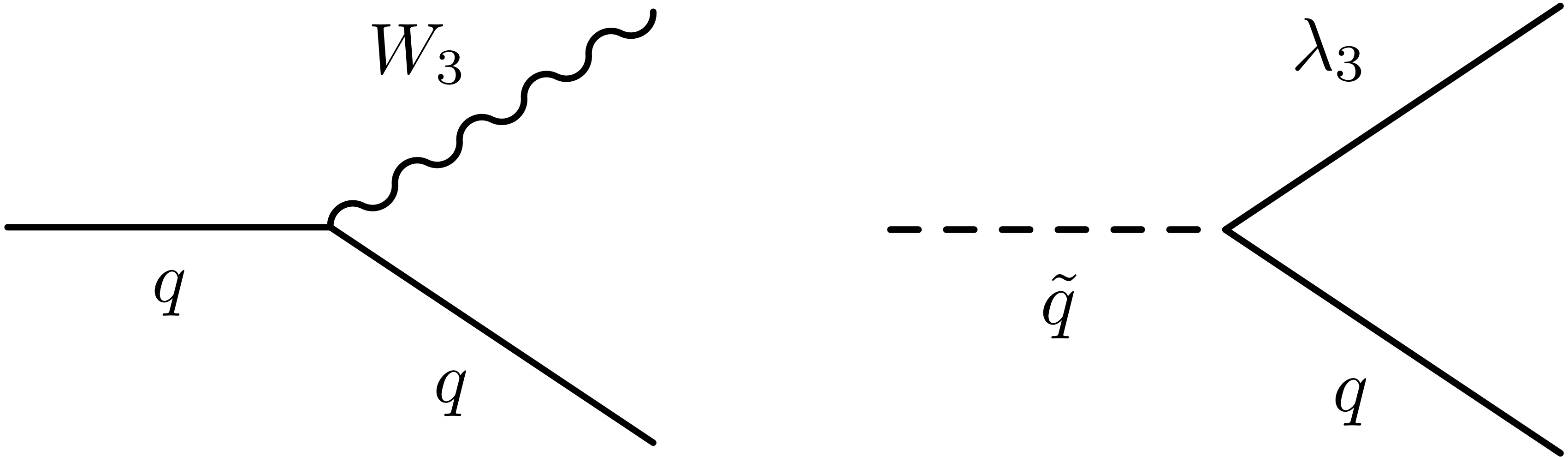
Castaño, Piard, Ramond (1994) + Errata
Dedes, Lahanas, Tamvakis (1996)

Deriving the RGEs including Thresholds

- Use RGE for general theory from Machacek & Vaughn, and Luo et al.
- Converted to 4-component form with complex scalars and both Majorana and Dirac fermions $\Rightarrow$ phenomenologist friendly.
- For correct decoupling must write Lagrangian in mass basis.

Machacek & Vaughn (1983) I,II,III
Luo, Wang, Xiao (2003)

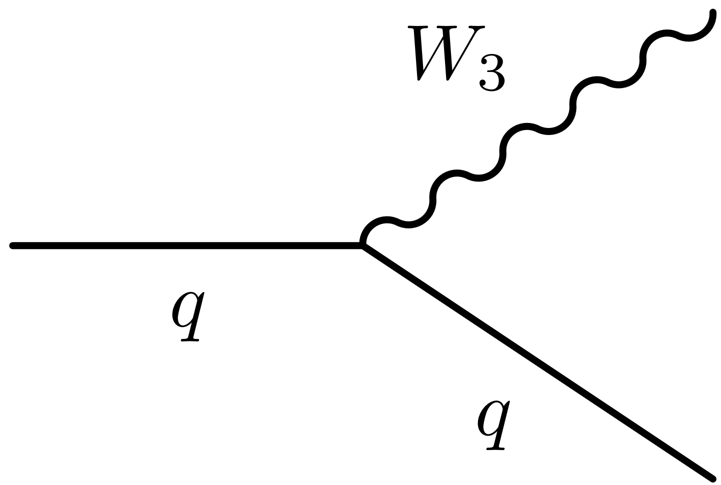
Additional couplings when SUSY is broken

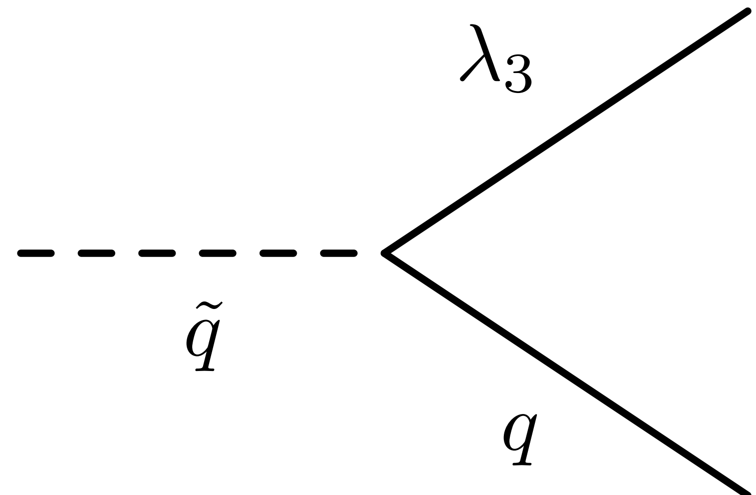


Equal in the SUSY limit. RGE for g depends only on g.

Can conceptually be different when some SUSY particles have decoupled from the theory.

Additional couplings when SUSY is broken





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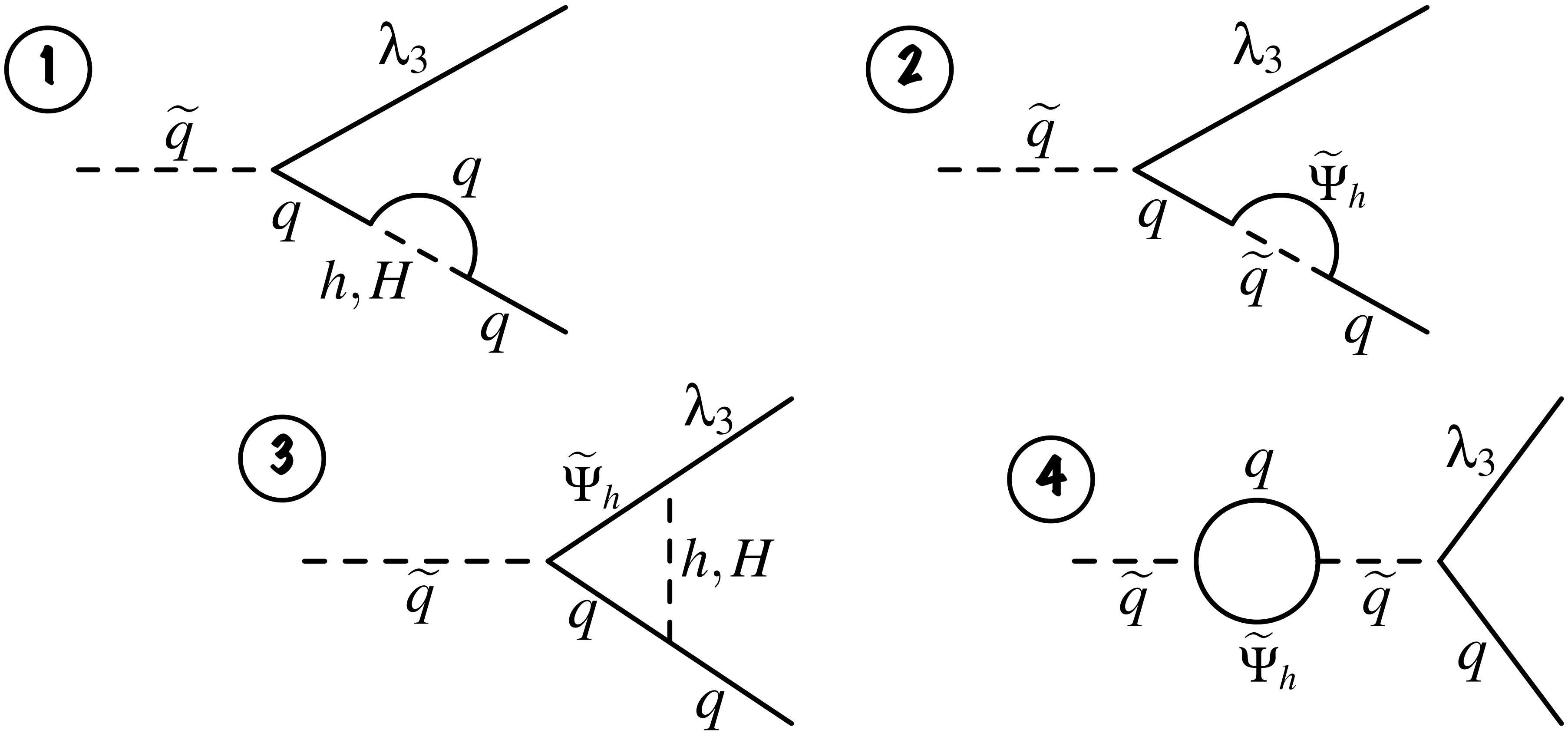
Can conceptually be different when some SUSY particles have decoupled from the theory.

New squark-quark-gaugino coupling: $\left(\tilde{g}^{Q}\right)_{ij}$.

Example: Case where $(\tilde{g}^Q)_{ij}$ is not equal to $g(\mathbb{1})_{ij}$.



These terms appear in the RGE as:

1 + 2 - 4x 3 + 2x 4 = zero

If one of the exchanged particles disappears from the theory, $\tilde{g}$ can depend on f, $\tilde{f}$, g and $\tilde{g}$.

Sample RGE: f_u

$$
\begin{aligned}
(4\pi)^2 \frac{d(s\mathbf{f}_u)}{dt} = & \frac{s}{2}\left\{3\left[s^2\theta_h + c^2\theta_H\right](\mathbf{f}_u)(\mathbf{f}_u)^\dagger + \left[c^2\theta_h + s^2\theta_H\right](\mathbf{f}_d)(\mathbf{f}_d)^\dagger + 4c^2\left[-\theta_h + \theta_H\right](\mathbf{f}_d)(\mathbf{f}_d)^\dagger\right\}(\mathbf{f}_u) \\
& + s(f_u)_{ik}\left[\theta_{\tilde h}\theta_{\tilde Q_l}(\tilde f_u^Q)^\dagger_{kl}(\tilde f_u^Q)_{lj} + \frac{4}{9}\theta_{\tilde B}\theta_{\tilde u_l}(\tilde g'_{u_R})^*_{kl}(\tilde g'_{u_R})^T_{lj} + \frac{4}{3}\theta_{\tilde g}\theta_{\tilde u_l}(\tilde g^s_{u_R})^*_{kl}(\tilde g^s_{u_R})^T_{lj}\right] \\
& + \frac{s}{4}\Big[2\theta_{\tilde h}\theta_{\tilde u_k}(\tilde f_u^{u_R})_{ik}(\tilde f_u^{u_R})^\dagger_{kl} + 2\theta_{\tilde h}\theta_{\tilde d_k}(\tilde f_d^{d_R})_{ik}(\tilde f_d^{d_R})^\dagger_{kl} + 3\theta_{\tilde W}\theta_{\tilde Q_k}(\tilde g_Q)^T_{ik}(\tilde g_Q)^*_{kl} \\
& + \frac{1}{9}\theta_{\tilde B}\theta_{\tilde Q_k}(\tilde g'_Q)^T_{ik}(\tilde g'_Q)^*_{kl} + \frac{16}{3}\theta_{\tilde g}\theta_{\tilde Q_k}(\tilde g^s_Q)^T_{ik}(\tilde g^s_Q)^*_{kl}\Big](f_u)_{lj} \\
& + s\theta_{\tilde h}\theta_{\tilde Q_k}\left[-3\theta_{\tilde W}(\tilde g_{h_u})^*(\tilde g_Q)^T_{ik} + \frac{1}{3}\theta_{\tilde B}(\tilde g'_{h_u})^*(\tilde g'_Q)^T_{ik}\right](\tilde f_u^Q)_{kj} \\
& - \frac{4}{3}\theta_{\tilde B}\theta_{\tilde h}\theta_{\tilde u_k}s(\tilde g'_{h_u})^*(\tilde{\mathbf{f}}_u^{u_R})_{ik}(\tilde g'_{u_R})^T_{kj} + s(\mathbf{f}_u)\Big[\left(s^2\theta_h + c^2\theta_H\right)\mathrm{Tr}\left\{3(\mathbf{f}_u)^\dagger(\mathbf{f}_u)\right\} \\
& + c^2\left(\theta_h - \theta_H\right)\mathrm{Tr}\left\{3(\mathbf{f}_d)^\dagger(\mathbf{f}_d) + (\mathbf{f}_e)^\dagger(\mathbf{f}_e)\right\}\Big] \\
& + \frac{s}{2}\theta_{\tilde h}(\mathbf{f}_u)\Big\{3\theta_{\tilde W}\left[(\tilde g_{h_u})^2\left(s^2\theta_h + c^2\theta_H\right) + (\tilde g_{h_d})^2\left(c^2\theta_h - c^2\theta_H\right)\right] \\
& + \theta_{\tilde B}\left[(\tilde g'_{h_u})^2\left(s^2\theta_h + c^2\theta_H\right) + (\tilde g'_{h_d})^2\left(c^2\theta_h - c^2\theta_H\right)\right]\Big\} - s(\mathbf{f}_u)\left[\frac{17}{12}g'^2 + \frac{9}{4}g_2^2 + 8g_3^2\right]
\end{aligned}
$$

- Full threshold RGE must reduce to:
  - SM RGE with SUSY (and extra Higgs) thetas = 0.
  - MSSM RGE with all thetas = 1.

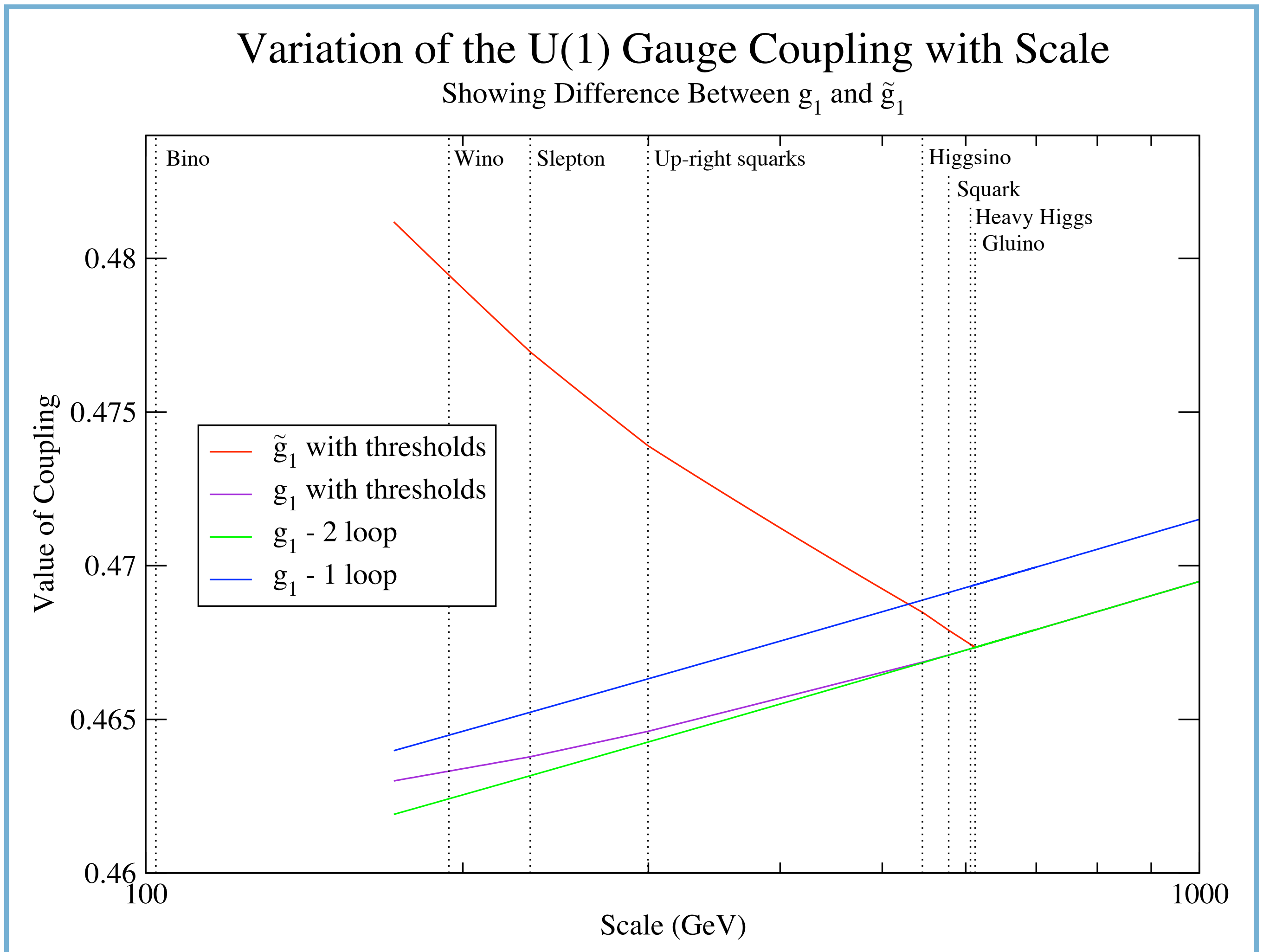
Unexpected Dependences

- Gaugino mass RGEs display possible dependence on μ when $\theta_H \to 0$, for example:

$$(4\pi)^2 \frac{dM_2}{dt} \ni 4sc\left(-\theta_h + \theta_H\right)\theta_{\tilde{h}}(\tilde{g}_{h_u})\mu^*(\tilde{g}_{h_d})$$

- Have obtained full system of g, $\tilde{g}$, f & $\tilde{f}$ RGEs.

- Soft parameters are under control but not completed.



- Since threshold effects can be similar in magnitude to two-loop effects

  ➡ Must be included for true two-loop accuracy.

- Lagrangian now contains $\tilde{g}$ and $\tilde{f}$ for SUSY couplings

  ➡ Important implications for phenomenology of sparticles.

Consider $\tilde{t}_1 \to c\widetilde{Z}_1$.

- $\tilde{t}_1$ is mass eigenstate - not flavour eigenstate.
- Contains $\tilde{c}_L$, $\tilde{c}_R$, ... ⇒ can decay to $c\,\widetilde{Z}_1$.
- Previously estimated by Hikasa & Kobayashi using single step approximation:

$$f(WEAK) = f(GUT) - \ln\left(\frac{M_{GUT}}{M_{WEAK}}\right)[RGE(Q)]$$

- With RGE solution the result changes significantly.

Hikasa & Kobayashi (1987)

General Comparison of RGE Result With One-Step Approx.

Method	Width ($\times 10^{-11}$ GeV)
Hikasa-Kobayashi	~55
1-loop (all thresholds at m_H)	2.72
2-loop (all thresholds at m_H)	3.11
2-loop (general thresholds)	2.93
2-loop (general thresholds with ~ terms)	3.21

For mSUGRA point:

$m_0=150$
$m_{1/2}=250$
$A_0=-850$
$\tan\beta=4$
$\mathrm{sign}(\mu)=+1$
$M_t=172$

$\Rightarrow m_{Z_1}=99$ $m_{t_1}=190$

- The Hikasa-Kobayashi approximation underestimates the width by more than a factor of 15.
- Using ~-terms to calculate decay is as important as two loop vs one loop.
- Will have a large effect on the branching ratio in the case that a number of decay rates are of similar order.

Plan to incorporate code into Isajet.

- Generally, flavour effects may depend on more than Yukawa sources - i.e. Soft Masses and A-parameters.

- In mSUGRA, all flavour violation comes from the Yukawas and is determined solely by KM matrix.

- Even if the Yukawas are the only source of flavour violation, non-universal soft parameters may result in flavour violation which is not solely determined by the KM matrix.

Rotation Dependence

As example, take all mSUGRA inputs except for m_U^2

In diagonal quark basis, use GUT scale condition:

$$m_U^2 = m_0^2 \mathbb{1} + m_0^2 \times diag\,(a, b, c)$$

In this basis there is no explicit flavour mixing for squarks

Choice of rotation	Width (x10^{-11} GeV)
$V_L(u)=KM^\dagger$; $V_R(u)=V_R(d)=\mathbb{1}$.	3.21
$V_L(u)=V_R(u)=KM^\dagger$; $V_R(d)=KM$	6.43

Summary

- To correctly include thresholds, new SUSY couplings, $\tilde{g}$ and $\tilde{f}$ must be introduced.
- The new couplings must be used when calculating flavour violating effects.
- This has significant phenomenologial results as evidenced by stop decay.

RGE boundary conditions

- Weak scale Yukawa couplings:
  - Related to weak scale quark masses.
  - Rotated to current basis by V_L, V_R.
- GUT scale soft SUSY-breaking parameters:
  - mSUGRA or non-universal inputs.

Fixed General RGE

Using 2-component fermions, with:

$$\mathcal{L} \ni -\left(\frac{1}{2} Y^a_{pq} \psi_p^T \zeta \psi_q \phi_a + \mathrm{hc}\right),$$

the RGE is:

$$(4\pi)^2 \, \boldsymbol{\beta}^a_Y|_{1-loop} = \tfrac{1}{2}\left[\mathbf{Y}_2^T(F)\mathbf{Y}^a + \mathbf{Y}^a\mathbf{Y}_2(F)\right] + 2\mathbf{Y}^b\mathbf{Y}^{\dagger a}\mathbf{Y}^b$$
$$+ \mathbf{Y}^b \,\mathrm{Tr}\left\{\tfrac{1}{2}\left(\mathbf{Y}^{\dagger b}\mathbf{Y}^a + \mathbf{Y}^{\dagger a}\mathbf{Y}^b\right)\right\} - 3g^2\left\{\mathbf{C}_2(F), \mathbf{Y}^a\right\}$$

Transition from MSSM to SM running

- Can remove $\sin\beta$ ($\cos\beta$) from f_u (f_d) RGE for MSSM and define $\lambda = \sin\beta\, f_u$ at the point that the Heavy Higgs particles decouple from the theory.
- Some terms are not proportional to $\sin\beta\, f_u$. These are so-called $\sim$-terms.

By how much does $\tilde{f}$ differ from f?

RGE for $\tilde{f}$ contains terms like $f f^2$ and $f \tilde{f}^2$ so the difference between f and $\tilde{f}$ is roughly (with RGE $\sim f^3$):

$$f_2 \left(f_2 - \tilde{f}_2 \right) \sim 2 f^2 \Delta f \sim 2 f^2 \frac{t}{16\pi^2} \mathrm{RGE}$$

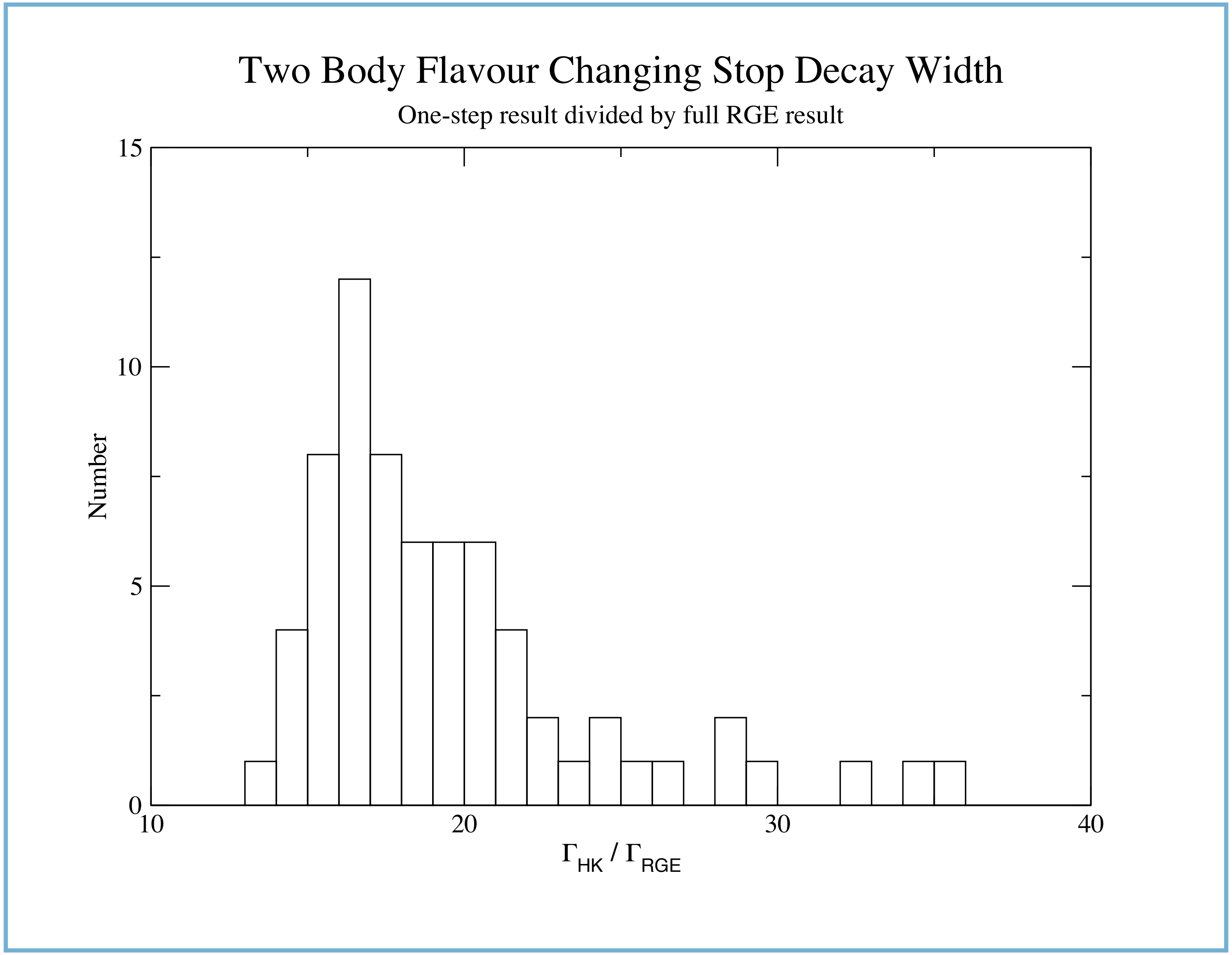
Similarly, the difference between two-loop and one-loop running is:

$$f_2 \left(f_2 - f_1 \right) \sim \frac{1}{16\pi^2} f^5 T$$

Since $t \sim 1$ and $T \sim 35$, the ratio between these two is roughly:

$$\frac{f_2 \left(f_2 - \tilde{f}_2 \right)}{f_2 \left(f_2 - f_1 \right)} \sim \frac{16\pi^2}{35} \sim 4.5$$

Various mSUGRA points with light stop



Stop mass ≥ 100 GeV