LHC DAYS

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Improved $Z \to \gamma\gamma$ decay in the renormalizable gauge sector of the noncommutative standard model

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Introduction

Example of noncommutativity: Heisenberg algebra

$$[\hat{x}^\mu, p^\nu] = i\hbar\delta^{\mu\nu}, \quad [p^\mu, p^\nu] = 0$$

Constructing models on non-commutative space-time

* The star product: $[x^\mu \stackrel{\star}{,} x^\nu] = x^\mu \star x^\nu - x^\nu \star x^\mu = i\theta^{\mu\nu}$.

$$(f \star g)(x) = e^{-\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}} f(x)g(y)|_{y\to x}$$

* Non-commutative coordinates:

$$x^\mu \to \hat{x}^\mu \Longrightarrow [\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}, \quad [\theta^{\mu\nu}, \hat{x}^\rho] = 0,$$

θ - constant, antisymetric and real: $\theta^{\mu\nu} = c^{\mu\nu}/\Lambda^2_{\rm NC}$

* Seiberg-Witten map (SW):

  In principle SW map express noncommutative functionals(parameters and functions of fields) spanned on the noncommutative space as a local functionals spanned on commutative space.

There are two essential points in which non-commutative gauge theories differ from standard gauge theories:

* The breakdown of Lorentz invariance with respect to a fixed non-zero $\theta^{\mu\nu}$ background field (which obviously fixes preferred directions)
* The appearance of new interactions and the modification of standard ones. For exapmle, triple–neutral–gauge boson, 2 fermion–2 gauge bosons, photon–neutrino interactions, etc.

Both properties have a common origin and appear in a number of phenomena

AT VERY HIGH ENERGIES AND/OR VERY SHORT DISTANCES.

SEARCH FOR THE SIGNAL OF NONCOMMUTATIVITY IN

DECAYS: 1$\rightarrow$ 2
SM forbidden – induced by the NC space-time:
The gauge sector:
* $Z \rightarrow \gamma\gamma, gg$

Neutrino sector:
* $\gamma_{\mathrm{pl}} \rightarrow \nu\bar{\nu}$

The hadron sector (neutral currents):
* $J/\psi \rightarrow \gamma\gamma$, $\Upsilon \rightarrow \gamma\gamma$

The hadron sector (flavour-changing currents):
* $K \rightarrow \pi\gamma$, $D \rightarrow (\pi, K)\gamma$, $B \rightarrow (\pi, K, D)\gamma$

SCATTERINGS: 2$\rightarrow$ 2
SM allowed – modified by the NC space-time:
* Moller scattering: $e^-e^- \rightarrow e^-e^-$
* Bhabha scattering: $e^+e^- \rightarrow e^+e^-$
* Annihilation: $e^+e^- \rightarrow \gamma\gamma$
* Photon-photon production: $\gamma\gamma \rightarrow \gamma\gamma$, $Z\gamma$, ZZ, $\bar{f}f$
* Fermion pair annihilation at LHC: $\bar{f}f \rightarrow Z\gamma$

NEUTRINO PROPERTIES: $\left(\mathrm{d}^{\mathrm{el}}_{\mathrm{mag}}\right)_\nu$, $\langle r^2_\nu \rangle$
* In the ν–mass extended SM allowed via 1–loop $\longrightarrow$
* Modified by the point-like NC interaction

CONSTRUCTING NCSM
VIA MOYAL-WEYL - $\star$ PRODUCT

[M. Chaichian at al. Eur. Phys. J. C**29** (2003) 413]

* Only U(N) gauge groups
* Matter content restricted to the (anti-)fundamental and adjoint rep.
* Charge quantization problem of NC Abelian GT
* Problems with UV/IR mixing; ($\frac{1}{|p\theta|^2}$ terms)
* NCQED signal in $2 \to 2$ processes: $e^+e^- \to \gamma\gamma$

[J.L. Hewett et al, Phys. Rev. **D64**, 075012 (2001)]

$$\frac{d\sigma}{d\cos\theta d\phi} = \frac{\alpha^2}{s}\frac{1+\cos^2\theta}{1-\cos^2\theta}[1-\sin^2\theta\ \ \sin^2\Delta_{NC}]$$

$$\Delta_{NC} = \frac{-s}{4\Lambda_{NC}^2}(c_{01}\sin\theta\cos\phi + c_{02}\sin\theta\sin\phi + c_{03}\cos\theta)$$

Experimental signatures of non-commutativity:

* Collider physics: $\Lambda_{NC} \sim \text{few TeV's}$
* Low-energy non-accelerator experiments: $\Lambda_{NC} \sim 10^8$ TeV

Limits rest on the assumptions, which may have to be modified

* θ is constant across large distances with respect to the NC scale
* Unrealistic gauge groups
* Non-commutativity down to low-energy scales

* See figures in: The OPAL Collaboration: Test of non-commutative QED in the process $e^+e^- \to \gamma\gamma$; hep-ex/0303035
* $\Longrightarrow$ $\Lambda_{NC} > 141\text{GeV}$ for all parameters.

CONSTRUCTING NCSM VIA SEIBERG-WITTEN MAP

[N. Seiberg and E. Witten; String theory and non-commutative geometry, JHEP **9909**, 032 (1999)]

[J. Madore, S. Schraml, P. Schupp and J. Wess; Gauge theory on noncommutative spaces, Eur. Phys. J. C**16** (2000) 161]

[B. Jurčo, S. Schraml, P. Schupp and J. Wess; Enveloping algebra valued gauge transformations for non-Abelian gauge groups on non-commutative spaces, Eur. Phys. J. C **17**, 521 (2000)

[X. Calmet, B. Jurčo, P. Schupp, J. Wess and M. Wohlgenannt; The standard model on non-commutative space-time, EPJ C**23** (2002) 363]

[W. Behr, N. G. Deshpande, G. Duplančić, P. Schupp, J.T. and J. Wess; The Z $\to$ gamma gamma, g g decays in the non-commutative standard model, Eur. Phys. J. C **29**, 441 (2003)]

[G. Duplančić, P. Schupp and J. Trampetić; Comment on triple gauge boson interactions in the non-commutative electroweak sector, Eur. Phys. J. C**32** (2003) 141]

[B. Melić, K. Passek-Kumerički, J.T., P. Schupp and M. Wohlgenannt; The Standard Model on Non-Commutative Space-Time: Electroweak Currents and Higgs Sector, EPJ C**24** (2005) 483 ibid. 499]

[F. Brandt, C.P. Martín and F. Ruiz Ruiz; Anomaly freedom in Seiberg-Witten noncommutative gauge theories JHEP **07** (2003) 068]

[M. Buric, D. Latas and V. Radovanovic, Renormalizability of noncommutative SU(N) gauge theory; JHEP **0602** (2006) 046]

[M. Buric, V. Radovanovic and J.T., The one-loop renormalization of the gauge sector in the noncommutative standard model; hep-th/0609073]

[M. Buric, D. Latas, V. Radovanovic and J.T., Improved Z $\to \gamma\gamma$ decay in the renormalizable NCSM; work in progress]

* Based on the Seiberg-Witten mapping
* Expansion in power series in θ $\rightarrow$ new vertices
* Any gauge groups
* Arbitrary matter representation
* No charge quantization problem
* No UV/IR mixing due to θ expansion
* Unitarity: θ^{ij}-OK, θ^{0i}-may/may not be OK;

carefull cannonical quantization produces always unitary theory:

(*Friedenhagen: Time in S matrix treated in form of slices*; very hard to calculate)

* Assumption:

$$\begin{aligned}\theta_{\mu\nu}\theta^{\mu\nu} &= -\theta_{\mu\nu}\theta^{\nu\mu} = -\theta^2 \\ &= \frac{2}{\Lambda^4_{\rm NC}}[\sum_{i,j=1\,;\,i<j}^{3}(c^{ij})^2 - \sum_{i=1}^{3}(c^{0i})^2] > 0\end{aligned}$$

to avoid potential difficulties with unitarity in non-commutative gauge field theories

* Construction of covariant Yukawa couplings OK
* One-loop renormalizable gauge sector at the first order in noncommutative parameter θ
* NCSM constructed as an effective, anomaly free and partly renormalizable theory

NC gauge transformation

Consider infinitesimal NC local gauge transformation $\widehat{\delta}$ of a fundamental matter field that carries a representation ρ_Ψ

$$\widehat{\delta}\widehat{\Psi} = i\rho_\Psi(\widehat{\Lambda}) \star \widehat{\Psi}$$

In Abelian case ρ_Ψ fixed by the hypercharge.

Covariant coordinates in NC theory introduced in analogy to covariant derivatives in ordinary theory

$$\widehat{x}^\mu = x^\mu + \theta^{\mu\nu}\widehat{A}_\nu$$

Locality

A $\star-$ product of ordinary functions f, g, determined by a Poisson tensor $\theta^{\mu\nu}(x)$, is local function of f, g with finite number of derivatives at each order in θ:

$$f \star g = f \cdot g + \frac{i}{2}\theta^{\mu\nu}(x)\partial_\mu f \cdot \partial_\nu g + \mathcal{O}(\theta^2)$$

Seiberg–Witten map express the non-commutative fields and parameters as local functions of the ordinary fields and parameters

$$\begin{aligned}
\widehat{\Lambda} &= \Lambda + \Lambda^{\theta}[V] + \Lambda^{\theta^2}[V] + \mathcal{O}(\theta^3) \\
\widehat{\psi}[\psi, V] &= \psi + \psi^{\theta}[\psi, V] + \psi^{\theta^2}[\psi, V] + \mathcal{O}(\theta^3) \\
\widehat{V}_\mu[V] &= V_\mu + V_\mu^{\theta}[V] + V_\mu^{\theta^2}[V] + \mathcal{O}(\theta^3),
\end{aligned}$$

NC field strengt $\widehat{F}_{\mu\nu} = \partial_\mu \widehat{V}_\nu - \partial_\nu \widehat{V}_\mu - i[\widehat{V}_\mu \stackrel{\star}{,} \widehat{V}_\nu]$

Gauge equivalence, and consistency conditions

Ordinary gauge transformations $\delta A_\mu = \partial_\mu \Lambda + i[\Lambda, A_\mu]$ and $\delta\Psi = i\Lambda\cdot\Psi$ induce non-commutative gauge transformations of the fields $\widehat{A}$, $\widehat{\Psi}$ with gauge parameter $\widehat{\Lambda}$

$$\delta\widehat{A}_\mu = \widehat{\delta}\widehat{A}_\mu \qquad \delta\widehat{\Psi} = \widehat{\delta}\widehat{\Psi}$$

Consistency require that any pair of non-commutative gauge parameters $\widehat{\Lambda}$, $\widehat{\Lambda'}$ satisfy

$$[\widehat{\Lambda} \stackrel{\star}{,} \widehat{\Lambda'}] + i\delta_\Lambda \widehat{\Lambda'} - i\delta_{\Lambda'}\widehat{\Lambda} = \widehat{[\Lambda, \Lambda']}.$$

Enveloping algebra-valued gauge transformation

The commutator

$$\begin{aligned}[\widehat{\Lambda} \stackrel{\star}{,} \widehat{\Lambda}'] &= \frac{1}{2}\{\Lambda_a(x) \stackrel{\star}{,} \Lambda'_b(x)\}[T^a, T^b] \\ &+ \frac{1}{2}[\Lambda_a(x) \stackrel{\star}{,} \Lambda'_b(x)]\{T^a, T^b\}\end{aligned}$$

of two Lie algebra-valued NC gauge parameters $\widehat{\Lambda} = \Lambda_a(x)T^a$ and $\widehat{\Lambda}' = \Lambda'_a(x)T^a$ does not close in the Lie algebra. For NC SU(N) & Lie algebra traceless condition incompatible with commutator. We have to consider enveloping algebra-valued NC gauge parameters

$$\widehat{\Lambda} = \Lambda^0_a(x)T^a + \Lambda^1_{ab}(x) : T^aT^b : + \Lambda^2_{abc}(x) : T^aT^bT^c : + \ldots$$

and fields. (The : : denotes ordering of the Lie algebra generators.)

THE FIRST AND SECOND ORDER SW MAPS

$$\Lambda^{\theta}[V] = \frac{1}{4}\theta^{\mu\nu}\{V_\nu, \partial_\mu \Lambda\}$$

$$\Lambda^{\theta^2}[V] = \frac{1}{32}\theta^{\mu\nu}\theta^{\kappa\lambda}\Big(\{V_\mu, \{\partial_\nu V_\kappa, \partial_\lambda \Lambda\}\} + \{V_\mu, \{V_\kappa, \partial_\nu\partial_\lambda \Lambda\}\}$$
$$+\{\{V_\mu, \partial_\nu V_\kappa\}, \partial_\lambda \Lambda\}\} - \{\{F_{\mu\kappa}, V_\nu\}, \partial_\lambda \Lambda\} - 2i[\partial_\mu V_\kappa, \partial_\nu\partial_\lambda \Lambda]\Big)$$

$$\psi^{\theta}[\psi, V] = -\frac{1}{2}\theta^{\alpha\beta}\left(V_\alpha\,\partial_\beta - \frac{i}{4}[V_\alpha, V_\beta]\right)\psi$$

$$\psi^{\theta^2}[\psi, V] = \frac{1}{32}\theta^{\mu\nu}\theta^{\kappa\lambda}\times$$
$$\Big(-4i\partial_\kappa V_\mu \partial_\nu \partial_\lambda + 4V_\kappa V_\mu \partial_\nu\partial_\lambda - 4\partial_\kappa V_\mu V_\nu \partial_\lambda + 4F_{\kappa\mu}V_\nu\partial_\lambda$$
$$-4V_\nu\partial_\kappa V_\mu\partial_\lambda + 8V_\nu F_{\kappa\mu}\partial_\lambda - 8iV_\mu V_\kappa V_\nu \partial_\lambda + 4iV_\mu V_\nu V_\kappa \partial_\lambda$$
$$-2\partial_\kappa V_\mu \partial_\lambda V_\nu + 2i\partial_\kappa V_\mu V_\lambda V_\nu - 2iV_\nu V_\lambda \partial_\kappa V_\mu$$
$$-i[[\partial_\kappa V_\mu, V_\nu], V_\lambda] - 4iV_\nu F_{\kappa\mu}V_\lambda + V_\kappa V_\lambda V_\mu V_\nu - 2V_\kappa V_\mu V_\nu V_\lambda\Big)\psi$$

$$V_\mu^{\theta}[V] = \frac{1}{4}\theta^{\alpha\beta}\{\partial_\alpha V_\mu + F_{\alpha\mu}, V_\beta\}$$

$$V_\mu^{\theta^2}[V] = \frac{1}{64}\theta^{\alpha\beta}\theta^{\gamma\delta}\times$$
$$\Big(8\{V_\alpha, \{F_{\mu\gamma}, F_{\beta\delta}\}\} + 8\{V_\alpha, \{\partial_\beta F_{\mu\gamma}, V_\delta\}\} + 2i\{V_\alpha, \{\partial_\mu V_\beta, V_\gamma V_\delta\}\}$$
$$-2\{V_\alpha, \{\partial_\beta\partial_\mu V_\gamma, V_\delta\}\} - \{V_\mu, \{F_{\alpha\gamma}, F_{\beta\delta}\}\} - 4\{V_\mu, \{\partial_\alpha V_\gamma, \partial_\delta V_\beta\}\}$$
$$+2\{V_\mu, \{V_\alpha V_\beta, V_\gamma V_\delta\}\} + 2\{\partial_\alpha V_\gamma, \{V_\beta, \partial_\mu V_\delta\}\} + 4\{\partial_\alpha V_\gamma, \{V_\mu, \partial_\delta V_\beta\}\}$$
$$+8\{\partial_\alpha V_\mu, \{\partial_\gamma V_\beta, V_\delta\}\} + 2\{\partial_\mu V_\alpha, \{F_{\beta\gamma}, V_\delta\}\} - 2\{V_\alpha V_\beta, \{V_\mu, V_\gamma V_\delta\}\}$$
$$-4\{V_\alpha V_\gamma, \{V_\mu, V_\beta V_\delta\}\} + 8\{V_\alpha V_\mu V_\gamma, V_\beta V_\delta\} + 8i[\partial_\alpha\partial_\gamma V_\mu, \partial_\beta V_\delta]$$
$$-2i[\partial_\mu F_{\alpha\gamma}, F_{\beta\delta}] - 4i[\partial_\alpha\partial_\mu V_\gamma, \partial_\delta V_\beta] - 4V_\alpha\partial_\beta V_\gamma \partial_\mu V_\delta$$
$$+4F_{\alpha\gamma}V_\mu F_{\beta\delta} - 4\partial_\mu V_\alpha \partial_\gamma V_\beta V_\delta + 2iV_\alpha V_\gamma(\partial_\beta V_\delta)V_\mu$$
$$-4iV_\alpha V_\gamma(\partial_\mu V_\beta)V_\delta - 2iV_\alpha(\partial_\beta V_\gamma)V_\delta V_\mu - 2iV_\alpha(\partial_\gamma V_\beta)V_\delta V_\mu$$
$$+4iV_\alpha(\partial_\mu V_\gamma)V_\beta V_\delta - 2iV_\mu V_\alpha V_\gamma(\partial_\beta V_\delta) + 2iV_\mu V_\alpha(\partial_\beta V_\gamma)V_\delta$$
$$+2iV_\mu V_\alpha(\partial_\gamma V_\beta)V_\delta - 2iV_\mu(\partial_\alpha V_\gamma)V_\delta V_\beta + 2i(\partial_\alpha V_\gamma)V_\delta V_\beta V_\mu\Big)$$

[L. Moller; Second order of the expansions of action functionals of the noncommutative standard model, JHEP **0410** (2004) 063]
[M. Wohlgenannt,... and J.T. work in progress]

The non-commutative Higgs field $\widehat{\Phi}$ is given by the hybrid SW map

$$\widehat{\Phi} \equiv \widehat{\Phi}[\Phi, V, V'] = \Phi + \Phi^{\theta}[V, V'] + \Phi^{\theta^2}[V, V'] + \mathcal{O}(\theta^3)$$

$\widehat{\Phi}$ is a functional of two gauge fields V and V' and transforms covariantly under gauge transformations:

$$\delta\widehat{\Phi}[\Phi, V, V'] = i\widehat{\Lambda} * \widehat{\Phi} - i\widehat{\Phi} * \widehat{\Lambda}'$$

$\widehat{\Lambda}$ and $\widehat{\Lambda}'$ are the corresponding gauge parameters. Hermitian conjugation yields $\widehat{\Phi}[\Phi, V, V']^{\dagger} = \widehat{\Phi}[\Phi^{\dagger}, V', V]$.

$$\widehat{D}_{\mu}\widehat{\Phi} = \partial_{\mu}\widehat{\Phi} - i\,\widehat{V}_{\mu} * \widehat{\Phi} + i\,\widehat{\Phi} * \widehat{V}'_{\mu}.$$

The precise representations of the gauge fields V and V' in the Yukawa couplings are inherited from the fermions on the left ($\bar{\psi}$) and on the right side (ψ) of the Higgs field, respectively.

The hybrid Seiberg-Witten map for the Higgs boson up to second order is not unique. One solution is given by

$$\Phi^{\theta}[\Phi, V, V'] = \frac{1}{2}\theta^{\alpha\beta} \times$$
$$\left[V_{\beta}\left(\partial_{\alpha}\Phi - \frac{i}{2}(V_{\alpha}\Phi - \Phi V'_{\alpha})\right) + \left(\partial_{\alpha}\Phi - \frac{i}{2}(V_{\alpha}\Phi - \Phi V'_{\alpha})\right) V'_{\beta}\right]$$

[M. Wohlgenannt,... and J.T. work in progress]

$$
\begin{aligned}
&\Phi^{\theta^2}[\Phi, V, V'] = -\frac{i}{32}\theta^{\alpha\beta}\theta^{\gamma\delta} \times \\
&\Bigg(V_\alpha \Big[V_\beta \Big(V_\gamma \Big(4\partial_\delta\Phi - 3iV_\delta\Phi + 4i\Phi V'_\delta \Big) + \Big(-4\partial_\gamma\Phi - 2i\Phi V'_\gamma \Big) V'_\delta \Big) \\
&+V_\gamma \Big[4i\partial_\beta\partial_\delta\Phi + V_\beta \Big(-4\partial_\delta\Phi + 2i\Big(V_\delta\Phi - 2\Phi V'_\delta \Big)\Big) + V_\delta \Big(4\partial_\beta\Phi + 4i\Phi V'_\beta \Big) \\
&+3\partial_\beta V_\delta\Phi - 4\partial_\beta\Phi V'_\delta - 4\partial_\delta\Phi V'_\beta + \Phi\Big(4\Big(-2\partial_\beta V'_\delta + \partial_\delta V'_\beta + i\Big(V'_\beta V'_\delta - 2V'_\delta V'_\beta \Big)\Big)\Big)\Big] \\
&+\partial_\beta V_\gamma \Big(8i\partial_\delta\Phi + 5V_\delta\Phi - 8\Phi V'_\delta \Big) + \partial_\gamma V_\beta \Big(-4i\partial_\delta\Phi - 3V_\delta\Phi \Big) \\
&+\partial_\gamma\Phi\Big(4\Big(-i\partial_\beta V'_\delta + i\partial_\delta V'_\beta + V'_\beta V'_\delta + V'_\delta V'_\beta \Big)\Big) + \Big(-8i\partial_\beta\partial_\gamma\Phi + 4\partial_\beta\Phi V'_\gamma \Big) V'_\delta \\
&+\Phi\Big(V'_\gamma \Big(4\partial_\beta V'_\delta - 4\partial_\delta V'_\beta - 4iV'_\beta V'_\delta + 4iV'_\delta V'_\beta \Big) + \Big(8\partial_\gamma V'_\beta + 4iV'_\beta V'_\gamma \Big) V'_\delta \Big)\Big] \\
&+\partial_\alpha V_\gamma \Big[4\partial_\beta\partial_\delta\Phi + V_\beta \Big(-4i\partial_\delta\Phi + 4\Phi V'_\delta \Big) + V_\delta \Big(-V_\beta\Phi - 4\Phi V'_\beta \Big) + 4i\partial_\beta\Phi V'_\delta \\
&-2i\partial_\delta V_\beta\Phi - 4i\partial_\delta\Phi V'_\beta + \Phi\Big(4i\partial_\delta V'_\beta - 4V'_\beta V'_\delta + 8V'_\delta V'_\beta \Big)\Big] \\
&+\partial_\alpha\Phi\Big[V'_\gamma \Big(-4i\partial_\delta V'_\beta + 4V'_\beta V'_\delta - 4V'_\delta V'_\beta \Big) + \Big(-4i\partial_\beta V'_\gamma + 8i\partial_\gamma V'_\beta - 4V'_\beta V'_\gamma \Big) V'_\delta \Big] \\
&+\partial_\alpha\partial_\gamma\Phi\Big(-4\partial_\delta V'_\beta + 4iV'_\delta V'_\beta \Big) + \Phi\Big[V'_\alpha \Big[V'_\gamma \Big(\partial_\beta V'_\delta + 2iV'_\beta V'_\delta \Big) \\
&+\Big(3\partial_\beta V'_\gamma - 5\partial_\gamma V'_\beta - 3iV'_\beta V'_\tau \Big) V'_\nu \Big] + \partial_\alpha V'_\gamma \Big(-2i\partial_\delta V'_\beta - 3V'_\delta V'_\beta \Big)\Big]\Bigg)
\end{aligned}
$$

Note that above Eqs., representing SW mapps up to second order in θ for fermion and Higgs fields respectively, in the case of $V' = 0$ and for two fields only, are identical.

[M. Wohlgenannt,... and J.T. work in progress]

NCSM ACTIONS

$$S_{\text{NCSM}} = S_{\text{fermions}} + S_{\text{gauge}} + S_{\text{Higgs}} + S_{\text{Yukawa}}$$

$$\begin{aligned} S_{\text{fermions}} &= \int d^4x \sum_{i=1}^{3} \Big(\overline{\widehat{L}}_L^{(i)} \star (i\widehat{\not{D}}\, \widehat{L}_L^{(i)}) + \overline{\widehat{Q}}_L^{(i)} \star (i\widehat{\not{D}}\, \widehat{Q}_L^{(i)}) \\ &+ \overline{\widehat{e}}_R^{(i)} \star (i\widehat{\not{D}}\, \widehat{e}_R^{(i)}) + \overline{\widehat{u}}_R^{(i)} \star (i\widehat{\not{D}}\, \widehat{u}_R^{(i)}) + \overline{\widehat{d}}_R^{(i)} \star (i\widehat{\not{D}}\, \widehat{d}_R^{(i)}) \Big) \end{aligned}$$

$$S_{\text{gauge}} = -\frac{1}{2} \int d^4x \sum_{\mathcal{R}} c_{\mathcal{R}} \mathbf{Tr} \Big(\mathcal{R}(\widehat{F}_{\mu\nu}) \star \mathcal{R}(\widehat{F}^{\mu\nu}) \Big)$$

$\mathcal{R}$ – unitary, irreducible and inequivalent representations of a gauge group. Real coefficients $c_{\mathcal{R}}$ that are subject to the constraints.

$$\frac{1}{g_{SM}^2} = \sum_{\mathcal{R}} c_{\mathcal{R}} \mathbf{Tr} \Big(\mathcal{R}(T_{SM}^a) \mathcal{R}(T_{SM}^a) \Big).$$

$$\begin{aligned} S_{\text{Higgs}} &= \int d^4x \Big(h_0^\dagger (\widehat{D}_\mu \widehat{\Phi}) \star h_0 (\widehat{D}^\mu \widehat{\Phi}) - \mu^2 h_0^\dagger (\widehat{\Phi}) \star h_0 (\widehat{\Phi}) \\ &- \lambda h_0^\dagger (\widehat{\Phi}) \star h_0 (\widehat{\Phi}) \star h_0^\dagger (\widehat{\Phi}) \star h_0 (\widehat{\Phi}) \Big) \end{aligned}$$

$$\begin{aligned} S_{\text{Yukawa}} &= -\int d^4x \sum_{i,j=1}^{3} \\ &\times \Big(G_e^{(ij)} (\overline{\widehat{L}}_L^{(i)} \star h_e(\widehat{\Phi}) \star \widehat{e}_R^{(j)}) + G_e^{\dagger\,(ij)} (\overline{\widehat{e}}_R^{(i)} \star h_e(\widehat{\Phi})^\dagger \star \widehat{L}_L^{(j)}) \\ &+ G_u^{(ij)} (\overline{\widehat{Q}}_L^{(i)} \star h_u(\widehat{\Phi}_c) \star \widehat{u}_R^{(j)}) + G_u^{\dagger\,(ij)} (\overline{\widehat{u}}_R^{(i)} \star h_u(\widehat{\Phi}_c)^\dagger \star \widehat{Q}_L^{(j)}) \\ &+ G_d^{(ij)} (\overline{\widehat{Q}}_L^{(i)} \star h_d(\widehat{\Phi}) \star \widehat{d}_R^{(j)}) + G_d^{\dagger\,(ij)} (\overline{\widehat{d}}_R^{(i)} \star h_d(\widehat{\Phi})^\dagger \star \widehat{Q}_L^{(j)}) \Big) \end{aligned}$$

GAUGE SECTOR

The gauge-invariant action for gauge fields:

$$S_{gauge} = -\frac{1}{2}\int d^4x \mathbf{Tr}\frac{1}{\mathbf{G}^2}\widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu}, \quad \frac{1}{g_{SM}^2} = \mathbf{Tr}\frac{1}{\mathbf{G}^2}T^a_{SM}T^a_{SM},$$

The trace $\mathbf{Tr}$ is over all representations
$\mathbf{G}$ – operator that commutes with all generators T^a_{SM} and encodes the couplings of the theory

$$\begin{aligned} S &= -\frac{1}{2}\mathrm{Tr}\int d^4x\, F_{\mu\nu}F^{\mu\nu} \\ &\quad +\theta^{\rho\sigma}\mathrm{Tr}\int d^4x\left[\left(\frac{1}{4}F_{\rho\sigma}F_{\mu\nu} - F_{\rho\mu}F_{\sigma\nu}\right)F^{\mu\nu}\right] + \mathcal{O}(\theta^2) \end{aligned}$$

Seiberg-Witten freedom:

$$\begin{aligned} \Delta S &= -2b\,\theta^{\rho\sigma}\,\mathrm{Tr}\int d^4x\, F^{\mu\nu}D_\mu D_\nu F_{\rho\sigma} \\ &= (\frac{-1}{4} + \frac{a}{4})\,\theta^{\rho\sigma}\,\mathrm{Tr}\int d^4x\, F^{\mu\nu}F_{\mu\nu}F_{\rho\sigma}. \end{aligned}$$

V_μ is the whole gauge potential for the gauge group $G_{SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\begin{aligned} V^\mu &= g'\mathcal{A}^\mu(x)Y + g\sum_{a=1}^{3}B^\mu_a(x)T^a_L + g_s\sum_{b=1}^{8}G^\mu_b(x)T^b_S \\ F_{\mu\nu} &= \partial_\mu V_\nu - \partial_\nu V_\mu - i[V_\mu, V_\nu] \end{aligned}$$

The choice of the trace corresponds to the choice of the representation of the gauge group:
The simplest choice is a sum of three traces over the SM gauge group: $\Rightarrow$ minimal NCSM
Choosing a trace over all particle with different quantum numbers in the model that have covariant derivative acting on them: $\Rightarrow$ non-minimal NCSM

Gauge sector: minimal NCSM

The mNCSM gauge action is given by

$$S_{\rm gauge}^{\rm mNCSM} = -\frac{1}{2}\int d^4x \left(\frac{1}{g'^2}{\rm Tr}_1 + \frac{1}{g^2}{\rm Tr}_2 + \frac{1}{g_s^2}{\rm Tr}_3\right) \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu} .$$

In the definition of $\mathrm{Tr}_{\mathbf{1}}$:

$$Y = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The fundamental representations for $SU(2)$ and $SU(3)$ generators in $\mathrm{Tr}_{\mathbf{2}}$ and $\mathrm{Tr}_{\mathbf{3}}$, respectively. In terms of physical fields, the action then reads

$$\begin{aligned} S_{\rm gauge}^{\rm mNCSM} = & -\frac{1}{2}\int d^4x \left(\frac{1}{2}\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu} + {\rm Tr}\,\mathcal{B}_{\mu\nu}\mathcal{B}^{\mu\nu} + {\rm Tr}\,G_{\mu\nu}G^{\mu\nu}\right) \\ & + \frac{1}{4} g_s\, d^{abc}\, \theta^{\rho\sigma}\int d^4x \left(\frac{a}{4}G^a_{\rho\sigma}G^b_{\mu\nu} - G^a_{\rho\mu}G^b_{\sigma\nu}\right) G^{\mu\nu,c} + \mathcal{O}(\theta^2) \end{aligned}$$

where $\mathcal{A}_{\mu\nu}$, $\mathcal{B}_{\mu\nu}(= B^a_{\mu\nu}T^a_L)$ and $G_{\mu\nu}(= G^a_{\mu\nu}T^a_S)$ denote the $U(1)$, $SU(2)_L$ and $SU(3)_c$ field strengths, respectively:

$$\begin{aligned} \mathcal{A}_{\mu\nu} &= \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu , \\ \mathcal{B}^a_{\mu\nu} &= \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g\, \epsilon^{abc} B^b_\mu B^c_\nu , \\ G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s\, f^{abc} G^b_\mu G^c_\nu . \end{aligned}$$

For adjoint representation $\Rightarrow$

* NO NEW NEUTRAL EW TGB INTERACTIONS
* FOR $a = 1, 3$ nmNCSM GAUGE SECTOR IS ONE-LOOP RENORMALIZABLE AT FIRST ORDER IN NC PARAMETER θ

Gauge sector: non-minimal NCSM

The action $S_{\text{gauge}}^{\text{nmNCSM}}$ up to linear order in θ:

$$S_{\text{gauge}}^{\text{nmNCSM}} = -\frac{1}{2}\int d^4x\, \mathbf{Tr}\frac{1}{\mathbf{G}^2}F_{\mu\nu}F^{\mu\nu} + \theta^{\rho\sigma}\int d^4x\, \mathbf{Tr}\frac{1}{\mathbf{G}^2}\left[\left(\frac{a}{4}F_{\rho\sigma}F_{\mu\nu} - F_{\rho\mu}F_{\sigma\nu}\right)F^{\mu\nu}\right] + \mathcal{O}(\theta^2)$$

where $\mathbf{Tr}\frac{1}{\mathbf{G}^2}$ is trace over all particle multiplets with different quantum numbers (5 multiplets for each generation of fermions and one Higgs multiplet given in Table)

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_Q$	T_3
$e_R^{(i)}$	$\mathbf{1}$	$\mathbf{1}$	-1	-1	0
$L_L^{(i)} = \begin{pmatrix} \nu_L^{(i)} \\ e_L^{(i)} \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$
$u_R^{(i)}$	$\mathbf{3}$	$\mathbf{1}$	$2/3$	$2/3$	0
$d_R^{(i)}$	$\mathbf{3}$	$\mathbf{1}$	$-1/3$	$-1/3$	0
$Q_L^{(i)} = \begin{pmatrix} u_L^{(i)} \\ d_L^{(i)} \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$
$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$1/2$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$
W^+, W^-, Z	$\mathbf{1}$	$\mathbf{3}$	0	$(\pm 1, 0)$	$(\pm 1, 0)$
A	$\mathbf{1}$	$\mathbf{1}$	0	0	0
G^b	$\mathbf{8}$	$\mathbf{1}$	0	0	0

The SM fields. Here $i \in \{1,2,3\}$ denotes the generation index. The electric charge is given by the Gell-Mann-Nishijima relation $Q = (T_3 + Y)$. The physical electroweak fields A, W^+, W^- and Z are expressed through the unphysical $U(1)_Y$ and $SU(2)$ fields $\mathcal{A}$ and B_a $(a \in \{1,2,3\})$. The gluons G^b $(b \in \{1,2,\ldots,8\})$ are in the octet representation of $SU(3)_C$.

producing the following action:

$$S^{\text{nmNCSM}}_{\text{gauge}} =$$

$$-\frac{g'^2}{2}\left(\frac{1}{g_1^2}+\frac{1}{2g_2^2}+\frac{4}{3g_3^2}+\frac{1}{3g_4^2}+\frac{1}{6g_5^2}+\frac{1}{2g_6^2}\right)\int d^4x f_{\mu\nu}f^{\mu\nu}$$

$$-\frac{g^2}{2}\left(\frac{1}{g_2^2}+\frac{3}{g_5^2}+\frac{1}{g_6^2}\right)\int d^4x\,\text{Tr}\,(F_{\mu\nu}F^{\mu\nu})$$

$$-\frac{g_s^2}{2}\left(\frac{1}{g_3^2}+\frac{1}{g_4^2}+\frac{2}{g_5^2}\right)\int d^4x\,\text{Tr}\,(G_{\mu\nu}G^{\mu\nu})$$

$$+g_s^3\left(\frac{1}{g_3^2}+\frac{1}{g_4^2}+\frac{2}{g_5^2}\right)\theta^{\rho\tau}\int d^4x\,\text{Tr}\left(\frac{a}{4}G_{\rho\tau}G_{\mu\nu}-G_{\mu\rho}G_{\nu\tau}\right)G^{\mu\nu}(=0)$$

$$+g'^3\kappa_1\theta^{\rho\tau}\int d^4x\,\left(\frac{a}{4}f_{\rho\tau}f_{\mu\nu}-f_{\mu\rho}f_{\nu\tau}\right)f^{\mu\nu}$$

$$+g'g^2\kappa_2\,\theta^{\rho\tau}\int d^4x\sum_{a=1}^{3}\left[(\frac{a}{4}f_{\rho\tau}F^a_{\mu\nu}-f_{\mu\rho}F^a_{\nu\tau})F^{\mu\nu,a}+c.p.\right]$$

$$+g'g_s^2\kappa_3\,\theta^{\rho\tau}\int d^4x\sum_{b=1}^{8}\left[(\frac{a}{4}f_{\rho\tau}G^b_{\mu\nu}-f_{\mu\rho}G^b_{\nu\tau})G^{\mu\nu,b}+c.p.\right]$$

$$\begin{aligned}
\kappa_1 &= -\frac{1}{g_1^2}-\frac{1}{4g_2^2}+\frac{8}{9g_3^2}-\frac{1}{9g_4^2}+\frac{1}{36g_5^2}+\frac{1}{4g_6^2},\\
\kappa_2 &= -\frac{1}{4g_2^2}+\frac{1}{4g_5^2}+\frac{1}{4g_6^2},\\
\kappa_3 &= +\frac{1}{3g_3^2}-\frac{1}{6g_4^2}+\frac{1}{6g_5^2}.
\end{aligned}$$

$f_{\mu\nu}$ $\quad F^a_{\mu\nu}$ $\quad G^b_{\mu\nu}$ – physical field strengths

$\Longleftrightarrow$ $\quad U(1)_Y$ $\quad SU(2)_L$ $\quad SU(3)_C$ – gauge group

$\kappa_1,\ \kappa_2,\ \kappa_3$ – parameters of the model

Matching the SM action at zeroth order in θ, three consistency conditions are imposed

$$
\begin{aligned}
\frac{1}{g'^2} &= \frac{2}{g_1^2}+\frac{1}{g_2^2}+\frac{8}{3g_3^2}+\frac{2}{3g_4^2}+\frac{1}{3g_5^2}+\frac{1}{g_6^2},\\
\frac{1}{g^2} &= \frac{1}{g_2^2}+\frac{3}{g_5^2}+\frac{1}{g_6^2},\\
\frac{1}{g_s^2} &= \frac{1}{g_3^2}+\frac{1}{g_4^2}+\frac{2}{g_5^2}.
\end{aligned}
$$

giving final expression for TGB action

$$
\begin{aligned}
S_{gauge} &= -\frac{1}{4}\int d^4x f_{\mu\nu}f^{\mu\nu}-\frac{1}{2}\int d^4x\,\mathrm{Tr}\,(F_{\mu\nu}F^{\mu\nu})\\
&- \frac{1}{2}\int d^4x\,\mathrm{Tr}\,(G_{\mu\nu}G^{\mu\nu})\\
&+ g'^2\kappa_1\theta^{\rho\tau}\int d^4x\,\left(\frac{a}{4}f_{\rho\tau}f_{\mu\nu}-f_{\mu\rho}f_{\nu\tau}\right)f^{\mu\nu}\\
&+ g'g^2\kappa_2\,\theta^{\rho\tau}\int d^4x\sum_{a=1}^{3}\left[(\frac{a}{4}f_{\rho\tau}F^a_{\mu\nu}-f_{\mu\rho}F^a_{\nu\tau})F^{\mu\nu,a}+c.p.\right]\\
&+ g'g_s^2\kappa_3\,\theta^{\rho\tau}\int d^4x\sum_{b=1}^{8}\left[(\frac{a}{4}f_{\rho\tau}G^b_{\mu\nu}-f_{\mu\rho}G^b_{\nu\tau})G^{\mu\nu,b}+c.p.\right]
\end{aligned}
$$

Above three consistenty conditions together with the requirement that $1/g_i^2>0$ define a 3D pentahedron in the six-dimensional moduli space spanned by $1/g_1^2,\ldots,1/g_6^2$

The interactions $\mathcal{L}^\theta$ in terms of physical fields (A, Z, W, G)

$$\mathcal{L}^\theta_{\gamma\gamma\gamma} = \frac{e}{4}\sin 2\theta_W\ \mathsf{K}_{\gamma\gamma\gamma}\theta^{\rho\tau}A^{\mu\nu}\left(aA_{\mu\nu}A_{\rho\tau} - 4A_{\mu\rho}A_{\nu\tau}\right)$$

$$\mathsf{K}_{\gamma\gamma\gamma} = \frac{1}{2}\,gg'(\kappa_1 + 3\kappa_2)$$

$$\mathcal{L}^\theta_{Z\gamma\gamma} = \frac{e}{4}\sin 2\theta_W\,\mathsf{K}_{Z\gamma\gamma}\,\theta^{\rho\tau}\,[2Z^{\mu\nu}\left(2A_{\mu\rho}A_{\nu\tau} - aA_{\mu\nu}A_{\rho\tau}\right) + 8Z_{\mu\rho}A^{\mu\nu}A_{\nu\tau} - aZ_{\rho\tau}A_{\mu\nu}A^{\mu\nu}]$$

$$\mathsf{K}_{Z\gamma\gamma} = \frac{1}{2}\left[g'^2\kappa_1 + \left(g'^2 - 2g^2\right)\kappa_2\right]$$

$$\mathcal{L}^\theta_{WW\gamma} = \frac{e}{4}\sin 2\theta_W\,\mathsf{K}_{WW\gamma}\,\theta^{\rho\tau}\left\{A^{\mu\nu}\left[2\left({W^+}_{\mu\rho}{W^-}_{\nu\tau} + {W^-}_{\mu\rho}{W^+}_{\nu\tau}\right) - a\left({W^+}_{\mu\nu}{W^-}_{\rho\tau} + {W^-}_{\mu\nu}{W^+}_{\rho\tau}\right)\right] + 4A_{\mu\rho}\left(W^{+\mu\nu}{W^-}_{\nu\tau} + W^{-\mu\nu}{W^+}_{\nu\tau}\right) - aA_{\rho\tau}{W^+}_{\mu\nu}W^{-\mu\nu}\right\}$$

$$\mathsf{K}_{WW\gamma} = -\frac{g}{g'}\left[g'^2 + g^2\right]\kappa_2$$

$$\mathcal{L}^\theta_{WWZ} = \mathcal{L}_{WW\gamma}(A \leftrightarrow Z)$$

$$\mathsf{K}_{WWZ} = -\frac{g'}{g}\,\mathsf{K}_{WW\gamma}$$

$$\mathcal{L}^\theta_{ZZ\gamma} = \mathcal{L}_{Z\gamma\gamma}(A \leftrightarrow Z)$$

$$\mathsf{K}_{ZZ\gamma} = \frac{-1}{2gg'}\left[g'^4\kappa_1 + g^2\left(g^2 - 2g'^2\right)\kappa_2\right]$$

$$\mathcal{L}^\theta_{ZZZ} = \mathcal{L}_{\gamma\gamma\gamma}(A \to Z)$$

$$\mathsf{K}_{ZZZ} = \frac{-1}{2g^2}\left[g'^4\kappa_1 + 3g^4\kappa_2\right]$$

$$\mathcal{L}^\theta_{Zgg} = \mathcal{L}_{Z\gamma\gamma}(A \to G^b)$$

$$\mathsf{K}_{Zgg} = \frac{g_s^2}{2}\left[1 + (\frac{g'}{g})^2\right]\kappa_3$$

$$\mathcal{L}^\theta_{\gamma gg} = \mathcal{L}_{Zgg}(Z \to A)$$

$$\mathsf{K}_{\gamma gg} = \frac{-g_s^2}{2}\left[\frac{g}{g'} + \frac{g'}{g}\right]\kappa_3$$

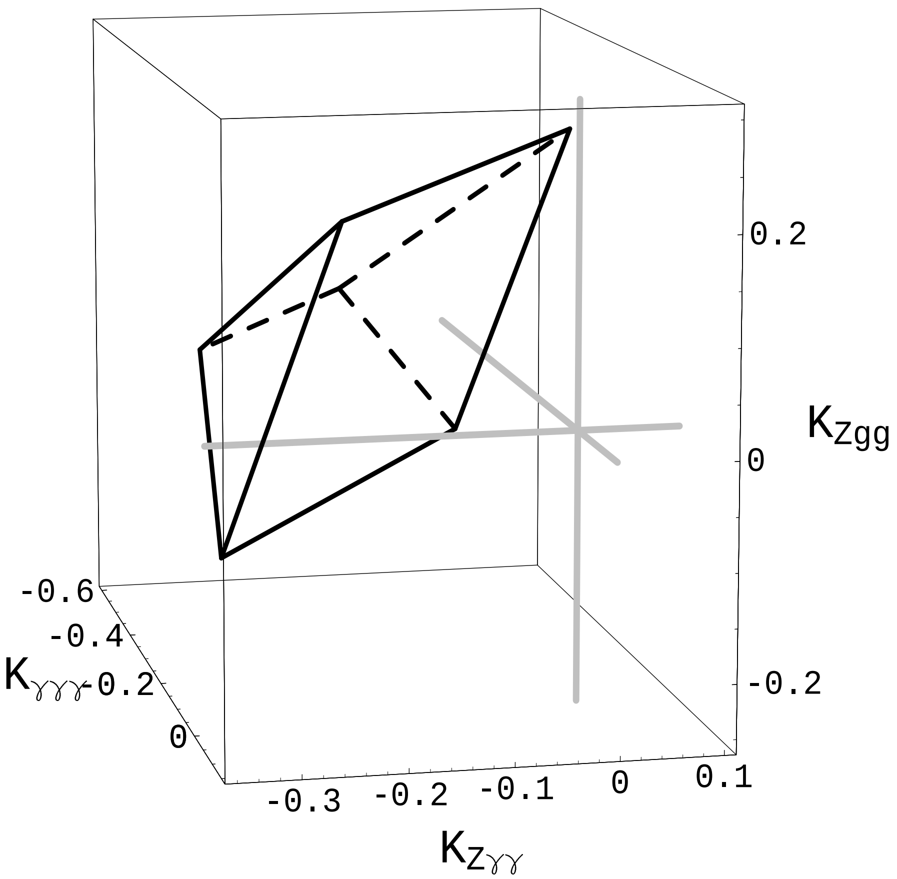
where $A_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu, \ldots$

$$\frac{2\mathrm{K}_{\gamma\gamma\gamma}}{gg'} = -\frac{1}{g_1^2}-\frac{1}{g_2^2}+\frac{8}{9g_3^2}-\frac{1}{9g_4^2}+\frac{7}{9g_5^2}+\frac{1}{g_6^2},$$

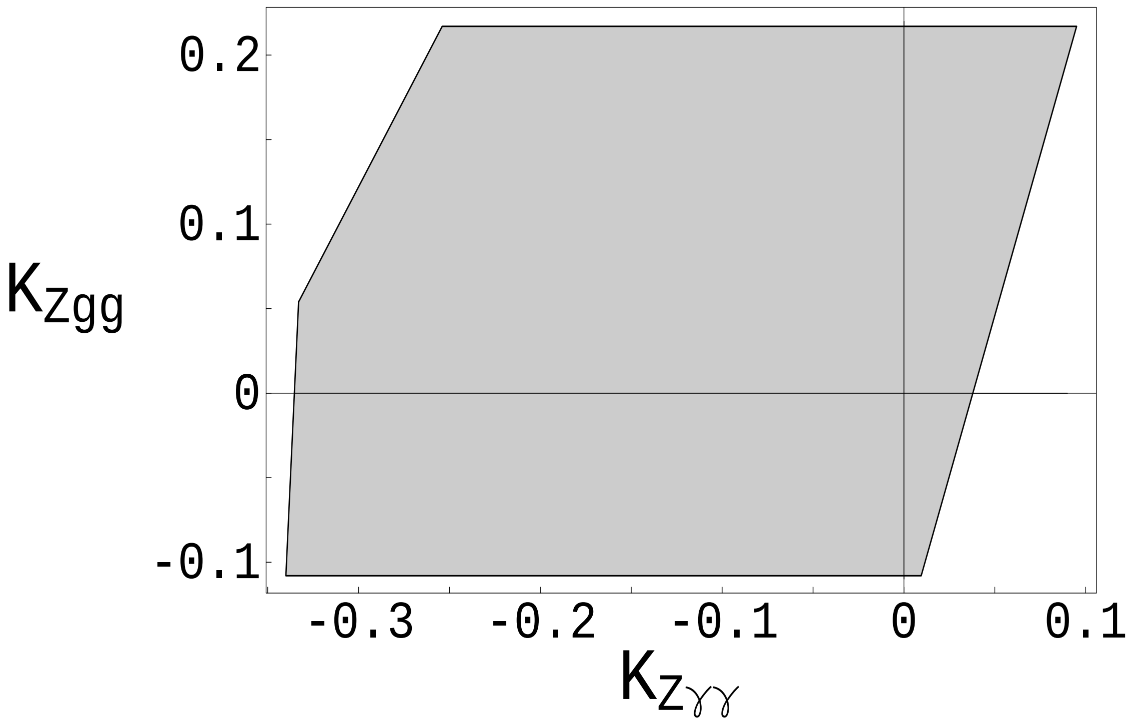
$$\frac{2\mathrm{K}_{Z\gamma\gamma}}{g'^2} = -\frac{1}{g_1^2}-\left(1-(\frac{g}{g'})^2\right)\frac{1}{2g_2^2}+\frac{8}{9g_3^2}-\frac{1}{9g_4^2}$$

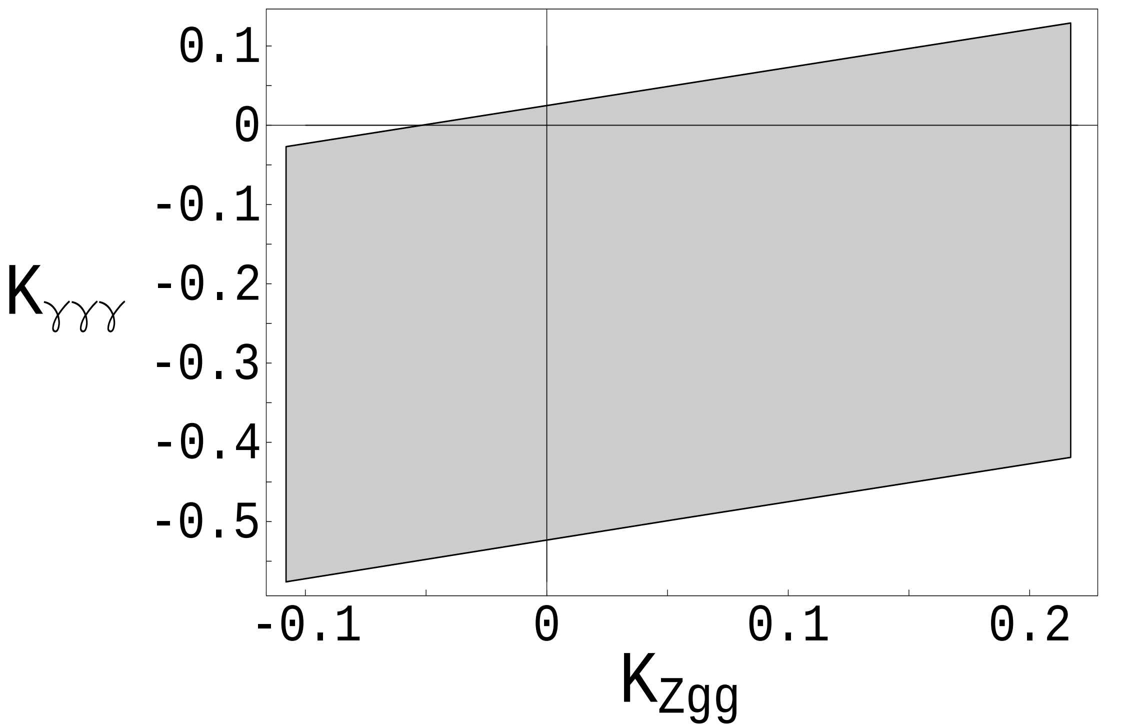
$$+\left(5-9(\frac{g}{g'})^2\right)\frac{1}{18g_5^2}+\left(1-(\frac{g}{g'})^2\right)\frac{1}{2g_6^2},$$

$$\frac{2\mathrm{K}_{Zgg}}{g_s^2} = \left(1+(\frac{g'}{g})^2\right)\left(\frac{1}{3g_3^2}-\frac{1}{6g_4^2}+\frac{1}{6g_5^2}\right).$$

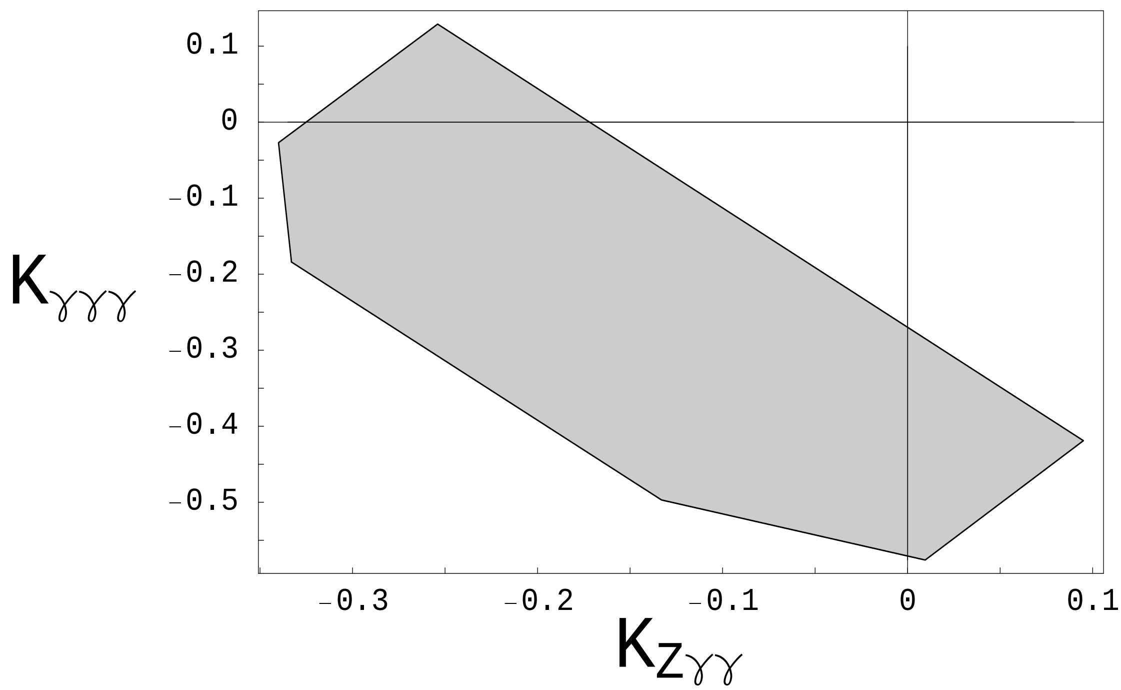


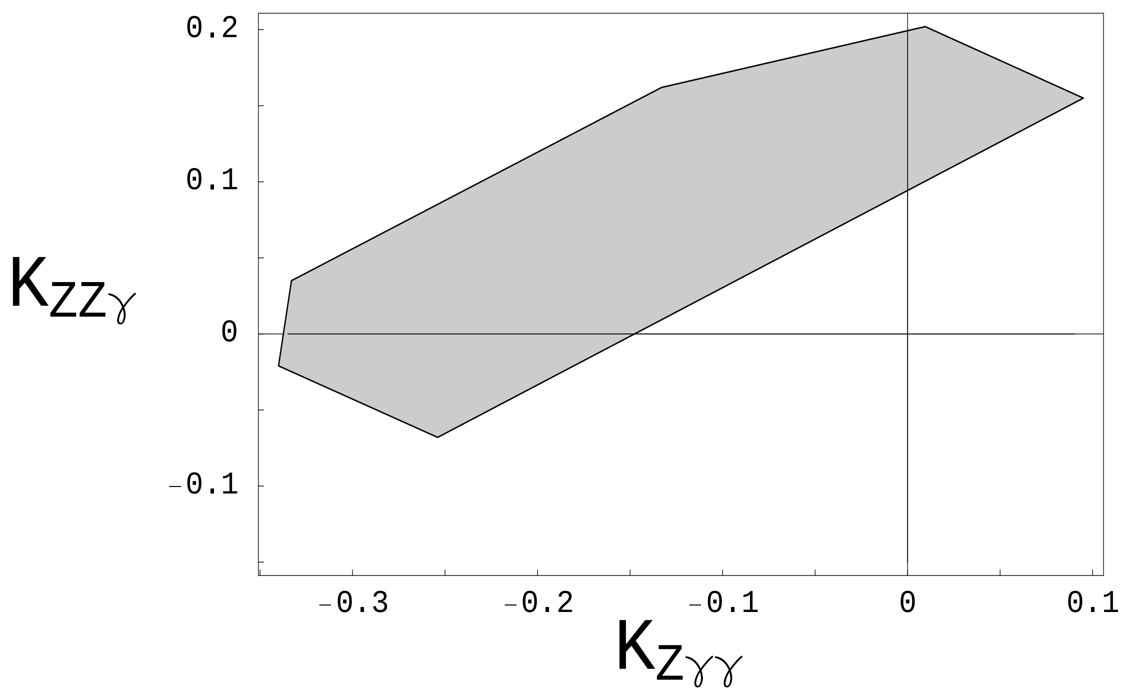
$\mathrm{K}_{\gamma\gamma\gamma}$	$\mathrm{K}_{Z\gamma\gamma}$	K_{Zgg}	$\mathrm{K}_{ZZ\gamma}$	K_{ZZZ}	$\mathrm{K}_{\gamma gg}$
−0.184	−0.333	0.054	0.035	−0.213	−0.098
−0.027	−0.340	−0.108	−0.021	−0.337	0.197
0.129	−0.254	0.217	−0.068	−0.362	−0.396
−0.576	0.010	−0.108	0.202	0.437	0.197
−0.497	−0.133	0.054	0.162	0.228	−0.098
−0.419	0.095	0.217	0.155	0.410	−0.396

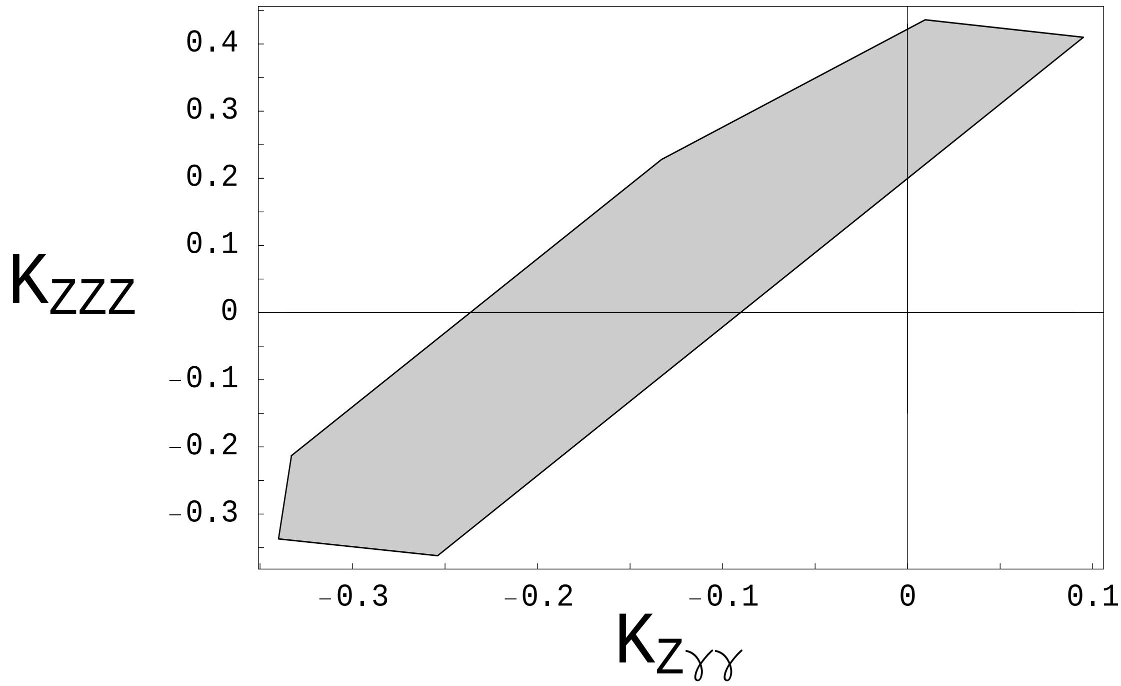

0.2
0.1
K_{Zgg}
0
-0.1
-0.3
-0.2
-0.1
0
0.1
$K_{Z\gamma\gamma}$

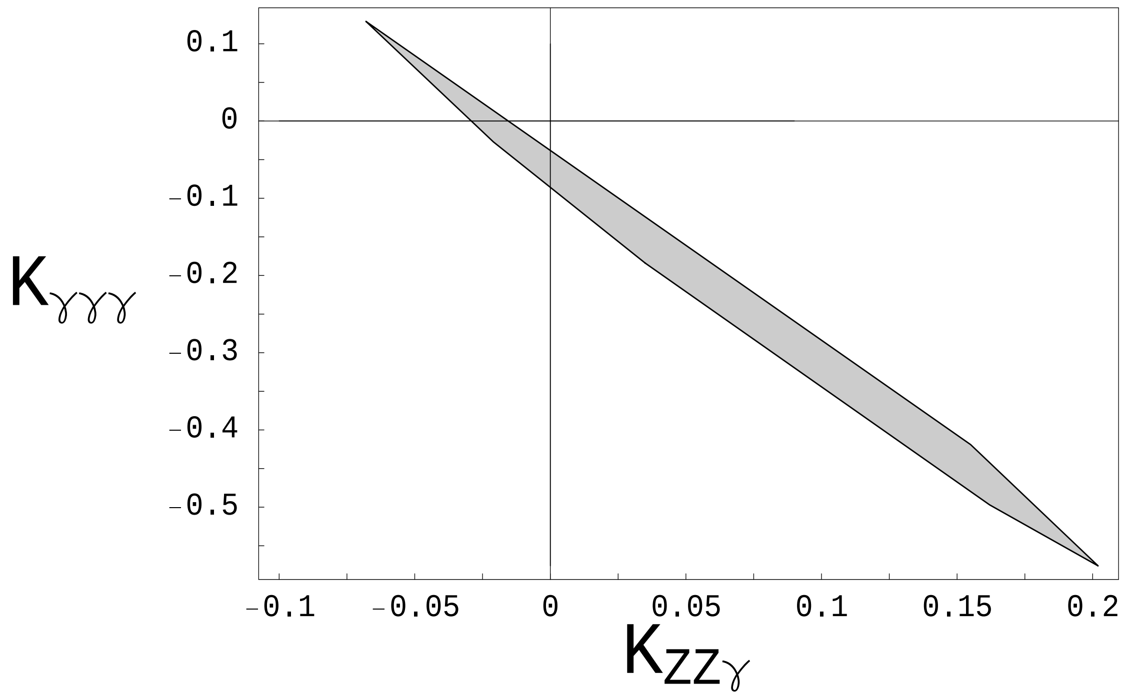

0.1
0
-0.1
$K_{\gamma\gamma\gamma}$
-0.2
-0.3
-0.4
-0.5
-0.1
0
0.1
0.2
K_{Zgg}

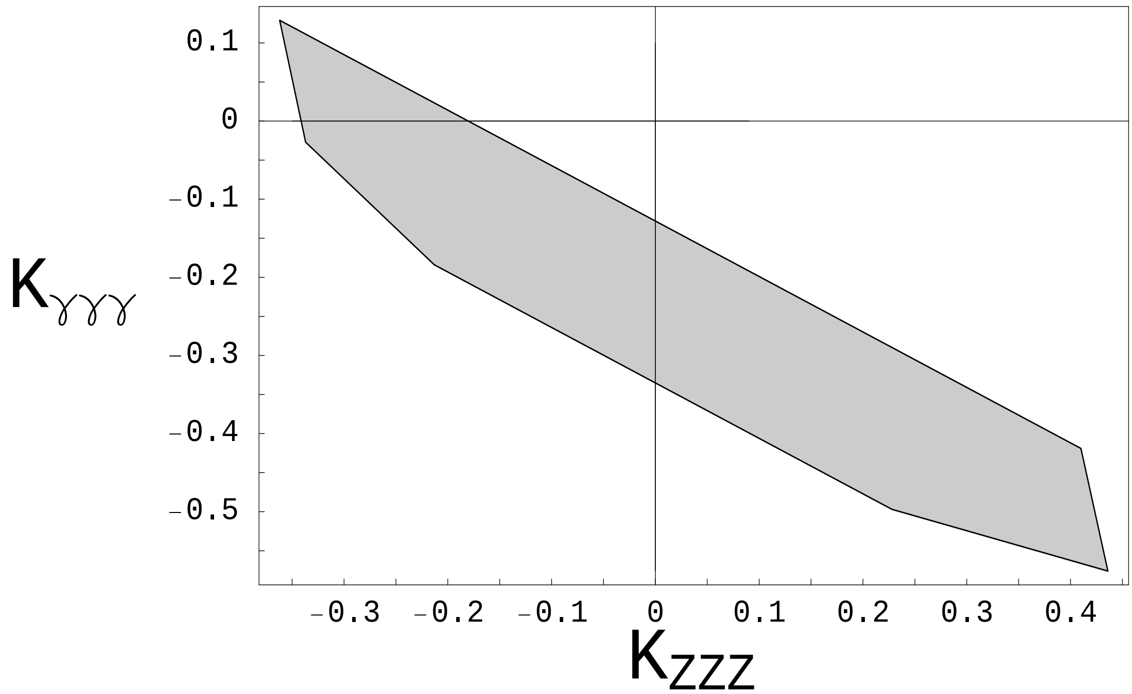
[G. Duplančić, P. Schupp and J. Trampetić; Comment on triple gauge boson interactions in the non-commutative electroweak sector, Eur. Phys. J. C**32** (2003) 141]



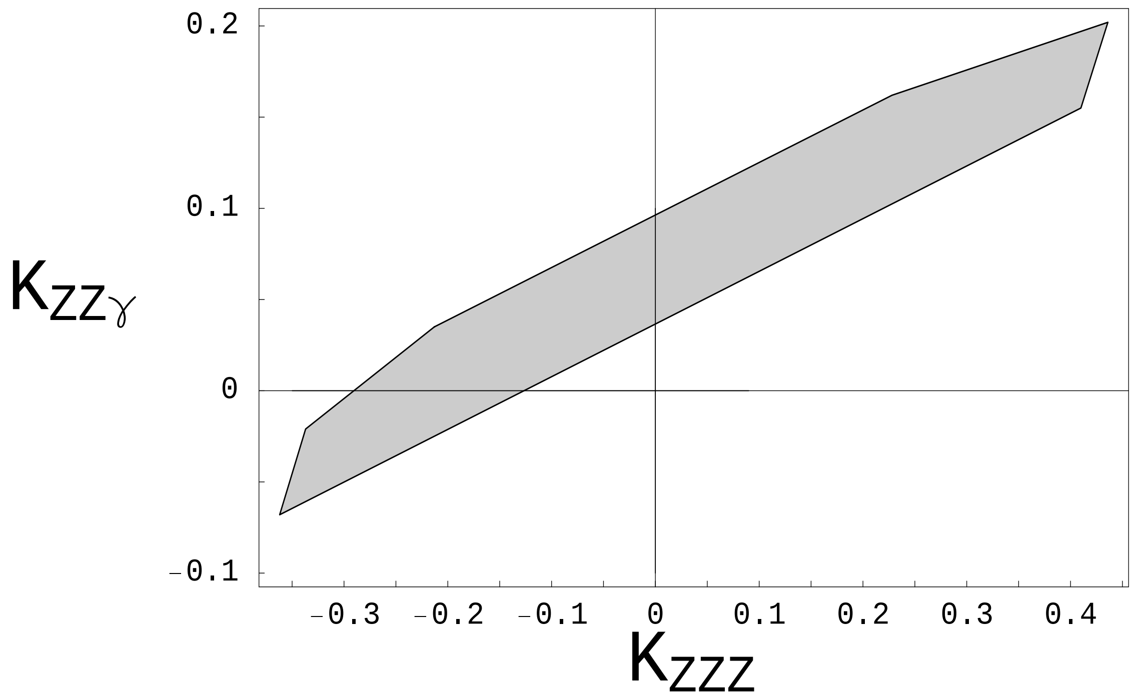



0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
K_{ZZZ}
-0.3
-0.2
-0.1
0
0.1
$K_{Z\gamma\gamma}$


0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
$K_{\gamma\gamma\gamma}$
-0.1
-0.05
0
0.05
0.1
0.15
0.2
$K_{ZZ\gamma}$

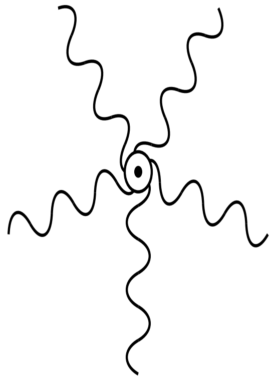


0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
$K_{\gamma\gamma\gamma}$
−0.3
−0.2
−0.1
0
0.1
0.2
0.3
0.4
K_{ZZZ}

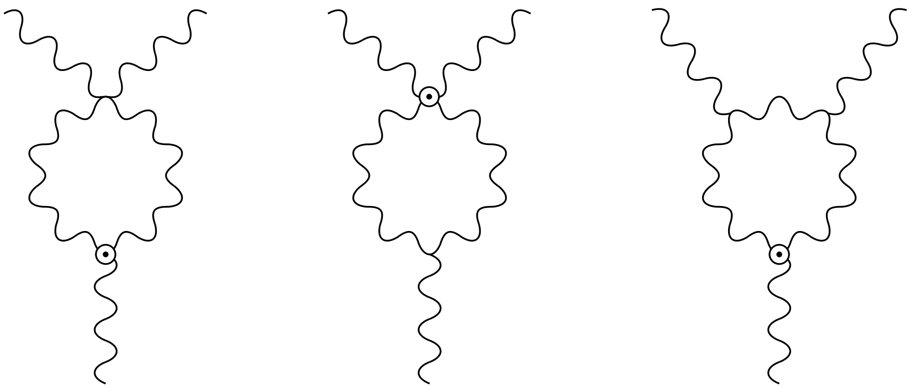


0.2
0.1
0
−0.1
$K_{ZZ\gamma}$
−0.3
−0.2
−0.1
0
0.1
0.2
0.3
0.4
K_{ZZZ}

Gauge Sector: Renormalizability



The NC couplings → additional vertices.
The lines are gauge fields $\mathcal{A}_\mu$, B^i_μ and G^a_μ



One-loop divergent corrections to the θ-3-vertex. also contains the contributions to the θ-4-vertex and θ-5-vertex.

Renormalizability: minimal NCSM

$$\Gamma^{(1)}_{\mathrm{div}} = \frac{N}{(4\pi)^2\epsilon}\theta^{\mu\nu}d^{abc}\int \mathrm{d}x\left(-\frac{25a-3}{48}G^a_{\mu\nu}G^b_{\rho\sigma}+\frac{a+21}{12}G^a_{\mu\rho}G^b_{\nu\sigma}\right)G^{c\rho\sigma},$$

$$\left(-\frac{25a-3}{48}\right):\left(\frac{a+21}{12}\right)=\frac{a}{4}:(-1)$$

Solutions exist for: $a = 1, 3$.

Renormalizability: nonminimal NCSM

$$\begin{aligned}
\Gamma^{(1)}_{\mathrm{div}} &= \frac{11}{3(4\pi)^2\epsilon}\int d^4x B^i_{\mu\nu}B^{\mu\nu i} + \frac{11}{2(4\pi)^2\epsilon}\int d^4x G^a_{\mu\nu}G^{\mu\nu a} \\
&+ \frac{4}{3(4\pi)^2\epsilon}g'g^2\kappa_2(3-a)\theta^{\mu\nu}\int d^4x\left(\frac{1}{4}f_{\mu\nu}B^i_{\rho\sigma}B^{\rho\sigma i} - f_{\mu\rho}B^i_{\nu\sigma}B^{\rho\sigma i}\right) \\
&+ \frac{6}{3(4\pi)^2\epsilon}g'g_S^2\kappa_3(3-a)\theta^{\mu\nu}\int d^4x\left(\frac{1}{4}f_{\mu\nu}G^a_{\rho\sigma}G^{\rho\sigma a} - f_{\mu\rho}G^a_{\nu\sigma}G^{\rho\sigma a}\right).
\end{aligned}$$

$\Gamma^{(1)}_{\mathrm{div}}$:
computed using background field method;
divergent part calculated in momentum representation by dimensional regularization.

FORBIDDEN DECAYS

GAUGE SECTOR: *TGB couplings and $Z \to \gamma\gamma$*

[W. Behr, N. G. Deshpande, G. Duplančić, P. Schupp, J.T. and J. Wess; The Z $\to \gamma\gamma$, g g decays in the non-commutative standard model, Eur. Phys. J. C **29**, 441 (2003)]

[G. Duplančić, P. Schupp and J. Trampetić; Comment on triple gauge boson interactions in the non-commutative electroweak sector, Eur. Phys. J. C**32** (2003) 141]

[M. Buric, V. Radovanovic and J.T., The one-loop renormalization of the gauge sector in the noncommutative standard model; hep-th/0609073]

[M. Buric, D. Latas, V. Radovanovic and J.T., Improved Z $\to \gamma\gamma$ decay in the renormalizable NCSM; work in progress]

From $\mathcal{L}_{Z\gamma\gamma} \Rightarrow$ the gauge-invariant amplitude $\mathcal{A}_{Z\to\gamma\gamma}$

$$\begin{aligned}\mathcal{A}^{\theta}(Z\to\gamma\gamma) &= -2e\sin 2\theta_W \,\mathrm{K}_{Z\gamma\gamma}\Theta_3^{\mu\nu\rho}(a;k_1,-k_2,-k_3)\\ &\times\ \epsilon_\mu(k_1)\epsilon_\nu(k_2)\epsilon_\rho(k_3);\end{aligned}$$

$k_1+k_2+k_3=0$;

$$\begin{aligned}\Theta_3^{\mu\nu\rho}(a;k_1,k_2,k_3) &= -(k_1\theta k_2)\\
&\times[(k_1-k_2)^\rho g^{\mu\nu}+(k_2-k_3)^\mu g^{\nu\rho}+(k_3-k_1)^\nu g^{\rho\mu}]\\
&-\theta^{\mu\nu}\,[k_1^\rho\,(k_2k_3)-k_2^\rho\,(k_1k_3)]\\
&-\theta^{\nu\rho}\,[k_2^\mu\,(k_3k_1)-k_3^\mu\,(k_2k_1)]\\
&-\theta^{\rho\mu}\,[k_3^\nu\,(k_1k_2)-k_1^\nu\,(k_3k_2)]\\
&+(\theta k_2)^\mu\,\left[g^{\nu\rho}\,k_3^2-k_3^\nu k_3^\rho\right]+(\theta k_3)^\mu\,\left[g^{\nu\rho}\,k_2^2-k_2^\nu k_2^\rho\right]\\
&+(\theta k_3)^\nu\,\left[g^{\mu\rho}\,k_1^2-k_1^\mu k_1^\rho\right]+(\theta k_1)^\nu\,\left[g^{\mu\rho}\,k_3^2-k_3^\mu k_3^\rho\right]\\
&+(\theta k_1)^\rho\,\left[g^{\mu\nu}\,k_2^2-k_2^\mu k_2^\nu\right]+(\theta k_2)^\rho\,\left[g^{\mu\nu}\,k_1^2-k_1^\mu k_1^\nu\right]\\
&+\theta^{\mu\alpha}(ak_1+k_2+k_3)_\alpha\,\left[g^{\nu\rho}\,(k_3k_2)-k_3^\nu k_2^\rho\right]\\
&+\theta^{\nu\alpha}(k_1+ak_2+k_3)_\alpha\,\left[g^{\mu\rho}\,(k_3k_1)-k_3^\mu k_1^\rho\right]\\
&+\theta^{\rho\alpha}(k_1+k_2+ak_3)_\alpha\,\left[g^{\mu\nu}\,(k_2k_1)-k_2^\mu k_1^\nu\right]\,.\end{aligned}$$

$$\sum_{\text{spins}} |\mathcal{A}_{Z\to\gamma\gamma}|^2 = -\theta^2 + (5a^2 - 22a + 25)\frac{(p\theta^2 p)}{M_Z^2}$$
$$-(a^2 + 2a - 3)\frac{(k\theta^2 k + k'\theta^2 k')}{M_Z^2}$$
$$-4(a-3)(3a-5)\frac{(k\theta k')^2}{M_Z^4}\,.$$

For $a = 3$

Z-boson at rest

$$\Gamma_{Z\to\gamma\gamma} = \frac{\alpha}{12}\frac{M_Z^5}{\Lambda_{\text{NC}}^4}\sin^2 2\theta_W \mathrm{K}^2_{Z\gamma\gamma}$$
$$\times \left[\frac{1}{6}(9a^2 - 58a + 63)\vec{E}_\theta^2 + \frac{a}{3}(a+2)\vec{B}_\theta^2\right],$$

$$\vec{E}_\theta = (c^{01}, c^{02}, c^{03}),\ \ \vec{B}_\theta = (c^{23}, c^{13}, c^{12})$$

$$-\theta^2 = -(\theta^2)^\mu_\mu = -\theta_{\mu\nu}\theta^{\nu\mu} = \theta_{\mu\nu}\theta^{\mu\nu} = -\frac{2}{\Lambda_{\text{NC}}^4}\left(\vec{E}_\theta^2 - \vec{B}_\theta^2\right)$$
$$\equiv \frac{2}{\Lambda_{\text{NC}}^4}\left(\sum_{i,j=1\,;\,i<j}^{3}(c^{ij})^2 - \sum_{i=1}^{3}(c^{0i})^2\right) > 0$$

For $a = 1$

Z-boson at rest

$$\Gamma_{Z\to\gamma\gamma} = \frac{\alpha}{12}\frac{M_Z^5}{\Lambda_{\rm NC}^4}\sin^2 2\theta_W \mathsf{K}_{Z\gamma\gamma}^2\left[\frac{7}{3}\vec{E}_\theta^2 + \vec{B}_\theta^2\right]$$

Z-boson at rest and polarized along the 3-axis

$$\Gamma_{Z^3\to\gamma\gamma} = \frac{\alpha}{4}\frac{M_Z^5}{\Lambda_{\rm NC}^4}\sin^2 2\theta_W\, \mathsf{K}_{Z\gamma\gamma}^2 \times\left[\frac{2}{5}\left((\theta^{01})^2 + (\theta^{02})^2\right) + \frac{23}{15}(\theta^{03})^2 + (\theta^{12})^2\right]$$

The same Lorentz structure of $\mathcal{L}_{Z\gamma\gamma}$ and $\mathcal{L}_{Zgg}$:

$$\frac{\Gamma_{Z\to gg}}{\Gamma_{Z\to\gamma\gamma}} = \frac{\Gamma_{Z^3\to gg}}{\Gamma_{Z^3\to\gamma\gamma}} = 8\frac{\mathsf{K}_{Zgg}^2}{\mathsf{K}_{Z\gamma\gamma}^2}.$$

The factor of eight in the above ratios is due to color.

For $a = 3$

Z-boson at rest

$$\Gamma_{Z\to\gamma\gamma} = \frac{5\alpha}{24} M_Z^5 \sin^2 2\theta_W \mathsf{K}_{Z\gamma\gamma}^2\left(\theta_{\mu\nu}\theta^{\mu\nu}\right)$$
$$= \frac{5\alpha}{12}\frac{M_Z^5}{\Lambda_{\rm NC}^4}\sin^2 2\theta_W \mathsf{K}_{Z\gamma\gamma}^2\left(\vec{B}_\theta^2 - \vec{E}_\theta^2\right)$$

Experimental situation

Decay mode: $Z \to \gamma\gamma$

$$BR = \frac{\Gamma(Z \to \gamma\gamma)}{\Gamma_{tot}(Z)} \begin{cases} < 5.2 \times 10^{-5} & \text{L3} & 1995 \\ < 5.5 \times 10^{-5} & \text{DELPHI} & 1994 \\ < 1.4 \times 10^{-4} & \text{OPAL} & 1991 \end{cases}$$

$e^+e^- \to \gamma\gamma$ near Z resonance is an ideal process to test QED. The present statistic enables comparison of data with the QED up to $\mathcal{O}(\alpha^3)$.

Deviation of the experimentaly measured cross sections from the QED prediction
$\longrightarrow$ evidence for $Z \to \gamma\gamma$ (SM forbidden) and $Z \to \pi^0\gamma\,/\,\eta\gamma$.

Decay mode: $Z \to gg$
could be observed through $Z \to 2\,\text{jets}$ processes.

Taking into account discrepancy between the experimentally observed hadronic width for Z boson and the theoretical SM estimate, we estimate the upper bound for any new hadronic mode to be of the order 10^{-3} GeV.

$$\Gamma_{Z\to gg} < 10^{-3}\ \text{GeV}$$

The forbidden decay $Z \to \gamma\gamma$ and the real decays $Z \to \pi^0\gamma \,/\, \eta\gamma$ would have the same experimental signature as the SM forbidden process

$$e^+e^- \to Z^* \to \gamma\gamma$$

Rare decays at high energies, the two photons from π^0 or η decays are very close seen in EM calorimeter as a single high energy photon:

$$e^+e^- \to Z^* \to (\pi^0, \eta)\gamma \to (\gamma\gamma)\gamma$$

The measurement of the total cross section as a function of center of mass energy ($\sqrt{s}$) can be used to set limits on (L3 COLLABORATION – 1995):

$$\begin{aligned} BR(Z \to \pi^0\gamma) &< 5.2 \times 10^{-5} \\ BR(Z \to \eta\gamma) &< 7.6 \times 10^{-5} \\ BR(Z \to \gamma\gamma) &< 5.2 \times 10^{-5} \end{aligned}$$

Theoretical estimates $Br(Z \to \pi^0\gamma \,/\, \eta\gamma) \sim 10^{-10}$.

(Arnellos at al. Nucl.Phys.B 196 (1982) 378)

Discusion: GAUGE SECTOR

The range of the scale of non-commutativity:

$$1 \geq \Lambda_{\rm NC}/{\rm TeV} \geq 0.25$$

– For central value $|{\rm K}_{Z\gamma\gamma}| \simeq 0.1$ we have

$$4 \times 10^{-8} \lesssim BR^{\rm nmNCSM}(Z \to \gamma\gamma) \lesssim 10^{-5}$$

– For pure electroweak part of nmNCSM (extracted from pentahedron), any pair of electroweak couplings never vanish simultaneously – Figures from:

[G. Duplančić, P. Schupp and J.T.; Comment on triple gauge boson interactions in the non-commutative electroweak sector, Eur. Phys. J. C**32** (2003) 141]

– From L3 Collboration (1995) experiment:

$$BR(Z \to \gamma\gamma) \;<\; 5.2 \times 10^{-5}$$

and for the central value $|{\rm K}_{Z\gamma\gamma}| \simeq 0.1$ we found the following bound on the scale of noncommutativity

$$a = 1 \quad (\vec{E}_\theta^2 \simeq \vec{B}_\theta^2 \simeq 1) \;\Rightarrow\; \Lambda_{\rm NC} > 162 \text{ GeV}$$

$$a = 3 \quad (\vec{E}_\theta^2 = 0;\; \vec{B}_\theta^2 \simeq 1) \;\Rightarrow\; \Lambda_{\rm NC} > 180 \text{ GeV}$$

DISCUSION

Limits on Λ_{NC} from theory and experiment

DECAYS: 1→ 2

* $Z \to \gamma\gamma \quad \Rightarrow \Lambda_{NC} > \binom{162}{180}$ GeV, [Duplančić,...];[Burić,...]

* $\gamma_{pl} \to \nu\bar{\nu} \quad \Rightarrow \Lambda_{NC} >$ 81 GeV, [Schupp, JT, Wess, Raffelt]

* $J/\psi \to \gamma\gamma \quad \Rightarrow \Lambda_{NC} >$ 9 GeV, [Melic, Passek, J.T.]

* $K \to \pi\gamma \quad \Rightarrow \Lambda_{NC} >$ 43 GeV, [Melic, Passek, J.T.]

SCATTERINGS: 2→ 2

* $e^+e^- \to \gamma\gamma \Rightarrow \Lambda_{NC} >$ 141 GeV, [OPAL Coll. (2003)]

* $\gamma\gamma \to \bar{f}f \quad \Rightarrow \Lambda_{NC} >$ 200 GeV, [T. Ohl et al.]

* $\bar{f}f \to Z\gamma \quad \Rightarrow \Lambda_{NC} >$ 1 TeV, [T. Ohl et al.]

NEUTRINO DIPOLE MOMENTS:

* $(d_{mag})^{Dirac} \quad \Rightarrow \Lambda_{NC} >$ 1.8 TeV, [Minkowski et al.]

* $(d^{el}_{mag})^{Majorana} \Rightarrow \Lambda_{NC} >$ 150 TeV, [Minkowski et al.]

CONCLUSION

* The SM forbidden decay signals are clean due to the absence of the SM contributions.

* Deviations from SM, due to the NC contributions, of the differential cross sections for $2 \longrightarrow 2$ transitions could be significant and potentialy measurable.

* The sensitivity to the noncommutative parameter $\theta^{\mu\nu}$ could be in a range of the next generation of linear colliders with a c.m.e. around a few TeV's.

* Experimental discovery of the SM forbidden decays and/or SM deviations in the scattering processses would indicate physics beyond the SM.

* To determine if above signals are coming from the NC or some other sources would require large amount of extra theoretical and experimental work.