New State of Matter in White Dwarfs

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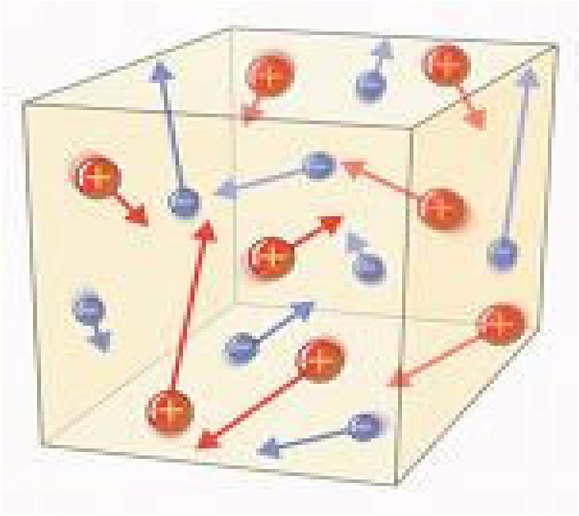
with R. A. Rosen, JCAP, JHEP 08,09
D. Pirtskhalava, JCAP 09



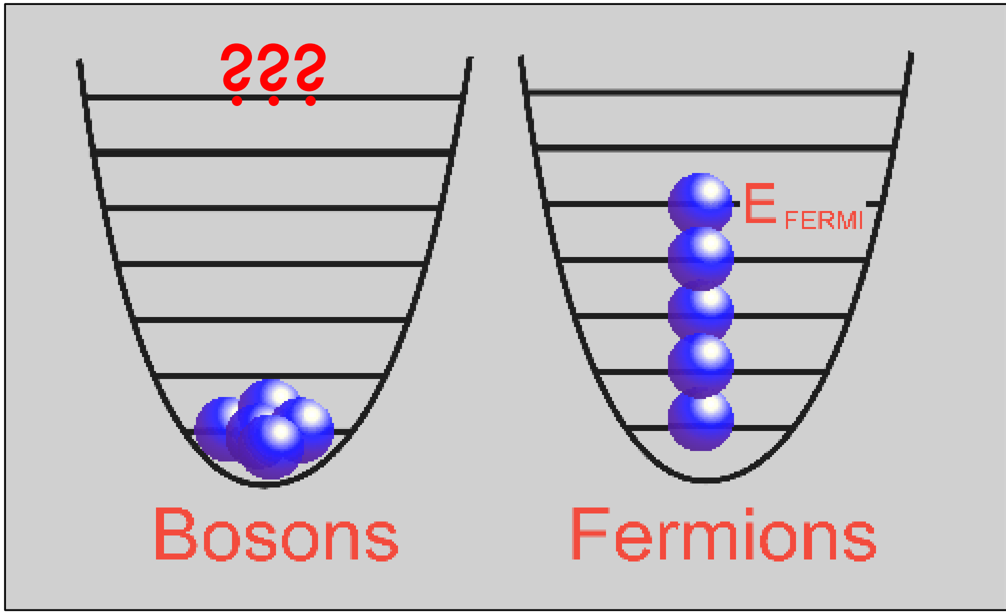
He or C nuclei and Electrons

d -- average interparticle separation

1 Fermi << d << 1 Angström

$$\rho \simeq 10^6 \frac{\mathrm{g}}{\mathrm{cm}^3}$$

10 eV < T << 10 KeV

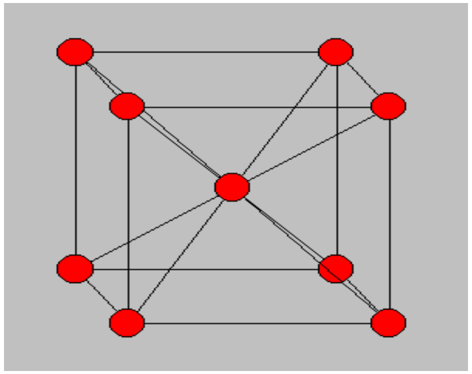


Crystallization temperature

$$\Gamma \equiv \frac{E_{\mathrm{Coulomb}}}{2E_{\mathrm{Thermal}}/3} = \frac{(Ze)^2/4\pi d}{k_B T}$$

$$\Gamma \simeq 180$$

To crystallize



$$T_{\mathrm{cryst}} \simeq 0.8 \cdot 10^3 \rho^{\frac{1}{3}} Z^{\frac{5}{3}} \ K$$

Critical Temperature

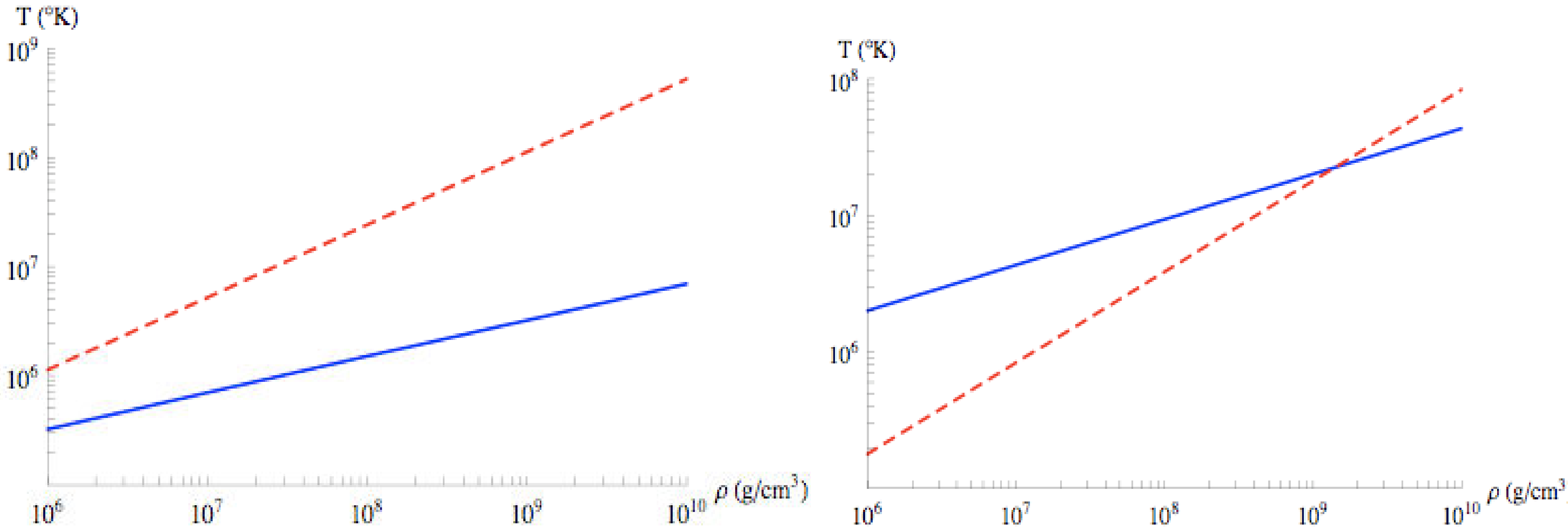
Temperature at which thermal de Broglie wavelengths begin to overlap

$$T_c \simeq \frac{4\pi^2}{3 m_H d^2}$$

Uncertainties in particle position as large as inter-particle separation

$$T_c = \rho^{2/3} \left(\frac{3.5 \cdot 10^2}{Z^{5/3}} \right) K$$

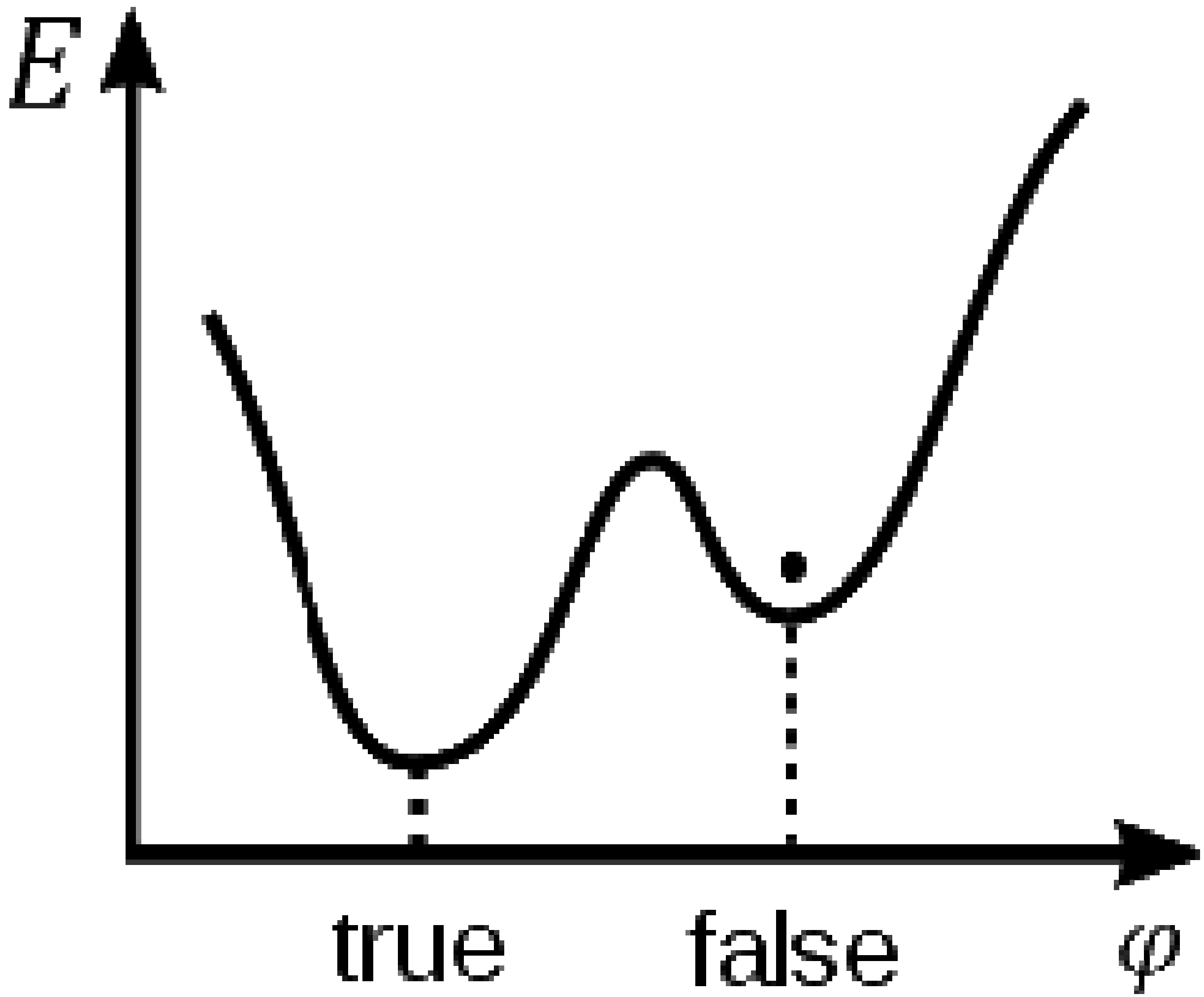
Critical T > Crystallization T => Charged Condensation!



(a) Helium nuclei

(b) Carbon nuclei

Figure 2.1: Crystallization temperatures (blue, solid line) and condensation temperatures (red, dashed line) as a function of mass density for systems of helium and carbon nuclei.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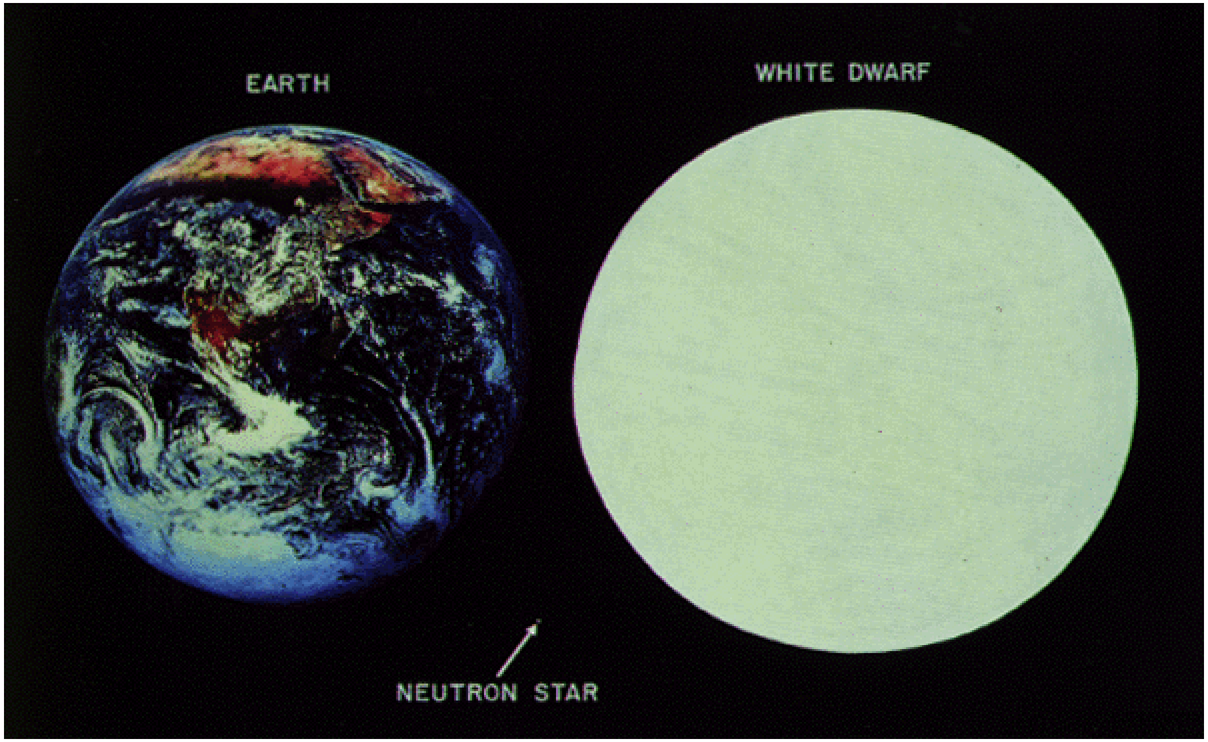
Relatively Light Stars End Up As White Dwarfs

Heavier Stars End Up Either As Neutron Stars Or As Black Holes



Typical density: $\rho \simeq 10^6 \frac{\mathrm{g}}{\mathrm{cm}^3}$

What Keeps White Dwarfs Stable?

Electrons -

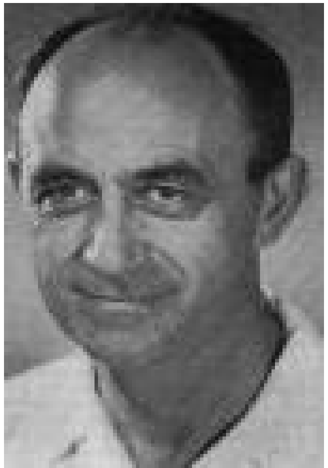
Outward Directed
Electron Pressure

$$N\sqrt{k_F^2 + m_e^2}$$

Nuclei +

Inward Directed
Gravitational Pull

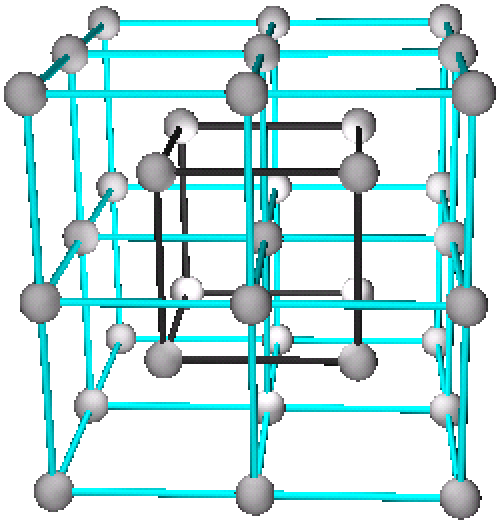
$$-\frac{GM^2}{R}$$

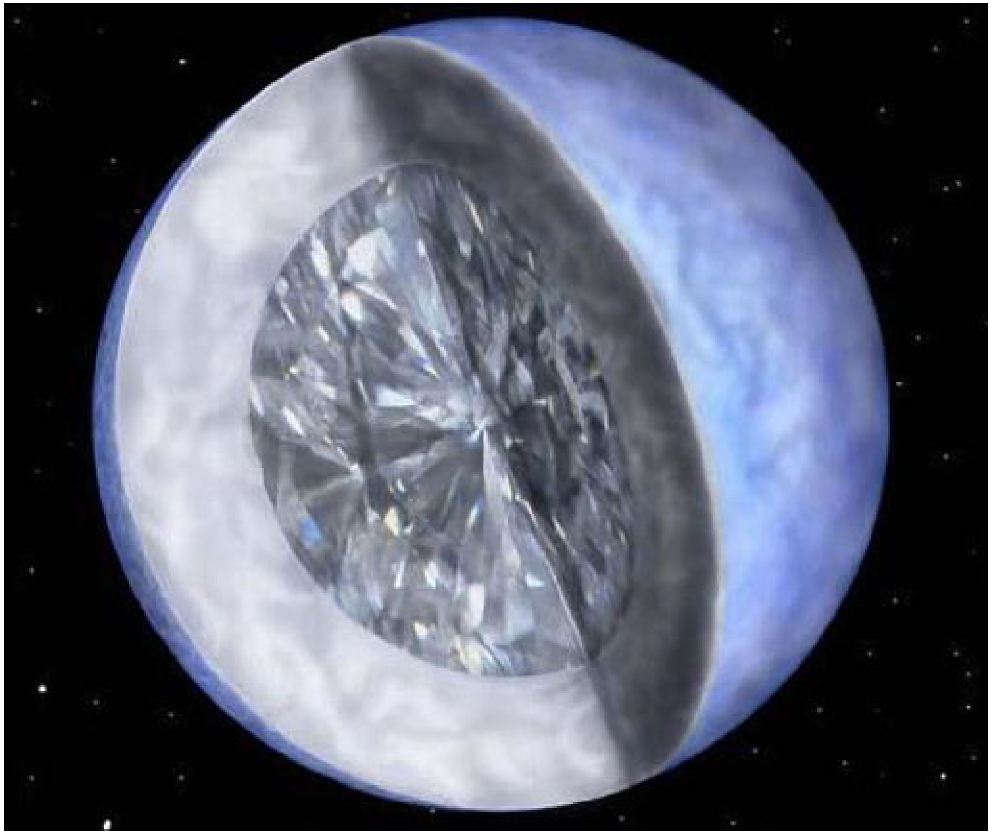


versus

Standard Theory: Carbon, Oxygen, …,
Containing WD's Crystallize As they Cool Down

(Mestel, Ruderman)





Specific heat dominated by the crystal phonon contribution

$$C_V \sim T^3$$

Cooling time: ~10 Gyr

Helium White Dwarfs In Binary Systems

0.2-0.5 solar mass dwarfs with helium core

Some characteristics for helium white dwarfs

Physical quantity	Numerical value
Electron density	$(0.13\ MeV)^3$
(Nonrelativistic) electron Fermi energy	$0.16\ MeV$
Separation between atomic nuclei	$1.1 \cdot 10^3\ fm$
Crystallization temperature	$2.5 \cdot 10^5\ K$
Debye temperature	$4 \cdot 10^6\ K$
Critical temperature	$1.1 \cdot 10^6\ K$

Average carbon white dwarf

Physical quantity	Numerical value
Electron density	$(0.4\ MeV)^3$
Relativistic electron Fermi energy	$1.3\ MeV$
Separation between atomic nuclei	$5.2 \cdot 10^2\ fm$
Crystallization temperature	$5.0 \cdot 10^6\ K$
Debye temperature	$2.2 \cdot 10^7\ K$
Critical temperature	$1.6 \cdot 10^6\ K$

Building up effective field theory

- Order parameter: complex scalar
- U(1) symmetries for scalars and fermions
- Rotational symmetry
- Galilean invariance
- Schrödinger equation in the lowest order

Effective Order Parameter Lagrangian for Charged Condensation

$$\mathcal{L}_{eff} = \mathcal{P}\left(\frac{i}{2}(\Phi^* D_0\Phi - (D_0\Phi)^*\Phi) - \frac{|D_j\Phi|^2}{2m_H}\right) + \bar{\psi}(i\gamma^\mu D^f_\mu - m_f)\psi$$

$$\mathcal{P}(x) = x + C_2 x^2 + \dots .$$

Greiter, Wilczek,Witten
in BCS superconductivity

Solution

$$Z|\Phi|^2 = J_0\,, \quad A_\mu = 0, \quad \mathcal{P}'(0) = 1$$

Small perturbations (simplified)

$$\mathcal{L}_2 = -\frac{1}{4} f_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu \tau)^2 + \frac{1}{2} m_\gamma^2 b_\mu^2 + 2 m_H m_\gamma b_0 \tau$$

$$\mathcal{H} = \frac{\pi_j^2}{2} + \frac{f_{ij}^2}{4} + \frac{(\partial_j \pi_j - 2 m_H m_\gamma \tau)^2}{2 m_\gamma^2} + \frac{P_\tau^2 + (\partial_j \tau)^2}{2} + \frac{m_\gamma^2 b_j^2}{2}$$

Spectrum of small perturbations

Two transverse modes:

$$\omega^2 = \mathbf{k}^2 + m_\gamma^2, \qquad m_\gamma^2 \equiv \frac{Ze^2 J_0}{m_H}$$

One longitudinal mode – phonon

$$\omega^2 \simeq m_\gamma^2 + \frac{\mathbf{k}^4}{4m_H^2}$$

Mass gap, Landau condition, super(fluidity)conductivity!

Phonon contribution to specific heat: $\exp(-m_\gamma/T)$

Impurity Interactions (H, He3,...)

Charge screening:

$$G(\omega = 0, \mathbf{k}) \simeq \left(\mathbf{k}^2 + m_0^2 + \frac{4M^4}{\mathbf{k}^2} \right)^{-1}$$

$$= \frac{1}{\mathbf{k}^2 + m_0^2} - \frac{1}{\mathbf{k}^2 + m_0^2 + \mathbf{k}^2(\mathbf{k}^2 + m_0^2)^2/4M^4} .$$

$$V = \frac{\alpha_{\rm em} Q_1 Q_2 e^{-Mr}}{r} \cos(Mr) \, , \quad M \equiv (2e^2 m_H J_0)^{1/4} \, .$$

Gap-less Fermion Excitations

Friedel potential:

$$V_{stat} = \alpha_{\rm em} Q_1 Q_2 \left(\frac{e^{-m_0 r}}{r} + \frac{\alpha_{\rm em}}{4\pi} \frac{\sin(2k_F r)}{k_F^3 r^4} \right)$$

Long-range sign-indefinite interactions!

RKKY (Ruderman-Kittel-Kazuya-Yosida)

Kohn-Luttinger effect: attraction, superconductivity.

Gap-less Fermions and Phonon

Generalised Friedel potential:

$$V_{stat} = \alpha_{\rm em} Q_1 Q_2 \left(\frac{e^{-Mr}}{r} \cos(Mr) + \frac{4\alpha_{\rm em}}{\pi} \frac{k_F^5 \sin(2k_F r)}{M^8 r^4} \right) .$$

Generalised Kahn-Luttinger effect:

$$\exp\left(-\frac{1}{e_{eff}^2}\right), \quad e_{eff}^2 \propto e^2 \left(\mu_f/m_H\right)^2 .$$

Supercondustivity at too low temperature: $T_{sc} \sim exp(-400)$.

White dwarf cooling

$$-c_v \frac{dT}{dt} = L = CAm_u T^{3.5}$$

State	Specific heat
Classical Bose gas	$\frac{3}{2} k_B$
Quantum crystal	$\frac{16\pi^4}{5} \left(\frac{T}{\theta_D}\right)^3 k_B$
Nonrelativistic Fermi Gas	$\frac{\pi^2}{2} \frac{k_B T}{E_F} k_B$
Relativistic Fermi Gas	$\frac{(3\pi^2)^{\frac{2}{3}}}{3} \frac{k_B T}{J_0^{1/3}} k_B$

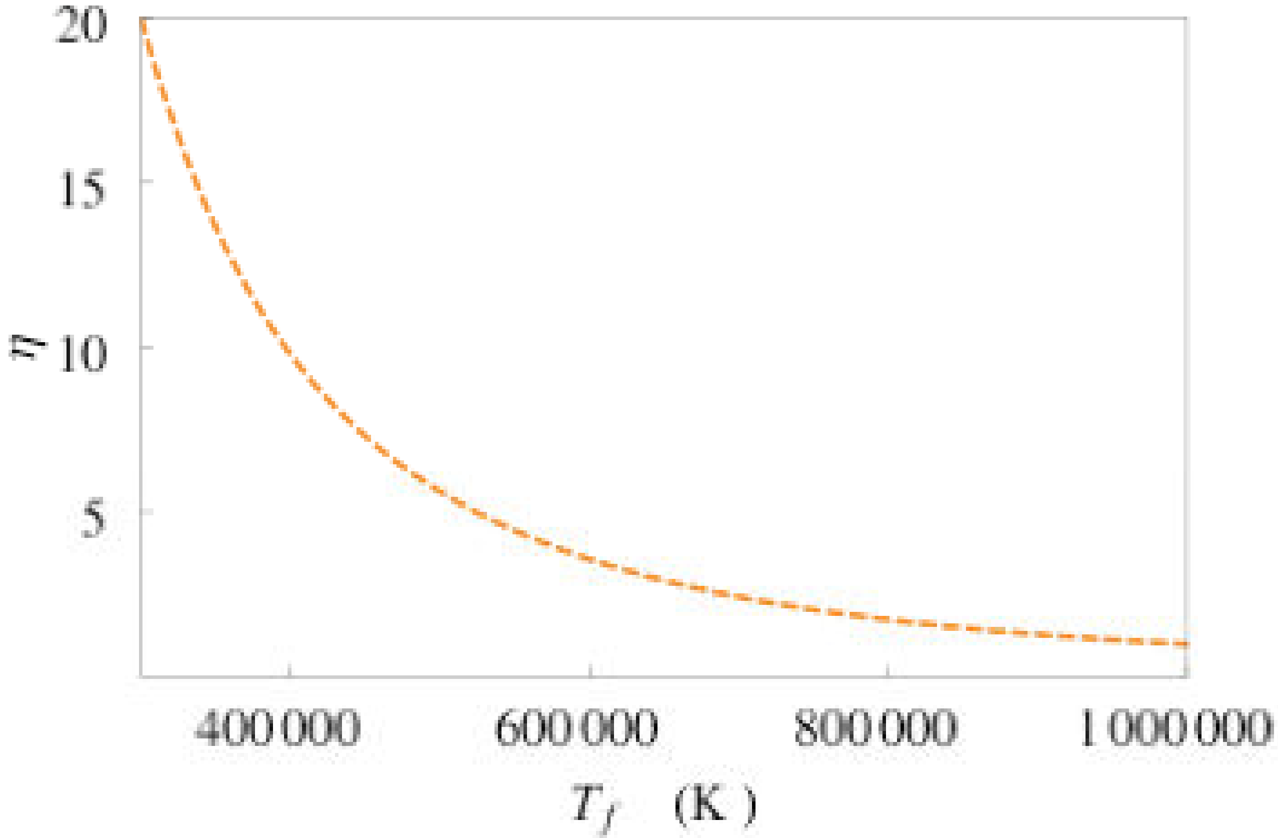
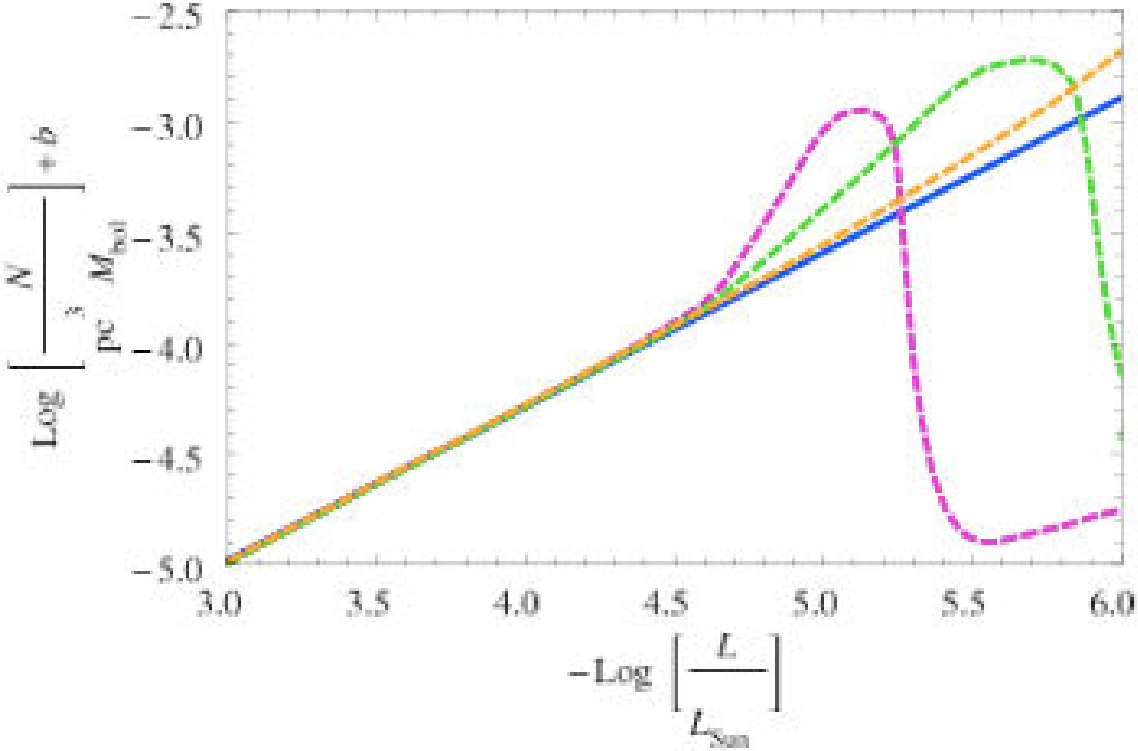


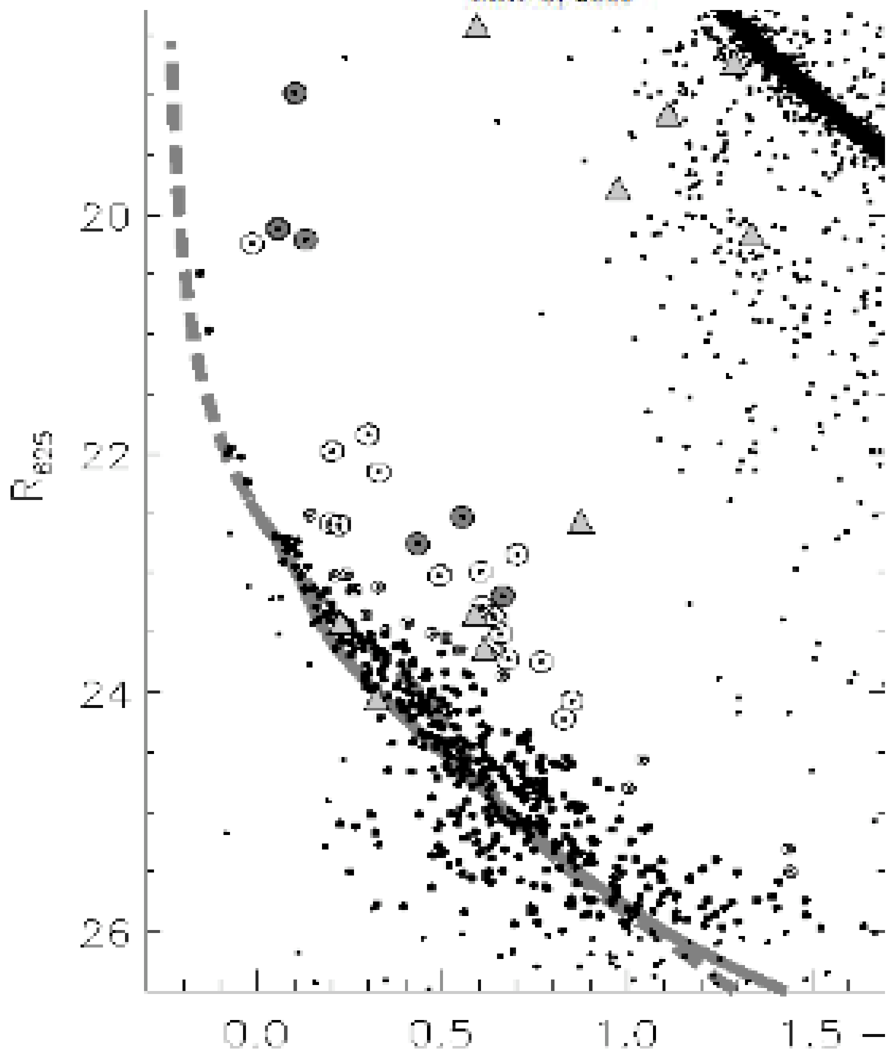
Figure 1. The ratio of the ages, as a function of final temperature, for two identical helium dwarf stars, with and without the interior condensation.


−2.5
−3.0
−3.5
−4.0
−4.5
−5.0
3.0
3.5
4.0
4.5
5.0
5.5
6.0
$\mathrm{Log}\left[\frac{N}{\mathrm{pc}^3\ M_{bol}}\right]+b$
$-\mathrm{Log}\left[\frac{L}{L_{Sun}}\right]$

HELIUM-CORE WHITE DWARFS IN THE GLOBULAR CLUSTER NGC 6397

Rachel R. Strickler[1,2], Adrienne M. Cool[2], Jay Anderson[3], Haldan N. Cohn[4], Phyllis M. Lugger[4], Aldo M. Serenelli[4]

May 3, 2009



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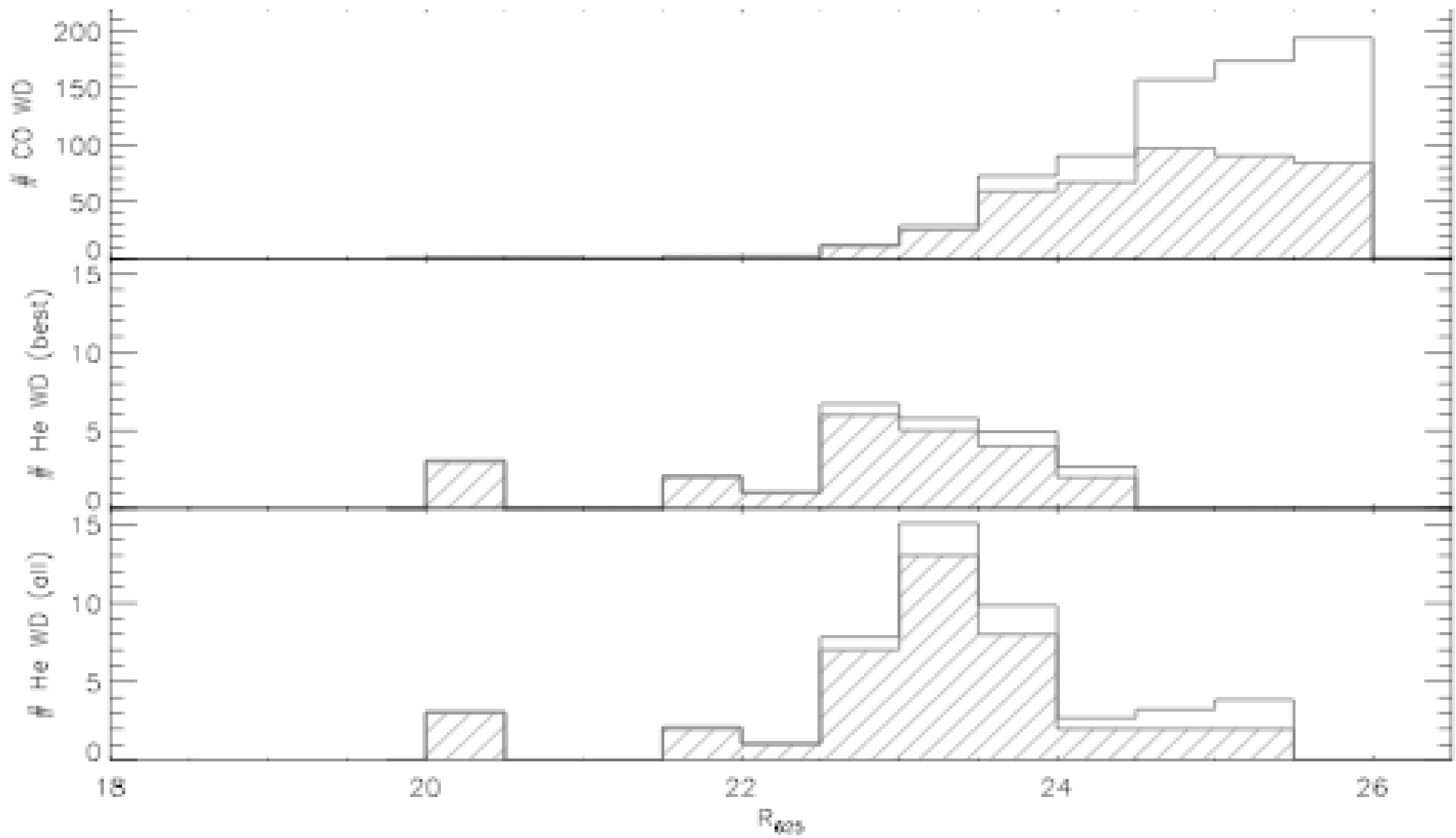
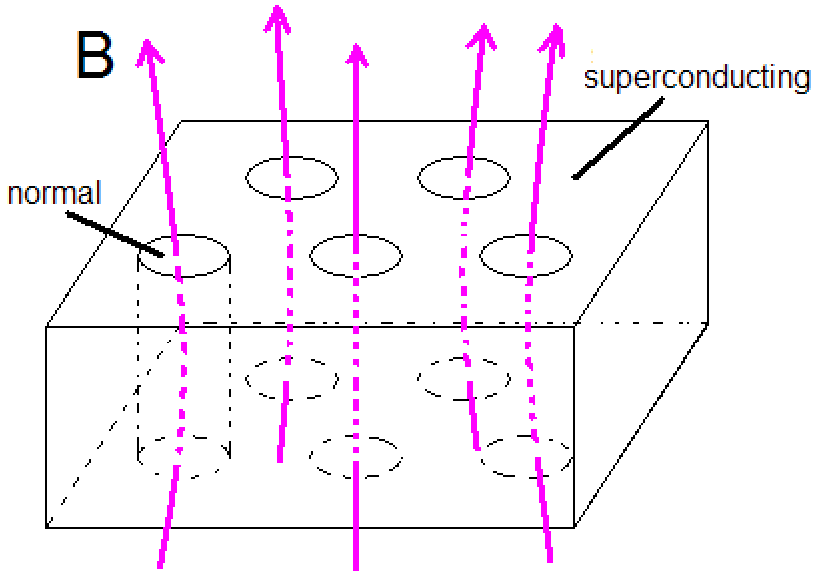
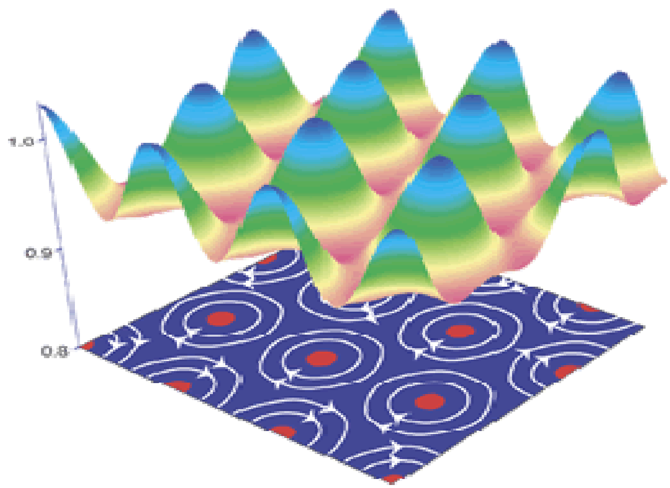


Fig. 6.— Results of artificial star tests. The top panel shows the fraction of stars recovered in artificial star tests for stars on the WD sequence as a function of R_{625} magnitude. We used these completeness statistics to infer the numbers of CO WDs (panel 2) and He WDs (best 24 only: panel 3; all: panel 4) in the cluster down to $R_{625} = 26$. Shaded histograms indicate numbers observed; white histograms include the completeness corrections. From the two bottom panels we conclude that, in contrast to the CO WD sequence, the He WD sequence ends before the magnitude limit is reached.

Magnetic Vortices in Superconductors and White Dwarfs

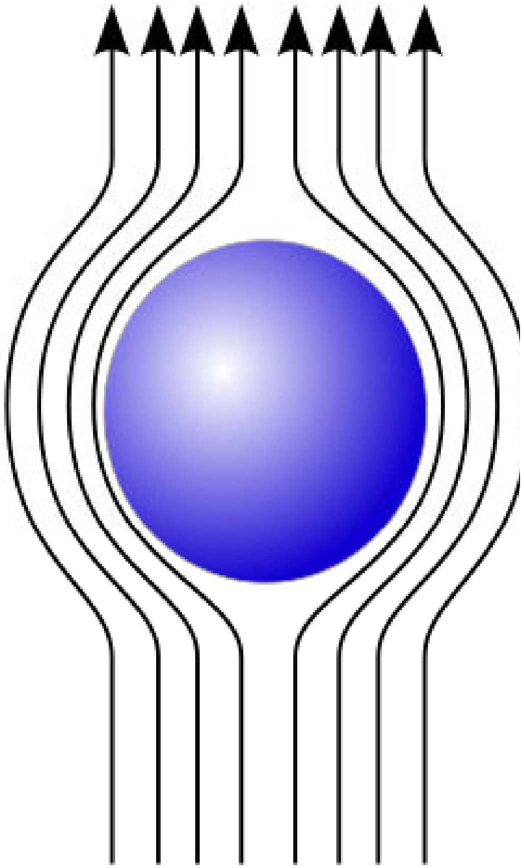


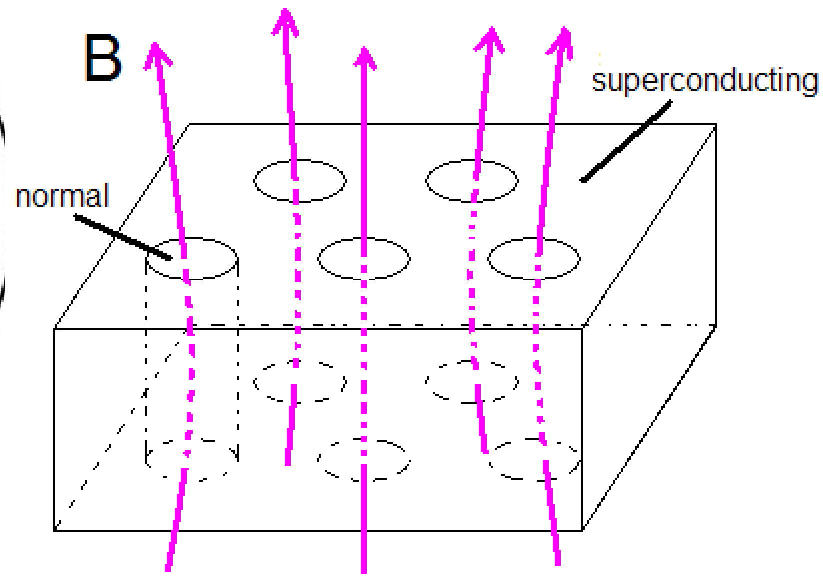


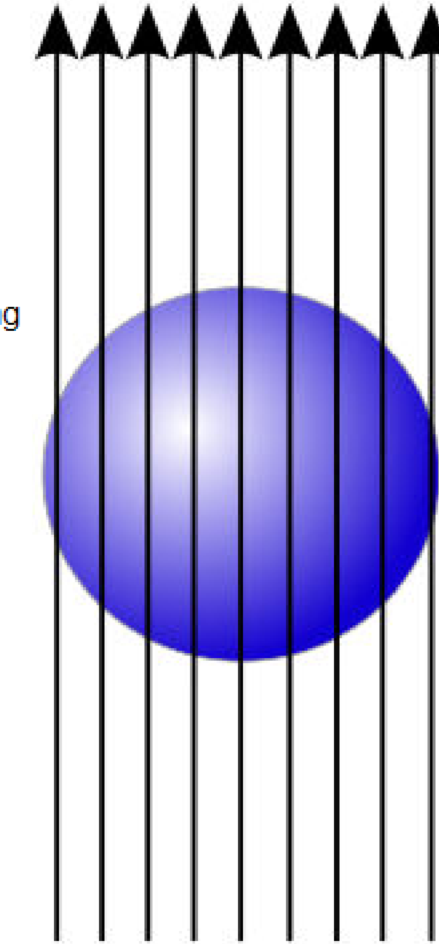
$$H_{c1} < H < H_{c2}$$

Rotation and Magnetic Properties of Helium White Dwarfs

Magnetic flux lines





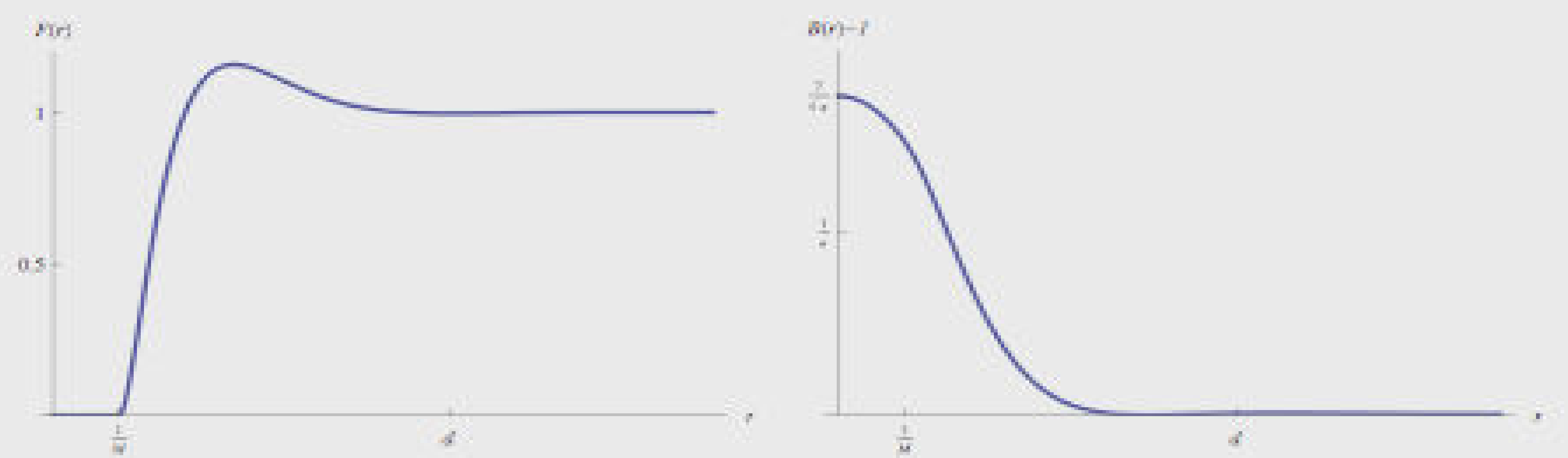


$$H_{c1} < H < H_{c2}$$

Magnetic Flux Lines

$$\Phi = \oint \mathbf{A} \cdot \mathrm{d}\mathbf{l} = \oint A_\theta r \mathrm{d}\theta = \frac{2\pi n}{2e}$$

$$\begin{aligned} x \frac{d}{dx}\left(\frac{1}{x}\frac{dA}{dx}\right) &= F^2(A-1)\,, \\ -\frac{1}{x}\frac{d}{dx}\left(x\frac{dF}{dx}\right) &= \left[\kappa^2(B^2-1) - \frac{n^2}{x^2}(A-1)^2\right] F \\ \frac{1}{x}\frac{d}{dx}\left(x\frac{dB}{dx}\right) &= F^2 B - 1\,. \end{aligned}$$



(a) Scalar field as a function of radius

(b) Potential as a function or radius

Critical Magnetic Field

$$H_{c1} = \frac{m_\gamma^2}{8e}\left(\log(\kappa) + 14\right)$$

$$H_{c1} = 10^7 - 10^9\ Gauss$$

Disruption of Condensate

$$H_{c2} = \frac{M^2}{e}$$

$$H_{c2} \simeq 10^{15}\ \text{Gauss}$$

London Magnetic Field

$$\nabla \times \mathbf{v} = \frac{2e}{m_H} \mathbf{H}$$

$$\mathbf{H} = \frac{2m_H}{2e} \boldsymbol{\Omega} = \frac{2eJ_0}{m_\gamma^2} \boldsymbol{\Omega}$$

$$H \simeq 10^{-4} \text{ Gauss}$$

Conclusions and Outlook

Charge Condensate – new state of matter in the Universe

Fast cooling of helium white dwarfs (super-massive carbon dwarfs ?)

Magnetic field in flux tubes, high critical field; small London field

Other possible applications: neutron star crust,…???