Methodical Accelerator Design
Highlights on Solenoid for FCC-ee

Laurent Deniau

CERN – Beams Department

laurent.deniau@cern.ch

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Track and Twiss

Tracking through the solenoid element involves the following steps:

1. Misalign: align the element by moving the particle(s) from beam to aligned element frames using $-errors$. (`tmalign1`).
2. Tilt: rotate the particle(s) by $-tilt$ from beam to element frames.
3. Collimation: check for aperture and survival on entry (track only).
4. Radiation: radiate for first half of the length (**NYI**).
5. Transport: apply the solenoid transport map (`tmsol, trsol`).
6. Radiation: radiate for second half of the length (**NYI**).
7. Collimation: check for aperture and survival on exit (**NYI**).
8. Tilt: rotate the particle(s) by $tilt$ from element to beam frames.
9. Misalign: misalign the element by moving particle back from element to beam frame including survey. (`tmalign2`).

☞ Second order terms are not allowed in Matrix by the Track module.

Solenoid

Solenoid field potential and strength:

$$A_x = -\frac{1}{2}B_s y, \quad A_y = \frac{1}{2}B_s x, \quad A_s = 0, \quad k_s = \frac{qB_s}{2P_0}.$$

Solenoid Hamiltonian (quadratic, expanded, $\rho_{\text{sol}} \ll L$, thin fringe):

$$\mathcal{H}_{\text{sol}} = \frac{\eta\delta_p}{\beta_0} + \frac{1}{2}\left((p_x + k_s y)^2 + (p_y - k_s x)^2 + \frac{p_t^2}{\beta_0^2\gamma_0^2}\right).$$

Solenoid transport map (linear):

$$M_{\text{sol}} = M_{\text{fringe}}^{(\text{exit})} M_{\text{body}} M_{\text{fringe}}^{(\text{entry})}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -k_s & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ k_s & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{S}{2k_s} & 0 & \frac{1-C}{2k_s} & 0 & 0 \\ 0 & C & 0 & S & 0 & 0 \\ 0 & \frac{1-C}{2k_s} & 1 & \frac{S}{2k_s} & 0 & 0 \\ 0 & -S & 0 & C & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\beta_0^2\gamma_0^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & k_s & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -k_s & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

where $C = \cos(2k_s L)$ and $S = \sin(2k_s L)$.

Solenoid – *cont'd*

Solenoid field potential and strength:

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Solenoid Hamiltonian (quadratic, expanded, $r \ll l$, thin fringe):

$$\mathcal{H}_{\text{sol}} = \frac{\eta\delta_p}{\beta_0} + \frac{1}{2}\left((p_x + k_s y)^2 + (p_y - k_s x)^2 + \frac{p_t^2}{\beta_0^2\gamma_0^2}\right).$$

Solenoid transport map (linear):

$$M_{\text{sol}} = M_{\text{fringe}}^{(\text{exit})} M_{\text{body}} M_{\text{fringe}}^{(\text{entry})} = M_{\text{foc}} R_{\text{xy}} = R_{\text{xy}} M_{\text{foc}}$$

$$= \begin{pmatrix} C & 0 & S & 0 & 0 & 0 \\ 0 & C & 0 & S & 0 & 0 \\ -S & 0 & C & 0 & 0 & 0 \\ 0 & -S & 0 & C & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C & \frac{1}{k_s}S & 0 & 0 & 0 & 0 \\ -k_s S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & C & \frac{1}{k_s}S & 0 & 0 \\ 0 & 0 & -k_s S & C & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\beta_0^2\gamma_0^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

where $C = \cos k_s L$ and $S = \sin k_s L$.

MAD-X – Solenoid thick map

Solenoid strength K_s:

$$k_s = \frac{K_s b_v}{2(1+\delta_p)}, \quad C = \cos k_s L, \quad S = \sin k_s L.$$

Solenoid transport map (linear):

$$M_{\text{sol}}^{\text{(thick)}} = \begin{pmatrix} C^2 & \frac{1}{k_s}CS & SC & \frac{1}{k_s}S^2 & 0 & 0 \\ -k_s CS & C^2 & -k_s S^2 & SC & 0 & 0 \\ -SC & -\frac{1}{k_s}S^2 & C^2 & \frac{1}{k_s}CS & 0 & 0 \\ k_s S^2 & -SC & -k_s CS & C^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\beta_0^2\gamma_0^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\Delta t = L\frac{\eta\delta_p}{\beta_0}. \qquad \text{(phase slip factor)}$$

Solenoid matrix $M_{\text{sol}}^{\text{(thick)}}$ is symplectic, i.e. $M^T S M = S$.

MAD-X – Solenoid thin map

Solenoid strength K_s, K_{si}:

$$k_s = \frac{K_s b_v}{2}, \quad k_{si} = \frac{K_{si} b_v}{2(1+\delta_p)}, \quad C = \cos k_{si}, \quad S = \sin k_{si}.$$

Solenoid transport map (linear):

$$M_{\text{sol}}^{\text{(thin)}} = M_{\text{sol}}^{\text{(thick)}} \left(L \to 0 \Rightarrow C_{\text{foc}} \to 1, S_{\text{foc}} \to k_{si}, \frac{S_{\text{foc}}}{k_s} \to 0 \right)$$

$$= \begin{pmatrix} C & 0 & S & 0 & 0 & 0 \\ -k_s k_{si} C & C & -k_s k_{si} S & S & 0 & 0 \\ -S & 0 & C & 0 & 0 & 0 \\ k_s k_{si} S & -S & -k_s k_{si} C & C & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

MAD-X – Solenoid track map

Solenoid strength K_s, K_{si}:

$$k_s = \frac{K_s b_v}{2}, \quad k_{si} = \frac{K_{si} b_v}{2(1+\delta_p)}, \quad C = \cos k_{si}, \quad S = \sin k_{si},$$

where for each particle:

$$1 + \delta_p = \sqrt{1 + \frac{2p_t}{\beta_0} + p_t^2}.$$

Solenoid tracking map (thin map):

$$\begin{aligned}
p_x^f &\leftarrow p_x - k_s k_{si} x, \quad p_y^f \leftarrow p_y - k_s k_{si} y, \\
x &\leftarrow \quad xC + yS, \qquad p_x \leftarrow \quad p_x^f C + p_y^f S, \\
y &\leftarrow -xS + yC, \qquad p_y \leftarrow -p_x^f S + p_y^f C, \\
t &\leftarrow t - \frac{(1/\beta_0 + p_t) k_{si}}{(1+\delta_p)^2} \left(\frac{k_s}{2}(x^2 + y^2) + y p_x^f - x p_y^f \right).
\end{aligned}$$

Solenoid + antisolenoids

Solenoid and antisolenoid strengths $\pm K_s$:

$$k_s = \pm \frac{K_s b_v}{2(1+\delta_p)}.$$

Solenoid and antisolenoids transport map (linear):

$$M_{\frac{\text{asol}}{2}\text{-sol-}\frac{\text{asol}}{2}} = [M_{\text{foc}} R_{\text{xy}}]\left(-k_s \frac{L}{2}\right)[M_{\text{foc}} R_{\text{xy}}](k_s L)[M_{\text{foc}} R_{\text{xy}}]\left(-k_s \frac{L}{2}\right)$$

$$= M_{\text{foc}}(k_s 2L) = \begin{pmatrix} C & \frac{1}{k_s}S & 0 & 0 & 0 & 0 \\ -k_s S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & C & \frac{1}{k_s}S & 0 & 0 \\ 0 & 0 & -k_s S & C & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{2L}{\beta_0^2\gamma_0^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

where $C = \cos k_s 2L, \quad S = \sin k_s 2L.$

Y-Rotation

Rotation angle ϕ:

$$C = \cos\phi, \quad S = \sin\phi, \quad T = S/C.$$

Y-rotation map (PhyG eq. 5.100, wrong):

$$M_{\text{yrot}} = \begin{pmatrix} C & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/C & 0 & 0 & 0 & -T/\beta_0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ S/\beta_0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \Delta p_x = T.$$

Y-rotation map (PTC, as lim of sbend):

$$p_z = \sqrt{1 + \frac{2p_t}{\beta_0} + p_t^2 - p_x^2 - p_y^2} = \sqrt{(1+\delta_p)^2 - p_x^2 - p_y^2},$$

$$x \leftarrow \frac{x p_z}{C(p_z - T p_x)}, \quad p_x \leftarrow C p_x + S p_z,$$

$$y \leftarrow y + \frac{T x p_y}{p_z - T p_x}, \quad t \leftarrow t - \frac{(1/\beta_0 + p_t) T x}{p_z - T p_x}.$$

MAD-X – Y-Rotation map

Rotation angle ϕ:

$$C = \cos\phi, \quad S = \sin\phi, \quad T = S/C.$$

Track map (wrong):

$$\begin{aligned} x &\leftarrow xC + tS, & p_x &\leftarrow p_x C + p_t S, \\ t &\leftarrow -xS + tC, & p_t &\leftarrow -p_x S + p_t C. \end{aligned}$$

Twiss map (almost correct?):

$$M_{\text{yrot}} = \begin{pmatrix} 1/C & 0 & 0 & 0 & 0 & 0 \\ 0 & C & 0 & 0 & 0 & -S/\beta_0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ T/\beta_0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \qquad \text{(also in MAD8)}$$

$$\Delta p_x = -S.$$

TiltX on beam around solenoid + antisolenoids

TiltX(L, ϕ) at position $s + L$ by angle ϕ (from HB slide 7):

$$x \leftarrow x + \phi L \left(1 - \frac{p_t}{\beta_0}\right), \quad p_x \leftarrow p_x + \phi, \quad t \leftarrow t + \frac{\phi L}{2\beta_0}(p_x - \phi).$$

Comparing to DriftX (PhyG eq. 5.5):

$$x \leftarrow x + p_x L \left(1 - \frac{p_t}{\beta_0}\right), \quad t \leftarrow t + \frac{\eta \delta_p L}{\beta_0} + \frac{p_t}{\beta_0^2 \gamma_0^2} + \frac{p_x L}{2\beta_0} p_x.$$

Comparing to DriftX map (Track, PTC):

$$p_z = \sqrt{1 + \frac{2p_t}{\beta_0} + p_t^2 - p_x^2 - p_y^2} = \sqrt{(1 + \delta_p)^2 - p_x^2 - p_y^2},$$

$$x \leftarrow x + \frac{p_x L}{p_z}, \quad t \leftarrow t - \left(\frac{1}{\beta_0} + p_t\right) \frac{L}{p_z} + \frac{L}{\beta_0}.$$