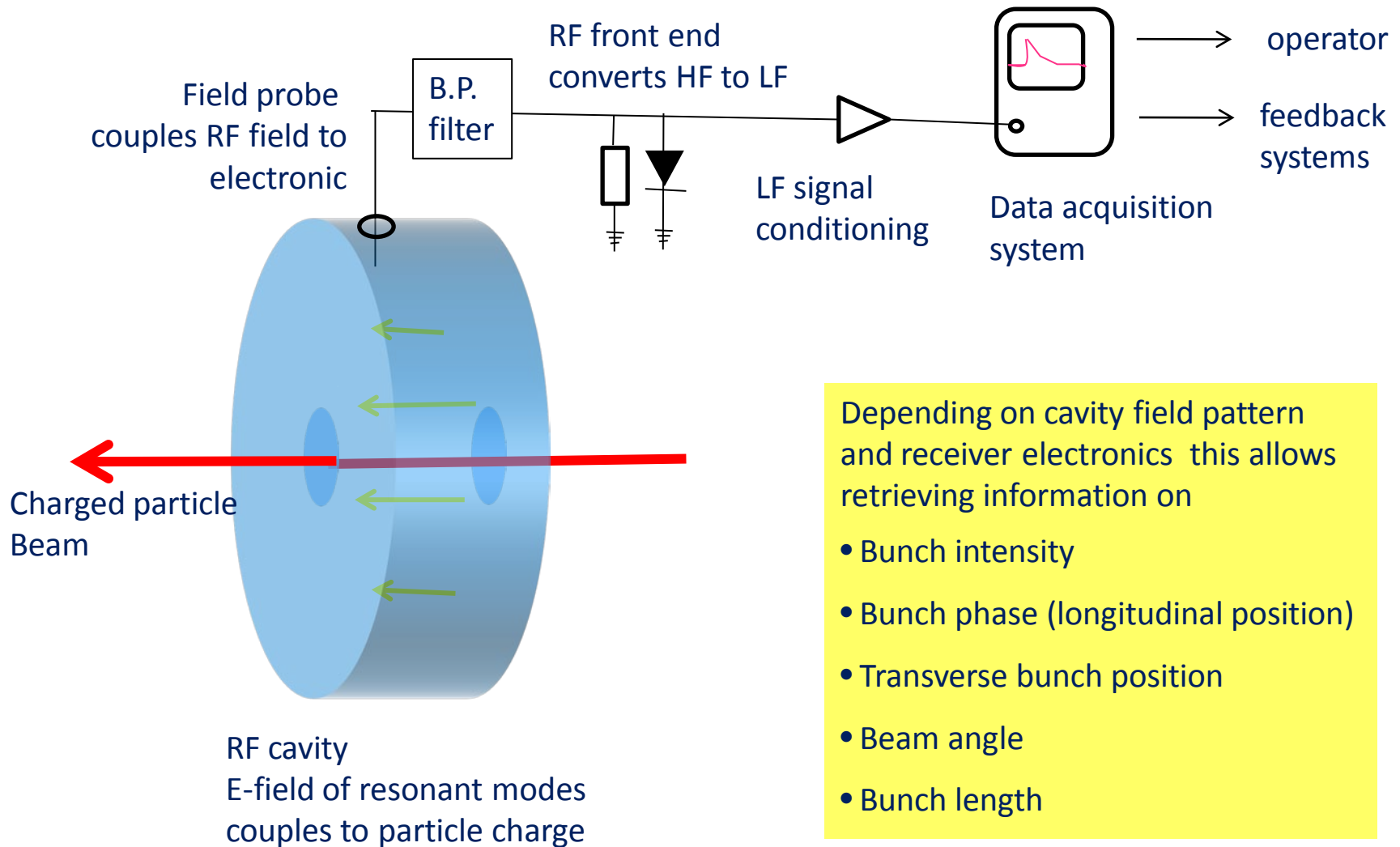


RF beam diagnostics

- *Basics of beam monitoring with RF cavities*
- *Example devices*
- *Quantitative analysis*
- *Measuring rms bunch length*

Basics of RF cavities for beam measurements



Energy transfer from beam to cavity

Electric field of a resonant cavity mode

$$\vec{E}(\vec{x}, t) = A\vec{E}(\vec{x})e^{i\omega t}$$

Power flow between cavity and a traversing charge $\frac{dW_q}{dt} = q \frac{d\vec{x}}{dt} \cdot \vec{E}(\vec{x})e^{-i\omega t}$

Beams are in good approximation paraxial

$$\frac{d\vec{x}}{dt} \approx \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$$

$$\frac{dW_q}{dq} = \int E_z(\vec{x})e^{-i\frac{\omega}{v}z} dz = V$$

$$dW_q = -dW_{cav}$$

$$W_{cav} = \iiint \epsilon_0 \vec{E}^2 dV$$

Energy transfer from beam to cavity

Voltage induced by charge

$$\frac{dV}{dq} = \frac{dV}{dW_{cav}} \frac{dW_{cav}}{dq}$$

$$= \frac{V}{2W_{cav}} \cdot V = \frac{V^2}{2W_{cav}} = \frac{\left(\int E_z(\vec{x}) e^{-i\frac{\omega}{v}z} dz \right)^2}{2\epsilon_0 \iiint \vec{E}(\vec{x})^2 dV}$$

Definition loss factor

$$k_{loss} \equiv \frac{dV}{2dq} = \frac{V^2}{4W_{cav}} = \frac{R_{shunt} \omega}{4Q_0} = \frac{\left(\int E_z(\vec{x}) e^{-i\frac{\omega}{v}z} dz \right)^2}{4\epsilon_0 \iiint \vec{E}(\vec{x})^2 dV}$$

Voltage induced by traversing charge $V = 2k_{loss}q$

Energy deposited in empty cavity $W = k_{loss}q^2$

k_{loss} depends only on field distribution $\vec{E}(\vec{x})$,

beam position and particle velocity v !

Energy transfer for finite bunchlength

For a particle bunch of finite length with

$$\frac{dq}{dt} = i_b(t) = q_b f(t)$$

each time slice contributes with different phase to the induced voltage of a resonant mode with frequency ω

The induced voltage is in this case

$$V = 2k_{loss} q_b F_b$$

and the energy deposited in an initially empty cavity

$$W = k_{loss} q_b^2 F_b^2$$

with the "bunch Formfactor"

$$F_b = \left| \int f(t) \cdot e^{i\omega t} dt \right|$$

For a gaussian bunch with

$$i_b(t) = \frac{q_b}{\sqrt{2\pi\sigma_b}} \exp\left(\frac{-t^2}{2\sigma_b^2}\right)$$

$$F_b = \exp\left(\frac{-\omega^2\sigma_b^2}{2}\right)$$

Example: $\omega=2\pi \cdot 3 \text{ GHz}$, $\sigma_b=10 \text{ ps} \Rightarrow F_b=0.982$

Dissipation of energy in cavity

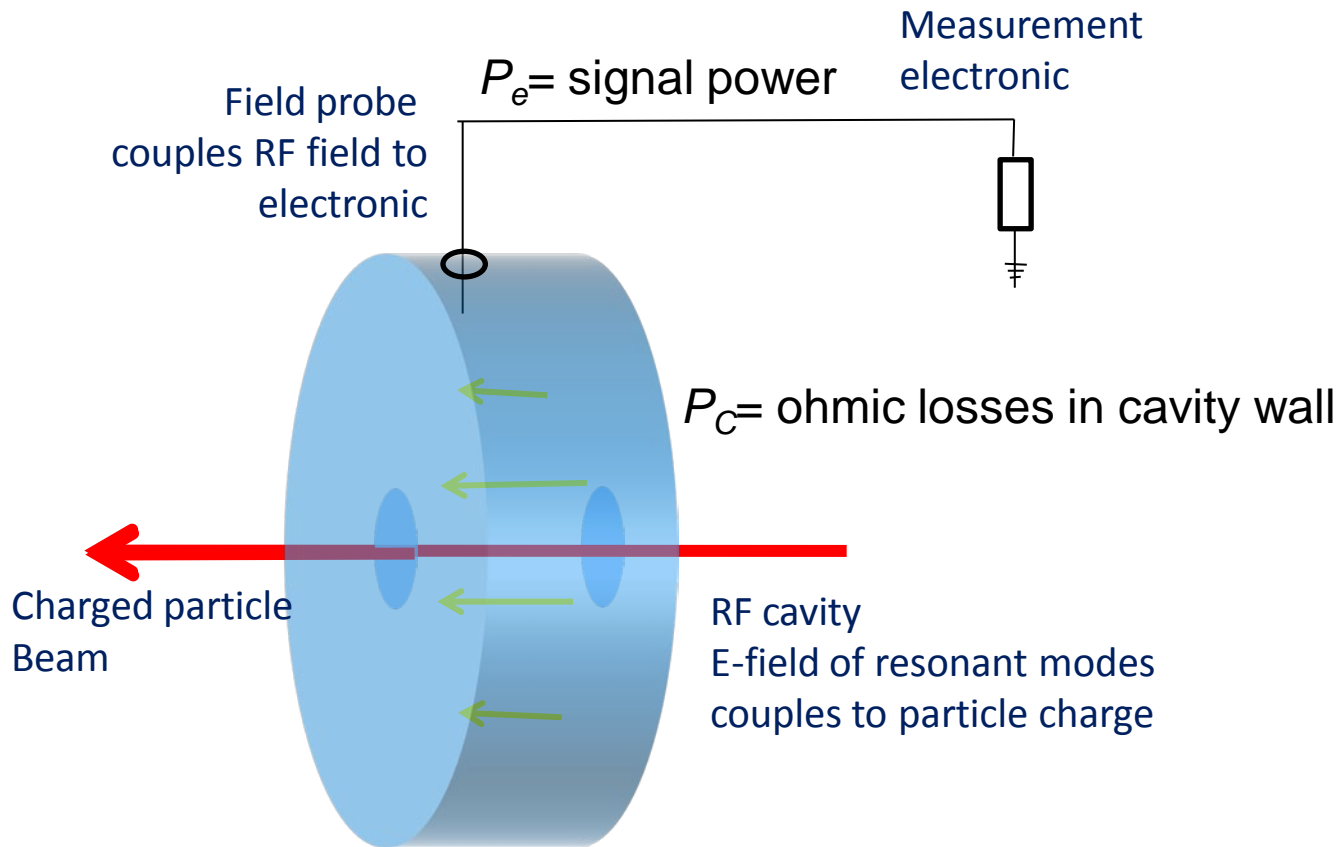
Power loss in RF cavity $P_C = \frac{R_{Surf}}{2\mu_0^2} \iint_S B^2 dS$

with $R_{Surf} = \frac{1}{\sigma \delta} = \sqrt{\frac{\mu_0 \omega}{2\sigma}}$

Cavity quality factor $Q_0 \equiv \frac{\omega W}{P_C}$

$$W(t) = W(0) \exp\left(-\frac{\omega}{Q_0} t\right)$$
$$= k_{loss} q_b^2 F_b^2 \exp\left(-\frac{\omega}{Q_0} t\right)$$

Energy flow



Dissipation of energy in cavity and external measurement electronic

Dissipated power $P_t = P_C + P_E$

coupling factor $\beta \equiv \frac{P_E}{P_C}$

Loaded quality factor $Q_L \equiv \frac{\omega W}{P_C + P_E} = \frac{Q_0}{1 + \beta}$

$$W(t) = W(0) \exp\left(-\frac{\omega}{Q_L} t\right) = k_{loss} q_b^2 F_b^2 \exp\left(-\frac{\omega(1 + \beta)}{Q_0} t\right)$$

Signal power $P_E(t) = \frac{\omega W(t)}{Q_L \left(1 + \frac{1}{\beta}\right)} = \frac{\omega k_{loss} q_b^2 F_b^2}{Q_0} \beta \exp\left(-\frac{\omega(1 + \beta)}{Q_0} t\right)$

Cavity driven by continuous bunch train

Steady state voltage in cavity $V = \frac{R_{Shunt} I_b F_b}{(1 + \beta)} = \frac{4Q_0 k_{loss} I_b F_b}{\omega(1 + \beta)}$

External signal power $P_E = R_{Shunt} I_b^2 F_b^2 \frac{\beta}{(1 + \beta)^2} = \frac{4Q_0 k_{loss}}{\omega} I_b^2 F_b^2 \frac{\beta}{(1 + \beta)^2}$

An error between bunch frequency ω and resonant cavity frequency ω_0

leads to a phase error ψ of the induced voltage with

$$\tan \psi = \frac{-2Q_0}{1 + \beta} \frac{\omega - \omega_0}{\omega_0}$$

a reduced voltage amplitude

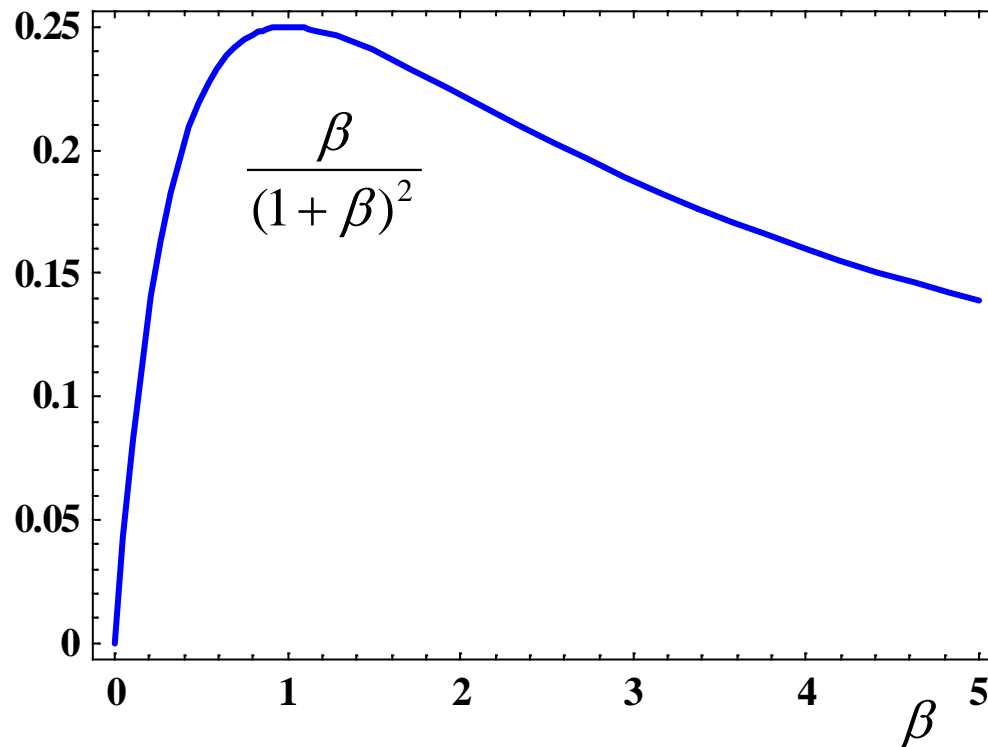
$$V = \frac{R_{Shunt} I_b F_b}{(1 + \beta)} \cos \psi$$

and a reduced external signal power

$$P_E = R_{Shunt} I_b^2 F_b^2 \frac{\beta}{(1 + \beta)^2} \cos^2 \psi$$

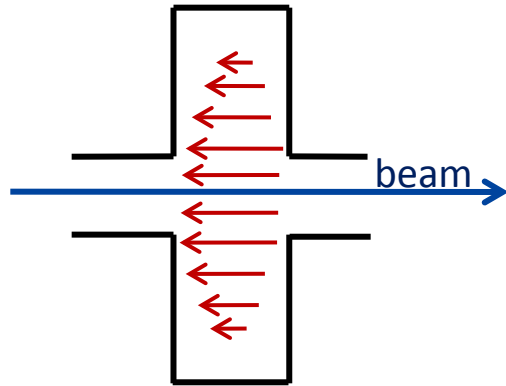
Signal power vs. coupling factor β for c.w. beams

External signal power $P_E = R_{Shunt} I_b^2 F_b^2 \frac{\beta}{(1 + \beta)^2} = \frac{4Q_0 k_{loss}}{\omega} I_b^2 F_b^2 \frac{\beta}{(1 + \beta)^2}$



Beam measurement types

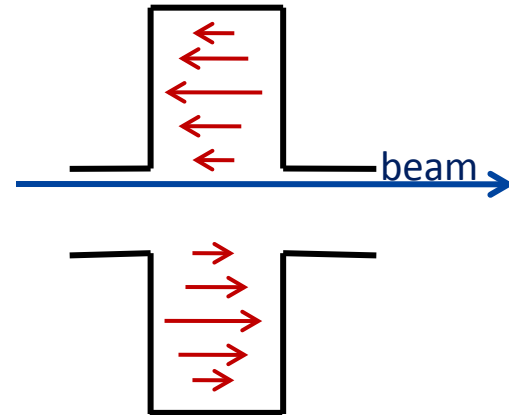
Measurement of beam intensity or beam timing/phase relative to some external RF reference



Cavity mode with rotational symmetry and electric field maximum on beam axis

“Monopole mode”, “ TM_{010} like mode”

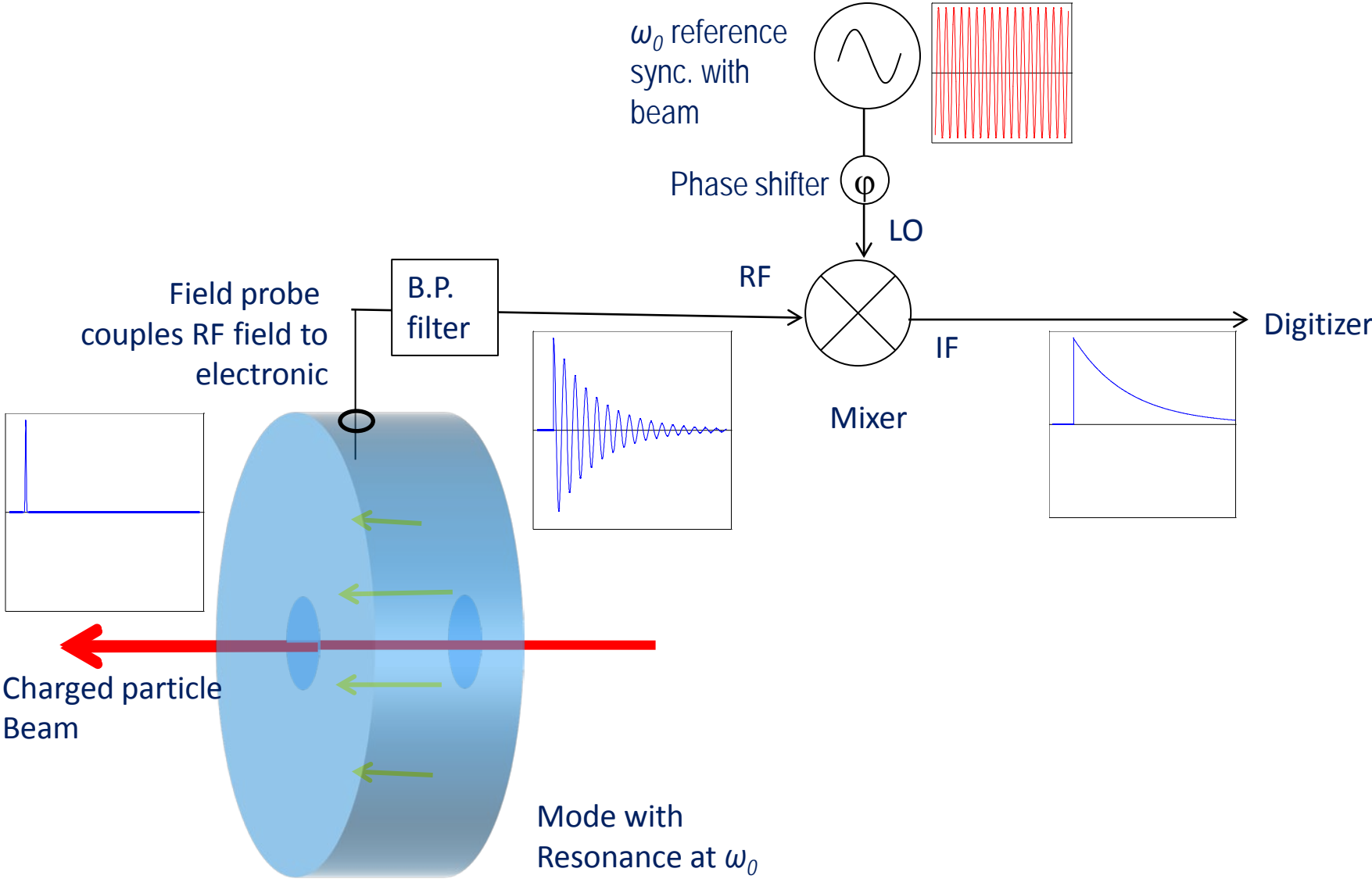
Measurement of beam position



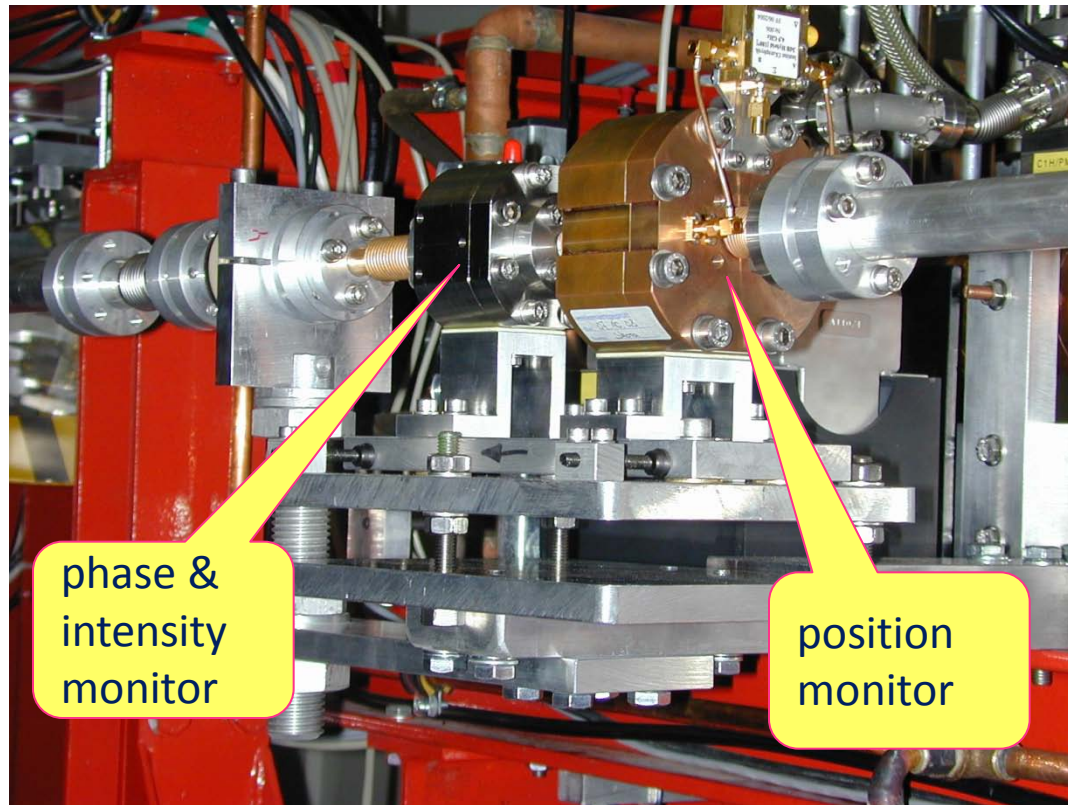
Cavity mode with Zero electric field on axis, azimuthal field dependence like $\cos(\theta)$ and field strength dependence on beam position approximately linear with displacement r

“Dipole mode”, “ TM_{110} ” like mode

Signal path in RF frontend

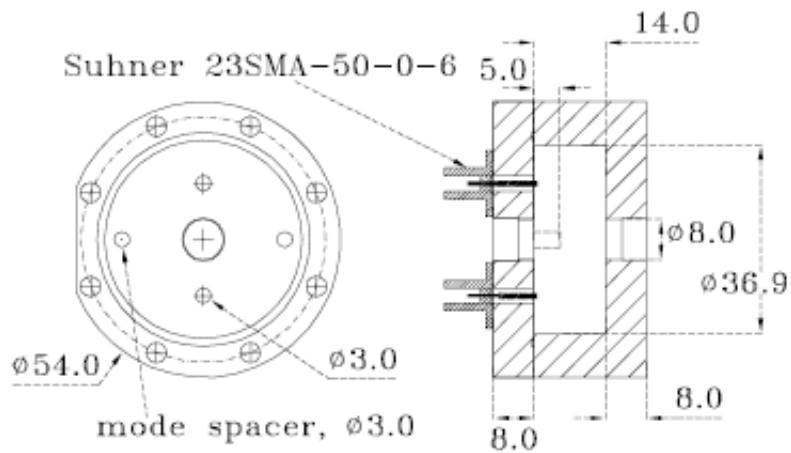
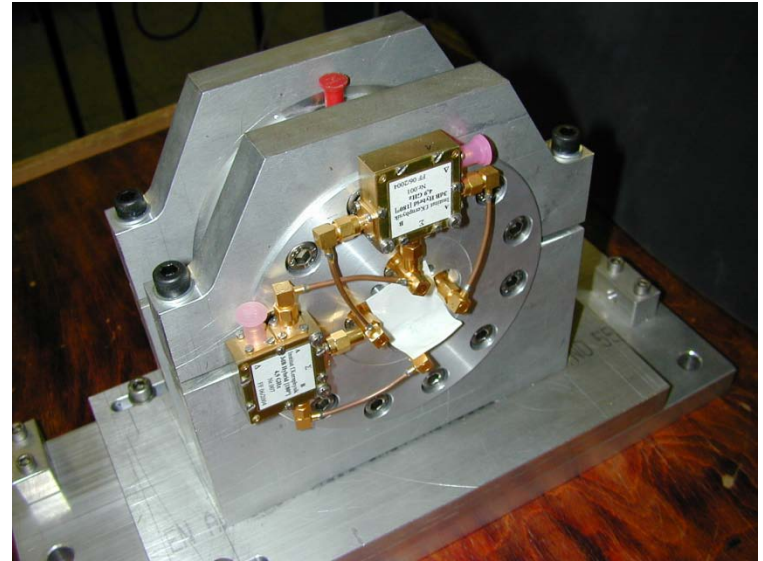


Example of RF monitors in MAMI



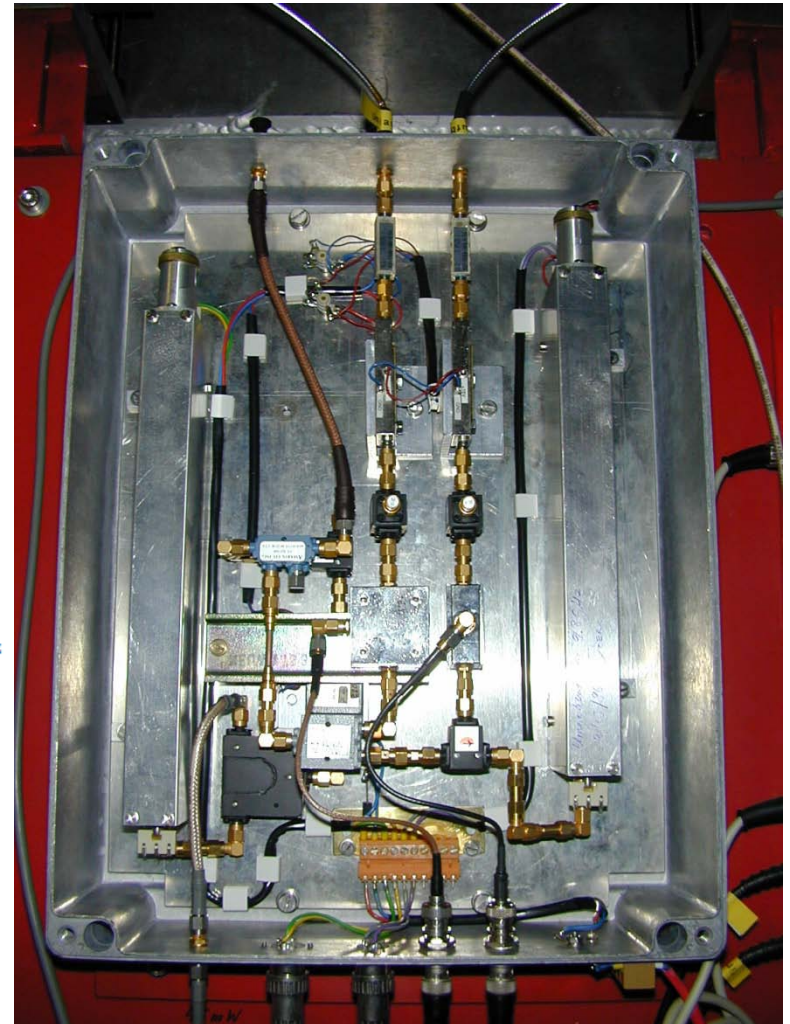
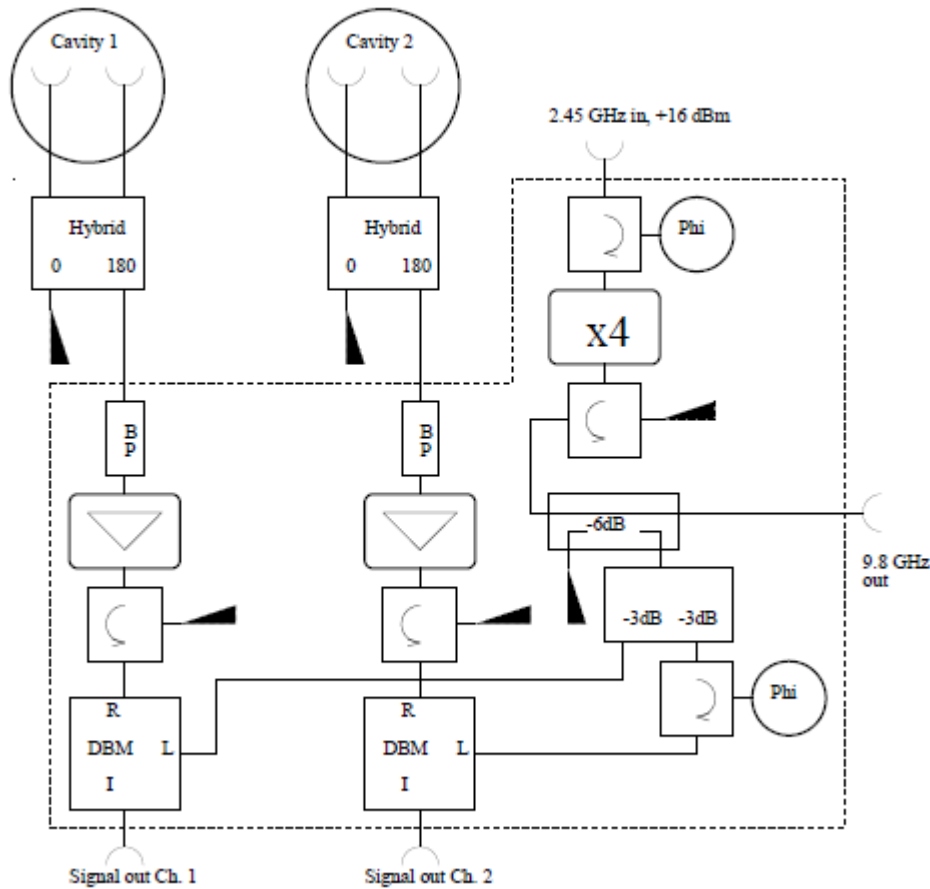
4.9 GHz Phase and Position Resonators in MAMI double sided Microtron (at Mainz University)

MAMI monitors cont.



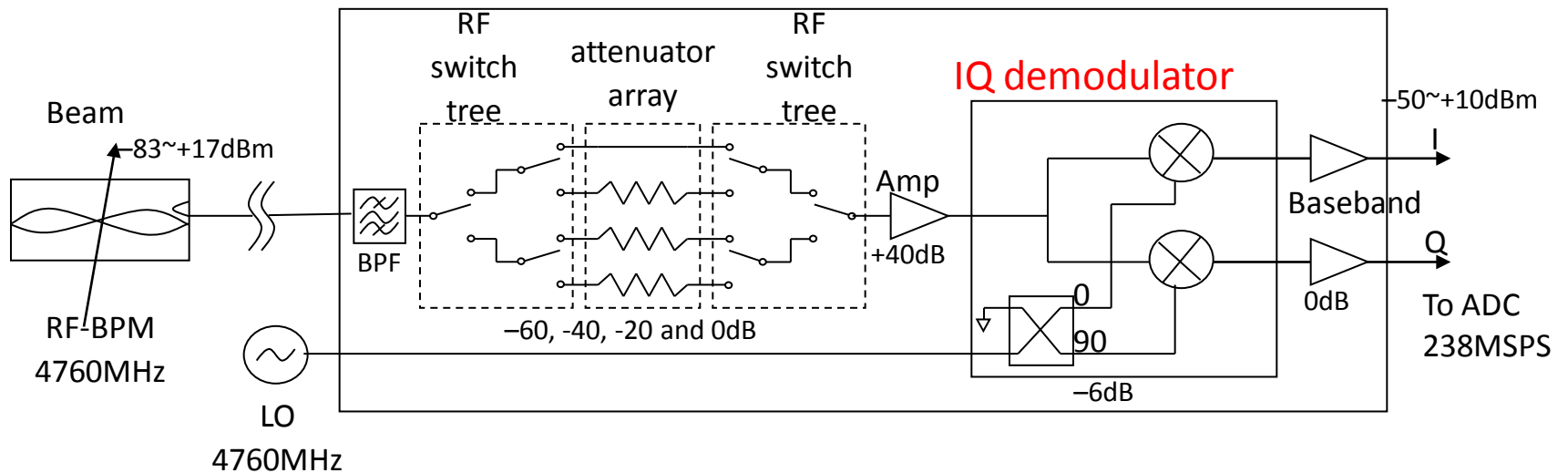
Drawing of a similar cavity
(but at 9.8GHz)

RF electronic for 9.8 GHz RF BPM's in MAMI

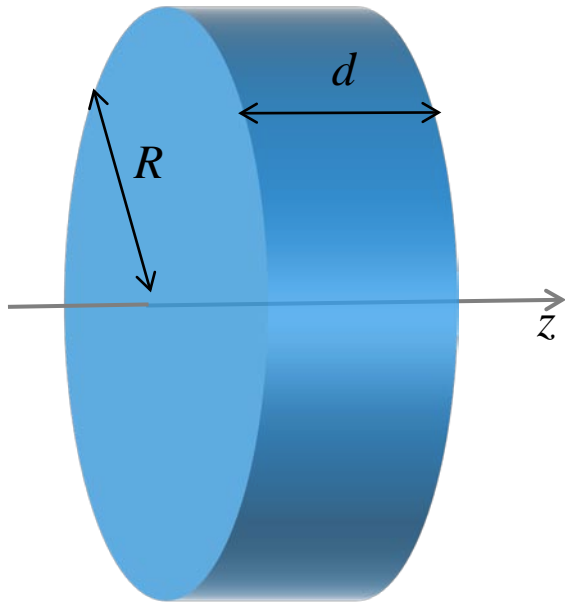


Courtesy H. Euteneuer and T. Doerk

Example of RF front with IQ demodulation and switchable gain



Properties of “Pillbox” cavity TM_{mn0} modes



$$E_z = A J_m(kr) \cos(m\vartheta) e^{i\omega t}$$

$$B_r = A \frac{-i}{\omega r} J_m(kr) \sin(m\vartheta) e^{i\omega t}$$

$$B_\vartheta = A \frac{-i}{2c} (J_{m-1}(kr) - J_{m+1}(kr)) \cos(m\vartheta) e^{i\omega t}$$

$$E_\vartheta = E_r = B_z = 0$$

$$\text{with } k \equiv \frac{\omega}{c}, \quad \omega = \frac{c X_{mn}}{R} \text{ and } J_m(X_{mn}) = 0$$

Properties of “Pillbox” cavity TM_{mn0} modes cont.

$$V(r, \vartheta) = \int_{-d/2}^{d/2} E_z(\vec{x}) e^{-i\frac{\omega}{v}z} dz$$

$$= A \frac{2v}{\omega} J_m(kr) \cos(m\vartheta) \sin\left(\frac{\omega}{2v}d\right)$$

$$W = \frac{\epsilon_0}{2} \int_{-d/2}^{d/2} \int_0^{2\pi R} E_z^2 r dr d\vartheta dz$$

$$= A^2 \frac{\pi\epsilon_0 R^2 d}{2} J_1(X_{0n})^2 \quad \text{for } m = 0$$

$$= A^2 \frac{\pi\epsilon_0 R^2 d}{4} J_{m-1}(X_{mn}) \cdot J_{m+1}(X_{mn}) \quad \text{for } m > 0$$

$$k_{loss}(r, \vartheta) = \frac{V^2}{4W}$$

$$= \frac{2v^2}{\pi\epsilon_0 R^2 \omega^2} \frac{J_0(kr)^2}{J_1(X_{0n})^2} \frac{\sin^2\left(\frac{\omega}{2v}d\right)}{d} \quad \text{for } m = 0$$

$$= \frac{4v^2}{\pi\epsilon_0 R^2 \omega^2} \frac{-J_m(kr)^2}{J_{m-1}(X_{mn}) \cdot J_{m+1}(X_{mn})} \frac{\sin^2\left(\frac{\omega}{2v}d\right)}{d} \cos^2(m\vartheta) \quad \text{for } m > 0$$

Properties of “Pillbox” cavity TM_{mn0} modes cont.

Power loss in RF cavity $P_C = \frac{R_{Surf}}{2\mu_0^2} \iint_S B^2 dS$

with $R_{Surf} = \frac{1}{\sigma \delta} = \sqrt{\frac{\mu_0 \omega}{2\sigma}}$

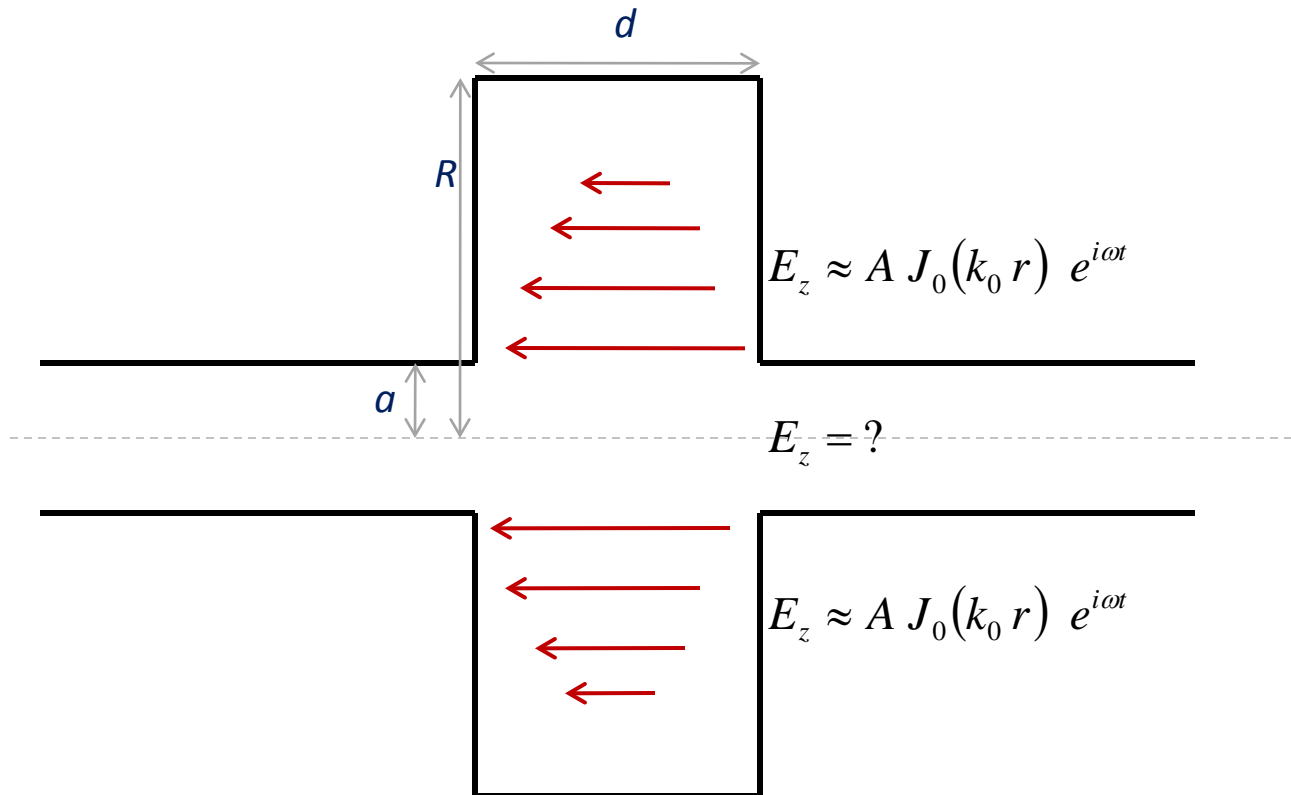
Cavity quality factor $Q_0 = \frac{\omega W}{P_C}$

$$Q_0 = \frac{\mu_0 \omega}{2} \frac{d R}{R_{Surf} d + R}$$

Shunt impedance $R_{Shunt} = \frac{4}{\pi \epsilon_0^2 c^2} \frac{v^2}{R_{Surf} \omega^2 R(d+R)} \frac{J_0(kr)^2}{J_1(X_{0n})} \sin^2\left(\frac{\omega}{2v} d\right)$ for $m = 0$

Shunt impedance $R_{Shunt} = \frac{8}{\pi \epsilon_0^2 c^2} \frac{v^2}{R_{Surf} \omega^2 R(d+R)} \frac{J_m(kr)^2}{J_{m-1}(X_{mn}) \cdot J_{m+1}(X_{mn})} \sin^2\left(\frac{\omega}{2v} d\right) \cos^2(m\vartheta)$ for $m > 0$

Pillbox cavity with beam pipe, monopole mode



Energy transfer for off axis beam, monopole modes

$$E_z(r, z, t) = \int_{-\infty}^{\infty} A(k_z) J_0(k_r r) e^{i(k_z z - \omega t)} dk_z$$

$$k_r = \sqrt{k_0^2 - k_z^2} \quad \text{with} \quad k_0 \equiv \frac{\omega}{c}$$

$$= \int_{-\infty}^{\infty} A(k_z) J_0\left(\sqrt{k_0^2 - k_z^2} r\right) e^{ik_z z - \omega t} dk_z$$

$$V(r) = \int_{-\infty}^{\infty} E_z(r, z, t) dz = \int_{-\infty}^{\infty} E_z\left(r, z, \frac{z}{v}\right) dz$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(k_z) J_0\left(\sqrt{k_0^2 - k_z^2} r\right) e^{ik_z z - \frac{\omega}{v} z} dk_z dz$$

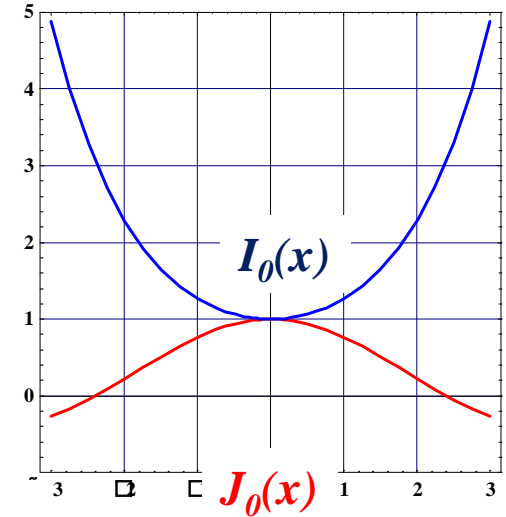
$$= \int_{-\infty}^{\infty} A(k_z) J_0\left(\sqrt{k_0^2 - k_z^2} r\right) \left(\int_{-\infty}^{\infty} e^{ik_z z - \frac{\omega}{v} z} dz \right) dk_z$$

$$= \int_{-\infty}^{\infty} A(k_z) J_0\left(\sqrt{k_0^2 - k_z^2} r\right) \delta\left(k_z - \frac{\omega}{v}\right) dk_z$$

$$= A\left(\frac{\omega}{v}\right) J_0\left(\sqrt{k_0^2 - \frac{\omega^2}{v^2}} r\right)$$

$$J_0(ix) = I_0(x)$$

$$V(r) = A\left(\frac{\omega}{v}\right) I_0\left(\sqrt{\frac{\omega^2}{v^2} - k_0^2} r\right)$$



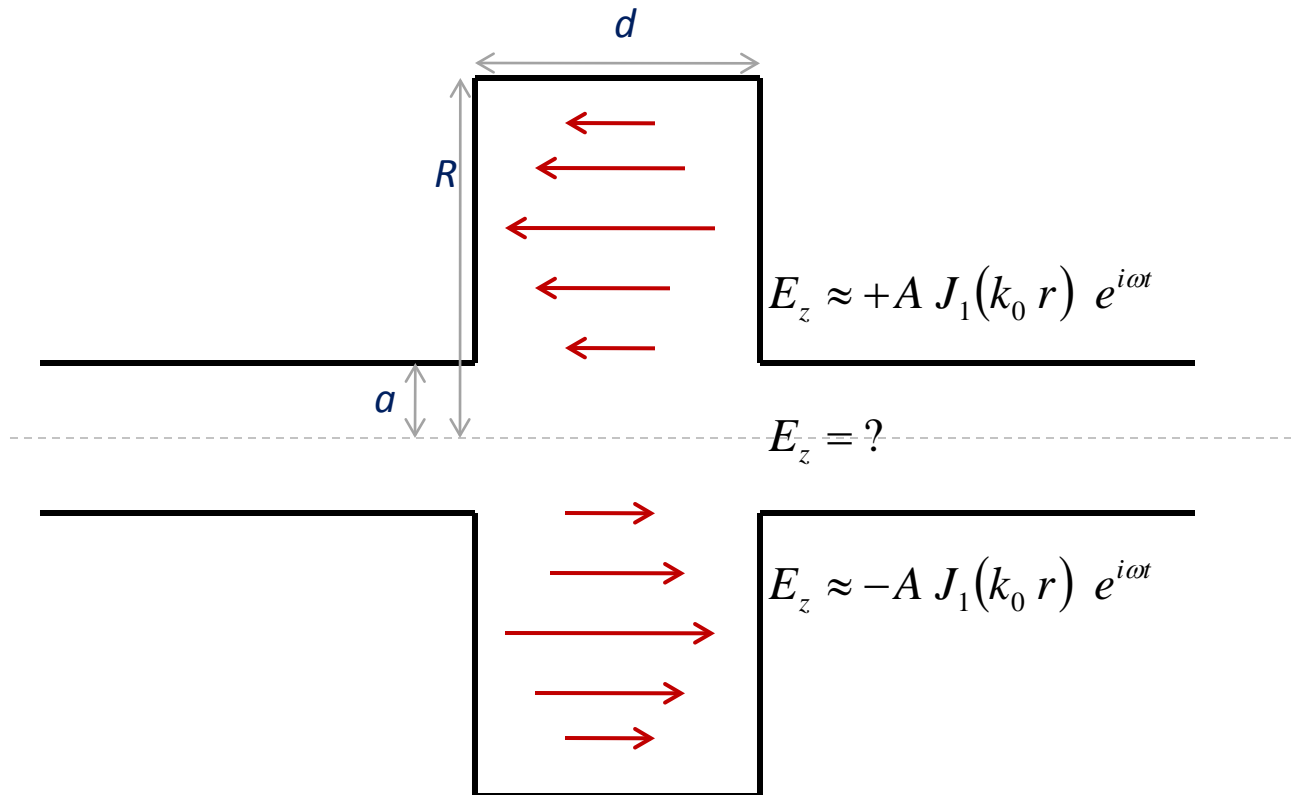
**valid for arbitrary cavity geometries
with rotational symmetry :**

$$V(r) \propto I_0\left(\frac{\omega}{c} \sqrt{\frac{1}{\beta_{rel}^2} - 1} r\right)$$

$$k_{loss}(r) \propto I_0\left(\frac{\omega}{c} \sqrt{\frac{1}{\beta_{rel}^2} - 1} r\right)^2$$

$$V(r) = \text{const.}, \quad k_{loss}(r) = \text{const.}, \quad \text{for } \beta \approx 1$$

Pillbox cavity with beam pipe, dipole mode



Energy transfer for off axis beam, dipole modes

$$E_z(r, \vartheta, z, t) = \int_{-\infty}^{\infty} A(k_z) J_1(k_r r) \cos(\vartheta) e^{i(k_z z - \omega t)} dk_z$$

$$k_r = \sqrt{k_0^2 - k_z^2} \quad \text{with} \quad k_0 \equiv \frac{\omega}{c}$$

$$= \int_{-\infty}^{\infty} A(k_z) J_1\left(\sqrt{k_0^2 - k_z^2} r\right) \cos(\vartheta) e^{ik_z z - \omega t} dk_z$$

$$V(r, \vartheta) = \int_{-\infty}^{\infty} E_z(r, \vartheta, z, t) dz = \int_{-\infty}^{\infty} E_z\left(r, \vartheta, z, \frac{z}{v}\right) dz$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(k_z) J_1\left(\sqrt{k_0^2 - k_z^2} r\right) e^{ik_z z - \frac{\omega}{v} z} dk_z dz$$

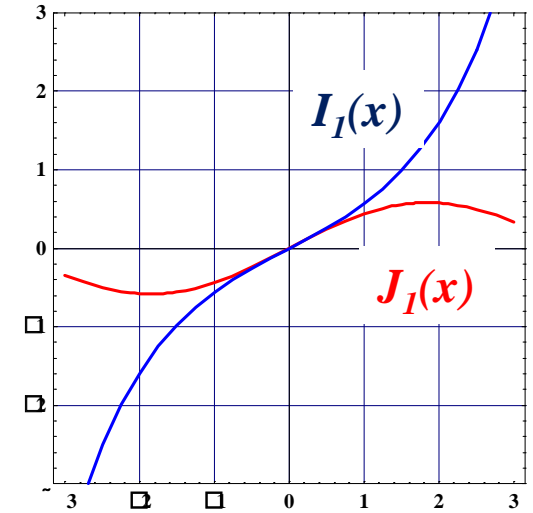
$$= \cos(\vartheta) \int_{-\infty}^{\infty} A(k_z) J_1\left(\sqrt{k_0^2 - k_z^2} r\right) \left(\int_{-\infty}^{\infty} e^{ik_z z - \frac{\omega}{v} z} dz \right) dk_z$$

$$= \cos(\vartheta) \int_{-\infty}^{\infty} A(k_z) J_1\left(\sqrt{k_0^2 - k_z^2} r\right) \delta\left(k_z - \frac{\omega}{v}\right) dk_z$$

$$= \cos(\vartheta) A\left(\frac{\omega}{v}\right) J_1\left(\sqrt{k_0^2 - \frac{\omega^2}{v^2}} r\right)$$

$$\cos(\vartheta) = \frac{x}{r}, \quad J_1(ix) = i I_1(x)$$

$$V(r, x) = \frac{x}{r} i A\left(\frac{\omega}{v}\right) I_1\left(\sqrt{\frac{\omega^2}{v^2} - k_0^2} r\right)$$



**valid for arbitrary cavity geometries
with rotational symmetry :**

$$V(x) \propto I_1\left(\frac{\omega}{c} \sqrt{\frac{1}{\beta_{rel}^2} - 1} r\right) \cdot \frac{x}{r}$$

$$k_{loss}(x) \propto I_1\left(\frac{\omega}{c} \sqrt{\frac{1}{\beta_{rel}^2} - 1} r\right)^2 \cdot \frac{x^2}{r^2}$$

for $\beta \approx 1$, $V(x) \propto x$, $k_{loss}(x) \propto x^2$

Loss factors for Pillbox cavities with beam aperture $\phi=2a$

$$k_{loss}(r, \vartheta) = \frac{2v^2}{\pi\epsilon_0 R^2 \omega^2} \frac{J_0(ka)^2}{J_1(X_{0n})^2} \frac{\sin^2\left(\frac{\omega}{2v}d\right)}{d} \frac{I_0\left(k\sqrt{\frac{c^2}{v^2}-1}r\right)^2}{I_0\left(k\sqrt{\frac{c^2}{v^2}-1}a\right)^2} \quad \text{for } m=0$$

$$k_{loss}(r, \vartheta) = \frac{4v^2}{\pi\epsilon_0 R^2 \omega^2} \frac{-J_m(ka)^2}{J_{m-1}(X_{mn}) \cdot J_{m+1}(X_{mn})} \frac{\sin^2\left(\frac{\omega}{2v}d\right)}{d} \frac{I_m\left(k\sqrt{\frac{c^2}{v^2}-1}r\right)^2}{I_m\left(k\sqrt{\frac{c^2}{v^2}-1}a\right)^2} \cos^2(m\vartheta) \quad \text{for } m > 0$$

for $v \approx c$

$$k_{loss}(r, \vartheta) = \frac{2v^2}{\pi\epsilon_0 R^2 \omega^2} \frac{J_0(ka)^2}{J_1(X_{0n})^2} \frac{\sin^2\left(\frac{\omega}{2v}d\right)}{d} \quad \text{for } m=0$$

$$k_{loss}(r, \vartheta) = \frac{4v^2}{\pi\epsilon_0 R^2 \omega^2} \frac{-J_m(ka)^2}{J_{m-1}(X_{mn}) \cdot J_{m+1}(X_{mn})} \frac{\sin^2\left(\frac{\omega}{2v}d\right)}{d} \frac{r^{2m}}{a^{2m}} \cos^2(m\vartheta) \quad \text{for } m > 0$$

Scaling of Cavity properties with frequency

Monopole mode cavities

$$k_{loss} \propto \omega$$

$$R_{Shunt} \propto \sqrt{\omega}$$

Dipole mode cavities

$$k_{loss} \propto \omega^3 r^2$$

$$R_{Shunt} \propto \omega^{\frac{3}{2}} r^2$$

Quality Factor

$$Q \propto \frac{1}{\sqrt{\omega}}$$

Always stay below cut-off frequency of lowest waveguide mode in beam-pipe (TE₁₁)



$$\omega < \frac{1.841 c}{R_{beampipe}}$$

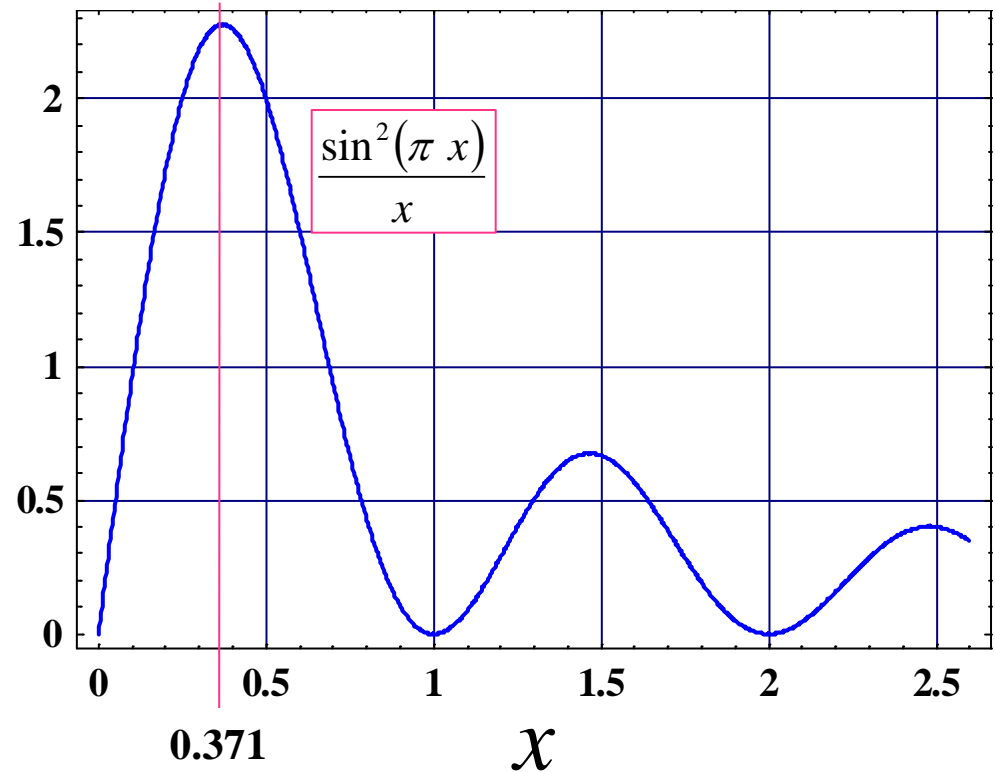
Example: $R_{beampipe} = 2$ cm
⇒ stay well below 4.4GHz

Optimum cavity length for single bunch BPM

$$k_{loss} \propto \frac{\sin^2\left(\frac{\omega}{2v}d\right)}{\frac{\omega}{2v}d}$$

$$\frac{\omega}{2v}d = \frac{\pi d}{\beta_{rel.}\lambda}$$

$$d_{opt} = 0.371 \cdot \frac{v}{c} \lambda$$

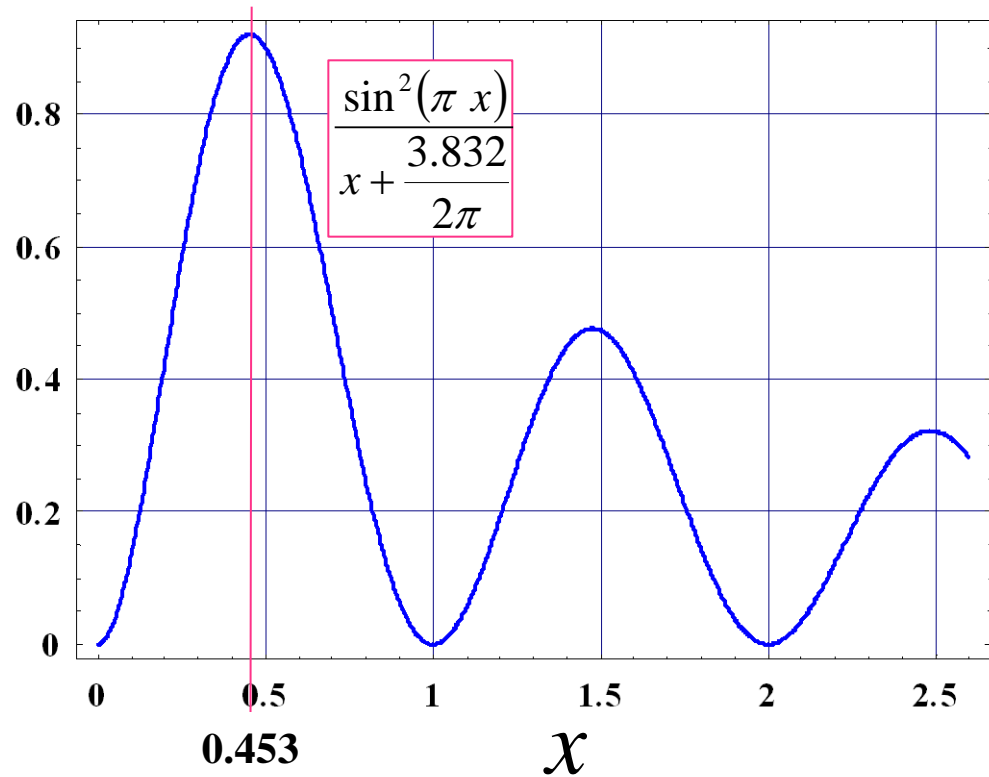


Optimum cavity length for c.w. beam BPM

$$R_{Shunt} \propto \frac{\sin^2\left(\frac{\omega}{2v}d\right)}{d + \frac{X_{mn}c}{\omega}} = \frac{\sin^2\left(\frac{\pi}{\beta_{rel.}} \frac{d}{\lambda}\right)}{\lambda \left(\frac{d}{\lambda} + \frac{X_{mn}}{2\pi}\right)}$$

$$X_{11} = 3.832$$

$$d_{opt} = 0.453 \cdot \frac{v}{c} \lambda$$



Numerical example

Position resolution and measurement range of a 3GHz RF BPM
with $a=15\text{mm}$, $d=25\text{mm}$, $\beta=5$
Material copper $\sigma=5.88 \cdot 10^7$

for single bunch beam with
 $q=100\text{pC}$
 $\sigma_t=3\text{ps}$
 $T=100\text{ MeV } (v \approx c)$.

RF front end can resolve
 $P_e = -50\text{dBm}$.

$$P_E(t) = \frac{\omega k_{\text{loss}} q_b^2 F_b^2}{Q_0} \beta \exp\left(-\frac{\omega(1+\beta)t}{Q_0}\right)$$

$$R = \frac{c}{\omega} X_{11} = \frac{c}{2\pi \cdot 3\text{GHz}} 3.83171 = 61\text{mm}$$

$$k_{\text{loss}} = \frac{4v^2}{\pi \epsilon_0 R^2 \omega^2} \frac{-J_1(ka)^2}{J_0(X_{11}) \cdot J_2(X_{11})} \frac{\sin^2\left(\frac{\omega}{2v} d\right)}{d} \frac{r^2}{a^2} \cos^2(m\vartheta)$$

$$= \frac{4c^2}{\pi \epsilon_0 61^2 \text{mm}^2 \omega^2} \frac{-J_1\left(\frac{2\pi \cdot 3\text{GHz}}{c} 15\text{mm}\right)^2}{J_0(3.83171) \cdot J_2(3.83171)} \frac{\sin^2\left(\frac{\pi \cdot 3\text{GHz}}{c} 25\text{mm}\right)}{25\text{mm}} \frac{x^2}{15^2 \text{mm}^2}$$

$$k_{\text{loss}} = 950 \cdot \frac{V}{\text{pC} \cdot \text{m}^2} x^2$$

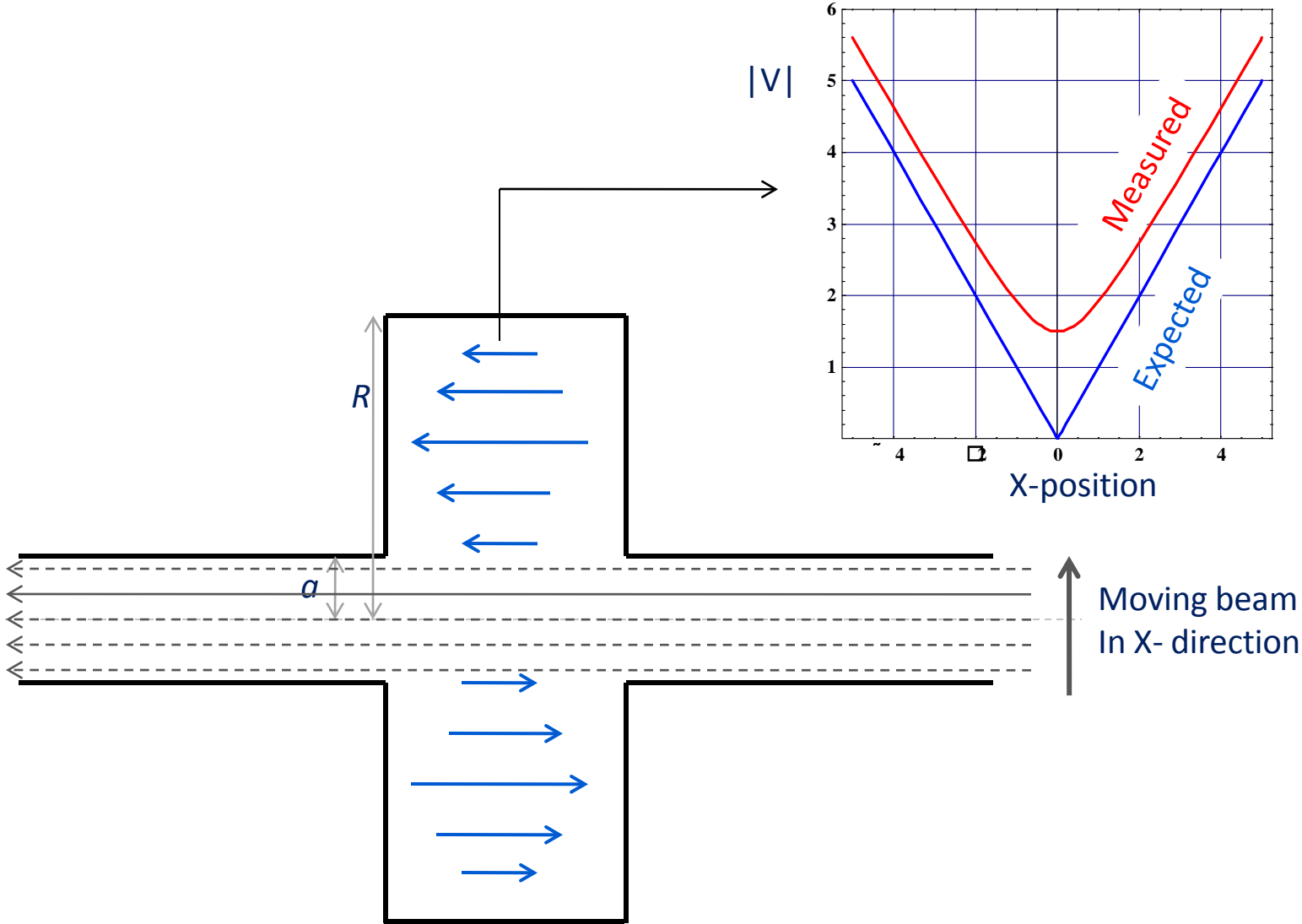
$$F_b = \exp\left(\frac{-\omega^2 \sigma_b^2}{2}\right) = \exp\left(\frac{-(2\pi \cdot 3\text{GHz} \cdot 3\text{ps})^2}{2}\right) = 0.998, \quad R_{\text{Surf}} = \sqrt{\frac{\mu_0 \omega}{2\sigma}}$$

$$Q_0 = \frac{\mu_0}{2} \frac{\omega}{R_{\text{Surf}}} \frac{d R}{d + R} = \frac{\mu_0}{2} \frac{2\pi \cdot 3\text{GHz}}{\sqrt{\frac{4\pi \cdot 10^{-7} \cdot 2\pi \cdot 3\text{GHz}}{2 \cdot 5.88 \cdot 10^7 \text{Sm}^{-1}}}} \frac{25\text{mm} \cdot 61\text{mm}}{25\text{mm} + 61\text{mm}} = 14795$$

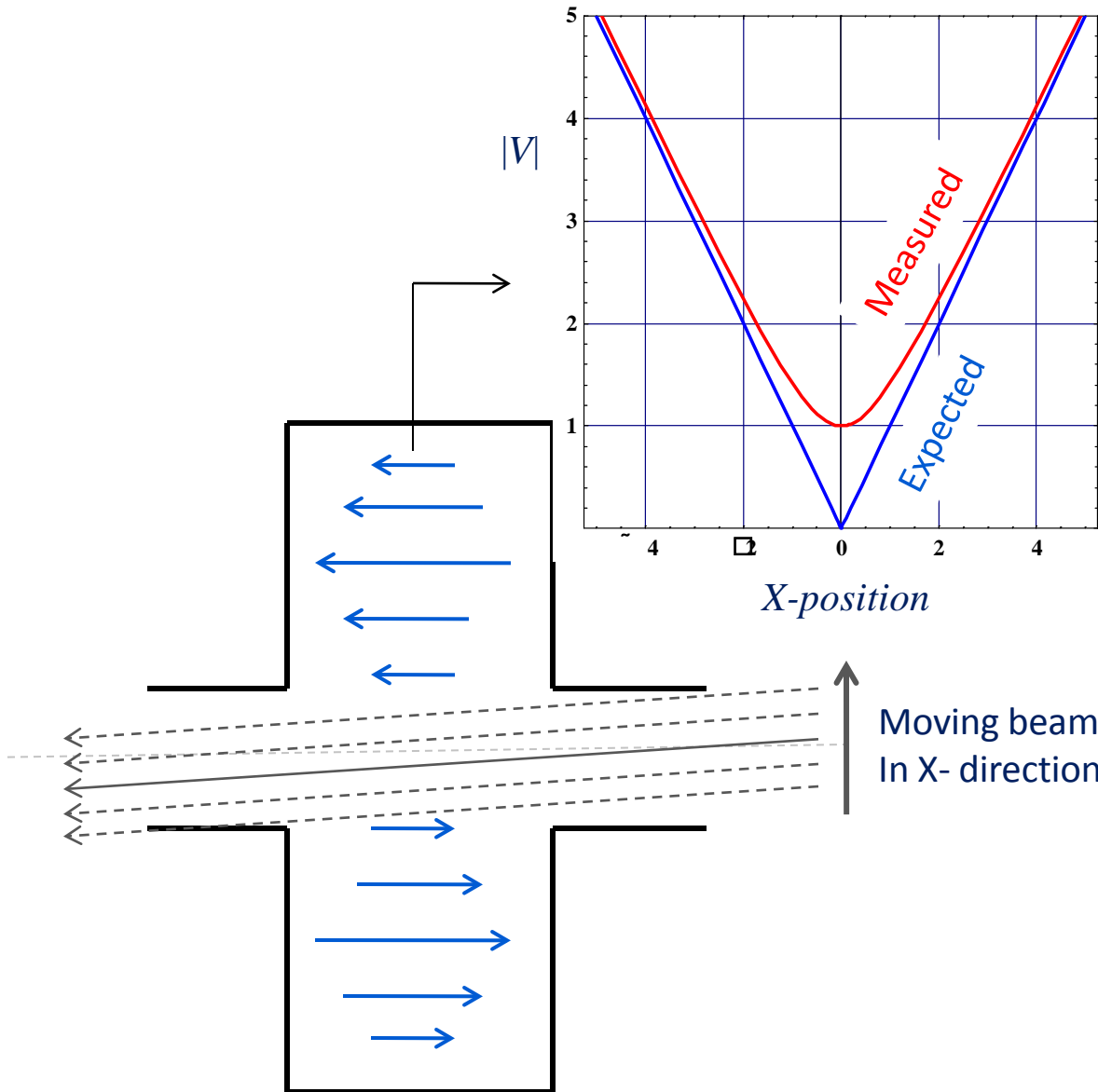
$$P_E(t) = 60.3\text{W } x^2 \exp\left(\frac{-t^2}{131\text{ns}}\right)$$

$$x_{\text{min}} = \sqrt{\frac{10^{-50\text{dBm}/10}}{60.3\text{W}}} = 13\mu\text{m}$$

Measurement troubles



Sensitivity to beam angle

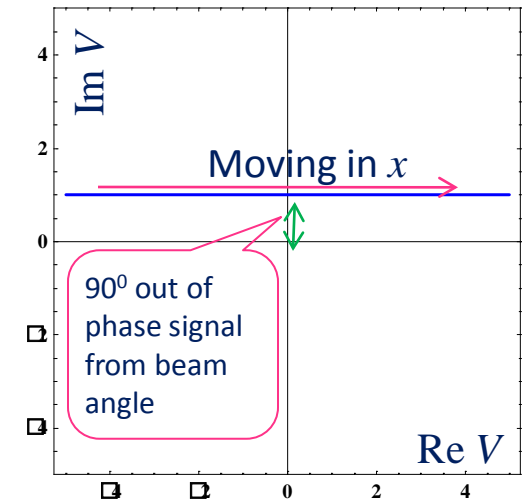


Reasons:

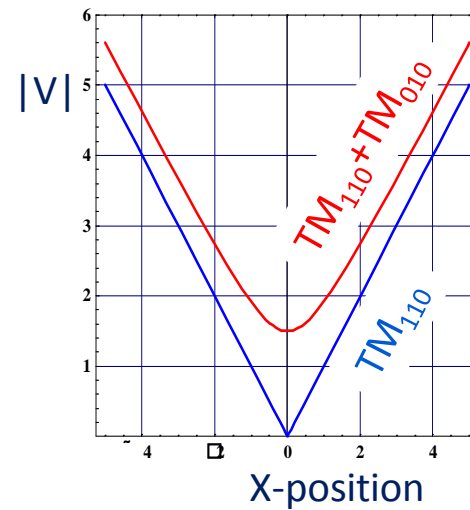
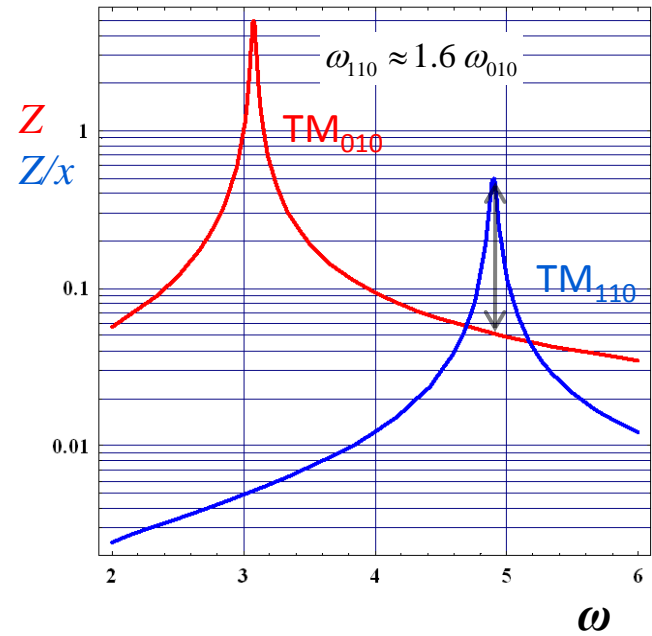
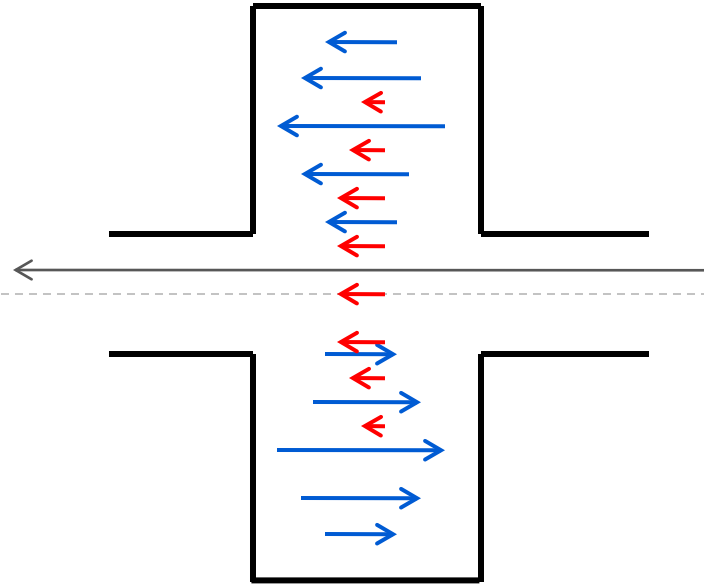
- Beam comes with an angle
- Cavity is tilted

Remedies

- Improve cavity angle alignment
- Shorten cavity length
(at the expense of reduced sensitivity)
- Use it as a feature
(requires IQ demodulation)



Common mode signal from monopole signals

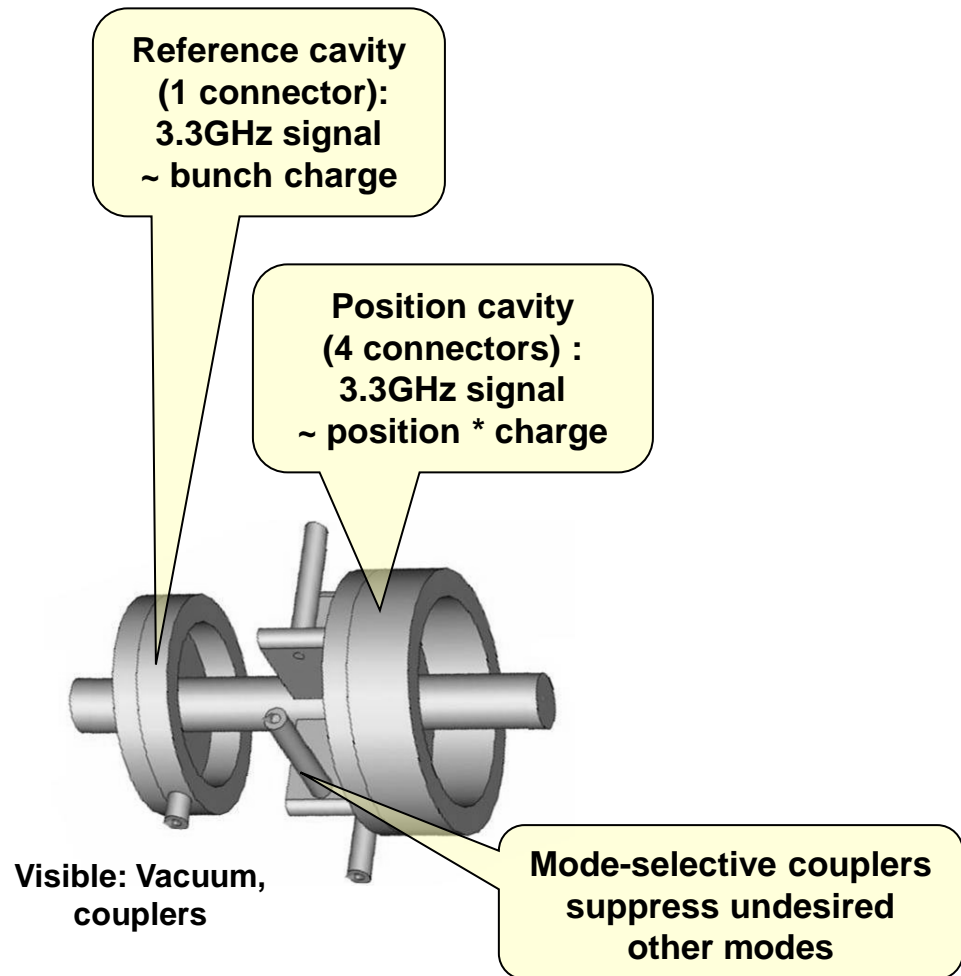
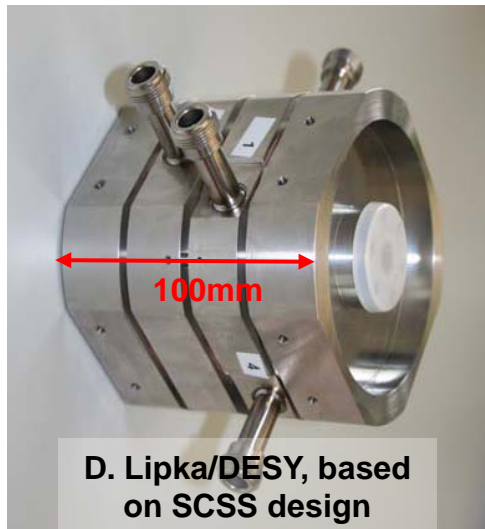


Remedies:

- Symmetric coupling with 180° Hybrid
- Mode selective couplers

RF-BPM (similar designs for SCSS, European-XFEL, SwissFEL)

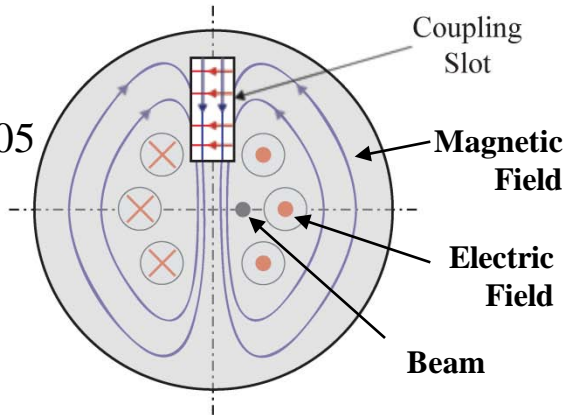
Dual-resonator,
coaxial connectors,
mode-selective
(E-XFEL, 3.3GHz)



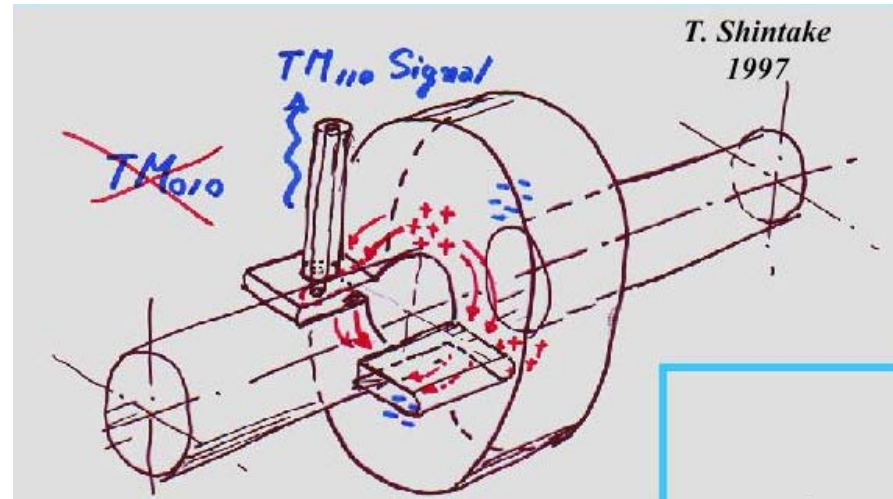
Beam Position = $k * (V_{Pos_Cav} / V_{Ref_Cav})$. Factor k: Not fixed, variable via attenuator.

Reject Monopole Mode

Ref: V. Vogel
Nanobeam 2005

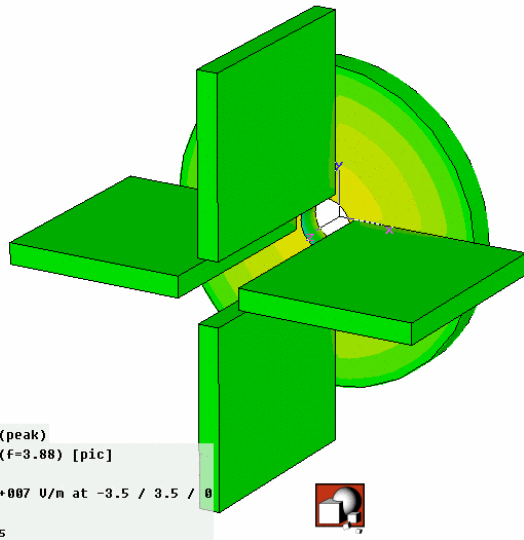


Coupling waveguides couples
to TM_{110} mode, not to TM_{010}

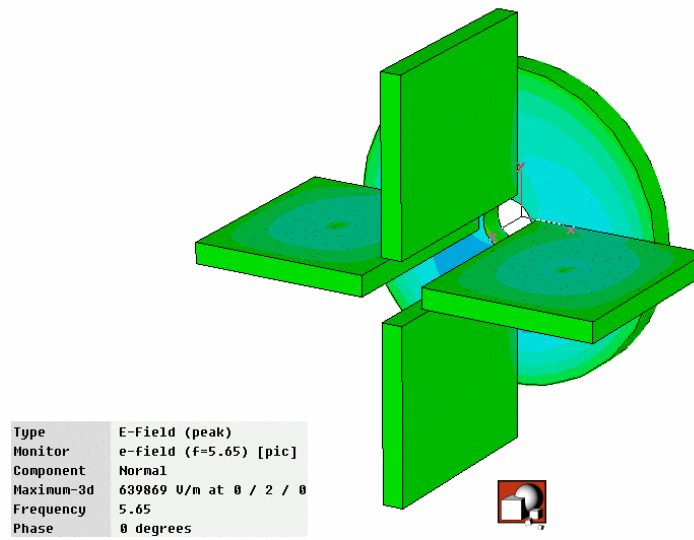


Reject Monopole Mode

Monopole Mode

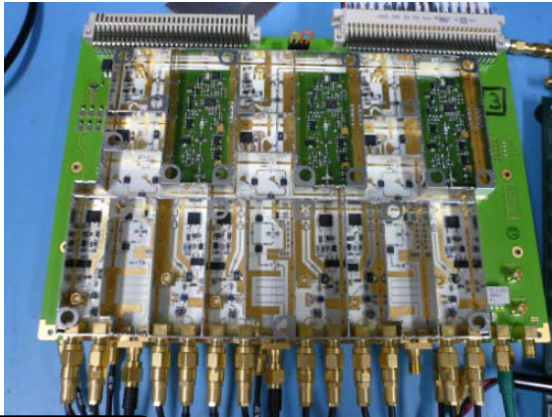


Dipole Mode

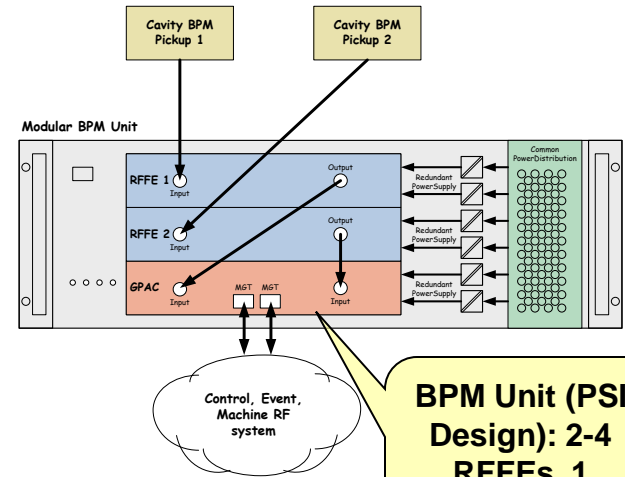


- propagation of dipole mode in waveguide
- monopole mode no propagation in waveguide

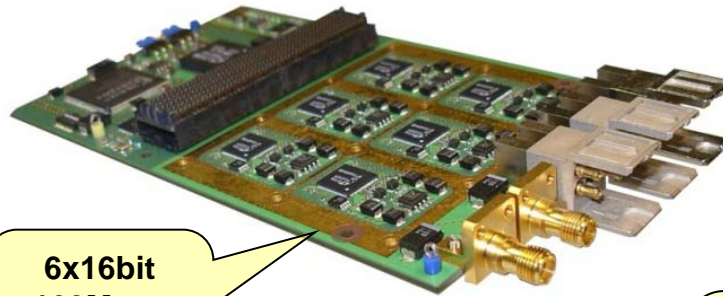
PSI Cavity BPM Electronics for Eu-XFEL and SwissFEL



3.3GHz
RFFE

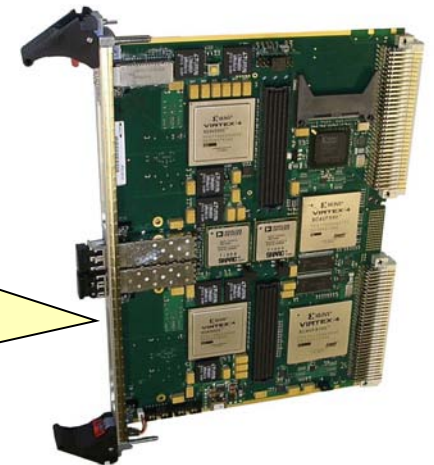


BPM Unit (PSI Design): 2-4 RFFEs, 1 FPGA Board.

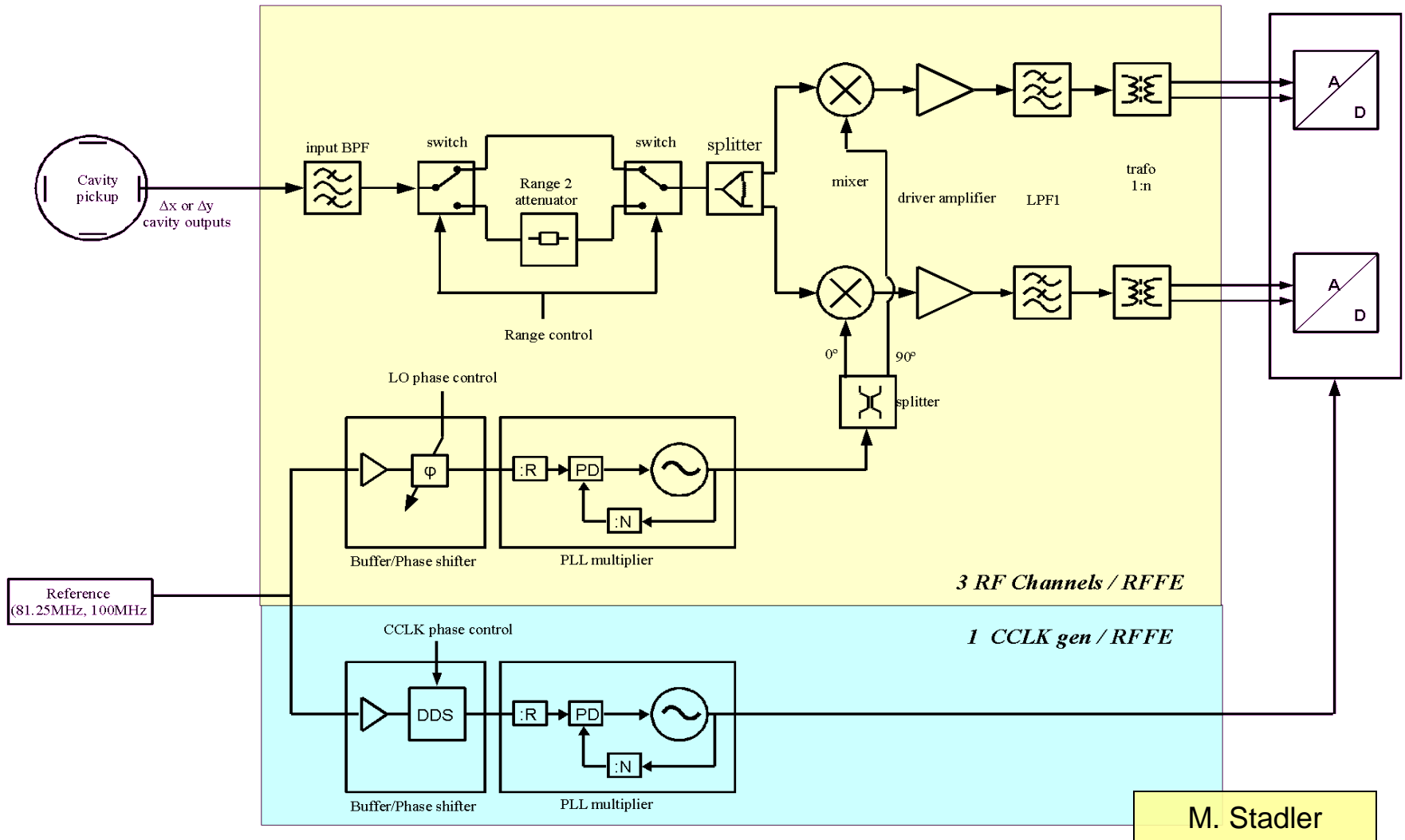


6x16bit
160Msps
ADC
Mezzanine

FPGA
Mezzanine
Carrier
Board
(IBFB
version)



Cavity BPM Electronics: RFFE



Cavity BPM Electronics: ADC Mezzanine Board

Six 16-bit 160Msps ADCs

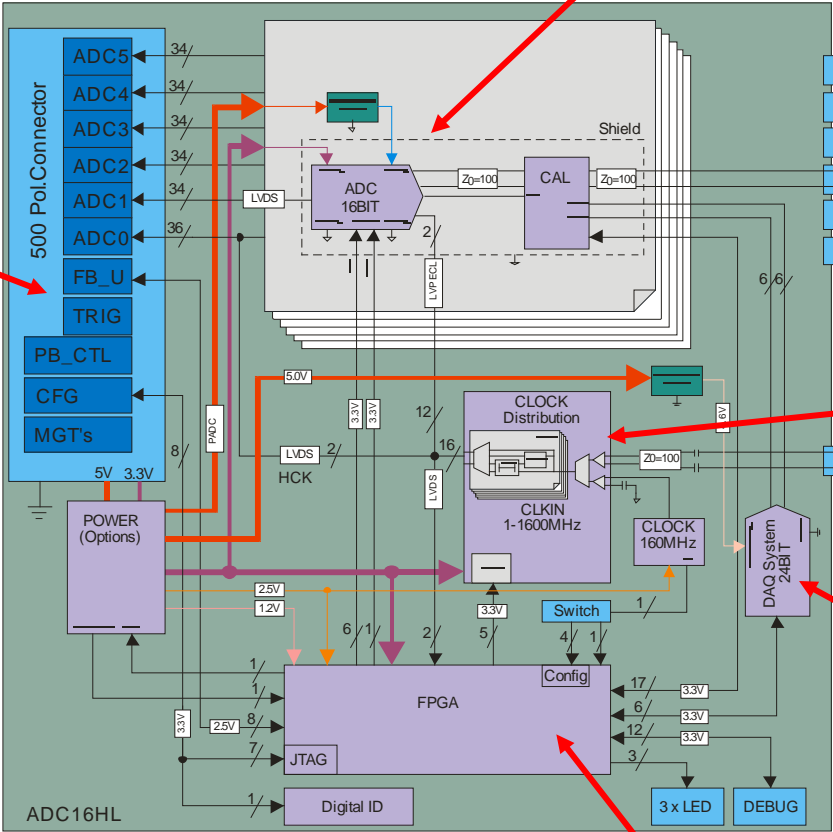
500pol. High Speed Connector (Carrier Board FPGA Interface)

Differential Inputs

Low Jitter Clock Distribution (80fs)

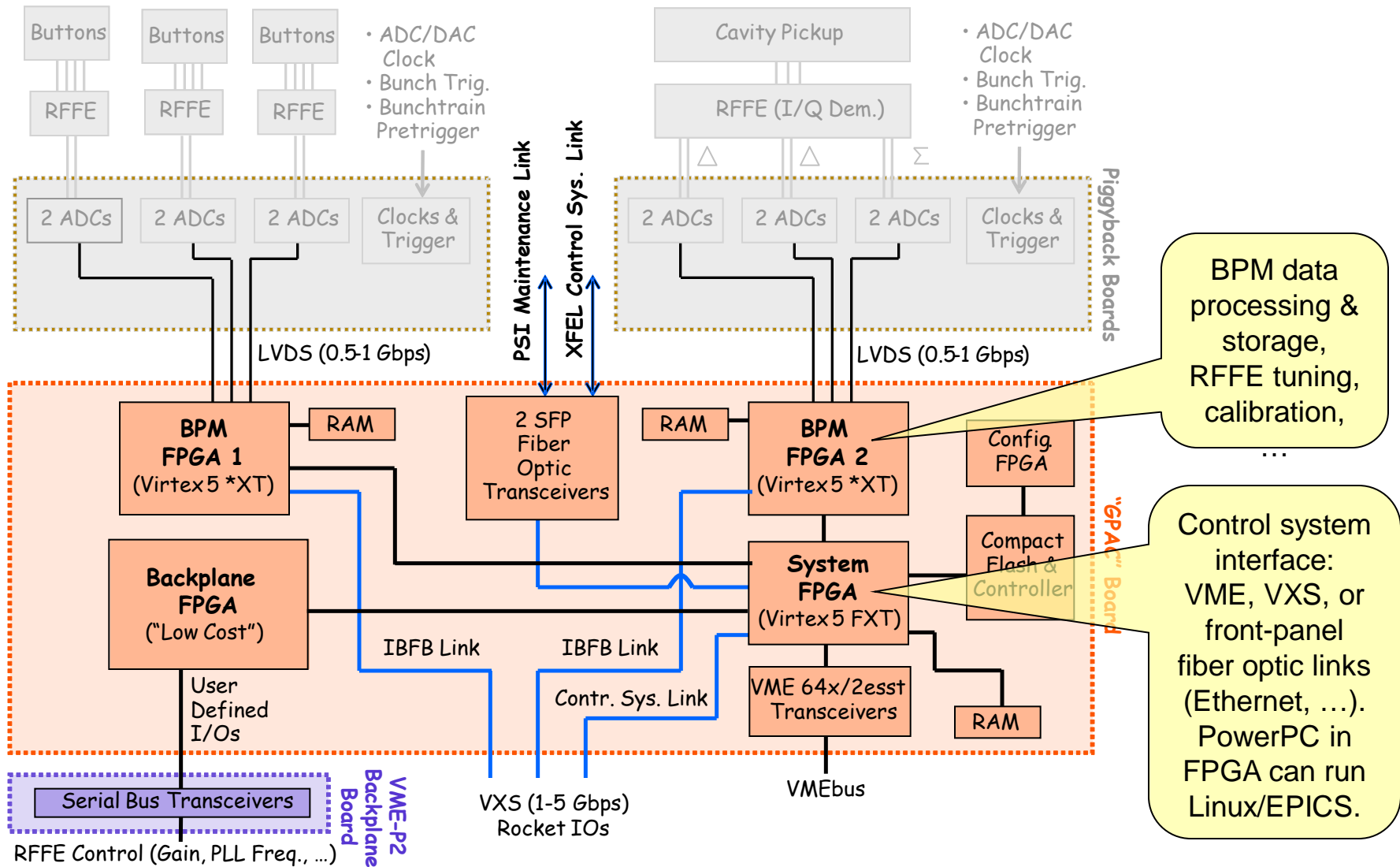
On-Board Gain and Offset Calibration

Control Unit

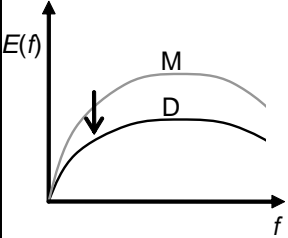
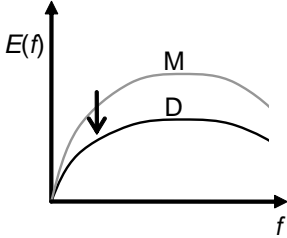
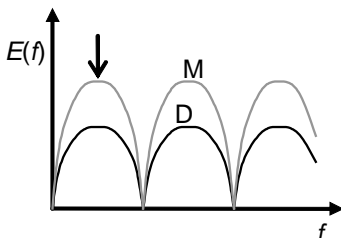
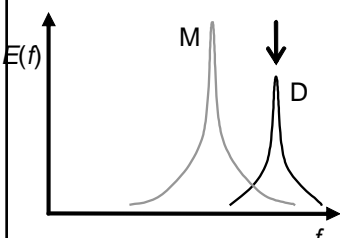
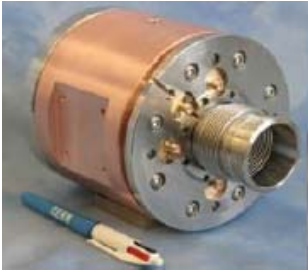





M. Roggli

BPM Electronics: Digital/FPGA Carrier Board



Comparison of different BPM types

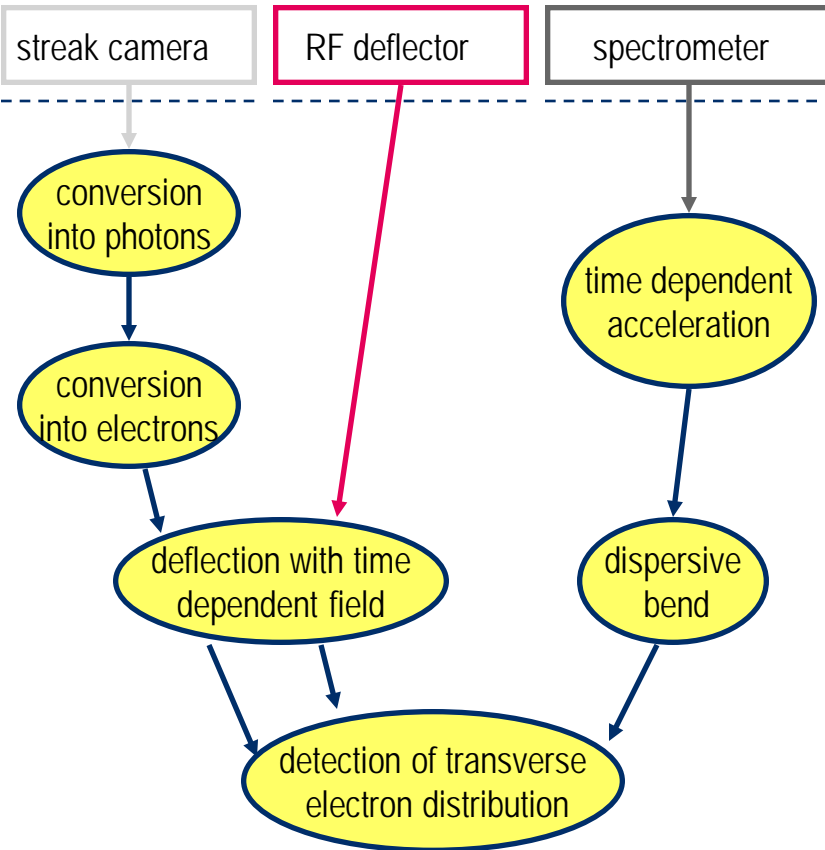
Pickup	Transformer	Button	Matched Stripline	RF Cavity
Spectrum				
Monopole Mode Suppression	Modal (hybrid) / electronics	Modal (hybrid) / electronics	Modal (hybrid) / electronics	Modal (coupler), frequency,
Typical RMS Noise, 10pC, *20mm pipe*	>50μm	>100μm	~60μm	<1μm
Typical Electronics Frequency	0.1...200MHz	300...800MHz	300...800MHz	1-12GHz
Pictures				

Comparison of different BPM types cont.

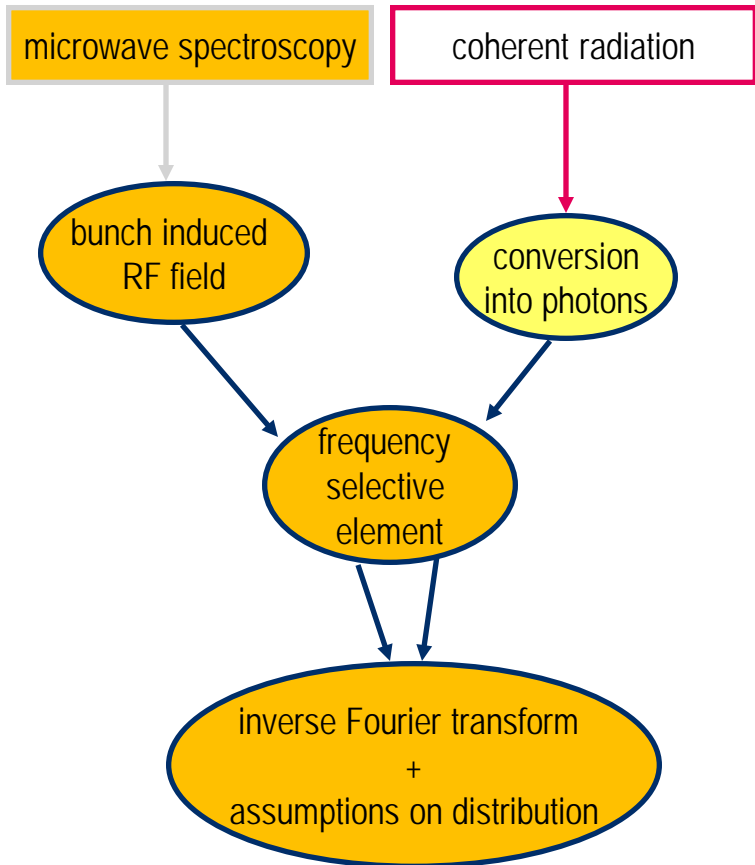
	Transformer	Button	Stripline	RF Cavity
flexibility bunch spacing, train length, bunch length	++	+	-	-
precision & resolution	+	+	+	++
sensitivity for low charges	+	-	+	++
difficulty of calibration	+	+	+	-
impedance (collective instabilities)	+	+	+	-
complexity of electronics	+	+	+	-
size	-	++	+	+

Measurement methods for short bunch length

Time Domain Methods



Frequency Domain Methods



Bunch Length Measurements with RF methods

For beam intensity and position measurement the bunch length affects the signal in proportion with bunch form factor

Induced voltage $V = 2k_{loss} q_b F_b$

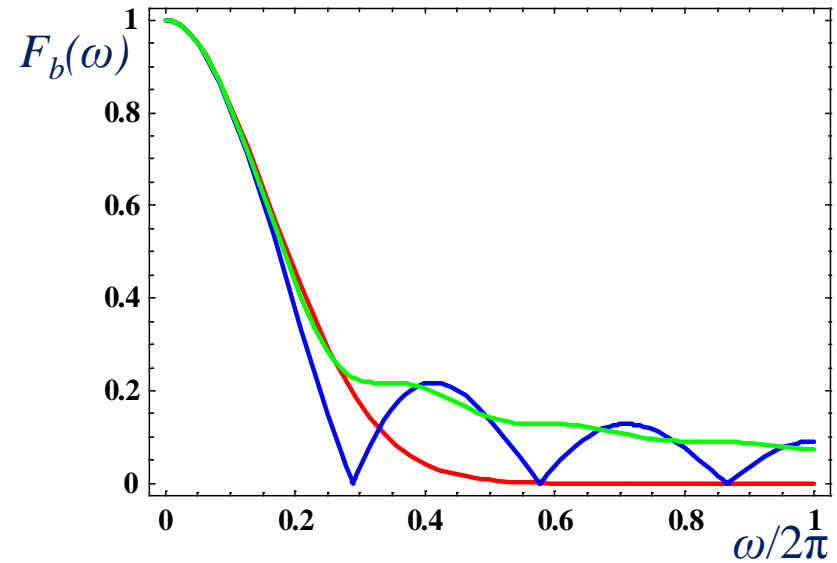
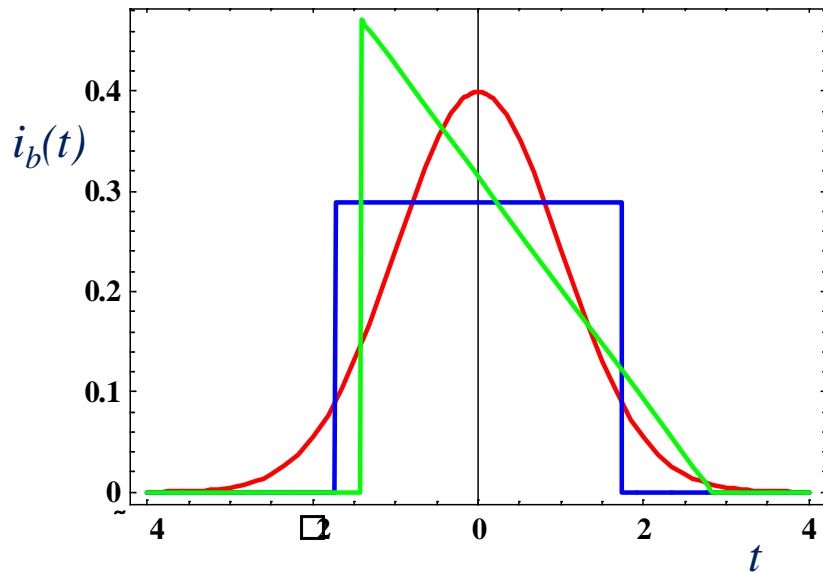
Energy deposited $W = k_{loss} q_b^2 F_b^2$

with "bunch Formfactor" $F_b = \frac{\left| \int i_B(t) \cdot e^{i\omega t} dt \right|}{\int i_B(t) dt} = \frac{\left| \int i_B(t) \cdot e^{i\omega t} dt \right|}{q_B}$

By sampling F_b at different frequencies information of bunch length and shape can be obtained !

Relation bunch spectrum / r.m.s. bunch length

Three bunch shapes with same q_b and σ_b



$$F_b(\omega) = \frac{1}{q_b} \left| \int_{-\infty}^{+\infty} i_b(t) e^{i\omega t} dt \right| = \frac{1}{q_b} \left| \int_{-\infty}^{+\infty} i_b(t) \left(1 + i\omega t - \frac{\omega^2 t^2}{2} + \dots \right) dt \right| = \frac{1}{q_b} \left(1 - \frac{\omega^2}{2} \int_{-\infty}^{+\infty} i_b(t) t^2 dt \right) + \dots$$

$$F_b(\omega) \approx \frac{1}{q_b} \left(1 - \frac{\omega^2}{2} \sigma_{rms}^2 \right) \quad \text{for low frequencies}$$

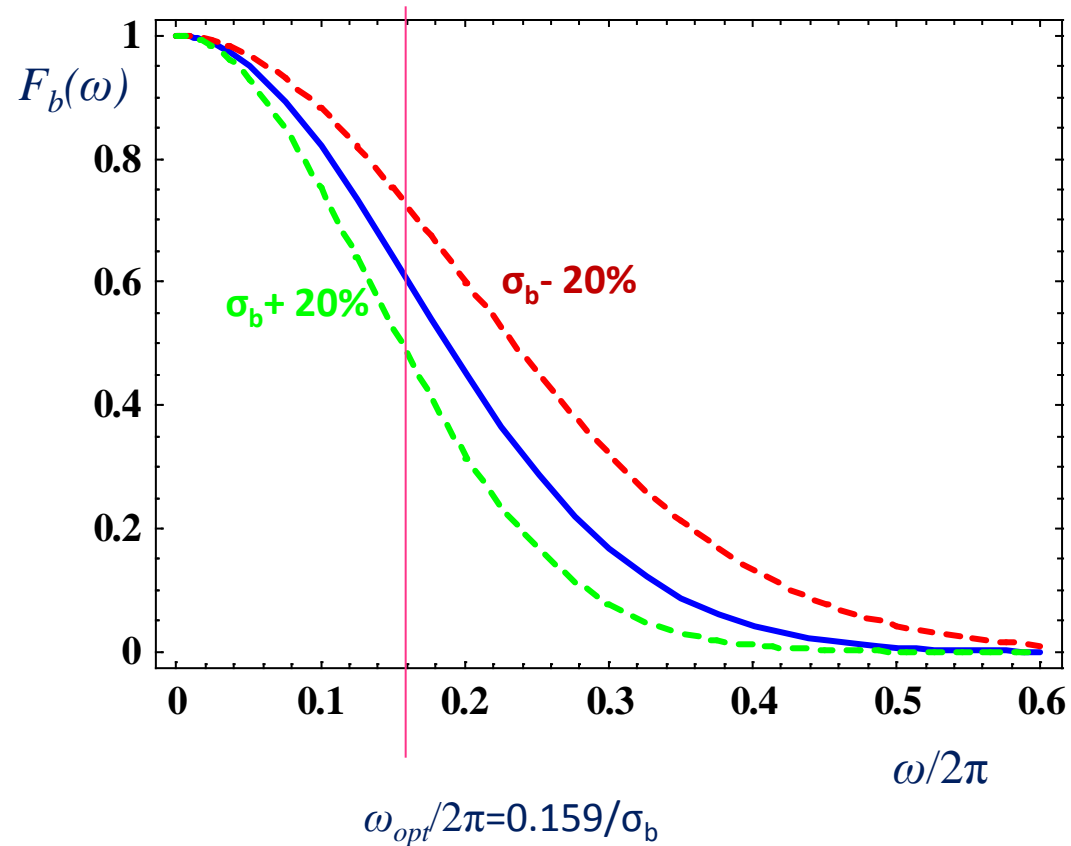
Best frequency choice for σ_b determination

$$i_b(t) = \frac{q_b}{\sqrt{2\pi\sigma_b}} \exp\left(\frac{-t^2}{2\sigma_b^2}\right)$$

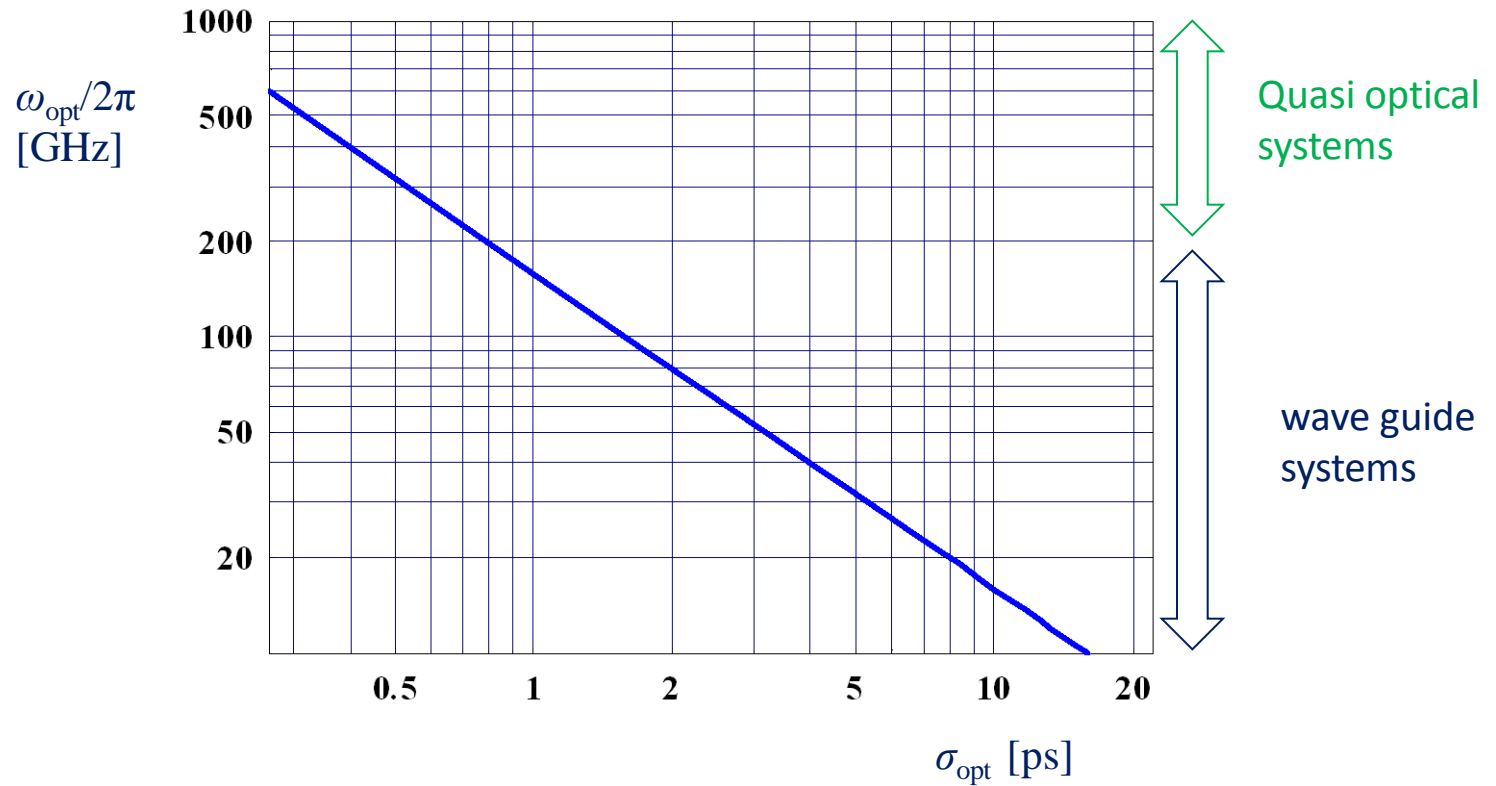
$$F_b(\omega) = \frac{q_b\sigma_b}{\sqrt{2\pi}} \exp\left(\frac{-\omega^2\sigma_b^2}{2}\right)$$

$$\frac{d^2 F_b(\omega)}{d\omega^2} = 0 \Rightarrow \omega_{opt} = \frac{1}{\sigma_b}$$

Best frequency for maximum sensitivity

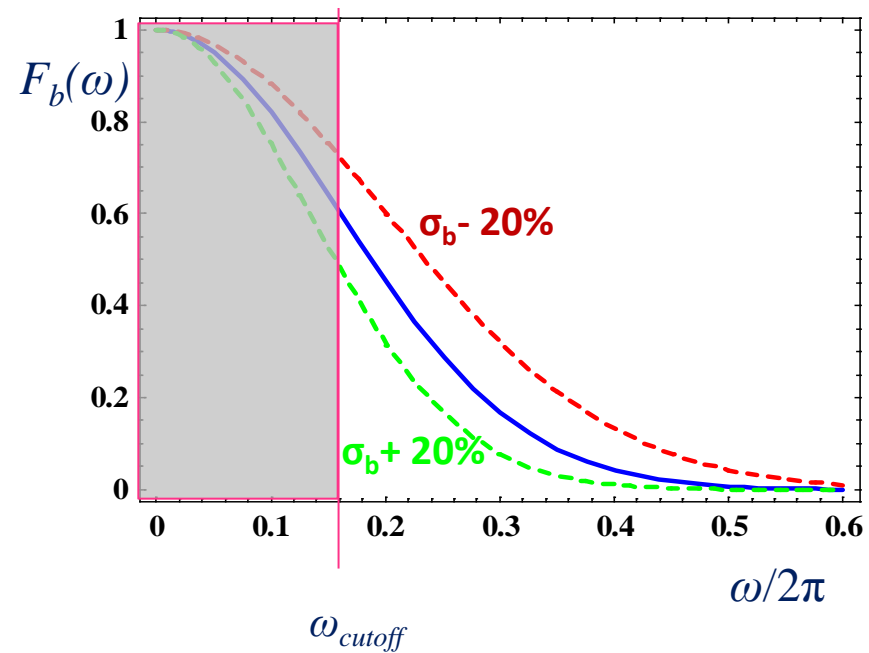
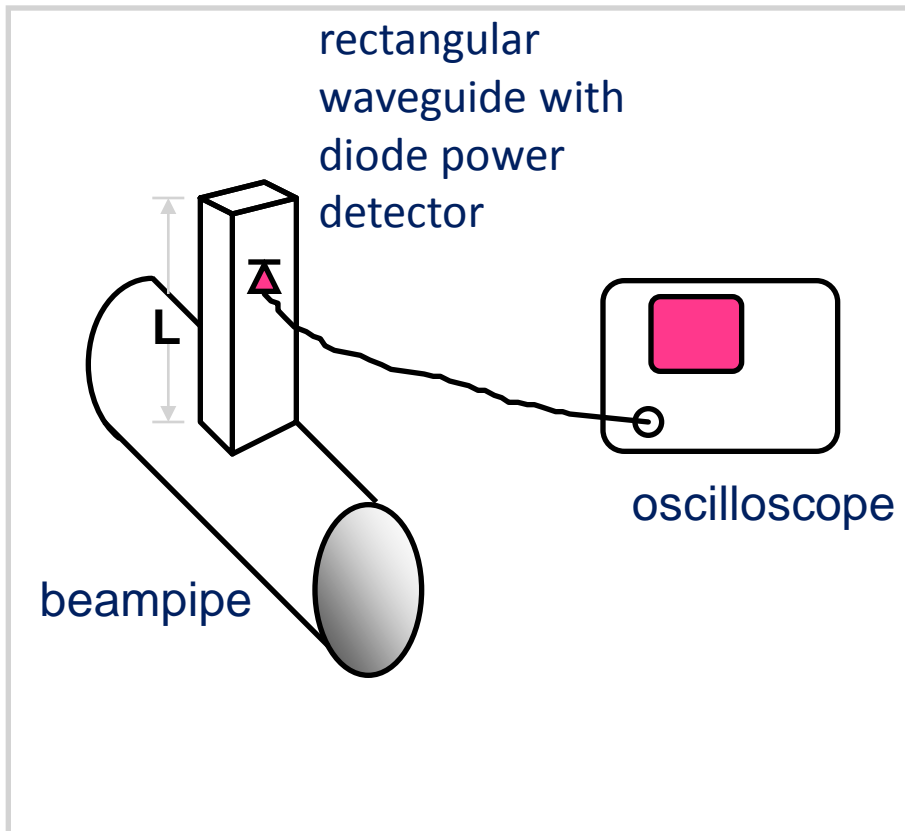


Range of microwave methods

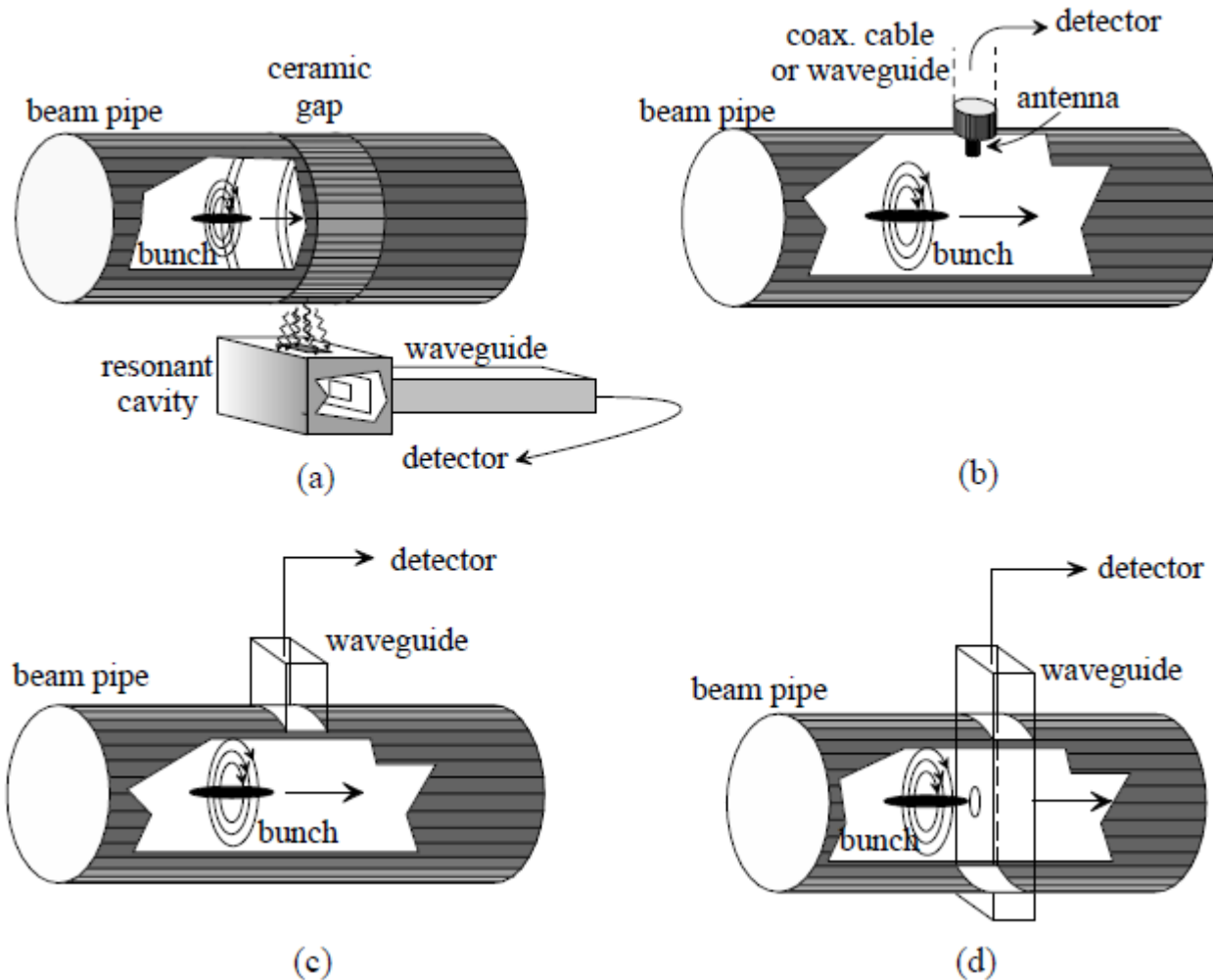


A very simple system

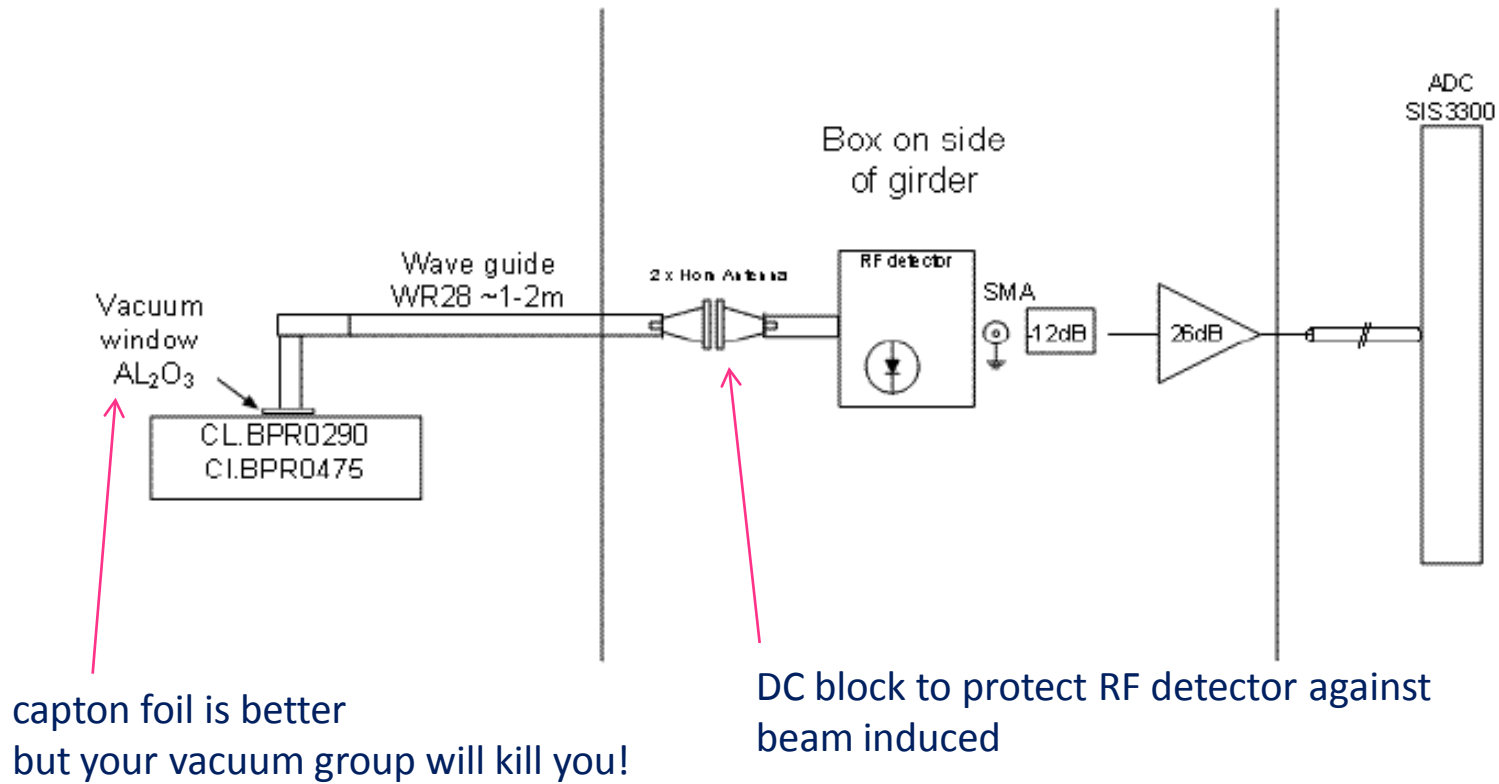
Choose rectangular waveguide with $\omega_{\text{cutoff}} \approx \omega_{\text{opt}}$. Connect waveguide to beampipe.
Detector will measure integrated spectrum integrated above ω_{cutoff}



Different configurations for coupling the electron bunch field

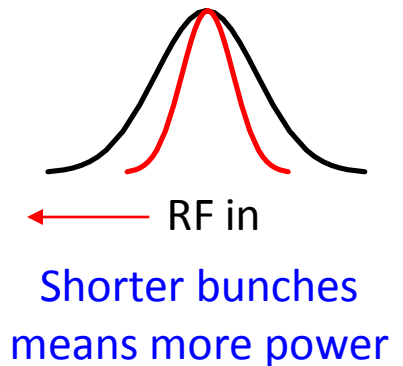
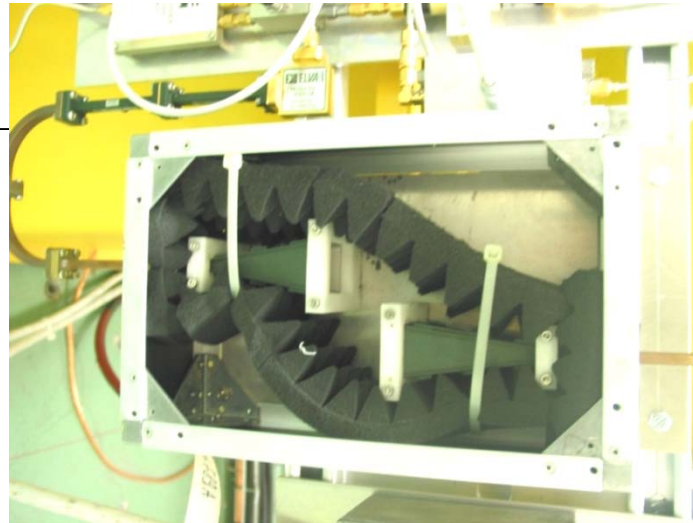
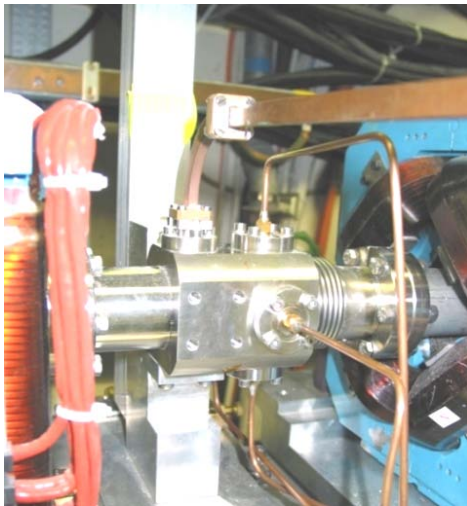
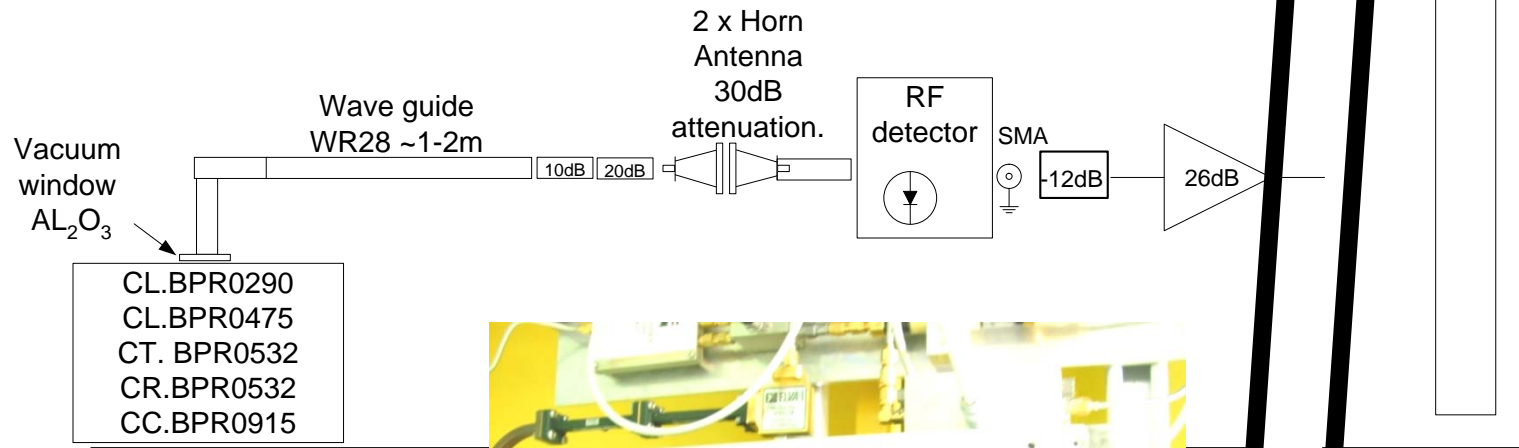


Waveguide pick-up system design



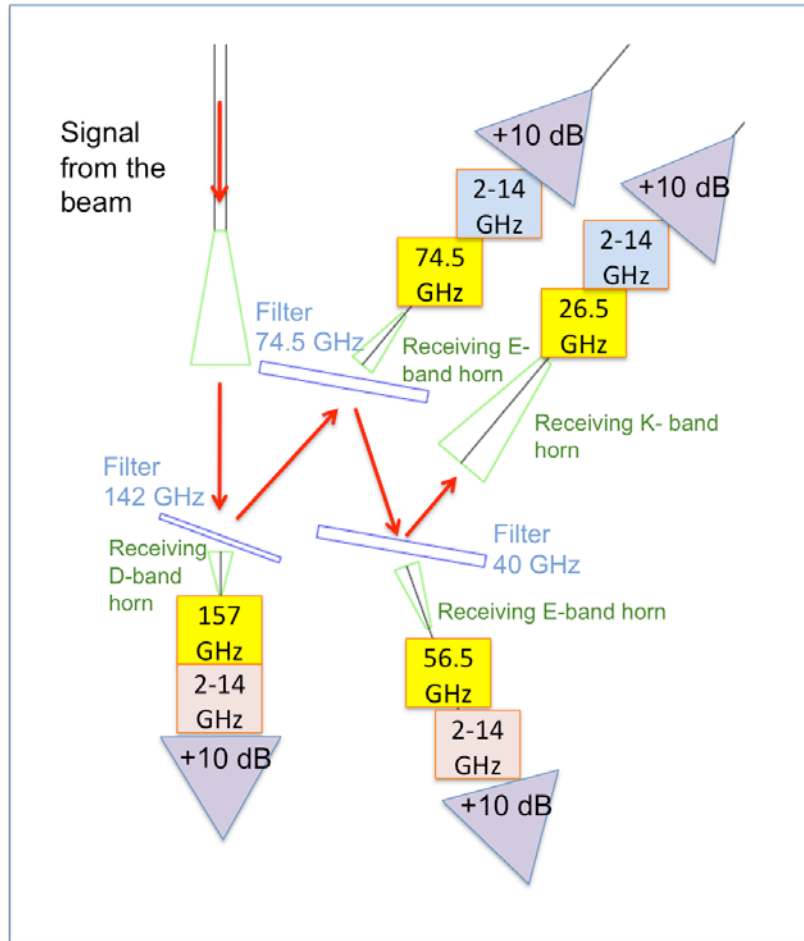
Waveguide pick-up of CTF3 at CERN

Non intercepting bunch length monitor

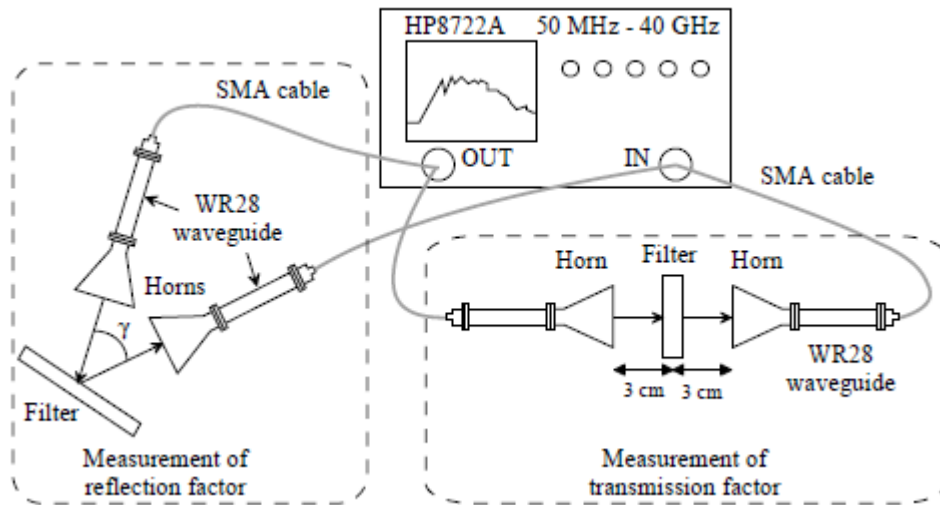


A more sophisticated system

CTF3(CERN) mm-wave spectrometer



Filter for mm-wave



Reflection=low pass

Transmission=high pass

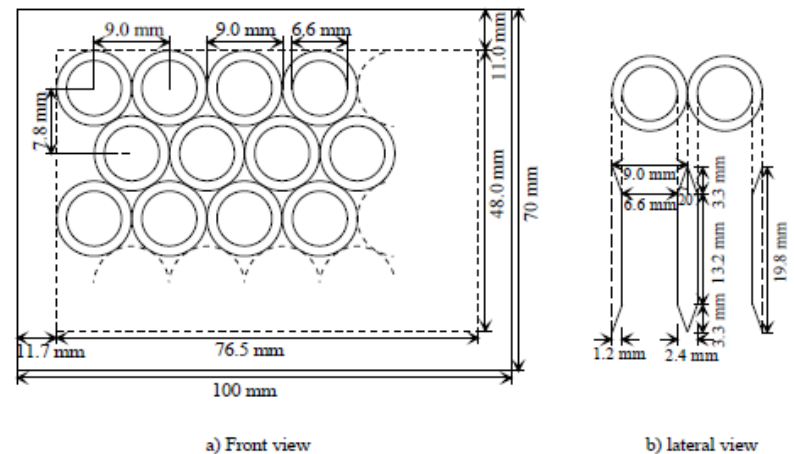


Figure 3.27: Filter with cut-off frequency at 26.5 GHz and conical edges at an angle of 20° .

Example CTF3 mm-wave spectrometer

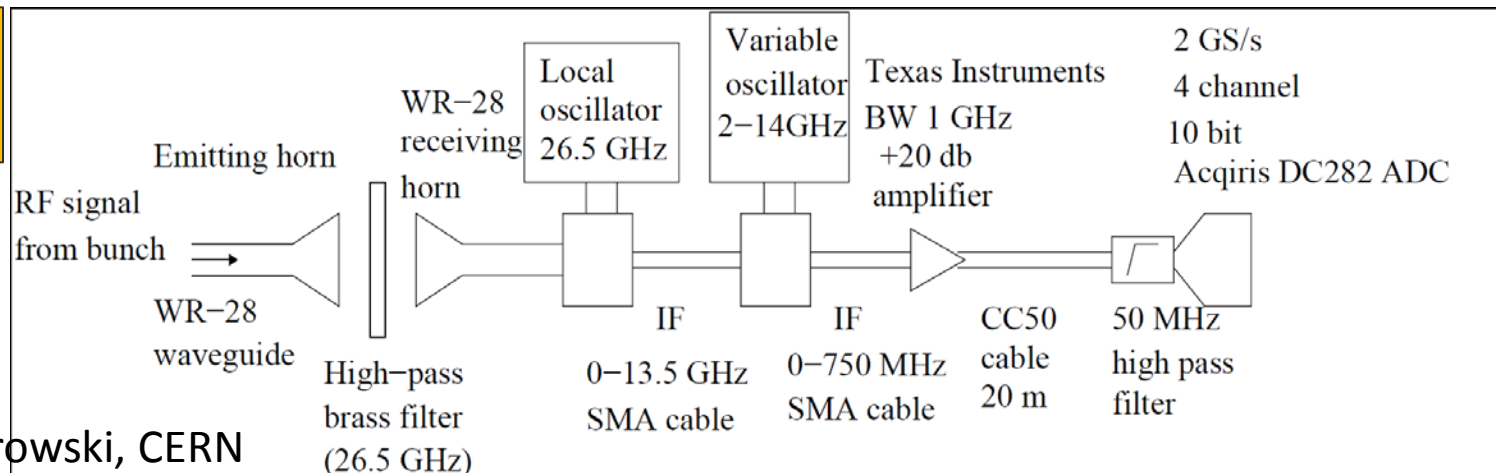
Example of one down mixing stage - RF-pickup

Example:

1. 33 GHz beam harmonic (11th of 3 GHz)
2. ADC is 2 GS/s, typically use 4000 points, 2 micro second time window, delta t = 0.5 ns
3. Depending on the period of the bunch length variations along the pulse & parasitic noise optimize the choice of the second LO mixing stage
4. choose to down mix to a high frequency LO signal, choose 716 MHz

Beam acceleration	Beam harmonic #	Beam harmonic	Fixed first Mixing	Variable Mixing	IF	IF (measured)
2.99855 GHz	11	32.984 GHz	26.5 GHz	7.2 GHz	716 MHz	735 MHz

Example Schema K-band down mixing scheme



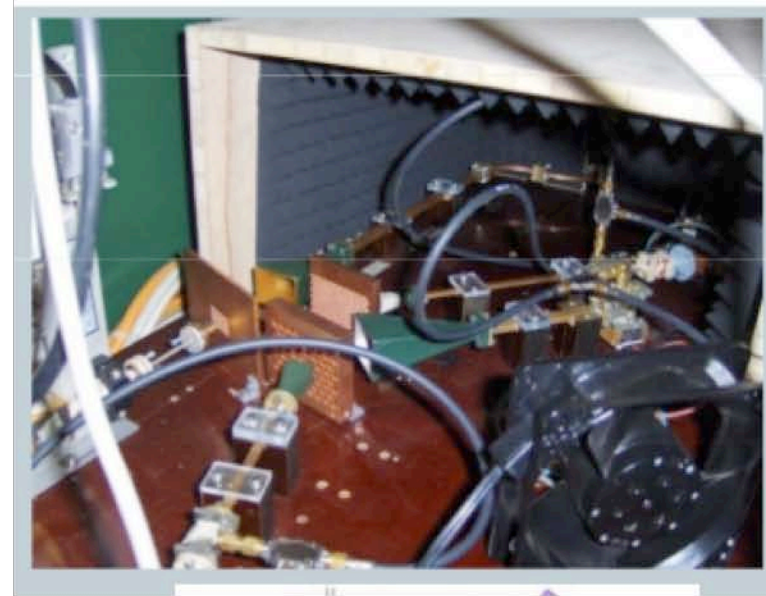
Courtesy A. Dabrowski, CERN

Example CTF3 mm-wave spectrometer



Power supplies for
the 3 mixing
stages & 4
amplifiers

4 Amplifier Channels

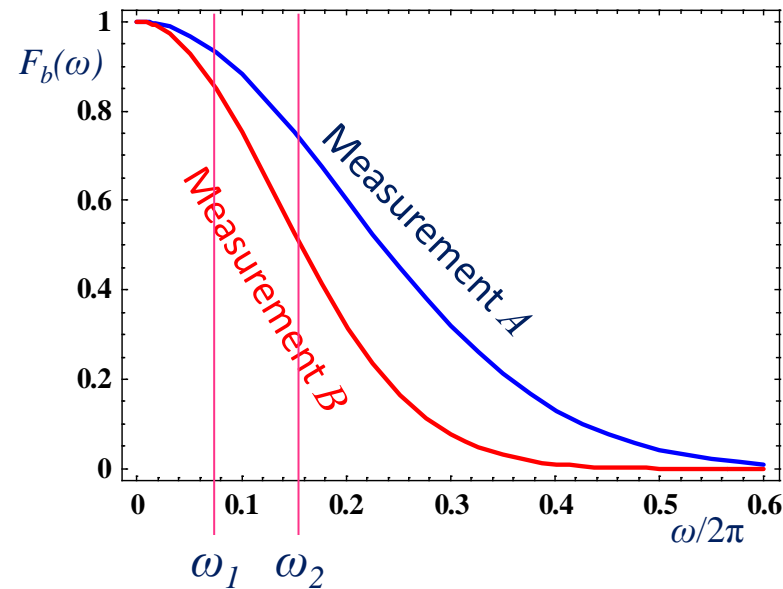


Self consistent calibration of spectral bunch length measurement

1. Measure RF signal S_1 and S_2 at two frequencies ω_1 and ω_2
2. Change machine setting to obtain a different (yet unknown) bunch-length
3. Measure again RF signal S_1 and S_2 at two frequencies ω_1 and ω_2
4. Compute response function R_1 and R_2 at ω_1 and ω_2 and bunch-lengths σ_A and σ_B for the two machine settings from knowledge of spectral shape
5. Store R_1 and R_2 for future measurements

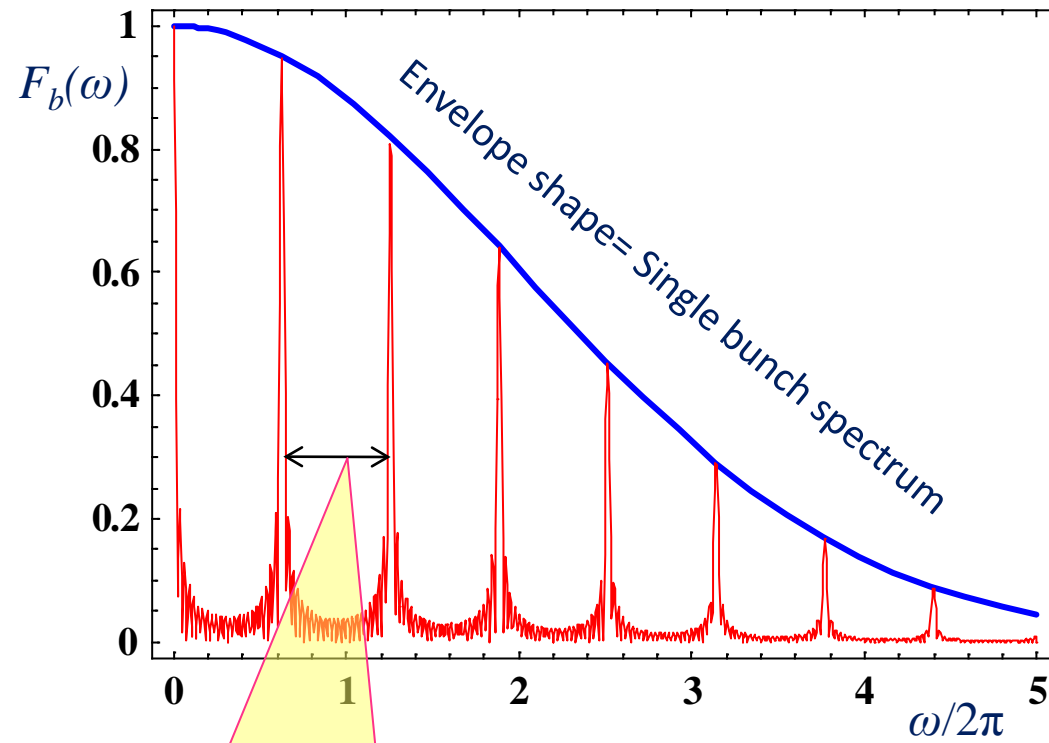
$$S_{1A} = R_1 \left(1 - \frac{\omega_1^2 \sigma_A^2}{2} \right) \quad S_{2A} = R_2 \left(1 - \frac{\omega_2^2 \sigma_A^2}{2} \right)$$

$$S_{1B} = R_1 \left(1 - \frac{\omega_1^2 \sigma_B^2}{2} \right) \quad S_{2B} = R_2 \left(1 - \frac{\omega_2^2 \sigma_B^2}{2} \right)$$



$$\Rightarrow \begin{aligned} R_1 &= \frac{\omega_2^2 (S_{1A} S_{2B} - S_{1B} S_{2A})}{(\omega_1^2 - \omega_2^2) (S_{2A} - S_{2B})} & \sigma_A &= \sqrt{\frac{2S_{1A} (S_{2A} - S_{2B}) \omega_1^2 - 2S_{2A} (S_{1A} - S_{1B}) \omega_2^2}{(S_{1B} S_{2A} - S_{1A} S_{2B}) \omega_1^2 \omega_2^2}} \\ R_2 &= \frac{\omega_1^2 (S_{1A} S_{2B} - S_{1B} S_{2A})}{(\omega_1^2 - \omega_2^2) (S_{1A} - S_{1B})} & \sigma_B &= \sqrt{\frac{2S_{2B} (S_{1A} - S_{1B}) \omega_2^2 - 2S_{1B} (S_{2A} - S_{2B}) \omega_1^2}{(S_{1A} S_{2B} - S_{1B} S_{2A}) \omega_1^2 \omega_2^2}} \end{aligned}$$

Spectrum of bunch trains



Line spacing=1/bunch spacing

Thank you for your attention !



& Have fun measuring your beam parameters!