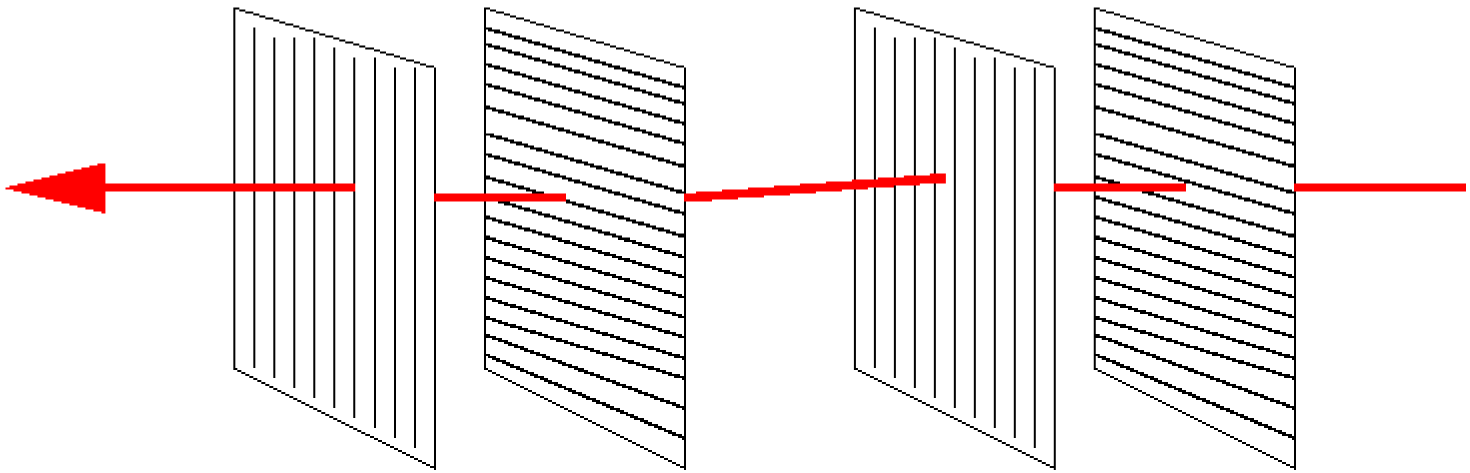
Physics of the Beam Telescope

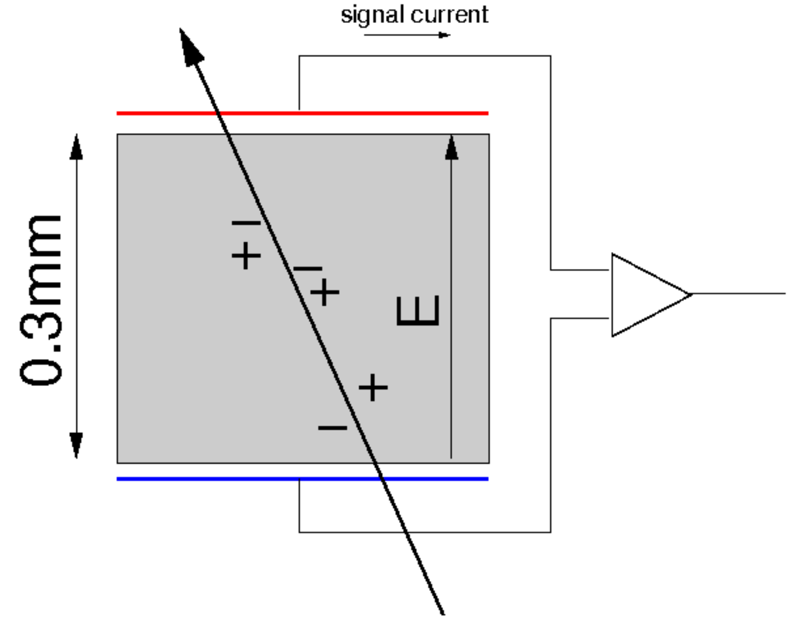
From the Particle to the Position Information

T. Rohe, Paul Scherrer Institut



Principle of particle detection

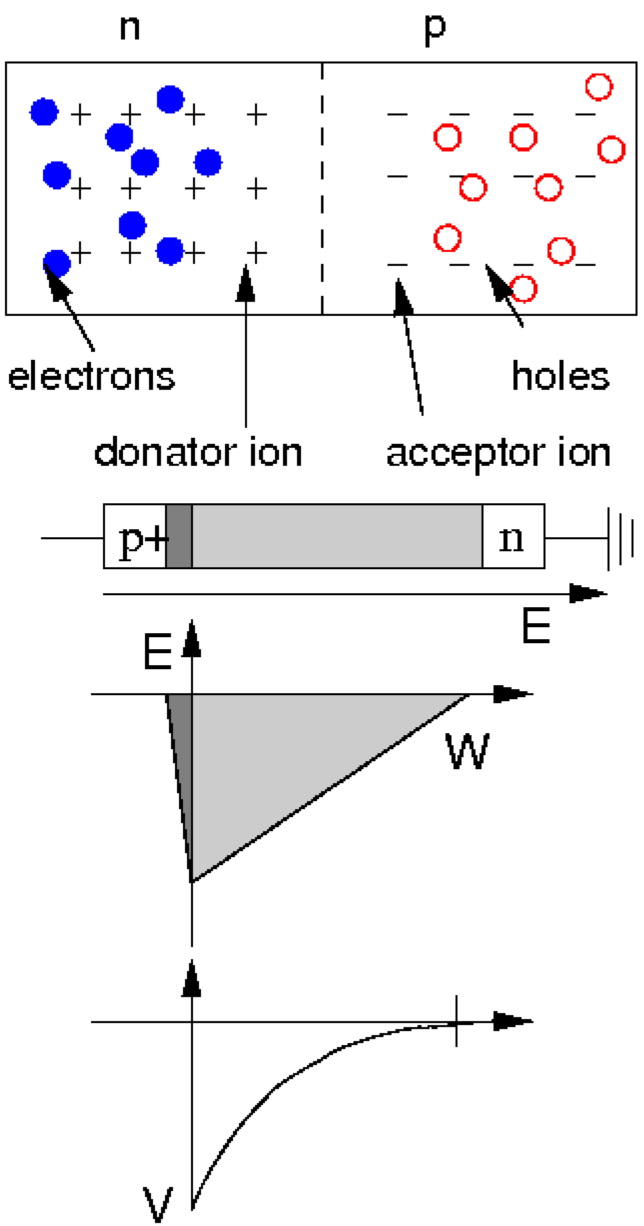
- A particle (or a photon) ionises the detection medium
- The charge carriers are separated by an E-field
- Their drift induce a charge on the collection electrodes
- Simple example
  - Signal current I_s = 4 fC/10ns = **400 nA**
  - Need an average field of $E=v/\mu=0.03cm/10ns/1400cm^2/Vs$ ~ 2100 V/cm or V=60V
  - Current through such a piece of silicon $I_R=V/R$ ~ 60V/300Ω ~ **200mA**
- Need to suppress leakage current
  → pn-junction



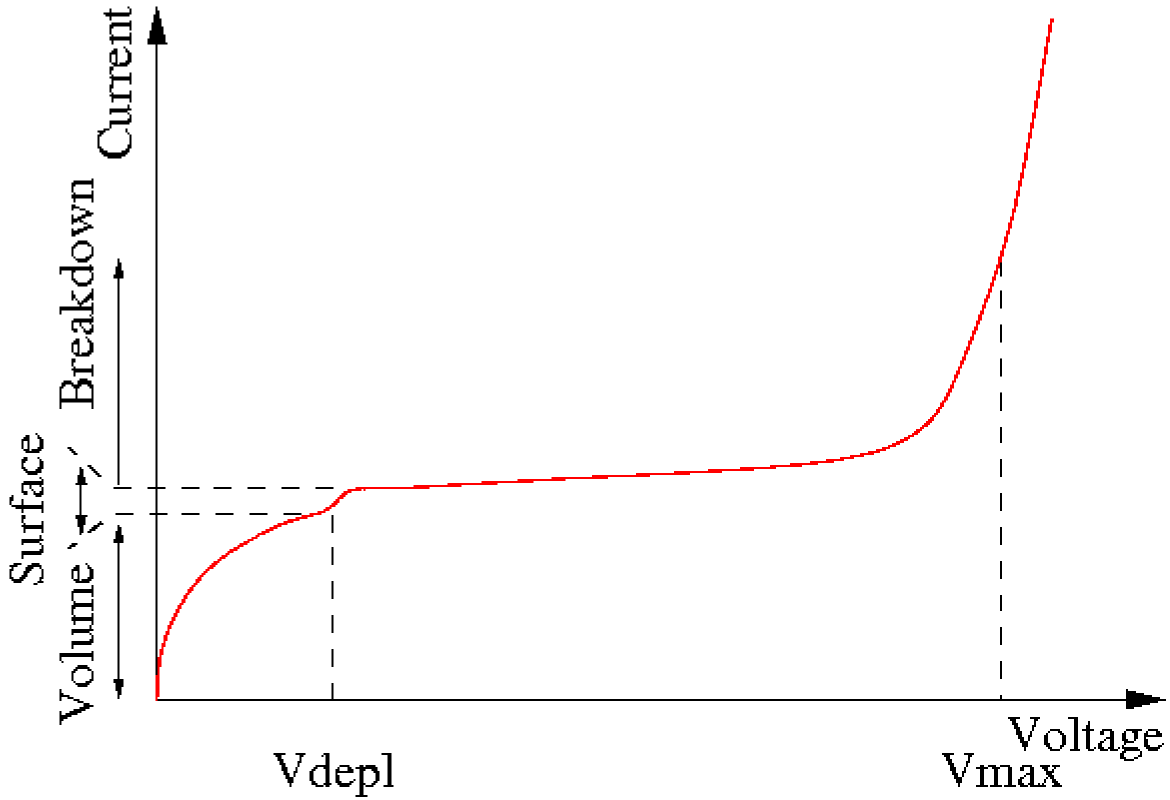
- Thickness: 0.3mm
- Area: $1cm^2$
- Resistivity: 10kΩcm
- Mobility (electrons): ~$1400cm^2/Vs$
- Collection time: ~10ns
- Charge released: ~25000 e^- ~ 4fC

pn-Junction

- If p and n-type silicon “touch”
  - Majority carriers diffuse to other side
    $\mathbf{J_{diff} = -D_n \nabla n}$ or $\mathbf{J_{diff} = D_p \nabla p}$
    with **$D = kT\mu/e$** (Einstein relation)
  - There they recombine with the majority carriers of this side and a “depleted” region built up.
  - The remaining acceptor/donator ions cause an electric field which counteracts the diffusion
    $\mathbf{J_{drift} = -e\, n\, \mu_n\, E}$ or $\mathbf{J_{drift} = e\, p\, \mu_p\, E}$
  - Equalising both currents and integrating over the depletion region gives for the so called built in voltage $V_{bi} \sim kT/e \ln(N_A N_D/n_i^2)$
  - The width of the depletion region can be increased by an external voltage V
    $W \sim \mathrm{sqrt}(2\varepsilon_0\varepsilon_{si}V/eN_D)$ if $N_D >> N_A$ and $V >> V_{bi}$
  - Field is maximum at junction and 0 at the back



IV-Curve

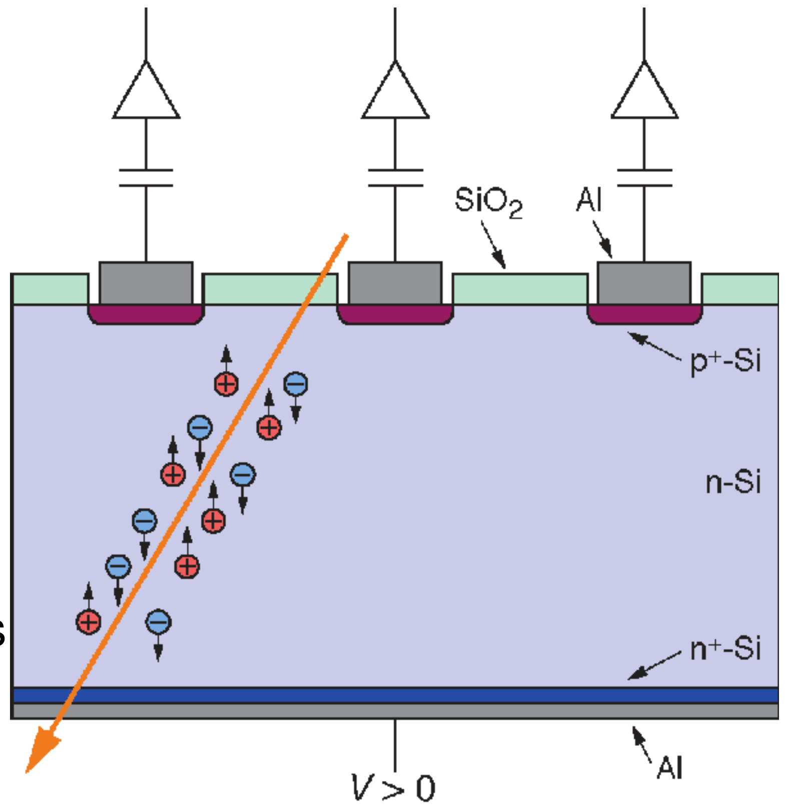


- Dark current in a reversely biased junction is caused by thermal generation of e-h-pairs in the space charge region
- $J_{vol} \approx -en_i W/\tau_g$ (τ_g: carrier generation life time)
- Temperature dependence
  $J_{vol} \sim T^2 \exp(-E_g(T)/2kT)$ or a factor 2 every 8K

- There are other components
  - Generation current from the interfaces/surfaces
  - Electrical avalanche breakdown
- **In forward direction a pn-junction shows exponential behaviour**

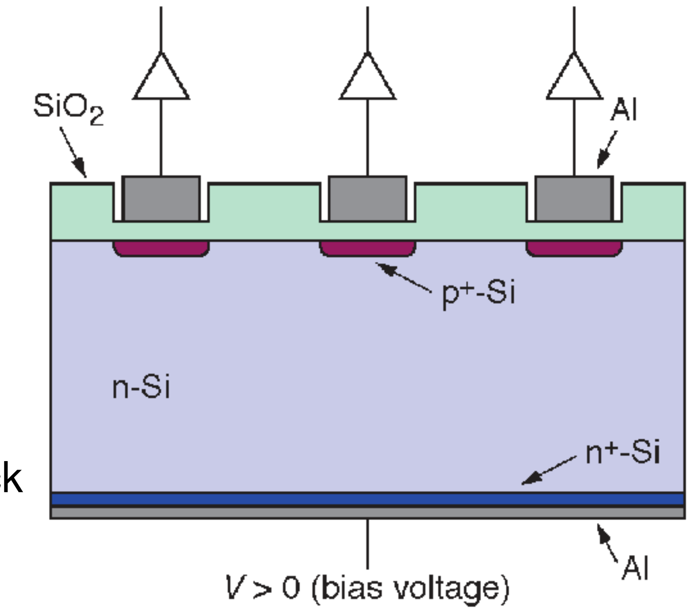
Strip detector (DC-coupled)

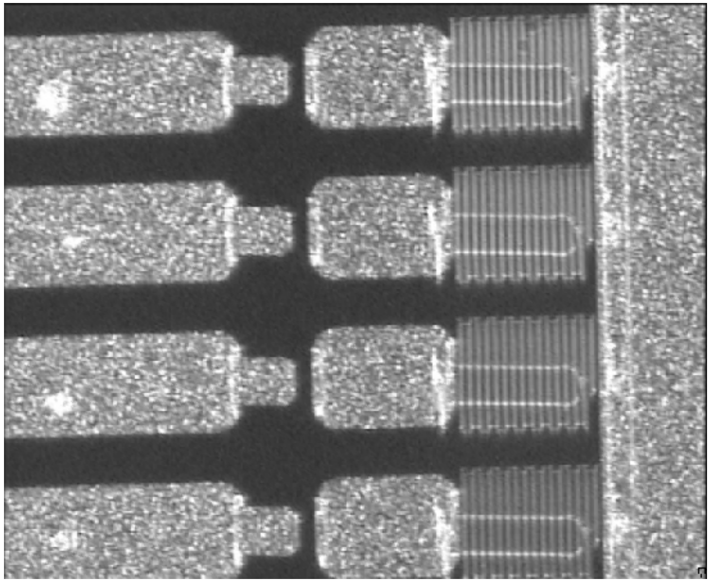
- Simplest possible realisation of a position sensitive Silicon detector
- Simulated in these course
- Strips are Boron implants
- Substrate is Phosphorous doped (~2-10 kΩcm) and ~300μm thick
  - $V_{fd} < 200V$
- Backside Phosphorous implant to establish ohmic contact and to prevent early breakdown
- Highest field close to the collecting electrodes where most of the signal is induced
- Use of n-substrate is purely historical, now p-type sensors are under consideration for sLHC



Strip detector (AC-coupled)

- Bias resistor and coupling capacitance are difficult to implement on an ASIC
- Implementation on sensor possible
  - Capacitor as SiO_2 layer
    - Due to the large strip size, a few 100nm thick layer possible
    - Stable up > 100V
  - Long poly resistor with R>1MΩ
- Capacitor yield might be problematic
  - Every capacitor has to be tested
  - Yield can be improved by applying a sandwich layer of Si_3N_4
  - Costs

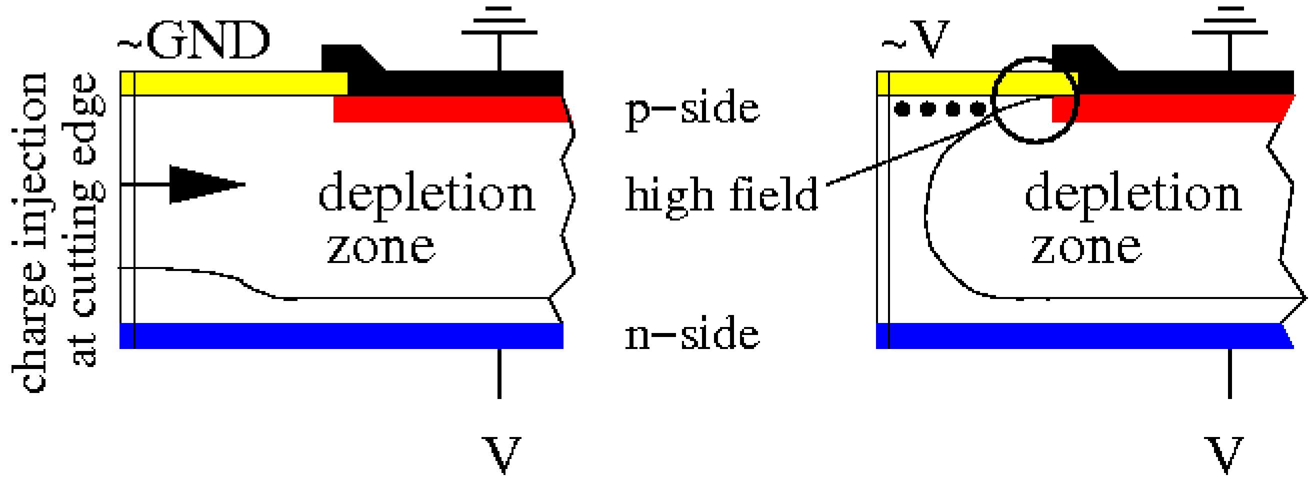


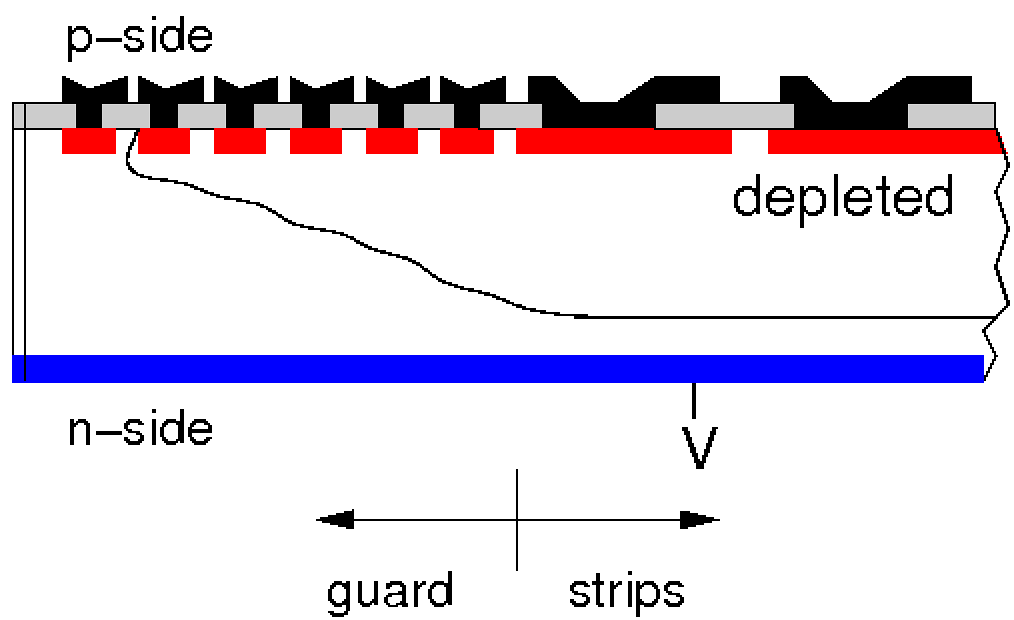


[M. Krammer]

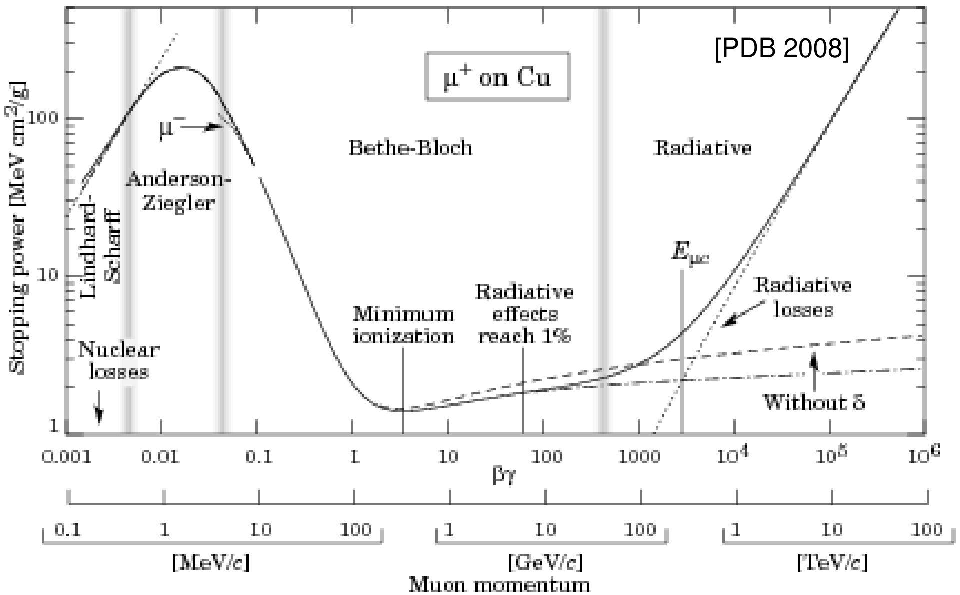
Excursion: Guard rings

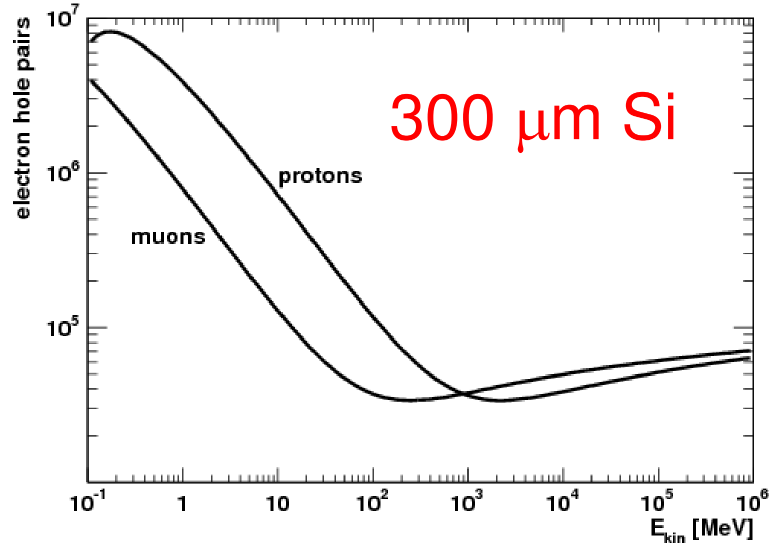
- Needed to prevent
  - Edge break down by reducing gently the potential between strips and edge
  - Injection by preventing the space charge region from reaching the scribe line





Charge generation

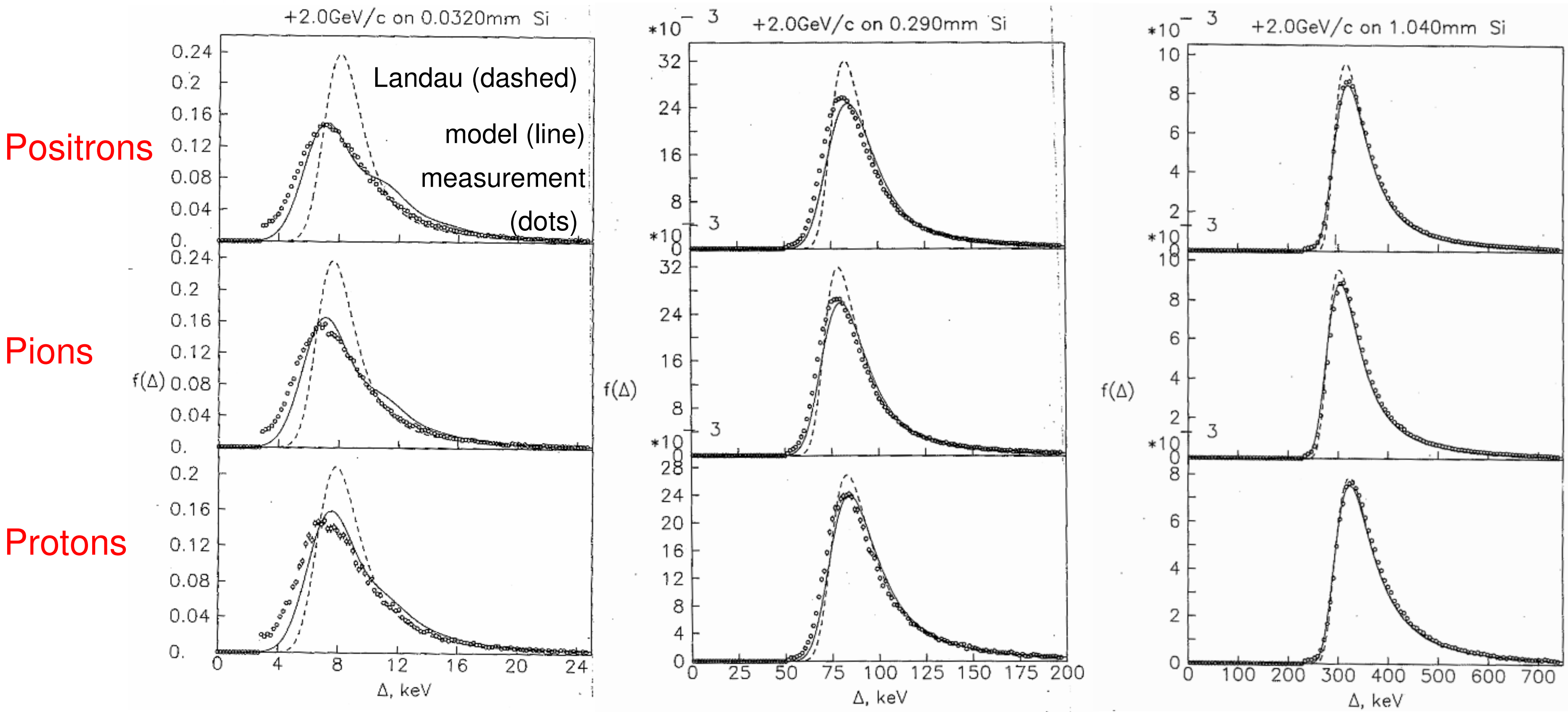




- dE/dx (energy loss of passing particle) is caused by Coulomb scattering with (quasi free) electrons
- Bethe-Bloch formula gives the **average**
  - $\beta < 0.1$: Particle velocity is in the same order of magnitude as the speed of the electrons in the absorbing medium.
  - $\beta < 0.96$: $\langle dE/dx \rangle \sim 1/\beta^2$
    - <dE/dx> can be used as a hint on the particle type
    - Leads to "Bragg peak"
  - **$\beta > 0.96$: Minimum ionising particle (mip)**

Landau distribution

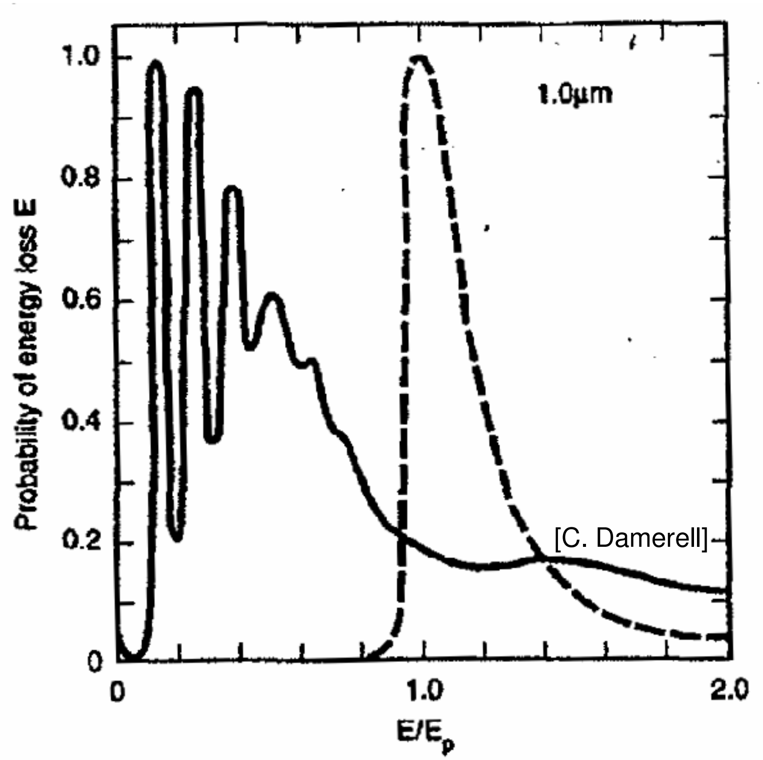
T. Rohe, PSI
2^{nd} Training event Jan. 2010
Physics of the beam telescope

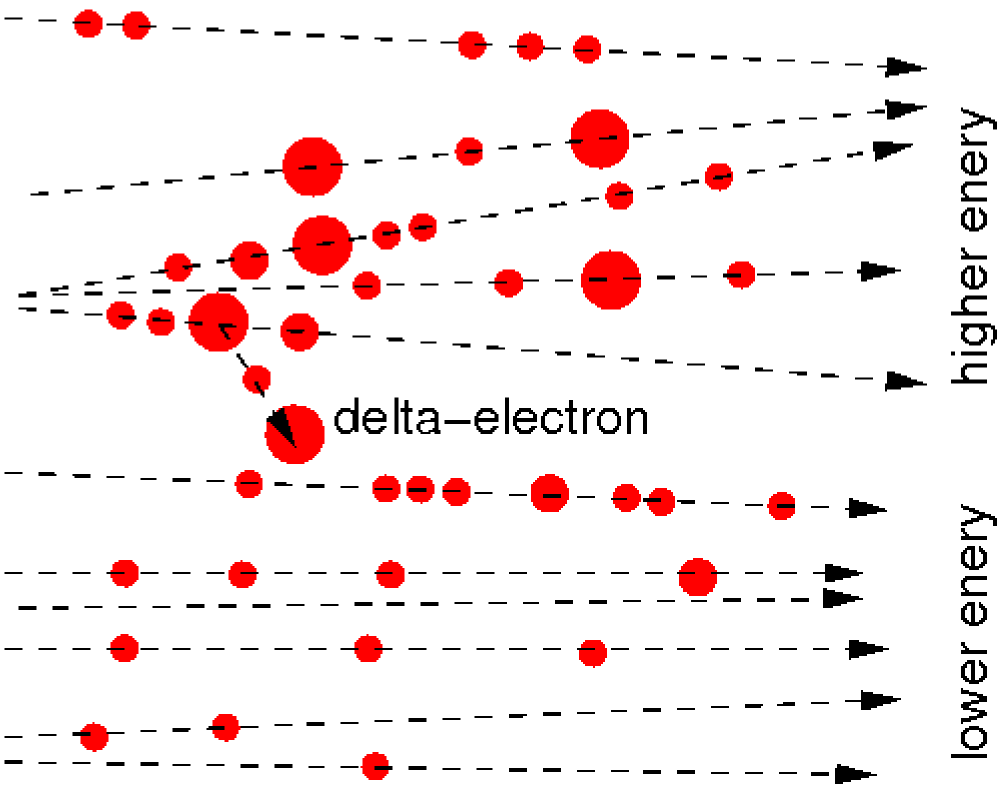


[Bak et al. Nucl. Phys. B288 (1987) 681-716]

- Large sensor thickness averages over more interactions
  - Much less fluctuations (distribution is better described by the Landau function)
  - Thin sensors have a “dangerous” tail towards low signals
- Tail at high signals are mainly from **δ-electrons**

Very thin sensors



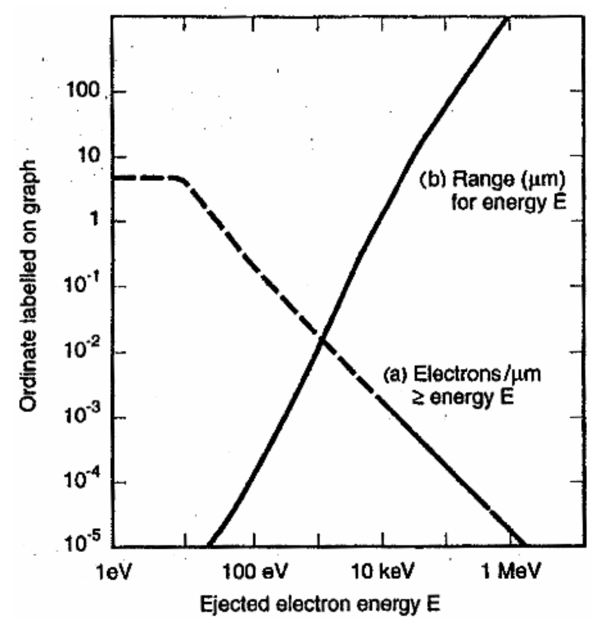


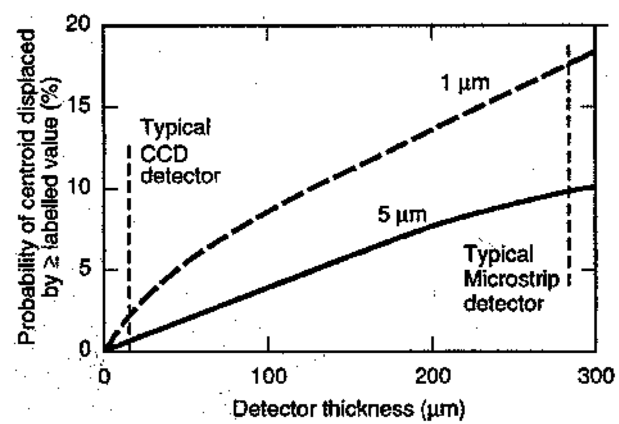
Energy deposition in a very thin silicon detector. Size of the circle is proportional to the primary energy deposition

- Very thin sensors(~1µm) show "quantisation" effects
  - Relevant for (surface) CCDs and partly for MAPS
- There is a probability of having no interaction at all
- **At a thickness of ~10µm peaks are completely averaged out**

High energetic secondaries

- Sometimes very high signals occur (tail of the pulse height distribution)
  - The secondary electron gains enough energy by the collision to become a ionising particle itself (**knock-on** or **δ–electron**)
  - A silicon nucleus is kicked out of the lattice
  - A silicon nucleus is destroyed
- High energetic secondary particles
  - Not parallel to primary particle
  - **Lead to a pull of the track during reconstruction**
- Can be suppressed by
  - Cut on pulse height (loss of efficiency)
  - (Very) thin detectors
  - Detailed analysis of cluster shape (esp. in pixel detectors)



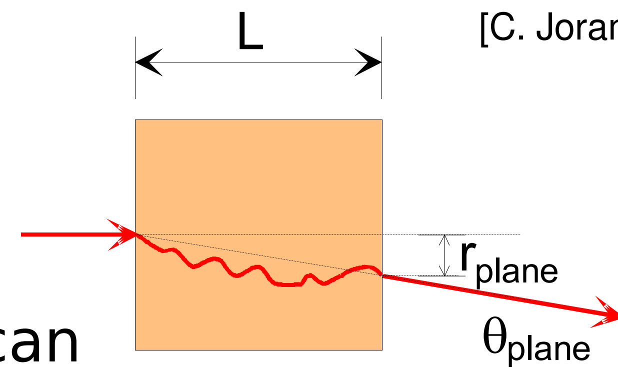


[C. Damerell]

Excursion: Multiple scattering

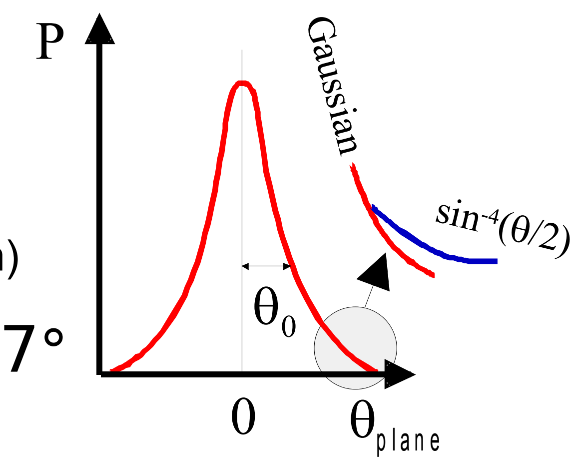
- Coulomb interaction with the Silicon nuclei
  - Transfer of transversal momentum
  - No significant energy loss (nuclei are much heavier than the incoming particle)
- The most probable scattering angle is 0°
- For the central ~98% the distribution can be described by a Gaussian with

[C. Joram]



$$\theta^{rms}_{plane} = \frac{13.6\,MeV}{\beta\, pc}\, z \sqrt{\frac{x}{X_0}}\left[1+0.038\ln\left(\frac{x}{X_0}\right)\right] \approx \frac{0.6\,MeV}{pc}$$

300µm Silicon ($X_{0,Si}$ = 9.36cm)

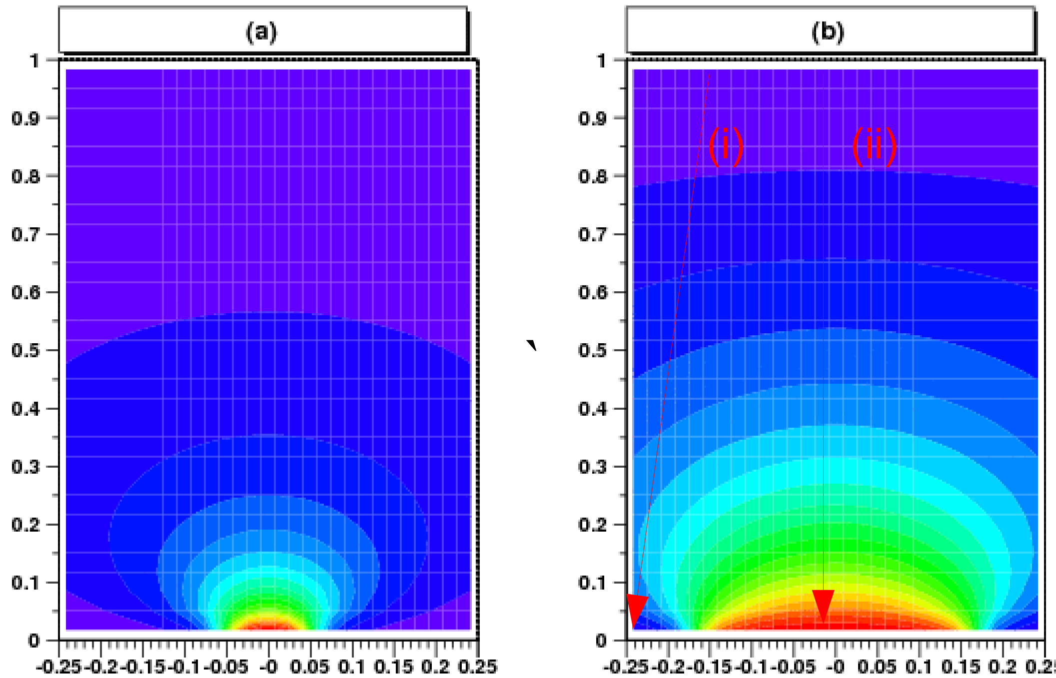


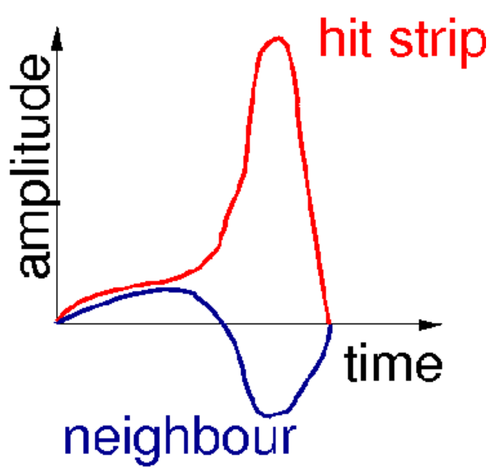
- 1 GeV: rms ~ 0.07°, 100 MeV: rms ~0.7°
- There are non Gaussian “Rutherford” tails
- **Multiple scattering limits spatial resolution of low energetic tracks** (e.g. no test beam possible in πE1: 200MeV/c pions).

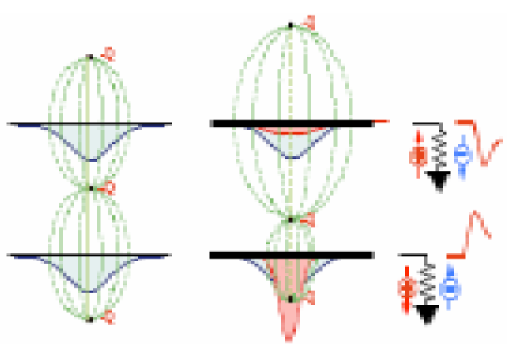
Signal formation

- Signal is induced by the **drift** of charges
- Begins **instantaneously**
- Mechanism can be described by (purely geometrical) **weighting field** (Ramo)
- Signal induced when charge q drifts from x_1 to x_2

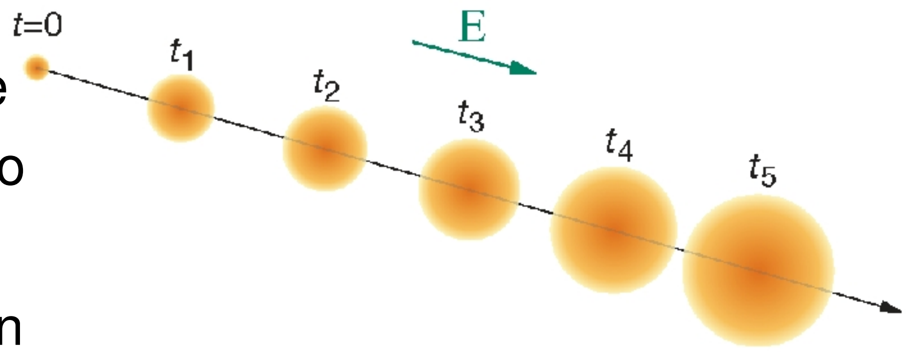
  $Q=q[\phi_w(x_1)-\phi_w(x_2)]$
- Field is calculated by
  - setting electrode under consideration to potential 1 and all others to 0
  - Solving the Poisson equation (without fixed charge)
- The smaller the electrodes the more the field is concentrated
- If q is collected by neighbour (i), a "bipolar" signal is introduced with the total 0
- Charge drifting along (ii) induces q



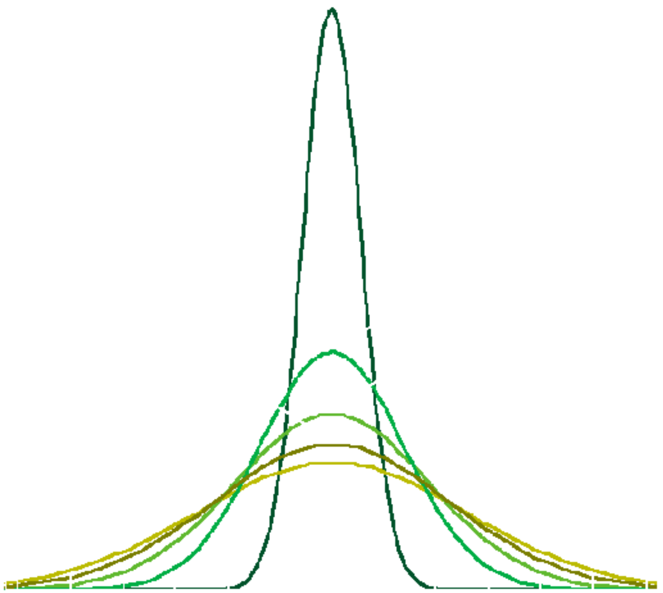




Diffusion

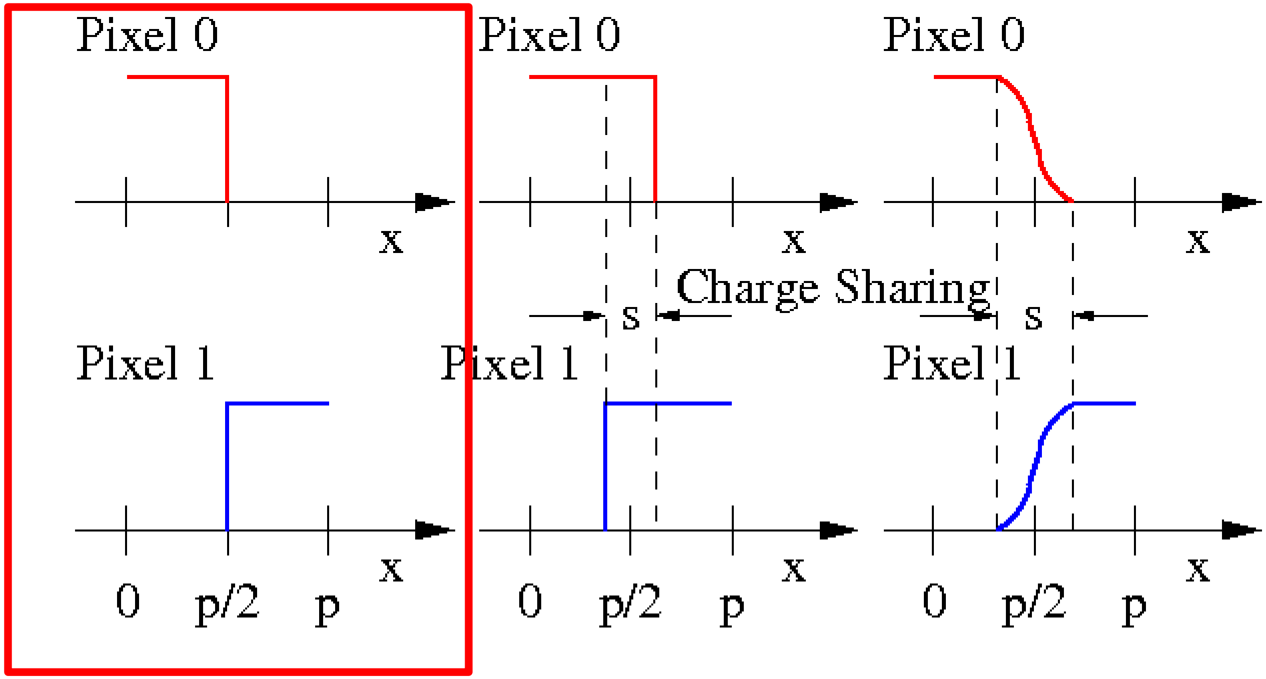


- Carriers in a charge cloud have thermal movement in addition to the drift (**Diffusion**)
- **σ_D = sqrt(2Dt)** with the diffusion constant **$D=kT\mu/e$**
  - t ~1/μ and D~μ
  - σ_D is equal for electrons and holes
- σ (t~10ns) < 10μm
- If σ~pitch, and the noise is low, diffusion increases spatial resolution (charge sharing)



[M. Krammer]

Position Reconstruction (1)



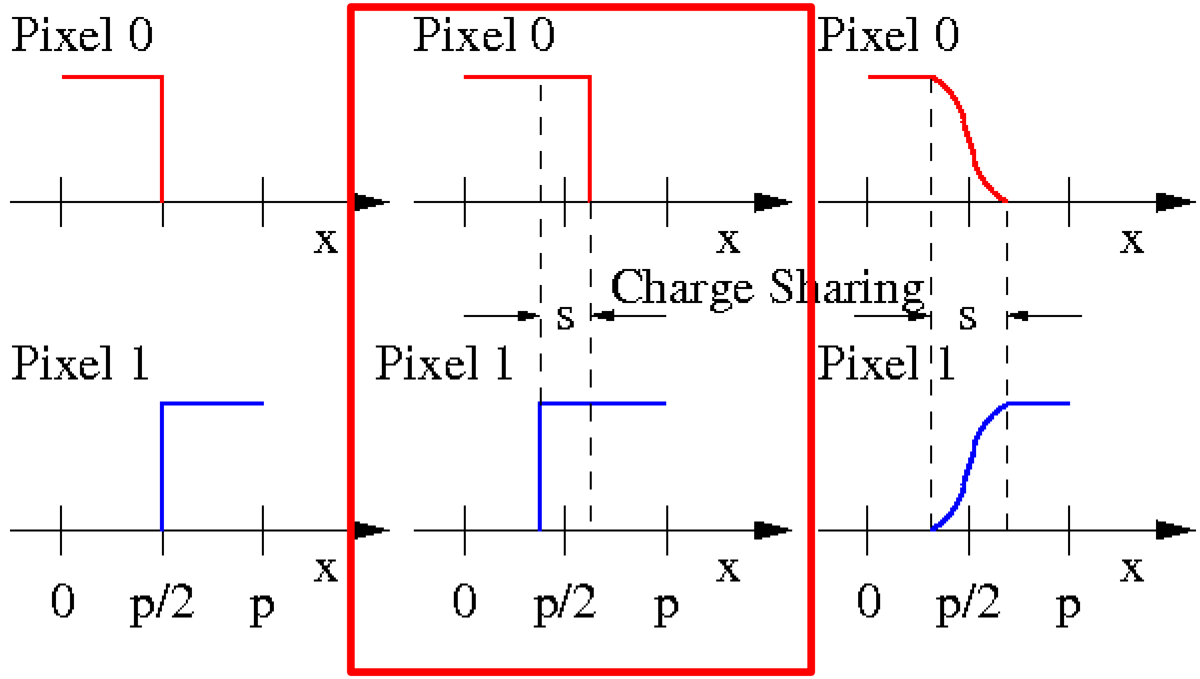
- Easiest option "**binary without charge sharing**"
  - Apply a signal threshold in a way that always (exactly) one strip fires and allocate the hit position to the strip centre
  - Usually done in photon counting devices
  - Average difference between "real" and reconstructed position:

$$\sigma^2 = \frac{\int_{-\frac{p}{2}}^{\frac{p}{2}} (x_r - x_m)^2 D(x_r)\, dx_r}{\int_{-\frac{p}{2}}^{\frac{p}{2}} D(x_r)\, dx_r} = \frac{p^2}{12}$$

D(x) =1 uniform distribution of tracks

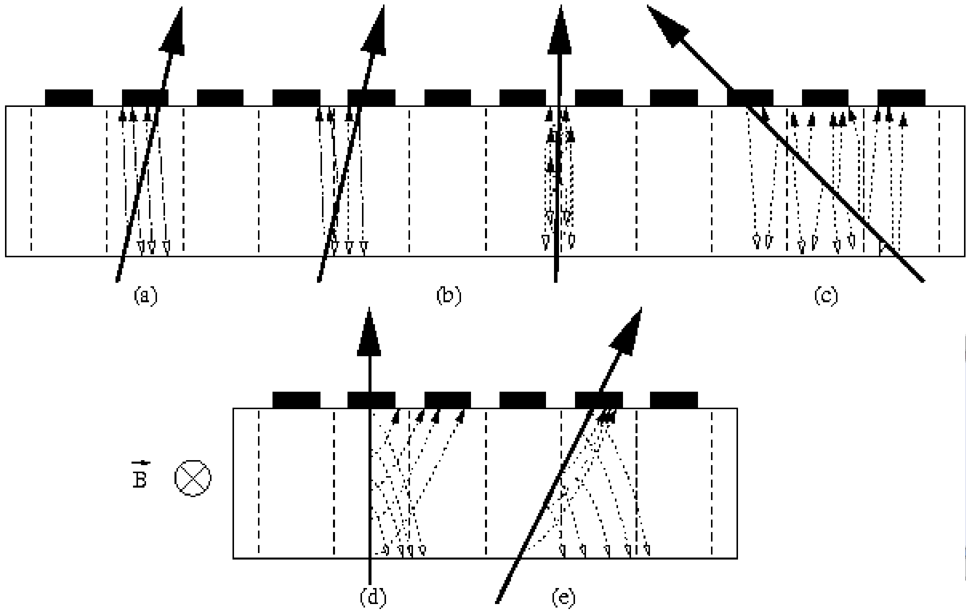
X_m=0 pixel centre

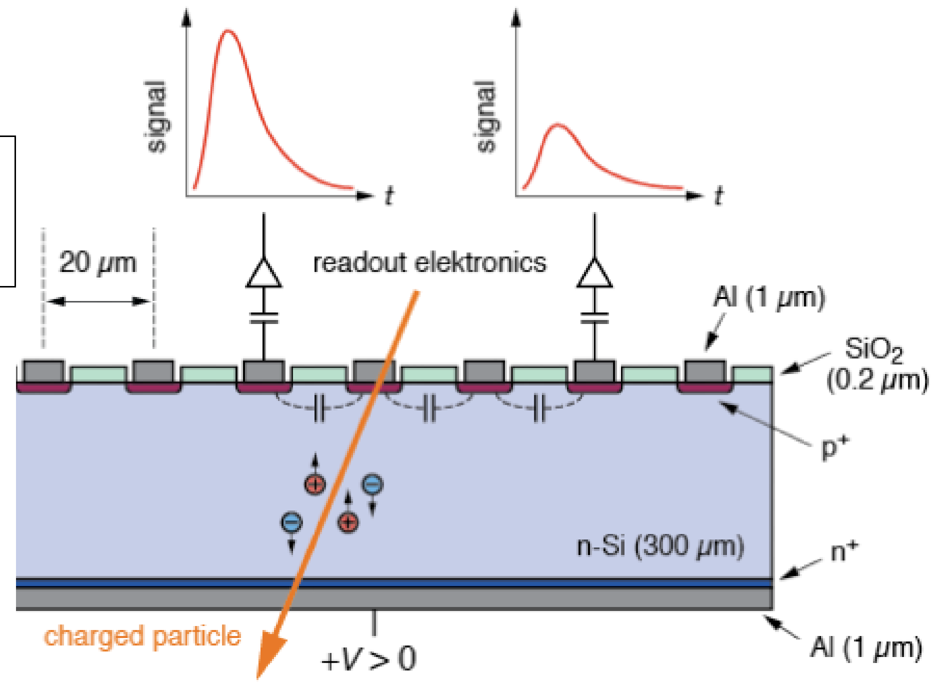
Position Reconstruction (2)



- **Binary with charge sharing**
  - Apply a signal threshold in a way that two neighbouring strips fire and allocate the hit position of those to the centre between strips
  - Average difference between "real" and allocated position:
    - σ = s/sqrt(12) double hits
    - σ = (p-s)/sqrt(12) single hits
  - In optimal case x = p/2 the pitch is **effectively halved**

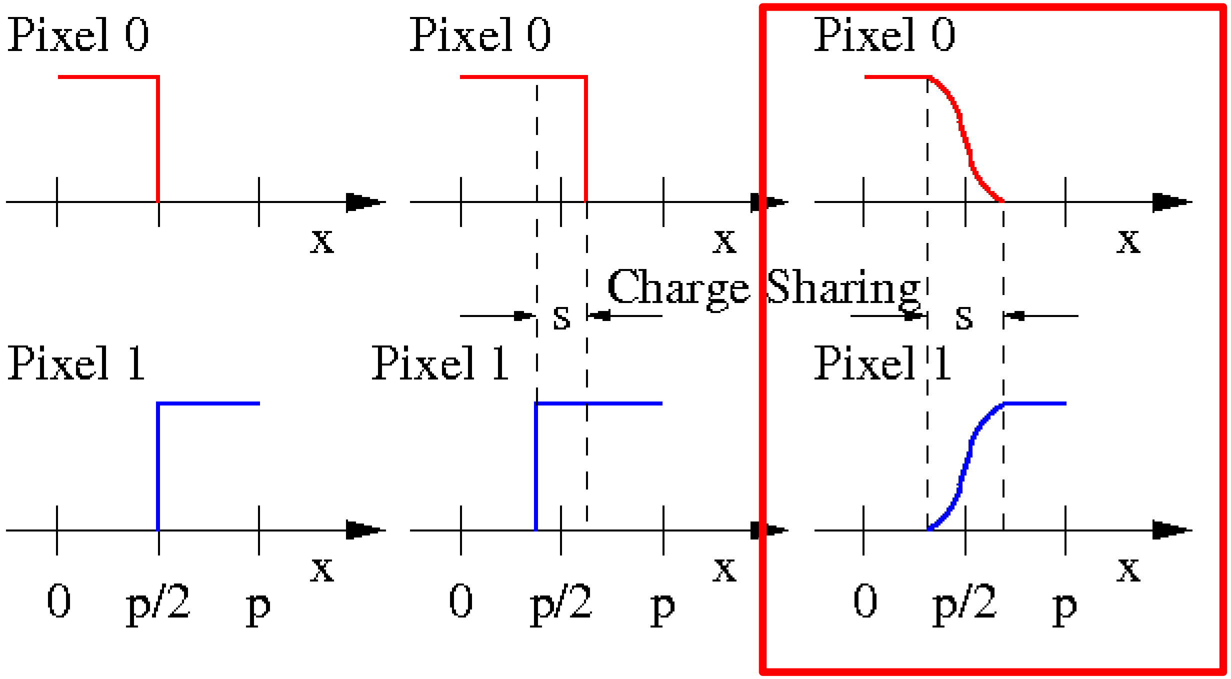
Charge sharing





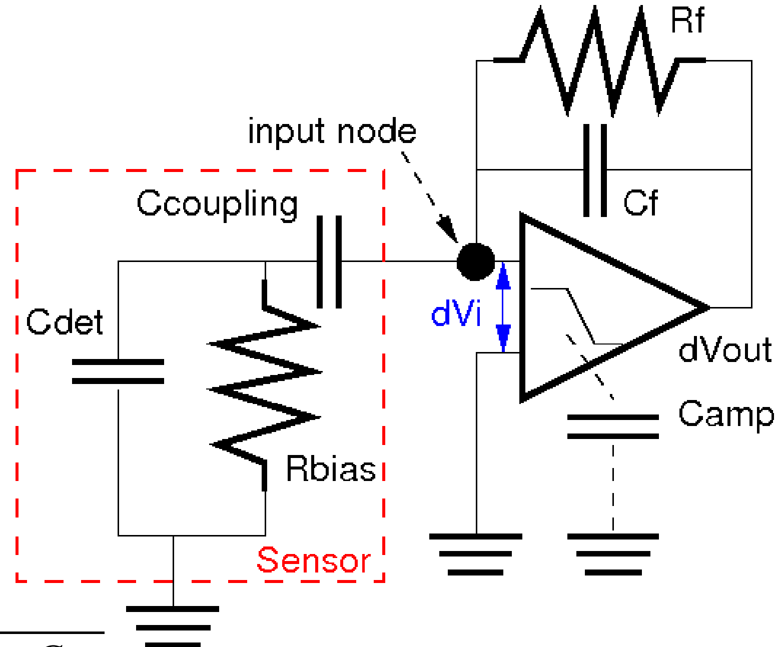
- Usually diffusion is too small to cause charge sharing
- Charge sharing can be caused by
  - **Magnetic field**
  - **Tilt tracks**
  - **Floating inter-strips**
  - δ-electrons (not wanted)

Position Reconstruction (3)



- **Analogue interpolation**
  - If signal charge is shared between neighbours use the pulse height to interpolate between the strips
  - Requires
    - Sufficient fraction of "double hits"
    - Analogue signal processing
    - Good signal to noise ratio

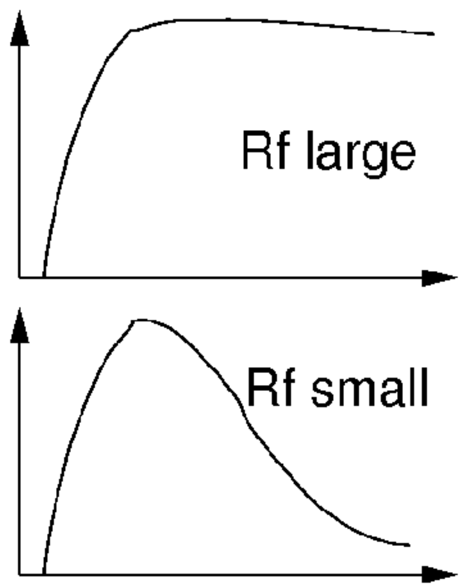
Charge sensitive amplifier



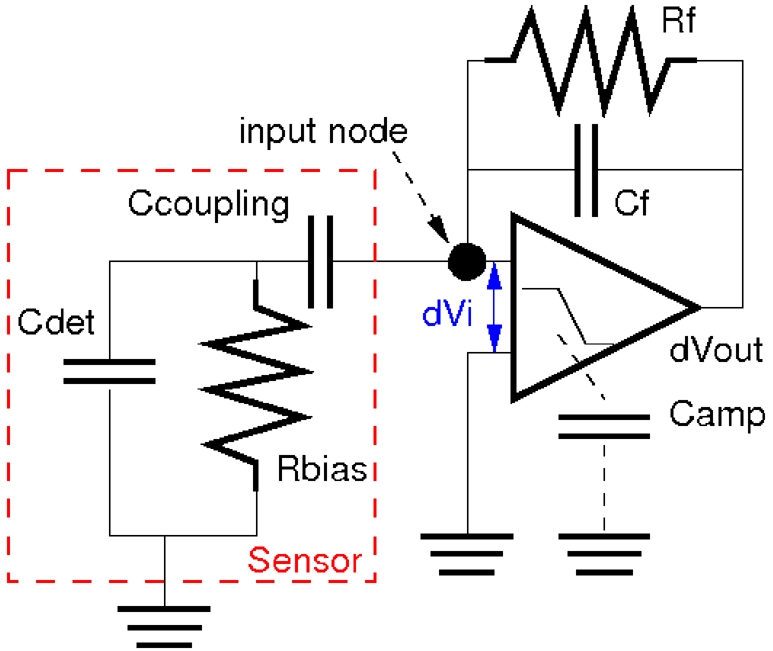
- Charge Q_i is deposited on the input node of an operational amplifier (op-amp)
- Amplifier will pull down the input node to "virtual ground" via feed back capacitor and the output to $-Q_i/C_f$ if gain is infinite
- For gain $-v_0$:

$$\Delta V_i = \frac{Q_i}{\underbrace{C_i + (1+v_0) C_f}_{C_{eff}}} \qquad \Delta V_{out} = \Delta V_i * v_0 = -\frac{Q_i}{C_f} * \frac{1}{1 + \frac{1}{v_0} + \frac{C_i}{v_0 C_f}}$$

- **In order to have a large amplitude one needs to have $C_{eff} >> C_i$ = sum of all other capacitances**
- C_i is usually dominated by the detector capacitance
- $C_{coupling}$ is very large and can be neglected
- R_f determines the return to base line



Amplifier noise



- ΔV_{out} has a noise component
  - Can be calculated (long + complicated)
  - Use the relation from last page to correlate the noise at the output to an equivalent noise charge

$$ENC = \underbrace{C_f \Delta V_{out} \left(1 + \frac{1}{v_0}\right)}_{a} + \underbrace{\frac{\Delta V_{out}}{v_0}}_{b} C_i$$

- **Noise depends not on C_i (which dominated by sensor) but ENC does**
  - C_i influences only the charge amplification
  - The translation from noise voltage into input charge depends on C_i
- **Is the dominant noise factor in strip detectors**
- Values for the CMS strip detector: a=400 e b=60 e/pF

Shot noise

- Leakage current is caused by thermally generated e-h pairs
- The statistical fluctuation of the current is a source of noise
- Assuming an integration time tp and the Poisson statistic:

$$ENC = \frac{e}{2}\sqrt{\frac{It_p}{e}} \approx 107\sqrt{I[nA]\,t_p[\mu s]}$$

- Usually this noise component can be neglected
- In case of radiation damage I increases dramatically
  - Cooling required
- **Long integration times give larger shot noise**

Resistive noise

The thermal movement of charge carriers in resistors results in a noise voltage

- **Bias resistor (R_p)**

$$ENC = \frac{e}{e}\sqrt{\frac{kTt_p}{2R_p}} \approx 772\sqrt{\frac{t_p[\mu s]}{R_p[M\Omega]}} \qquad T=300K$$

- Bias resistor should be high (also to have a long RC time)
- Technological limits

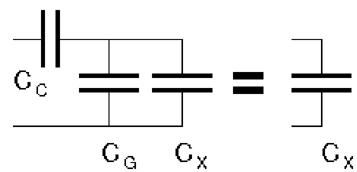
- **Resistance between "origin" of signal and amplifier input R_s**

$$ENC \approx 0.4\, C_i \sqrt{\frac{R_s}{t_p}}$$

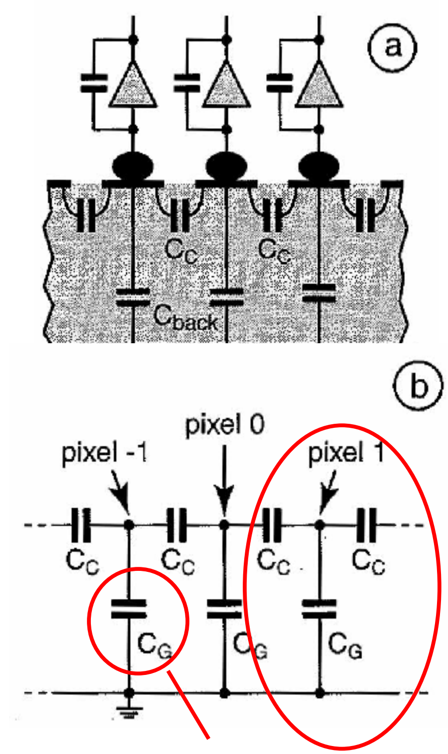
- Favours low resistance e.g. thick Aluminium lines

Cross talk

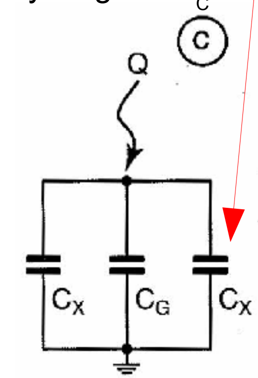
- Charge deposited on one channel can induce parasitic signal on neighbour (cross talk)
- Consider only next neighbours
- Using following approx:
  - $C_G \approx C_{eff}$, $C_G >> C_{back}$, $C_G >> C_C$
- Replace the infinite C-chain by C_X
  - Value can be calculated assuming:



- Release charge Q on node
  - $V_0=Q/(C_G+2C_X)$
  - The charge is divided between the capacitors
    - Fraction on C_G: $q_0 = C_G/(C_G+2C_X)$
    - Rest equally divided between the two C_X
  - Again replace C_X by adding an additional node
    - Fraction on CG in Channel 1: $q_1 = \frac{1}{2}(1-q_0)\frac{C_G}{C_G+C_X} \approx \frac{C_C}{C_{eff}}$
- Typical values of the cross talk are in the order of **2-5%**



C_G is C “to GND” and contains everything but C_C

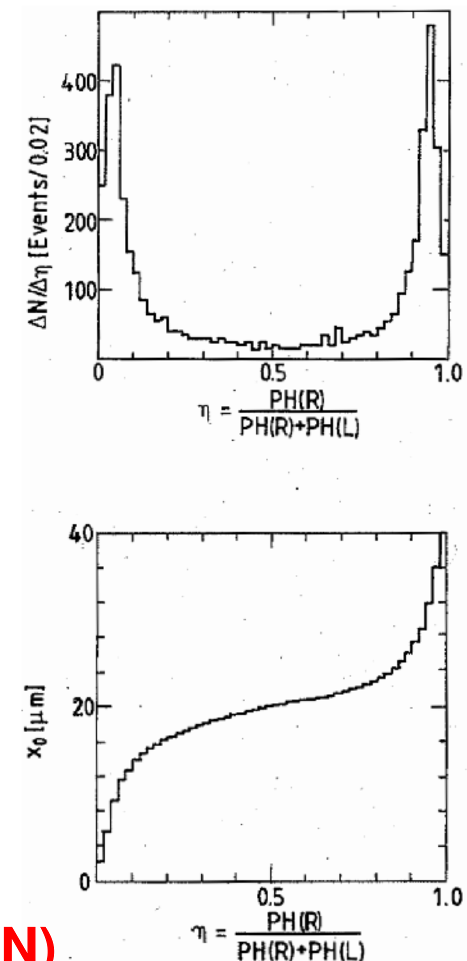


Analogue interpolation

- Define: $\eta = \frac{PH(R)}{PH(R) + PH(L)}$
- Centre of gravity (most simple)
  - x_0 = strip centre + η * pitch
  - Problem: **not all values of η have same probability**
- η-algorithm
  - Use the fact that all values of x_0 are equally probable: Integrate the upper histogram:

$$x_0 = \frac{Pitch}{Number\ of\ events} \int_0^{\eta} \frac{dN}{d\eta} d\eta - strip\ centre$$

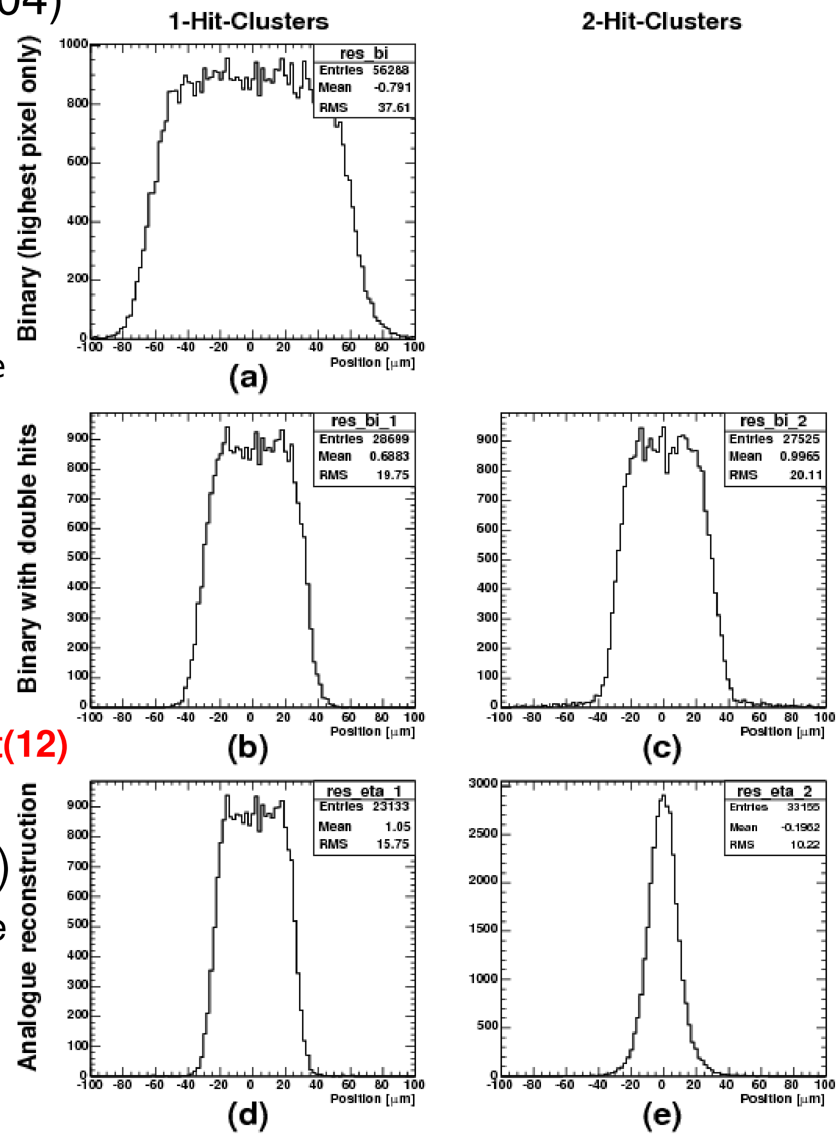
  - Need to loop over all events twice
  - **Resolution ~ pitch / signal to noise ratio (S/N)**



[Belau et al. NIM 214 (1983) 252-260]

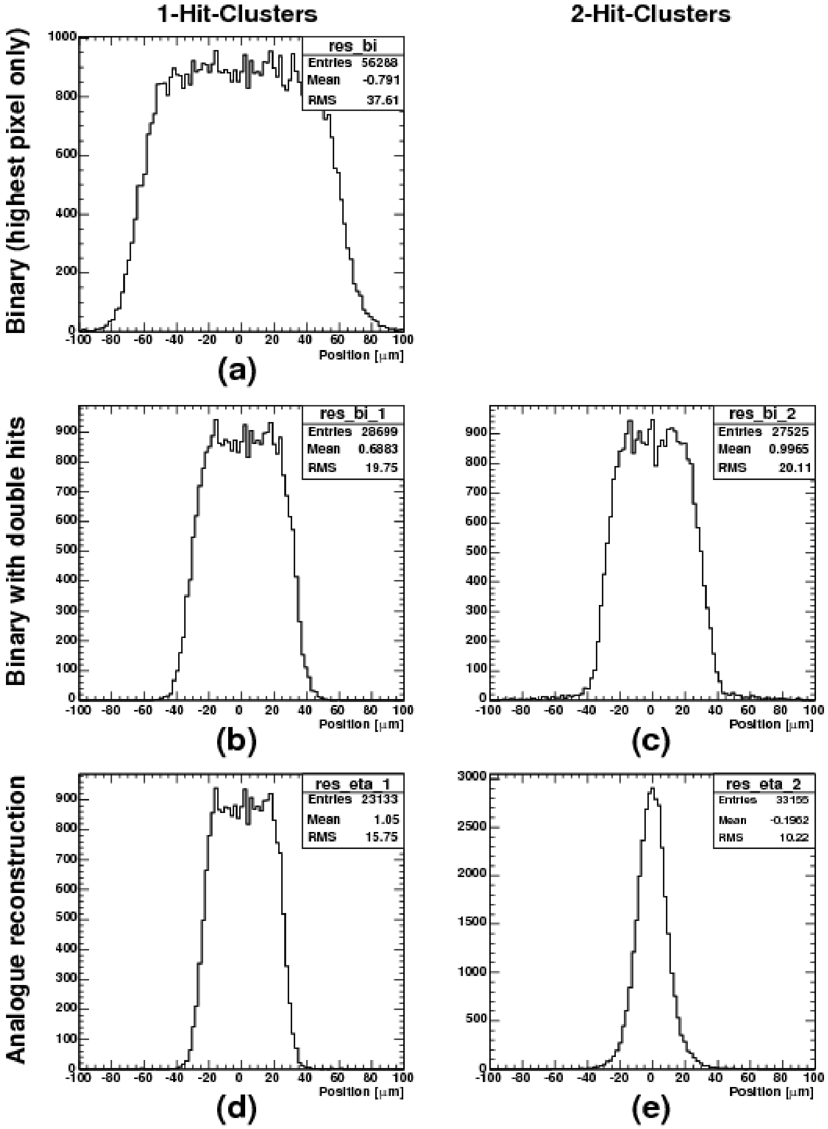
Measurement example

- Data from a test beam (CERN-SPS H2,2004)
  - Position of particle predicted with high precision ~2μm by a 4-layer beam telescope
  - DUT:
    - n-in-n silicon pixel detector
    - Pitch: 125μm, thickness: 285μm
    - Θ_L ~ 17° (B=3T)
    - Analogue ROC with thresholds applied off-line
- Plot difference between predicted and reconstructed position (**residuals**)
- Binary readout, no charge sharing (a)
  - Assign track to the centre of the channel with highest signal
  - **"Box" distribution with RMS 38μm~125μm/sqrt(12)**
  - Edges not sharp: "diffusion"
- Binary readout, with charge sharing (b)+(c)
  - Assign track to channel centre if one is hit, and the middle between channels if two are hit
  - Signal threshold ~3600e
  - # of single and double hit clusters ~equal
  - **Box distributions with RMS~20μm**

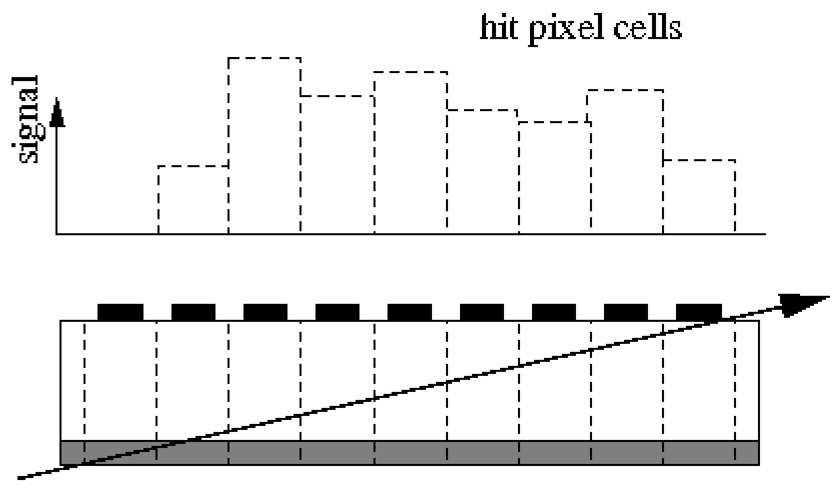


Measurement example (2)

- Analogue readout with η-algorithm (d)+(e)
  - Adjust threshold to minimum (~2000 e)
  - Fraction of double hit clusters reached ~60% (want more best 100% !)
  - Resolution of single hit clusters:
    - Box distribution with **RMS ≈15μm ≈ 0.4×125μm / sqrt(12)**
  - Resolution of double hit clusters
    - Used the η-algorithm
    - RMS ~ 10μm
    - **If Gaussian is fitted σ=9.2μm**
    - Latter neglects non-Gaussian tails mai form δ-electrons



Head-tail for tilted tracks



- In some regions of an experiment (e.g. Innermost pixel layers) have **very long clusters**
- Signal of central pixels of the cluster **does not contain spatial information**
- Fluctuation of signals even spoils position reconstruction
- Due to the long path of the particle in Silicon there is a high probability for δ's
- Use only the first and last channel for position reconstruction (CoG) and **ignore the centre** of the cluster (e.g. done in the CMS barrel pixel)

Summary (the ideal sensor)

- Thin
  - No multiple scattering
  - Little δ electrons (good spatial resolution)
- Thick
  - Large signal, good S/N ratio
  - Good spatial resolution
- Small pitch
  - Good spatial resolution
- Larger pitch
  - Cheap
- Narrow strips
  - Small capacitance (noise and cross talk)
  - Focus charge on one strip (detection efficiency)
- Wide strips/floating inter strips
  - Homogeneous drift field (break down behaviour)
  - Charge less focused (more charge sharing, spatial resolution)

Good luck !