CABLE EFFECTS ON SNR FOR XCAL UPGRADE ON LHCb

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Calorimeter Electronics Upgrade Meeting

OUTLINE

- Cable effects on SNR
- Skin effect
  - Introduction
  - Signal attenuation
  - Impedance after the cable
  - Cable resistance due to skin effect
  - Noise contribution
- Conclusions

Cable effects on SNR

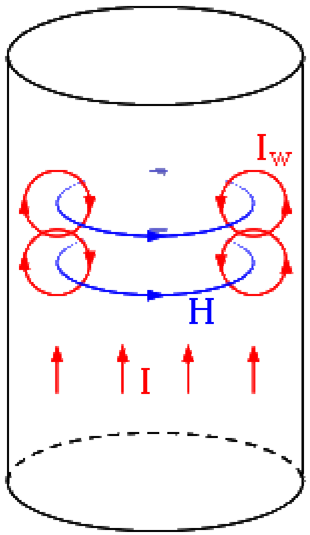
- Cables provide some constraints when fast shaping signals
- Cable effects on SNR:
  - Attenuation due to the skin effect
    - ⇒ long tail in the step response of the cable
    - ⇒ part of the signal is delayed and does not contribute
  - Resistance of the cables
    - ⇒ noise source distributed along the cable

Skin effect: introduction

- Skin effect:
  - Tendency of AC current to distribute itself within a conductor so that the current density near the surface of it is greater than at its core
  - f↑ → R↑
- Define penetration or "skin" depth δ as the distance over which the current falls 1/e of its original value:

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}}$$

- Skin effect is due to the circulating eddy currents cancelling the current flow in the center of a conductor and reinforcing it in the surface



Skin effect: signal attenuation

- Signal attenuation due to skin effect
  - Resistance per unit length R_S: $R_S = \frac{1}{\pi D}\sqrt{\frac{\omega\mu\rho}{2}}$
  - It can be shown that the internal cable impedance becomes:

$$Z_S = R_S + j\omega L_i = \ldots = \frac{\sqrt{\mu\rho}}{\pi D}\sqrt{j\omega}$$

  - Transmission line characteristic impedance for high ω :

$$Z_0 = \sqrt{\frac{Z_S + j\omega L}{j\omega C}} \approx \sqrt{\frac{L}{C}}$$

  - Transfer function of a length of line:

$$\frac{V(x+l,\omega)}{V(x,\omega)} = e^{-\gamma l} = e^{-\frac{\sqrt{\mu\rho}l}{2Z_0\pi D}\sqrt{j\omega}}e^{-\sqrt{LC}lj\omega}$$

Propagation constant $\gamma = \sqrt{(Z_S + j\omega L)j\omega C} \approx \frac{\sqrt{\mu\rho}}{2Z_0\pi D}\sqrt{j\omega} + \sqrt{LC}\,j\omega$

Skin effect: signal attenuation

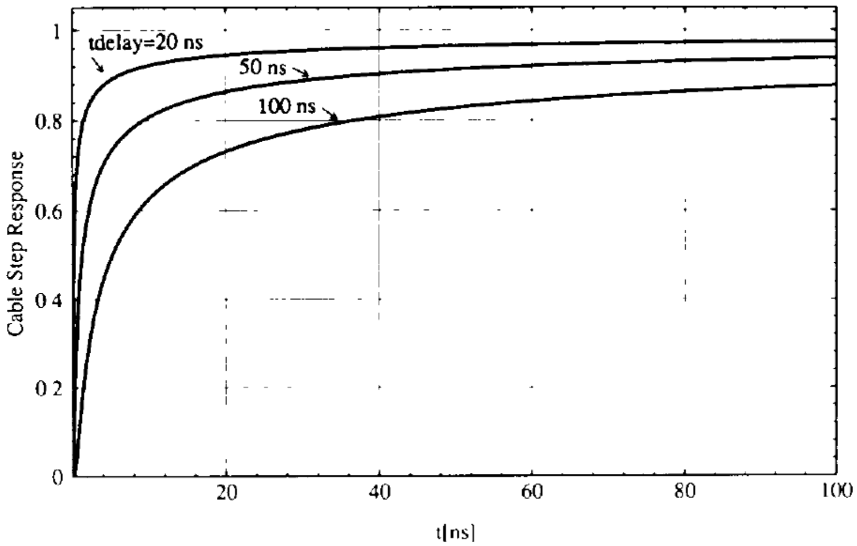
- Signal attenuation due to skin effect (continued)
  - Inverse Fourier transform:

$$h_1(t) = \frac{\tau_0}{2\sqrt{\pi}} t^{-\frac{3}{2}} e^{-\frac{\tau_0}{4t}} U(t) \qquad \text{with} \qquad \tau_0 = \frac{\sqrt{\mu\rho}\, l}{2 Z_0 W}$$

  - And the step response of the transmission line:

$$u_1(t) = erfc\left[\frac{1}{2}\sqrt{\frac{\tau_0}{t}}\right]$$

Introduction of long time constants → strong attenuation of the signal transmitted through the cable

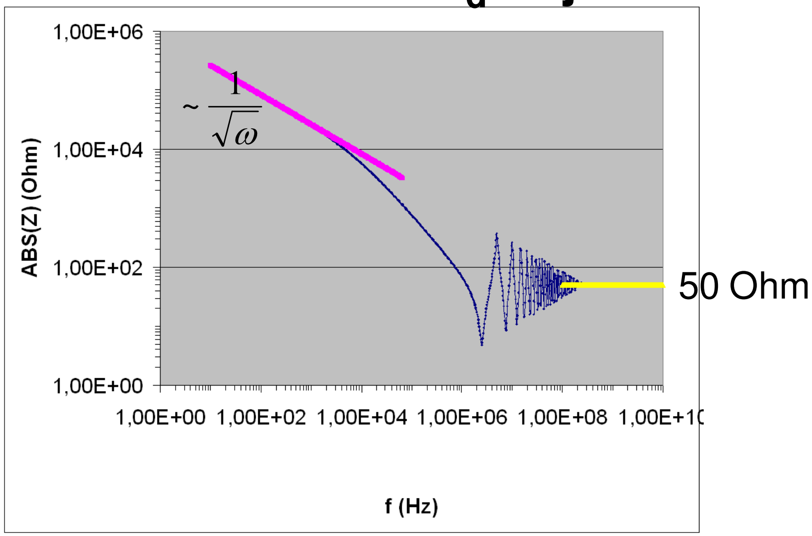


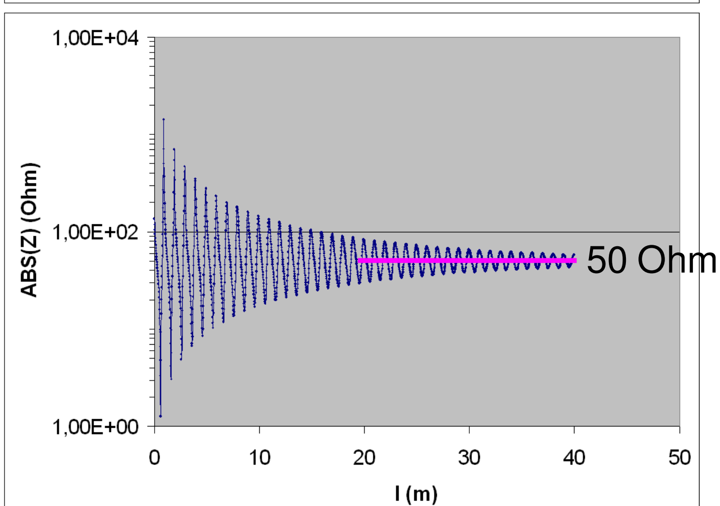
Skin effect: impedance after the cable

- Study two options for the detector impedance with different length cables and frequencies:
  - Detector capacitance: $\mathbf{Z_d=1/jCw}$
  - Clipping line at the PMT output: $\mathbf{Z_d=R_d}$

Skin effect: impedance after the cable

- Impedance seen after x m of cable towards the detector when $\mathbf{Z_d=1/jCw}$:





$$Z(x,\omega) = R_0 \frac{\frac{1}{j\omega C_d} + R_0 tgh(\gamma x)}{R_0 + \frac{1}{j\omega C_d} tgh(\gamma x)}$$

$$Z(x,\omega) \xrightarrow{\omega\to 0} \frac{R_0}{\alpha l} \sim \frac{1}{\sqrt{\omega}}$$

$$Z(x,\omega) \xrightarrow{\omega\to\infty} R_0$$

$$Z(x,\omega) \xrightarrow{x\to 0} \frac{1}{j\omega C_d}$$

$$Z(x,\omega) \xrightarrow{x\to\infty} R_0$$

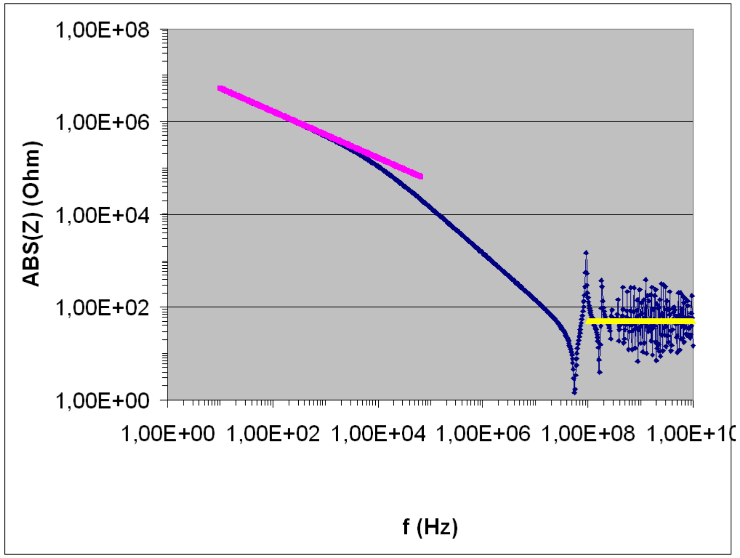
$$Z(x,\omega) \xrightarrow[\omega\to\infty]{x\to 0} \frac{1}{j\omega C_d}$$

$$Z(x,\omega) \xrightarrow[\omega\to 0]{x\to\infty} R_0$$

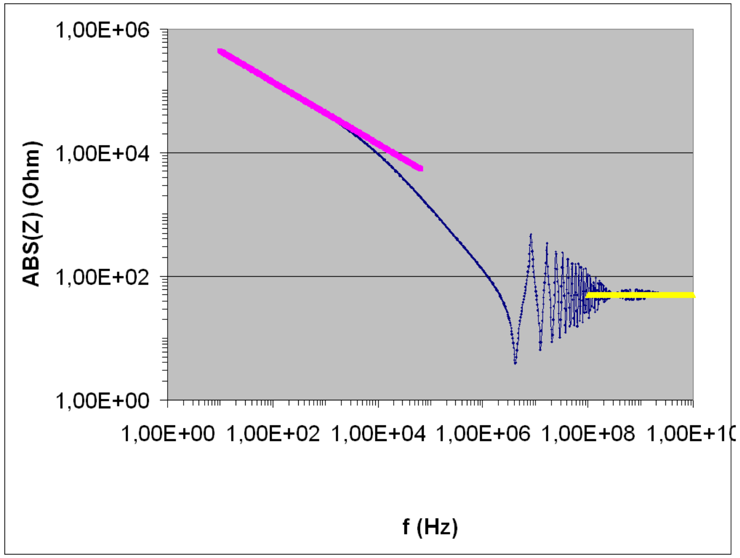
Skin effect: impedance after the cable

- Impedance seen after x m of cable towards the detector when $\mathbf{Z_d=1/jCw}$:

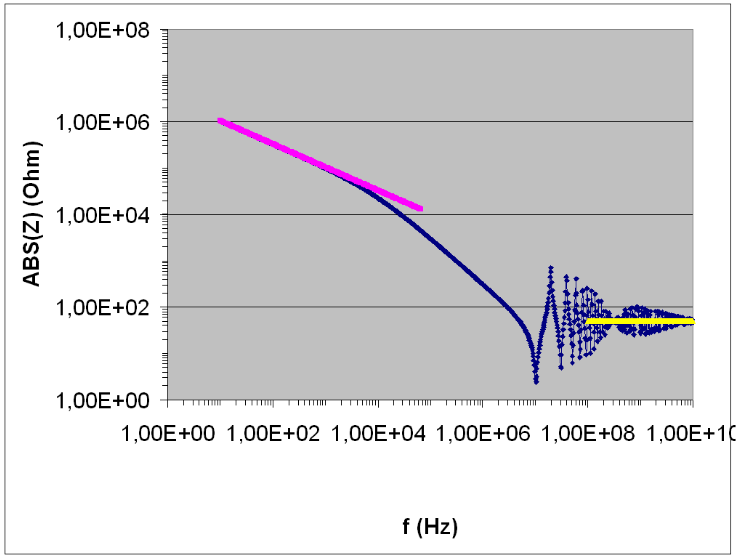
1 m



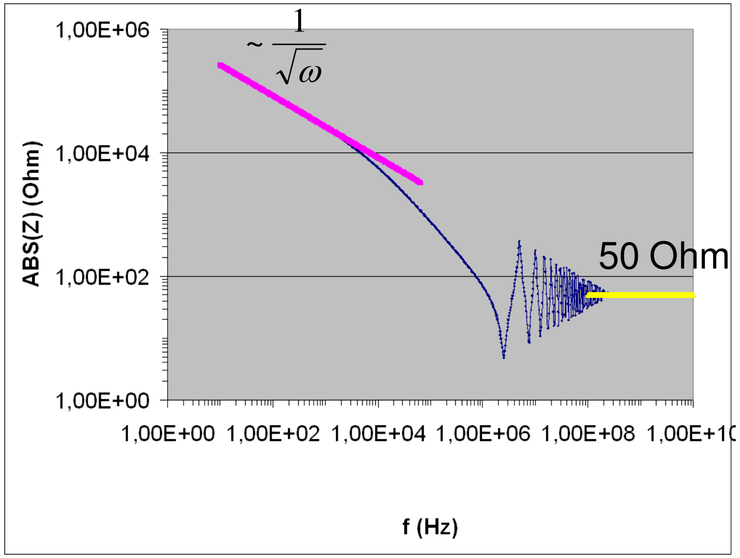
12 m



5 m



20 m



Skin effect: impedance after the cable

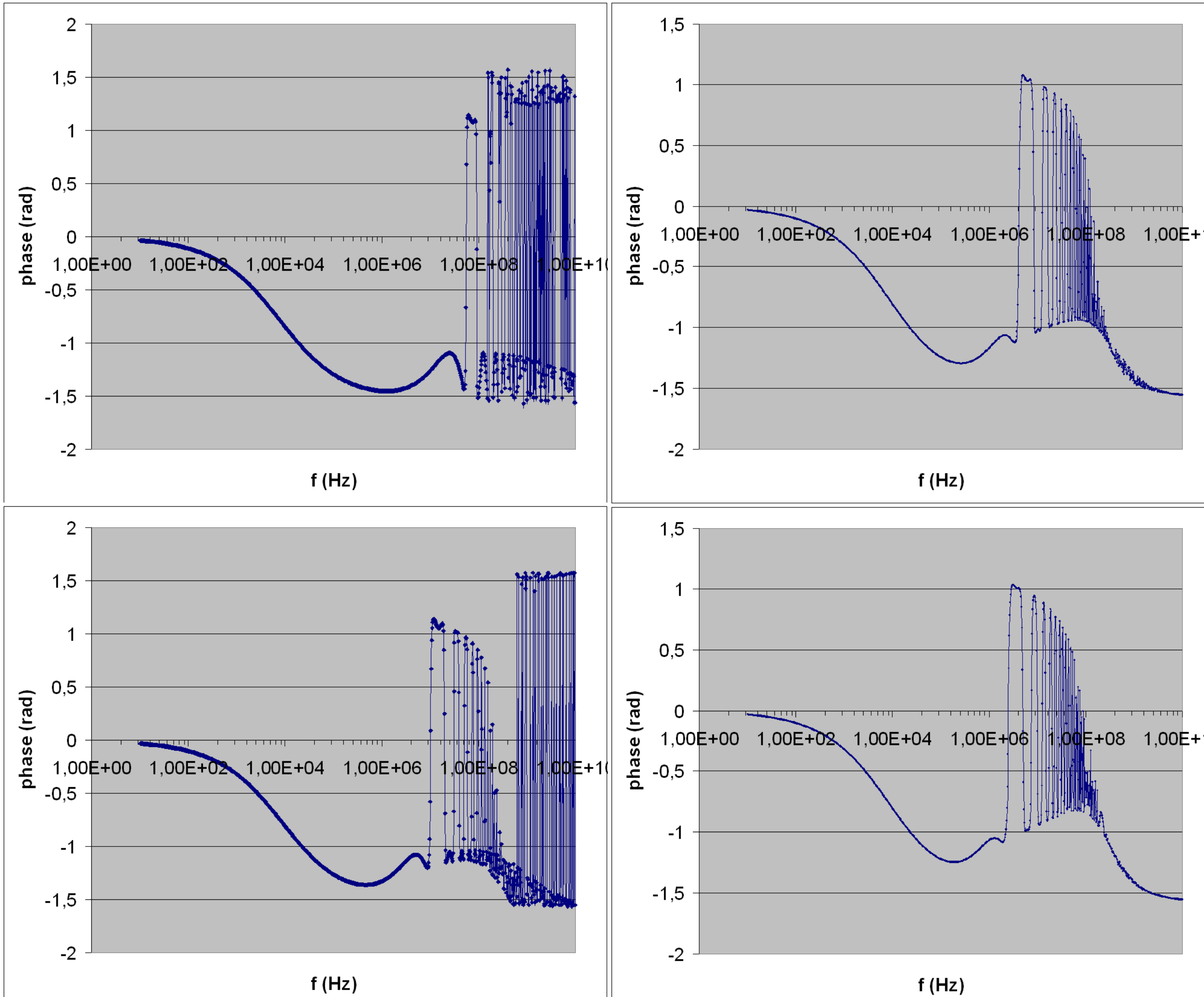
- Phase seen after x m of cable towards the detector when $\mathbf{Z_d=1/jC\omega}$:

1 m

12 m

5 m

20 m

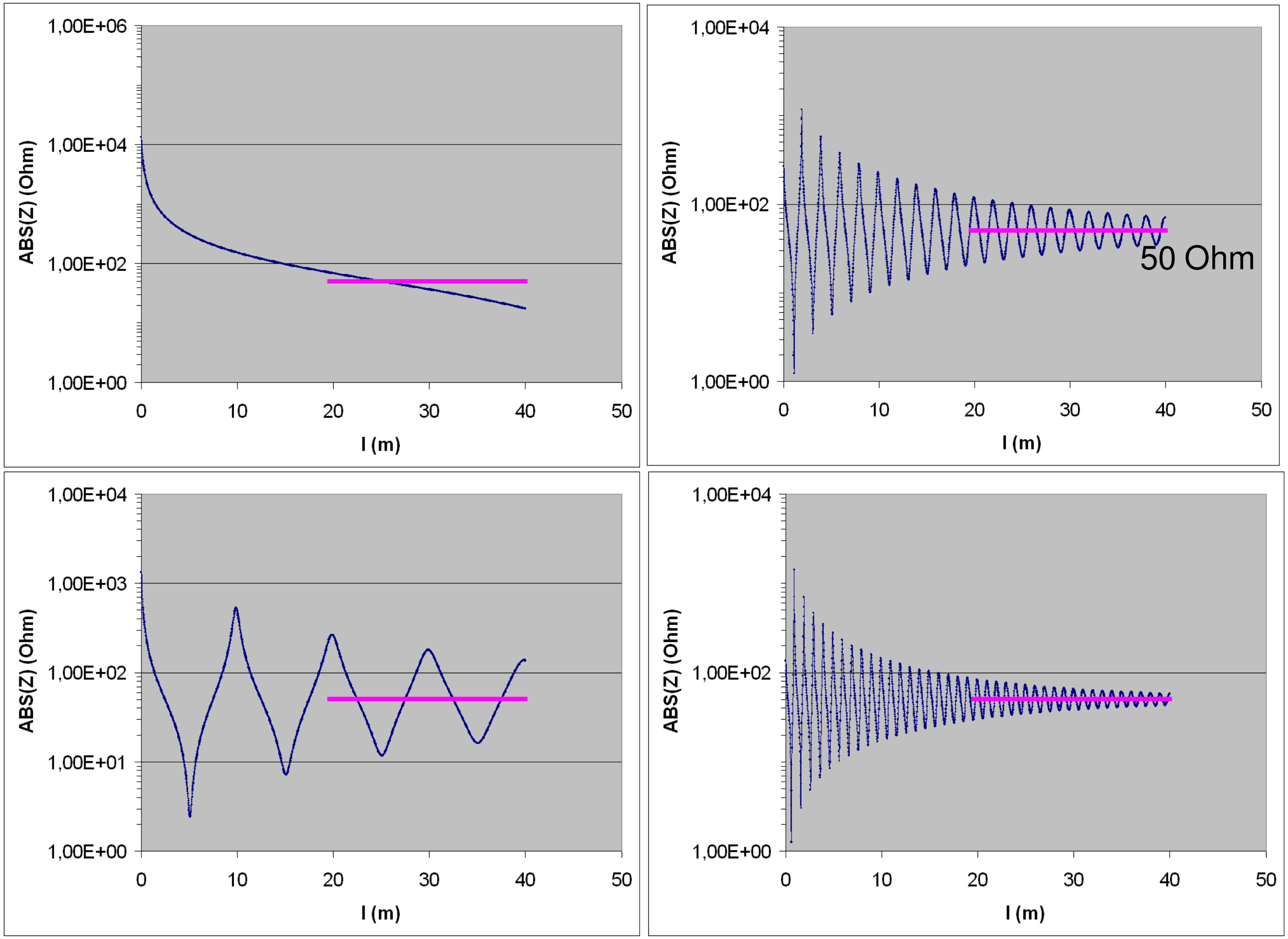


Skin effect: impedance after the cable

- Impedance seen after x m of cable towards the detector when $\mathbf{Z_d=1/jCw}$:

1 MHz

50 MHz



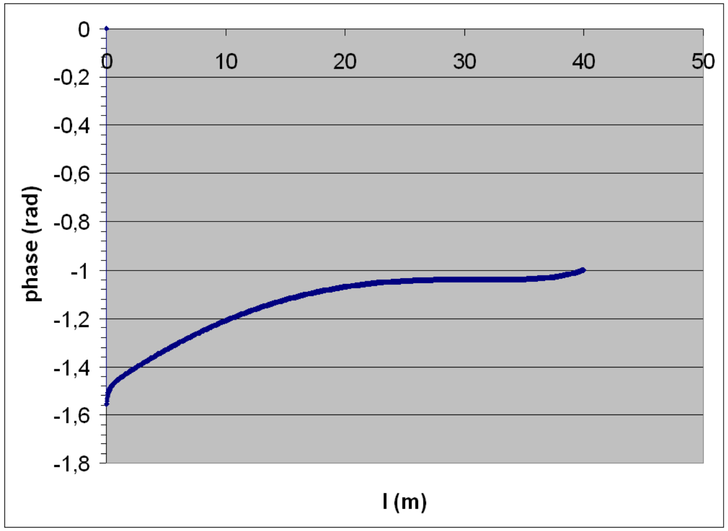
10 MHz

100 MHz

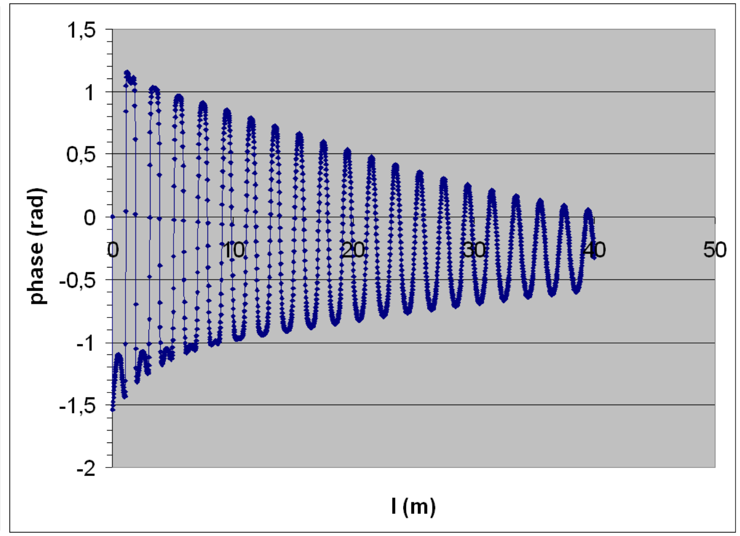
Skin effect: impedance after the cable

- Phase seen after x m of cable towards the detector when $\mathbf{Z_d=1/jCw}$:

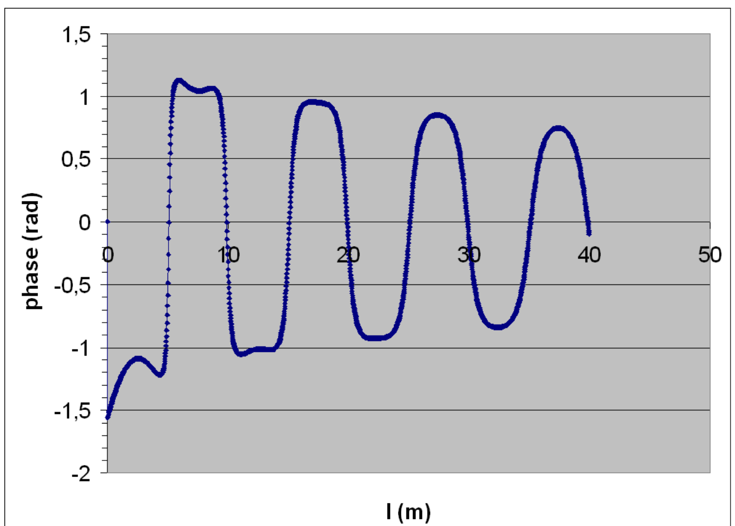
1 MHz



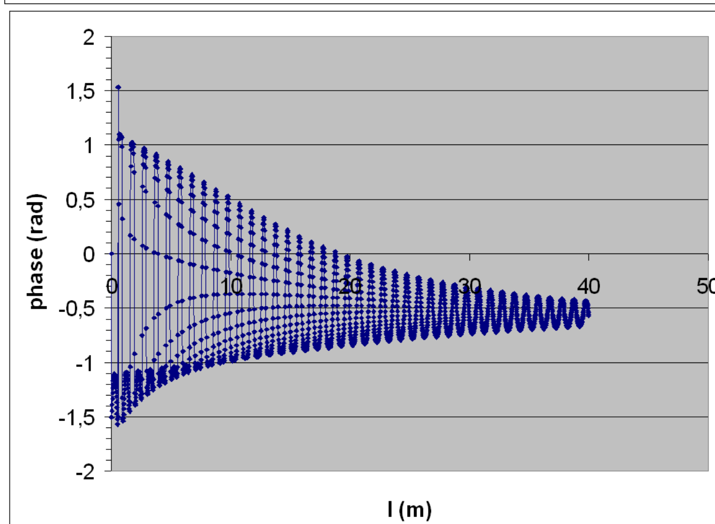
50 MHz



10 MHz

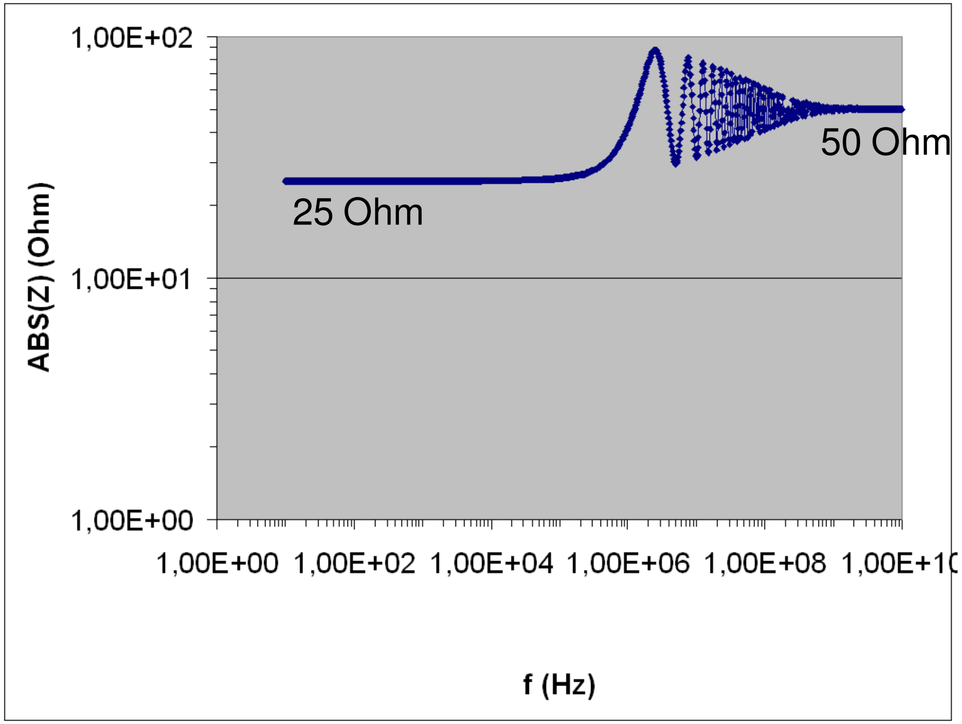


100 MHz



Skin effect: impedance after the cable

- Impedance seen after x m of cable towards the detector when $Z_d = R_d$:



$$Z(x,\omega) = R_0 \frac{R_d + R_0 tgh(\gamma x)}{R_0 + R_d tgh(\gamma x)}$$

$$Z(x,\omega) \xrightarrow{\omega \to 0} R_d$$

$$Z(x,\omega) \xrightarrow{\omega \to \infty} R_0$$

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$$Z(x,\omega) \xrightarrow[]{\substack{x \to \infty \\ \omega \to 0}} R_0$$

Skin effect: impedance after the cable

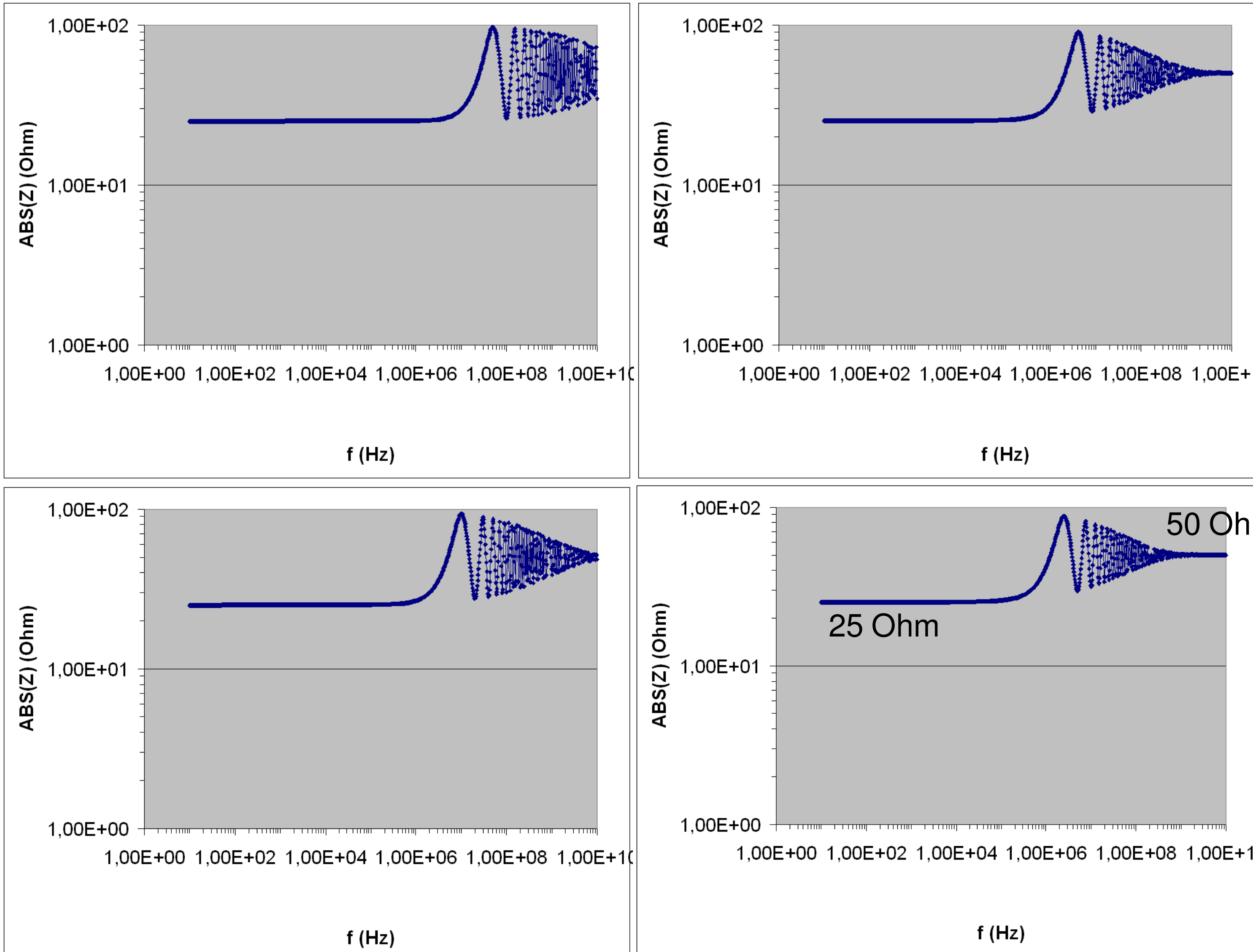
- Impedance seen after x m of cable towards the detector when $\mathbf{Z_d=R_d}$:

1 m

12 m

5 m

20 m



Skin effect: impedance after the cable

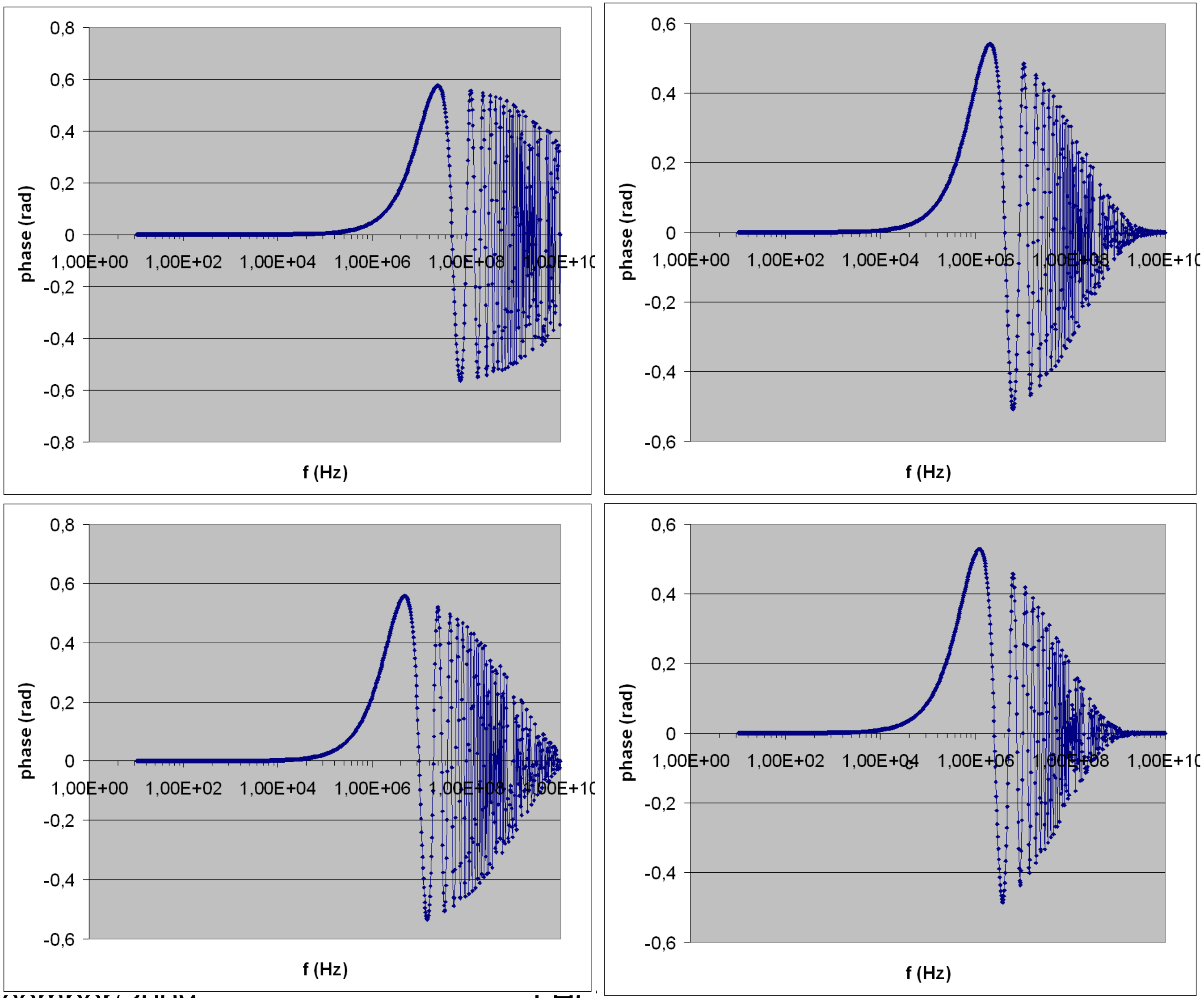
- Phase seen after x m of cable towards the detector when $Z_d = R_d$:

1 m

12 m

5 m

20 m



Skin effect: impedance after the cable

CONCLUSIONS:

- As expected, for
  - Very short cables, $Z \approx Z_d$
  - Long cables, $Z \approx Z_0$
- Impedance seen after 12 m of cable towards the detector when $\mathbf{Z_d=1/jCw}$:

  $f < 1$ kHz $\rightarrow |Z| \sim 1/\sqrt{\omega}$

  1 kHz $< f <$ 2-3 MHz $\rightarrow |Z| \sim 1/j\omega C_d$ (as without the cable)

  2-3 MHz $< f <$ 1 GHz $\rightarrow |Z|$ oscillates between 2 and 200Ω

  $f >$ 1 GHz $\rightarrow |Z| \sim 50\Omega$
- Impedance seen after 12 m of cable towards the detector when $\mathbf{Z_d= R_d}$:

  $f <$ 2-3 MHz $\rightarrow |Z| \sim R_d$ (as without the cable)

  2-3 MHz $< f <$ 1 GHz $\rightarrow |Z|$ oscillates between 25 and 100Ω

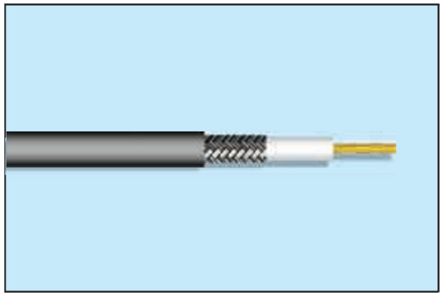
  $f >$ 1 GHz $\rightarrow |Z| \sim 50\Omega$

Skin effect: cable resistance

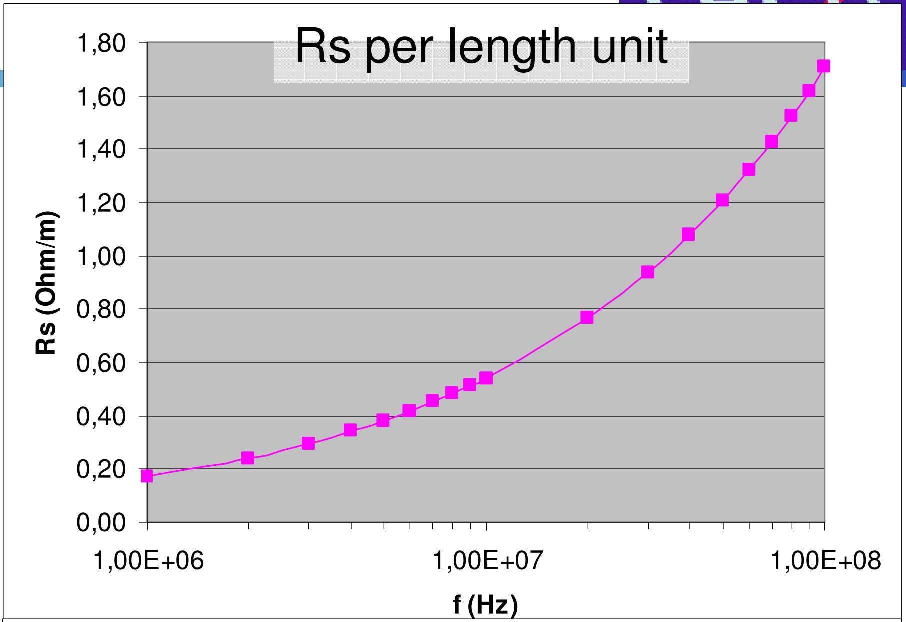
- Resistance per unit length R_S:

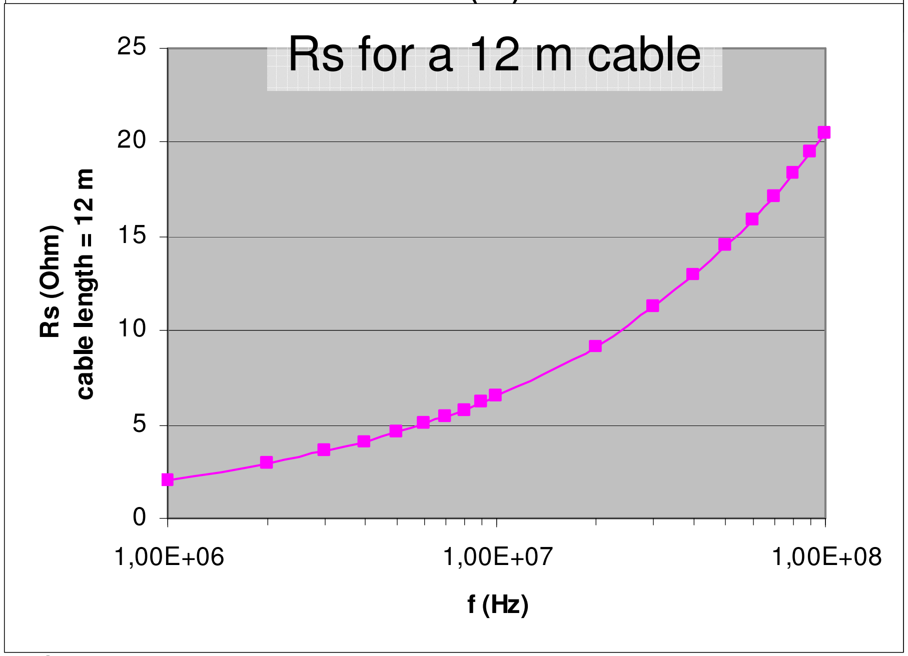
$$R_S = \frac{1}{\pi D}\sqrt{\frac{\omega\mu\rho}{2}}$$

- Skin effect resistor Rs values:
  - Freq high enough to suppose current only on the cable surface
  - D = 0.48 mm
  - $\mu_{Cu} \approx \mu_0 = 1.26 \cdot 10^{-6}$ H/m
  - $\sigma_{Cu} = 5.96 \cdot 10^{7}$ S/m



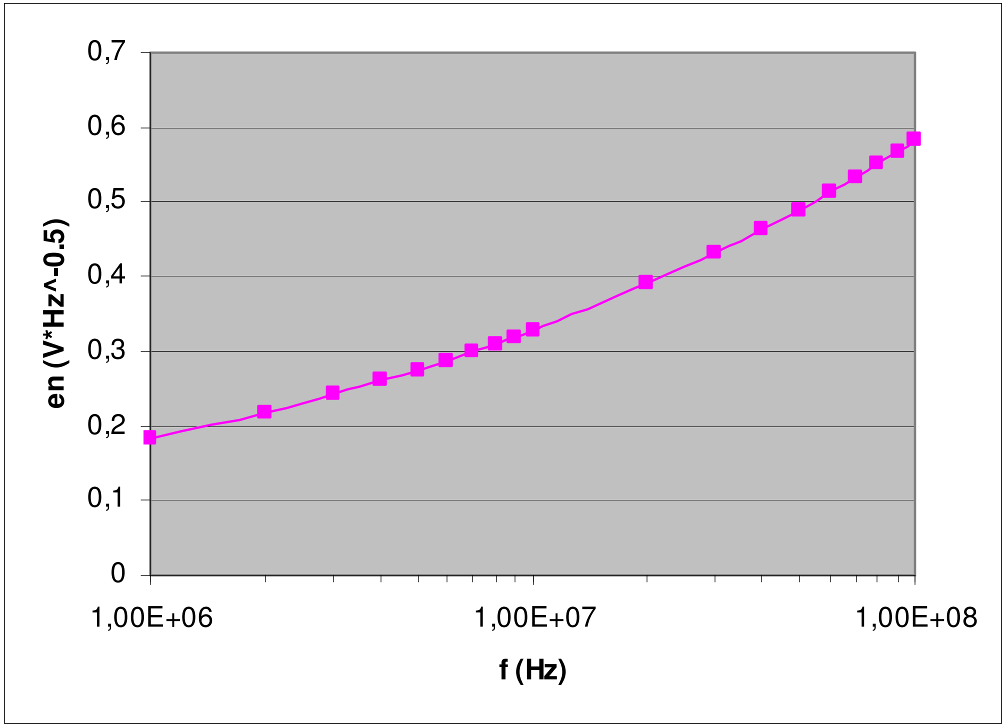
Cable used: coaxial KX3B





Skin effect: noise contribution

- Noise generator per unit length: $\hat{e}_n^{\,2}(f) = 4KTR_S(f)$
  - Propagation constant (rearranged): $\gamma = \dfrac{R_S(\omega)}{2R_0} + j\dfrac{\omega}{v_p}$



Plot e_n (nV/√Hz):

- Temperature: 300 K
- Cable length: 12 m
- Fast shaping times approximation:
  - Skin effect noise ~ single noise generator at preamp input
  - Aproximate R_S at preamp+shaper central frequency

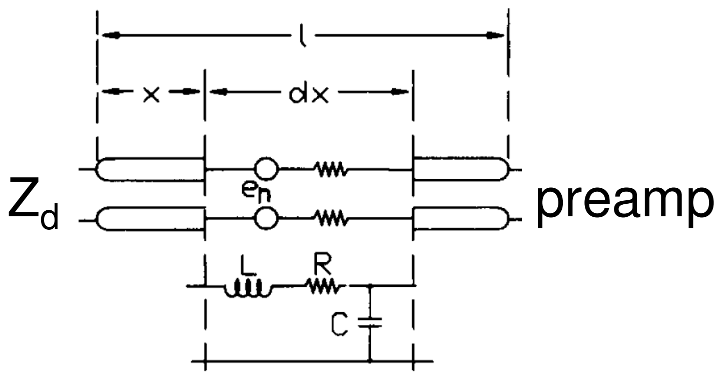
$\Rightarrow R_S \simeq 18\ \Omega$

Noise current

- The noise current per unit length at position x (^ means per unit length):

$$\hat{\imath}_n^{\,2}(x) = \hat{e}_n^{\,2} \frac{1}{\left|Z(x,\omega) + R_0\right|^2} \left|e^{-\gamma(l-x)}\right|^2$$

$$\begin{cases} \hat{e}_n^{\,2} = 4KTR_S \\ R_S = \dfrac{1}{\pi D}\sqrt{\dfrac{\omega\mu\rho}{2}} \\ \gamma = \dfrac{R_S}{2R_0} + j\dfrac{\omega}{v_p} \\ Z = R_0 \dfrac{Z_d + R_0 tgh\gamma x}{R_0 + Z_d tgh\gamma x} \end{cases}$$



  – From which we can obtain i_n^2 integrating over the length of all the cable from the detector to the preamp:

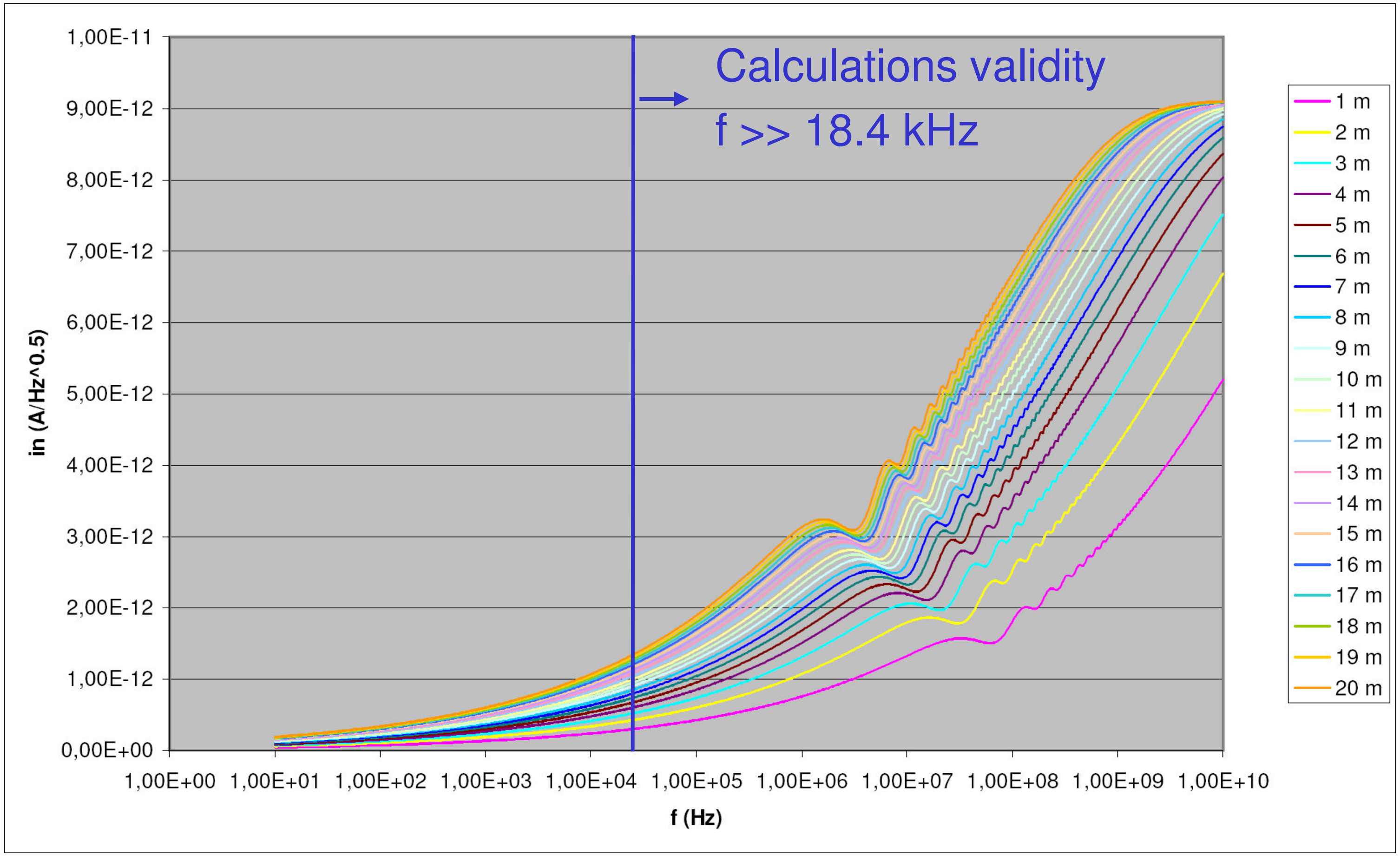
$$i_n^{\,2}(\omega) = \int_0^l \hat{\imath}_n^{\,2}(x)\,dx$$

  – In the case of the clipping line (Z_d=R_d):

$$i_n^{\,2} = \frac{\hat{e}_n^2}{4R_0^{\,2}(R_0+R_d)^2}\left[(R_0+R_d)^2 \frac{1-e^{-4\alpha l}}{2\alpha} + R_0R_d \frac{e^{-4\alpha l} - 2e^{-2\alpha l} + 1}{\alpha} + \left(R_0^{\,2} + R_d^{\,2}\right)e^{-2\alpha l}\frac{\sin 2\beta l}{\beta}\right]$$

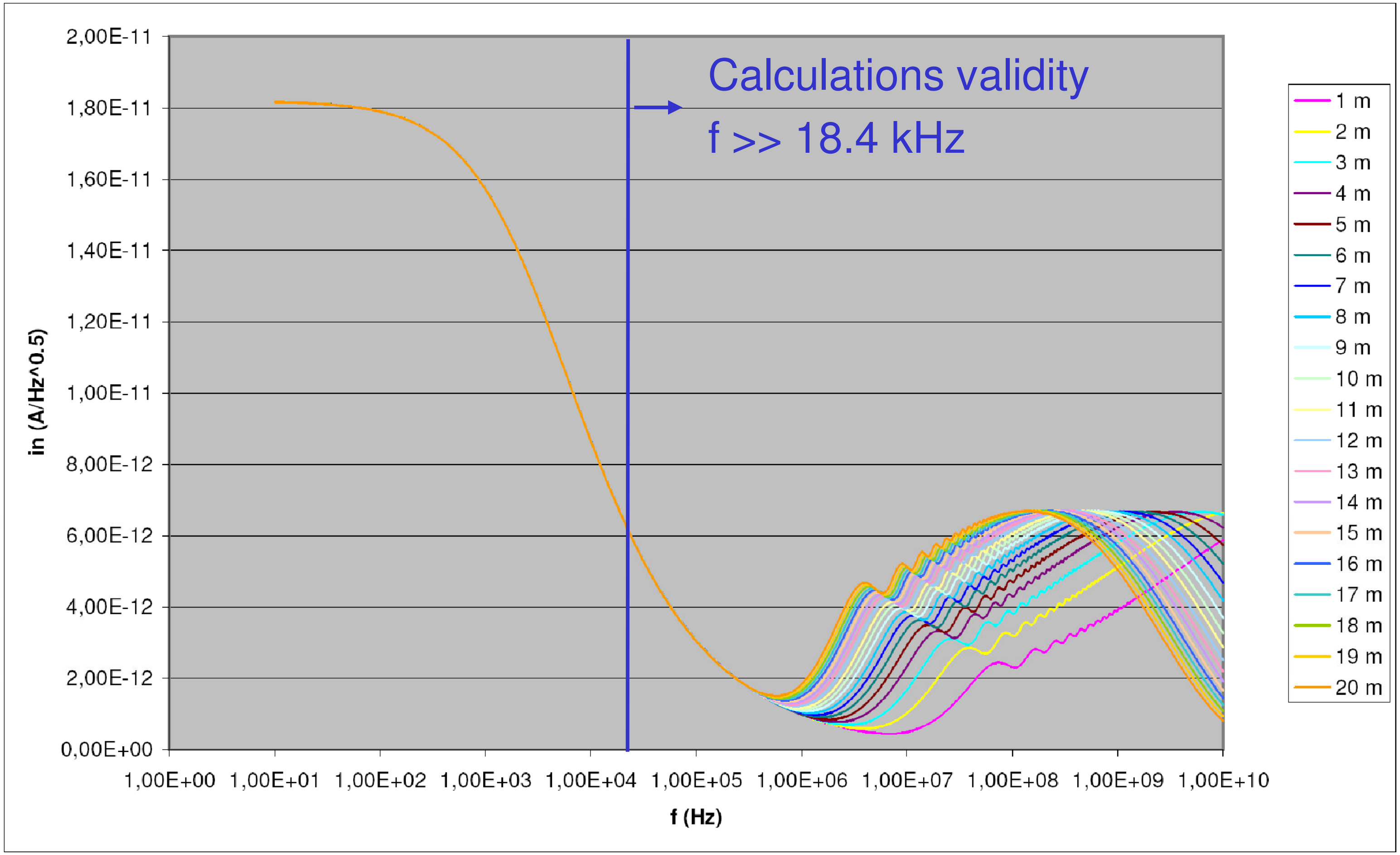
Calculated skin effect noise current

- Noise current generated by the cable In the case of the clipping line (Zd=Rd):



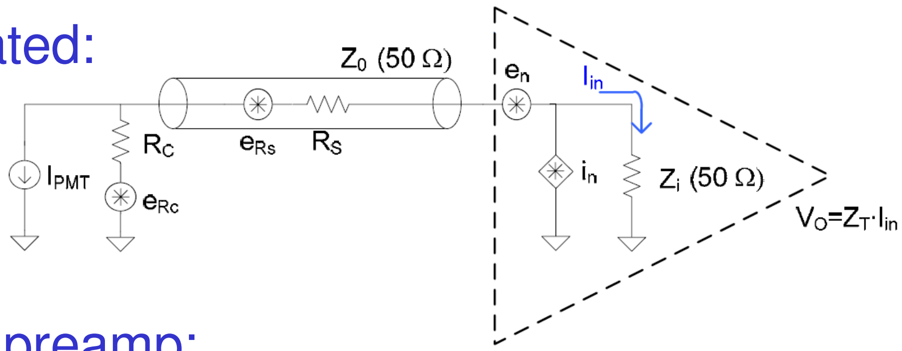
Calculated skin effect noise current

- Noise current generated by the cable in the case without clipping line (Zd=1/jCdw):



Skin effect: PSD calculation

- Case of clipping line and current amp
- On David's talk, all amp PSD is calculated:



$$i_{ni}^2 = i_{ni}^2\Big|_{e_{tRc}} + i_{ni}^2\Big|_{e_{tRs}} + i_{ni}^2\Big|_{e_n} + i_{ni}^2\Big|_{i_n} + \text{cov}\ldots$$

  - Transimpedance gain Z_T=500Ω
- PSD of the cable (skin effect) after the preamp:

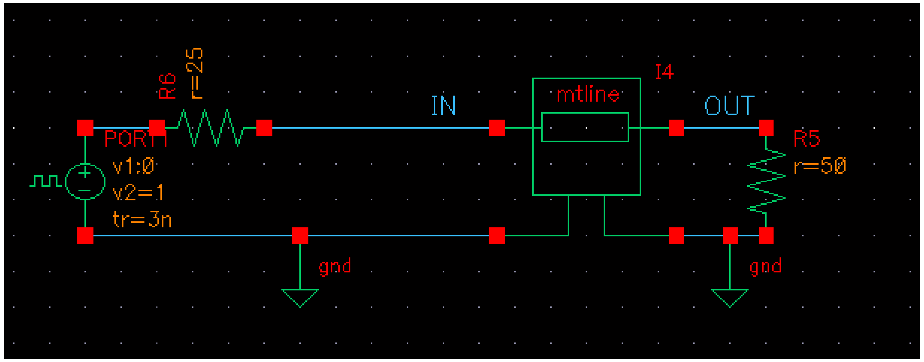
$$e_{no}(cable) = Z_T i_n$$

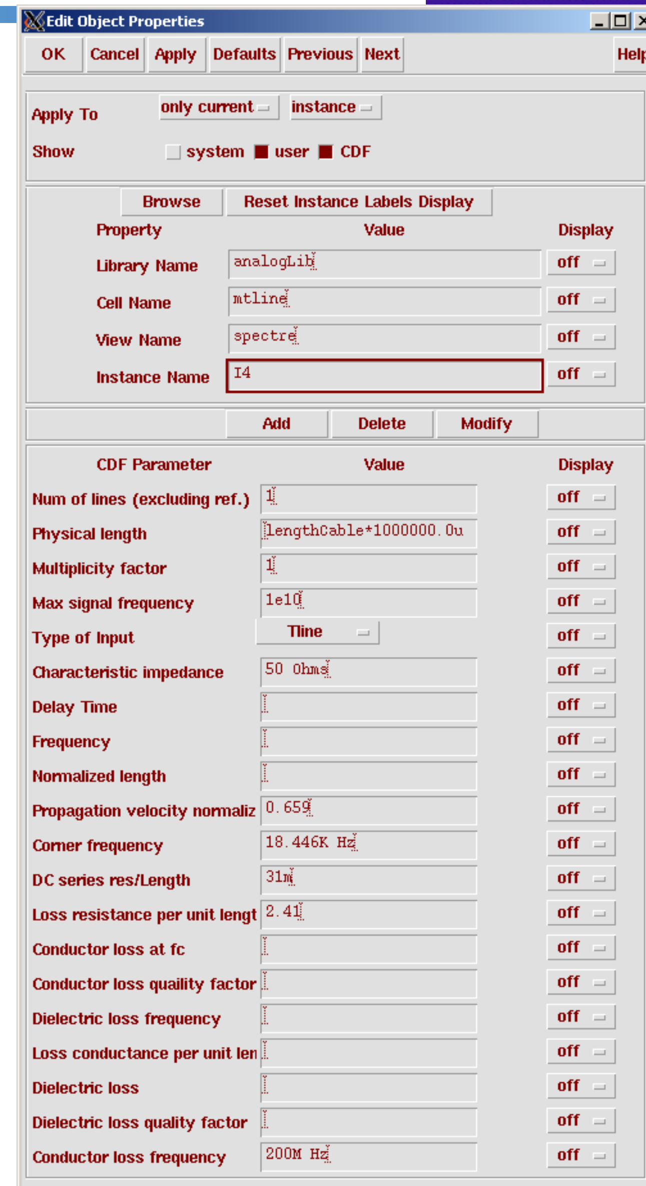
  - We can use the previous result (slide 16), or
  - A lumped resistor at about 18Ω:

$$i_{ni}{}^2\Big|_{e_{tRs}} = \frac{4KTR_S}{\left|R_S + Z_0\right|^2}$$

Skin effect simulated noise

Cadence Spectre simulation circuit with mtline:

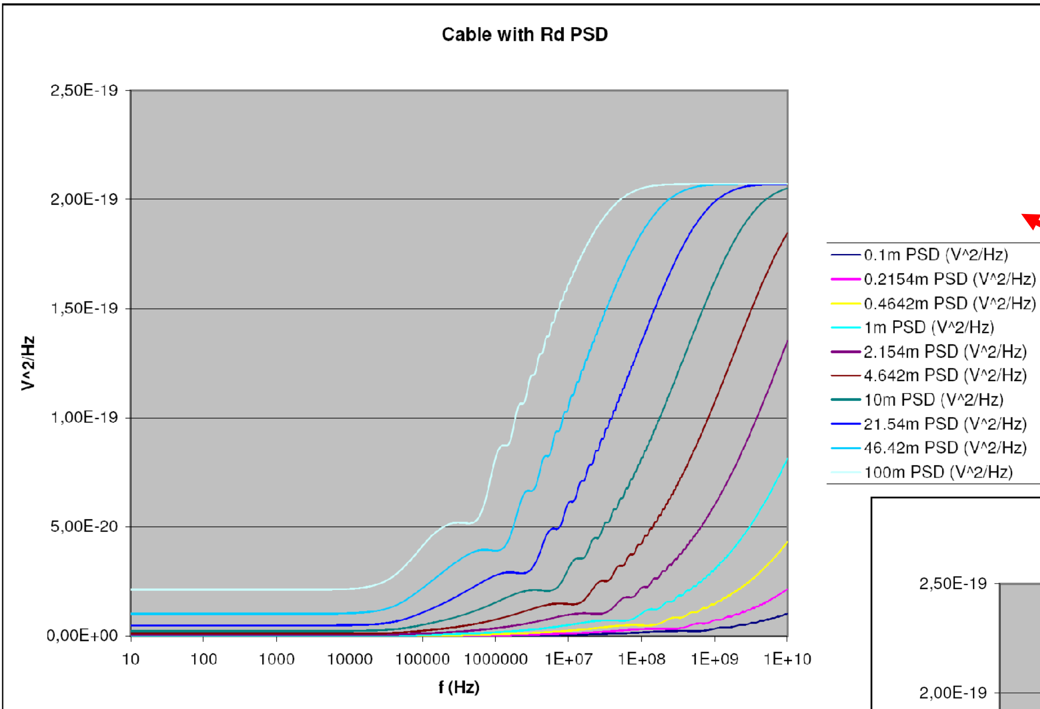


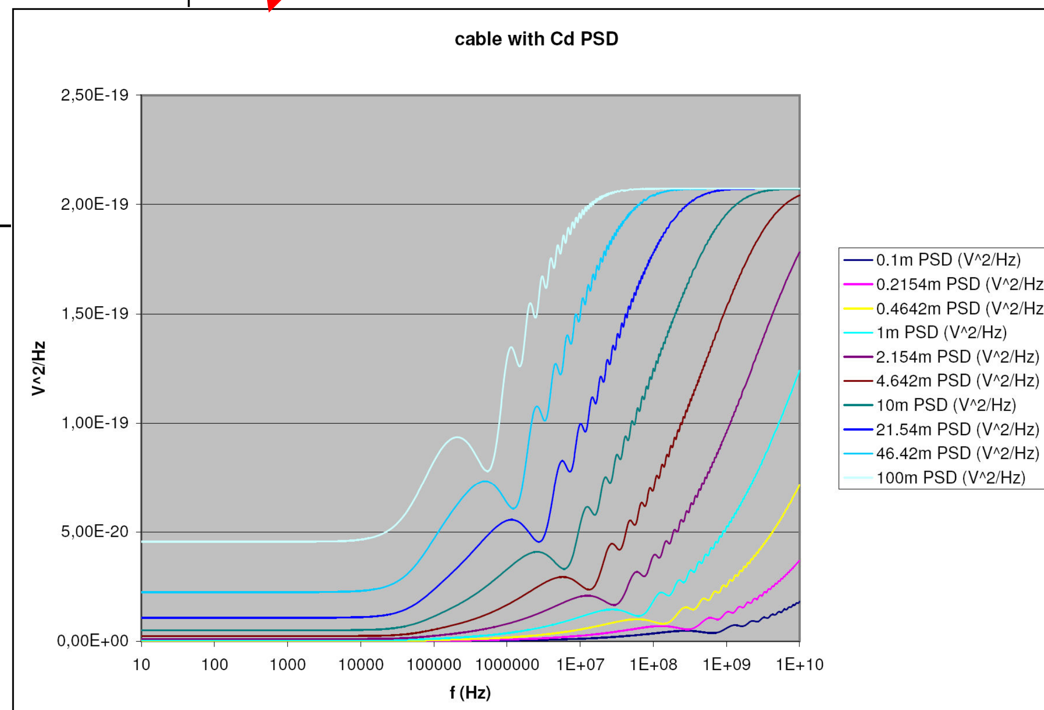


Parameter	Name	Value
Cable physical length	l	12 m
Normalized velocity	v	0.659 c
Corner frequency: f at which skin depth = conductor's width	f_{corner}	18.446 kHz
DC series resistance per unit length	R_{DC}	0.031 Ω/m
Conductor loss measurement frequency	f_c	200 MHz
Conductor series resistance per unit length at f_c	R_S	2.411 Ω/m

Skin effect simulated noise

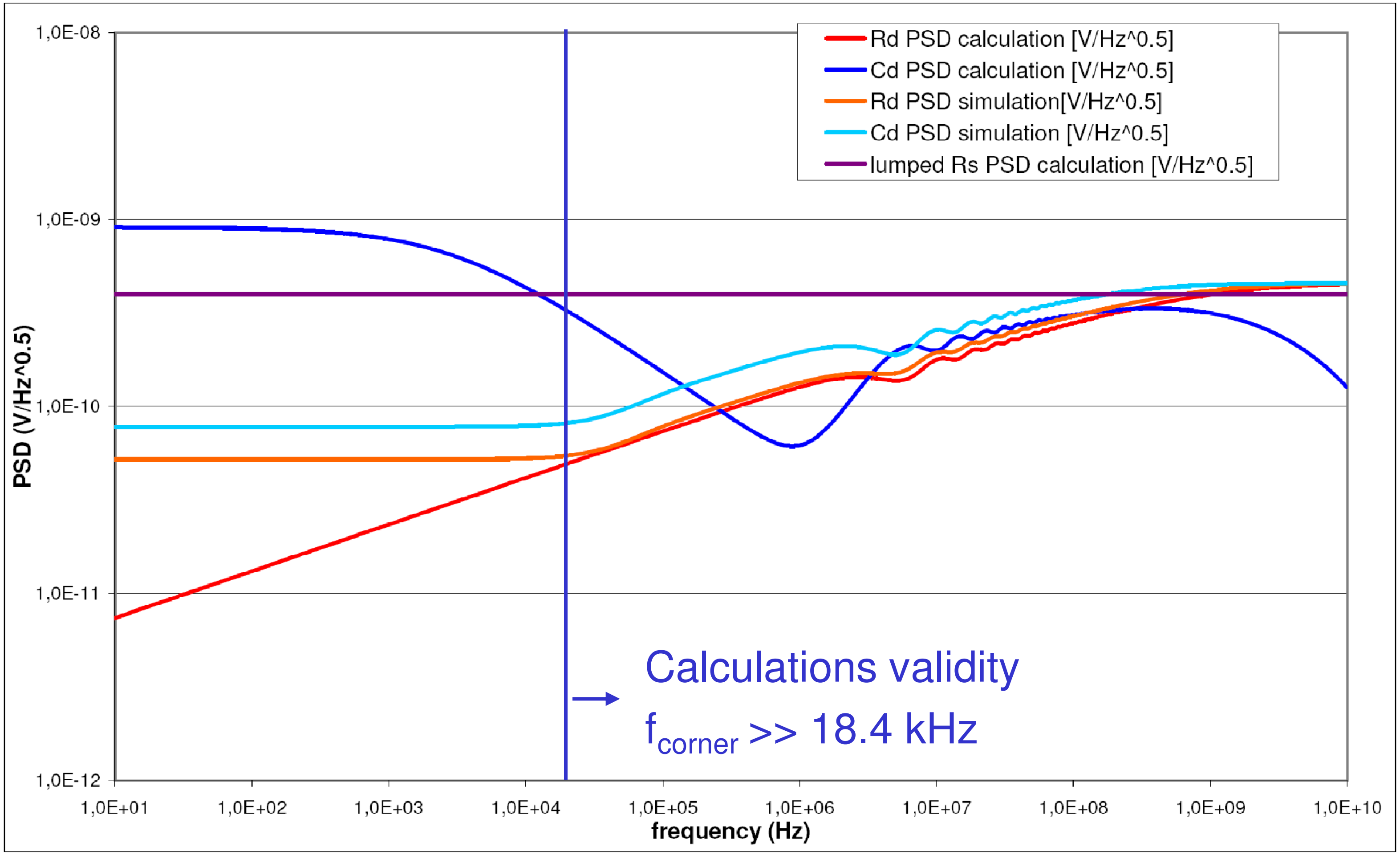
- Cadence Spectre simulation circuit with mtline and different lengthes (from 0.1 to 100 m):
  - with clipping line ($Z_d = R_d$)
  - without clipping line ($Z_d = 1/jC_d\omega$)





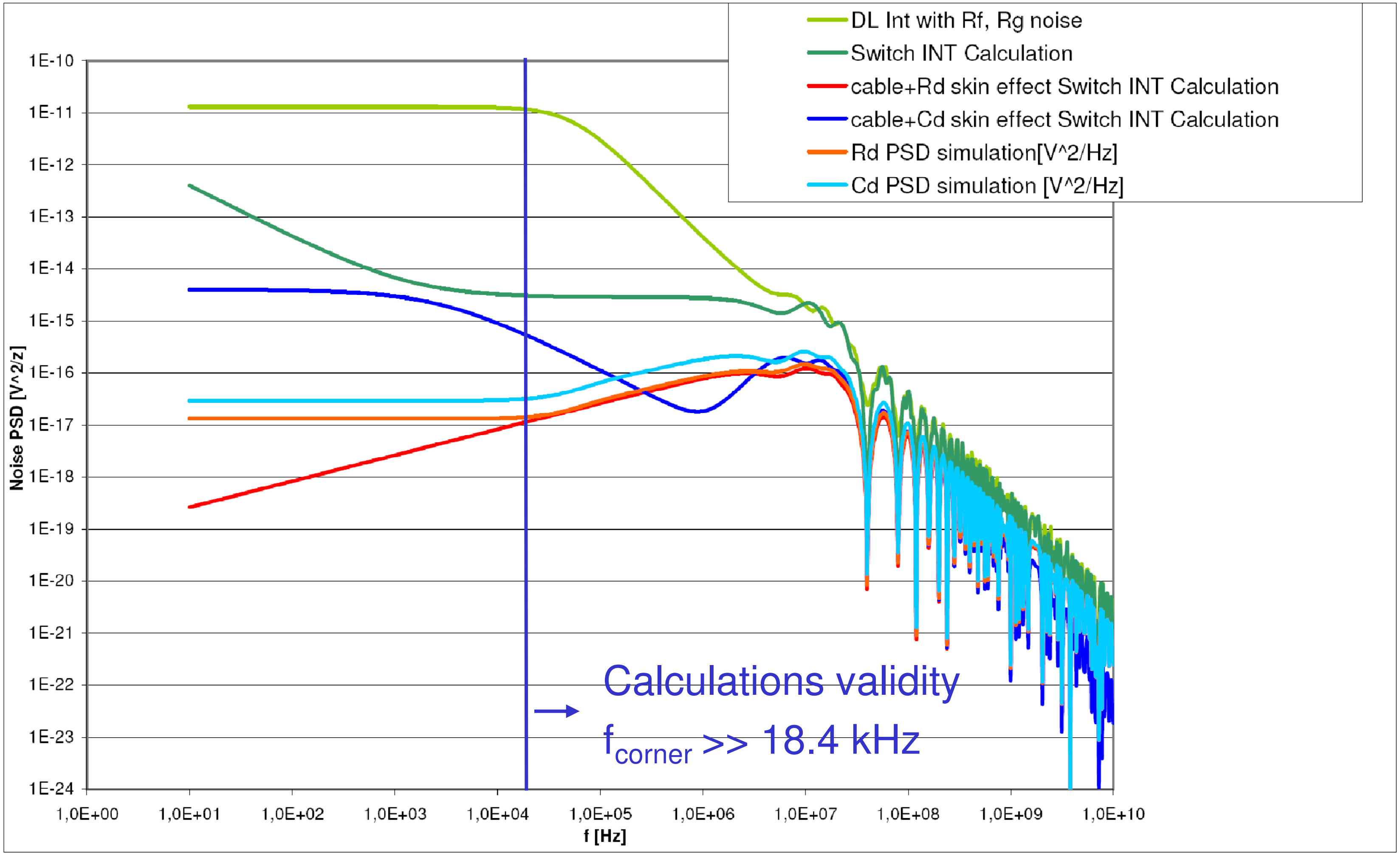
Skin effect generated noise

- Comparison between $Z_d=1/jC_d\omega$ and $Z_d=R_d$ for a cable of 12m calculation, simulation, and $R_S=18\Omega$ lumped approximation:



Skin effect generated noise

- Comparison between Zd=1/jCdw and Zd=Rd for a cable of 12m after integration:



Conclusions

- 2 main cable effects on SNR:
  - Attenuation due to the skin effect:
    - long tail in the step response of the cable
    - part of the signal is delayed and does not contribute
  - Increase of resistance of the cables
    - noise source distributed along the cable
- Impedance for a 12m cable at 2-3 MHz < f < 1 GHz
  - Zd=1/jCw: |Z| oscillates between 2 and 200 Ω
  - Zd= Rd : |Z| oscillates between 25 and 100 Ω
- Calculated and simulated skin effect generated noise offer more precision than the approximation with a lumped resistor at preamp input
- Noise calculations are valid for $f \gg f_{corner}$ = 18.4 kHz
- **Calculated and simulated noise due to skin effect is low enough**