The Higgs sector in the MSSM

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Outline

- The Higgs sector in the Standard Model
- Naturalness problem
- Supersymmetry and the MSSM
- The MSSM Higgs sector at tree level
- Decoupling limit
- Loop corrected Higgs mass
- Higgs searches/production at LEP&LHC

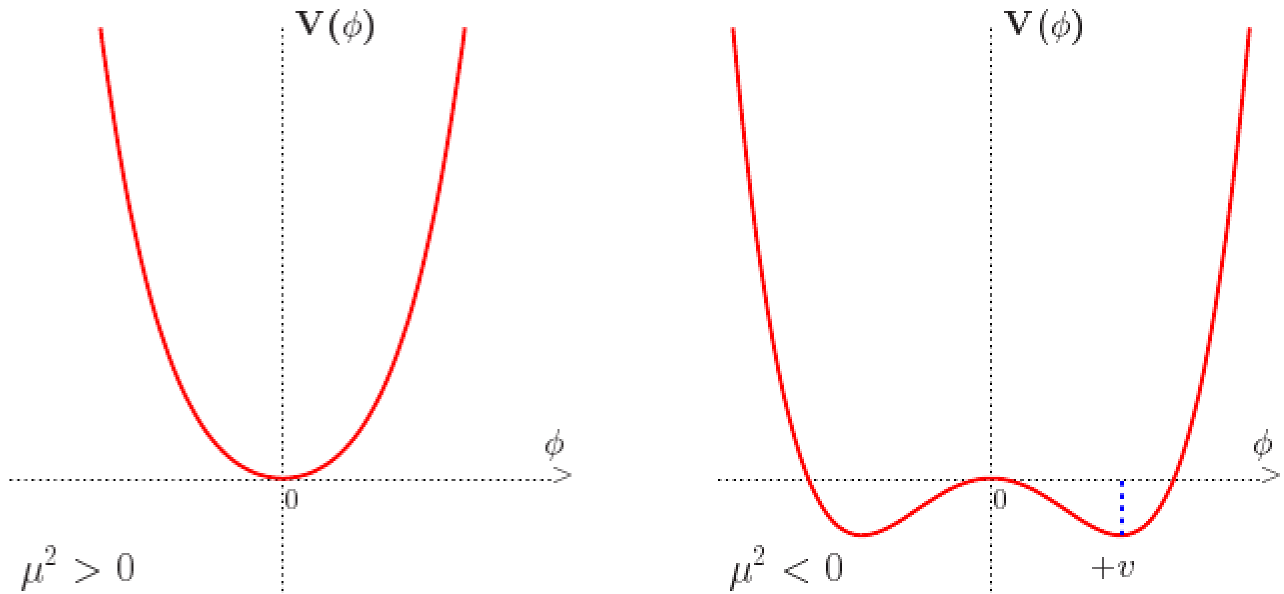
The Higgs sector in the Standard Model

- The Standard model contains one Higgs doublet, that gives masses to up, down – quarks and leptons.

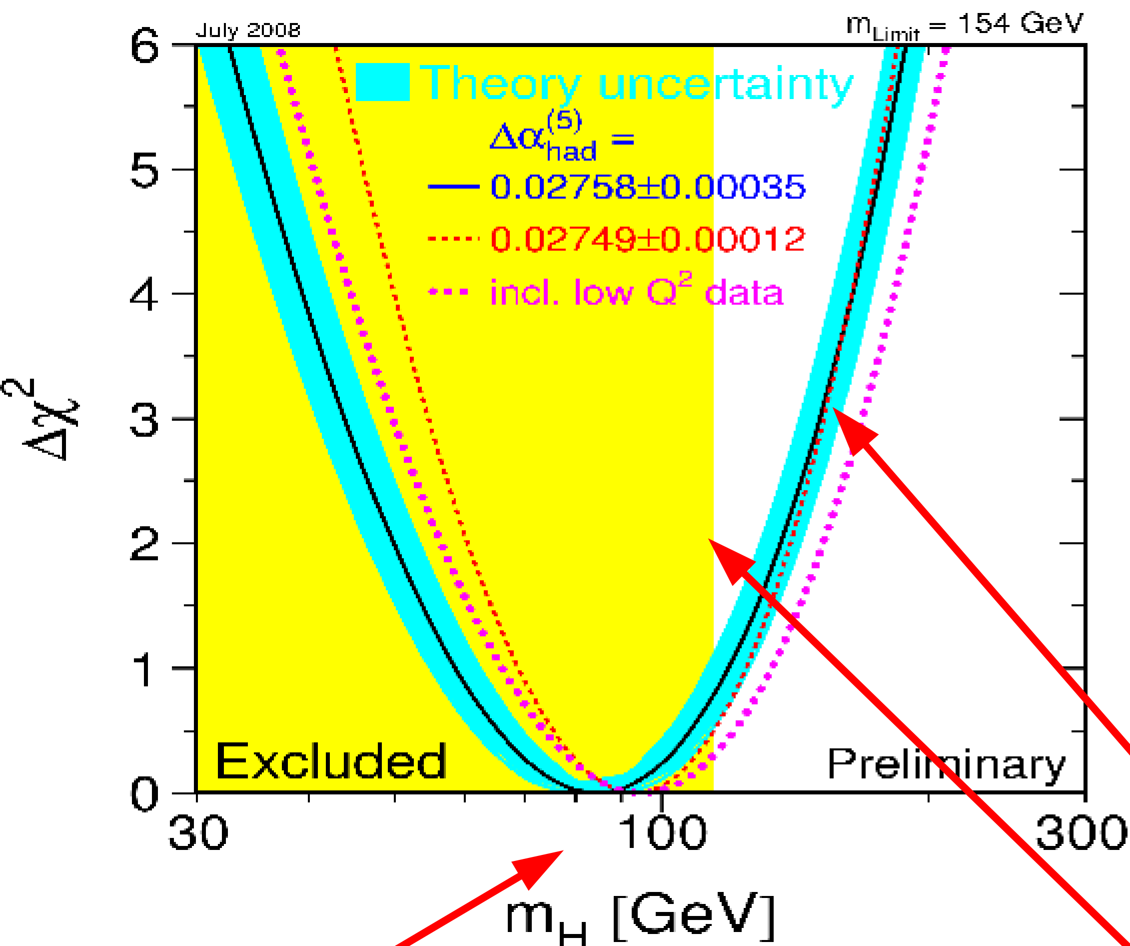
$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 - i\phi_2 \\ \phi_3 - i\phi_4 \end{pmatrix}$$

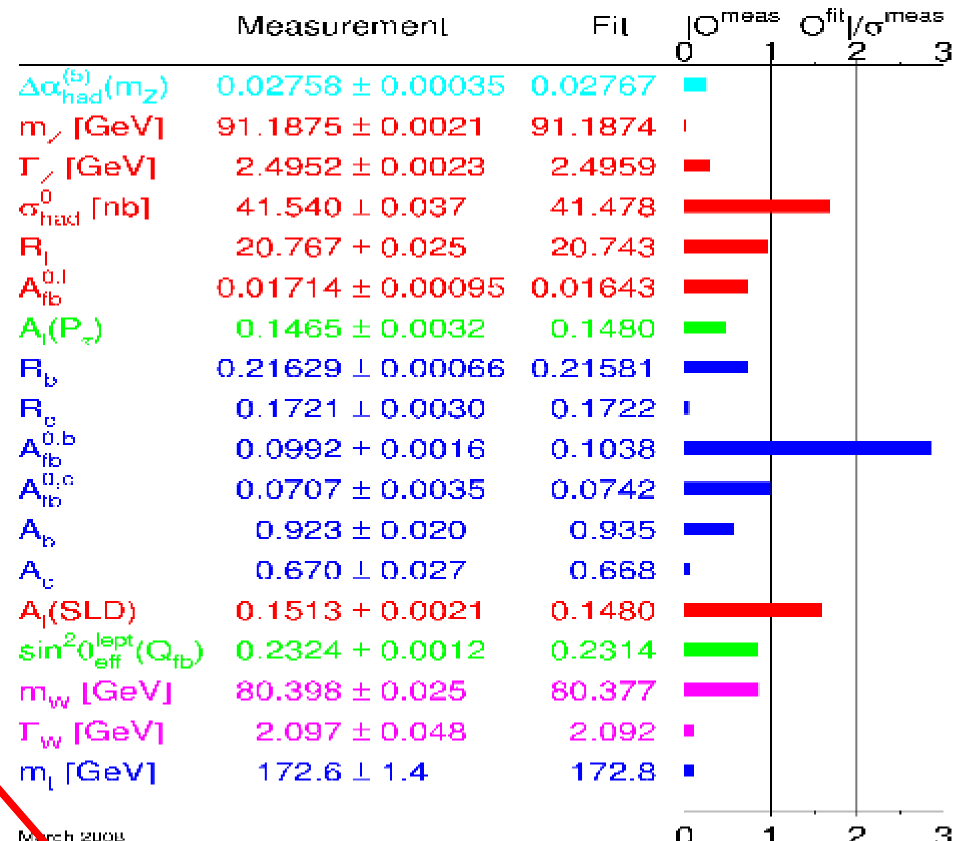
- $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ is spontaneously broken down to $U(1)_{\mathrm{EM}}$ electromagnetism

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \, \partial^\mu \phi - V(\phi) \; , \; V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$



The Higgs sector in the Standard Model





	Measurement	Fit	$\lvert O^{meas} - O^{fit}\rvert/\sigma^{meas}$
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02767	
m_Z [GeV]	91.1875 ± 0.0021	91.1874	
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959	
σ_{had}^{0} [nb]	41.540 ± 0.037	41.478	
R_l	20.767 ± 0.025	20.743	
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01643	
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480	
R_b	0.21629 ± 0.00066	0.21581	
R_c	0.1721 ± 0.0030	0.1722	
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038	
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742	
A_b	0.923 ± 0.020	0.935	
A_c	0.670 ± 0.027	0.668	
A_l(SLD)	0.1513 ± 0.0021	0.1480	
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314	
m_W [GeV]	80.398 ± 0.025	80.377	
Γ_W [GeV]	2.097 ± 0.048	2.092	
m_t [GeV]	172.6 ± 1.4	172.8	

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Uncertainties from higher ord

Mass limit from LEP

In the SM, the global fit suggests a light Higgs boson mass

Naturalness problem

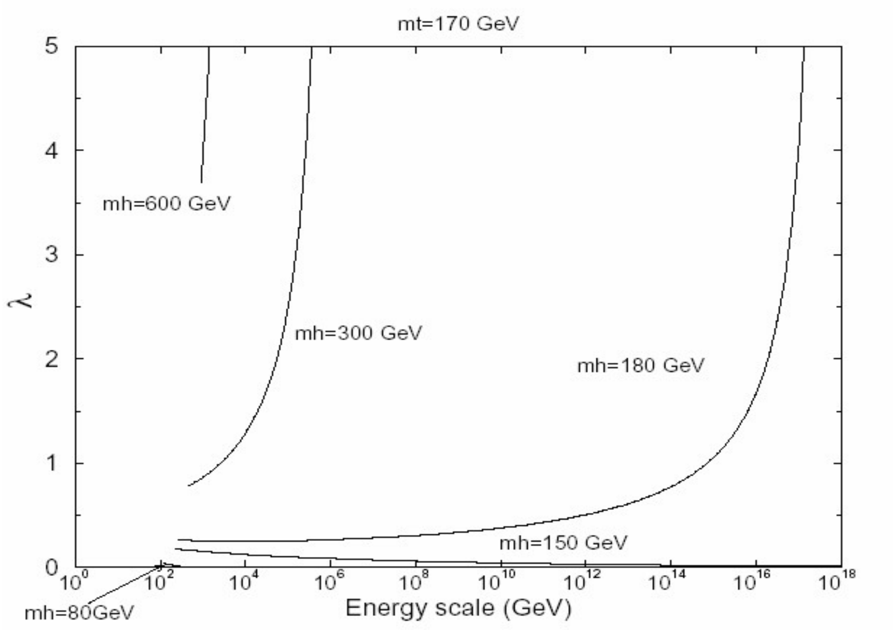
- Is the Standard Model valid up to the Planck scale ?
- Limits on the SM Higgs mass
- Three Higgs boson mass ranges

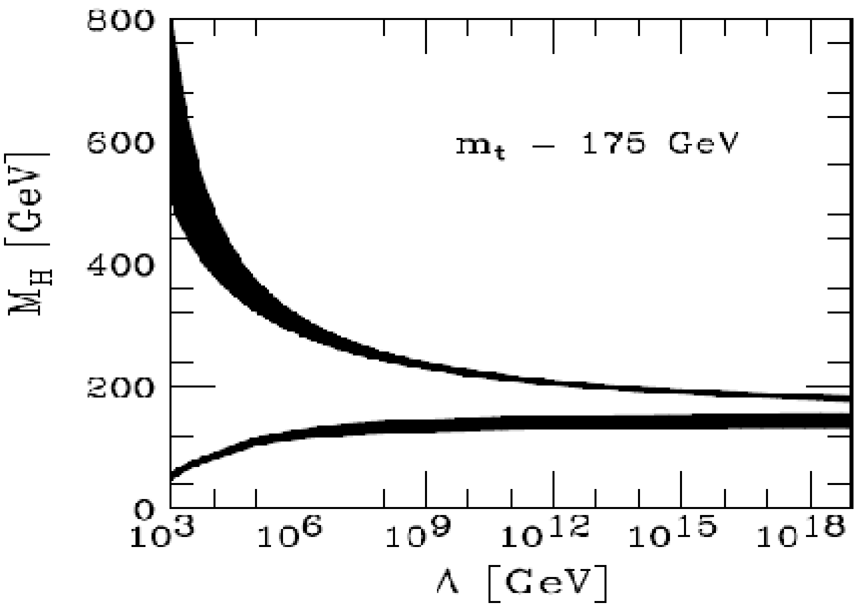
1.	110 GeV	$\leq$	$M_{H_{SM}}$	$\leq$	130 GeV	Λ	λ_H goes negative, destabilizes vacuum
2.	130 GeV	$\leq$	$M_{H_{SM}}$	$\leq$	180 GeV	M_{PL}	no new physics up to the Planck scale :(
3.	180 GeV	$\leq$	$M_{H_{SM}}$	$\leq$	190 GeV	Λ	λ_H blows up

RGE for the Higgs self-coupling constant in the SM

$$\mu\frac{d}{d\mu}\lambda = \frac{1}{16\pi^2}(24\lambda^2 + 12y_t\lambda - 6y_t^2 + \ldots)$$

$$v = \sqrt{\frac{M_{H_{SM}}}{2\lambda}} \quad v = 246\text{GeV}$$





Naturalness problem

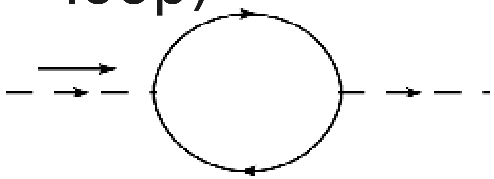
- Nothing restricts the SM to survive up to the Planck Scale if the Higgs mass is in the region from 130-180 GeV. However this is unnatural.
- If the cutoff scale Λ is very high, fine tuning of the Higgs boson mass is a serious problem. The squared Higgs boson mass depends quadratically on the cutoff scale Λ.

$$m^2_{phys} = m^2_0 + g^2\Lambda^2_{cutoff}$$

- We need to find a reason to keep the Higgs boson mass light. The mass of the Higgs boson should be located in the electroweak scale.
- Idea: There is a symmetry that protects the Higgs boson from receiving too large corrections.

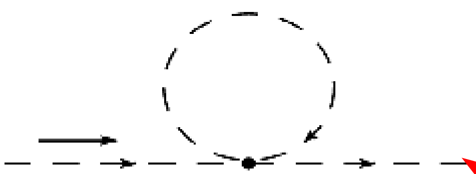
Supersymmetry

- The Higgs boson self-energy (main contribution comes form top-loop)



$$\Delta_{M_H^2} = -\frac{\lambda_f^2}{8\pi^2}\Lambda^2$$

energy cutoff



$$\Delta_{M_H^2} = \frac{\lambda_s}{16\pi^2}\Lambda^2$$

superpartner scalar

- Both contributions cancel if $\lambda_s = 2\lambda_f^2$

<u>matter superfield</u>

$$\begin{pmatrix} \Psi \\ \Phi \end{pmatrix}$$

weyl fermion DoF(=2)

complex scalar field DoF(=2)

<u>vector superfield</u>

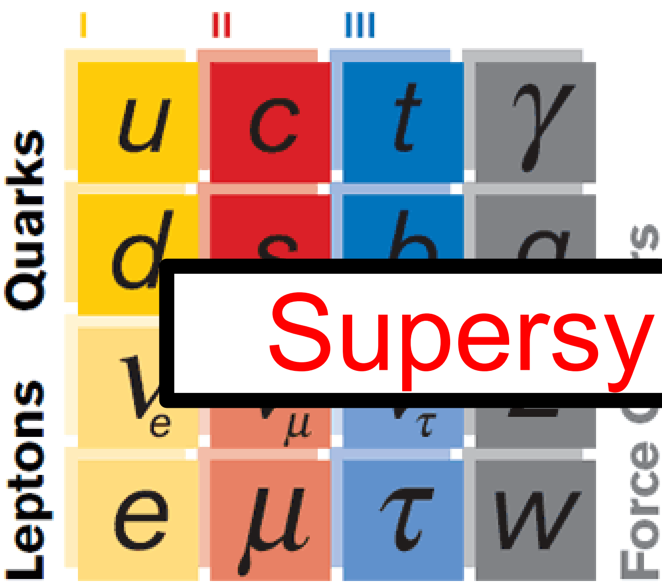
$$\begin{pmatrix} V \\ \lambda \end{pmatrix}$$

vector boson DoF(=2)

gaugino DoF(=2)

The MSSM

- The MSSM is the minimal supersymmetric extension of the SM
  - Minimal (=1) set of SUSY generators
  - Minimal (=2) set of Higgs doublets H_1, H_2



Three Generations of Matter

Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates
Higgs bosons	0	+1	$H_u^0\ H_d^0\ H_u^+\ H_d^-$	$h^0\ H^0\ A^0\ H^\pm$
squarks	0	−1	$\tilde{u}_L\ \tilde{u}_R\ \tilde{d}_L\ \tilde{d}_R$	(same)
			[illegible]	(same)
			[illegible]	[illegible]
sleptons	0	−1	$\tilde{e}_L\ \tilde{e}_R\ \tilde{\nu}_e$	(same)
			$\tilde{\mu}_L\ \tilde{\mu}_R\ \tilde{\nu}_\mu$	(same)
			$\tilde{\tau}_L\ \tilde{\tau}_R\ \tilde{\nu}_\tau$	$\tilde{\tau}_1\ \tilde{\tau}_2\ \tilde{\nu}_\tau$
neutralinos	1/2	−1	$\tilde{B}^0\ \widetilde{W}^0\ \tilde{H}_u^0\ \tilde{H}_d^0$	$\tilde{N}_1\ \tilde{N}_2\ \tilde{N}_3\ \tilde{N}_4$
charginos	1/2	−1	$\widetilde{W}^\pm\ \tilde{H}_u^+\ \tilde{H}_d^-$	$\tilde{C}_1^\pm\ \tilde{C}_2^\pm$
gluino	1/2	−1	$\tilde{g}$	(same)
goldstino (gravitino)	1/2 (3/2)	−1	$\tilde{G}$	(same)

Supersymmetry is broken in nature

The Higgs potential in the MSSM

- Three sources for the Higgs potential in the MSSM
  - D-terms from the superpotential
  - F-terms from the superpotential
  - Supersymmetry breaking Lagrangian
- Electroweak symmetry breaking → Supersymmetry breaking
- CP- conserving at tree-level, since all phases can be absorbed

$$V_D = \frac{g_2^2}{8}\left[4|H_1^\dagger \cdot H_2|^2 - 2|H_1|^2|H_2|^2 + (|H_1|^2)^2 + (|H_2|^2)^2\right] + \frac{g_1^2}{8}(|H_2|^2 - |H_1|^2)^2$$

$$V_F = \mu^2(|H_1|^2 + |H_2|^2)$$

$$V_{\text{soft}} = m_{H_1}^2 H_1^\dagger H_1 + m_{H_2}^2 H_2^\dagger H_2 + B\mu(H_2 \cdot H_1 + \text{h.c.})$$

Higgs sector at tree-level

- Two Higgs doublets are required in supersymmetric theories to generate mass for both "up" type and "down" type quarks and charged leptons.

$$H_d = \begin{pmatrix} (v_d + \phi_d^0 + i\chi_d^0)/\sqrt{2} \\ \phi_d^- \end{pmatrix} \quad H_u = \begin{pmatrix} \phi_u^+ \\ (v_u + \phi_u^0 + i\chi_u^0)/\sqrt{2} \end{pmatrix}$$

normalization: $v^2 \equiv v_d^2 + v_u^2 = 4m_W^2/g^2 = (246\,\mathrm{GeV})^2$

- The mass eigenstates

Neutral sector		Charged sector	
2 CP even Higgs bosons:	h, H	charged Higgs bosons:	$H^\pm$
1 CP odd Higgs boson:	A	charged Goldstone bosons:	$G^\pm$
1 Goldstone boson:	G^0		

Higgs sector at Tree-level

- The two Higgs fields expanded in mass eigenstates

$$H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}[H^- \sin\beta - G^- \cos\beta] \\ v_u + [H \cos\alpha - h \sin\alpha] + i[A \sin\beta + G^0 \cos\beta] \end{pmatrix}$$

$$H_d = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d + [H \sin\alpha + h \cos\alpha] + i[A \cos\beta - G^0 \sin\beta] \\ \sqrt{2}[H^+ \cos\beta + G^+ \sin\beta] \end{pmatrix}$$

- The angle beta is the ratio of the two VEVs

$$v_d = v\cos\beta, \qquad v_u = v\sin\beta, \qquad \Rightarrow \frac{v_u}{v_d} = \tan\beta, \quad \beta \epsilon (0, \tfrac{\pi}{2})$$

- Two additional parameters in the Higgs potential M_A, $\tan\beta$

- In particular $m^2_{H^\pm} = m^2_A + m^2_W$

Higgs sector at tree-level

- Masses of h^0, H^0 are Eigenvalues of the following matrix

$$\mathcal{M}^2_{h^0,H^0} = \begin{pmatrix} m^2_{A^0}\sin^2\beta + m^2_Z\cos^2\beta & -(m^2_{A^0}+m^2_Z)\sin\beta\cos\beta \\ -(m^2_{A^0}+m^2_Z)\sin\beta\cos\beta & m^2_{A^0}\cos^2\beta + m^2_z\sin^2\beta \end{pmatrix}$$

$$m^2_{h^0,H^0} = \frac{1}{2}\left(m^2_{A^0} + m^2_Z \mp \sqrt{(m^2_{A^0}+m^2_Z)^2 - 4m^2_Z m^2_{A^0}\cos^2 2\beta}\right)$$

- Alpha is the angle that diagonal $M^2_{h^0,H^0}$. Form the above results one obtains

$$\cos^2(\beta-\alpha) = \frac{m^2_{h^0}(m^2_Z - m^2_{h^0})}{m^2_{A^0}(m^2_{H^0} - m^2_{h^0})}$$

$$0 \le \beta \le \pi/2$$

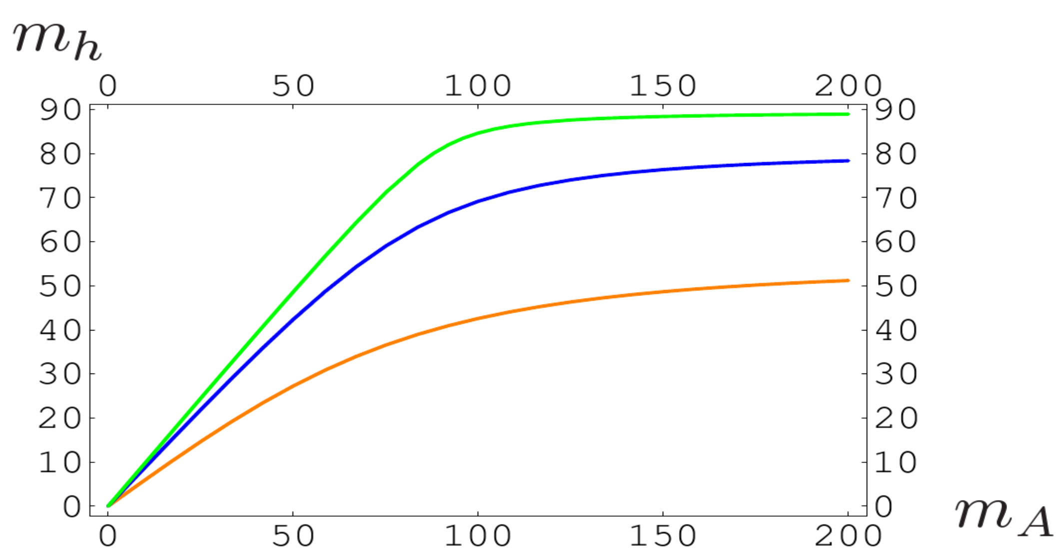
$$-\pi/2 \le \alpha \le 0$$

From the equation for m_{h0} one can derive an upper bound to the light CP even Higgs boson h^0

$$m_{h^0} = \frac{2m^2_Z m^2_{A^0}\cos^2 2\beta}{m^2_{A^0}+m^2_Z+\sqrt{(m^2_{A^0}+m^2_Z)^2-4m^2_Z m^2_{A^0}\cos^2 2\beta}} \le m^2_Z\cos^2 2\beta$$

Higgs sector at tree-level

m_{h0} as a function of m_{A0} and $\tan\beta$



$\tan\beta = 2$, $\tan\beta = 4$, $\tan\beta = 10$

- The SM does not constrain the valu $m^2_{h_{\rm SM}}$ at tree level
- Since $m^2_{h_{SM}} = 2\lambda v^2$ also the Higgs self-coupling λ is unknown
- In the MSSM the Higgs self-couplings are related to electroweak couplings.

Higgs sector at tree-level

Decoupling limit: $M_A >> M_Z$

- mh0 reaches its upper bound value

$$m_{h^0}^2 \simeq m_Z^2 \cos^2 2\beta$$

$$m_{H^0}^2 \simeq m_{A^0}^2 + m_Z^2 \frac{1 - 2\cos^2 2\beta}{2}$$

$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2$$

$$\cos^2(\beta - \alpha) \simeq \frac{m_Z^4 \sin^2 4\beta}{4 m_A^4}$$

- Other Higgs bosons become heavy and degenerate in mass

$$m_{A^0} \simeq m_{H^0} \simeq m_{H^\pm}$$

$$\cos(\beta - \alpha) \simeq 0$$

← Important parameter in Higgs coupling to fermion and vector bosons

Higgs couplings to gauge bosons

$$\begin{aligned}\mathscr{L} &= (\mathcal{D}^\mu H_u)^\dagger D_\mu H_u + (\mathcal{D}^\mu H_d)^\dagger D_\mu H_d \\ &= \frac{1}{2}|\partial_\mu \phi_u^0|^2 + \frac{1}{2}|\partial_\mu \phi_d^0|^2 + \left(\frac{g_Z^2}{8} Z_\mu Z^\mu + \frac{g^2}{4} W_\mu^+ W^{-\mu}\right)\left[(v_u + \phi_u^0)^2 + (v_d + \phi_d^0)^2\right]\end{aligned}$$

- Weak boson masses

$$m_W^2 = \frac{g^2 v^2}{4} \qquad m_Z^2 = \frac{g^2 + g'^2}{4} v^2 = \frac{m_W^2}{\cos^2\theta_W} \qquad v^2 = v_u^2 + v_d^2$$

- Gauge boson couplings

$$\begin{aligned} g_{h^0WW} &= g\ m_W \sin(\beta-\alpha) \\ g_{h^0ZZ} &= \frac{g}{\cos_{\theta_W}}\ m_Z \sin(\beta-\alpha)\end{aligned} \qquad \begin{aligned} g_{H^0WW} &= g\ m_W \cos(\beta-\alpha) \\ g_{H^0ZZ} &= \frac{g}{\cos_{\theta_W}}\ m_Z \cos(\beta-\alpha)\end{aligned}$$

$\cos(\beta-\alpha) \to 0 \Rightarrow H^0$ decouples from W, Z

$\sin(\beta-\alpha) \to 1 \Rightarrow h^0$ couples like SM Higgs boson

there are no tree level couplings of $A^0, H^\pm$ to VV (where $V = W, Z$)

Higgs couplings to gauge bosons

- Summary of the Higgs couplings to gauge bosons

$\cos(\beta-\alpha)$	$\sin(\beta-\alpha)$
$H^0W^+W^-$	$h^0W^+W^-$
H^0ZZ	h^0ZZ
ZA^0h^0	ZA^0H^0
$W^\pm H^\mp h^0$	$W^\pm H^\mp H^0$
$ZW^\pm H^\mp h^0$	$ZW^\pm H^\mp H^0$
$\gamma W^\pm H^\mp h$	$\gamma W^\pm H^\mp H^0$

Higgs couplings to fermions

$$-\mathscr{L} = h_t(\bar{t}P_L t H_u^0 - \bar{t}P_L b H_u^+) + h_b(\bar{b}P_L b H_d^0 - \bar{b}P_L t H_u^-) + \text{h.c.}$$

$$h_b = \frac{\sqrt{2}m_b}{v_d} = \frac{\sqrt{2}m_b}{v\cos\beta} \qquad h_t = \frac{\sqrt{2}m_t}{v_u} = \frac{\sqrt{2}m_t}{v\sin\beta}$$

- **Couplings relative to SM values**

decoupling limit

$$h^0\bar{b}b: \quad -\frac{\sin\alpha}{\cos\beta} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha) \quad \to 1$$

$$h^0\bar{t}t: \quad \frac{\cos\alpha}{\sin\beta} = \sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha) \quad \to 1$$

$$H^0\bar{b}b: \quad \frac{\cos\alpha}{\cos\beta} = \cos(\beta-\alpha) + \tan\beta\sin(\beta-\alpha) \quad \to \tan\beta$$

$$H^0\bar{t}t: \quad \frac{\sin\alpha}{\sin\beta} = \cos(\beta-\alpha) - \cot\beta\sin(\beta-\alpha) \quad \to -(\tan\beta)^{-1}$$

$$A^0\bar{b}b: \quad \gamma_5\tan\beta$$

$$A^0\bar{t}t: \quad \gamma_5\cot\beta$$

Loop-corrected Higgs mass

- Maximal mass of the CP-even Higgs boson h0 at one-loop level

$$m_h^2 \lesssim m_Z^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + x_t^2\left(1 - \frac{x_t^2}{12}\right)\right]$$

$$M_S^2 \equiv \frac{1}{2}(M_{\tilde{t}_1}^2 + M_{\tilde{t}_2}^2),\ x_t \equiv X_t/M_S$$

$$X_t \equiv A_t - \mu\cot\beta,\ X_b \equiv A_b - \mu\tan\beta$$

- m_t^4 dependence
- logarithmic sensitivity to stop quark masses
- dependence of Higgs mass on X_t, maximal value at $X_t = \sqrt{6}M_S$

Loop-corrected Higgs mass

- radiative corrections must be considered since, dominant effects arise from loops involving the 3rd generation quarks and squarks and are proportional to large Yukawa couplings

- Squark mixing matrix in interaction eigenstates

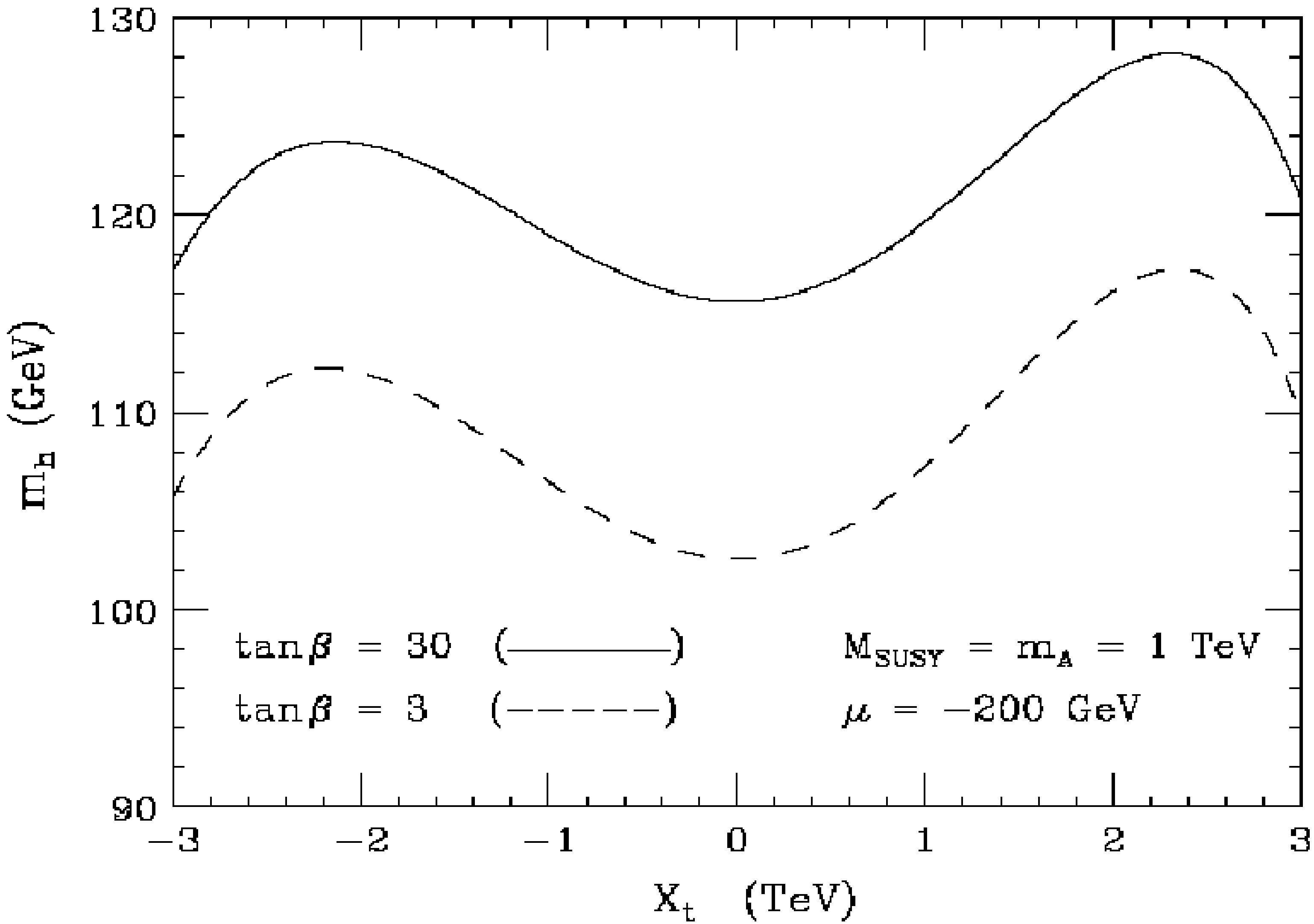
$$\begin{pmatrix} M_Q^2 + m_f^2 + D_L & m_f X_f \\ m_f X_f & M_R^2 + m_f^2 + D_R \end{pmatrix}$$

$$D_L \equiv (T_{3f} - e_f \sin^2\theta_W) m_Z^2 \cos 2\beta \qquad D_R \equiv e_f \sin^2\theta_W m_Z^2 \cos 2\beta$$

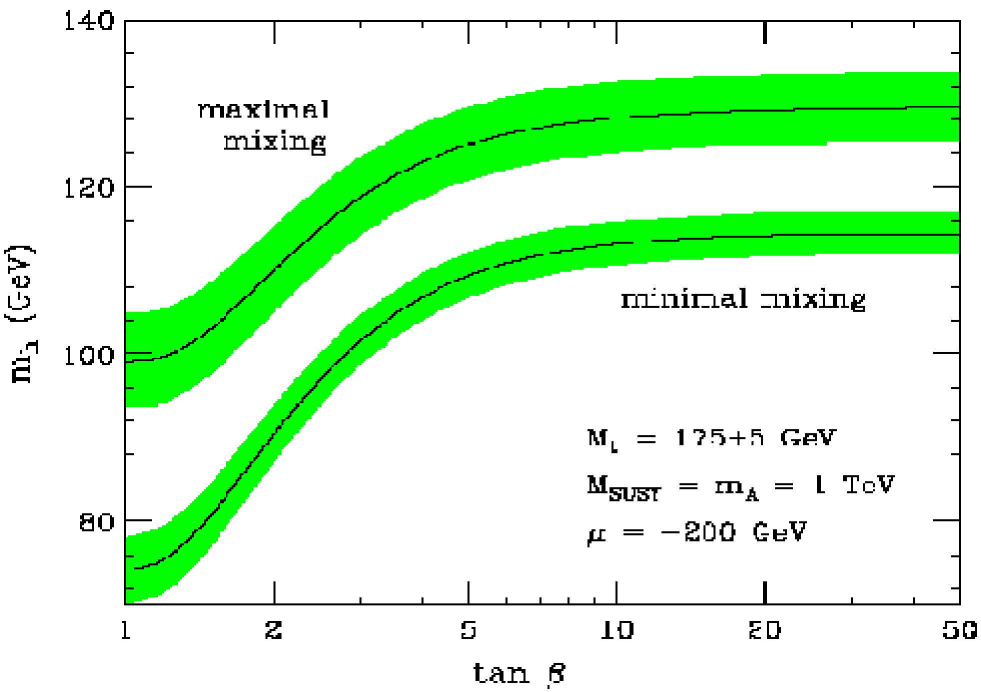
- Squark mixing matrix must be diagonalized to describe mass eigenstates $\tilde{q}_1, \tilde{q}_2$
- Squark mixing paramete $X_t \equiv A_t - \mu\cot\beta,\ X_b \equiv A_b - \mu\tan\beta$

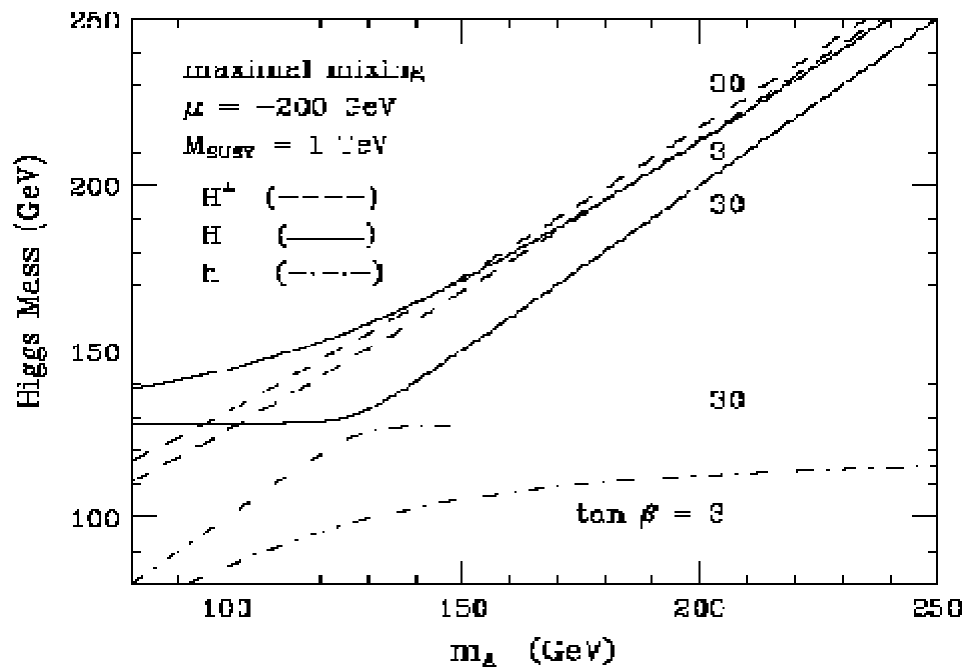
Loop-corrected Higgs mass

- Maximal Mixing vs Minimal Mixing X_t dependence



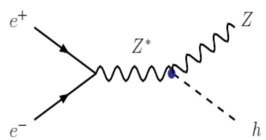
Loop-corrected Higgs mass



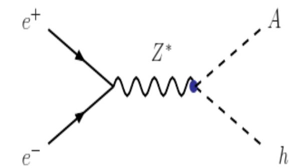


Higgs searches at LEP

- MSSM Higgs production at LE$\sqrt{s} \simeq 209$ GeV



$\sim \sin(\beta - \alpha)$

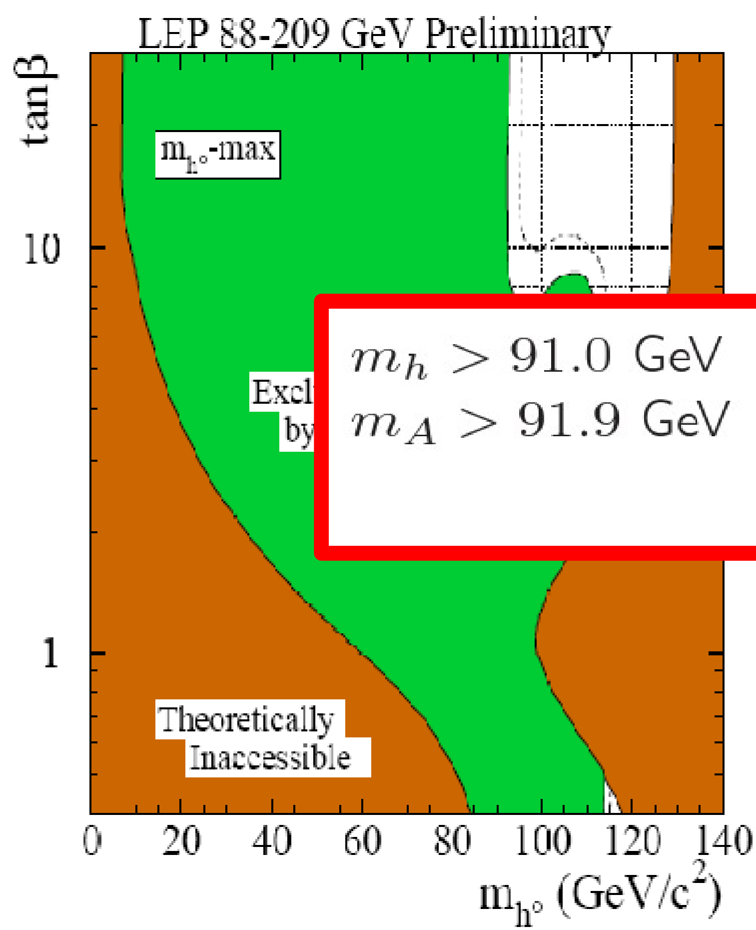


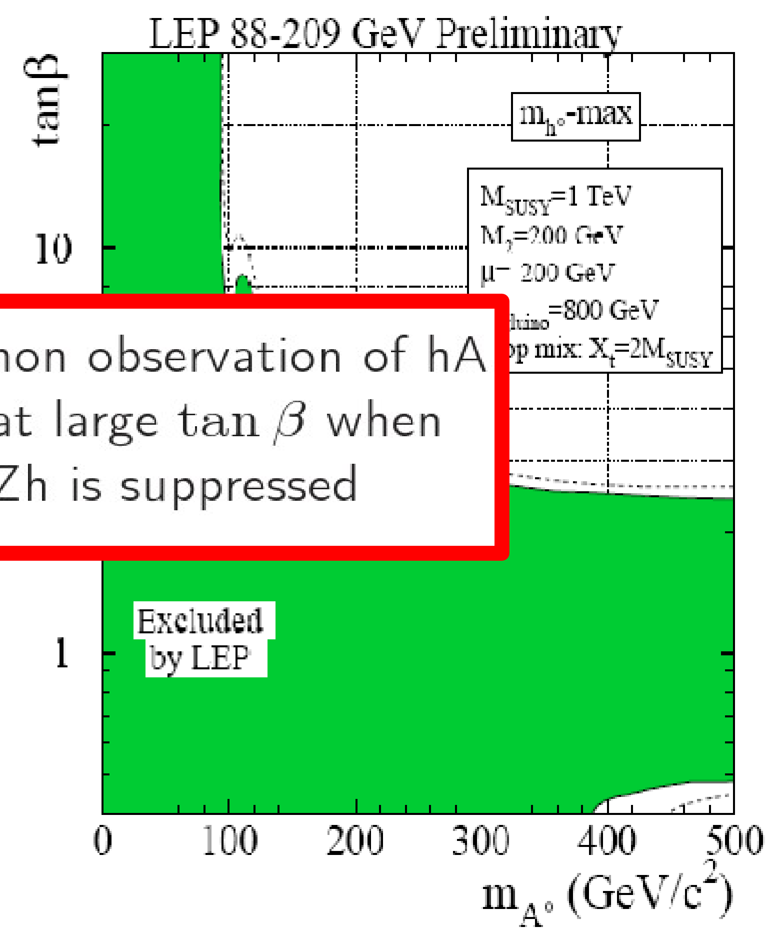
$\sim \cos(\beta - \alpha)$

large m_A: hZ searches

low m_A, large $\tan\beta$: hA searches

low m_A, low $\tan\beta$: both channels



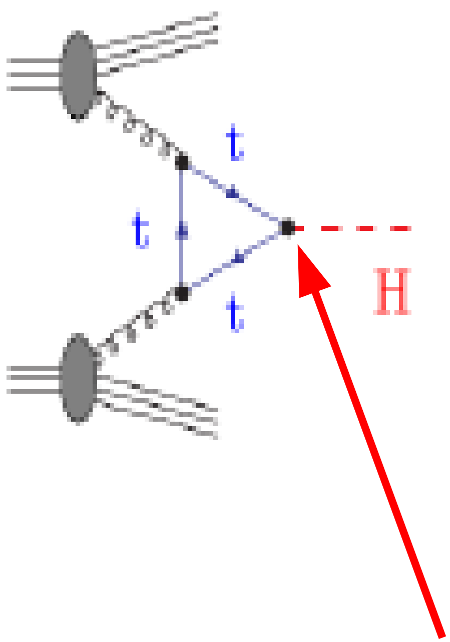


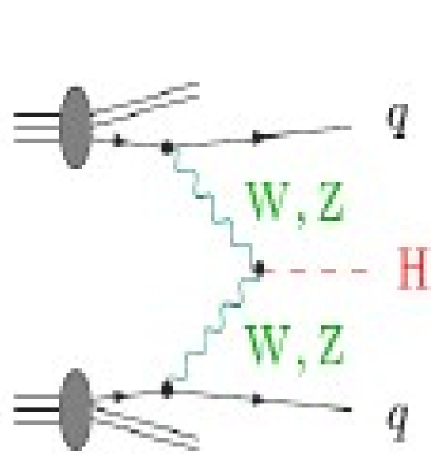
$m_h > 91.0$ GeV $\leftarrow$ non observation of hA

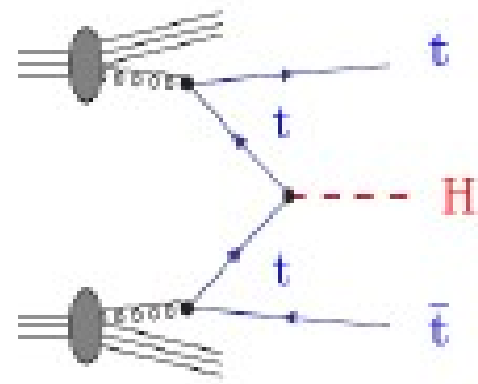
$m_A > 91.9$ GeV $\leftarrow$ at large $\tan\beta$ when Zh is suppressed

Higgs production at LHC

- Gluon fusion
- Vector boson fusion
- Associated production with $t\bar{t}$



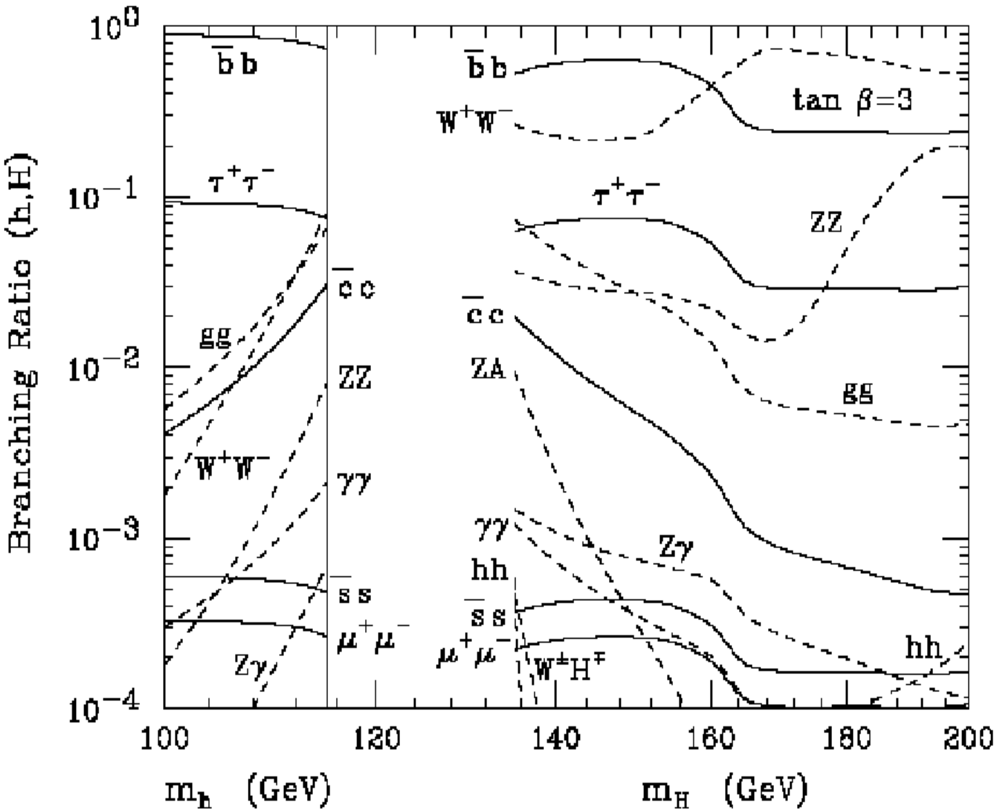


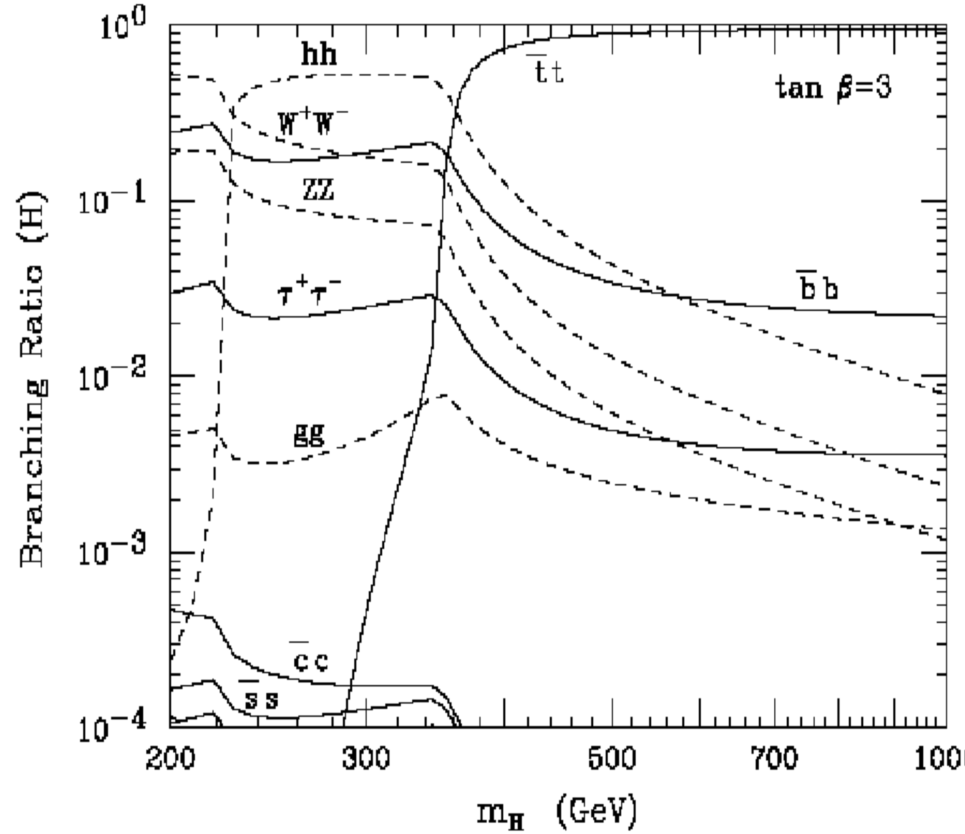


Large to yukawa coupling

MSSM Higgs branching ratios

Branching ratio of h^0 and H^0 for small $\tan\beta$





$M_{SUSY} = 1TeV$

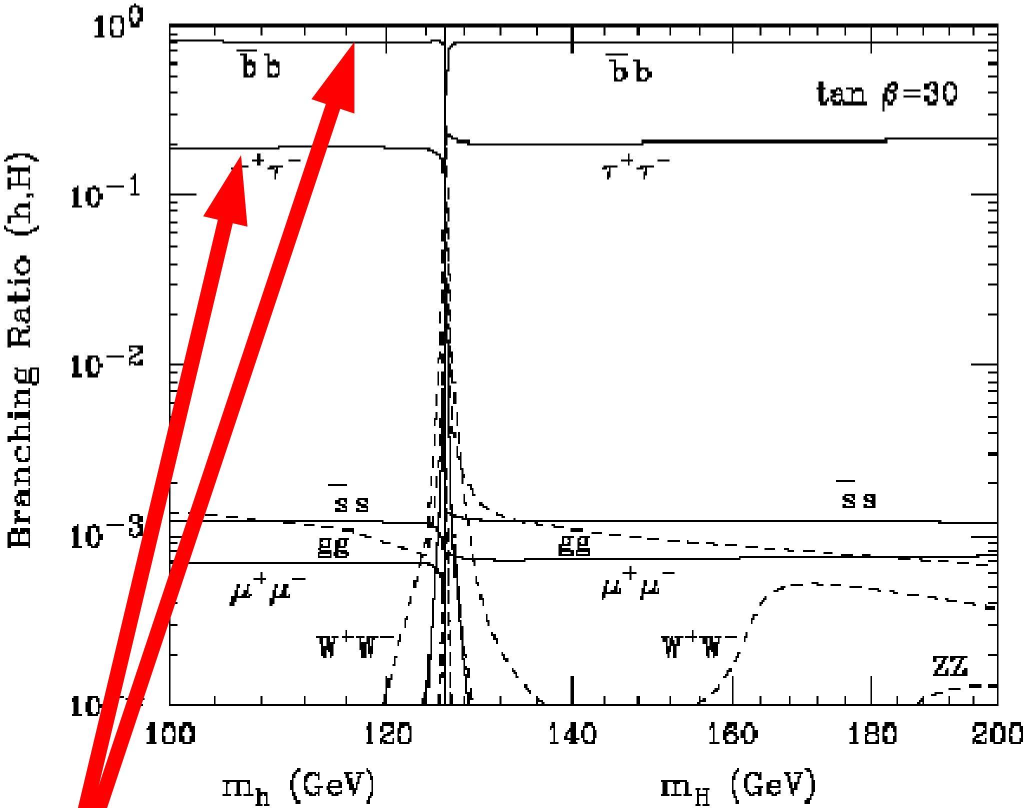
$X_t = 2.4 M_{SUSY}$

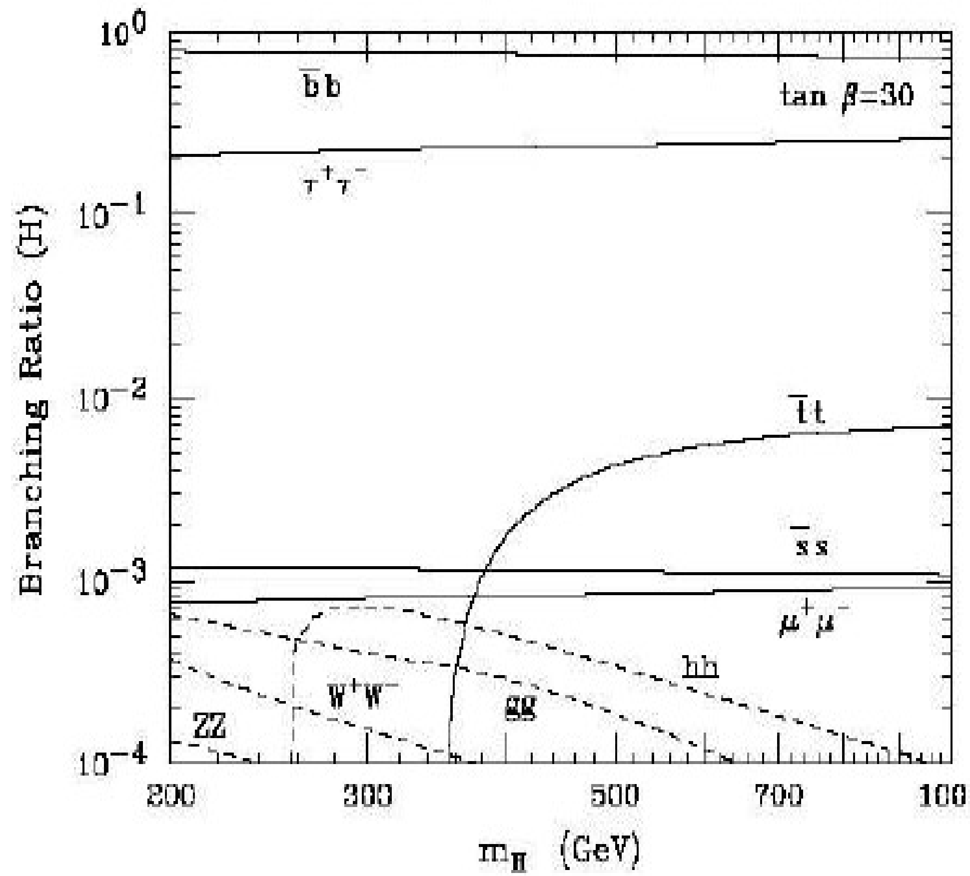
$\mu = M_2 = 1TeV$

$2M_1 \simeq 1TeV$

Higgs branching ratios

Branching ratio of h^0 and H^0 for large $\tan\beta$





$M_{SUSY} = 1TeV$

The couplings to down-like fermions $\tan\beta$ enhanced.

$X_t = 2.4 M_{SUSY}$

$\mu = M_2 = 1TeV$

$2M_1 \simeq 1TeV$

$h^0\bar{b}b$: $\tan\beta$ $H^0\bar{b}b$: $\tan\beta$

Summary

- Two Higgs doublets $\rightarrow h^0, H^0, A^0, H^\pm$
- Electroweak symmetry breaking $\rightarrow$ SUSY breaking
- Upper mass bound for h0 $\rightarrow$ one light Higgs boson $m_{h^0} \leq 135\mathrm{GeV}$