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Compound nuclear reactions induced by neutrons and the R-matrix formalism

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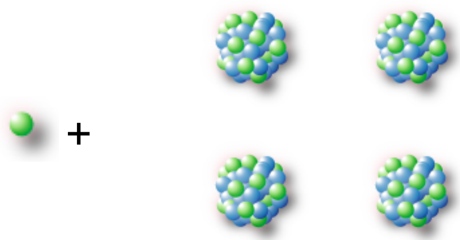
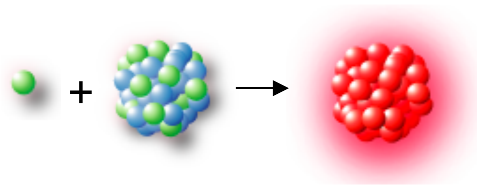
Neutron induced reactions

solid state



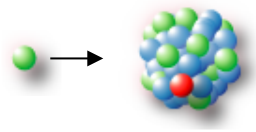
compound nucleus reactions



$\tau \sim 10^{-16}$

$E_n < 10$ MeV

direct reactions



$\tau \sim 10^{-22}$

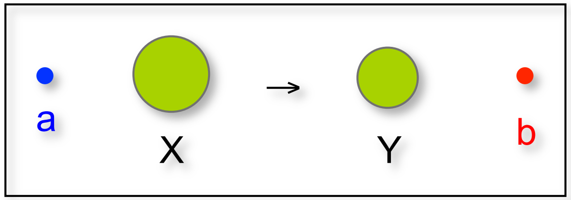
$E_n > 10$ MeV

1 nm — **neutron wave length** — 10 fm

1 meV — **neutron kinetic energy** — 10 MeV

Neutron-nucleus reactions

Reaction:
- X + a → Y + b
- X(a,b)Y



Cross section:
function of the kinetic energy of the particle a

$$\sigma(E_a) = \int\int \frac{d^2\sigma(E_a, E_b, \Omega)}{dE_b d\Omega} dE_b d\Omega$$

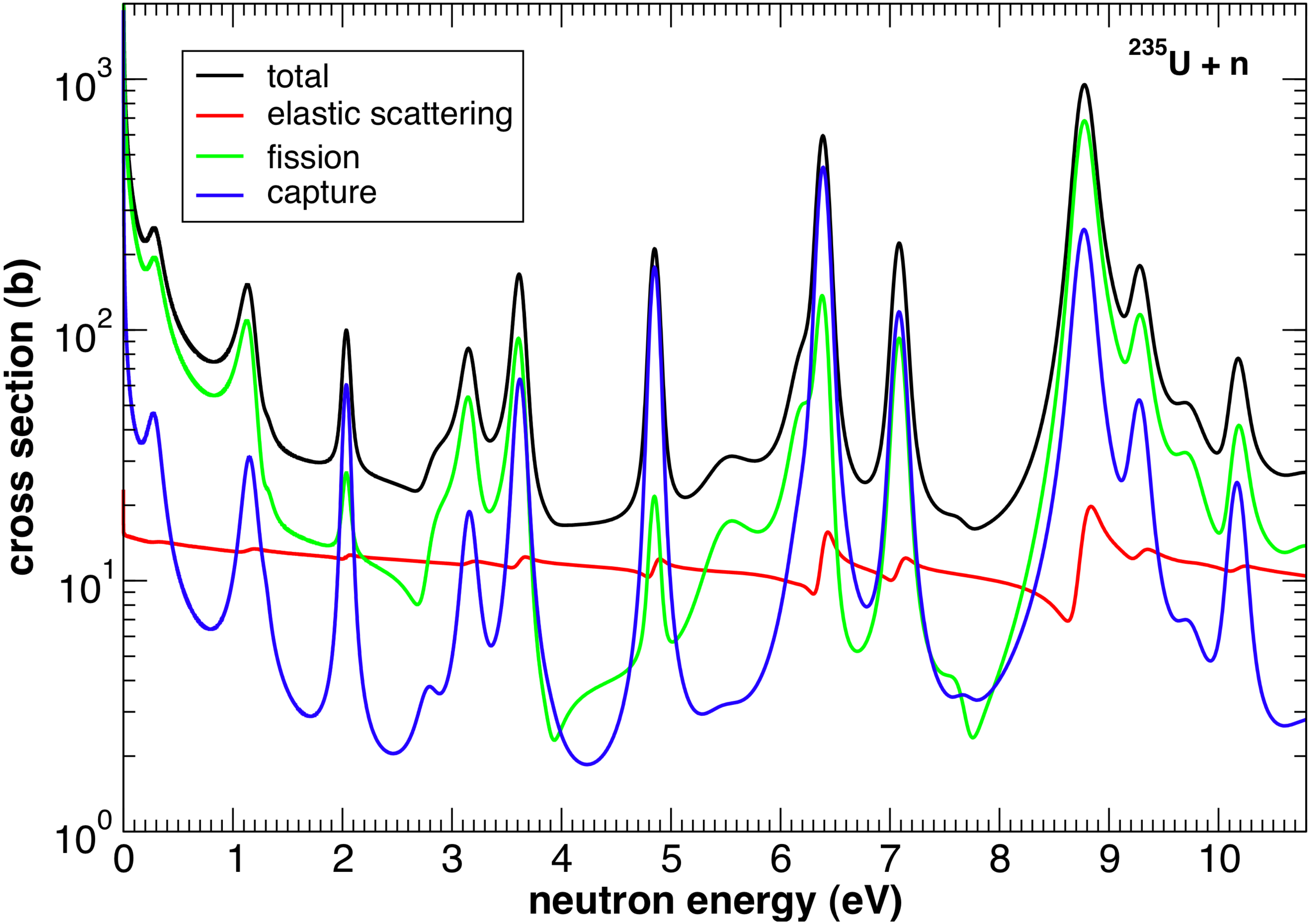
Differential cross section:
function of the kinetic energy of the particle a and function of the kinetic energy **or** the angle of the particle b

$$\frac{d\sigma(E_a, E_b)}{dE_b} \qquad \frac{d\sigma(E_a, \Omega)}{d\Omega}$$

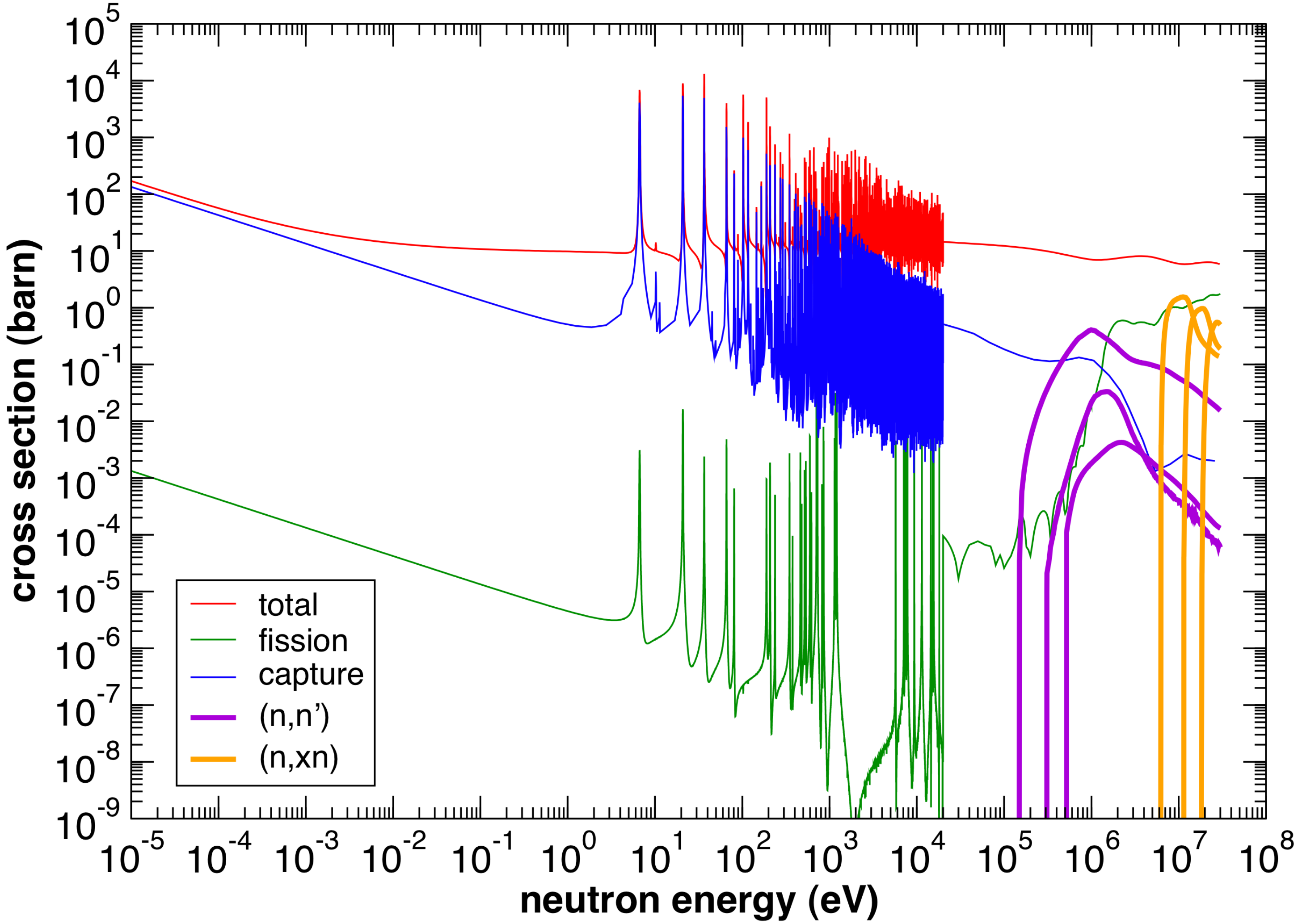
Double differential cross section:
function of the kinetic energy of the particle a and function of the kinetic energy **and** the angle of the particle b

$$\frac{d^2\sigma(E_a, E_b, \Omega)}{dE_b d\Omega}$$

Cross sections σ_T, σ_γ, σ_n et σ_f



Neutron fluxes and cross sections



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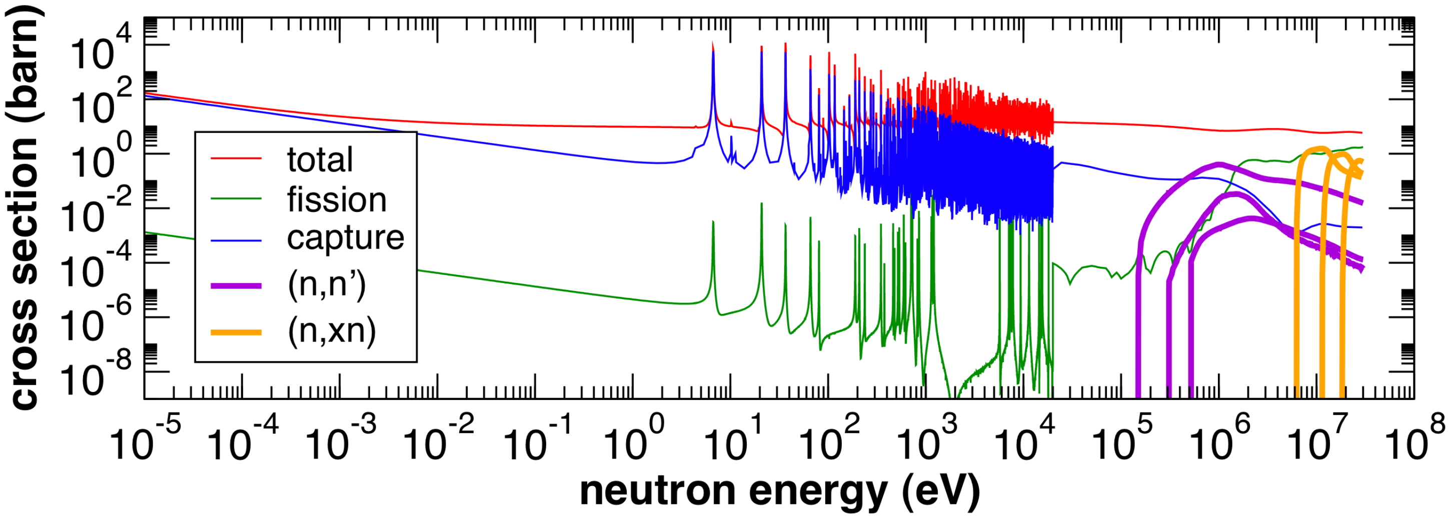

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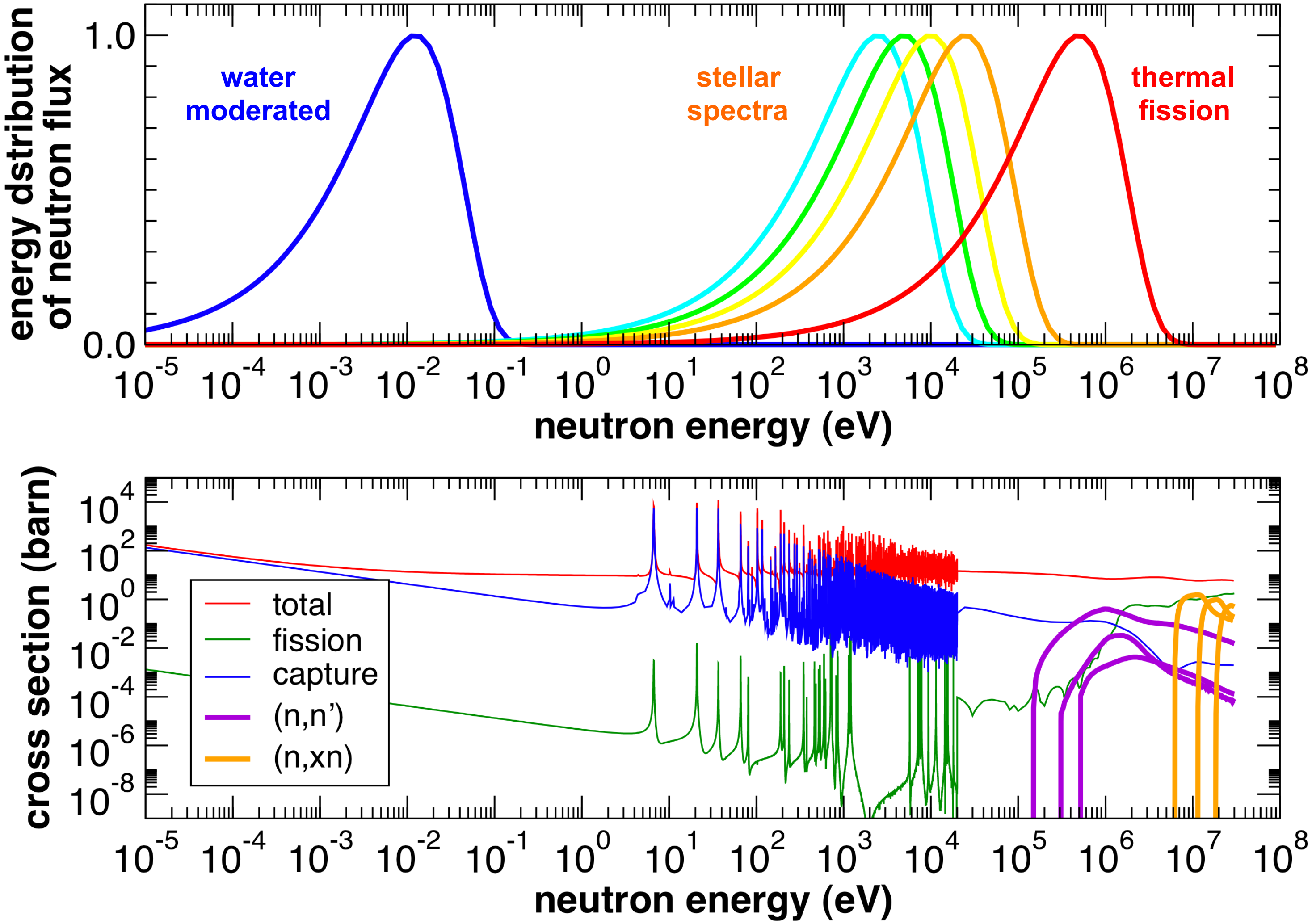

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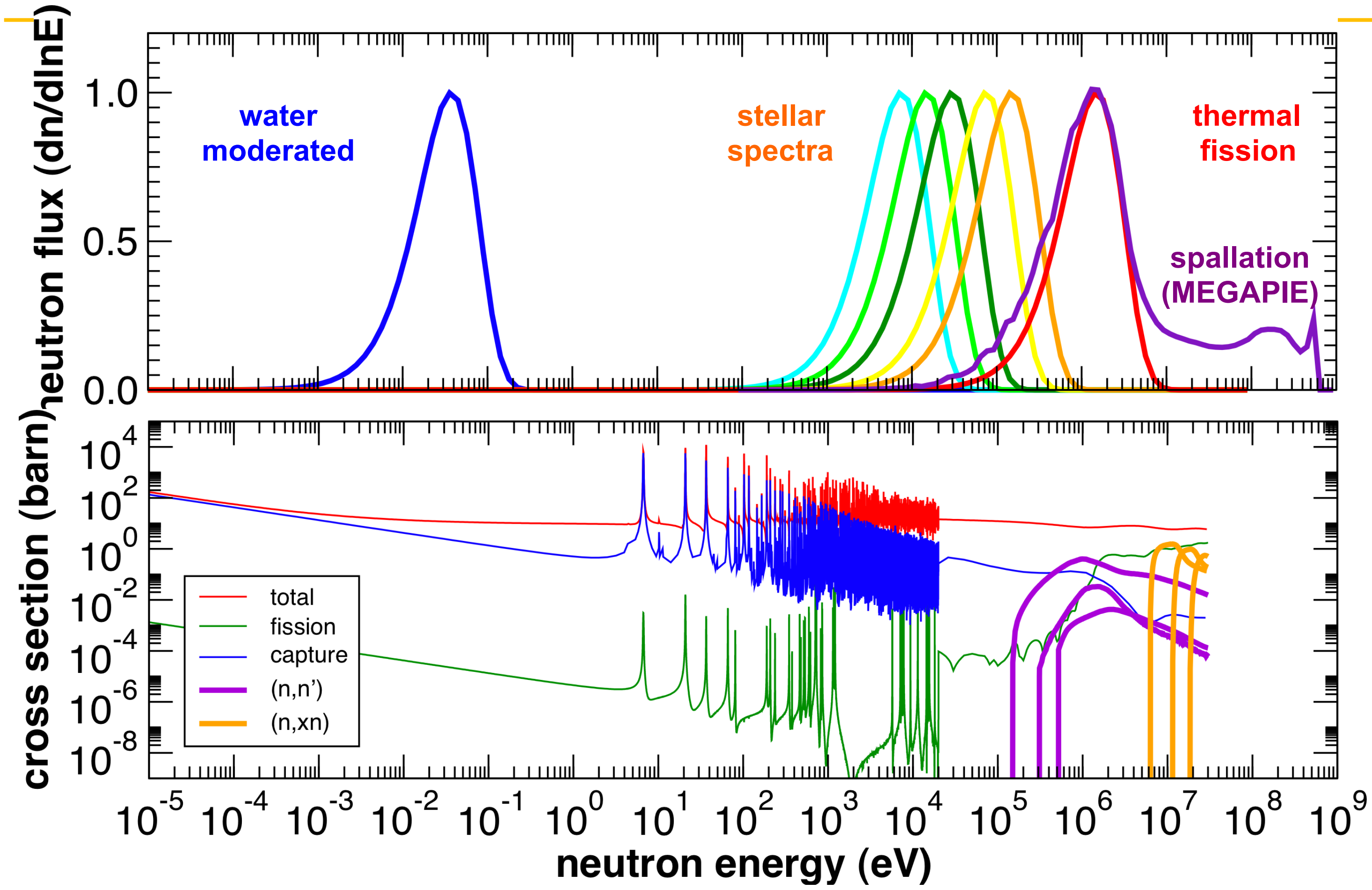

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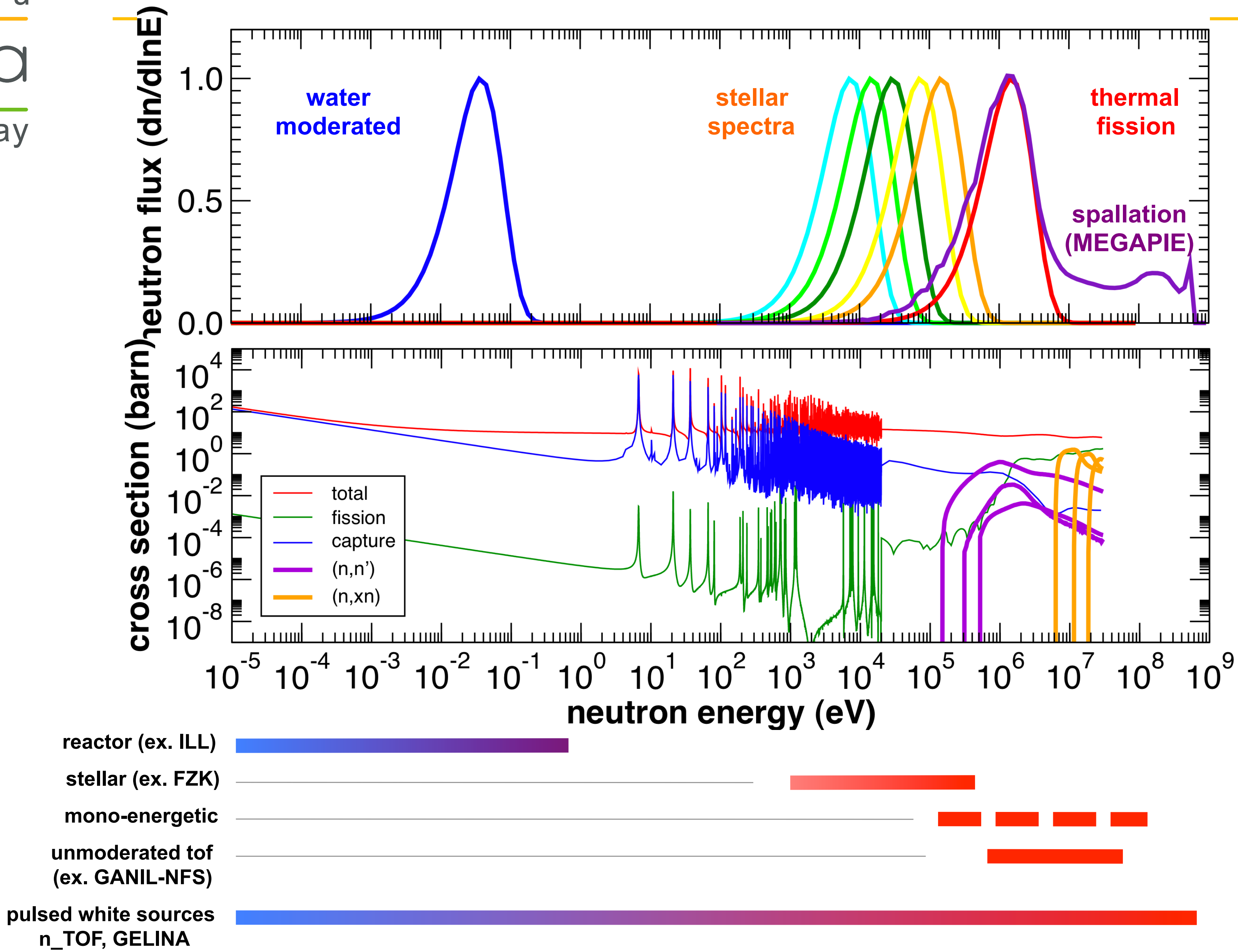
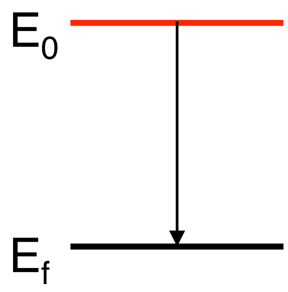


Decay of a nuclear state

state with a life time τ:

$$\Psi(t) = \Psi_0 e^{-iE_0 t/\hbar} e^{-t/2\tau}$$



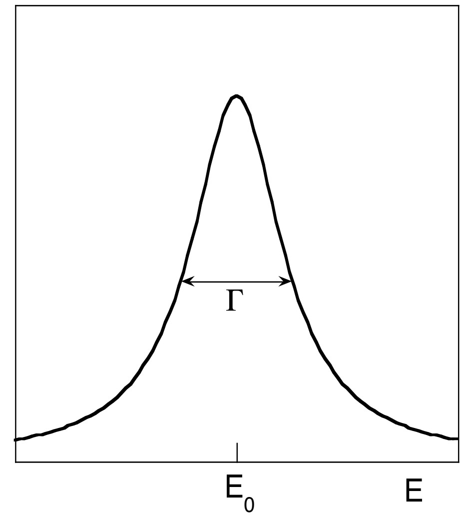
eigen state,
transitition $E_0 \rightarrow E_f$
life time τ

definition (Heisenberg):

$$\Gamma = \frac{\hbar}{\tau}$$

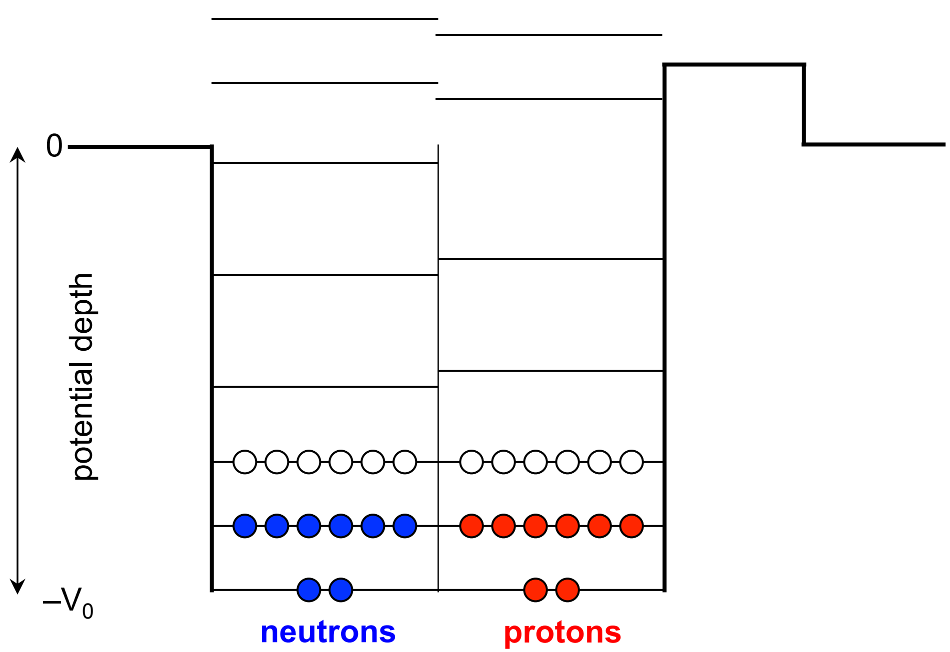
Fourier transform gives energy profile:

$$I(E) = \frac{\Gamma / 2\pi}{(E - E_0)^2 + \Gamma^2 / 4}$$

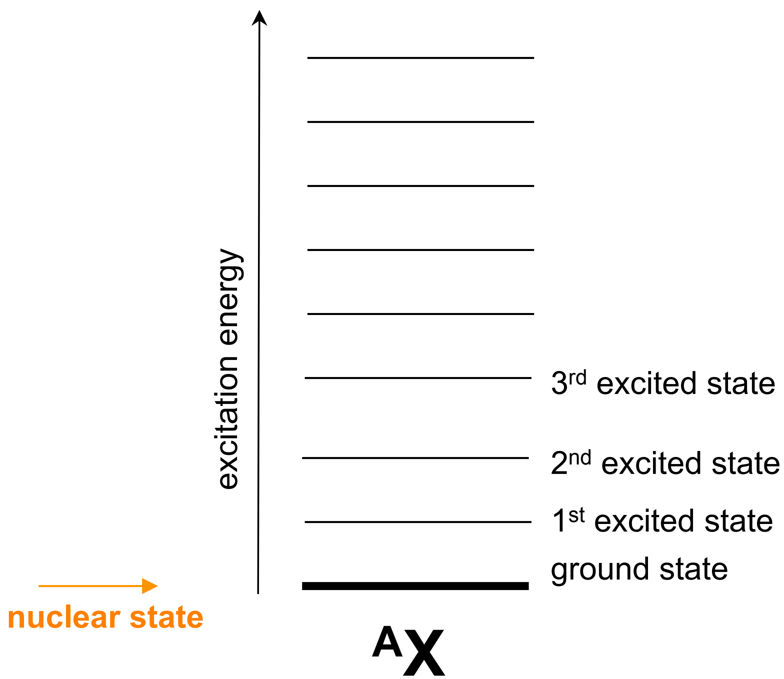


The nucleus as a quantum system

shell model representation:
configuration of nucleons in their potential



level scheme representation:
excited states of a nucleus
(shell model and other states)



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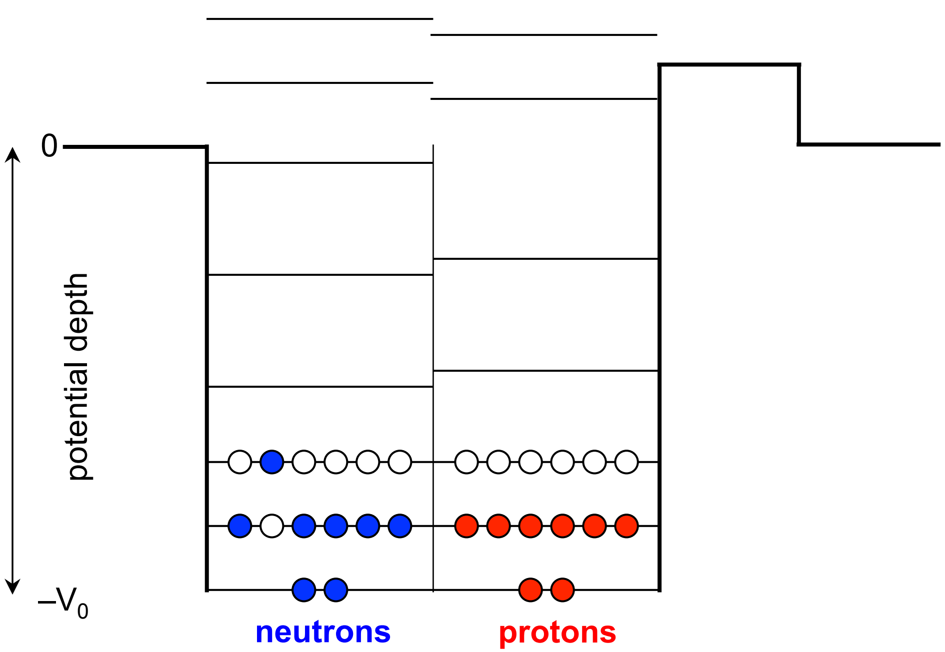

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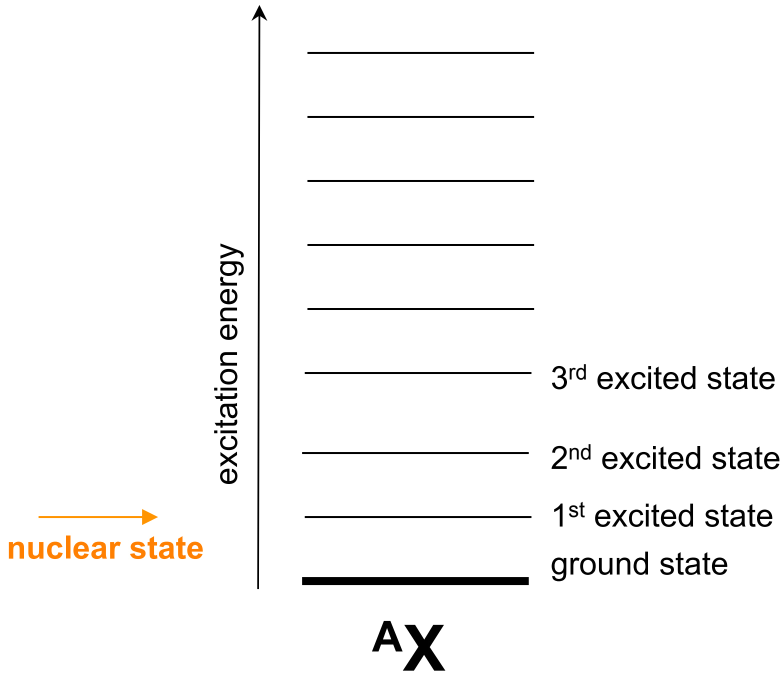

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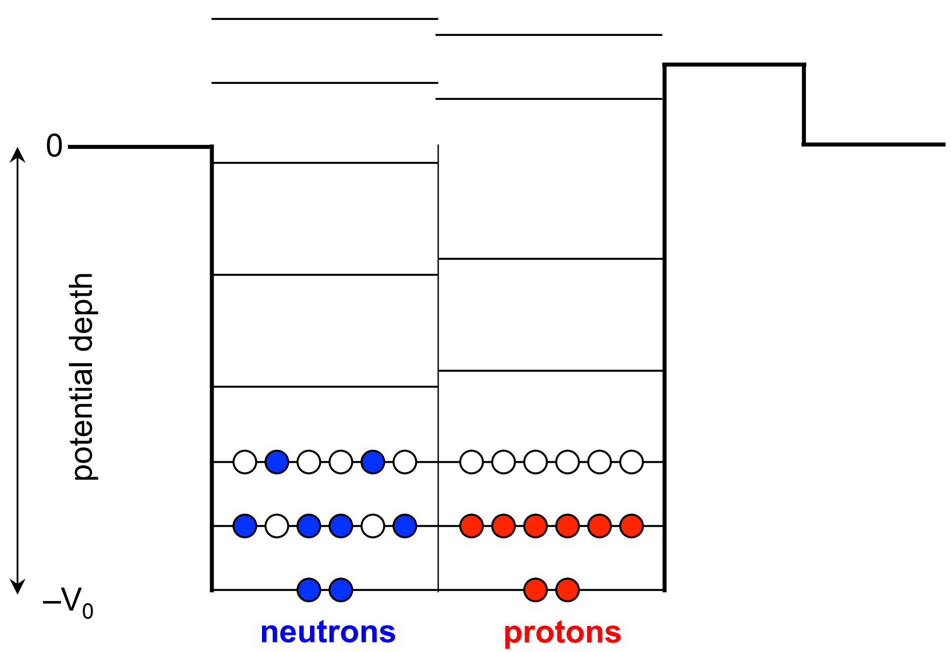

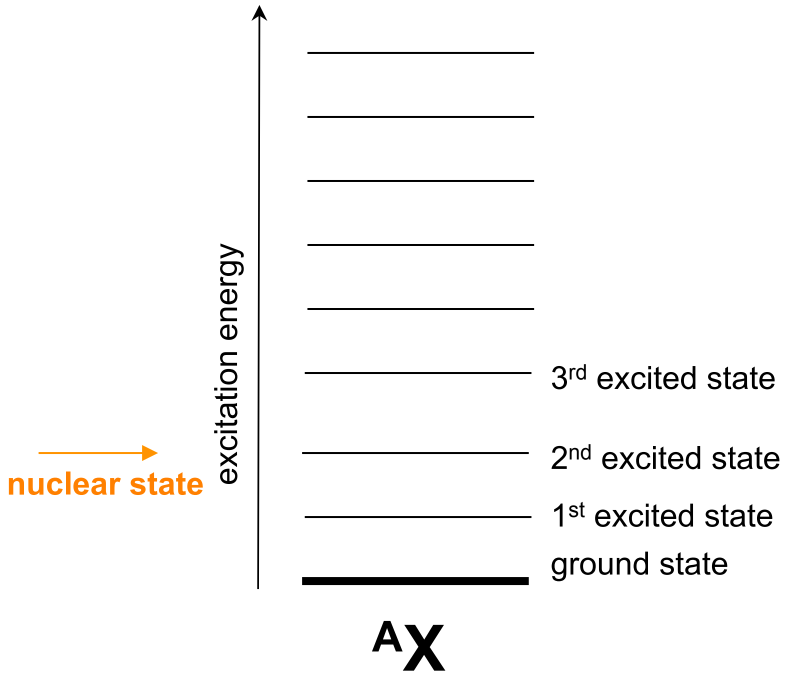

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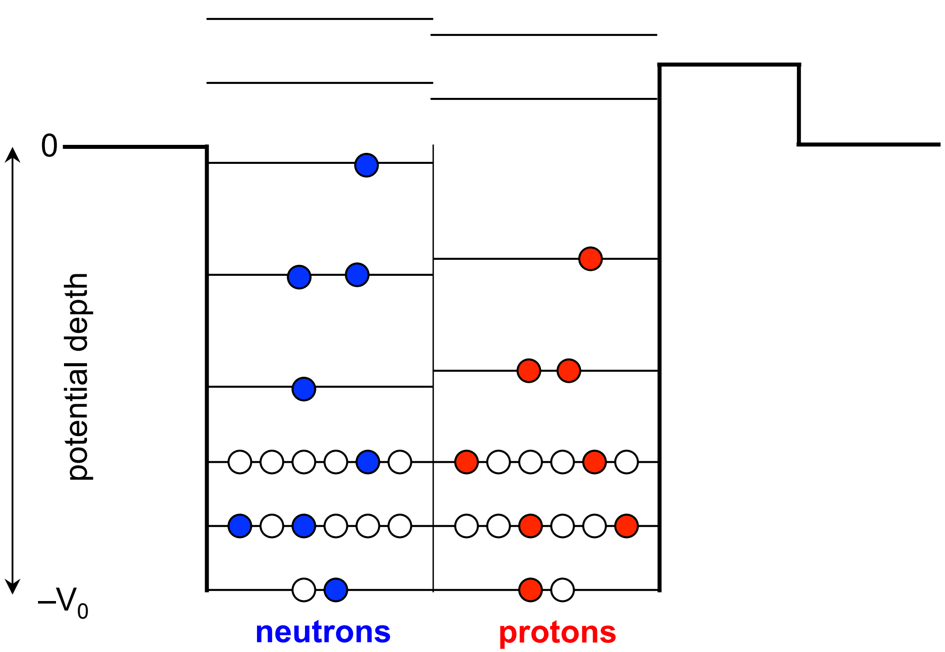
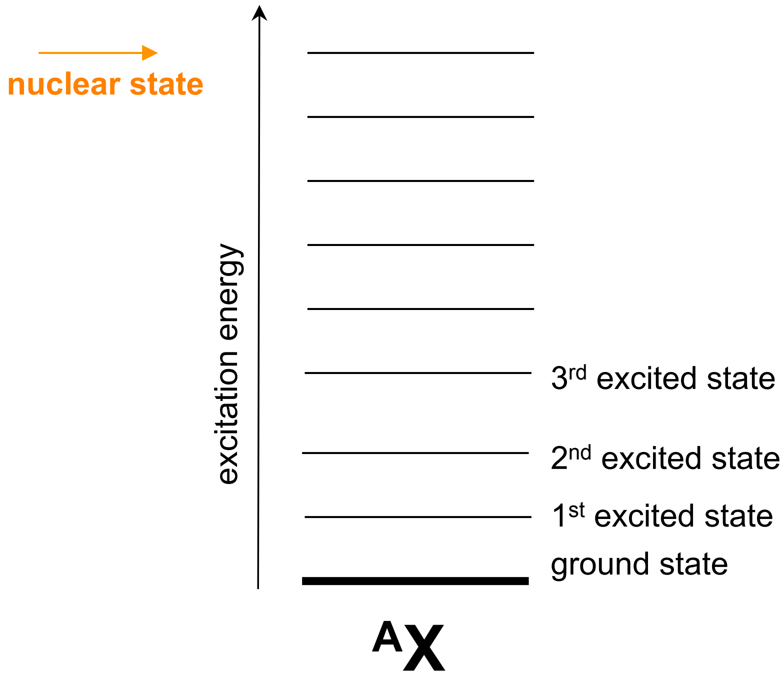
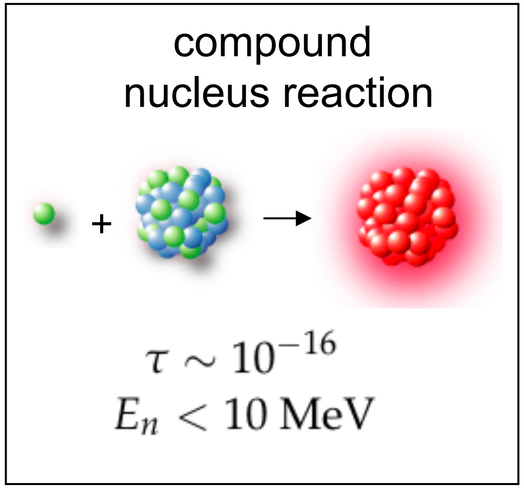
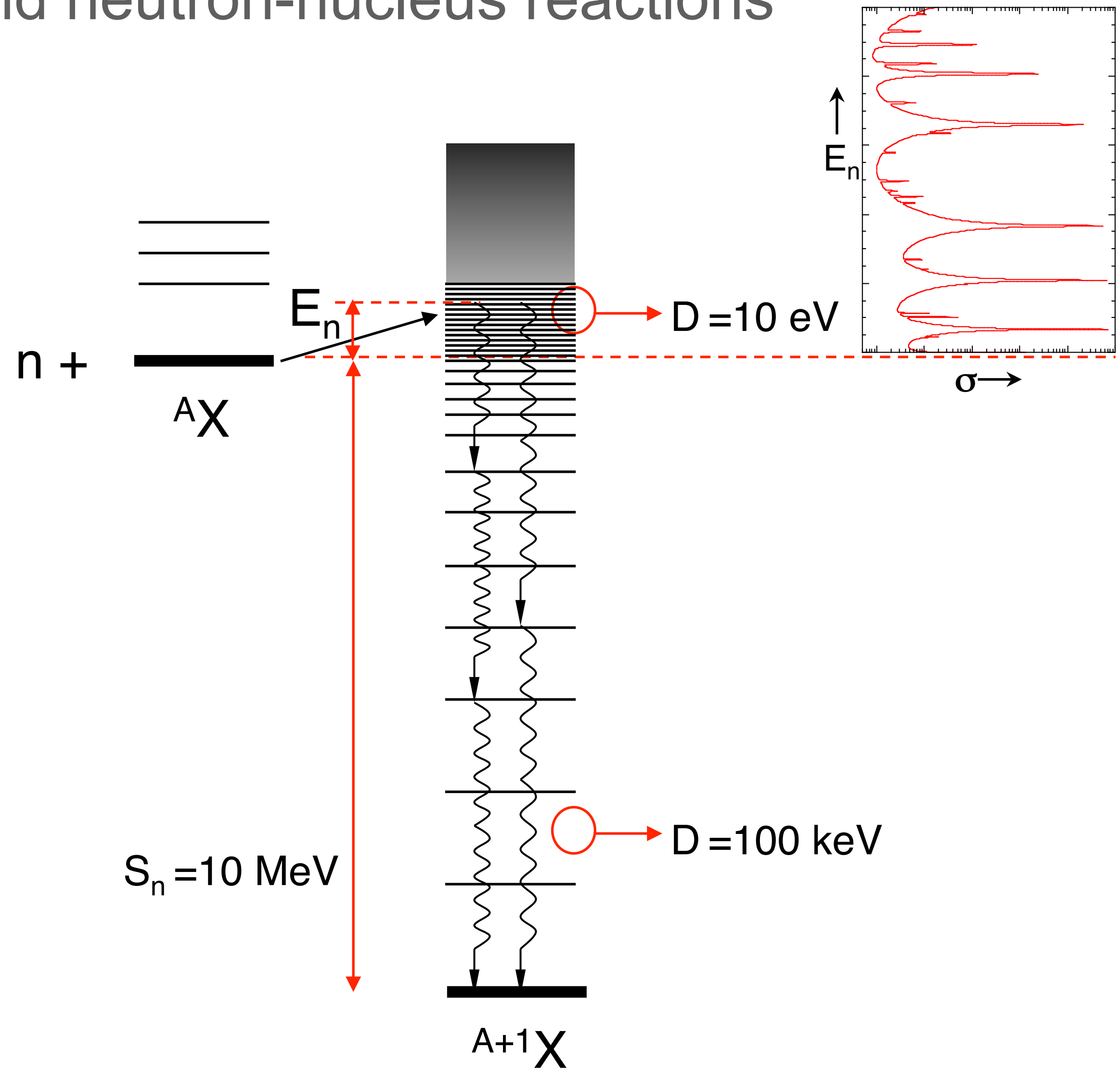


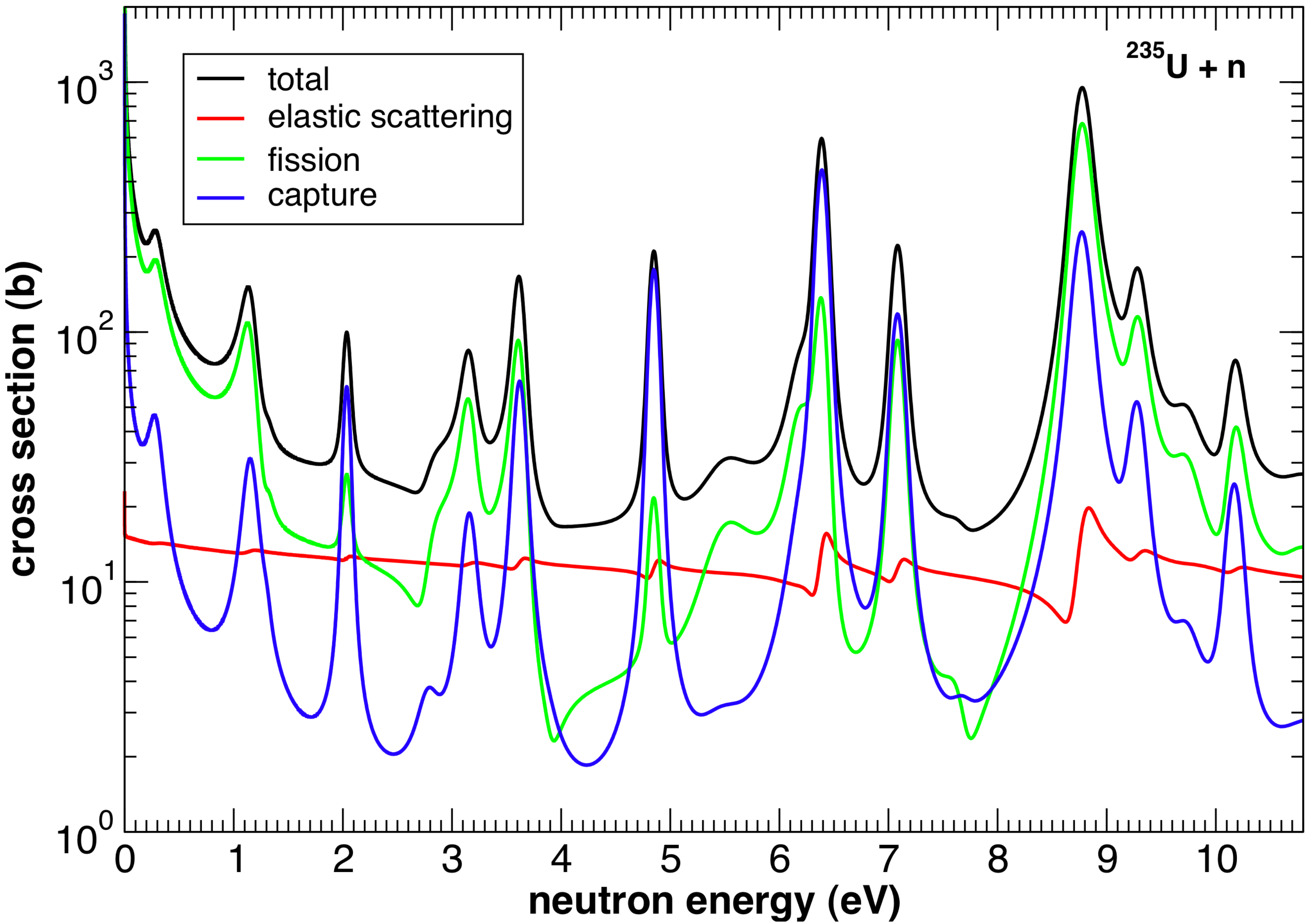


Compound neutron-nucleus reactions

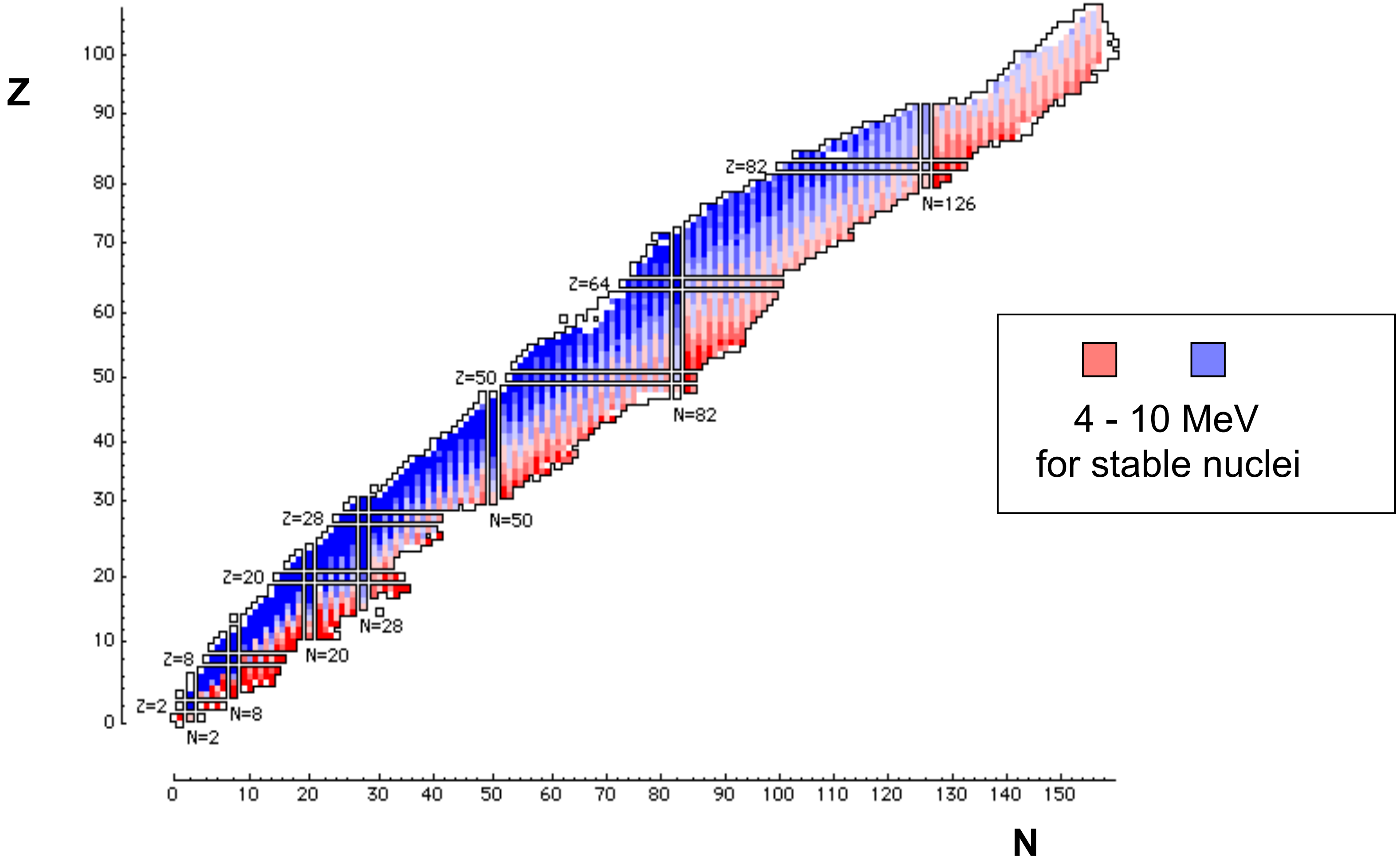




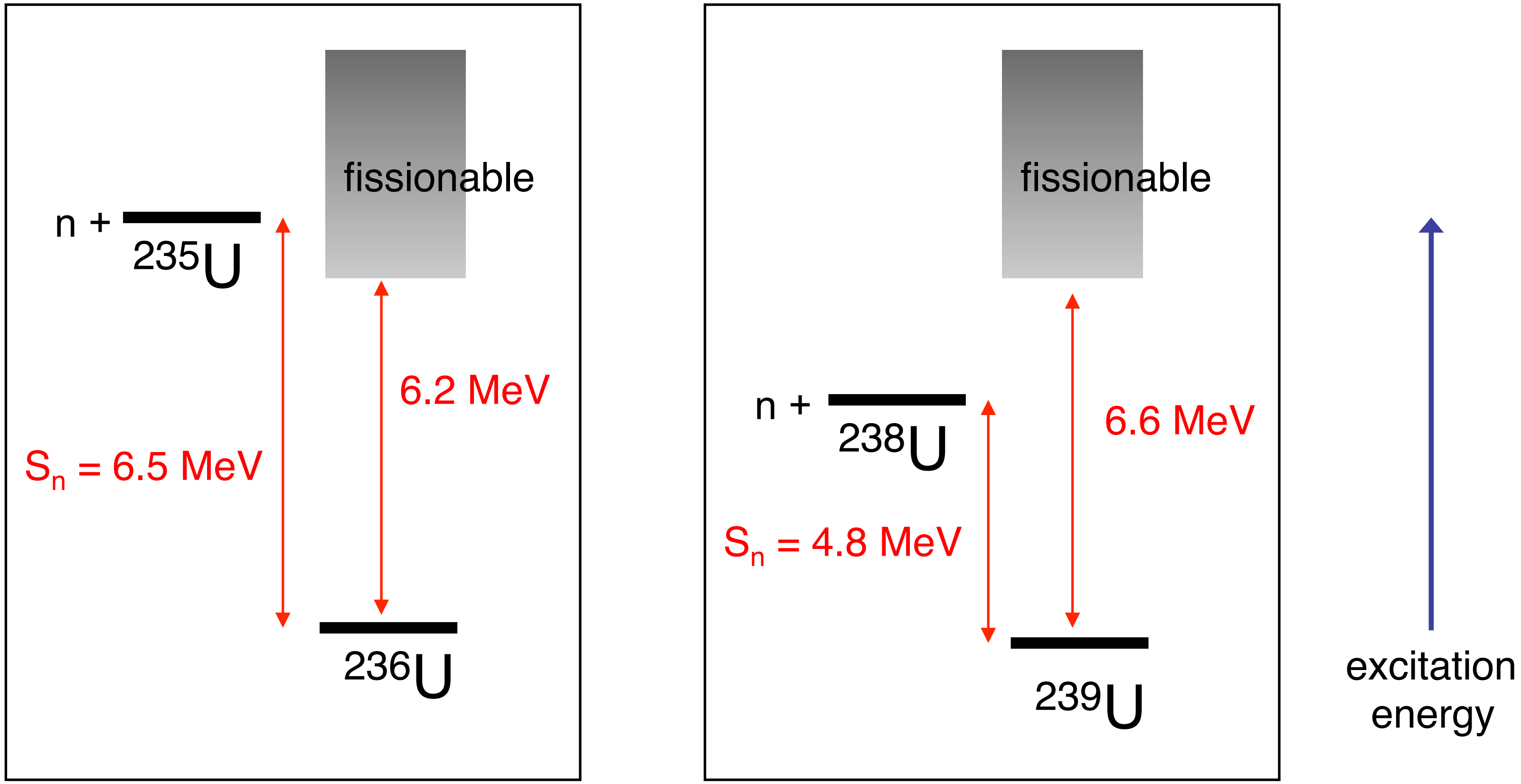
Cross sections σ_T, σ_γ, σ_n et σ_f



Neutron separation energy

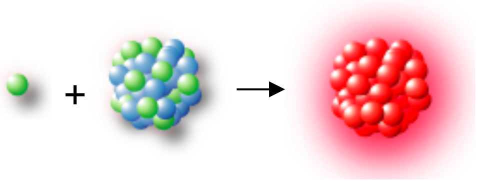


Fission of ^{235}U+n et ^{238}U+n

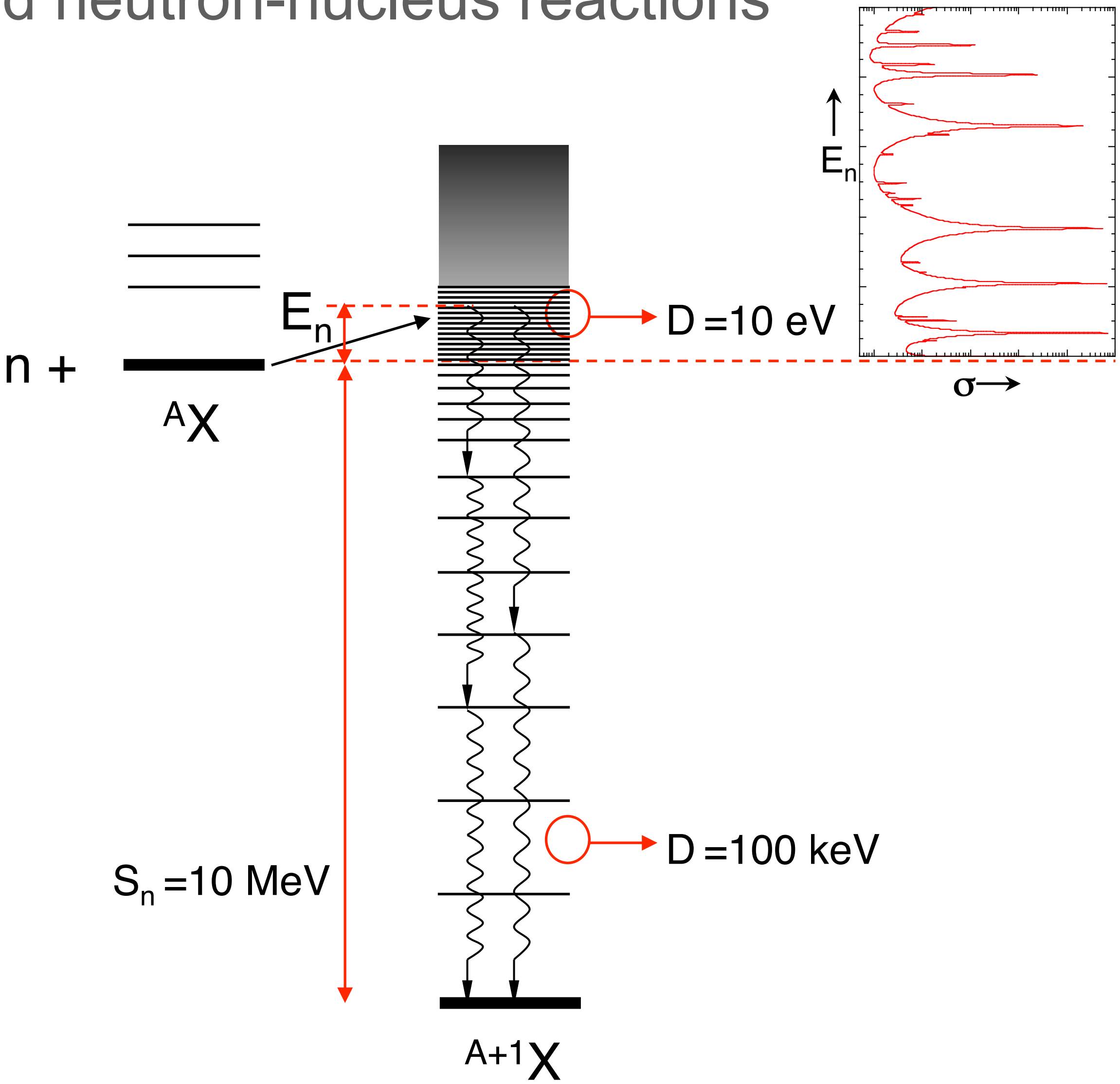


Compound neutron-nucleus reactions

compound
nucleus reaction



$$\tau \sim 10^{-16}$$
$$E_n < 10\ \mathrm{MeV}$$

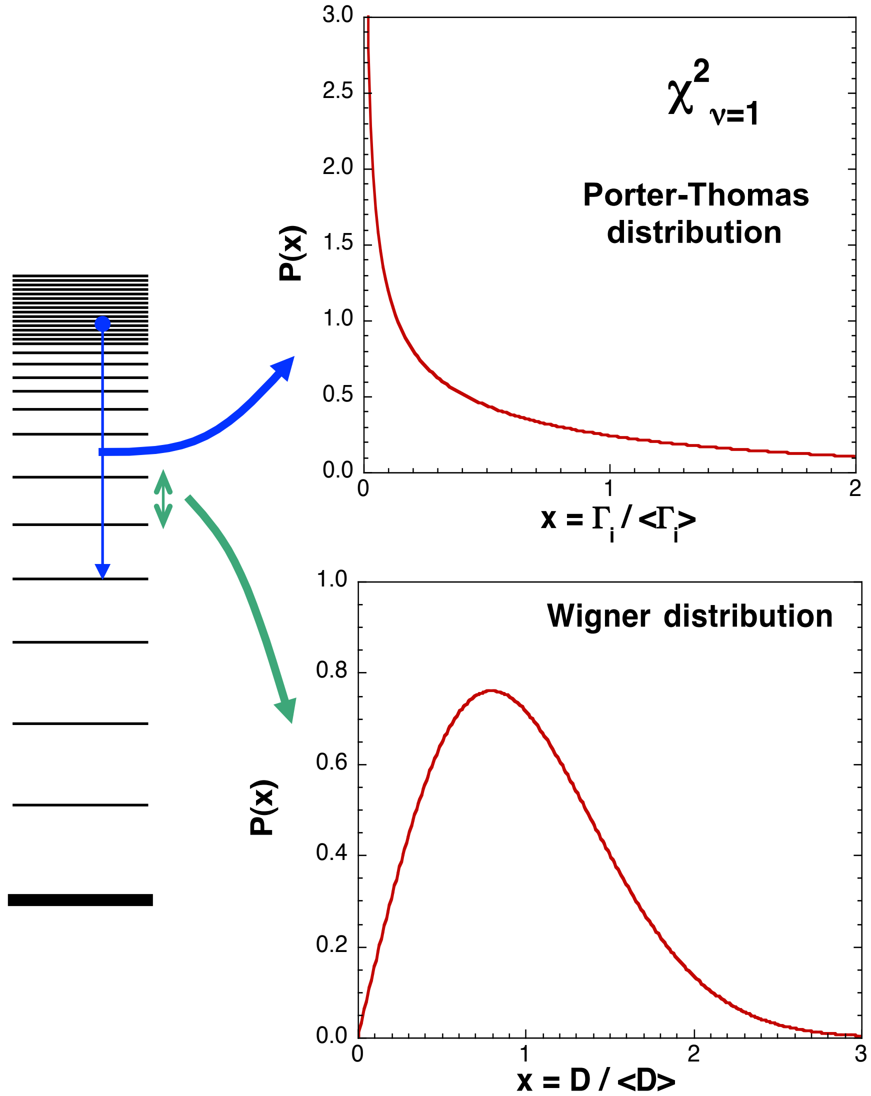


Statistical model

The nucleus at energies around S_n can be described by the
Gaussian Orthogonal Ensemble (GOE)

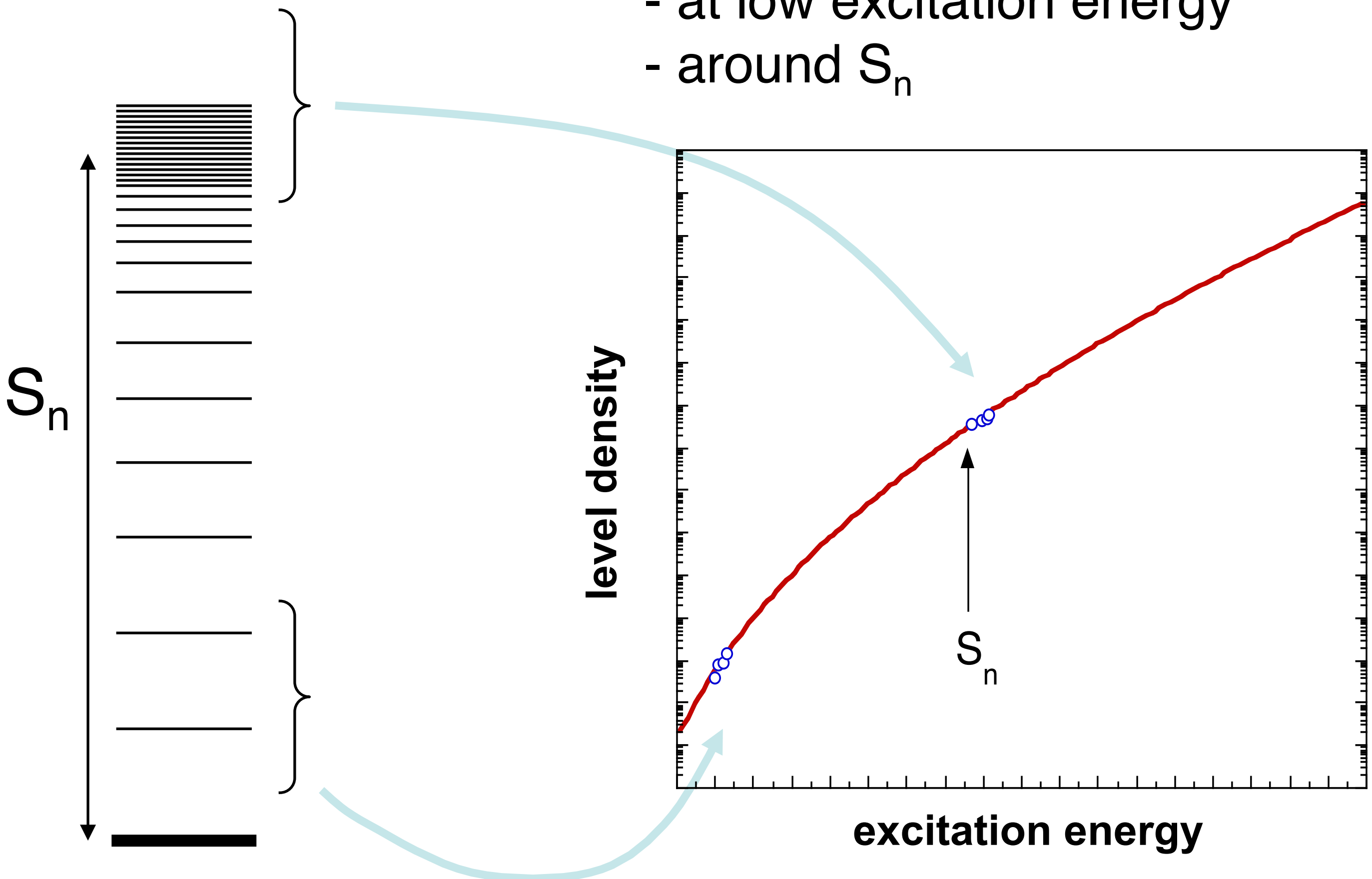
The matrix elements governing the nuclear transitions are random vairables with a Gaussian distribution.

- Consequences:
  - The partial width have a Porter-Thomas distribution
  - The spacing of levels with the same J^{π} have approximately a Wigner distribution.



Level density

Separated states only observable
- at low excitation energy
- around S_n



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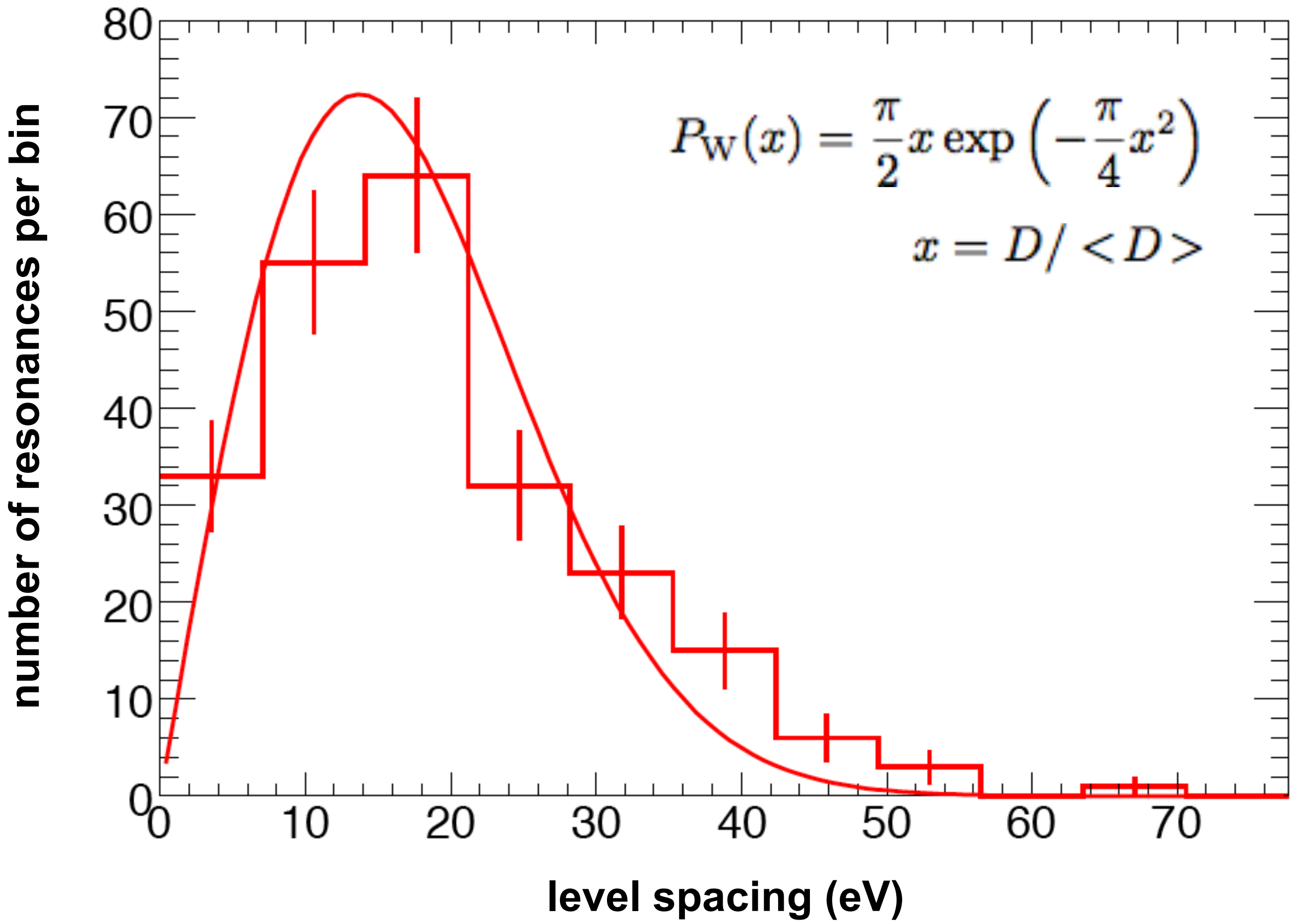
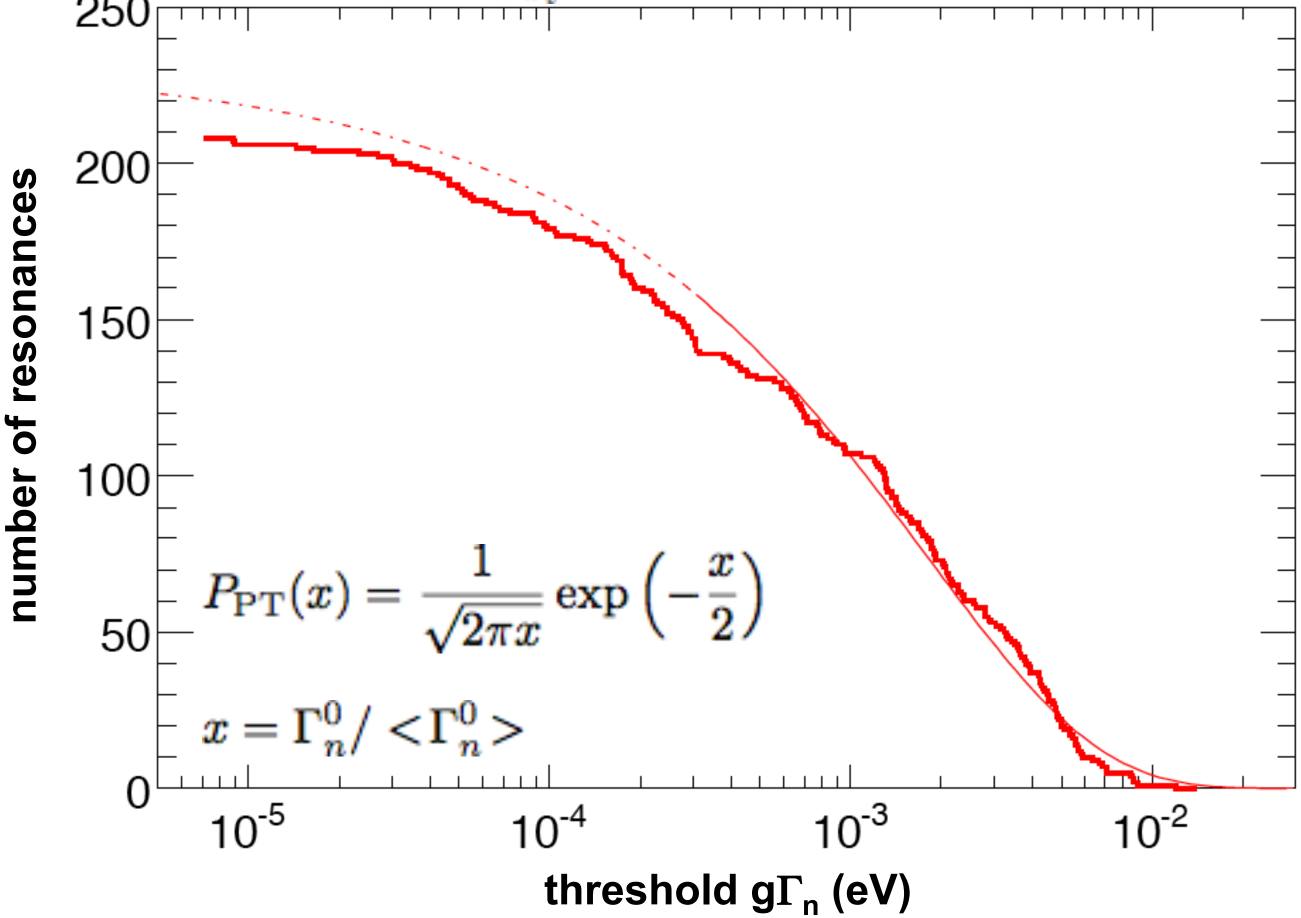
Spacing distribution of two consecutive levels



Neutron width distribution

$$N(x_t) = N_0 \int_{x_t}^{\infty} P_{\mathrm{PT}}(x)\,dx = N_0\left(1 - \mathrm{erf}\sqrt{x_t/2}\right)$$



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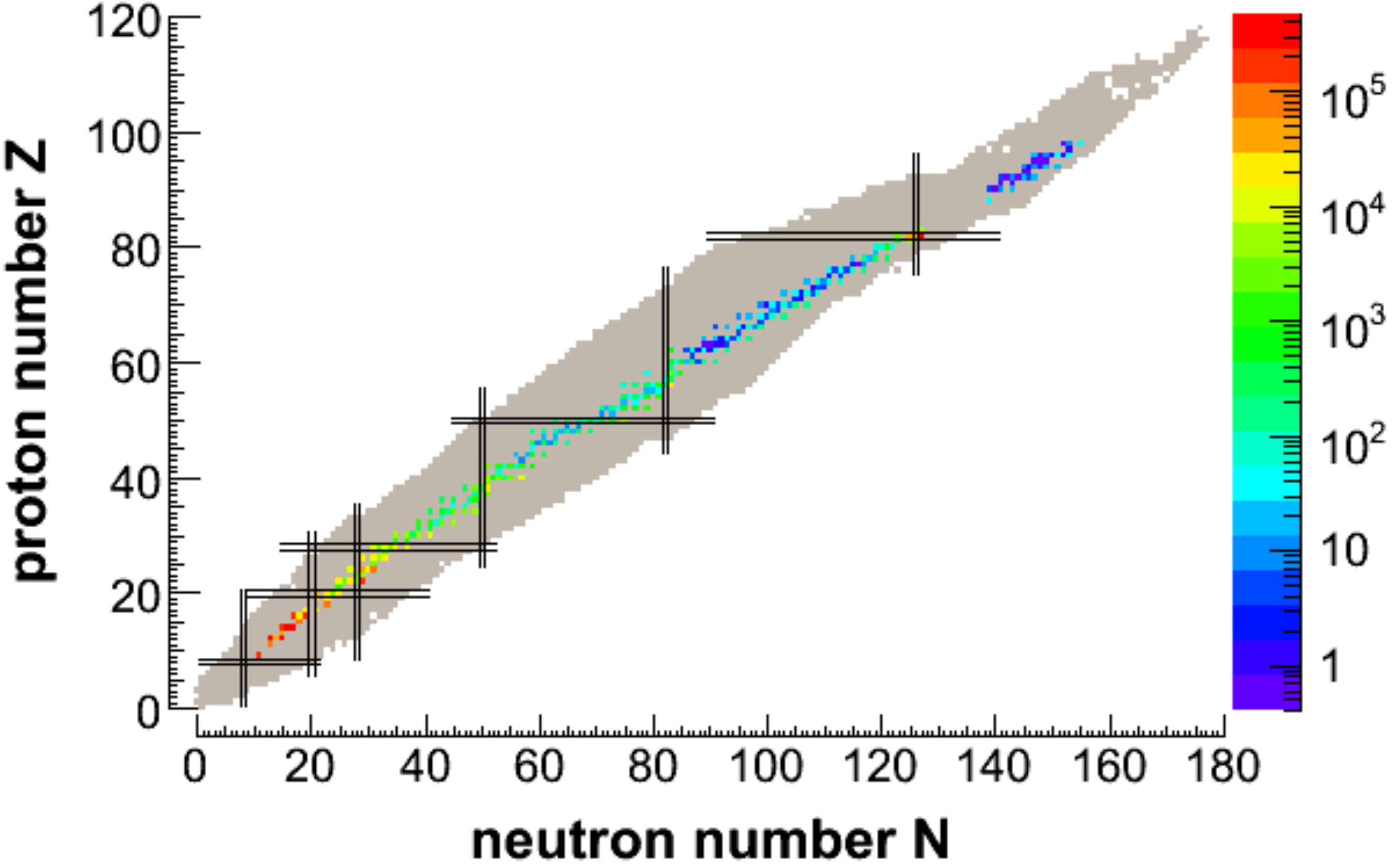
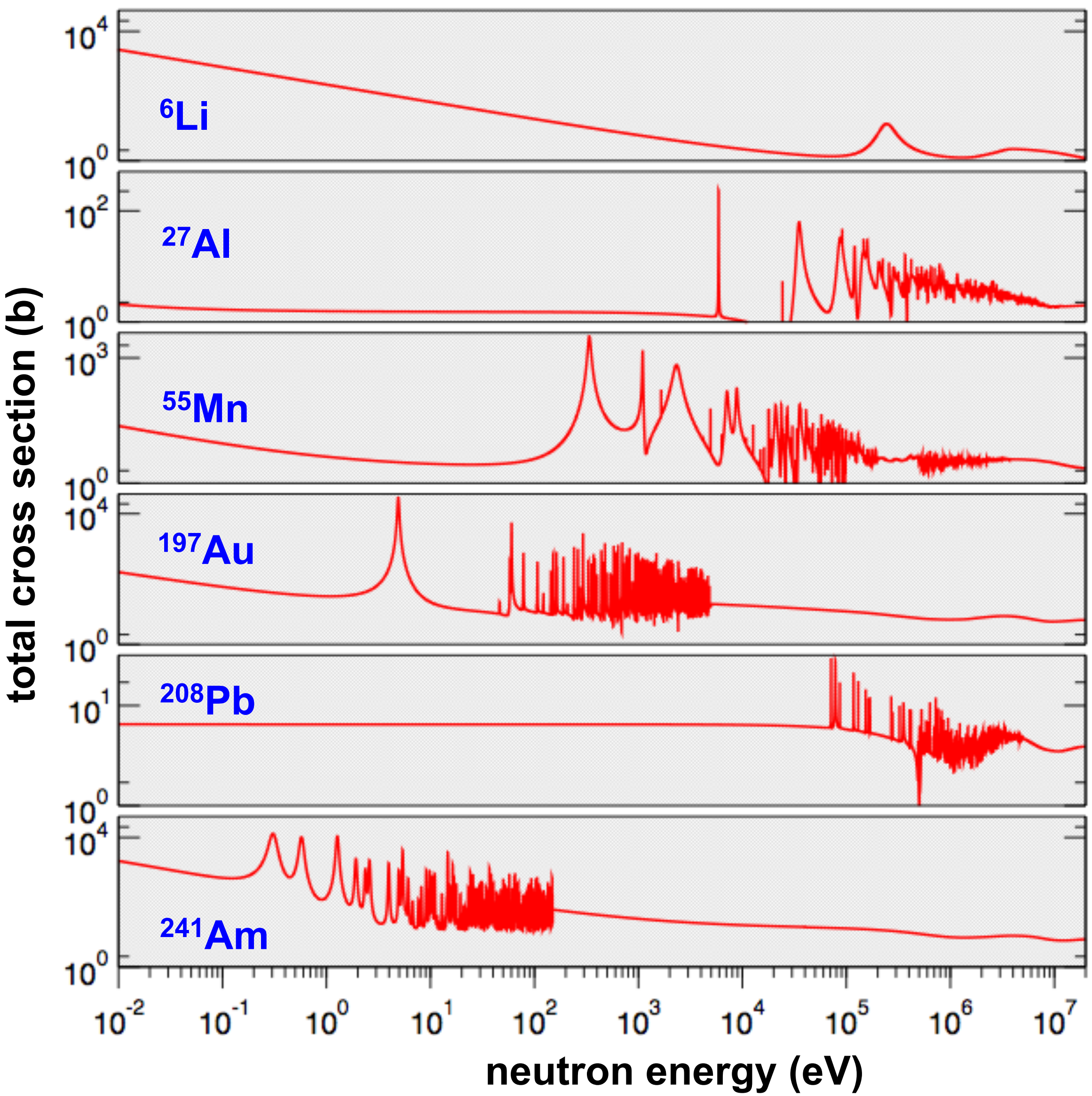
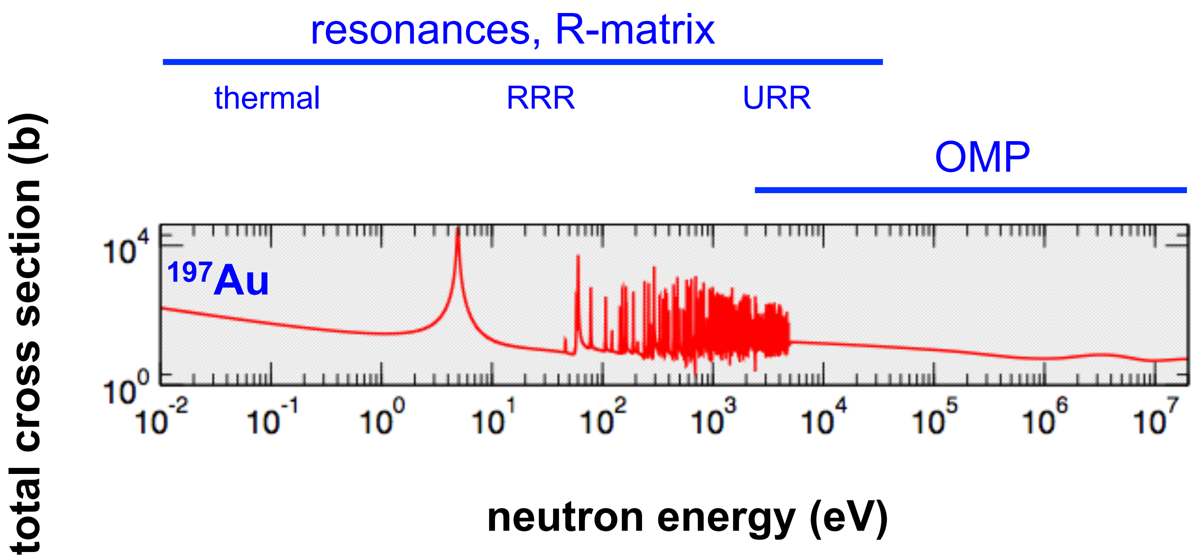
Level spacing D_0



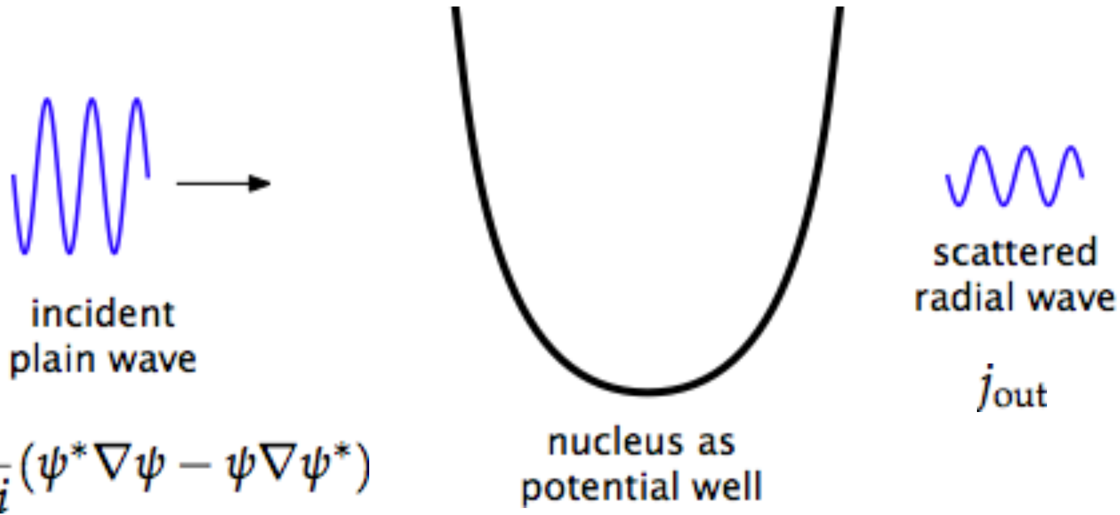


total cross section (b)
neutron energy (eV)
^{6}Li
^{27}Al
^{55}Mn
^{197}Au
^{208}Pb
^{241}Am
10^{4}
10^{0}
10^{2}
10^{0}
10^{3}
10^{0}
10^{4}
10^{0}
10^{1}
10^{0}
10^{4}
10^{0}
10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5} 10^{6} 10^{7}

Compound nucleus reactions



Neutron-nucleus reactions



$$j_{\text{inc}} = \frac{\hbar}{2mi}(\psi^* \nabla \psi - \psi \nabla \psi^*)$$

Conservation of probability density: $\sigma(\Omega) = \frac{r^2 j_{\text{out}}(r,\Omega)}{j_{\text{inc}}}$

Solve Schrödinger equation of system to get cross sections.
Shape of wave functions of in- and outgoing particles are known, potential is unknown. Two approaches:

- **calculate potential (optical model calculations, smooth cross section)**
- **use eigenstates (R-matrix, resonances)**

See: Lane and Thomas, RMP (1957)

The R-matrix formalism

partial incoming wave functions: $\mathcal{I}_c$

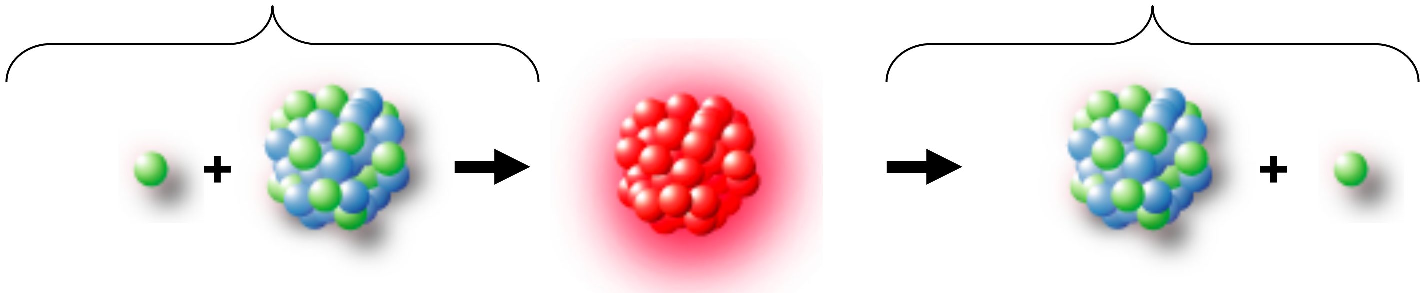
partial outgoing wave functions: $\mathcal{O}_{c'}$

related by collision matrix: $U_{cc'}$

cross section:

$$\sigma_{cc'} = \pi \lambda_c^2 |\delta_{c'c} - U_{c'c}|^2$$

External region ($r>a_c$, well separated particles):
- no interaction, Schrödinger equation solvable.



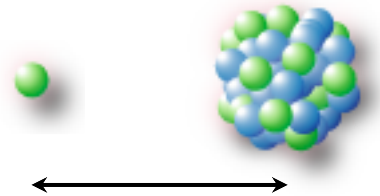
entrance channel
$c = \{\alpha, \ell, j, J, m_J\}$

compound nucleus

exit channel
$c' = \{\alpha', \ell', j', J', m'_J\}$

Internal region ($r<a_c$ compound nucleus):
- wave function is expansion of eigenstates λ.

The R-matrix formalism



r

separation distance

$r > a_c$ external region

$r < a_c$ internal region

$r = a_c$ match value and derivate of Ψ

$$\left[\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2m_c}{\hbar^2}(V(r) - E)\right] R(r) = 0$$

External region: solve Schrödinger equation

central force, separate radial and angular parts.

$$\psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi)$$

solution: solve Schrödinger equation of relative motion:

- Coulomb functions
- special case of neutron particles (neutrons): **Bessel functions**

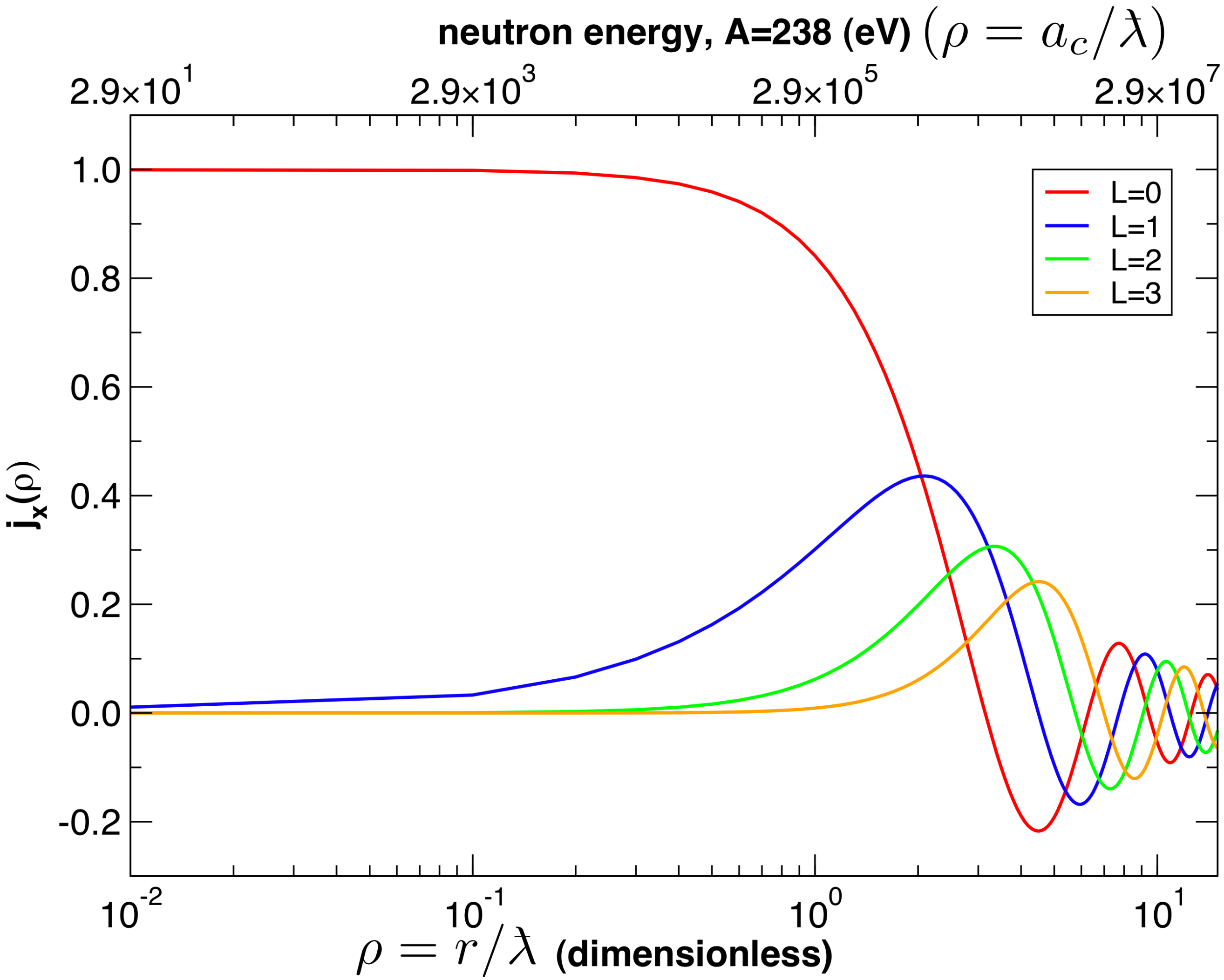
Internal region: Schrödinger equation cannot be solved directly

solution: expand the wave function as a linear combination of its eigenstates.

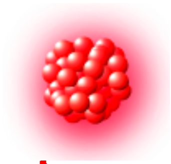
using the R-matrix:

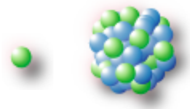
$$R_{cc'} = \sum_{\lambda} \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E}$$

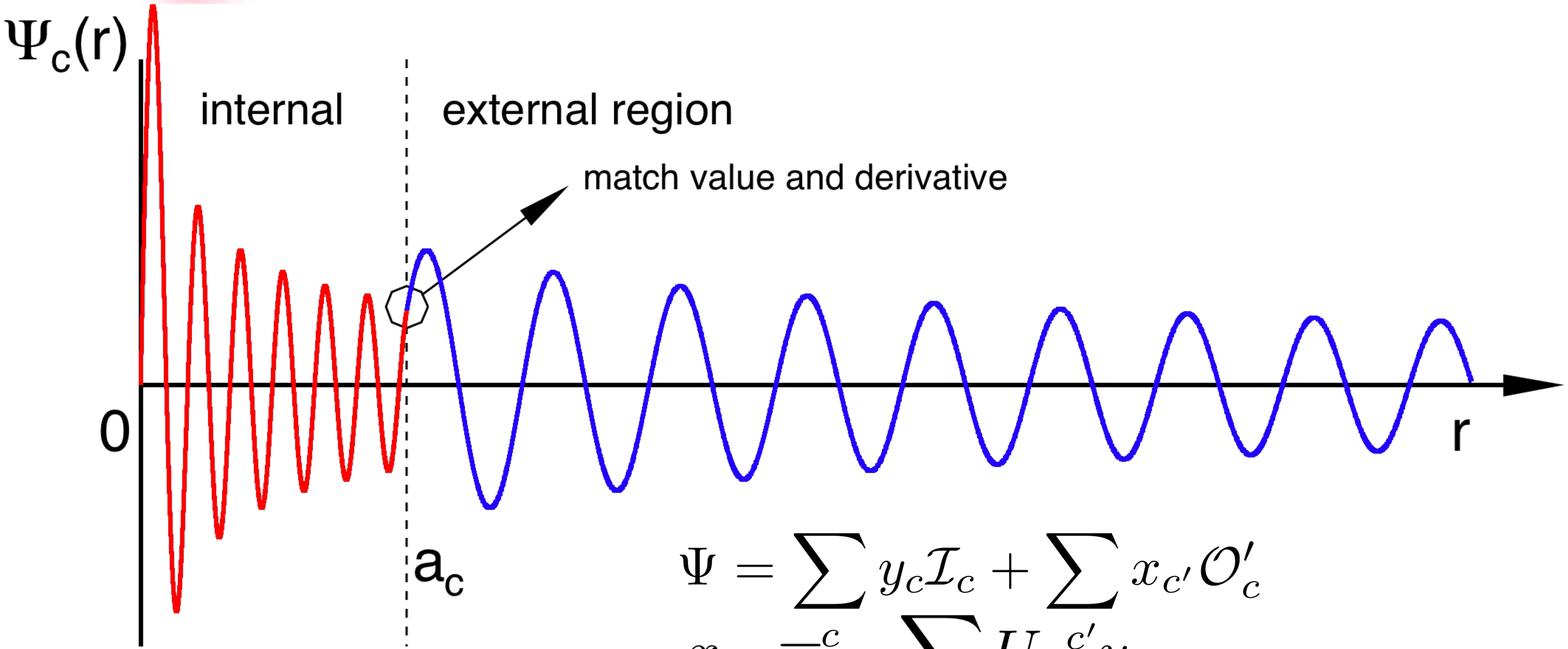
The R-matrix formalism



The R-matrix formalism







$$\Psi = \Psi(R_{cc'})$$

$$R_{cc'} = \sum_{\lambda} \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E}$$

$$\Psi = \sum y_c \mathcal{I}_c + \sum x_{c'} \mathcal{O}'_c$$

$$x_{c'} \equiv^{c} - \sum U_{c'c}^{c'} y_c$$

$$\mathcal{I}_c = I_c r^{-\epsilon 1} \varphi_c i^\ell Y^\ell_{m_\ell}(\theta,\phi)/\sqrt{v_c}$$

$$\mathcal{O}_c = O_c r^{-1} \varphi_c i^\ell Y^\ell_{m_\ell}(\theta,\phi)/\sqrt{v_c}$$

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The R-matrix formalism

The wave function of the system is a superposition of incoming and outgoing waves:

$$\Psi = \sum_c y_c \mathcal{I}_c + \sum_{c'} x_{c'} \mathcal{O}'_c$$

Incoming and outgoing wavefunctions have form:

$$\mathcal{I}_c = I_c r^{-1} \varphi_c i^\ell Y^\ell_{m_\ell}(\theta, \phi)/\sqrt{v_c}$$

$$\mathcal{O}_c = O_c r^{-1} \varphi_c i^\ell Y^\ell_{m_\ell}(\theta, \phi)/\sqrt{v_c}$$

The physical interaction is included in the collision matrix **U**:

$$x_{c'} \equiv -\sum_c U_{c'c} y_c$$

The wave function depends on the R-matrix, which depends on the widths and levels of the eigenstates.

$$\Psi = \Psi(R_{cc'})$$

$$R_{cc'} = \sum_\lambda \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E}$$

The R-matrix formalism

The relation between the R-matrix and the collision matrix:

$$\mathbf{U} = \mathbf{\Omega}\mathbf{P}^{1/2}[\mathbf{1} - \mathbf{R}(\mathbf{L} - \mathbf{B})]^{-1}[\mathbf{1} - \mathbf{R}(\mathbf{L}^* - \mathbf{B})]\mathbf{P}^{-1/2}\mathbf{\Omega}$$

with: $L_c = S_c + iP_c = \left(\frac{\rho}{O_c}\frac{dO_c}{d\rho}\right)_{r=a_c}$

The relation between the collision matrix and cross sections:

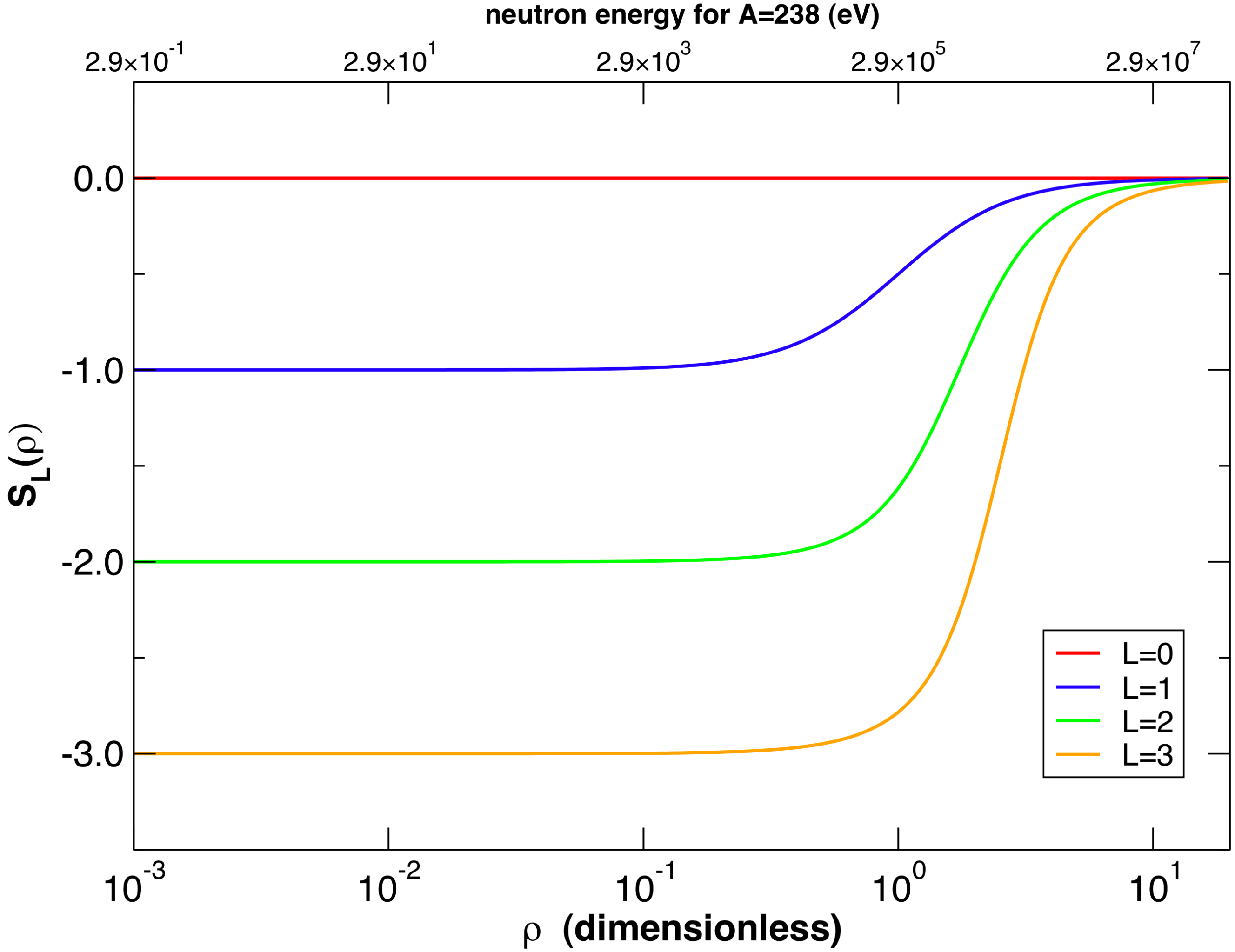
channel to one other channel: $\sigma_{cc'} = \pi\lambda\!\!\!^{-}{}_c^2|\delta_{c'c} - U_{c'c}|^2$

channel to any other channel: $\sigma_{cr} = \pi\lambda\!\!\!^{-}{}_c^2(1 - |U_{cc}|^2)$

channel to same channel: $\sigma_{cc} = \pi\lambda\!\!\!^{-}{}_c^2|1 - U_{cc}|^2$

channel to any channel (total): $\sigma_{c,T} = \sigma_c = 2\pi\lambda\!\!\!^{-}{}_c^2(1 - \mathrm{Re}\, U_{cc})$

The R-matrix formalism



The R-matrix formalism

The Breit-Wigner Single Level approximation:

total cross section:

$$\sigma_c = \pi \lambda\!\!\!^{-2}_c g_c \left(4 \sin^2 \phi_c + \frac{\Gamma_\lambda \Gamma_{\lambda c} \cos 2\phi_c + 2(E - E_\lambda - \Delta_\lambda)\Gamma_{\lambda c} \sin 2\phi_c}{(E - E_\lambda - \Delta_\lambda)^2 + \Gamma_\lambda^2/4} \right)$$

neutron channel: $c = n$

only capture, scattering, fission: $\Gamma_\lambda = \Gamma = \Gamma_n + \Gamma_\gamma + \Gamma_f$

other approximations: $\ell = 0 \quad \cos\phi_c = 1 \quad \sin\phi_c = \rho = k a_c \quad \Delta_\lambda = 0$

total cross section:

potential; interference; elastic; capture; fission

$$\sigma_T(E) = \underbrace{4\pi R'^2}_{\text{potential}} + \pi \lambda\!\!\!^{-2} g \left(\frac{\overbrace{4\Gamma_n (E - E_0) R'/\lambda\!\!\!^{-}}^{\text{interference}} + \overbrace{\Gamma_n^2}^{\text{elastic}} + \overbrace{\Gamma_n \Gamma_\gamma}^{\text{capture}} + \overbrace{\Gamma_n \Gamma_f}^{\text{fission}}}{(E - E_0)^2 + (\underbrace{\Gamma_n + \Gamma_\gamma + \Gamma_f +}_{\text{total width}})^2/4} \right)$$

total width

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The R-matrix formalism

The Reich-Moore approximation:

Use the fact that there are many photon channels, with Gaussian distributed amplitudes with zero mean:

$$< \gamma_{\lambda c}\gamma_{\mu c} >= \gamma^2_{\lambda c}\delta_{\lambda\mu}$$

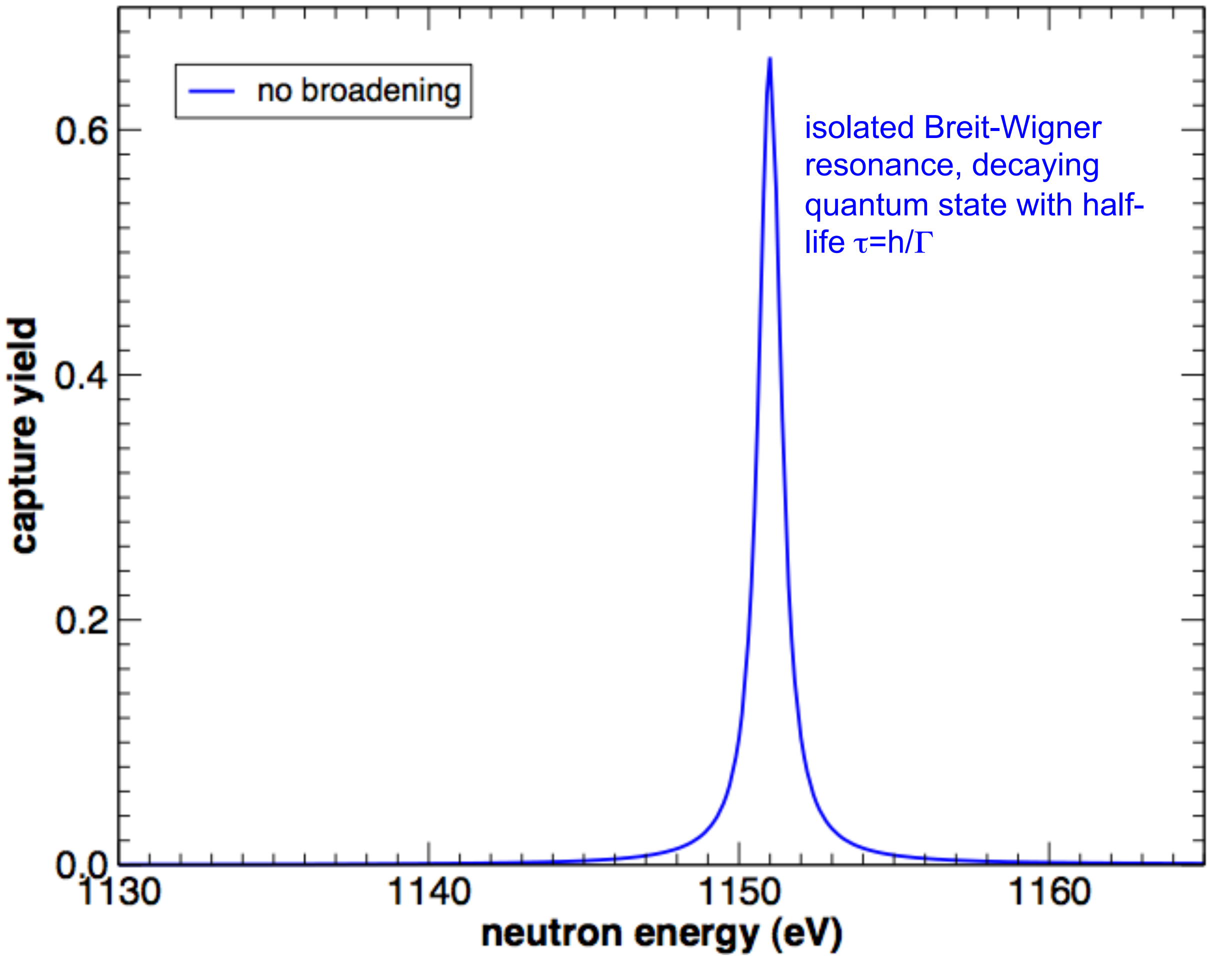
The sum over the amplitudes of the photon channels becomes then:

$$\sum_{c\in\text{photon}} \gamma_{\lambda c}\gamma_{\mu c} = \sum_{c\in\text{photon}} \gamma^2_{\lambda c}\delta_{\lambda\mu} = \Gamma_{\lambda\gamma}\delta_{\lambda\mu}$$

Then photon channels can be eliminated in the R-matrix:

$$R_{cc'} = \sum_{\lambda} \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E - i\Gamma_{\lambda\gamma}/2} \qquad c \notin \text{photon}$$

Measured reaction yield



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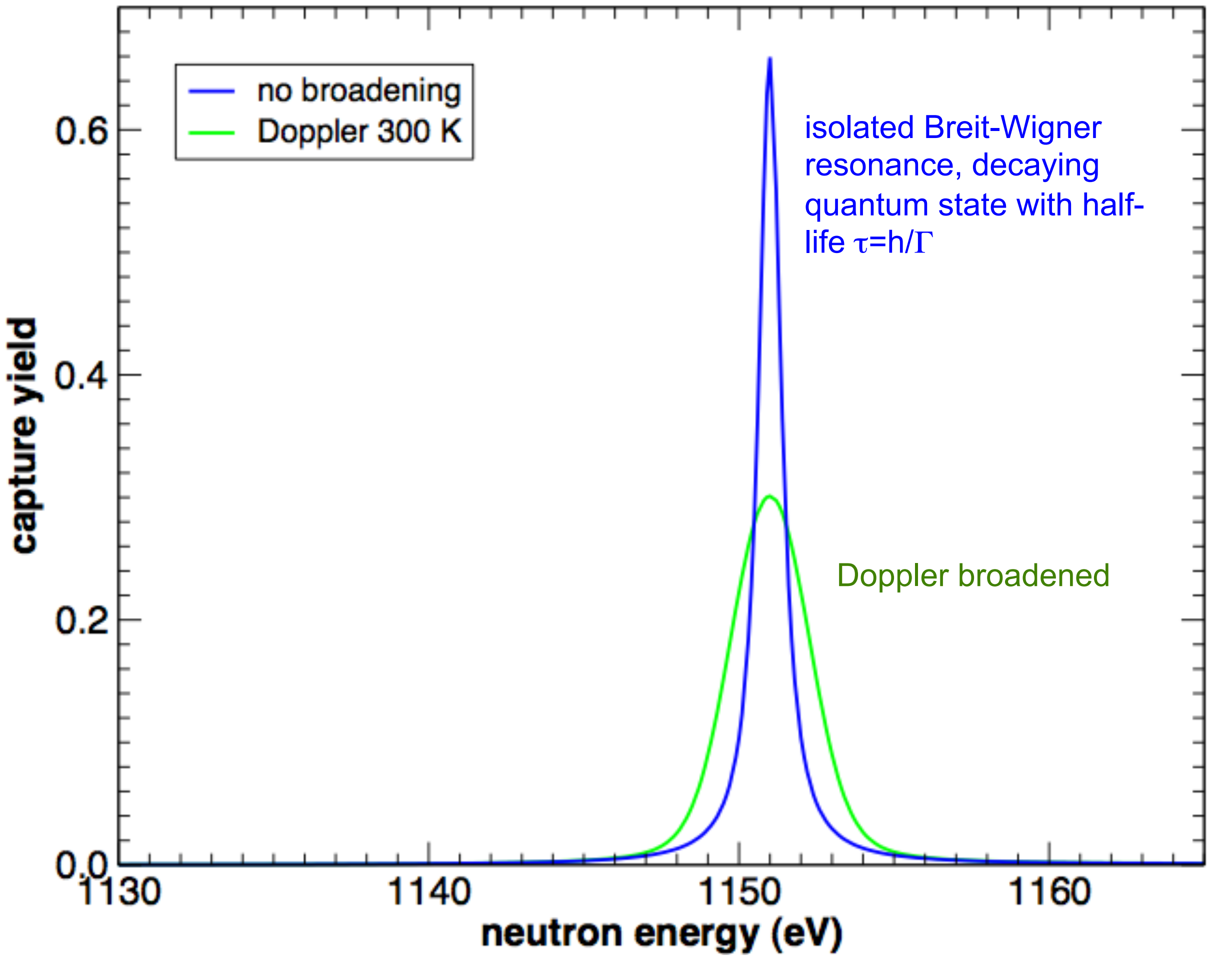

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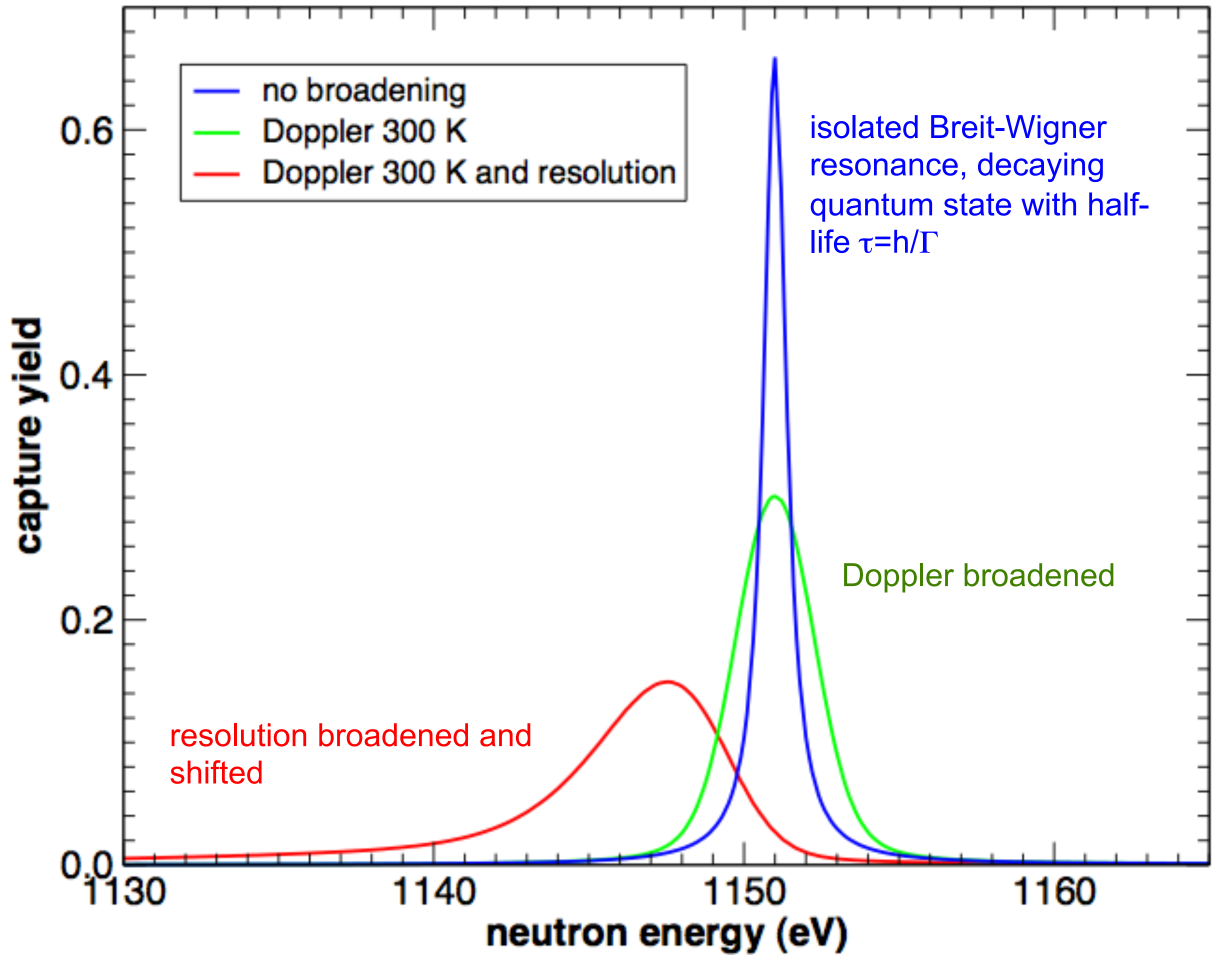

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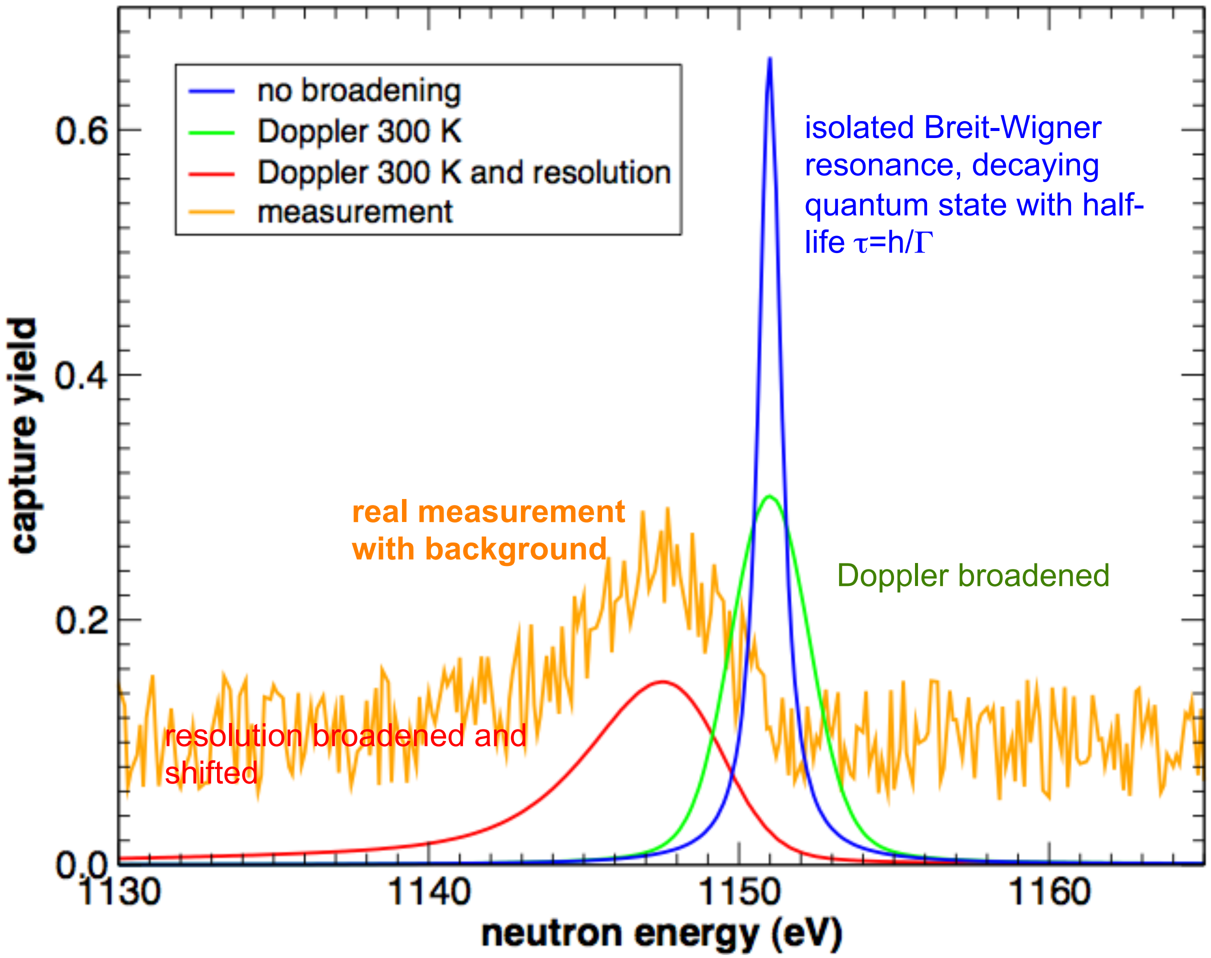
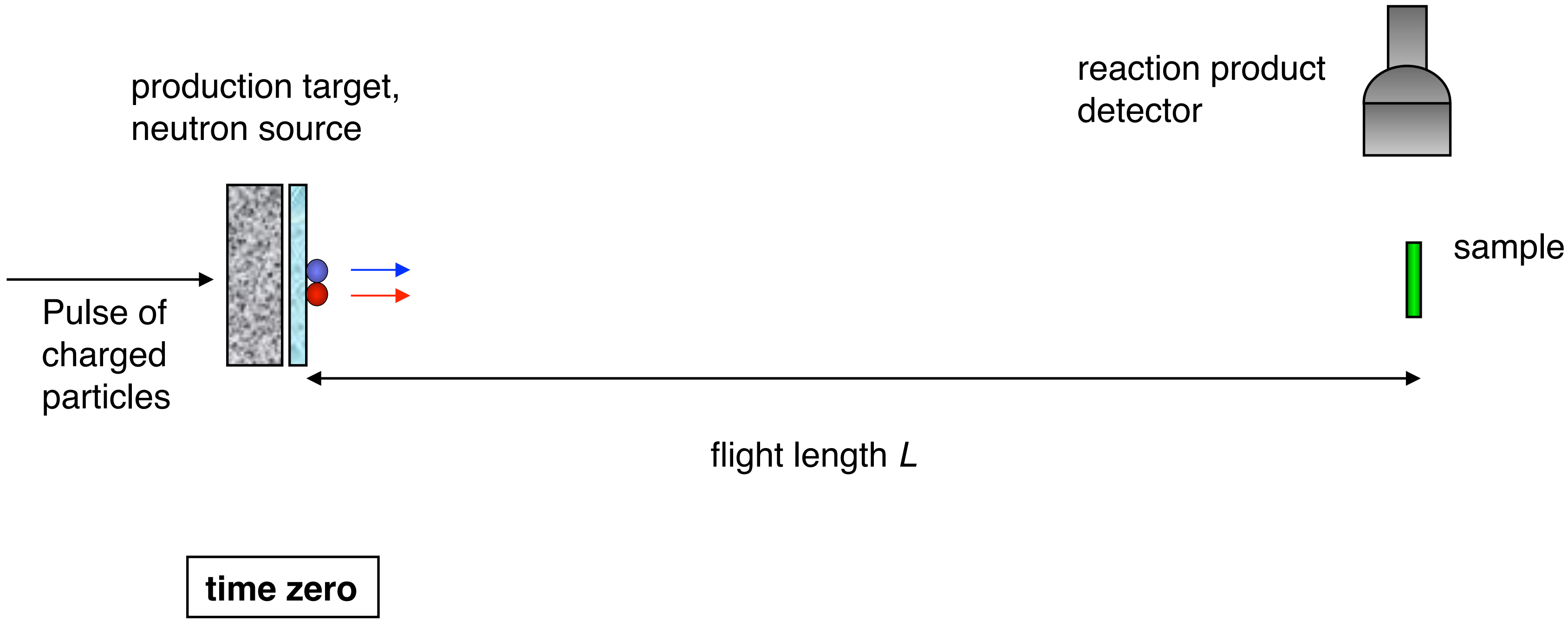
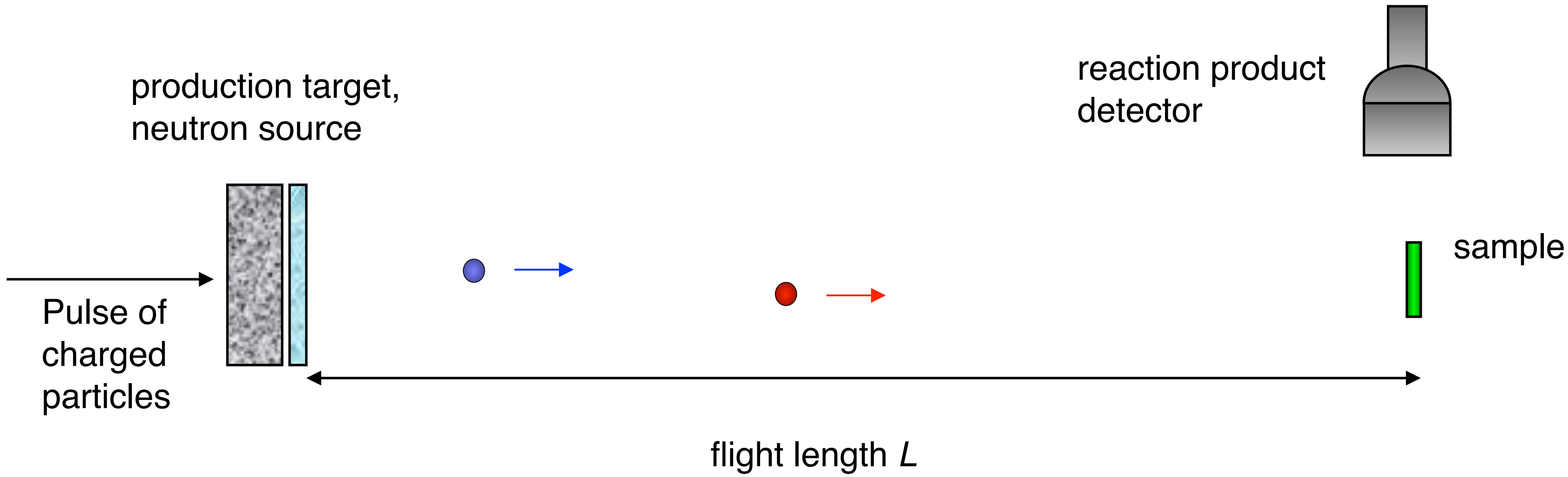


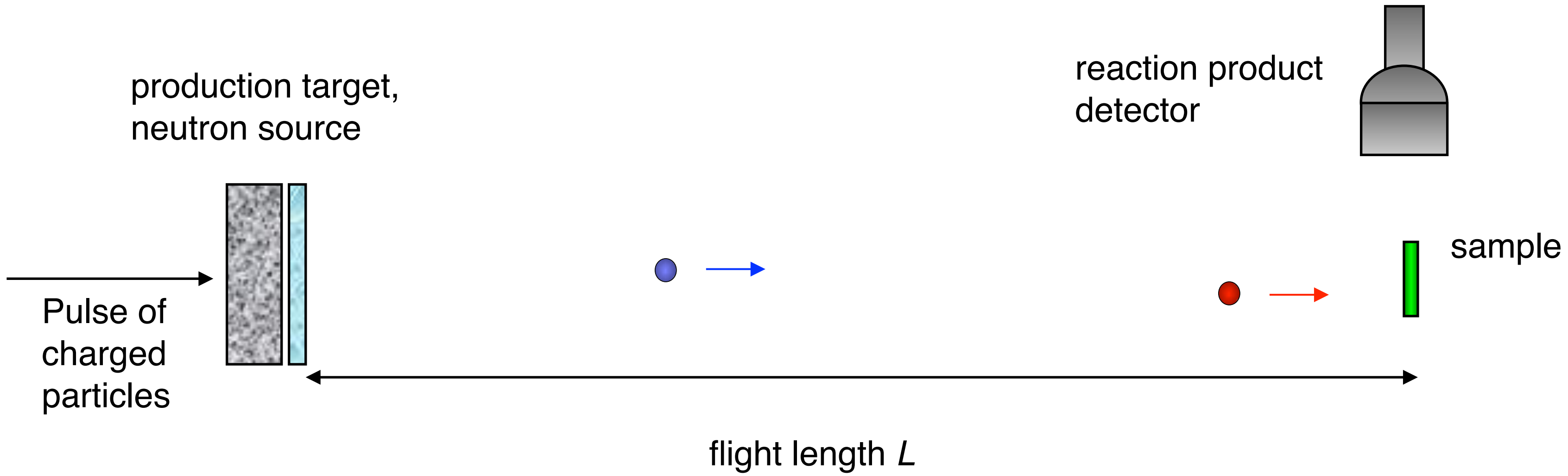
Measuring a reaction yield using the time-of-flight technique



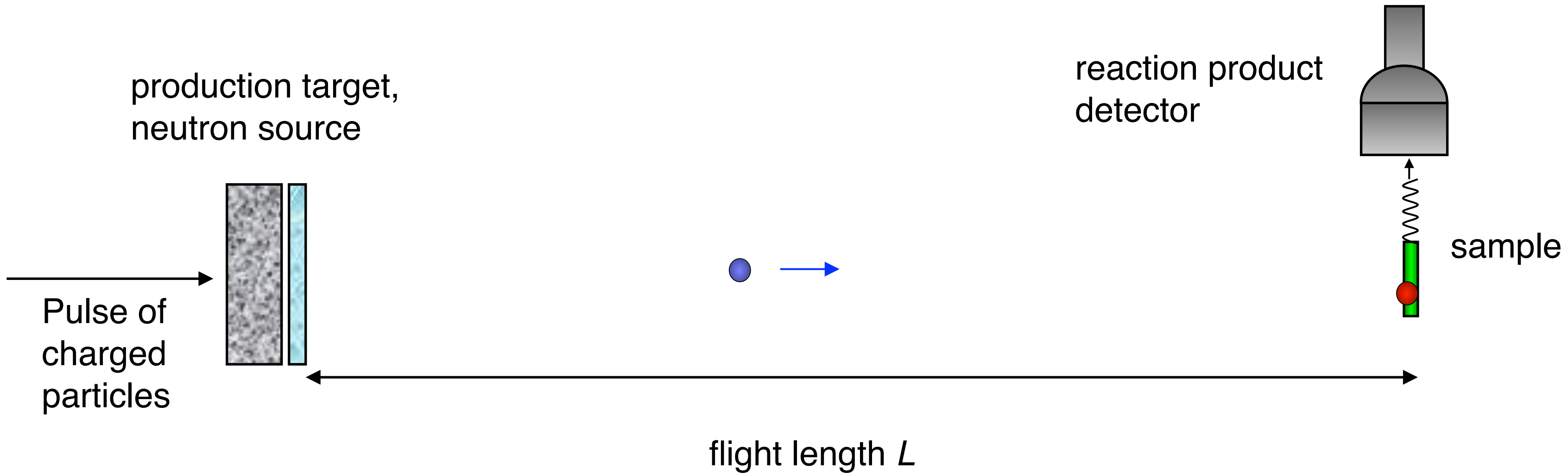
Measuring a reaction yield using the time-of-flight technique



Measuring a reaction yield using the time-of-flight technique

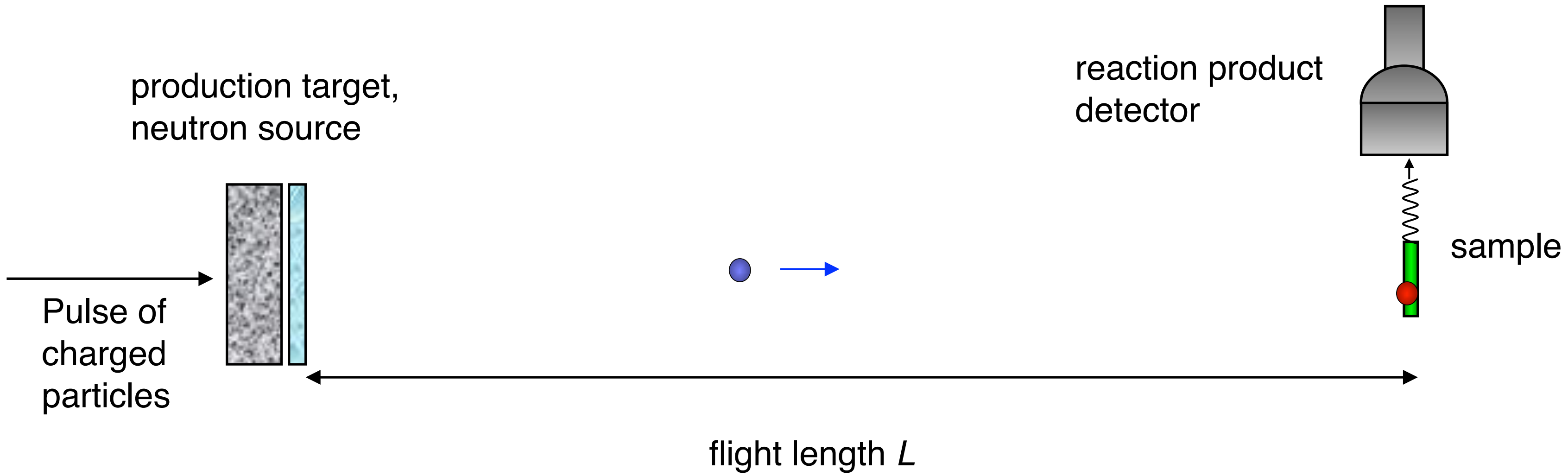


Measuring a reaction yield using the time-of-flight technique



time of flight *t*

Measuring a reaction yield using the time-of-flight technique



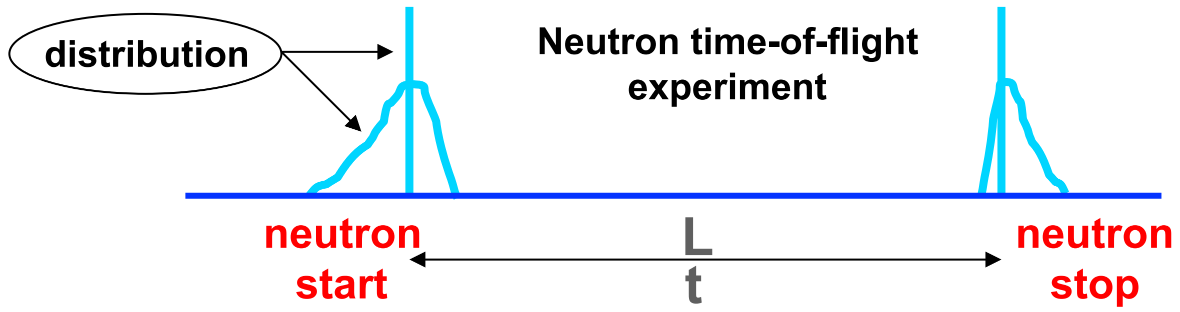
time of flight *t*

Kinetic energy of the neutron by time-of-flight

$$E_n = E_{tot} - mc^2 = c^2p^2 + m^2c^4 - mc^2 = mc^2(\gamma - 1) \qquad \gamma = (1 - v^2/c^2)^{-1/2}$$

$$E_n = \frac{1}{2}mv^2 = \alpha^2 \cdot \frac{L^2}{t^2}$$

Resolution



time-energy relation $\sqrt{E} = \alpha \frac{L}{t}$

- neutron time-of-flight: $t + \delta t$
- flight length: $L + \delta L$
- neutron kinetic energy: $E + \delta E$

The resolution can be expressed equivalenty in time, distance and energy:

$$R_t(\delta t)d\delta t = R_L(\delta L)d\delta L = R_E(\delta E)d\delta E$$

Comparing neutron time-of-flight facilities

- **facility characteristics**
  - instantaneous flux (neutrons per pulse)
  - average flux (neutrons per second)
  - resolution (resonance shape analysis)
  - background

- **facility equipment**
  - detectors
  - samples
  - data acquisition

irfu
cea
saclay

Comparing some neutron time-of-flight facilities

Facility	Location	Particle	Beam energy (MeV)	Neutron target	Pulse width (ns)	Beam power (kW)	Pulse frequency (Hz)	Flight path lengths (m)	Neutron production (n/pulse)
RPI	RPI, Troy, USA	e-	60	Ta	5	0.6	500	15–250	3.6×10^{9}
		e-	60	Ta	5,000	>10	300	15, 25	4.8×10^{11}
ORELA	ORNL, Oak Ridge, USA	e-	180	Ta	2–30	60	12–1,000	9–200	1×10^{12}
GELINA	EC-JRC-IRMM, Geel, Belgium	e-	100	U	1	10	40-800	5–400	4.3×10^{10}
nELBE	FZD, Rossendorf, Germany	e-	40	L-Pb	0.01	40	500,000	4	5.4×10^{7}
IREN	JINR, Dubna, Russia	e-	30	W	100	0.42	50	10–750	7.7×10^{10}
PNF	PAL, Pohang, Korea	e-	75	Ta	2,000	0.09	12	11	1.7×10^{10}
KURRI	Kumatori Japan	e-	46	Ta	2	0.046	300	10, 13, 24	2×10^{9}
		e-	30	Ta	4,000	6	100	10, 13, 24	8×10^{10}
LANSCE-MLNSC	LANL, Los Alamos, USA	p	800	W	135	80	20	7–60	7×10^{14}
LANSCE-WNR	LANL, Los Alamos, USA	p	800	W	0.2	1.44	13,900	8–90	8×10^{9}
n_TOF	CERN, Geneva, Switzerland	p	20,000	Pb	6	10	0.4	185	2×10^{15}
MLF-NNRI	J-PARC, Tokai, Japan	p	3,000	Hg	1,000	1,000	25	30	1.2×10^{17}

From: D. G. Cacuci (ed.), Handbook of Nuclear Engineering,
R. C. Block, Y. Danon, F. Gunsing, R. C. Haight
Chapter: Neutron Cross Section Measurements

Thank you for your attention.