Radiative neutron capture and photonuclear data in view of global information on electromagnetic strength and level density.

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> Giant dipole resonances and nuclear shapes Spreading width and electric dipole transitions Magnetic dipole transition strength Level densities and radiative capture Conclusions





## Dipole strength $f_1$

#### $f_{E1}$ is controlled by the <u>isovector giant dipole resonance GDR</u>.



Bartholomew et al., Advances in Nuclear Physics 7(1973) 229



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Axel, Phys. Rev. 126 (1962) 671





The E1-strength at low energy governs radiative capture processes; it is proportional to deviation from sum rule and to GDR width –

their determination needs special care !

P.Carlos et al., NPA 172 (1971) 437

Bohr & Mottelson, Nuclear Structure, Vol. II







Global parametrization for GDR splitting, width and strength

3 parameters – in addition to the B(E2)-values and 2 parameters determined by fit to masses – suffice to describe the GDR not only for the Nd-isotopes, but for all nuclei with A > 60.





GDR-energies as obtained from Lorentzian fits (1 or 2 poles, why not 3?)

average GDR-energies  $E_0$  are well predicted using mass fits (FRDM) and  $\underline{m^* = 874 \text{ MeV}}$ 

V.A. Pluiko, www-nds.iaea.org/RIPL-3/gamma/gdrparameters-exp.dat; subm. to ADNDT; R. Capote et al., NDS 110 (2009) 3107

W.D. Myers et al., PR C 15 (1977) 2032 A.Junghans et al., PLB 670 (2008) 200





## <u>Experimental</u> study (Coulex etc.) of >150 nuclei reveals close correlation between E2-collectivity $q_2$ and axiality $q_3$ (both are rotation invariant observables)









K. Kumar, Phys. Rev. Lett. 28, 249 (1972)







GDR-widths as obtained from Lorentzian fits (1 or 2 poles, why not 3?)

average GDR-widths are well predicted by hydrodynamics using wall formula  $\Gamma_0 = 0.05 \cdot E_0^{1.6}$ ; which leads for the 3 components to  $\Gamma_k = 0.05 \cdot E_k^{1.6}$ .

V.A. Pluiko, www-nds.iaea.org/RIPL-3/gamma/gdrparameters-exp.dat; subm. to ADNDT; R. Capote et al., NDS 110 (2009) 3107 A.

A. Junghans et al., PLB 670 (2008) 200

B. Bush and Y. Alhassid, NPA 531 (1991) 27





GDR-integrals as obtained from Lorentzian fits (1 or 2 poles, why not 3?)



M. Gell-Mann et al., PR 95 (1954) 1612



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subm. to ADNDT; R. Capote et al., NDS 110 (2009) 3107

#### **Triple Lorentzians (TLO) compared to dipole data for prolate and oblate nucleus** good description of photon-data in GDR and (n,y)-data below



S.F. Mughabghab, C.L. Dunford, PLB 487(00)155

A. Junghans et al., PLB 670 (2008) 200

G.M.Gurevich et al., NPA,351(81) 257 A.M.Goryachev,G.N.Zalesnyy, YF27(78)1479





#### *Triple Lorentzians* (TLO) *compared to dipole data for 'quasi-magic' nuclei; small* B(E2) - i.e. *small* $q_2$ - *leads to unsignificant deformation induced split.*





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R.D.Starr et al., PRC (82) 25 780 R.R.Harvey et al., PR136 (64) B126



#### Triple Lorentzians (TLO) compared to GDR for A=78 and A=238

with contradicting data



P. Carlos et al., NPA 258 (76) 365 A.M.Goryachev, G.N.Zalesnyy, VTYF 8 (82) 121

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A. Junghans et al., PLB 670 (2008) 200

Y. Birenbaum et al, PRC36(87)1293 A. Veyssière, et al., NPA 199 (73) 45 G.M.Gurevich et al., NPA 273 (76) 326





Simple Lorentzian fits to GDR data may result in too large  $f_{E1}$ 

because of no deformation induced split – or erroneous normalization of CEA-data







A good global descrption of the GDR shapes is possible (on absolute scale), when triaxial nuclei are treated as such (they are <u>not</u> rare) =>TLO.

The global TLO fit results in a smooth A-dependence of the spreading width and allows to obey the GGT sum rule (no need for pionic effects).
The resulting E1 strength can thus be considered reliable.

GDR peak shape does <u>not</u> have an impact on  $f_{E1}$  at low  $E_{\gamma}$ , but it optimizes global fit. Local fits with 3 resonances would need additional input.

Three questions remain, before this TLO- $f_{E1}$  is used for radiative processes:

1. What role play magnetic dipole transitions (M1)?

2. Can it be extrapolated to low energies (2-5 MeV)?

3. Can the effect of  $f_1$  be distinguished from the impact of the level density and its energy dependence?





Triple Lorentzian (TLO)  $-f_{E1}$  fitted globally to GDR's – inserted into TALYS

> = several exit channels well described simultaneously



M. Erhard et al., PRC81(10)034319

Calculations with TALYS, A.Koning et al., Rusev et al., PRC 79 (09) 061302 A. Junghans et al., PLB 670 (08) 200 H. Beil et al., Nucl. Phys. A 227 (74) 427





## E1 and M1 in the shell model

For the magic nucleus <sup>208</sup>Pb particle – hole calculations in a shell model basis are feasable. The resulting GDR (E1) has a Lorentzian shape.

Equivalent calculations of the M1 strength show a much narrower distribution – more like a Gaussian.

The widespread use of a Lorentzian also for M1 appears to be justified by analogy only. More M1 data are needed.



B.A.Brown, PRL 85 (2000) 5300 , cf. R.Schwengner et al., PRC 81, 054315 (2010)







M1 strength in heavy nuclei well described by 3 Gaussians with a total strength of  $< 0.2 A \mu_N^2$ .

K.Heyde et al., arxiv 1004-3429

radiative capture



⊳

A. Richter, Prog. Part. Nucl. Phys. 34 (1995) 261.



A sum of 3 Gaussians (orbital, isoscalar & isovector spin flip) is proposed for  $f_{M1,}$ with their poles and integrals adjusted to experiments specific on magnetic strength; this parametrization is in accord to old polarized neutron data for M1.



 $\gamma\gamma$  – coincidence data (two step cascade, TSC) yield information on  $\rho$  and  $f_1$  – but these two quantities are very strongly <u>anti</u>-correlated! **TLO-dipole strength** (together with CTM) is in reasonable agreement to data



The thin curves represent the best randomly selected functions of the density of intermediate cascade levels, reproducing I $\gamma\gamma$  with the same  $\chi^2$  values and are their mean values. Line 2 shows the prediction by Strutinsky's model with the parameter g depending on shell inhomogeneities of one-particle spectrum; line 3 shows the same model for g = const. CTM with T= 783 keV and T= 820 keV using D(S<sub>n</sub>) =15 eV.

Thin curves depict the best random functions reproducing Iyy with the same small  $\chi^2$  values and are their mean values. Solid black curve is the best approximation by model of Sukhovoj et al., dotted curve depicts strength function of KMF model.

## "Intensities of the two-step cascades can be reproduced with equal and minimal values of $\chi^2$ by infinite set of different level densities and radiative strength functions".

A. M. Sukhovoj, V.A.Khitrov (2008), submitted to Yadernaya Fizika

---: A. Junghans et al., PLB 670 (2008) 200 ---: id. + M1, K. Heyde et al., RMP (2010)





'Triple' dipole strength is in reasonable agreement to recent JINR-data for the low energy tail – which determines the radiative capture.

In contrast to  ${}^{96}Mo$ no information is given on the correlation between  $\rho$  and  $f_1$ 



---: A. Junghans et al., PLB 670 (2008) 200 ---: id. + M1, K. Heyde et al., RMP (2010)





#### Result of triple Lorentzian fits compared to results from 'Oslo method'



Oslo data are taken with He-projectiles, which excite nuclei in states with  $\ell \approx 3-5 \hbar$ =  $\triangleright$  thus the extraction of strength for low J depends on the spin dependence of the level density.

| H.T. Nyhus et al., PRC 81 (10) 024325;<br>http://ocl.uio.no/compilation | A. Junghans et al., PLB 670 (08) 200 | B.L.Berman et al., PR185 (69) 1576<br>R.Bergère et al., NPA 121 (68) 463 |
|---|--------------------------------------|--|
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#### Average photon width and radiative capture

$$\left\langle \Gamma_{R\gamma} \right\rangle = \left\langle \sum_{\mathbf{f}} \Gamma_{\gamma}(R \to \mathbf{f}) \right\rangle \quad f_1 E_{\gamma}^3 = \Gamma_{\gamma} \rho(E_R)$$

$$= 3 \int_{0}^{E_{R}} \frac{\rho(E_{f})}{\rho(E_{R})} E_{\gamma}^{3} f_{1}(E_{\gamma}) dE_{\gamma} \quad E_{R} = E_{f} + E_{\gamma}$$

$$= 3 \int_{0}^{E_{R}} \frac{E_{\gamma}^{3} f_{1}(E_{\gamma})}{e^{E_{\gamma}/T}} dE_{\gamma} \quad E_{R} = S_{n} + E_{n}$$

**CTM**  $\rightarrow$  average photon width depends only on  $E_{\gamma}$ ,  $f_1$ , T and <u>not</u> on  $\rho(S_n)$ , which cancels out. The factor **3** accounts for the decay statistics, it may be smaller.

$$\langle \sigma_R(n,\gamma) \rangle \approx 2(2\ell+1)\pi^2 \lambda_n^2 \rho(E_R) \langle \Gamma_{R\gamma} \rangle$$

the radiative capture cross section depends also on  $\rho(E_R)$ .



QRPA predicts onsiderably smaller  $f_1$ and thus also smaller radiative capture

CTM is good choice for  $\rho$ , as sensitivity on  $E_{\gamma}$  peaks at  $\approx 4$  MeV; it is taken as valid up to  $S_n$ , even if  $E_M < S_n$ .

G: S. Goriely et al., NPA 739 (2004) 331

![](_page_21_Picture_11.jpeg)

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J: A.Junghans et al., PLB 670 (2008) 200

![](_page_21_Picture_14.jpeg)

**TLO gives good description of both: photon absorption and radiative capture** EGLO fails – it starts from 1 or 2 Lorentzians (i.e.large  $\Gamma$ ) and reduces low energy strength by setting  $\Gamma \sim E_{\gamma}^2$ 

![](_page_22_Figure_1.jpeg)

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![](_page_22_Picture_4.jpeg)

## Average radiative width $\langle \Gamma \rangle$ and temperature T

![](_page_23_Figure_1.jpeg)

average radiative width  $\langle \Gamma \rangle$  and energy dependence of level density T show surprisingly similar trends in their dependence on mass number A; a slowly varying  $f_1$  is favoured.

> A.Koning *et al.*, Nucl. Phys. A **810** (08) 13 **T.Belgya, RIPL2 Handbook** (2006) IAEA-TECDOC-1506

A. Ignatyuk, IAEA-TECDOC-1506, RIPL-2

![](_page_23_Picture_5.jpeg)

![](_page_23_Picture_7.jpeg)

# Newly compiled (RIPL-3) average radiative widths $\langle \Gamma \rangle$ – compared to prediction of global TLO-fit for $f_1$ combined to T(A) $\sim A^{-2/3}$

![](_page_24_Figure_1.jpeg)

Average radiative widths  $\langle \Gamma \rangle$  in heavy nuclei are approximately reproduced in trend and absolute size; local effects like shell closure etc. are not and the influence of M1 is marginal.

An increase of 10% in T(A) rises  $\langle \Gamma \rangle$  by 70% – as compared to 10% for 10% change in  $f_1$ .

A. Ignatyuk in RIPL-3 (09), § 3 Resonances

![](_page_24_Picture_5.jpeg)

![](_page_24_Picture_7.jpeg)

## Average radiative widths $\langle \Gamma \rangle$ as measured in even-even nuclei compared to prediction of global TLO fit for $f_1$ combined to various T(A)

![](_page_25_Figure_1.jpeg)

**TLO** and the local fits for **T** reproduce the data much better than the global T(A)

![](_page_25_Picture_3.jpeg)

## **Conclusions**

The E1 strength  $f_{E1}$  as controlled by the isovector giant dipole resonance **GDR** has at  $E_{\gamma} \ll E_{GDR}$  a value proportional to (1) the spreading width  $\Gamma_{GDR}$  and (2) the ratio to the dipole sum rule. To extract both from GDR data the nuclear deformation has to be considered: The deviation from axial symmetry has an important effect, neglected up to now. Recent nuclear structure investigations show that triaxiality is (1) observed in very many nuclei and (2) anti-correlated to the dynamic quadrupole moment  $q_2$ . Any use of a Lorentzian for  $f_{E1}$  should be in accord to that; thus GDR data do <u>not</u> indicate (1) a strong deviation from the GMT sum rule (with  $m_{\pi}=0$ ) (2) a strong variation of  $\Gamma_{GDR}$  with A and Z.

Radiative neutron capture <u>strongly</u> depends on T(E) and <u>less</u> on  $f_1$  – on both for  $E_{\gamma} \approx 4$  MeV. Combined to local CTM fits, predictions with the triple Lorentzian TLO compare well to data – in dependence of A and on an absolute scale.

M1 strengh does <u>not</u> have Lorentzian shape – and it has a minor effect on radiative capture.

![](_page_26_Picture_4.jpeg)

![](_page_27_Figure_0.jpeg)

Fig. 1. Histogram — experimental intensity of two-step cascades for the levels  $E_f$  (summed over the intervals of 100 keV). Lines — variants of the calculation with random functions of level density and radiative strength functions presented in Figs. 2 and 3

A. M. Sukhovoj, V.A.Khitrov (2008), submitted to Yadernaya Fizika

![](_page_27_Picture_3.jpeg)

![](_page_27_Picture_5.jpeg)

![](_page_28_Figure_0.jpeg)

![](_page_29_Figure_0.jpeg)

### T depends on A<sup>-2/3</sup> and on shell structure

to test dipole strength for different A, Z and N the average photon width is calculated from globally averaged T

![](_page_29_Figure_3.jpeg)

A.Koning et al., Nucl. Phys. A 810 (08) 13

![](_page_29_Picture_5.jpeg)

![](_page_29_Picture_7.jpeg)

![](_page_30_Figure_0.jpeg)

![](_page_30_Picture_1.jpeg)

A. Ignatyuk, IAEA-TECDOC-1506, RIPL-2

![](_page_30_Picture_4.jpeg)

![](_page_31_Figure_0.jpeg)

![](_page_32_Figure_0.jpeg)

![](_page_33_Figure_0.jpeg)

![](_page_34_Figure_0.jpeg)

![](_page_34_Picture_2.jpeg)

![](_page_35_Figure_0.jpeg)

![](_page_35_Picture_1.jpeg)

![](_page_35_Picture_3.jpeg)

![](_page_36_Picture_0.jpeg)

![](_page_36_Picture_2.jpeg)

![](_page_37_Figure_0.jpeg)

![](_page_37_Picture_1.jpeg)

![](_page_37_Picture_2.jpeg)

![](_page_37_Picture_3.jpeg)

![](_page_38_Figure_0.jpeg)

![](_page_38_Picture_1.jpeg)

![](_page_38_Picture_3.jpeg)

![](_page_39_Figure_0.jpeg)

![](_page_39_Picture_1.jpeg)

![](_page_39_Picture_3.jpeg)

![](_page_40_Figure_0.jpeg)

![](_page_40_Picture_1.jpeg)

![](_page_40_Picture_3.jpeg)

## **Conclusions**

The E1 strength  $f_{E1}$  is controlled by the isovector giant dipole resonance GDR: At  $E_{\gamma} \ll E_{GDR}$  its value is proportional (1) to the spreading width  $\Gamma_{GDR}$  and (2) to the excess over the dipole sum rule. To extract both from GDR data the nuclear deformation has to be accounted for: The deviation from axial symmetry has an important effect, neglected up to now. Modern nuclear structure investigations show that triaxiality is (1) observed in very many nuclei and (2) anti-correlated to the quadrupole moment  $Q_i$ . Any use of a Lorentzian for  $f_{E1}$  has to be in accord to that; GDR data do <u>not</u> indicate (1) a strong deviation from the GMT sum rule (with  $m_{\pi}=0$ ) (2) a strong variation of  $\Gamma_{GDR}$  with A and Z.

Radiative neutron capture strongly depends on  $\rho$  and on the dipole strength  $f_1$  in the region 2 - 6 MeV. It is thus influenced by orbital magnetism (scissors mode) and the isoscalar spin flip M1 strengh. Respective data show that (1) they do <u>not</u> have Lorentzian shape (with  $\Gamma_{GDR}$ ) and (2) hitherto unobserved continua may increase their strength.

Very low energy strength (as predicted by KMF) is very difficult to be clearly identified experimentally.

![](_page_41_Picture_4.jpeg)

![](_page_41_Picture_6.jpeg)

![](_page_42_Figure_0.jpeg)

Mo dipole strength studies @ ELBE

and disagree to (<sup>3</sup>He,<sup>3</sup>He)-data from cyclotron @ Oslo.

![](_page_42_Figure_2.jpeg)

statistical analysis for <sup>92</sup>Mo (γ,γ) shows no collective strength (pigmy?),

only Porter – Thomas fluctuations.

Erhard et al., PRC81(10)034319

![](_page_42_Picture_6.jpeg)

Tonchev et al.; Siem et al., contrib. to PSF-workshop 2008

![](_page_42_Picture_8.jpeg)

## Dynamic rms Q-moment vs. static deformation

in relation to

![](_page_43_Figure_2.jpeg)

"vibrational" nucleus (y=0)

#### VS.

*"rotational" nucleus* (*γ*=0)

With the **rotation invariant observables** the the "traditional" mean deformation parameters  $Q_0$ ,  $\beta$  and  $\gamma$ are replaced by the root mean square (rms) averages  $Q_{rms}$ , d and  $\delta$ . The distiction between **spherical**, **vibrational**, **rotational** is lost; quantitative information is used to define **rms-values** and their **variance** instead. **Experiment** can only deliver **rms-information** about non-sphericity and non-axiality.

Fig. from V. Werner et al., PRC 78, 051303 (2008)

![](_page_43_Picture_8.jpeg)

![](_page_43_Picture_10.jpeg)

## Triaxiality in Nilsson-Strutinski calculations (FRDM-HFB)

![](_page_44_Figure_1.jpeg)

### and in calculations with the Thomas-Fermi plus Strutinsky integral (ETFSI) method, saying:

We are thus inclined to accept the widespread (>30%) occurrence of triaxiality...as being an essential feature of ETFSI calculations, if not of the real world ...albeit the associated reduction in energy, ...never exceeds 0.7MeV.

Fig. 4. The calculated ground state shape of  $^{106}$ Ru is triaxial, as is the case for several hundred other nuclei across the nuclear chart out of ~ 9000 studied.

P. Möller et al., PRL 97(06) 162502

![](_page_44_Picture_6.jpeg)

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A. K. Dutta et al., PRC 61(00)054303

![](_page_44_Picture_9.jpeg)

Various contributions to the sum of E2-strengths - i.e. the quadrupolar deformation

For most nuclei, the e.m. transition  $0_1^+ \rightarrow 2_1^+$ dominates the sum by ~ 95%.

![](_page_45_Figure_2.jpeg)

$$R^{(2)} \equiv \frac{\sum_{r=2,3,..}}{\sum_{r=1,2,3,..}} |\langle 2_r \parallel E2 \parallel 0 \rangle|^2$$

FIG. 1. Histogram of experimental  $R^{(2)}$  values for 101 nuclei from Zn to Fm obtained from all known  $B(E2:0_1^+ \rightarrow 2_i^+)$  values

But: in nearly sperical nuclei the transition from the g.s. to the high energy quadrupole mode GQR is comparable with  $0_1^+ \rightarrow 2_1^+$ . It corresponds to a fast oscillation and causes a respective increase of  $Q_{rms}$ 

![](_page_45_Picture_6.jpeg)

![](_page_45_Picture_7.jpeg)

![](_page_45_Picture_9.jpeg)

## **Observation vs. theory**

## The only "theoretical" assumptions to get d and $\delta$ are: reflection symmetry and equal distribution of charge and mass (i.e. protons and neutrons).

The radii  $\mathbf{R}_i$  of a non-axial ellipsoidal shape (assumed) and the dipole oscillation frequencies  $\boldsymbol{\omega}_i$  directly result from the observables  $\mathbf{K}_2$  and  $\mathbf{K}_3$ .

The  $K_3$  are well determined for ~ 100 nuclei only. For these the approximations needed have been proven to be OK:

$$\begin{split} K_{3} &= -\cos\left(3\delta\right) = \sqrt{\frac{7}{10K_{2}^{3}}} \sum_{r,s=1,\infty} \left\langle 0 \| \text{E2} \| 2_{r} \right\rangle \left\langle 2_{r} \| \text{E2} \| 2_{s} \right\rangle \left\langle 2_{s} \| \text{E2} \| 0 \right\rangle \\ &\approx \sqrt{\frac{7}{10K_{2}^{3}}} \left( \left\langle 0 \| \text{E2} \| 2_{1} \right\rangle \left\langle 2_{1} \| \text{E2} \| 2_{1} \right\rangle \left\langle 2_{1} \| \text{E2} \| 0 \right\rangle + 2 \left\langle 0 \| \text{E2} \| 2_{1} \right\rangle \left\langle 2_{1} \| \text{E2} \| 2_{2,3} \right\rangle \left\langle 2_{2,3} \| \text{E2} \| 0 \right\rangle \right) , \\ \left\langle \text{d}^{3} \cos\left(3\delta\right) \right\rangle \approx \left\langle \text{d}^{3} \right\rangle \left\langle \cos\left(3\delta\right) \right\rangle ; \quad \left\langle \text{d}^{3} \right\rangle \approx \left\langle \text{d}^{2} \right\rangle^{3/2} . \end{split}$$

 $K_3$  can also be derived from excitation energy information with nearly no experimental uncertainty, but often systematic problems due to theoretical approximations.

W. Andrejtscheff and P. Petkov, PRC 48 (93) 2531

C. Y. Wu and D. Cline, PRC 54 (96) 2356

V. Werner et al., PRC 71, 054314 (2005)

![](_page_46_Picture_9.jpeg)

![](_page_46_Picture_11.jpeg)

<u>Apparent GDR-width</u> – from  $(\gamma, n)$ -data taken at CEN Saclay

![](_page_47_Figure_1.jpeg)

c. splitting due to (static & dynamic) deformation not only for well deformed nuclei

![](_page_47_Picture_3.jpeg)

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Carlos et al., NP A 219 (1974) 61

![](_page_47_Picture_6.jpeg)

![](_page_48_Figure_0.jpeg)

Lepretre et al., NPA175(71)609

![](_page_48_Picture_2.jpeg)

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![](_page_49_Figure_0.jpeg)

A.Junghans et al., PLB 670 (2008) 200

![](_page_49_Picture_2.jpeg)

![](_page_49_Picture_4.jpeg)

![](_page_50_Figure_0.jpeg)

A.Junghans et al., PLB 670 (2008) 200

![](_page_50_Picture_2.jpeg)

![](_page_51_Figure_0.jpeg)

A.Junghans et al., PLB 670 (2008) 200

![](_page_51_Picture_2.jpeg)

![](_page_52_Figure_0.jpeg)

![](_page_52_Figure_1.jpeg)

S. Goriely et al., NPA 739 (2004) 331

![](_page_52_Picture_3.jpeg)

**EFNUDAT 2010 CERN** 

A.Junghans et al., PLB 670 (2008) 200

![](_page_52_Picture_6.jpeg)

![](_page_53_Figure_0.jpeg)

![](_page_53_Picture_1.jpeg)

![](_page_53_Picture_3.jpeg)

![](_page_54_Figure_0.jpeg)

![](_page_55_Figure_0.jpeg)

![](_page_55_Picture_1.jpeg)

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![](_page_56_Figure_0.jpeg)

![](_page_56_Picture_1.jpeg)

![](_page_56_Picture_3.jpeg)

![](_page_57_Figure_0.jpeg)

![](_page_57_Picture_1.jpeg)

![](_page_57_Picture_3.jpeg)

![](_page_58_Figure_0.jpeg)

![](_page_58_Picture_1.jpeg)

![](_page_58_Picture_3.jpeg)