Transverse-to-Longitudinal Emittance Exchange Using an RF Deflecting Cavity

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Motivation-1 (for X-Ray FEL)

$$\varepsilon_N < \gamma \frac{\lambda_r}{4\pi}$$

Transverse emittance must be < radiation wavelength (*e.g.*, $\varepsilon_N < 1\ \mu\text{m}$ at $\lambda_r \sim 1$ Å)

$$\sigma_\delta < \rho \approx \frac{1}{4}\left(\frac{1}{2\pi^2}\frac{I_{pk}}{I_A}\frac{\lambda_u^2}{\beta\varepsilon_N}\left(\frac{K}{\gamma}\right)^2\right)^{1/3}$$

Energy spread must be $< \rho$, the FEL parameter (*e.g.*, $\sigma_\delta < 0.04\%$)

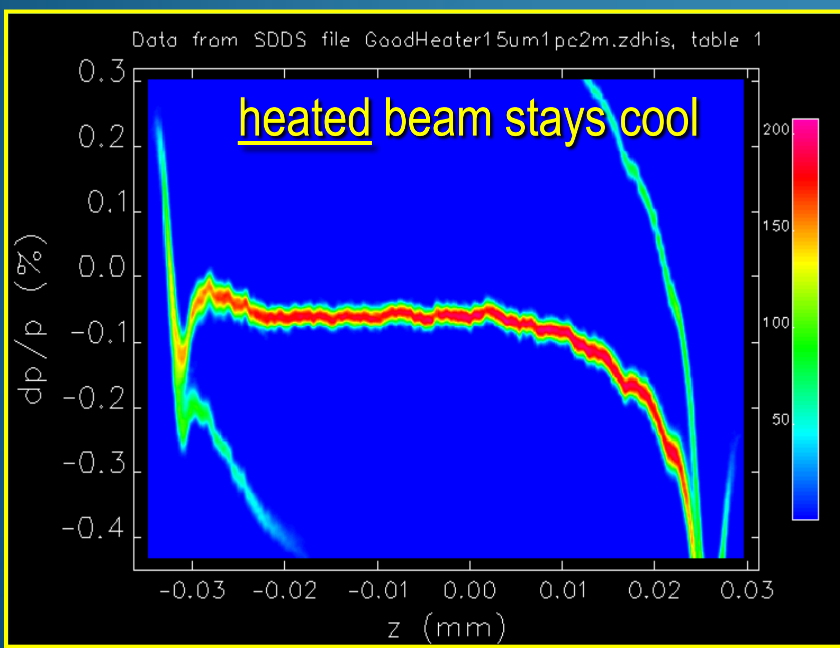
So we need $\gamma\varepsilon_x < 1\ \mu$m, and also $\gamma\sigma_z\sigma_\delta \equiv \gamma\varepsilon_z < 100\ \mu\text{m}$

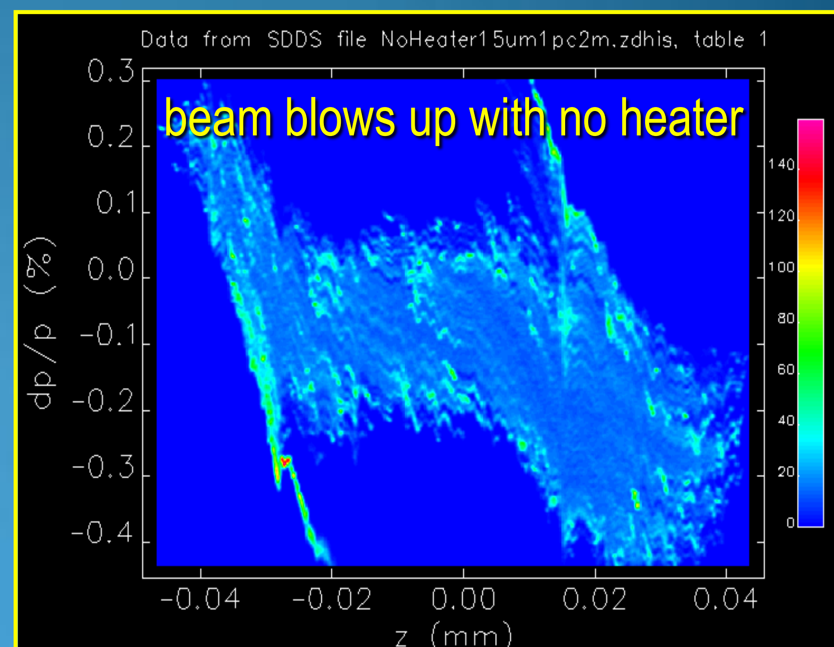
While RF guns produce $\gamma\varepsilon_x \sim \gamma\varepsilon_z \sim$ 1-3 μm

Can we reduce $\gamma\varepsilon_x$ at the expense of $\gamma\varepsilon_z$?

Motivation-2 (for X-Ray FEL)

- **Very bright beam from RF guns are susceptible to a μ-bunching instability (M. Borland, *et al.*).**
- **This can be controlled by increasing the intrinsic (slice) energy spread (Landau damped in bunch compressors) → requires a laser heater, but degrades 6D brightness!**

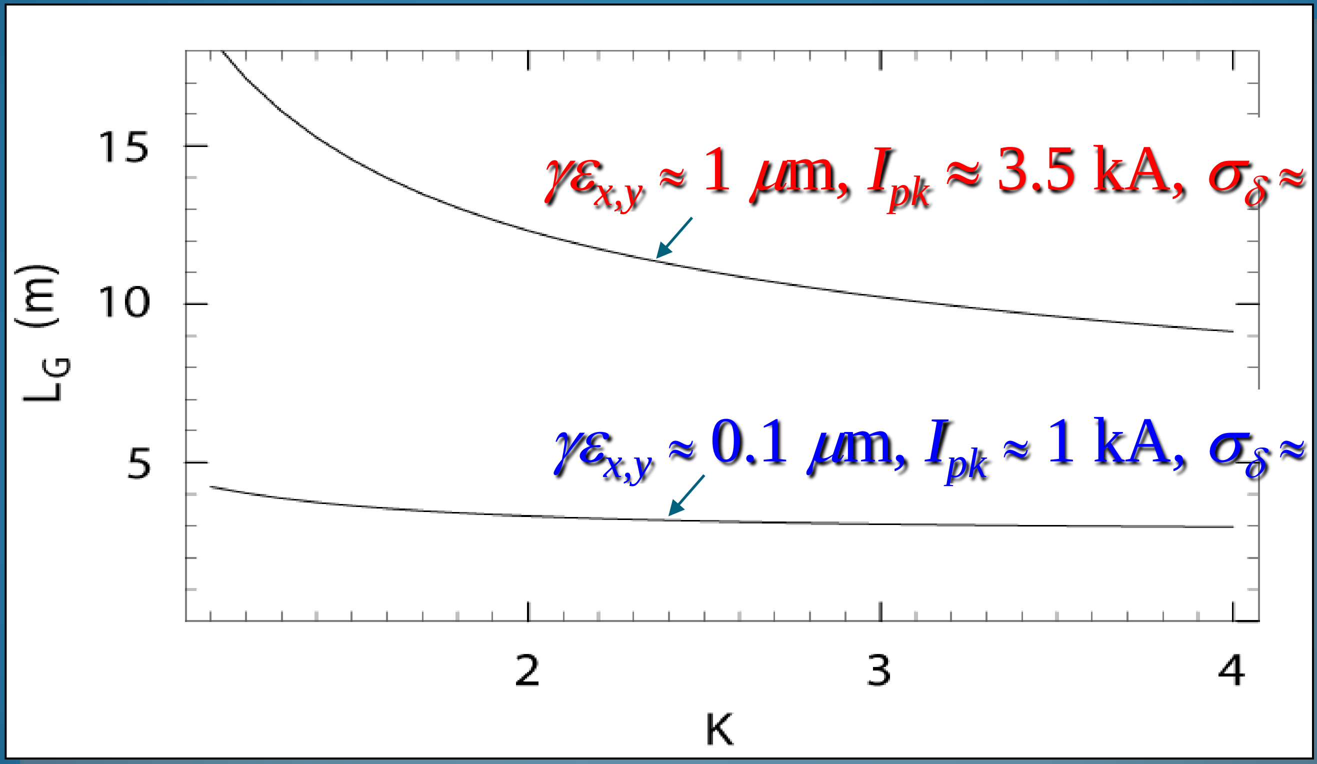




Final LCLS long. phase space at 14 GeV for initial modulation of 1% at λ_0 = 15 μm

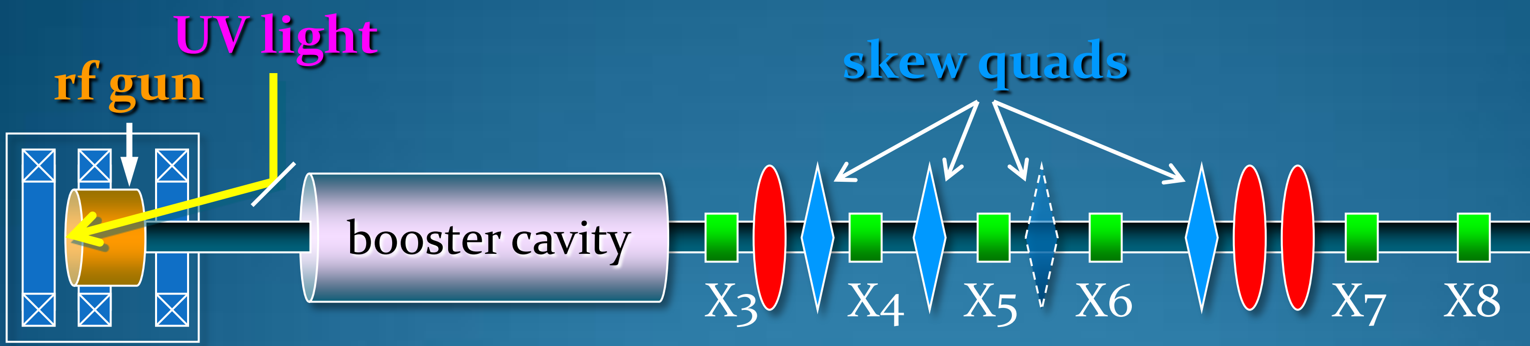
Strategy for FEL

- Use 'flat-beam injector' to generate a beam such as: $\gamma\varepsilon_x \sim 10$ μm, $\gamma\varepsilon_y \sim 0.1$ μm, and $\gamma\varepsilon_z \sim 0.1$ μm
- Exchange emittances (bends + RF deflector): $\gamma\varepsilon_x \leftrightarrow \gamma\varepsilon_z$
- Saturate FEL at 0.4 Å with no μ-bunching instability

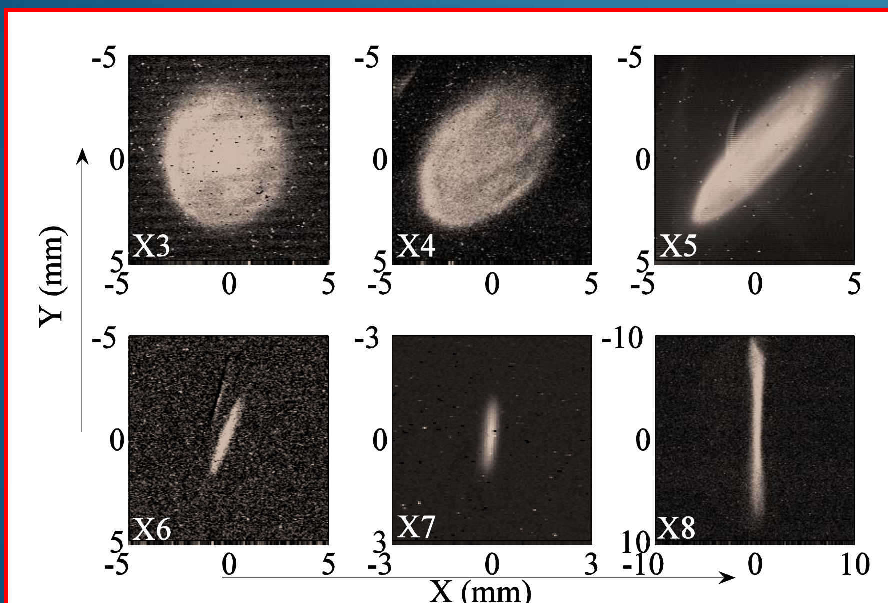


Flat Beam Injector (FNAL)

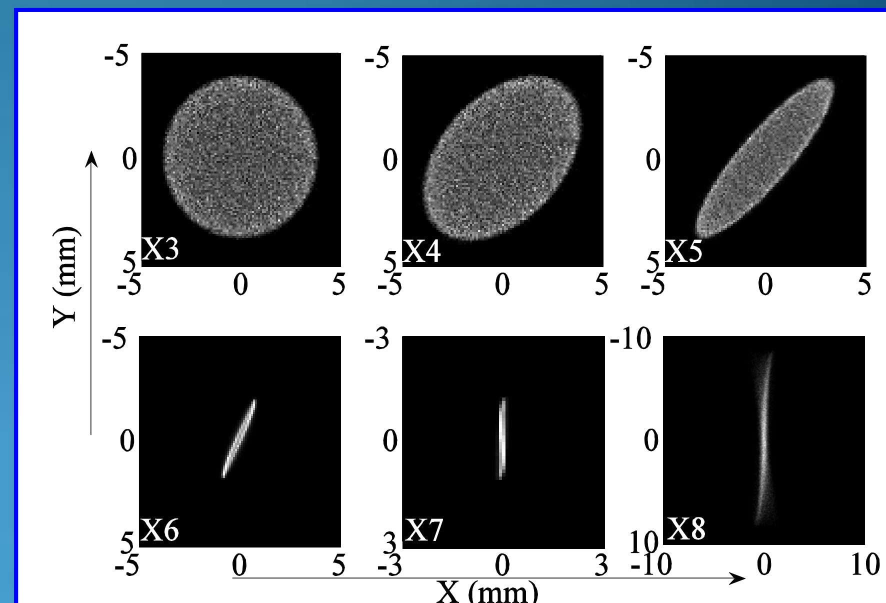
(Y. Derbenev), (R. Brinkmann, Y. Derbenev, K. Flöttmann), (D. Edwards …), (Y.-E Sun)



$\gamma\varepsilon_x \approx 0.4\ \mu m$
$\gamma\varepsilon_y \approx 40\ \mu m^*$



experiment



simulation

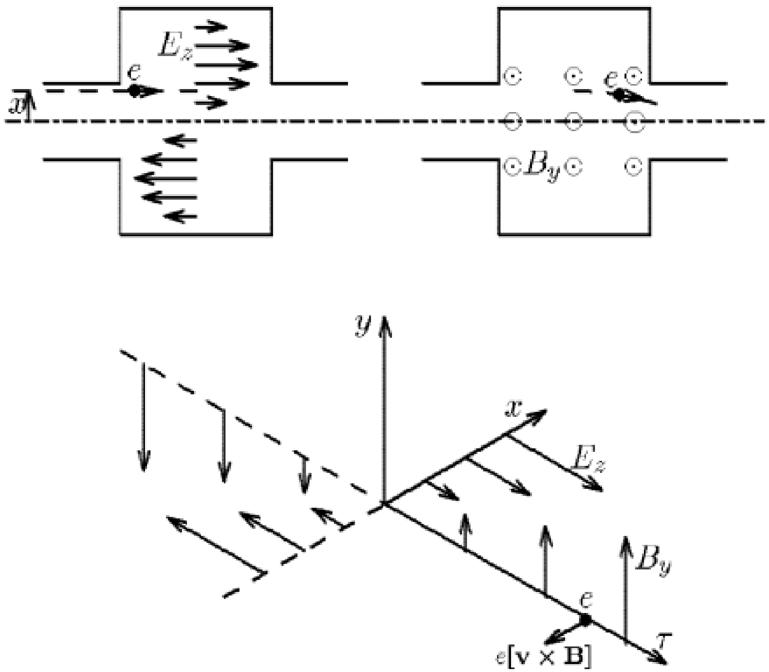
* Ph. Piot, Y.-E Sun, and K.-J. Kim, Phys. Rev. ST Accel. Beams 9, 031001 (2006)

Introduce Transverse RF Deflecting Cavity

Now introduce a rectangular transverse RF cavity operating in TM_{110} mode…

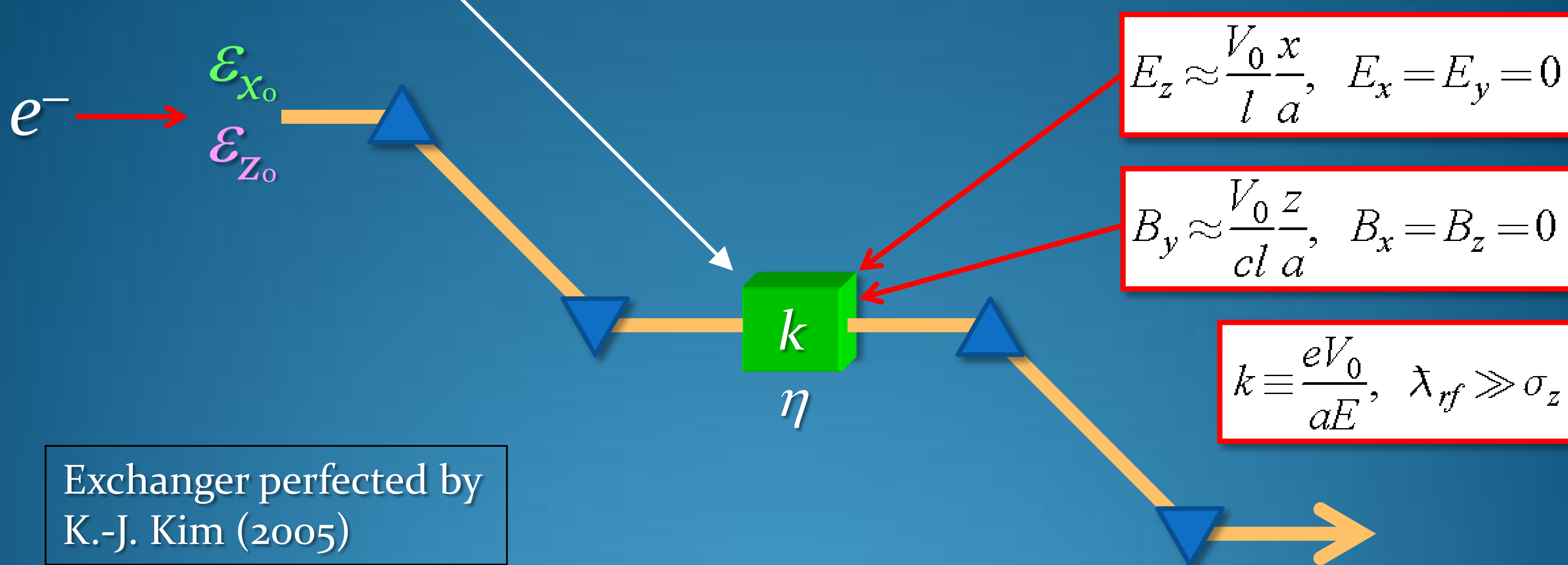
$$E_z = \frac{V_0}{l}\frac{x}{a}\cos\omega t \approx \frac{V_0}{l}\frac{x}{a},$$

$$B_y = \frac{V_0}{l}\frac{x}{a\omega}\sin\omega t \approx \frac{V_0}{cl}\frac{z}{a}.$$



Emittance Exchange Beamline

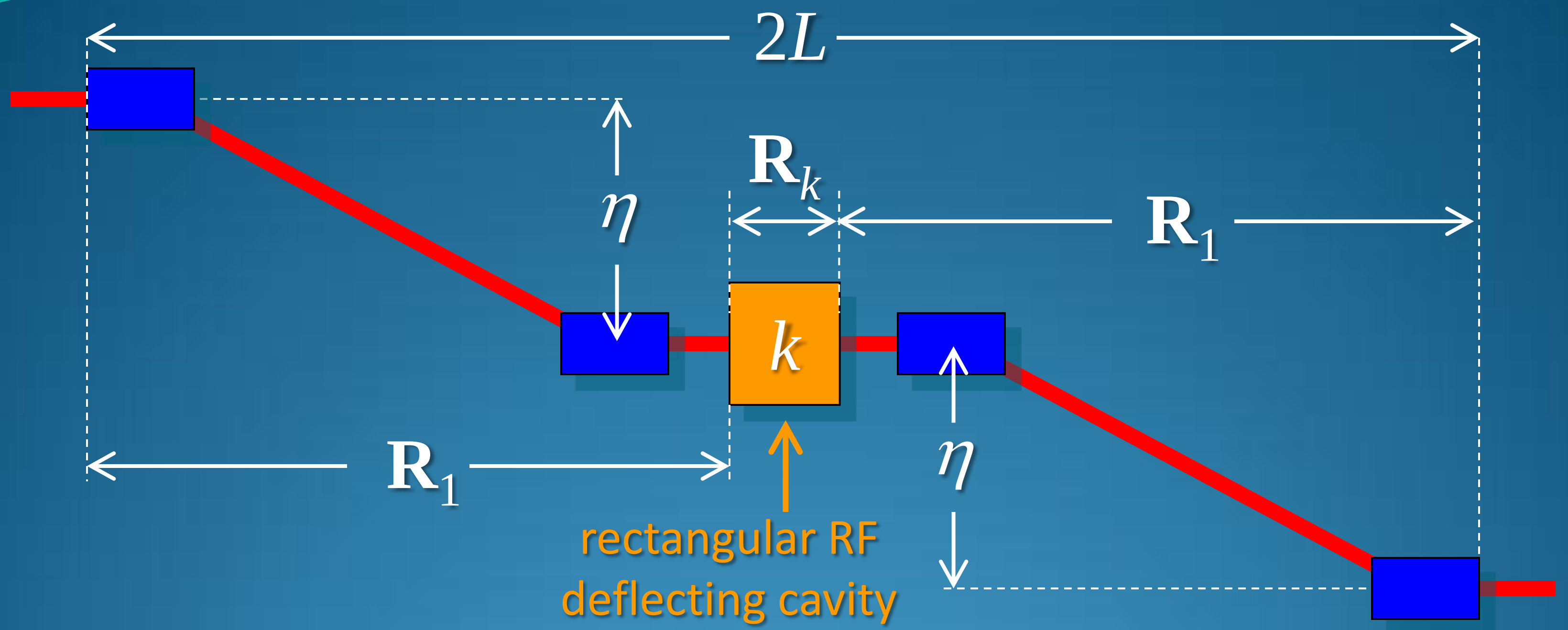
Transverse RF cavity (TM_{110}) in a double dog-leg…



- Particle at position x in cavity gets acceleration: $\Delta E/E \equiv \delta \approx kx$
- This energy deviation δ in cavity causes position change: $\Delta x = \eta\delta$
- Choose k to cancel initial position: $\Delta x \approx \eta k x = -x \rightarrow \boxed{\eta k = -1}$

Cornacchia, Emma, PRST AB **5**, 084001 (2002), & Emma, *et al.*, PRST AB **9**, 100702 (2006).

Emittance Exchanger Transfer Matrix



$\xi \equiv R_{56}$ of dog-leg

x, z mapping (ignore y coordinate here)

$$\mathbf{R}_1 = \begin{pmatrix} 1 & L & 0 & \eta \\ 0 & 1 & 0 & 0 \\ 0 & \eta & 1 & \xi \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{R}_k = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{R} = \mathbf{R}_1 \mathbf{R}_k \mathbf{R}_1$$

Full Emittance Exchanger

If RF deflector voltage is set to: $k = -1/\eta$

$$k \equiv \frac{eV_0}{aE},$$

$$\mathbf{R} = \begin{pmatrix} 0 & 0 & kL & \eta + kL\xi \\ 0 & 0 & k & k\xi \\ k\xi & \eta + kL\xi & 0 & 0 \\ k & kL & 0 & 0 \end{pmatrix}$$

…then transverse (bend-plane) and longitudinal emittances are completely exchanged.

$$\epsilon_x = \epsilon_{z0}$$
$$\epsilon_z = \epsilon_{x0}$$

Emittance Exchange Limitations

4x4 transfer matrix is four 2x2 blocks[1]:

$$\mathbf{R} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix},$$

symplectic conditions

$$|\mathbf{A}| + |\mathbf{C}| = \pm 1, \quad |\mathbf{A}| = |\mathbf{D}|, \quad |\mathbf{B}| = |\mathbf{C}|,$$

$$\boldsymbol{\sigma}_x = \varepsilon_{x_0} \mathbf{Q}_x \mathbf{Q}_x^T, \quad \mathbf{Q}_x \equiv \frac{1}{\sqrt{\beta_x}} \begin{pmatrix} \beta_x & 0 \\ -\alpha_x & 1 \end{pmatrix},$$

$$\lambda^2 \equiv tr\left\{\mathbf{U}\mathbf{U}^T\right\} = U_{11}^2 + U_{12}^2 + U_{21}^2 + U_{22}^2 \geq 0,$$

$$\mathbf{U} = \mathbf{Q}_x^{-1} \mathbf{A}^a \mathbf{B} \mathbf{Q}_z \, .$$

projected emittances are[2]...

$$\varepsilon_x^2 = |\mathbf{A}|^2 \varepsilon_{x_0}^2 + \left(1 - |\mathbf{A}|\right)^2 \varepsilon_{z_0}^2 + \varepsilon_{x_0} \varepsilon_{z_0} \lambda^2,$$

$$\varepsilon_z^2 = \left(1 - |\mathbf{A}|\right)^2 \varepsilon_{x_0}^2 + |\mathbf{A}|^2 \varepsilon_{z_0}^2 + \varepsilon_{x_0} \varepsilon_{z_0} \lambda^2.$$

Equal emittances remain equal. (if $\varepsilon_{x_0} = \varepsilon_{z_0}$ then $\varepsilon_x = \varepsilon_z$.)

Equal, uncoupled emittances cannot be generated from unequal, uncoupled emittances[3]. (Setting $|\mathbf{A}| = ½$ produces equal emittances, but then they are highly coupled with $\lambda^2 \neq 0$.)

[1] K.L. Brown, SLAC-PUB-2370, August 1980.

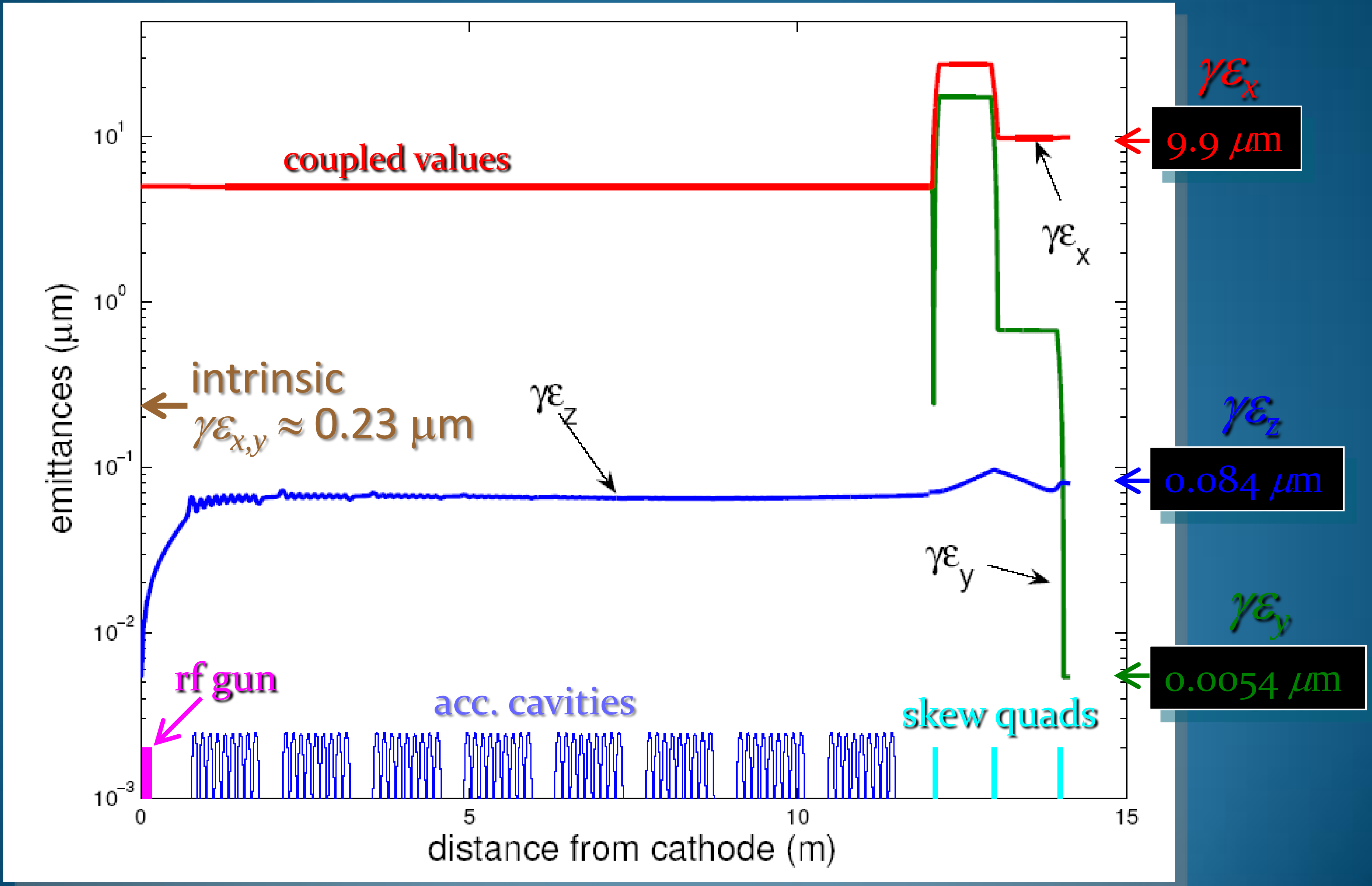
[2] Thanks to Bill Spence.

[3] E. Courant, in "Perspectives in Modern Physics...," R.E. Marshak, ed., Interscience Publishers, 1966.

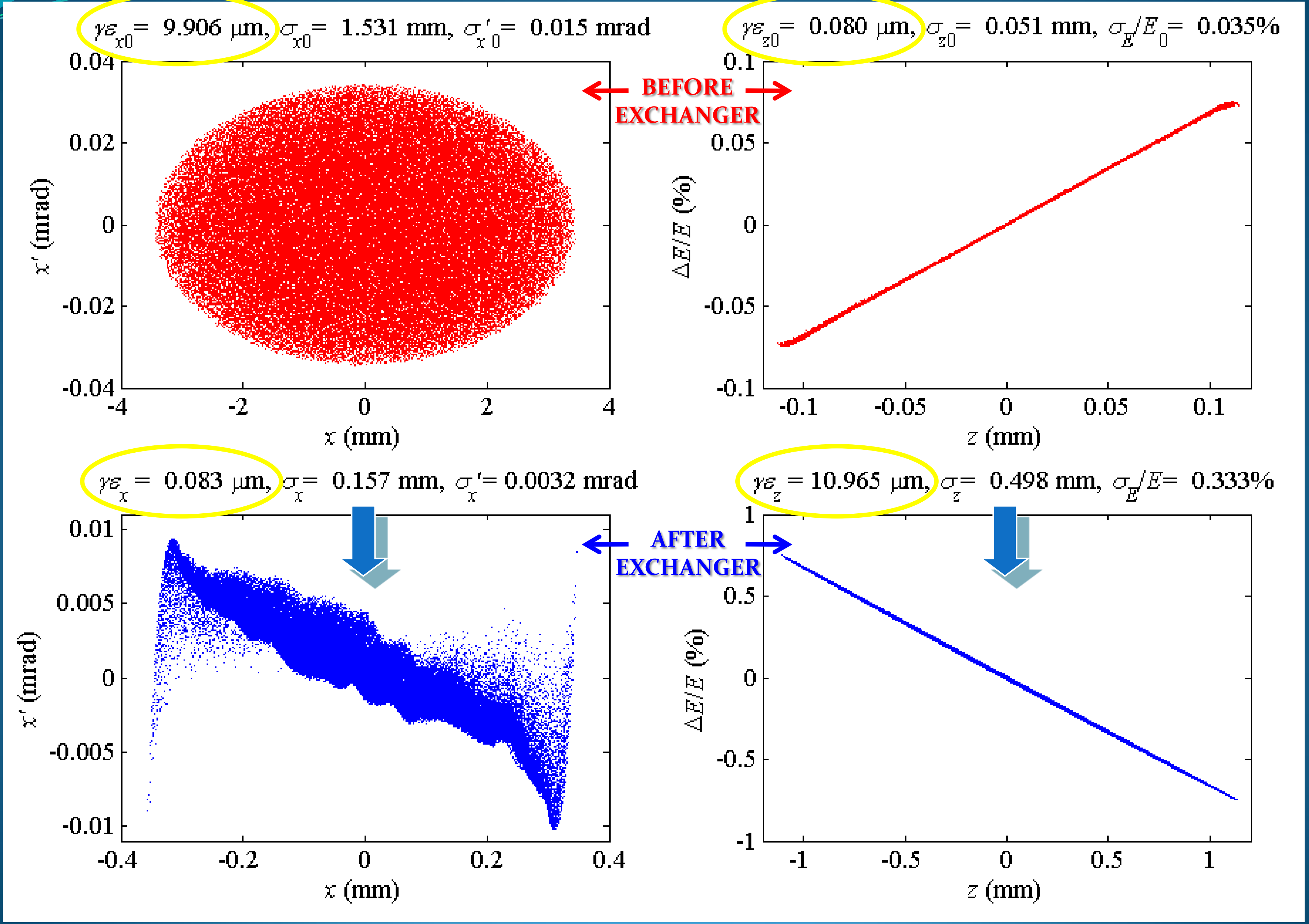
Emittance Exchange Simulation Parameters

Parameters (20 pC bunch charge)	symbol	value	unit
Electron energy	E	216	MeV
Dipole magnet length	L_B	20	cm
Drift length between dipole magnets	L	1	m
Bend angle per dipole magnet	θ	20	deg
Length of rec. RF cavity	L_c	30	cm
Initial horizontal norm. emittance	$\gamma\varepsilon_x$	9.92	μm
Initial longitudinal norm. emittance	$\gamma\varepsilon_z$	0.080	μm
Initial rms bunch length	σ_z^G	51	μm
Initial rms slice energy spread	σ_E^G	0.9	keV
Initial energy chirp (δ-z slope)	h	6.9	m^{-1}
Initial horizontal beta function	β_x	100	m
Initial horizontal alpha function	α_x	0	

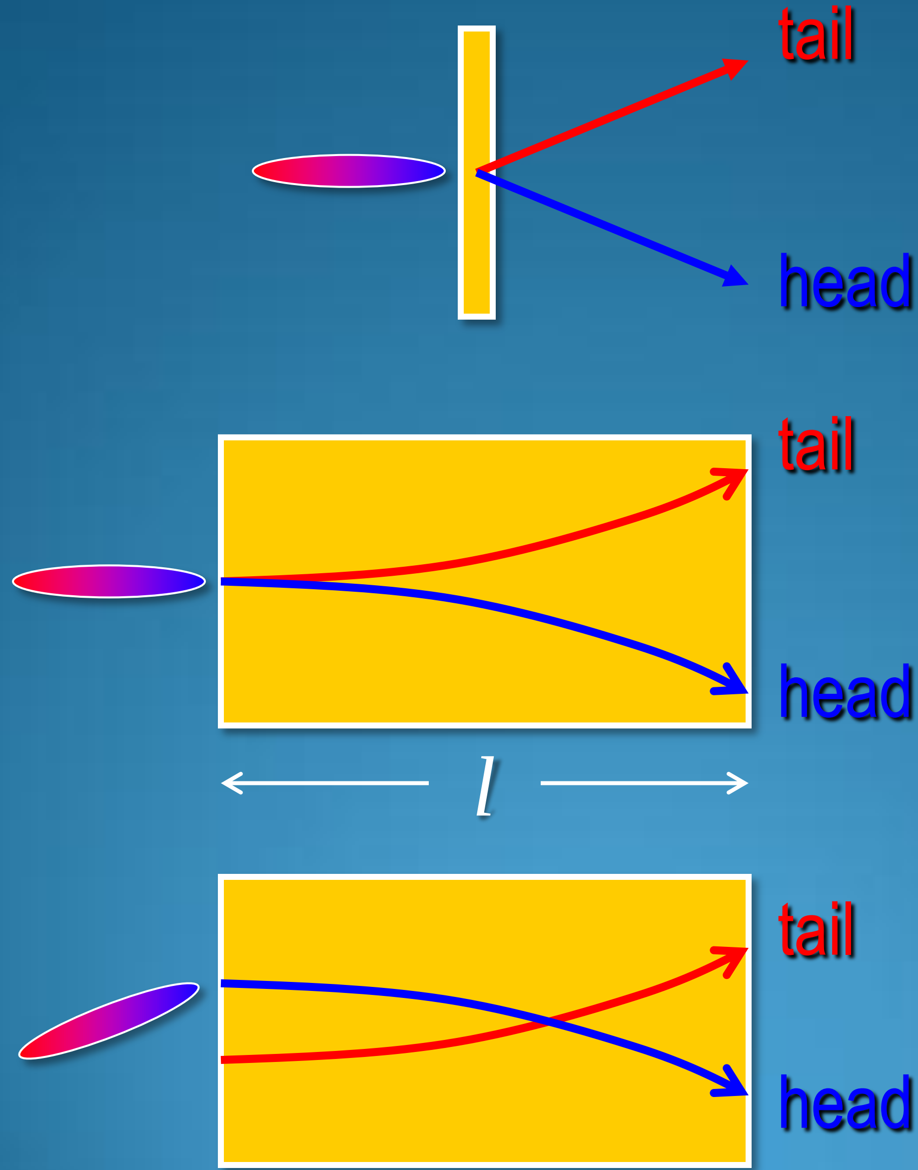
Evolution of Transverse Emittance Along Simulated Photo-Injector Beamline (to 216 MeV)



Transverse phase space (left two plots) and longitudinal phase space (right two plots) before (top) and after (bottom) emittance exchange.



Cavity Thick-Lens Effect Requires Some Attention

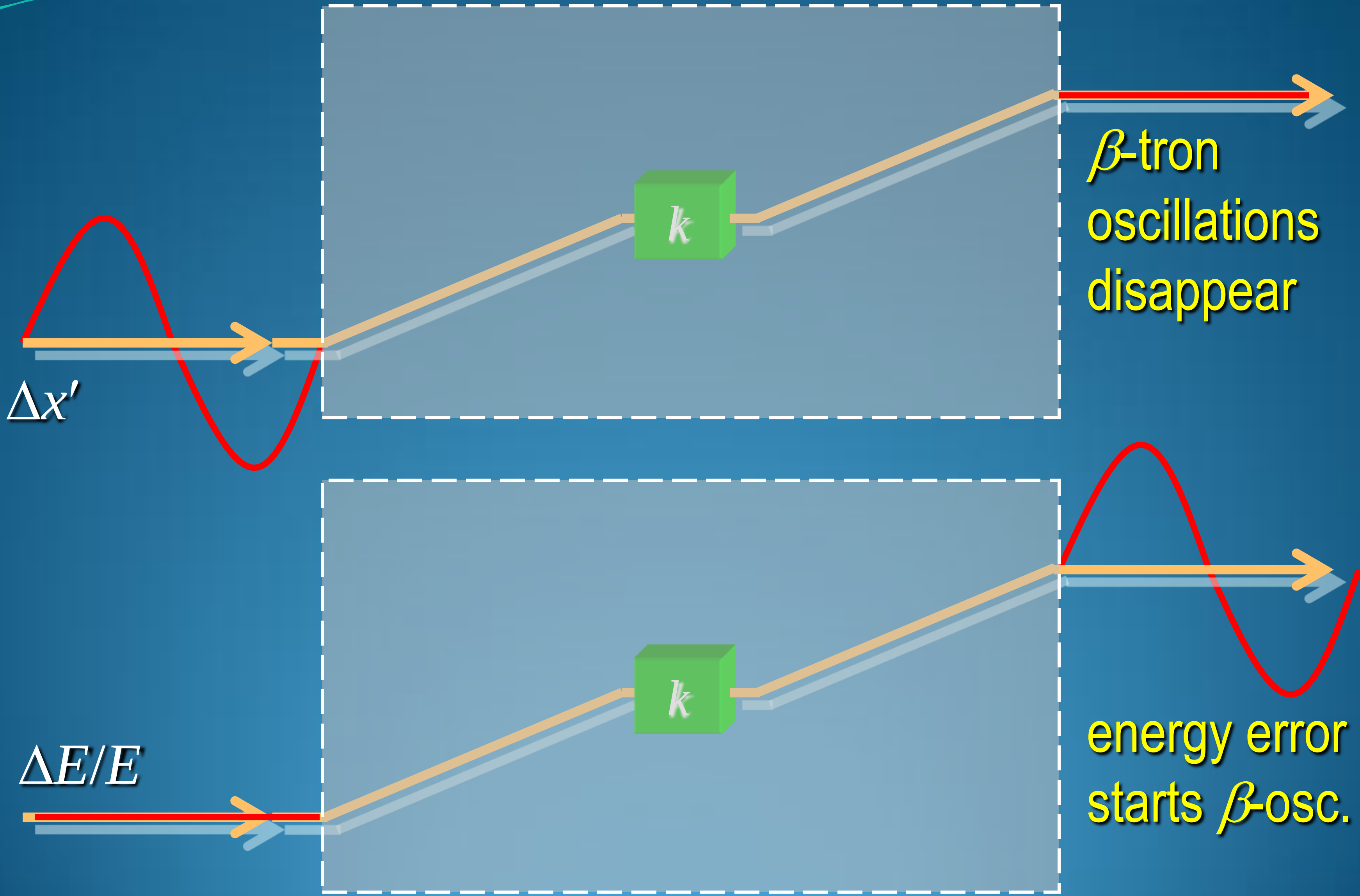


Thin-lens gives no x-offset in cavity

Thick-lens: B_y kick, and then x-offset changes δ

add 'chirp' to compensate: no mean x-offset

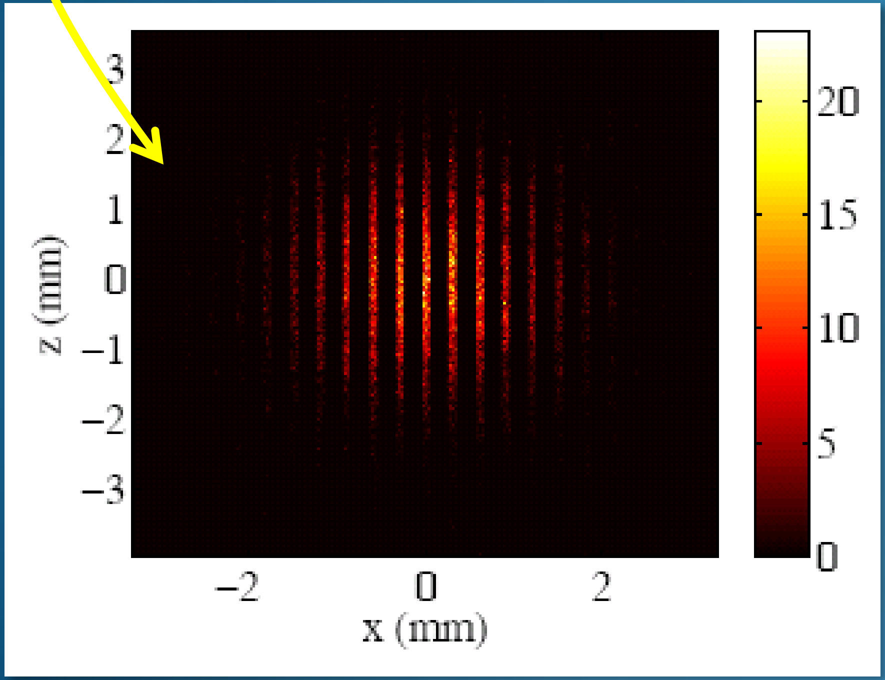
Unusual System Characteristics

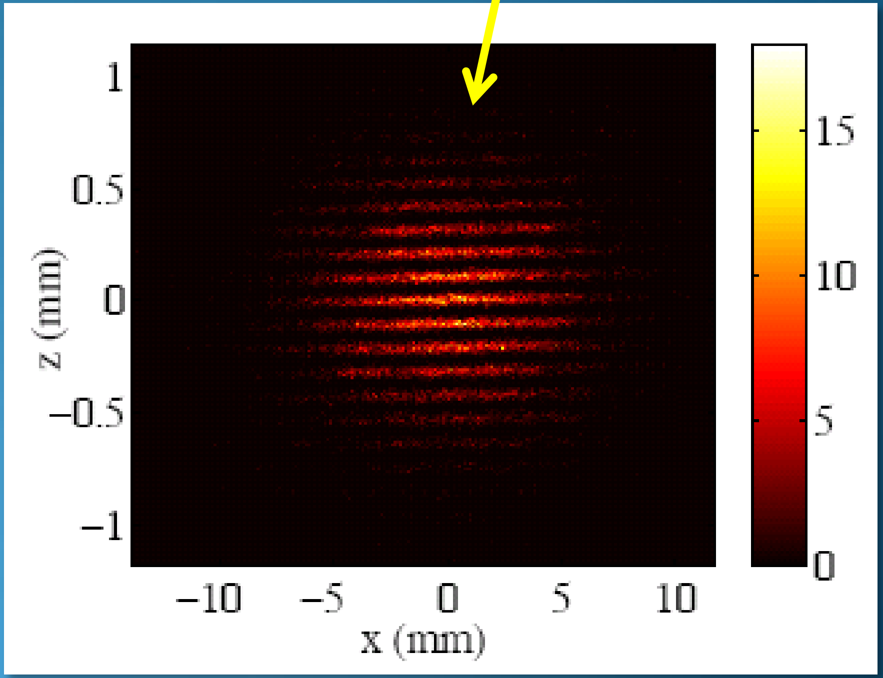


Need **extremely** stable energy (0.5×10^{-6} rms jitter ⇒ 10% x-beam size jitter)

ANOTHER IDEA: Femtosecond Bunch Trains from Phase Space Exchange Technique

- Multi-slit mask is used to establish a transverse modulation
- Exchanger maps this modulation into temporal



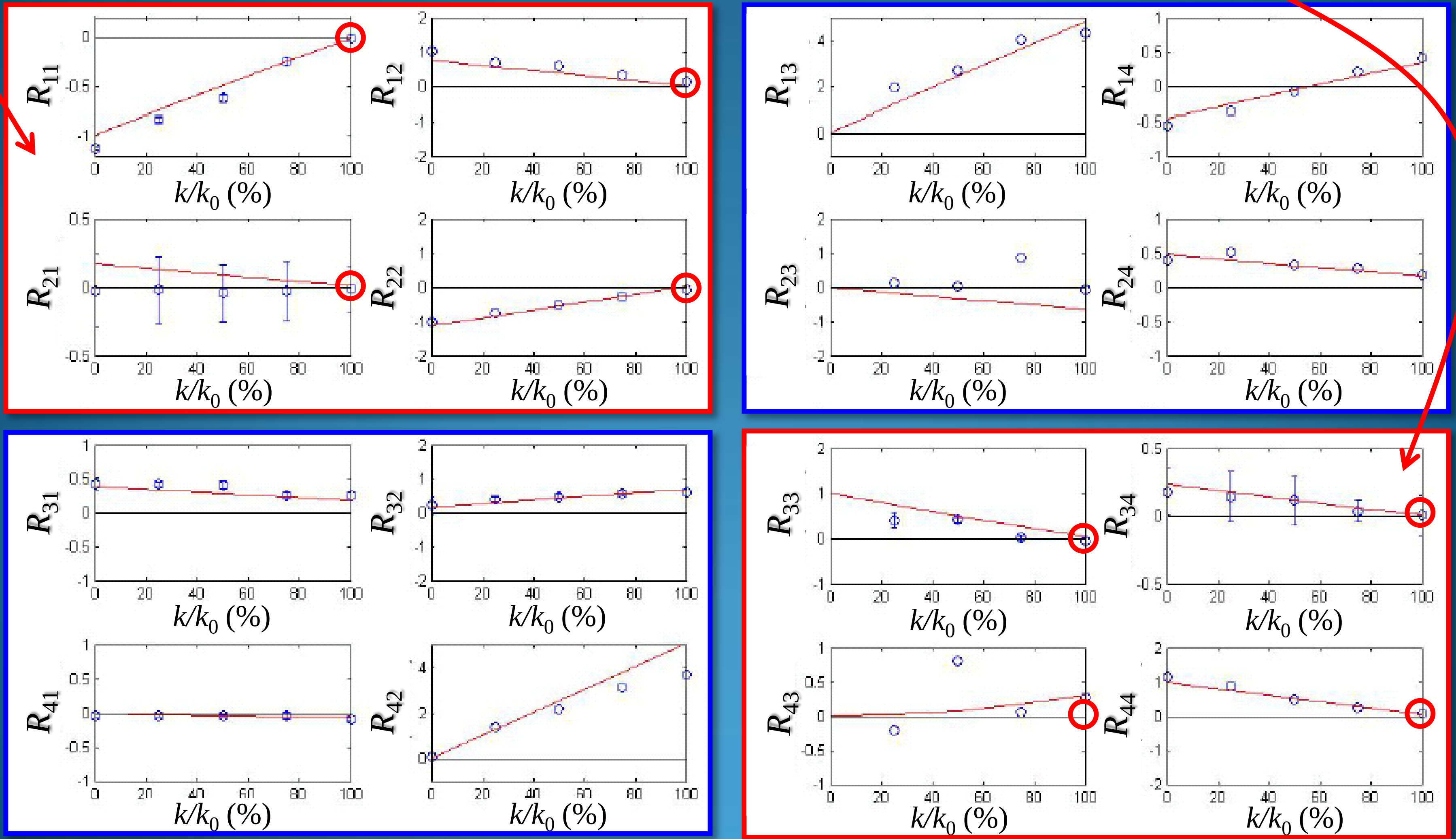


Yin-e Sun, Philippe Piot, Linac'08

Measurement of Exchange System (FNAL)

4×4 matrix measurement vs. RF power

Both on-diagonal 2×2 sub-matrices = 0 at full RF power (k/k_0 = 100%)



T. Koeth, Linac'08

Summary

- Simulations of flat-beam gun with emittance exchanger suggest possible levels of:
  $\gamma\varepsilon_z \approx 10\ \mu\text{m},\ \gamma\varepsilon_y \approx 0.005\ \mu\text{m},\ \gamma\varepsilon_x \approx 0.16\ \mu\text{m}$
- This beam allows shorter wavelength FELs and/or smaller, lower cost accelerators
- The resulting large *z*-emittance should also Landau-damp the micro-bunching instability
- Sensitivity to energy jitter may be Achilles heel
- Transverse RF opens great new potential !