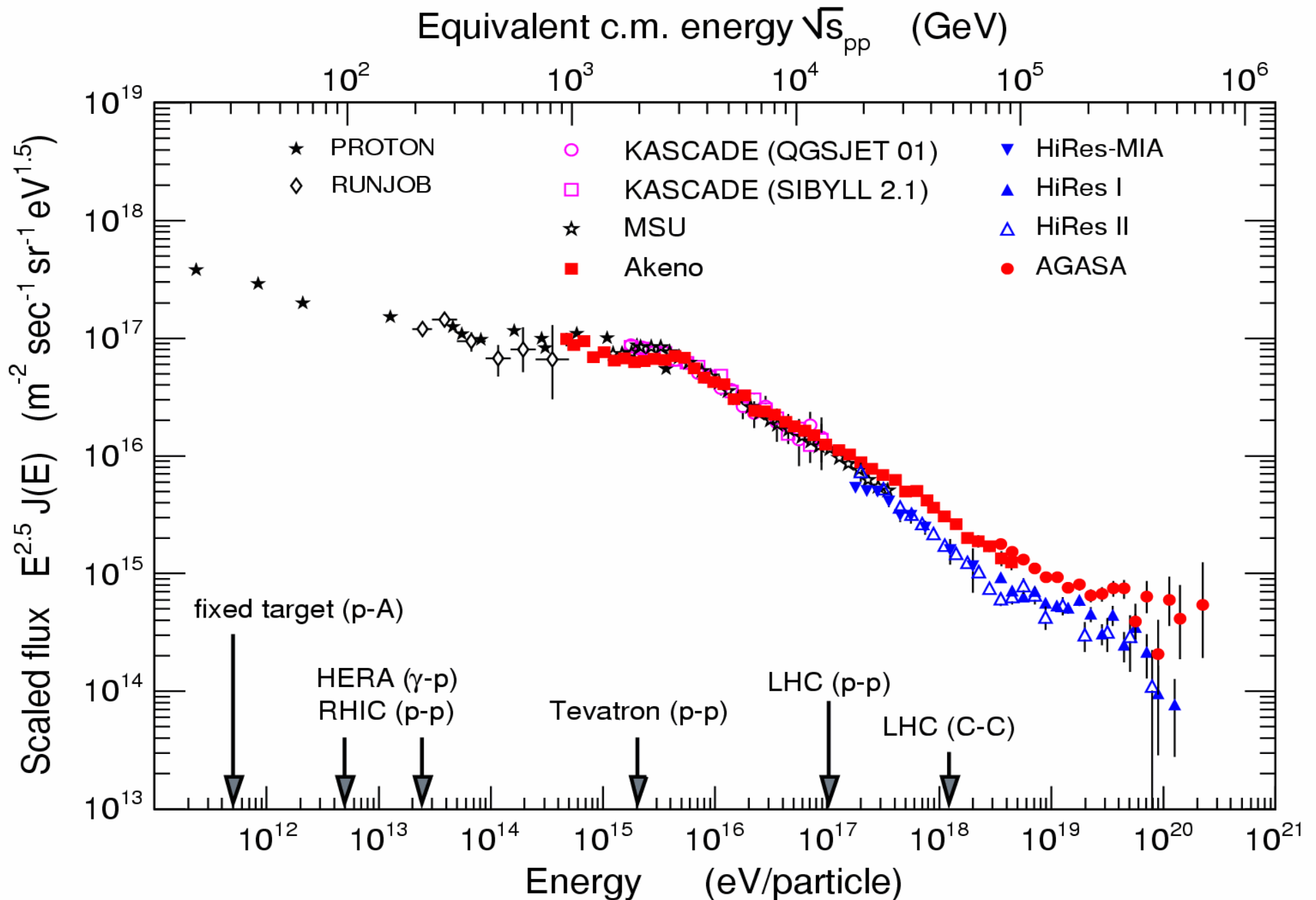


Modelling of hadronic showers

Ralph Engel

Forschungszentrum Karlsruhe, Germany

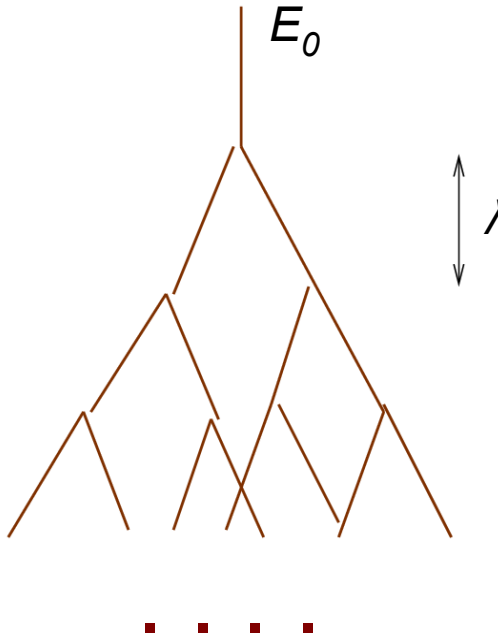
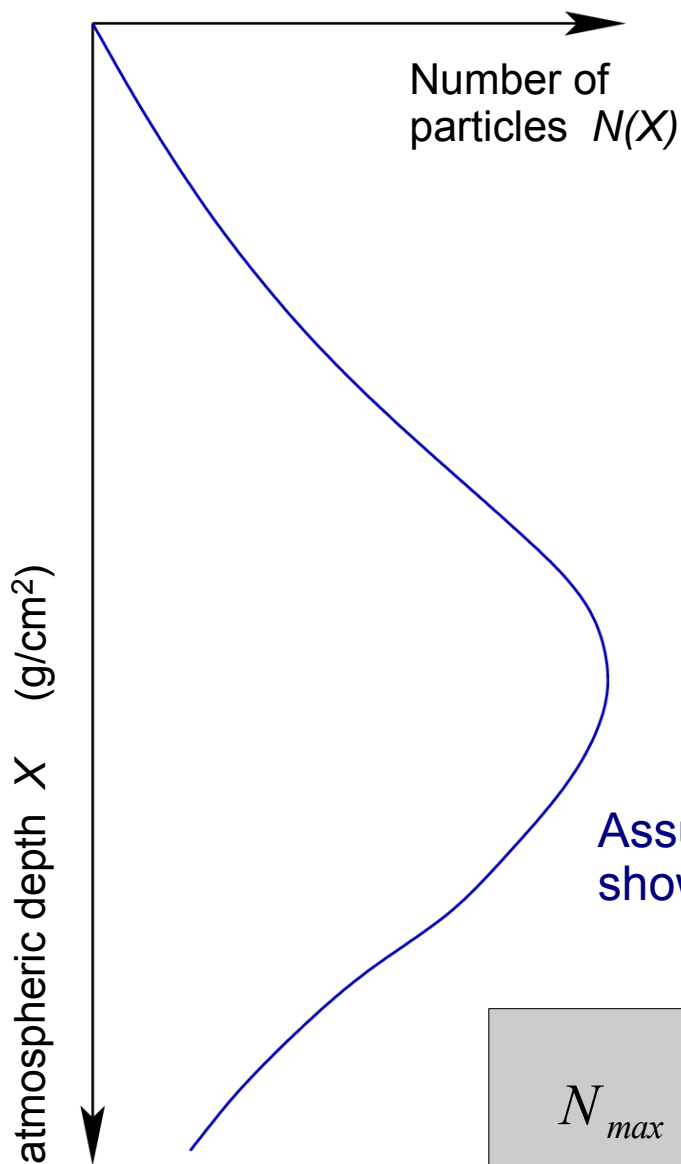
Comparison of energies



Outline

- Air showers and hadronic interactions
- Modelling of hadronic interactions
- Constraints from accelerator measurements
- Information from cosmic-ray data
- Possible interaction scenarios at ultra-high energy

Heitler's model of em. showers



Primary particle: photon

2^n particles after n interactions

$$n = X/\lambda$$

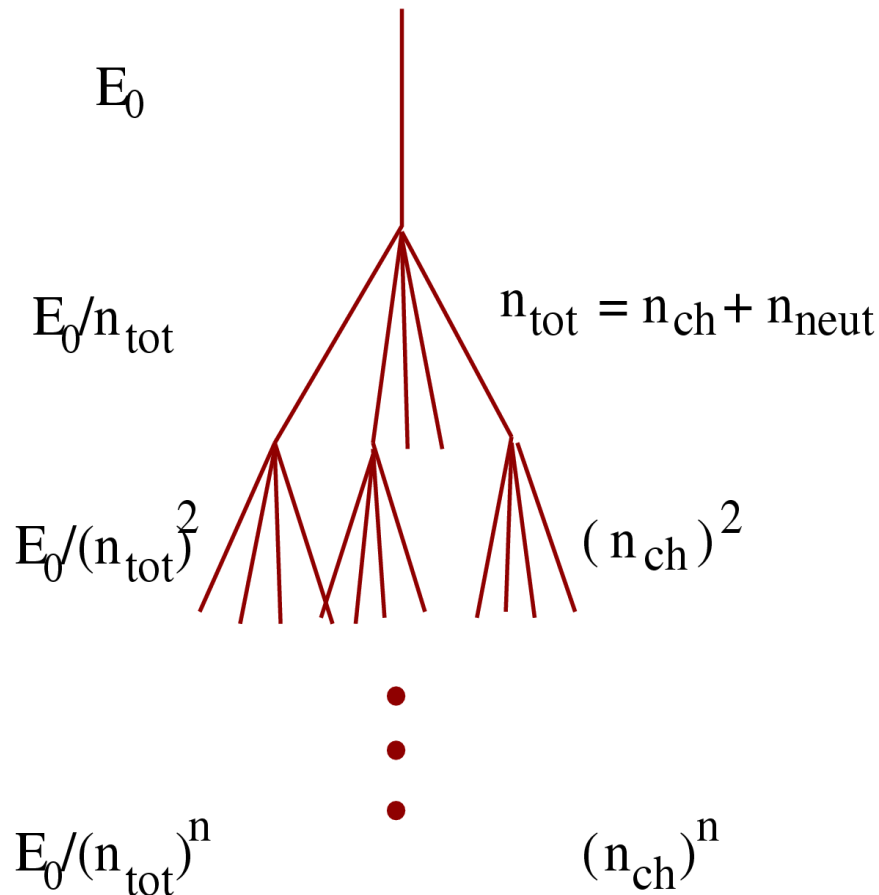
$$N(X) = 2^n = 2^{X/\lambda}$$

$$E(X) = E_0/2^{X/\lambda}$$

Assumption:
shower maximum reached if $E(X) = E_c$

$$N_{max} = E_0/E_c \qquad X_{max} \sim \lambda \ln(E_0/E_c)$$

Muon production in had. showers



Primary particle: proton

π^0 decay immediately

Only charged pions initiate new hadronic cascades

Cascade ends with decay at energy E_{dec}

$$E(X) = E_0 / (n_{tot})^n = E_{dec}$$

$$N_\mu = (n_{ch})^n$$

$$N_\mu = \left(\frac{E_0}{E_{dec}} \right)^\alpha, \quad \alpha = \frac{\ln n_{ch}}{\ln n_{tot}} \approx 0.82 \dots 0.95$$

Application: superposition model

Proton shower characteristics:

$$N_{max} = E_0 / E_c \qquad N_\mu = \left(\frac{E_0}{E_{dec}} \right)^\alpha$$
$$X_{max} = \lambda_e \ln(E_0)$$

Assumption:

nucleus of mass A and energy E_0 acts

like A independent nucleons with energy $E_n = E_0/A$

$$N_{max}^A = A E_n / E_c = E_0 / E_c$$

$$X_{max}^A \sim \lambda_e \ln(E_0/A)$$

$$N_\mu^A = A \left(\frac{E_0/A}{E_{dec}} \right)^\alpha = A^{1-\alpha} N_\mu$$

Toy model parameters

Hadronic interaction model

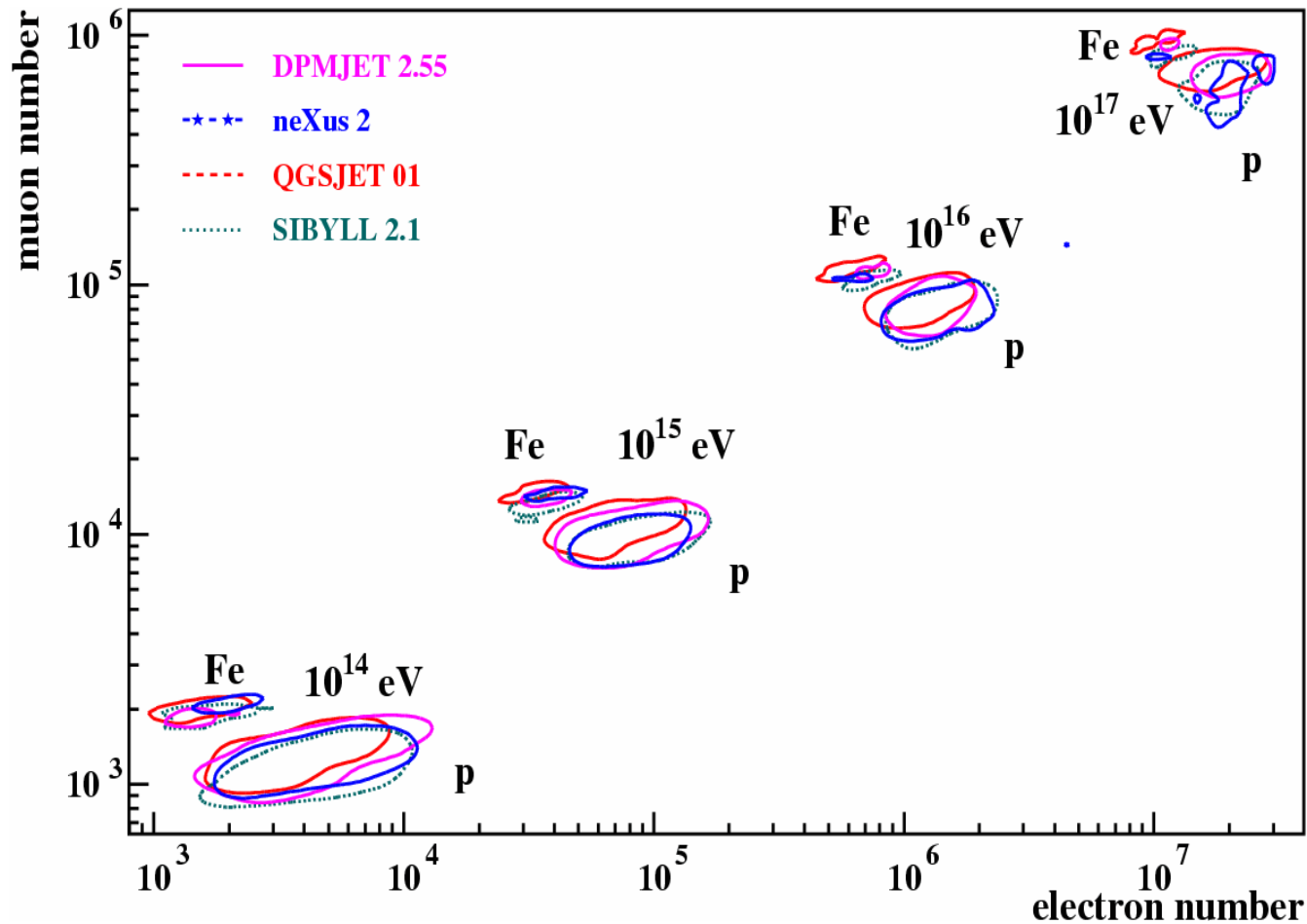
- interaction cross section
- multiplicity of secondary particles
- ratio of neutral to charged pion multiplicity

Atmosphere as target and calorimeter

- critical energy
- typical pion decay energy

Number of shower particles proportional to energy

Energy/composition: N_e - N_μ correlation



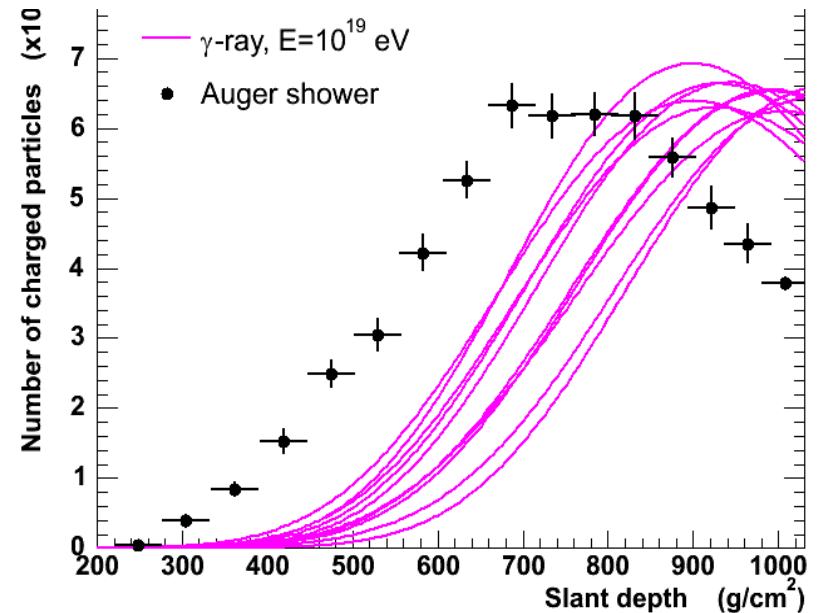
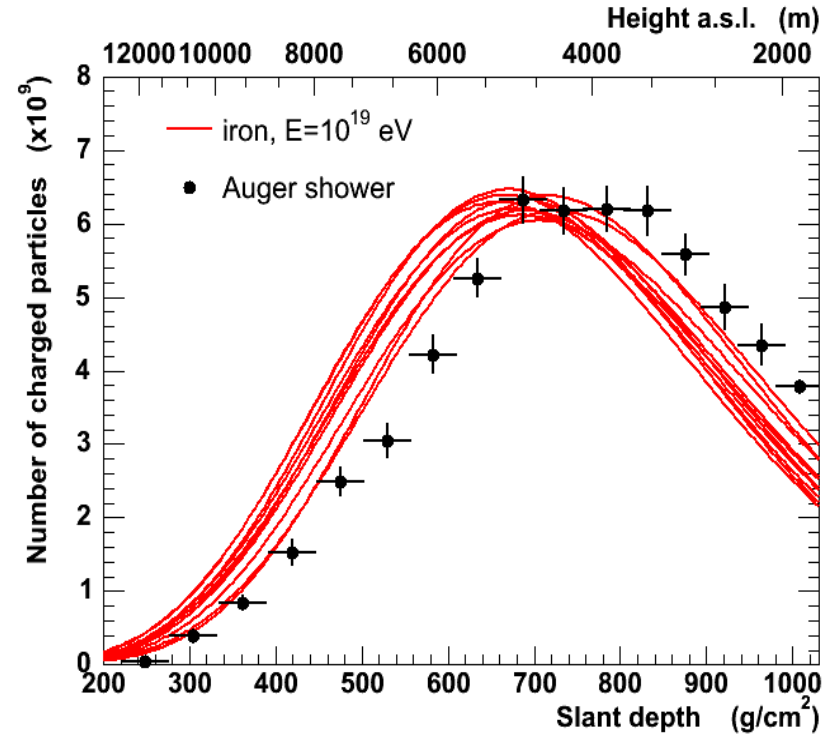
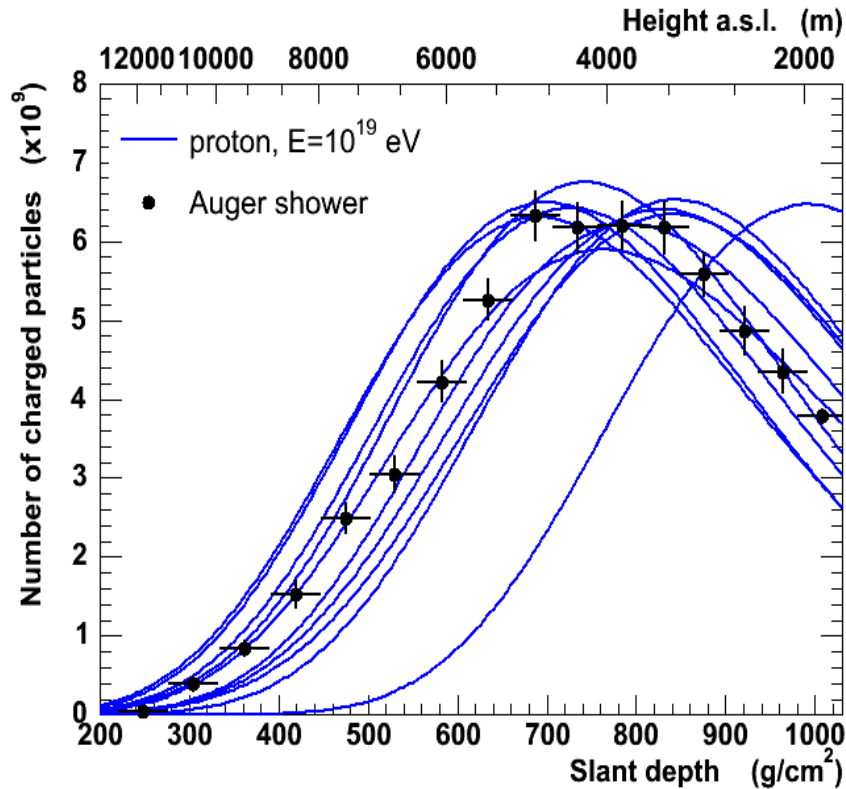
Standard method
for surface detector
arrays

$$N_\mu^A = A^{1-\alpha} \left(\frac{E_0}{E_{dec}} \right)^\alpha$$

Model dependence increasing with energy

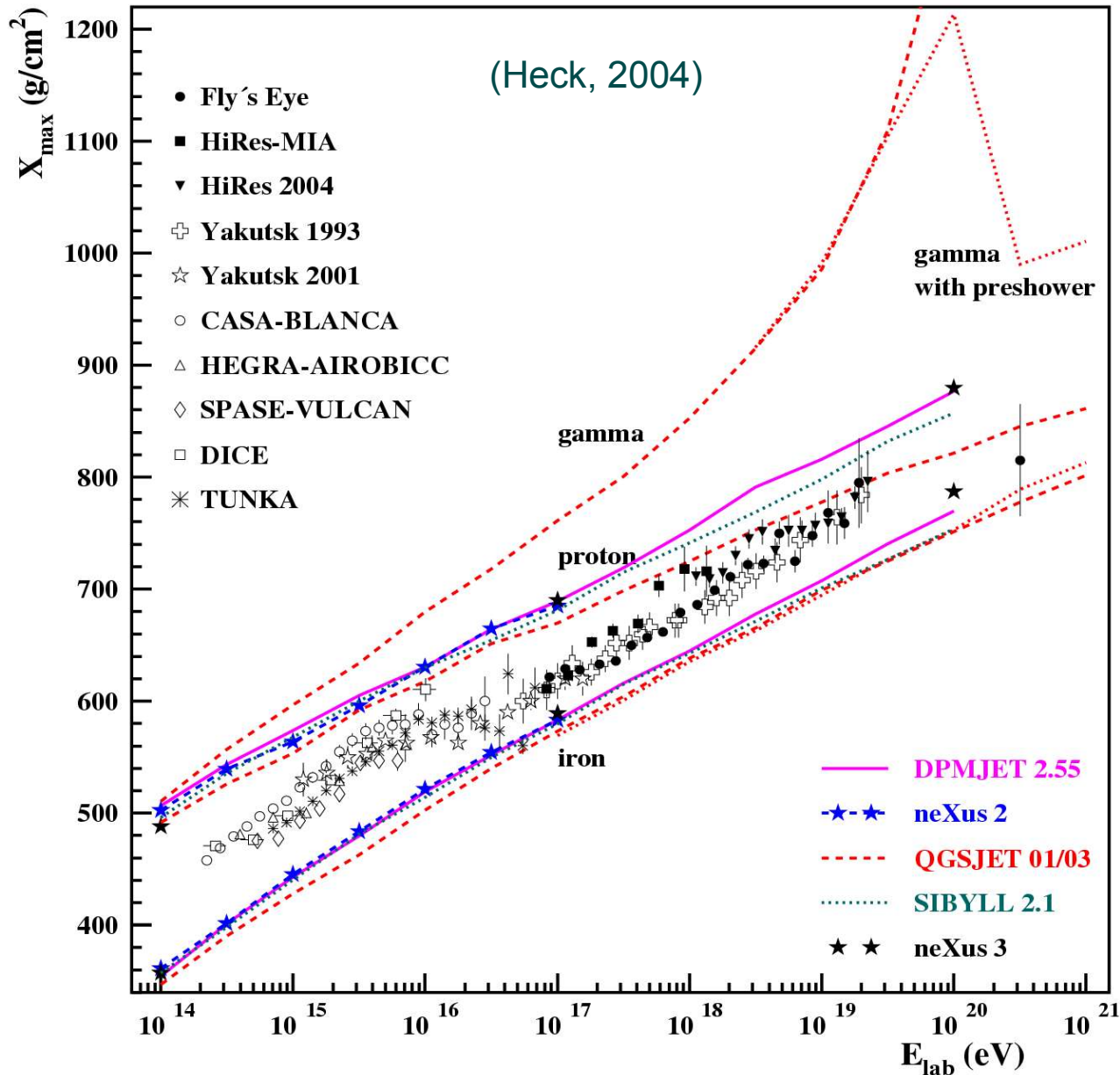
Energy/composition: shower profile

Detailed MC simulation: 10 showers
zenith angle 35°, QGSJET



$$N_{max}^A = N_{max}, \quad X_{max}^A \sim \lambda_e \ln(E_0/A)$$

Mean depth of shower maximum



Superposition model:

$$X_{max}^A \sim \lambda_e \ln(E_0/A)$$

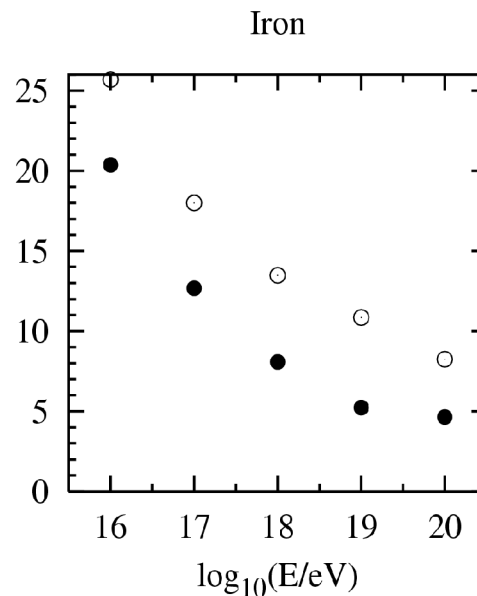
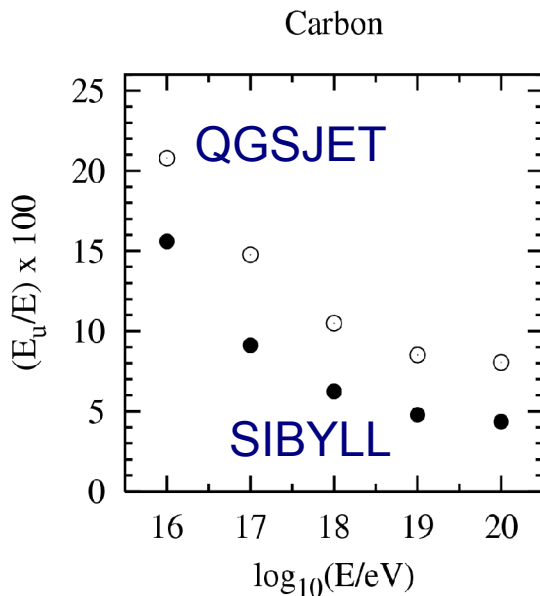
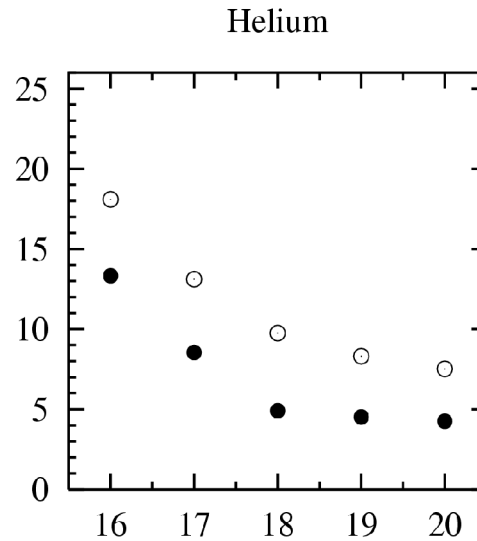
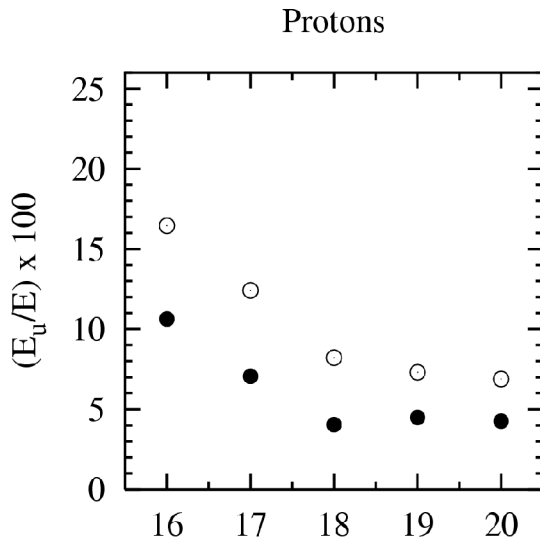
MC simulation (CORSIKA):

predictions depend on had. interaction model used for simulation

Energy reconstruction: fluorescence technique

Fraction of “unseen” energy E_u

$$E = E_{\text{cal}} + E_u$$



The higher the energy of the primary particle the smaller the fraction of unseen energy

(Alvarez-Muniz et al.,
PRD69 (2004) 103003)

Hadronic interaction models

Requirements:

- simulation of π , K, ρ , n,, Fe collisions with air nuclei (C,N,O, Ar)
- coverage of full energy range from production threshold to $\sqrt{s} \sim 400,000$ GeV
- minimum bias event simulation
 - central and peripheral collisions
 - diffractive and non-diffractive interactions
- optimal description of high-energy secondary particles
- tuned to existing fixed-target and collider data
- variable projectile/target combinations
- variable collision energy
- fast simulation

Cosmic ray hadronic interaction models

High energy models:

DPMJET II.5 and III (Ranft / Roesler, RE & Ranft)

neXus 2.0 and 3.0 (Drescher, Hladik, Ostapchenko, Pierog & Werner)

QGSJET 98 and 01 (Kalmykov & Ostapchenko)

SIBYLL 1.7 and 2.1 (Engel / RE, Fletcher, Gaisser, Lipari & Stanev)

- Gribov-Regge type models, minijets
- Parametrizations of data

Low/intermediate energy models:

GHEISHA (Fesefeldt)

Hillas' splitting algorithm (Hillas)

FLUKA (Fasso, Ferrari, Ranft & Sala)

UrQMD (Bass, Bleicher et al.)

TARGET (RE, Gaisser, Protheroe & Stanev)

HADRIN/NUCRIN (Hänßgen & Ranft)

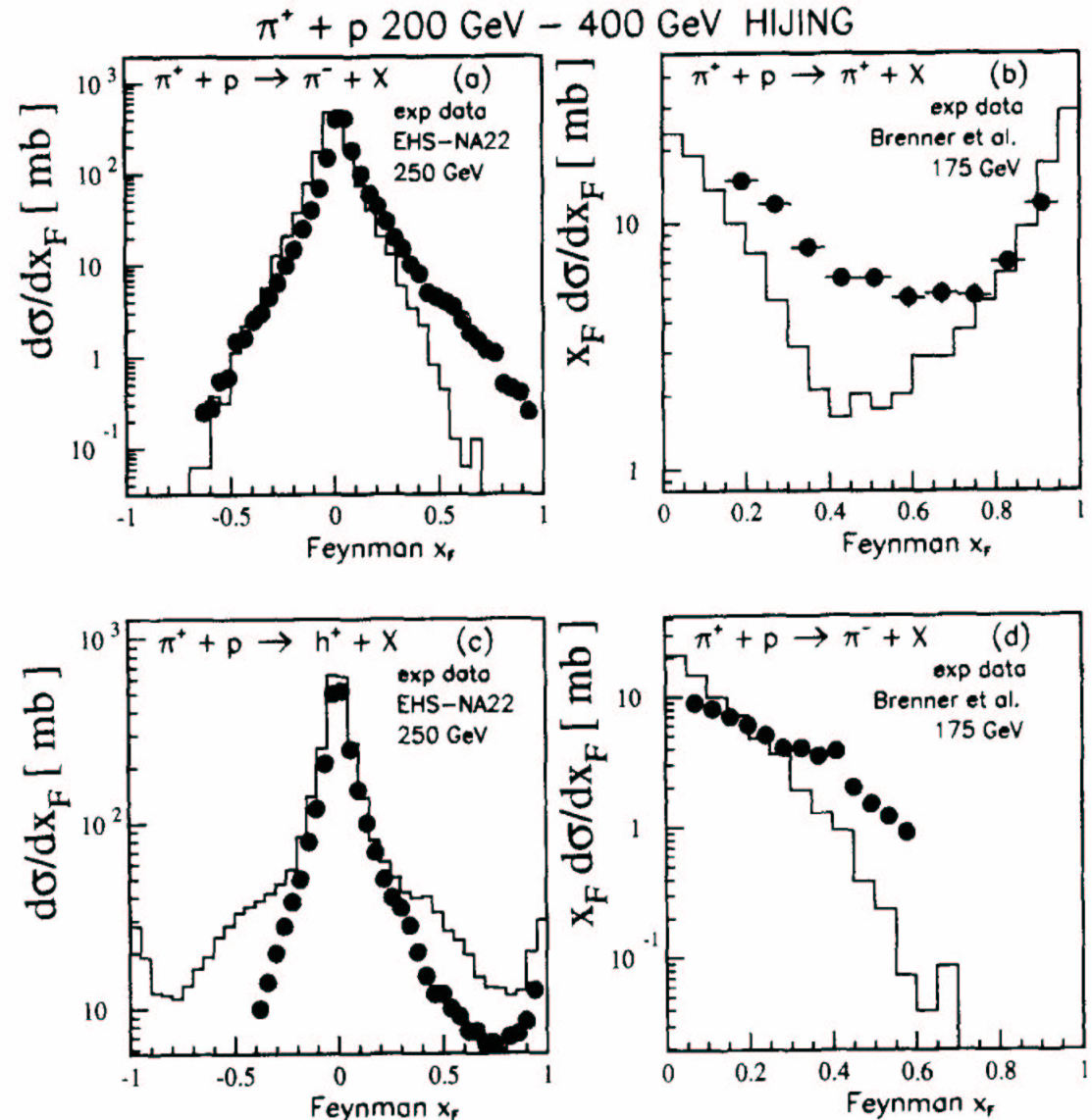
SOPHIA (Mücke, RE, Rachen, Protheroe, Stanev)

Why not PYTHIA, HERWIG, HIJING, ... ?

Most models not designed/tuned for simulating forward particle production

Most models cannot handle different projectiles/targets and energies

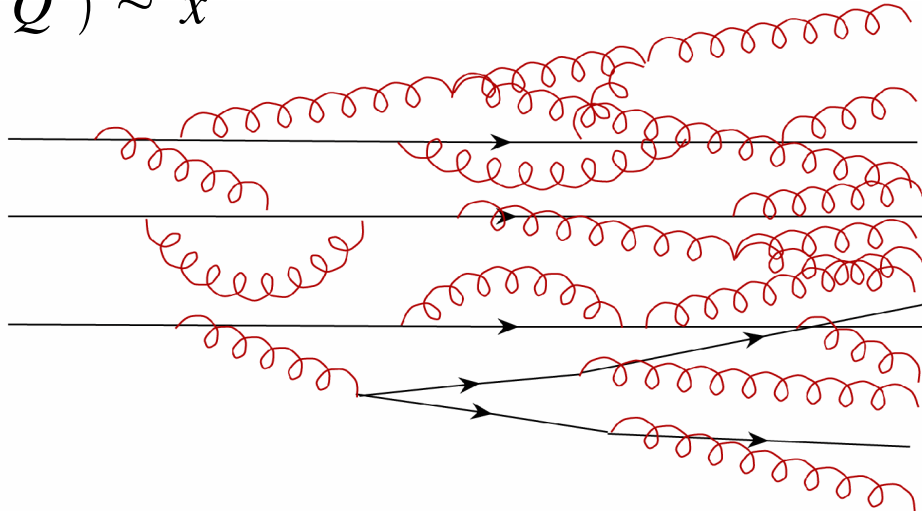
Example: comparison of HIJING to fixed target data



(Pop, Gyulassy & Rebel, Astropart. Phys. 10 (1999) 211)

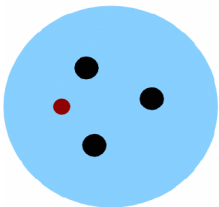
Geometric view of hadron

$$f_i(x, Q^2) \sim x^{-0.4}$$

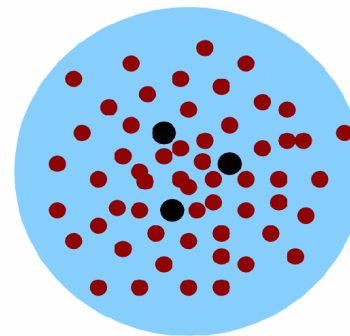


Total cross section:
number and spacial
distribution of gluons

Fast secondaries:
quarks/gluons with
large momentum
fraction x



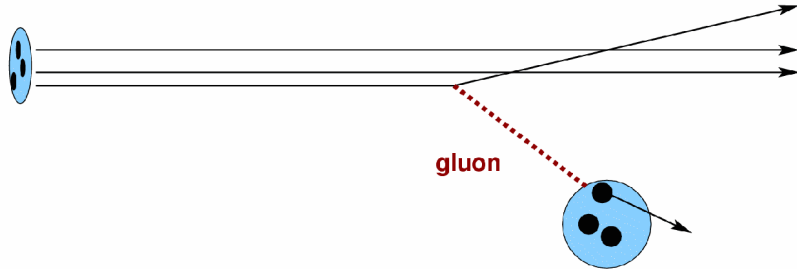
Hadron at low energy



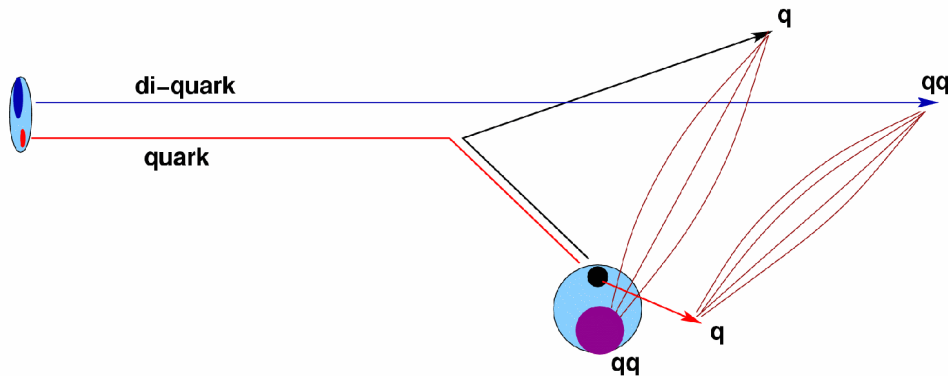
Hadron at high energy

Low energy: two-string models

Partonic view:



Color flow:



Basics:

- hadrons built up of quarks, gluons
- quarks, gluons carry color interactions lead to color transfer

String fragmentation from e+e- data

Regge parametrizations: Mueller diagrams

Momentum fraction of quark in nucleon:

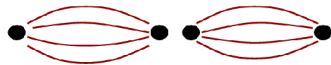
$$f_{\text{DPM}}(x_q) = x_q^{-1/2} (1 - x_q)^{3/2}$$

Momentum fraction of quark in pion:

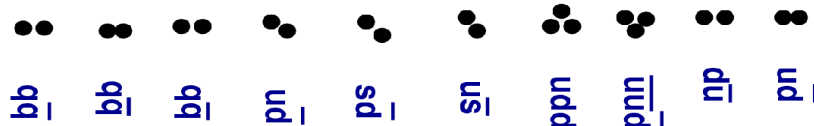
$$f_{\text{DPM}}(x_q) = x_q^{-1/2} (1 - x_q)^{-1/2}$$

String fragmentation process

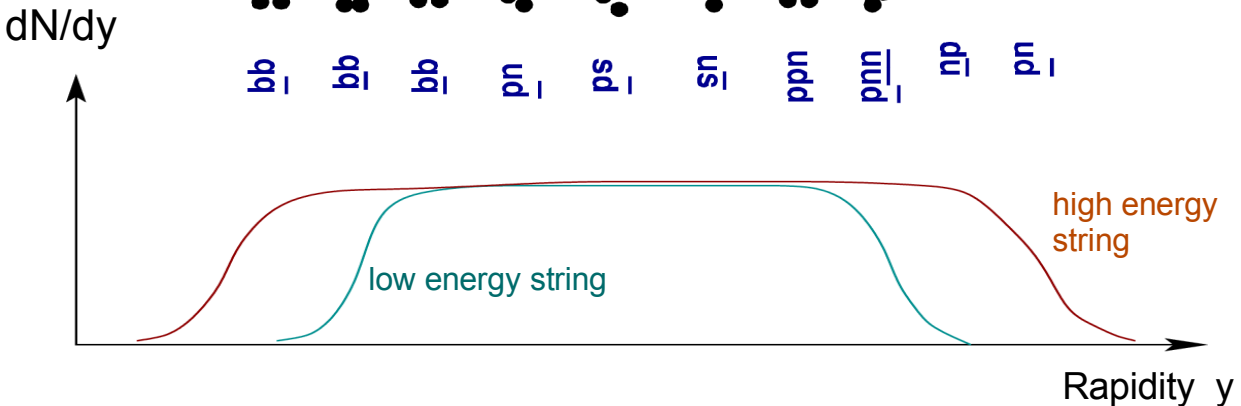
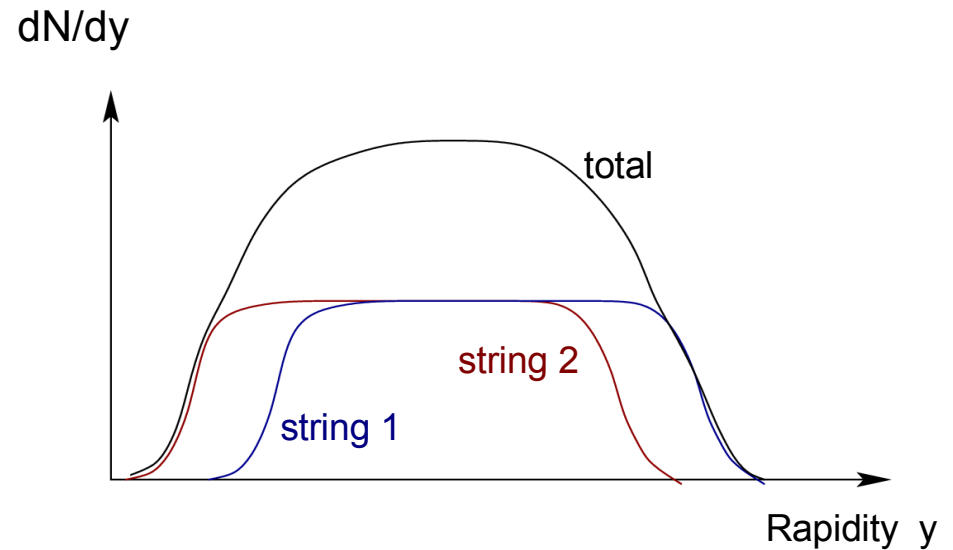
Example:
 $e^+ e^- \rightarrow q \bar{q}$



.....



Fragmentation:
 assumed to be universal



Rapidity plateau
 independent of energy:
Feynman scaling

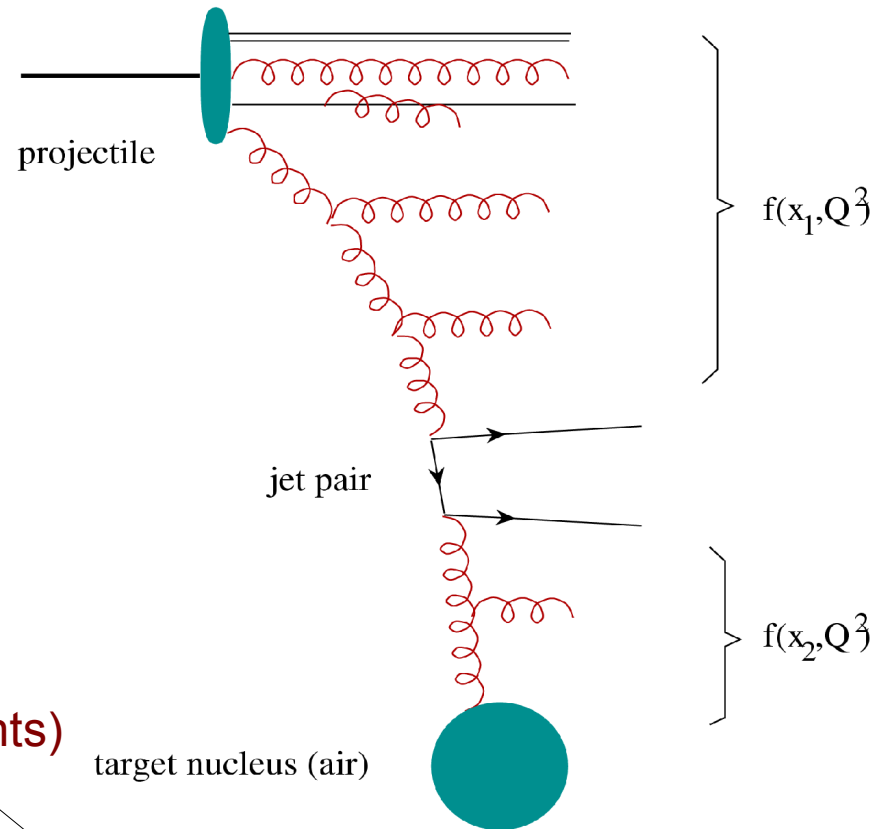
High energy: QCD minijet production

QCD parton model:

colinear factorization
DGLAP evolution Eqs.

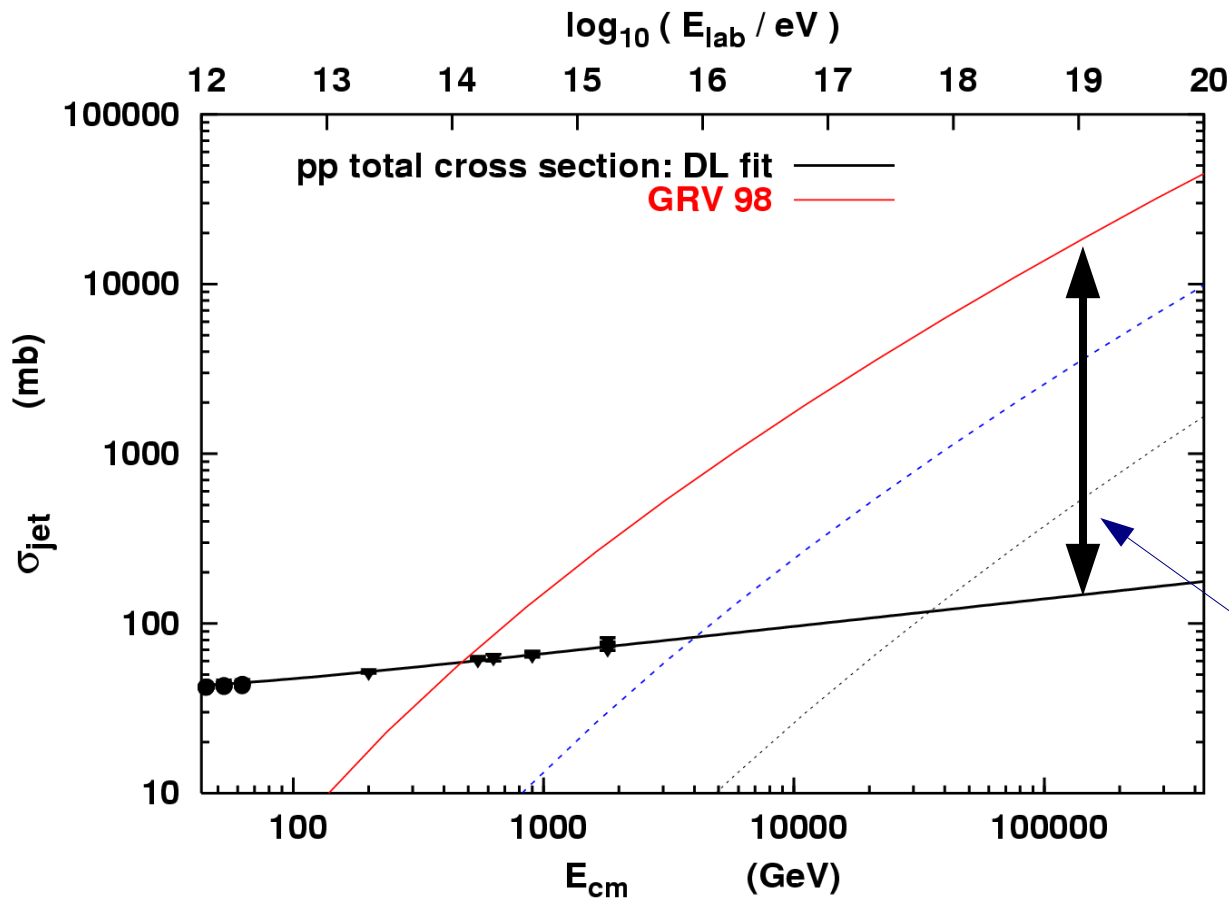
Transverse momentum
cutoff: $p_T > p_T^{\text{cutoff}}$

Parton densities
(HERA measurements)



$$\sigma_{\text{jet}} = \sum_{i,j,k,l} \int_{p_T^{\text{cutoff}}} dp_T \int dx_1 dx_2 f_i(x_1, Q^2) f_j(x_2, Q^2) \frac{d\sigma_{i,j \rightarrow k,l}}{dp_T}$$

Inclusive minijet cross section



$$p_T^{\text{cutoff}} = 2 \text{ GeV}$$

$$p_T^{\text{cutoff}} = 4 \text{ GeV}$$

$$p_T^{\text{cutoff}} = 8 \text{ GeV}$$

~150 jet pairs per
single p-p collision

$$\langle n_{\text{jet}} \rangle = \frac{\sigma_{\text{jet}}}{\sigma_{\text{ine}}}$$

Transverse momentum cutoff p_t^{cutoff} : QCD gives no prediction

(a) value at a given energy

(b) dependence on energy, projectile/target, ...

Transverse momentum cutoff

HERA data:
small $p_T^{\text{cutoff}} \sim 1.5$ GeV

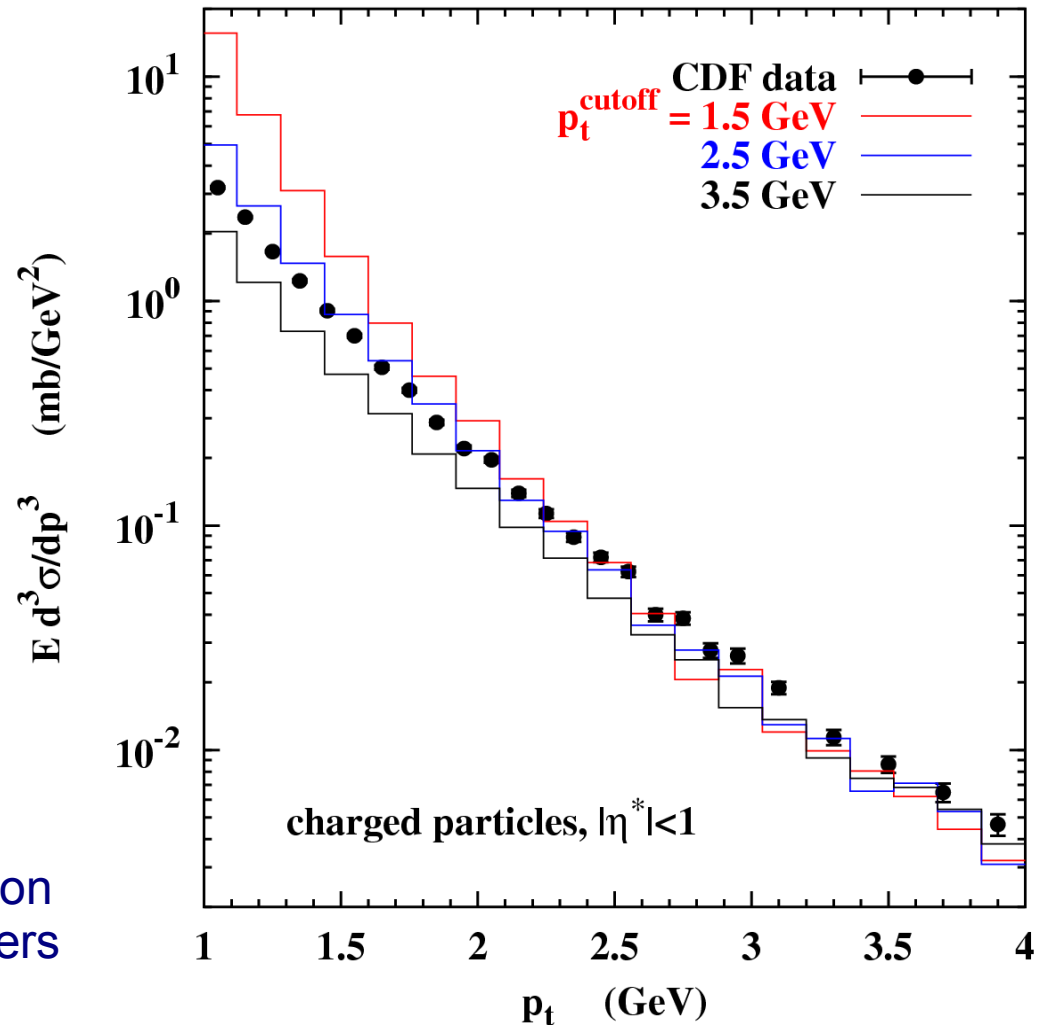
Tevatron data:
slightly larger $p_T^{\text{cutoff}} \sim 3$ GeV

Energy-dependence of
transverse momentum cutoff?

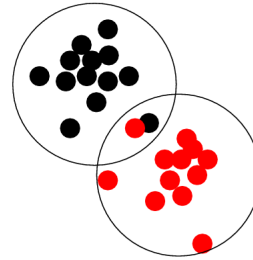
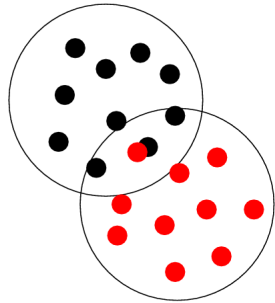
QGSJET II: energy-independent
transverse momentum cutoff, summation
of soft multi-pomeron graphs to all orders

SIBYLL: energy-dependent transverse
momentum cutoff

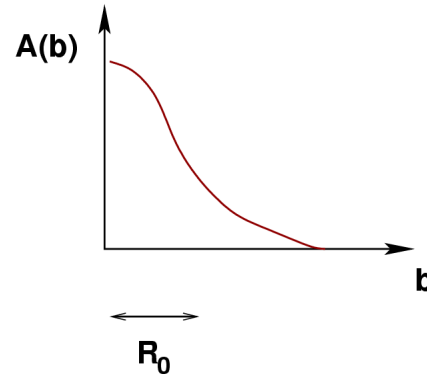
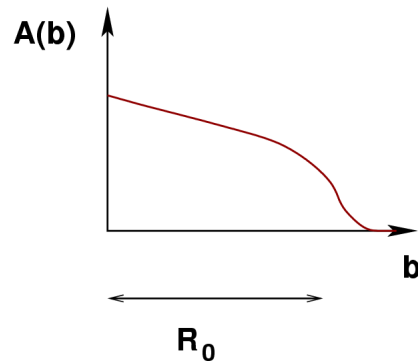
$$p_T^{\text{cutoff}} = p_T^0 + 0.065 \text{ GeV} \exp(0.9 \sqrt{\ln s})$$



Calculation of total cross section



QCD: prediction of inclusive cross section (number of partons)



Total cross section: profile function $A(b)$ needed

QGSJET: gaussian distribution

SIBYLL: em. form factor

Example: eikonal model

$$\sigma_{\text{ine}} = \pi \int db^2 \left(1 - \exp \left[-\sigma_{\text{jet}} A(b) - \sigma_{\text{soft}} A_s(b) \right] \right)$$

Resummation according to topologies

Analytical resummation
of inelastic cross
section (eikonal only):

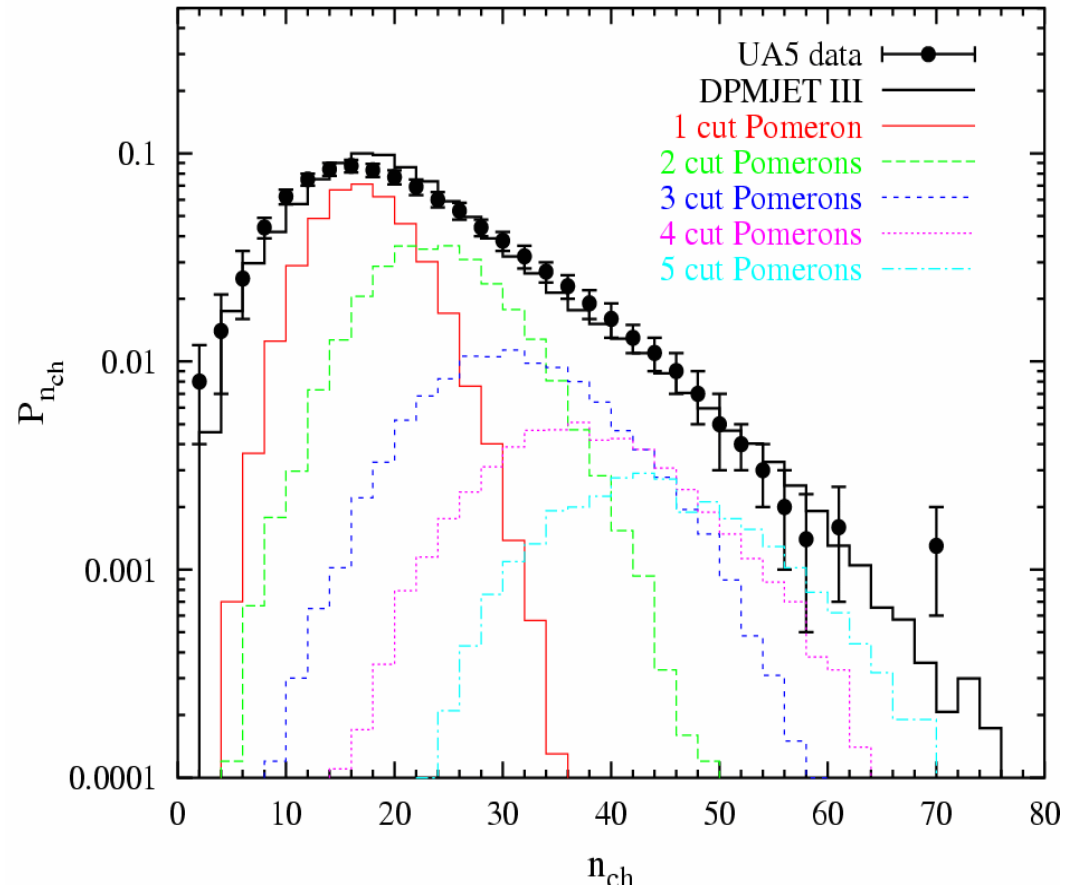
partial cross sections
for n hard/soft
partonic interactions
(cut pomerons)

$$\sigma_{\text{inel}}^{(n)} = \int d^2 b \frac{(2\chi)^n}{n!} \exp(-2\chi)$$

$$\chi = \chi_{\text{soft}} + \chi_{\text{hard}}$$

$$\chi_{\text{soft}} = \sigma_{\text{soft}} A_{\text{soft}}(s, b)$$

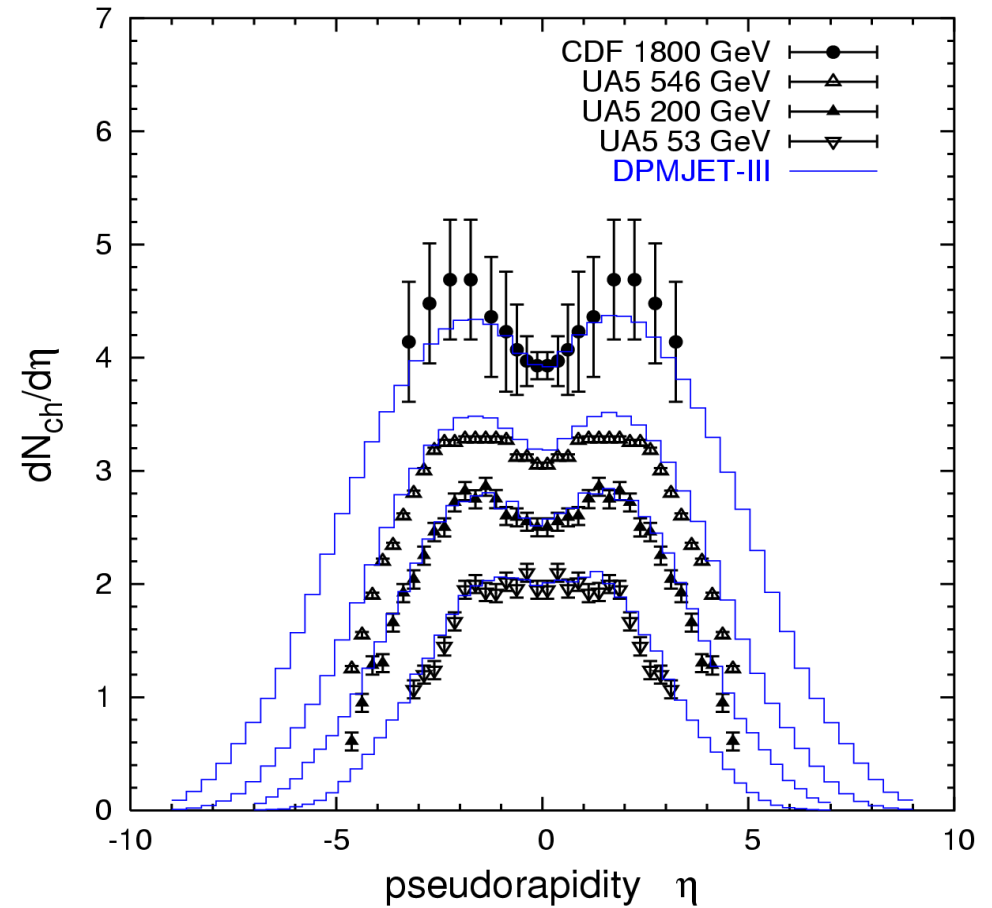
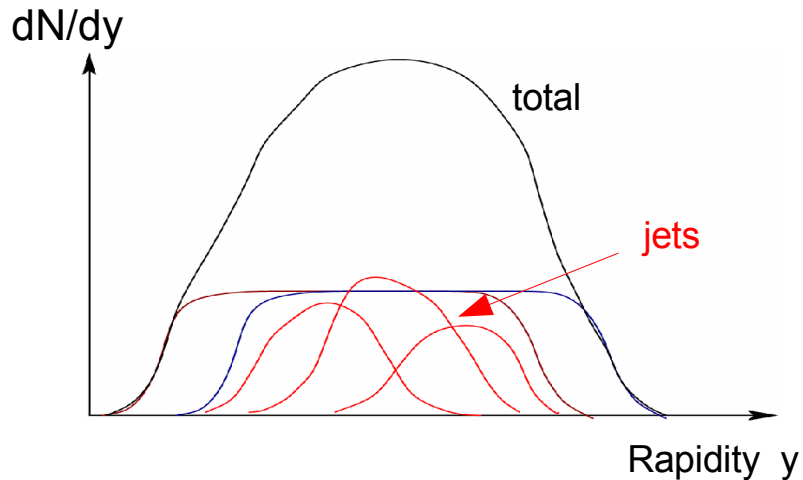
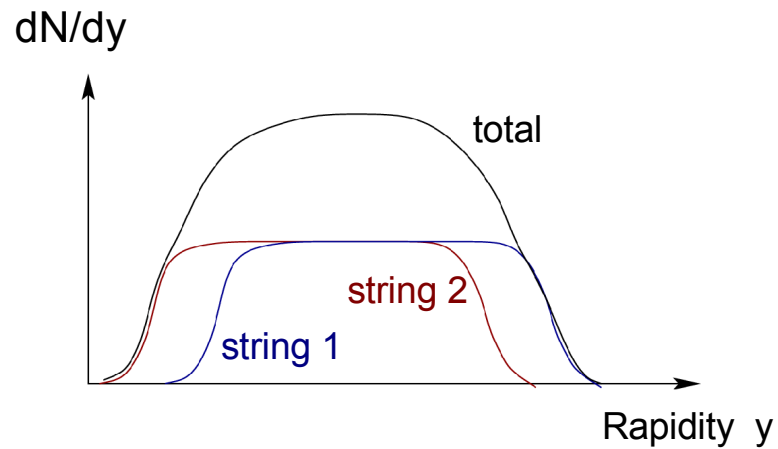
$$\chi_{\text{jet}} = \sigma_{\text{jet}} A_{\text{jet}}(s, b)$$



(AGK cutting rules)

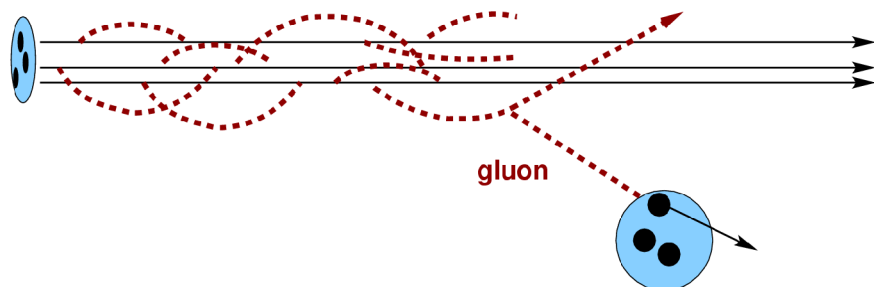
Violation of Feynman scaling

Multiple partonic interactions lead to increase of rapidity plateau



Leading particle production

Partonic view:



Parametrizations:

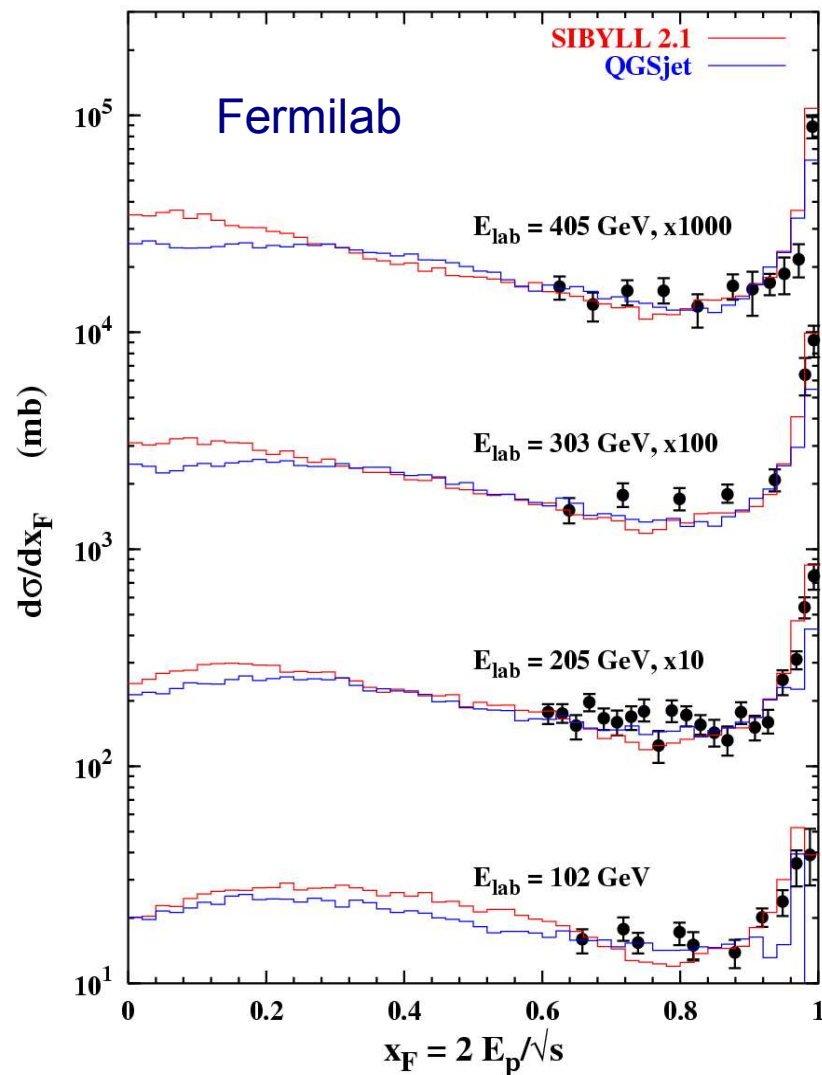
scaling assumed (energy-independent)

$$f_{\text{DPM}}(x_q) = x_q^{-1/2} (1 - x_q)^{3/2}$$

$$f_{\text{SIB}}(x_q) = (x_q + \mu^2/s)^{-1/4} (1 - x_q)^3$$

energy-momentum conservation

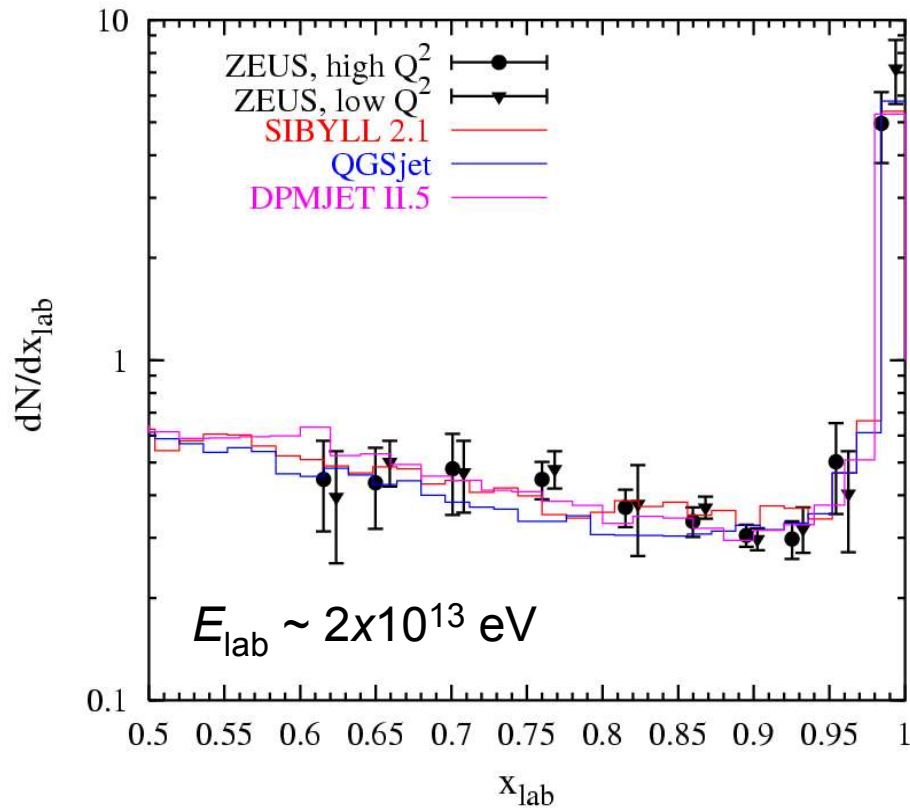
$p p \rightarrow p X$, NAL Hydrogen Bubble Chamber



$$E_{\text{lab}} \sim 4 \times 10^{11} \text{ eV}$$

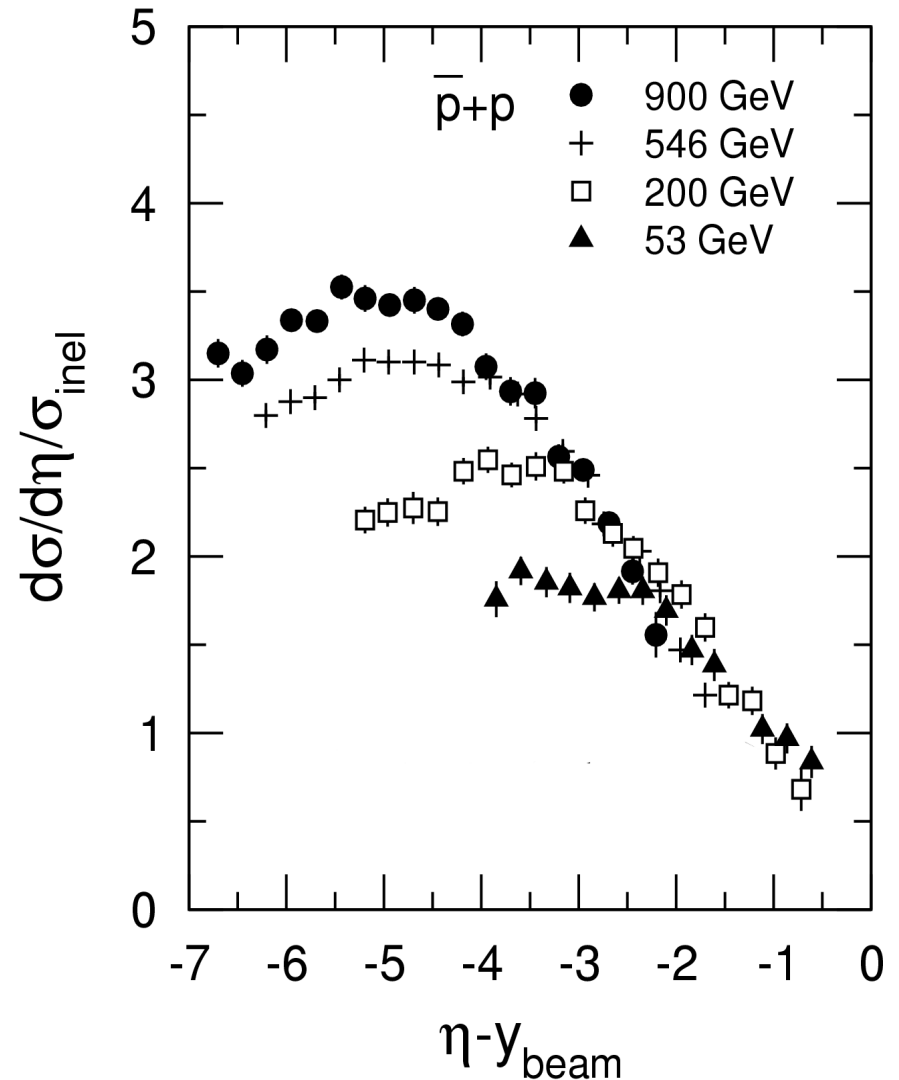
Extrapolation of leading particle production

HERA: $p\text{-}\gamma \rightarrow p/n X$



No indications of scaling violation of leading particle distributions

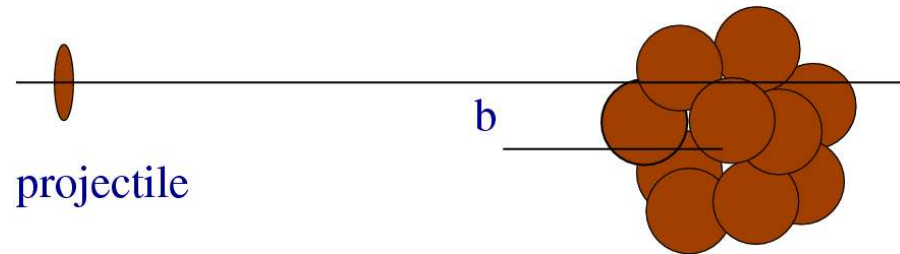
CERN: limiting fragmentation



Nuclear projectiles & targets

Nuclear effects:

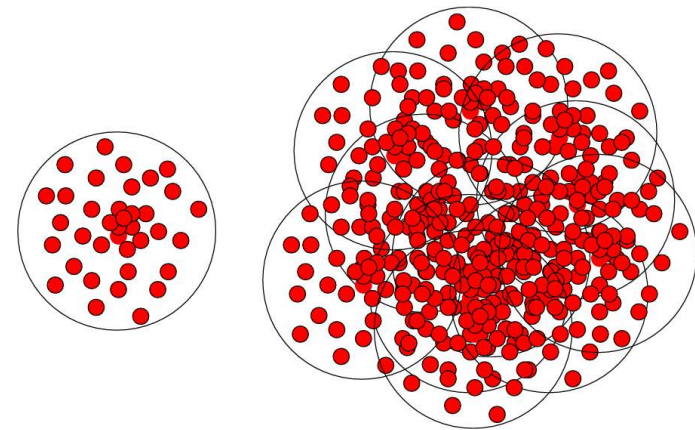
Gribov-Glauber approximation



$$\sigma_{\text{inel}}^{(pA)} \approx \pi \int db^2 \left(1 - \exp \left[-\sigma_{\text{tot}}^{(pp)} T_A(b) \right] \right)$$

Saturation effects expected to be even more important:

- String fusion and percolation
- Triple and multiple pomeron interactions
- Non-linear parton density evolution equations
- Color Glass Condensate Model

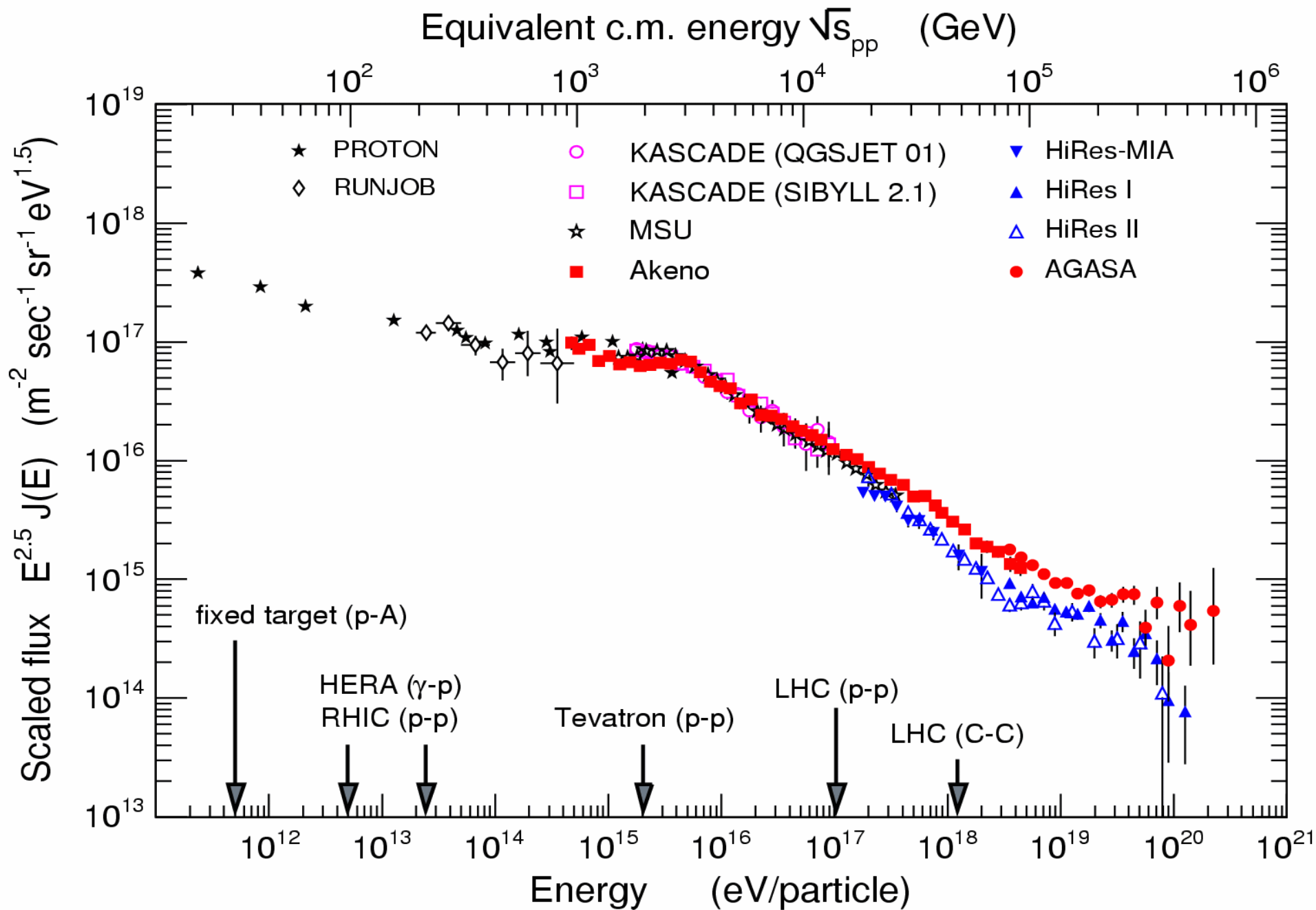


RHIC data!

Main sources of uncertainty

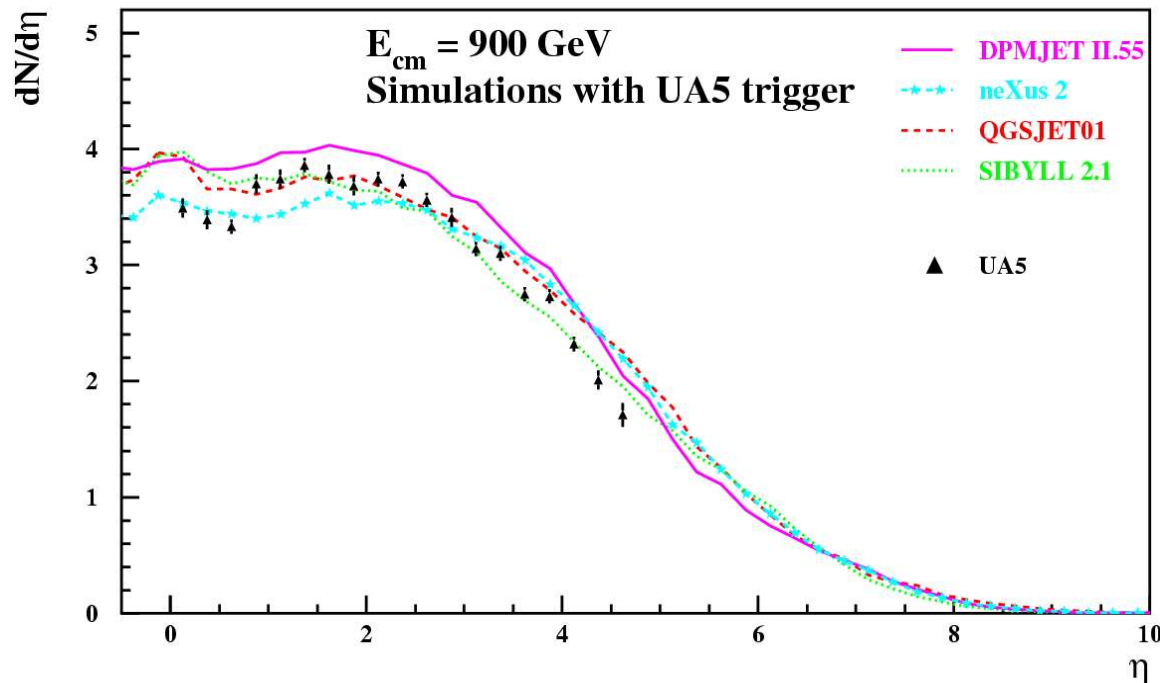
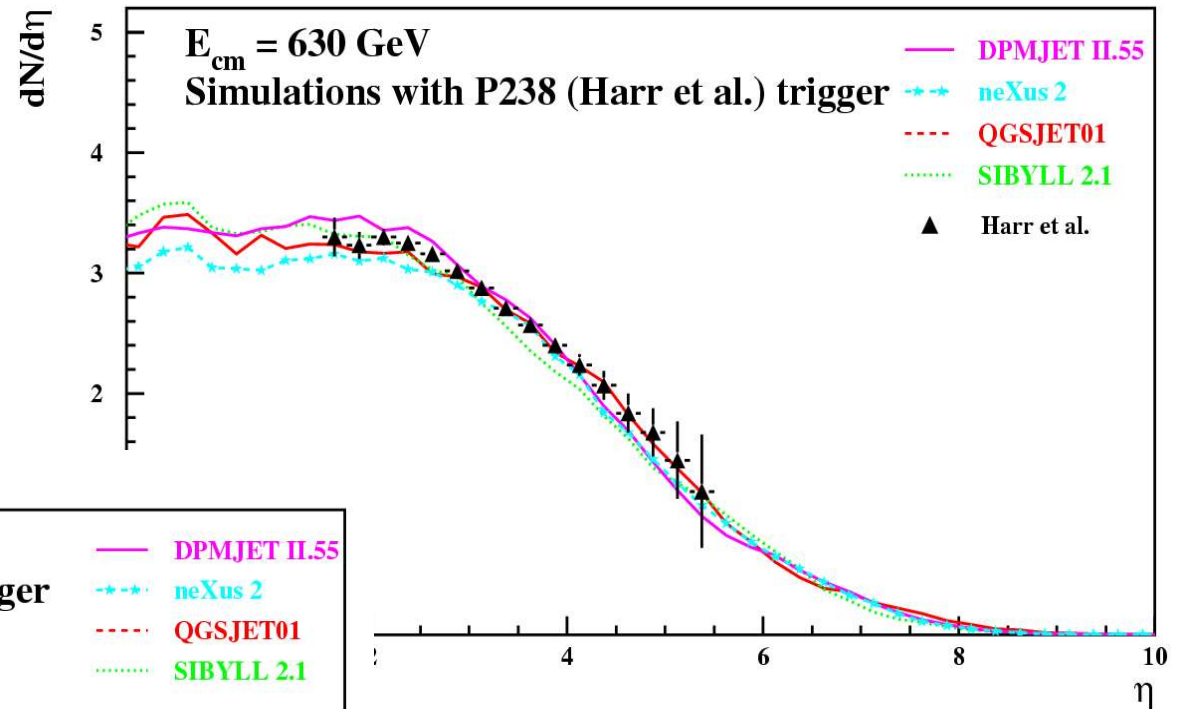
- Minijet cross section (parton densities, range of applicability)
- Transverse profile function (total cross section, multiplicity distribution)
- Energy dependence of leading particle distribution
- Role of nuclear effects (saturation, stopping power, QGP)

Tuning to accelerator measurements



Tuning to accelerator data (i)

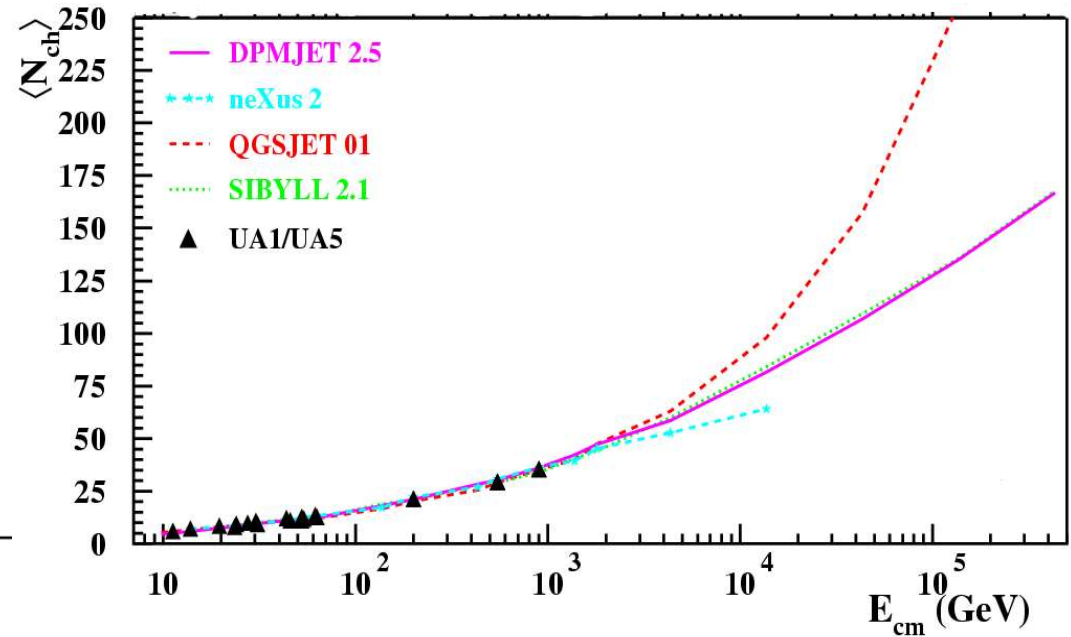
Proton-antiproton at
CERN SPS



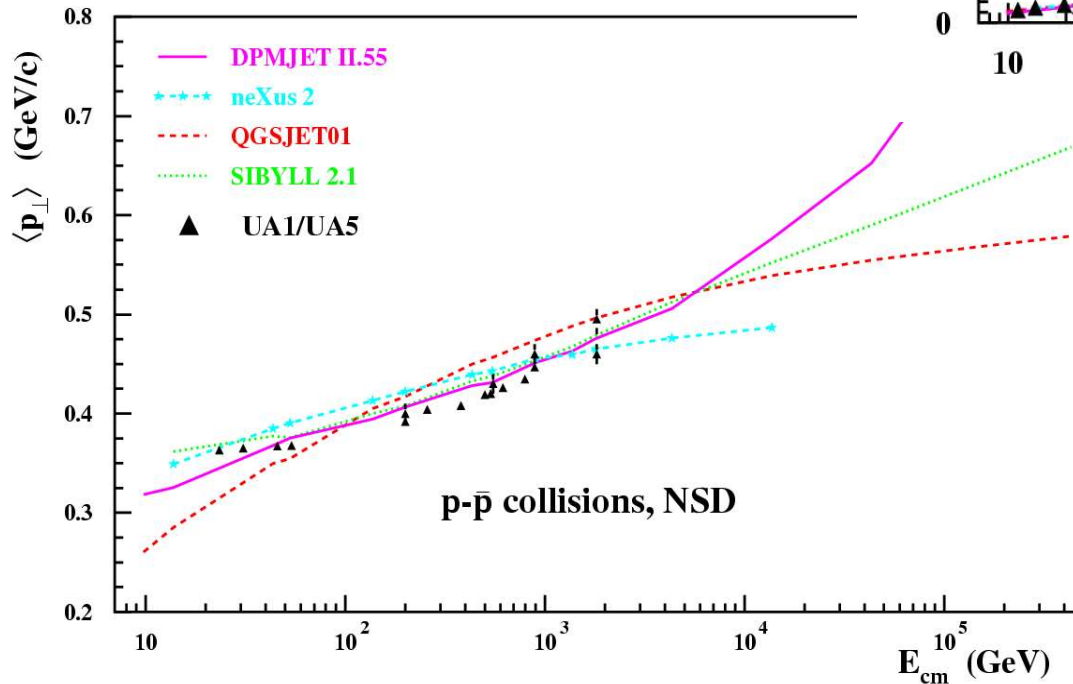
Pseudorapidity distribution
of charged particles

Tuning to accelerator data (ii)

Mean charged particle multiplicity

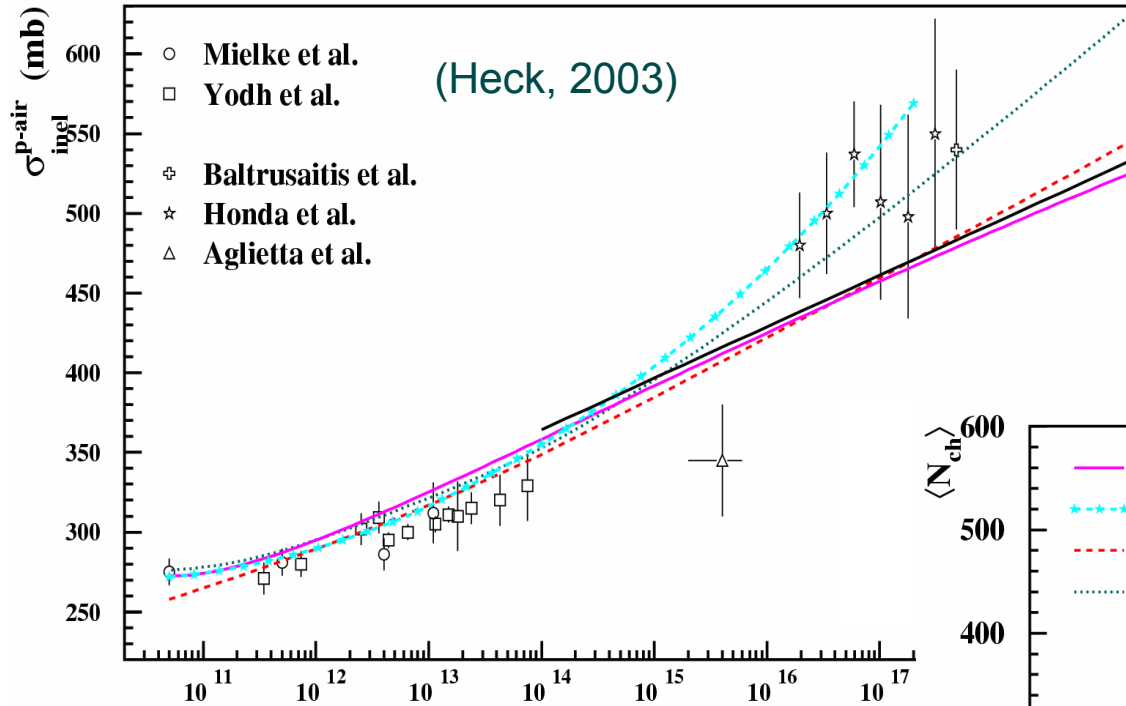


Mean transverse momentum



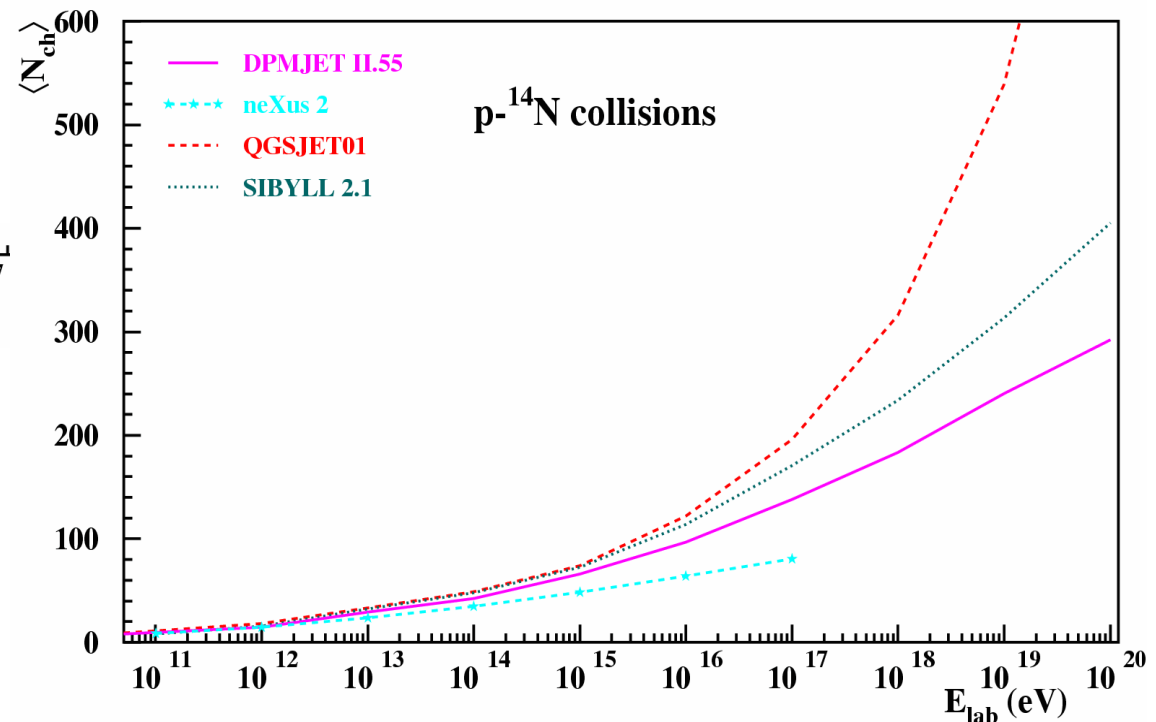
Proton-antiproton at
CERN SPS & Tevatron

Hadronic interaction model predictions



Production cross section p-air

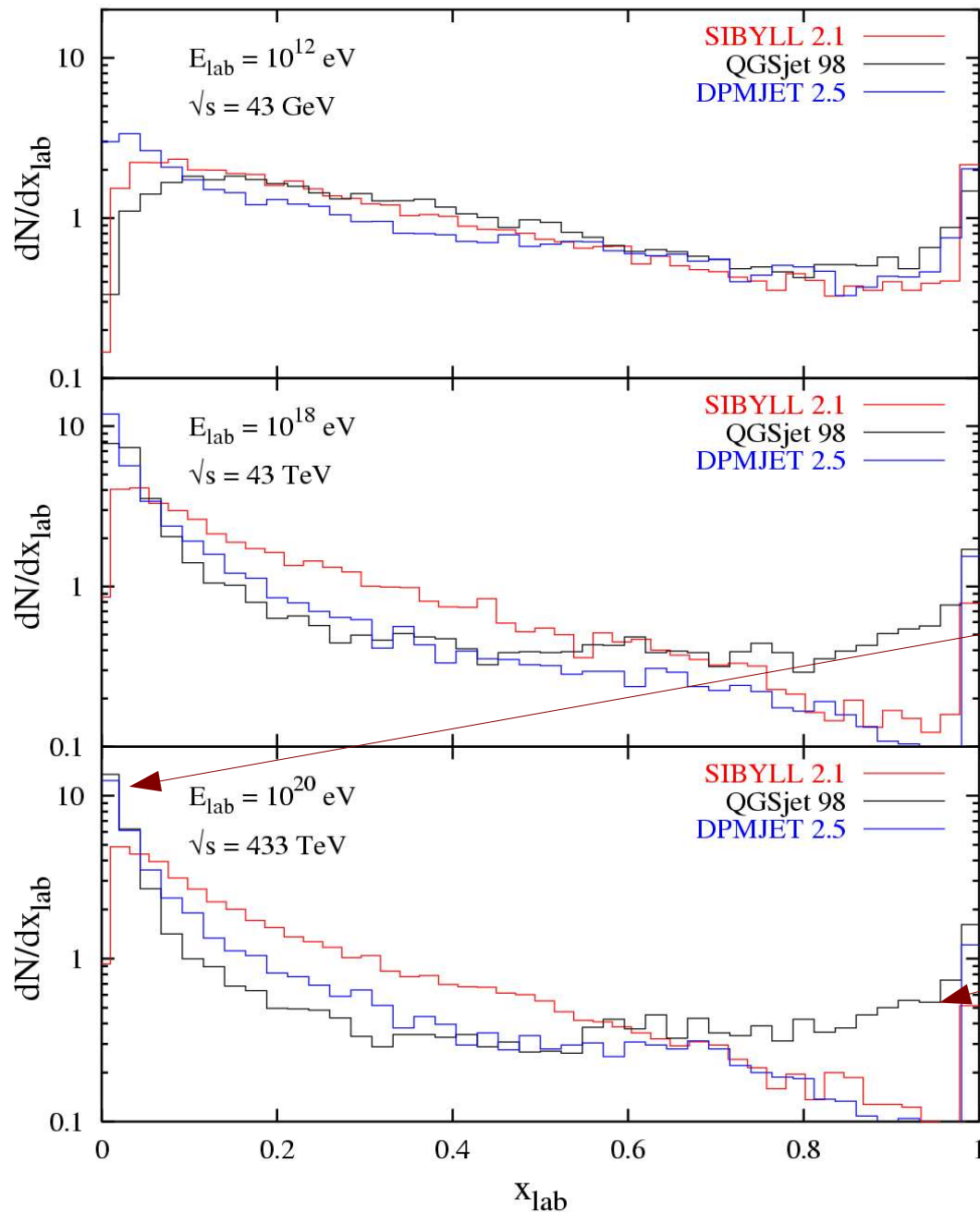
Secondary particle multiplicity (charged)



Wide range of predictions:

- Air showers rather insensitive
- Correlation of cross section and multiplicity: partial compensation

Extrapolation of leading particle production



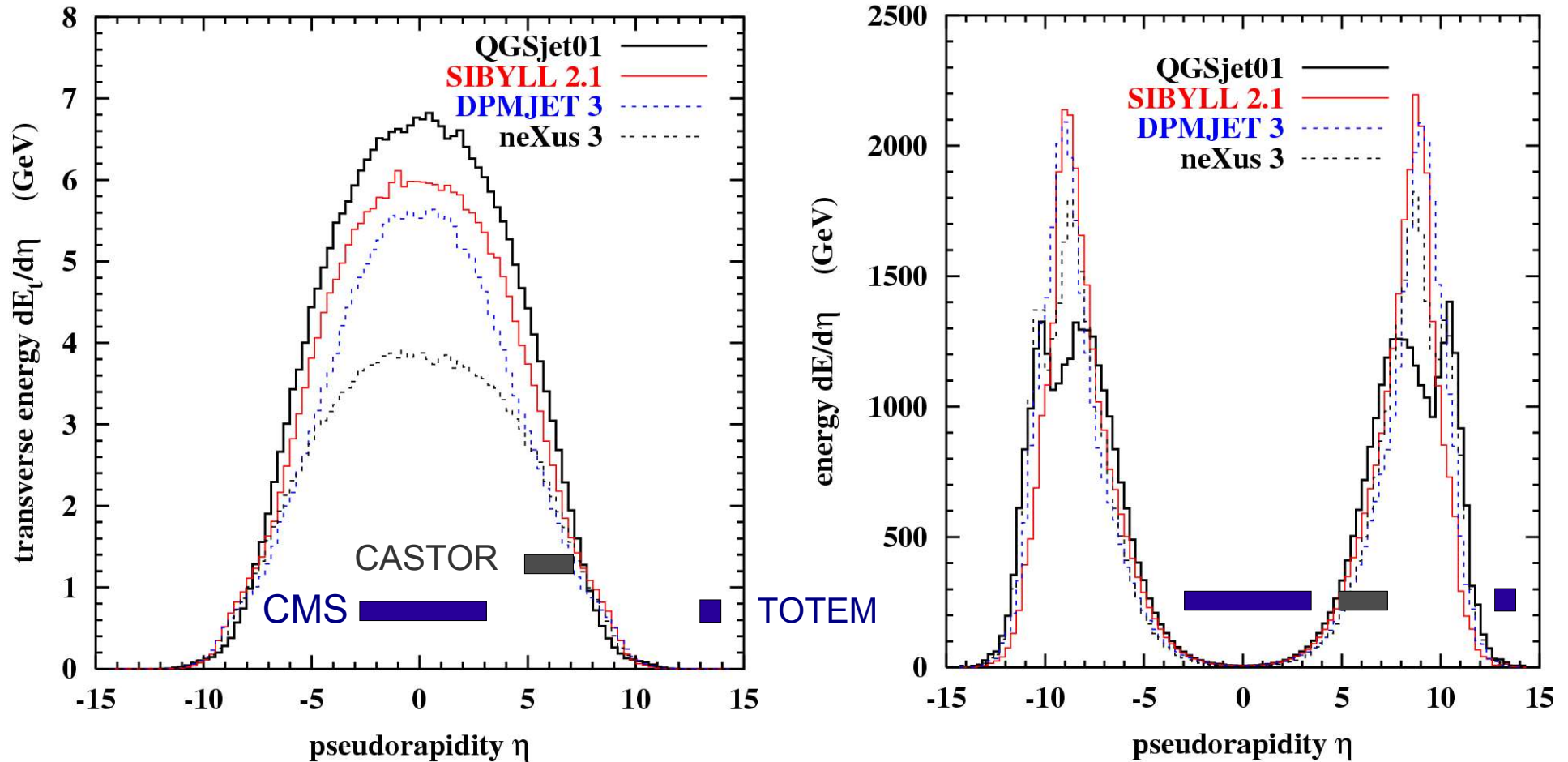
Distribution of momentum fraction of leading baryon

$p\text{-air} \rightarrow p/n X$

Extremely inelastic events

Nearly elastic events

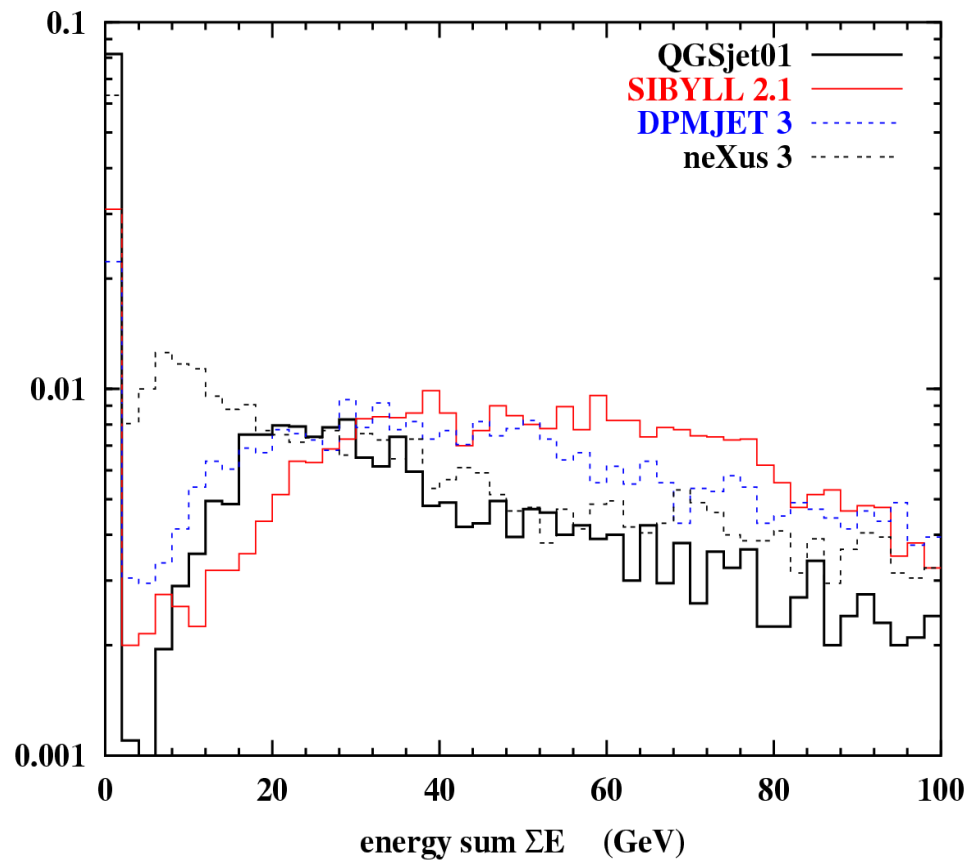
Discrimination potential of LHC (i)



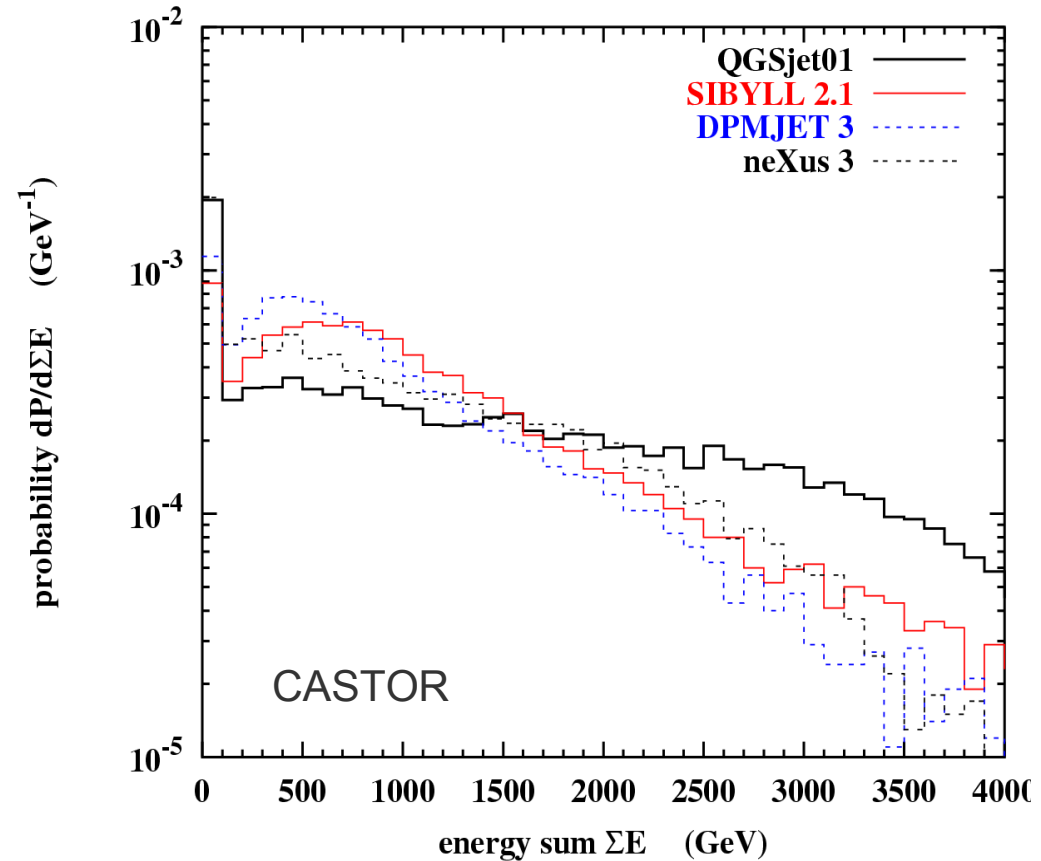
- p-p collisions at LHC at $\sqrt{s} = 14$ TeV
- major experiments consider to do CR relevant measurements (for example, CMS / CASTOR / TOTEM)

Discrimination potential of LHC (ii)

Total energy sum in different detector parts:



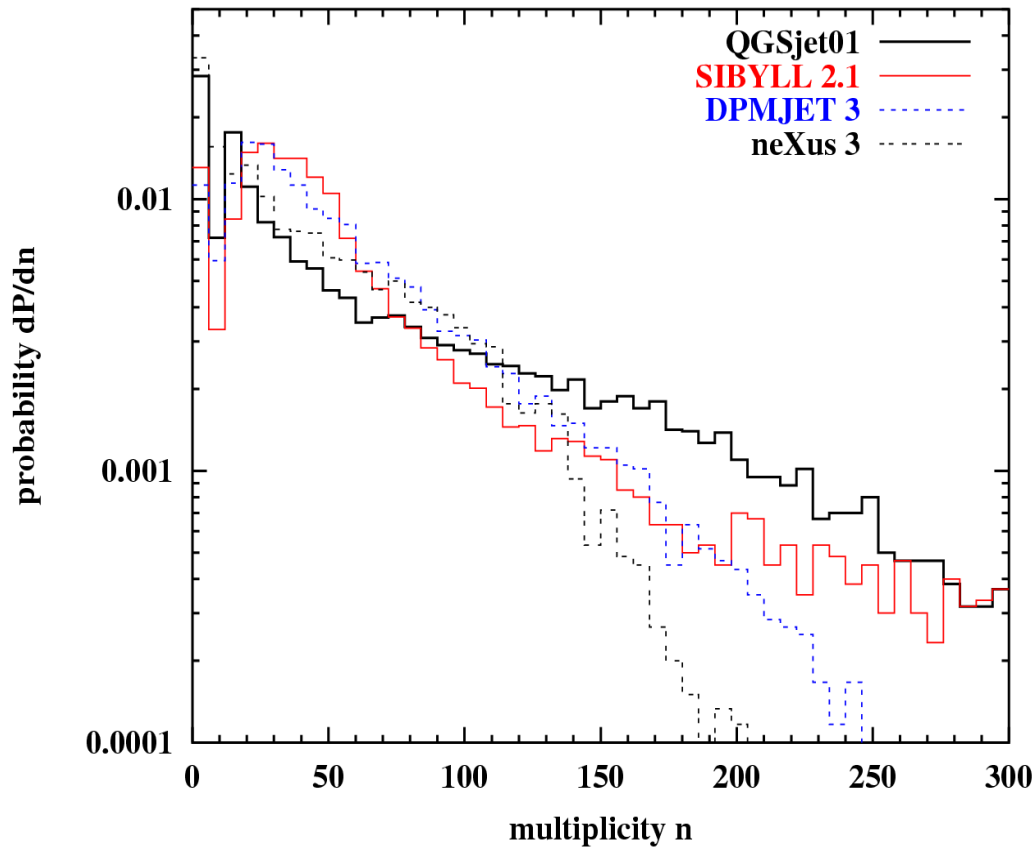
Central detector: $-3 < \eta < 3$



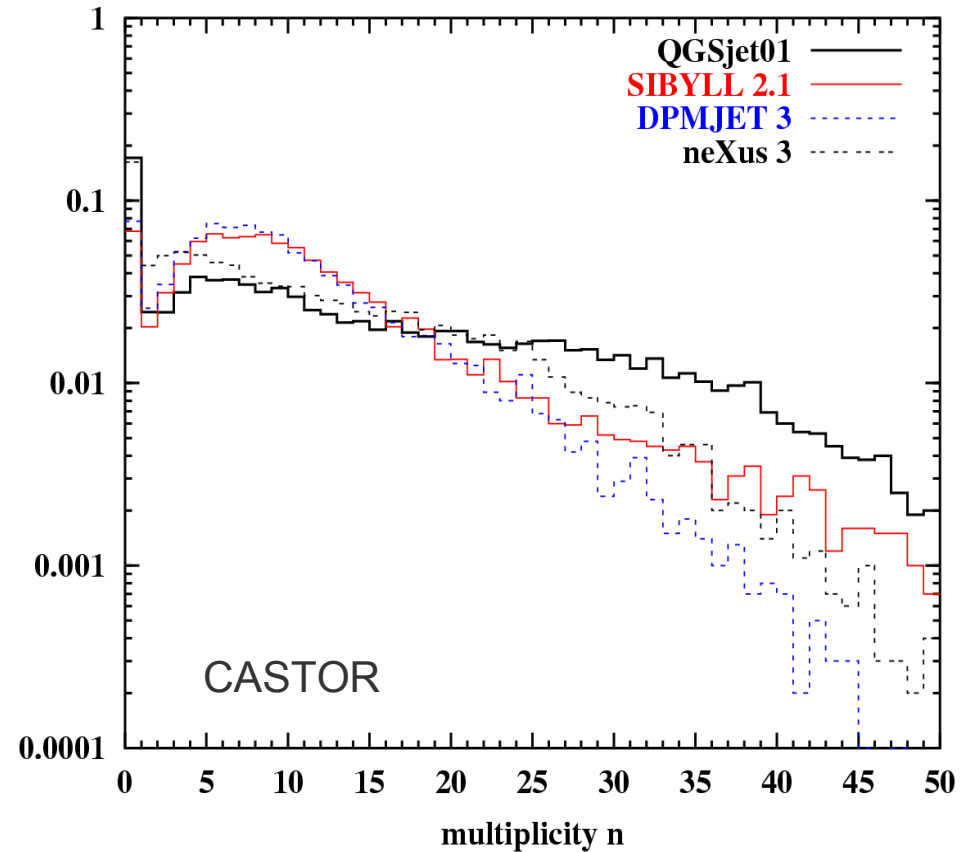
Forward detector: $5 < \eta < 7$

Discrimination potential of LHC (iii)

Total multiplicity in different detector parts:



Central detector: $-3 < \eta < 3$



Forward detector: $5 < \eta < 7$

Constraints from cosmic ray data

Difficulties with cosmic ray beams:

- no direct measurement of interaction
- primary energy unknown
- primary particle unknown

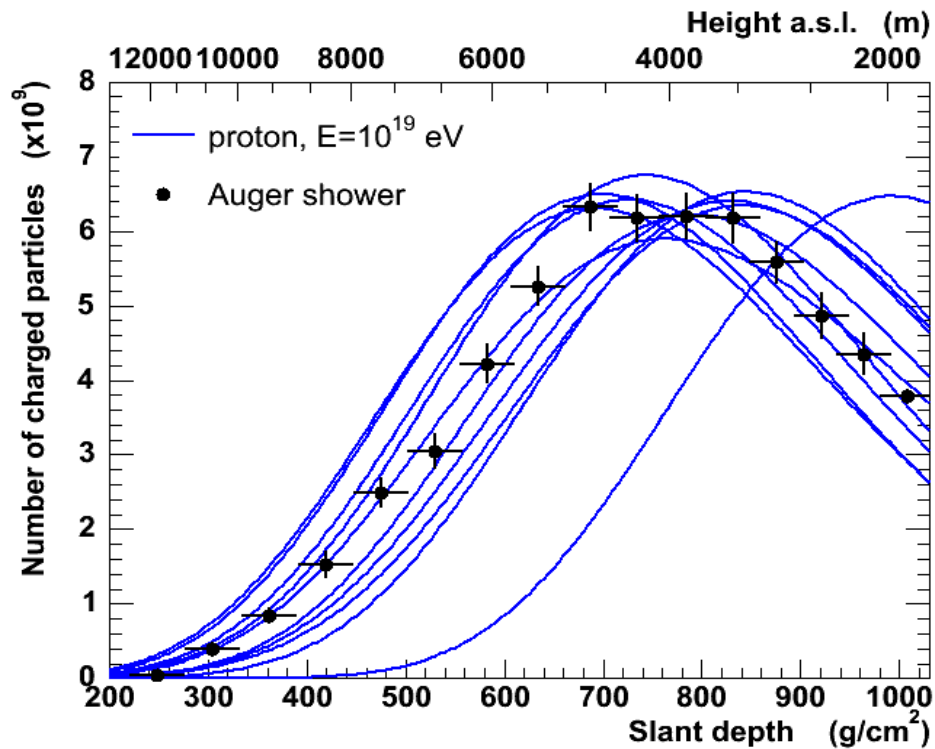
Possible methods of constraining models:

- comparison of measurements to simulated showers assuming a primary energy spectrum and composition
- consistency checks within limits given by expected primary composition
- multiparameter measurements: check of parameter correlations

Model-independent limits on interaction characteristics impossible

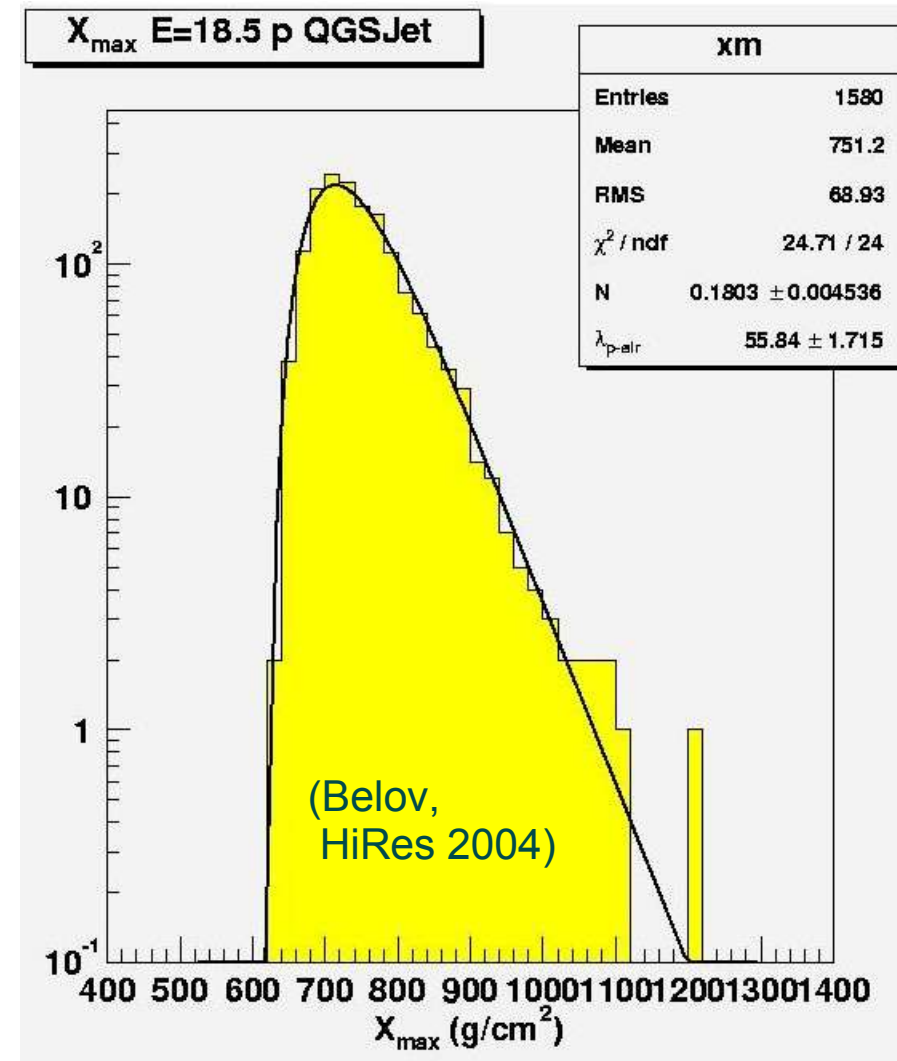
Cross section measurement

Correlation between first interaction point and depth of shower maximum



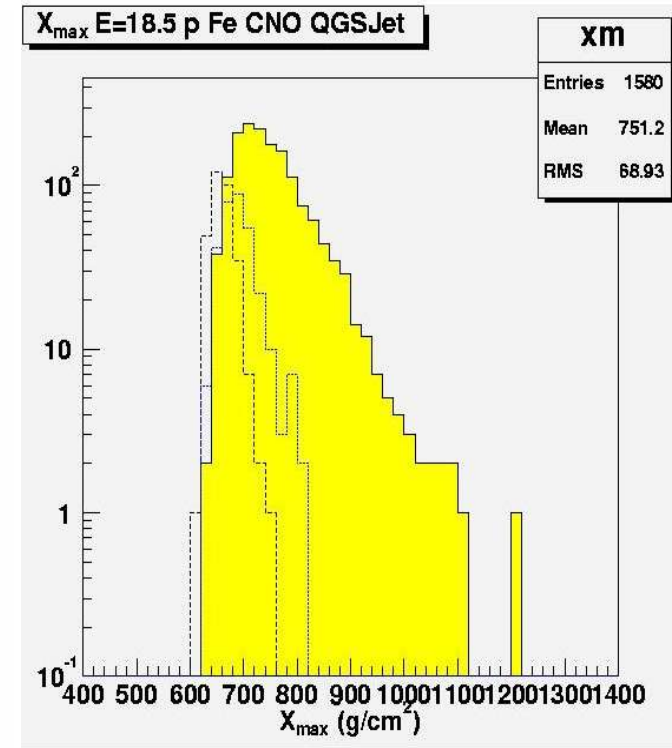
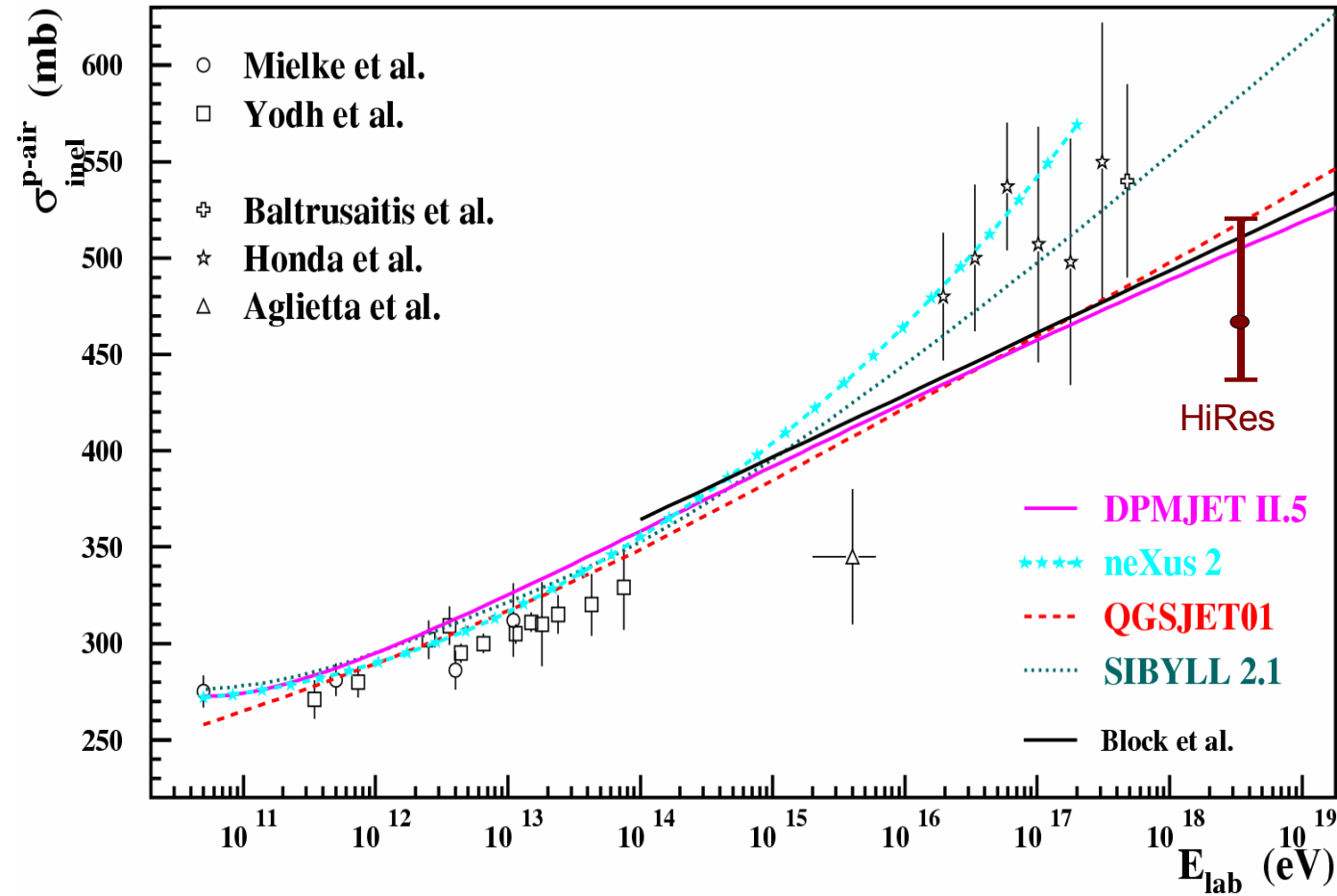
Slope in X_{max} distribution related to interaction length

- selection of proton showers
- selection by energy



HiRes cross section measurement

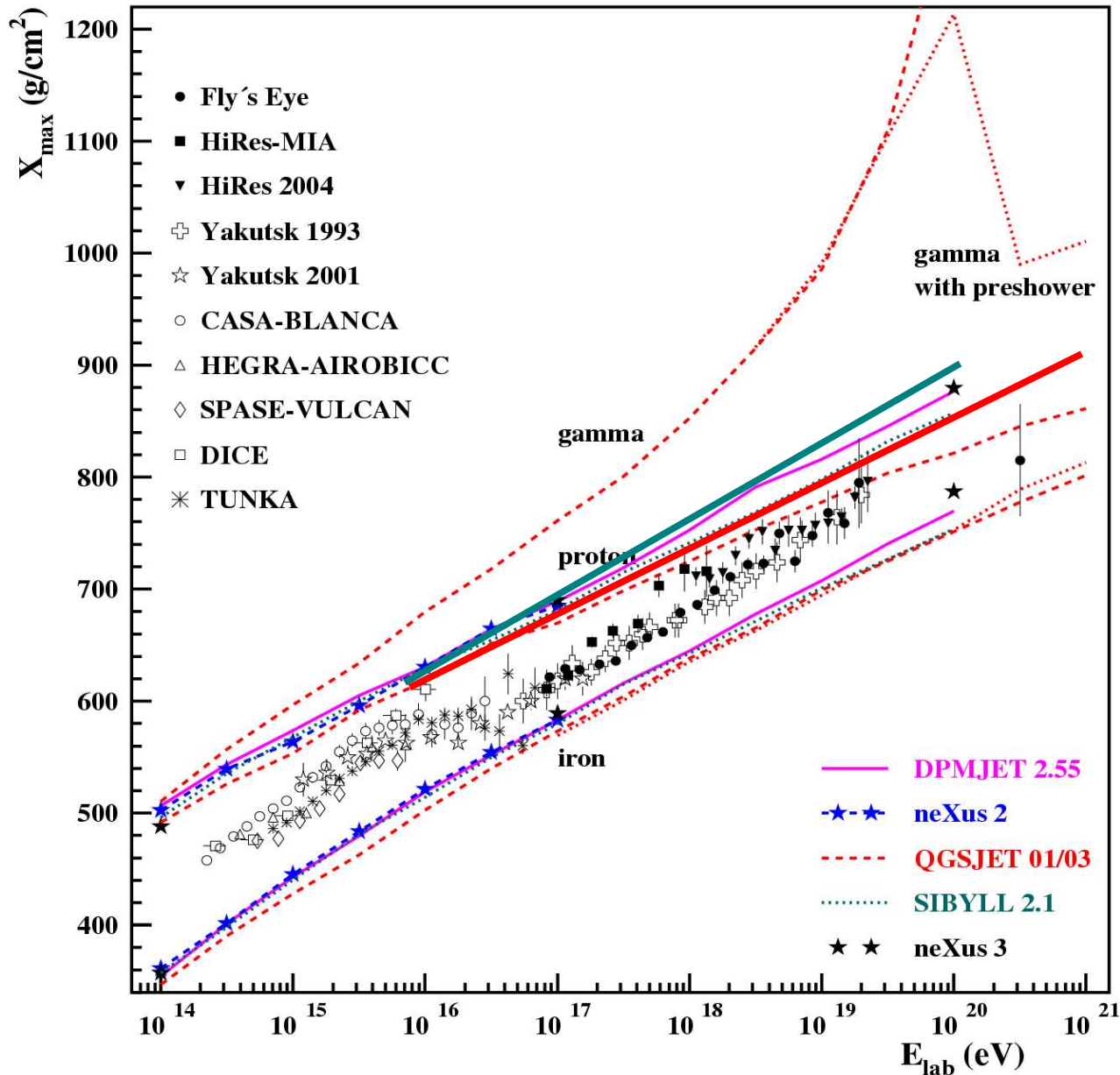
HiRes result on p-air production cross section



possible influence of heavier primaries and gamma-rays

$$\sigma_{in}^{p-Air} = 456 \pm 17 (stat) + 39 (sys) - 11 (sys) \text{ mb}$$

Implications of HiRes measurement

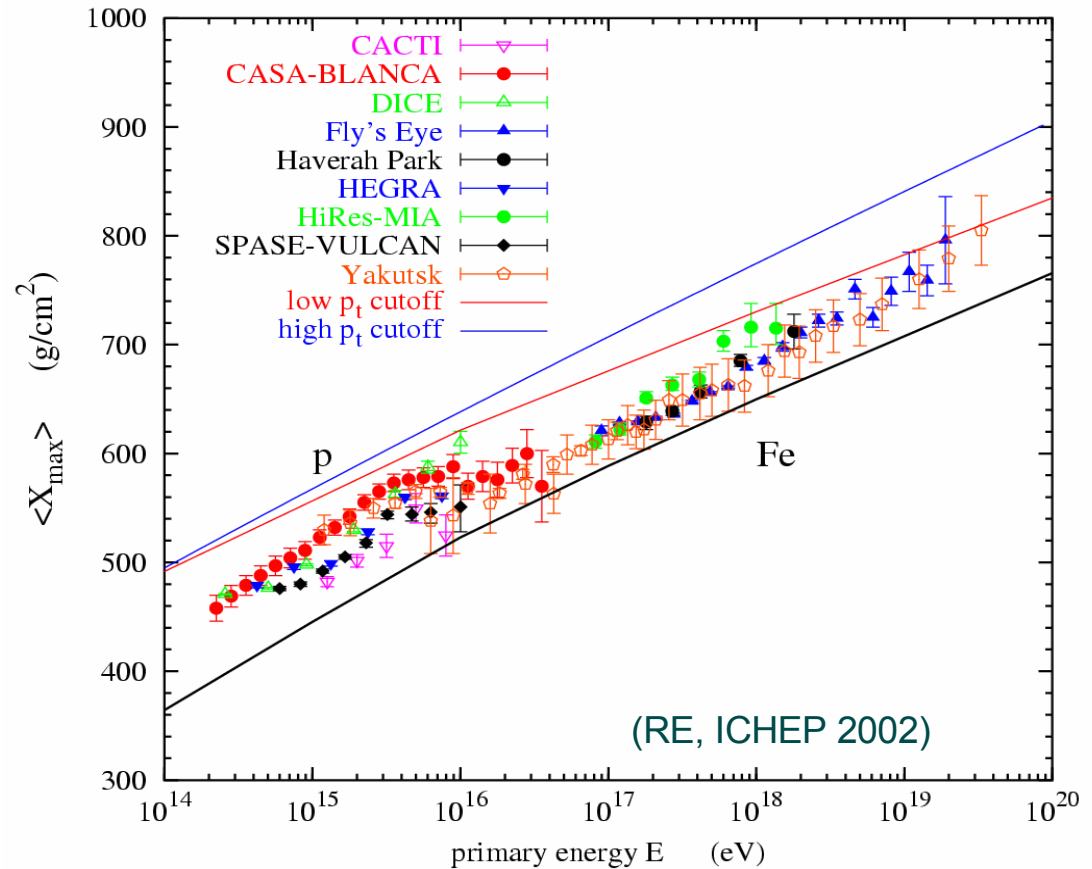
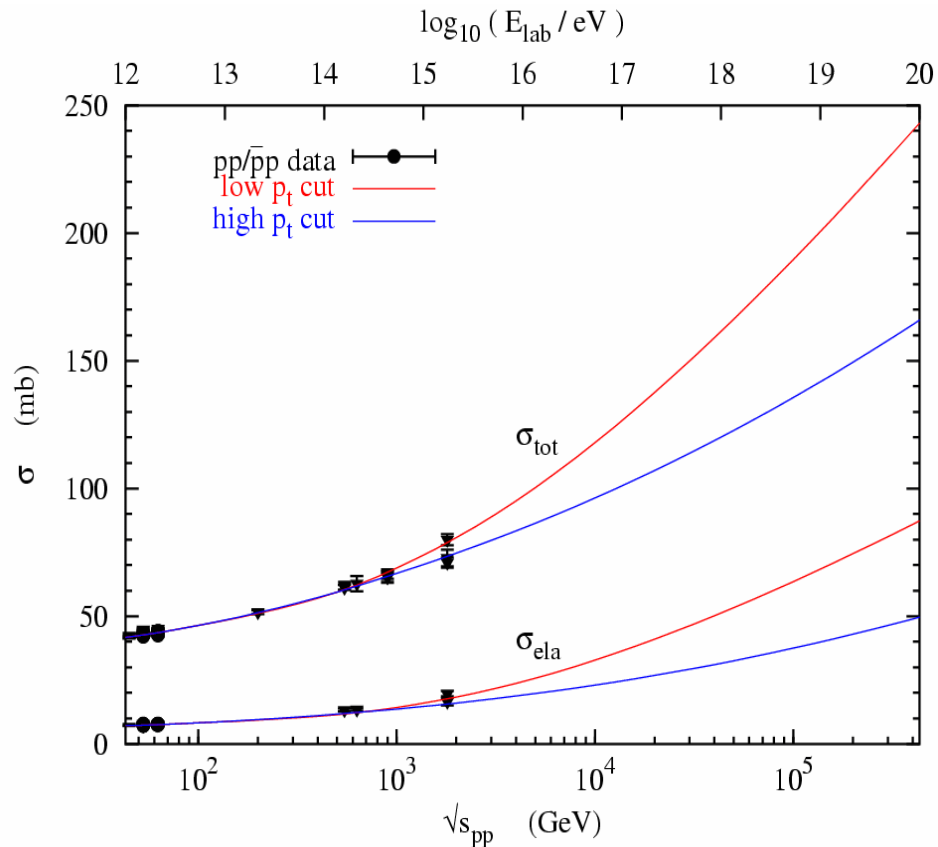


Toy model:

- only cross section of models is re-scaled to fit HiRes measurement
- depth of maximum of proton showers would increase by 25 – 35 g/cm²
- almost no change for iron showers
- data would correspond to mixed composition for all models

Upper cross section bound ?

Realistic model with steep cross section extrapolation (CDF-like)



- CDF-like extrapolation still consistent with data
- exclusion of extreme increase of cross section possible (but with caveats)

RHIC: parton density saturation?

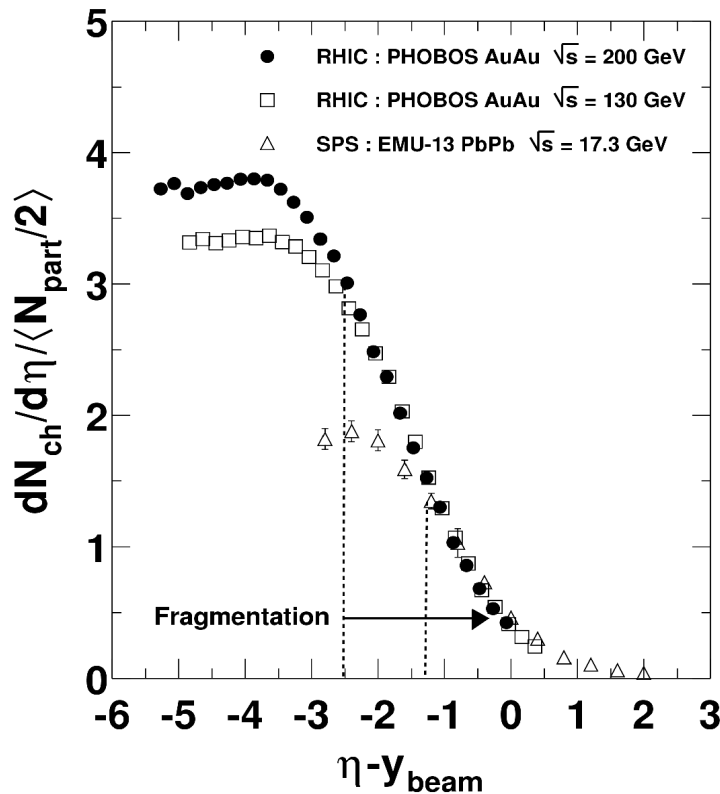
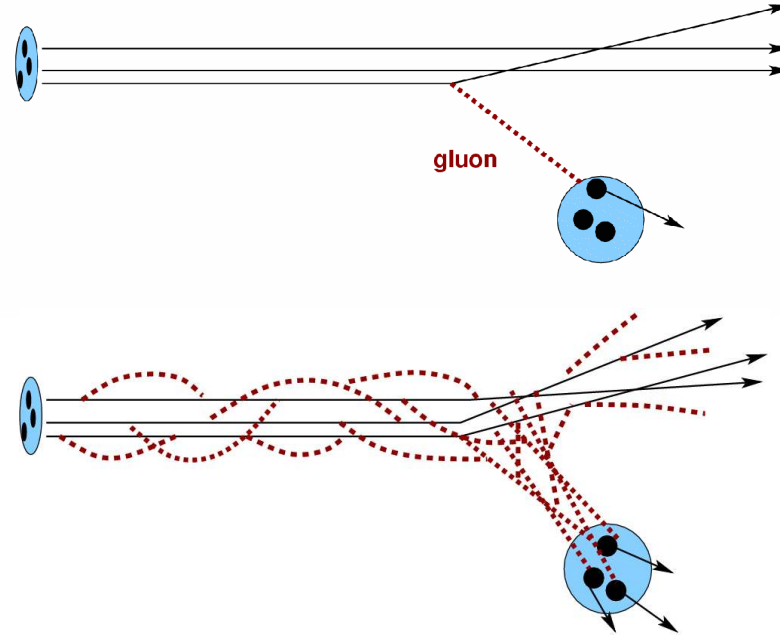
Low energy:

pronounced leading
particle effect

High energy & central collision:

black disk limit,
no leading particles

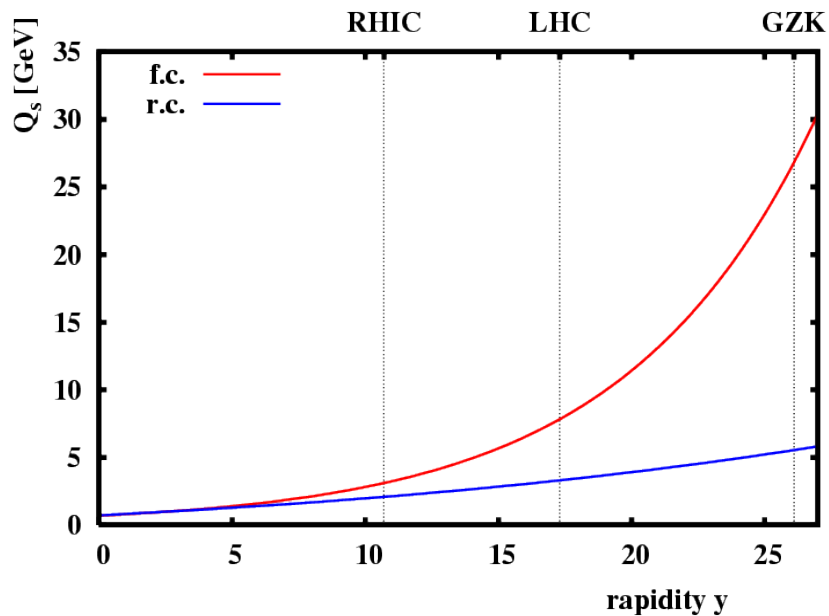
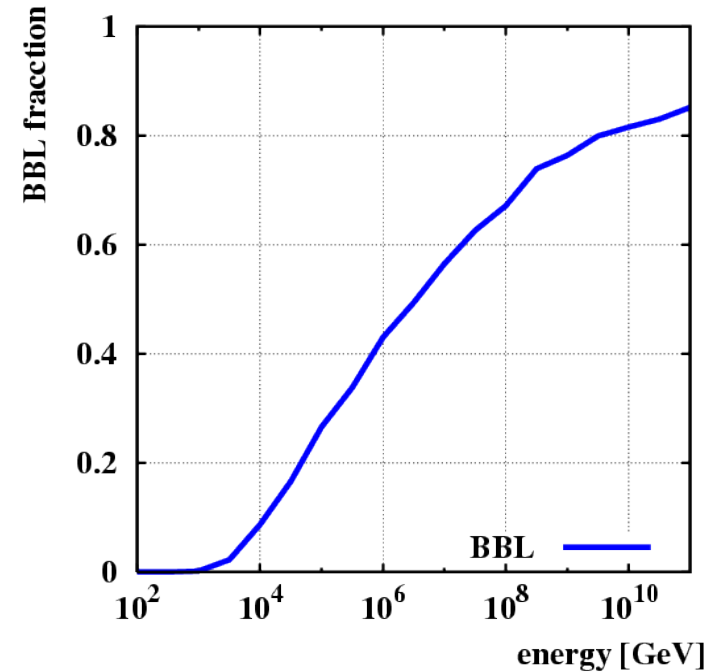
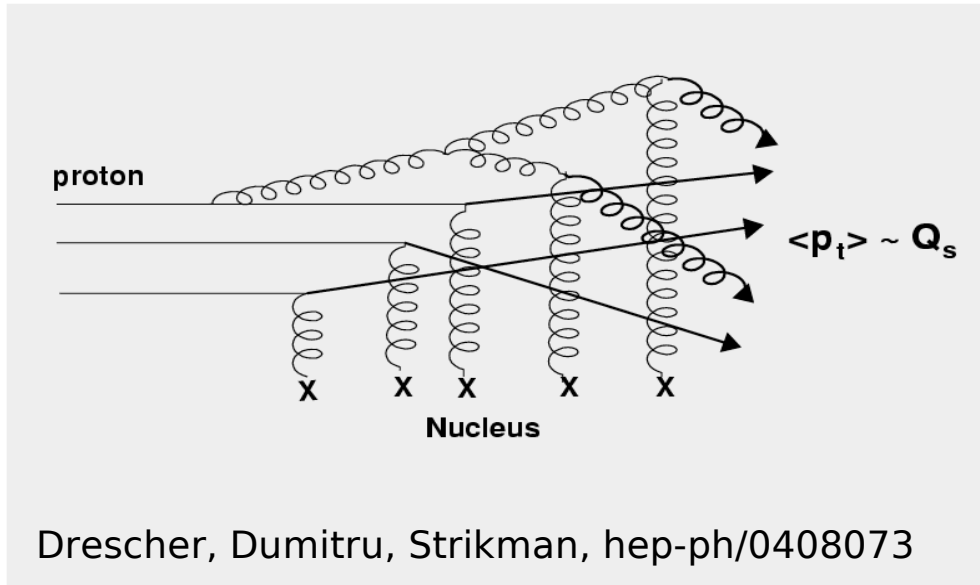
Partonic view:



Limiting fragmentation:

no scaling violations seen
in phase space region
accessible with RHIC detectors

Implications of black disk limit



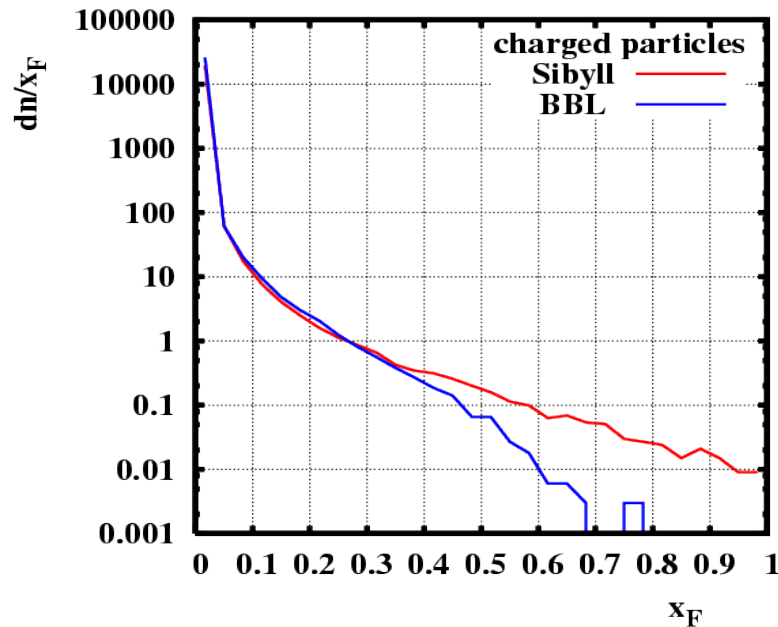
fixed-coupling BFKL:

$$Q_s^2(x, A) = Q_0^2(A) \left(\frac{x_0}{x} \right)^\lambda$$

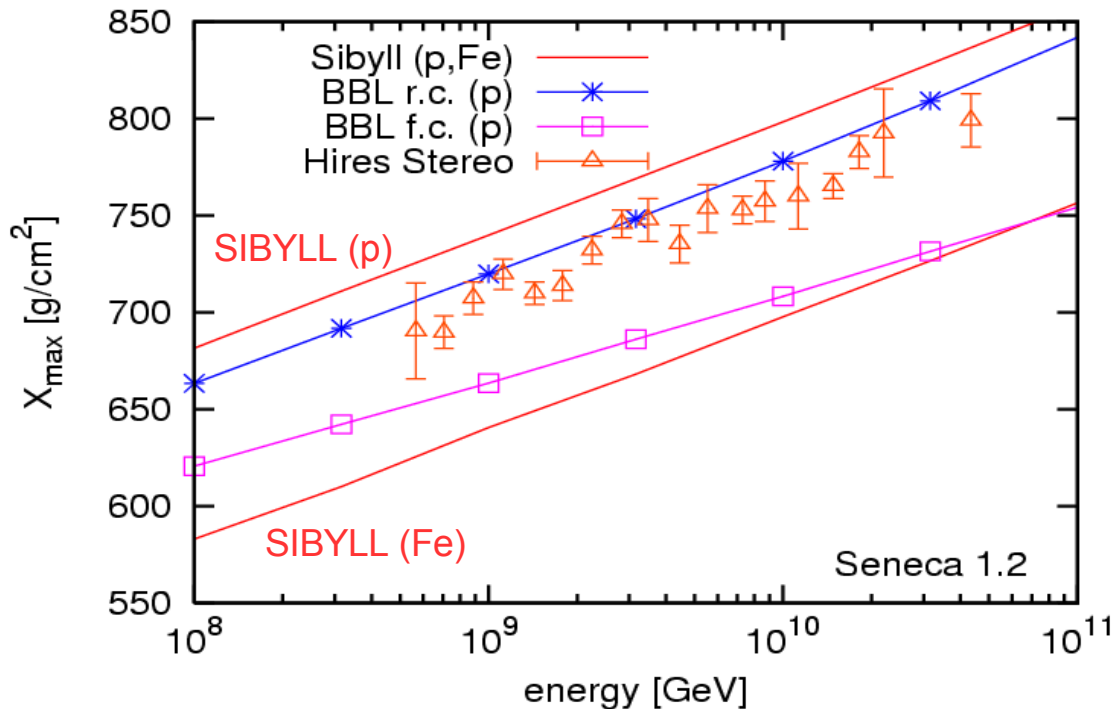
running coupling BFKL:

$$Q_s^2 = \Lambda^2 \exp(\log(Q_0^2/\Lambda^2) \sqrt{1+2c\alpha_s} y)$$

Different saturation scenarios



Leading particle distribution:
air shower predictions sensitive
to saturation scale mainly through
leading particle effect



running coupling BFKL:
moderate growth of
gluon density

fixed-coupling BFKL:
fast growth of gluon density:
contradiction to data

(Drescher, Dumitru,
Strikman, hep-ph/0408073)

Preview: new version of QGSJET

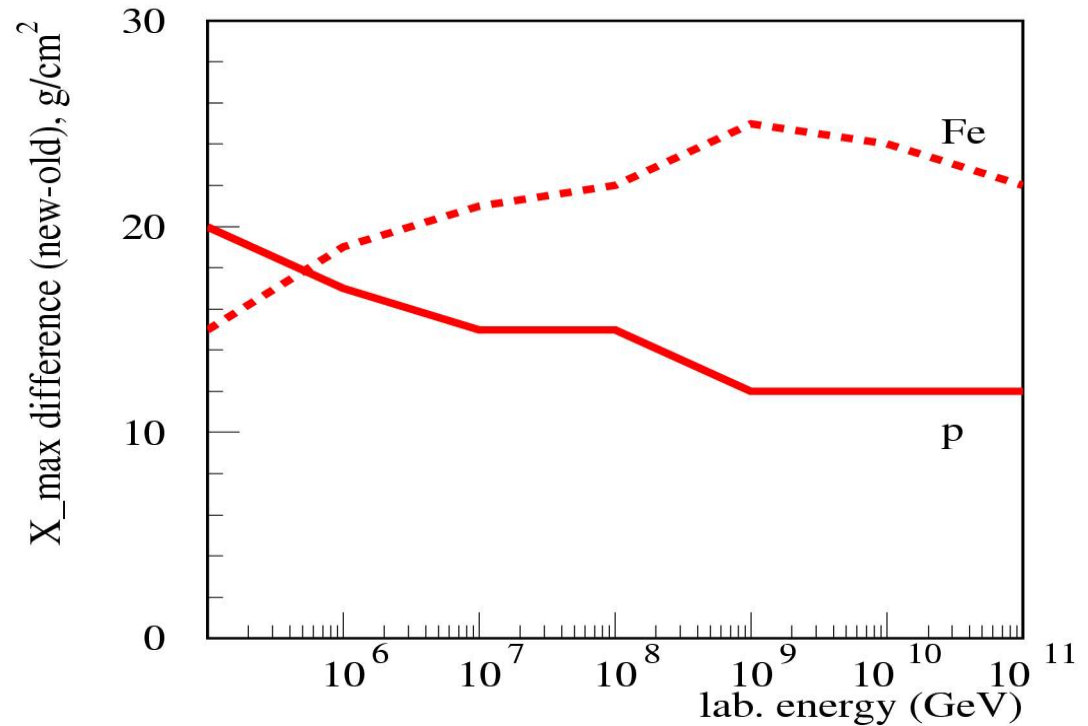
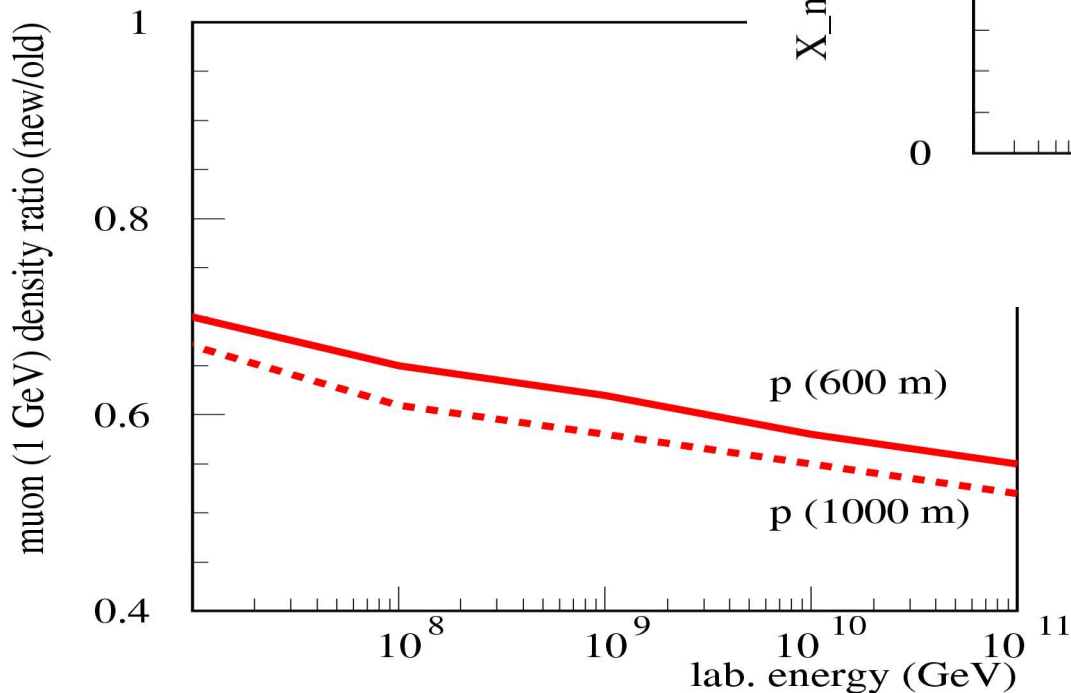
QGSJET I:

“standard candle” for simulations

QGSJET II:

- new parton densities
- summation of enhanced pomeron graphs

Muon density (> 1 GeV) ratio



Changes important:

- proton showers more like SIBYLL
- iron showers different
- no superposition model

Summary

- Cosmic rays and extensive air showers
 - air showers rather insensitive to many hadron production details
 - detailed predictions strongly model dependent
- Modelling of hadronic multiparticle production at high energy
 - attempt to formulate self-consistent models (unitarity)
 - large uncertainties due mainly to
 - minijet cross section (p_T cutoff, saturation, ...)
 - relation between inclusive vs. exclusive cross sections
 - leading particle distributions (scaling/scaling violation)
 - nuclear effects
- Accelerator measurements: very important, in particular forward direction
- Information from air shower data:
 - cross section measurement
 - indications for moderate growth of gluon density