

# Colour coherence in the fully unintegrated NLO evolution kernels of collinear factorization

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The Large Hadron Collider poses a challenge to theoretical calculations through:

- very precise measurements
- need of high accuracy QCD calculations also for *exclusive* (unintegrated) quantities
- huge range of energy scale requiring beyond LO calculations

A possible solution is the fully exclusive NLO Parton Shower Monte Carlo.

# NLO Parton Shower Monte Carlo for QCD Initial State Radiation

## Construction:

- based on the collinear factorisation (EGMPR, CFP),
- CFP=Curci-Furmanski-Petronzio scheme as a main guide (axial gauge,  $\overline{MS}$ ).

## The aims of the project:

- implementing *exactly* NLO DGLAP evolution,
- using new **exclusive NLO kernels**

## Mission statement:

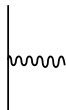
Single Feynman diagrams entering the NLO kernel are not gauge-invariant and may have soft singularities.

The construction of Monte Carlo requires good understanding of soft limit. In exclusive algorithm we have to do it diagram-by-diagram.

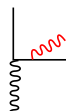
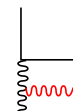
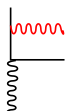
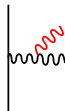
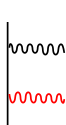
To obtain reasonable Monte Carlo weights we need to reproduce soft singularities exactly, up to NLO. This must be done analytically, today I will only show graphical analysis.

## Example real diagrams

qq and qg LO amplitudes:



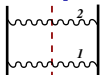
with additional gluon emissions (NLO corrections):



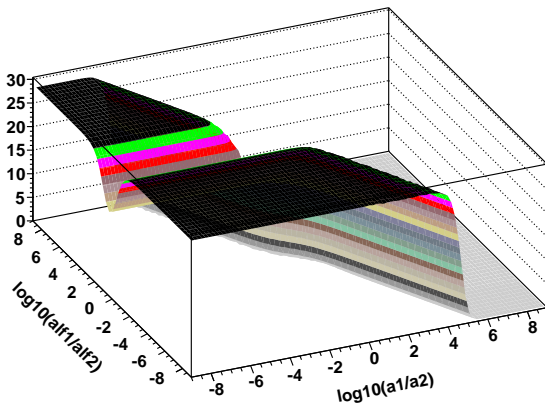
(non-singlet)

(singlet)

## Black plateau is eikonal double-log singularity



Canyon  $\sim P(z_1) \otimes P(z_2)|_{x=z_1 z_2}$ , Cliff at equal virtualities.



Two-particles phase space parametrised with Sudakov variables:

$$k_1 = \alpha_1 p + \alpha_1^- + k_{1\perp}, \quad k_2 = \alpha_2 p + \alpha_2^- + k_{2\perp}$$

I will use rapidity-related:  $a_i = \frac{k_{i\perp}}{\alpha_i}$  variables

On the plots:

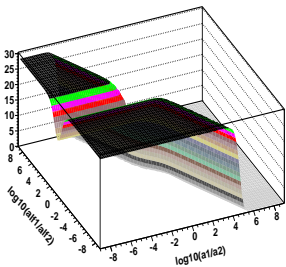
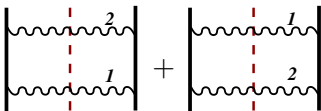
- normalisation to eikonal phase space  $d\Psi = \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \frac{da_1}{a_1} \frac{da_2}{a_2}$ ,
- $\alpha_1 + \alpha_2 = 1 - x$  fixed and non-zero  
(at least one emission is hard)
- $\max\{a_1, a_2\} = Q$  (if  $a_1 > a_2$ ,  $a_1 = Q$  and  $0 < a_2 < Q$ )
- use  $(\ln a_1/a_2, \ln \alpha_1/\alpha_2)$  variables

NOTATION: squared matrix elements = cut-diagrams:

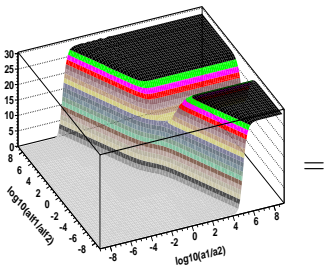
$$\left| \begin{array}{c} \text{Diagram 1} \\ + \\ \text{Diagram 2} \end{array} \right|^2 = \begin{array}{c} \text{Cut Diagram 1} \\ + \\ \text{Cut Diagram 2} \end{array} + 2 \begin{array}{c} \text{Crossed Cut Diagram} \end{array}$$

The diagrammatic equation shows the squared sum of two diagrams (left) equal to the sum of two cut diagrams (middle) plus twice a crossed cut diagram (right). The diagrams consist of wavy lines representing particles, with indices 1 and 2. Vertical dashed red lines indicate cuts in the diagrams.

Non-singlet diagrams  $\sim C_F^2$

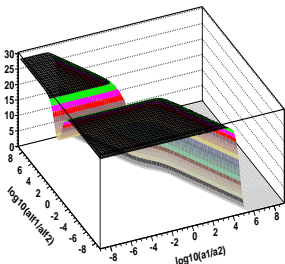
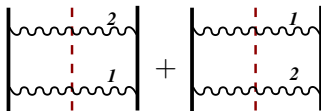


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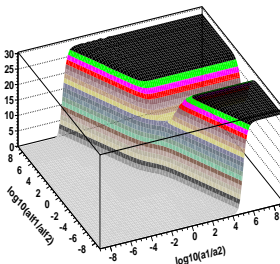


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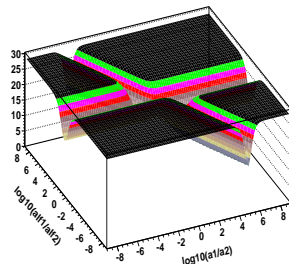


Non-singlet diagrams  $\sim C_F^2$ 

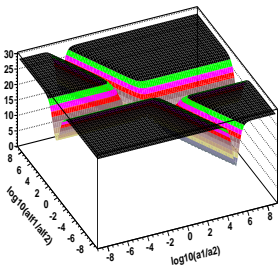
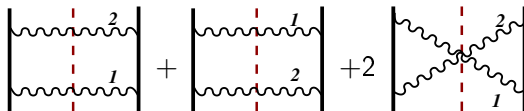
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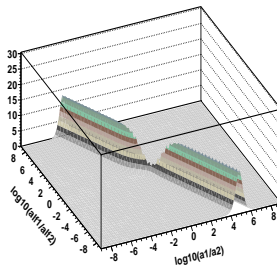
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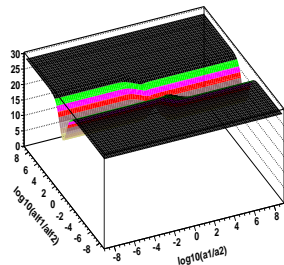
The remaining infinite canyon at equal virtualities spoils eikonal soft limit! (Cliffs are due to quark propagators.)

Non-singlet diagrams  $\sim C_F^2$ 

+2

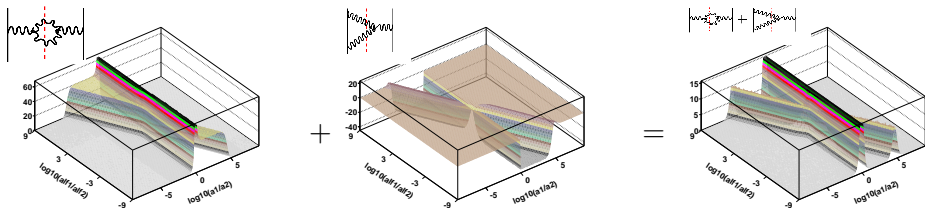


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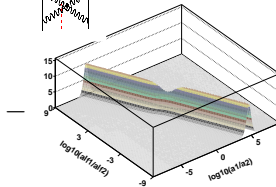
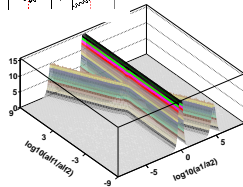
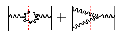
**Interference cures SOFT EIKONAL LIMIT, i.e. fills in bad canyon!**  
 Remaining canyon at  $\alpha_1 \sim \alpha_2$  due to  $\sim P(z_1) \otimes P(z_2)|_{x=z_1 z_2}$  is OK.

The ridge  $a_1 = a_2$  is the collinear singularity of the gluon pair  
**Triangular plateaux are double-log singularities**

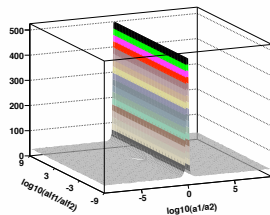


Plateaux disappear after adding interference, but a familiar ridge still spoils the soft limit.

# After adding both interferences

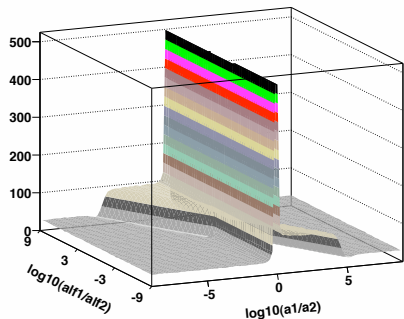


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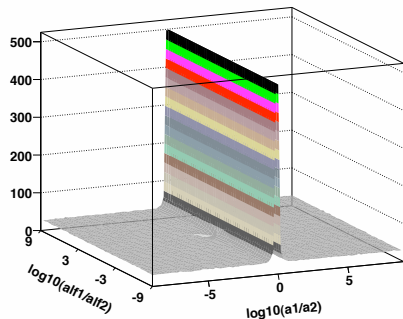


The soft limit is OK

# All non-singlet diagrams



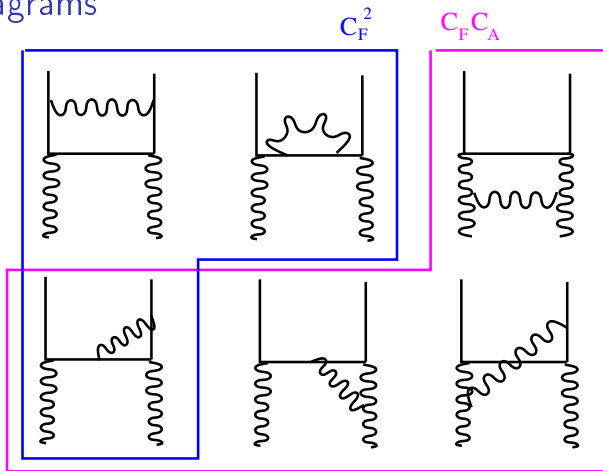
only "amplitude squared" diagrams



interferences added

Both double- and single- logs cancelled after adding interferences.

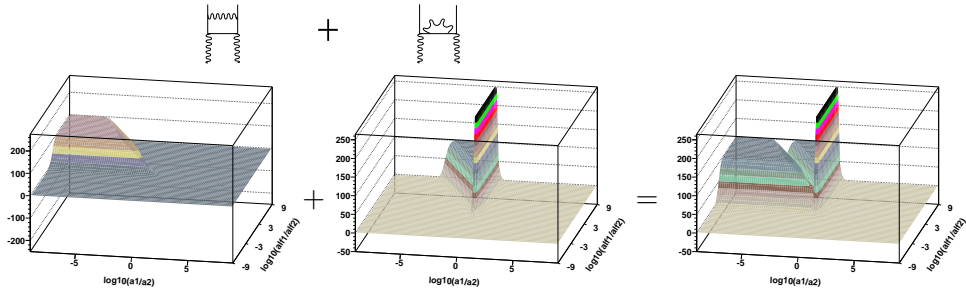
## Singlet diagrams



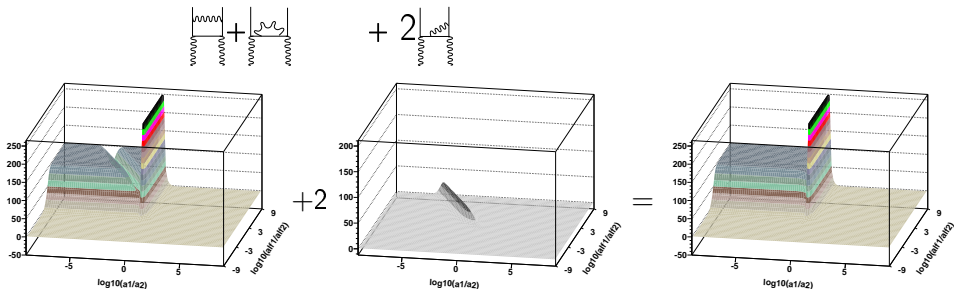
1- quark, 2 - gluon

Singlet diagrams  $\sim C_F^2$ 

Emission of a soft gluon from a quark. Plateaux bordered by equal rapidities – wrong virtuality ordering



The sum is a quadratic plateau with the canyon at equal virtualities.

Singlet diagrams  $\sim C_F^2$ 

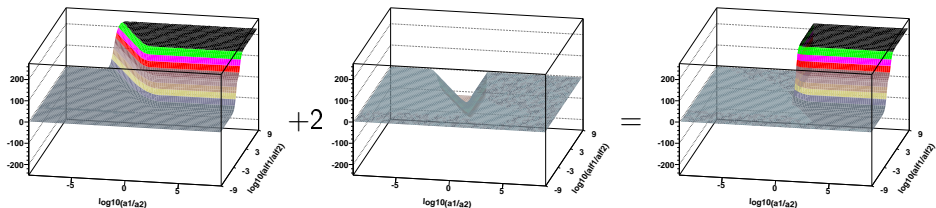
The canyon cancelled by the interference.

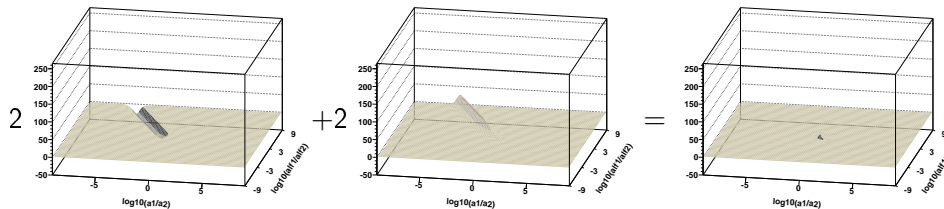
Uniform plateaux bordered by  $a_1 = a_2$

**Virtuality ordering turns out to be wrong!**

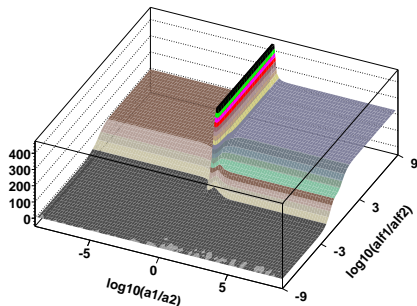


## Emission of a soft gluon from a quark.



Singlet diagrams  $\sim C_F C_A$ 

## All singlet diagrams



- 2 plateaux
  - brown - soft gluon emitted from quark  $\sim C_F^2$
  - navy-blue - soft gluon emitted from gluon  $\sim C_F C_A$
- borderline lies at  $a_1 = a_2$
- central ridge - collinear singularity

## Conclusions:

- Restoration of gauge invariance crucial (color coherence) in cancelling infra-red singularities
- Angular ordering everywhere!
- We understand the soft limits of NLO exclusive kernels
- Analytical formulae crucial for defining crude distributions for *exclusive NLO Monte Carlo*