Antenna Subtraction in pQCD at NNLO

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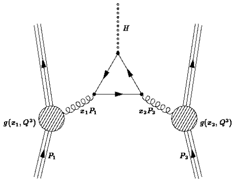
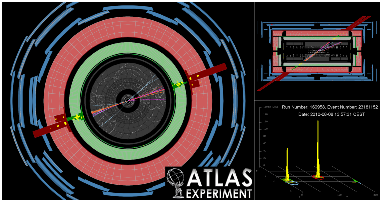
What To Expect From This Talk

- Motivation
- Theoretical framework
  - Matrix element factorization
  - Subtraction method
  - Antenna functions
  - Counterterm structure
- $q\bar{q} \rightarrow$ gggg
  - NNLO double real radiative counterterm
  - Numerical testing
- Summary and ongoing projects

Motivation - Why QCD & Why NNLO?

- QCD is the theory of strong interactions, the LHC is a strong physics collider





- Hadronic asymptotic states $\rightarrow$ Jet physics
- Phenomenology
- Search for new physics: backgrounds

Jet Cross Sections

The m jet cross section to

LO:

$$d\sigma_{LO} = \int_{d\Phi_m} d\sigma_B$$

NLO:

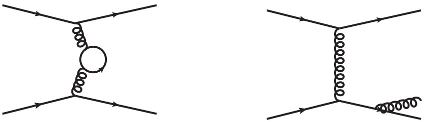
$$d\sigma_{NLO} = \int_{d\Phi_{m+1}} d\sigma^R_{NLO} + \int_{d\Phi_m} d\sigma^V_{NLO}$$

NNLO:

$$d\sigma_{NNLO} = \int_{d\Phi_{m+2}} d\sigma^{RR}_{NNLO} + \int_{d\Phi_{m+1}} d\sigma^{RV}_{NNLO} + \int_{d\Phi_m} d\sigma^{VV}_{NNLO}$$

Cross Section Pathologies

- Loop integration in DR $\rightarrow$ Laurent expansion in ϵ
  - Explicit UV poles in ϵ
  - Explicit IR poles in ϵ



- Phase space integration of real emission $\rightarrow$ kinematic poles
  - Vanishing kinematic invariants
  - Can't cancel directly against ϵ poles
  - Disallows numerical phase space integration

Cross Section Pathologies

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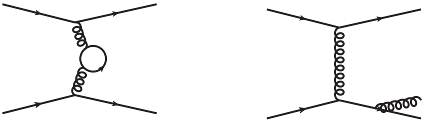
Cross Section Pathologies

- Loop integration in DR $\rightarrow$ Laurent expansion in ϵ
  - Explicit UV poles in ϵ absorbed by renormalization
  - Explicit IR poles in ϵ cancel with integrated real emission



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Single Unresolved Tree Factorization

- Soft gluon emission, $p_j \to 0$

$$|\mathcal{M}^0_{m+1}(\cdots p_i, p_j, p_k, \cdots)|^2 \to S_{ijk}\ |\mathcal{M}^0_{m}(\cdots, \tilde{p}_I, \tilde{p}_K \cdots)|^2$$

$$S_{ijk} = \frac{2s_{ik}}{s_{ij}s_{jk}}$$

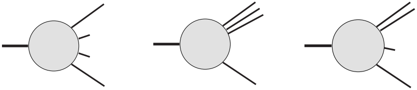
- Collinear limit $p_j || p_k$

$$|\mathcal{M}^0_{m+1}(\cdots p_i, p_j, p_k, p_l, \cdots)|^2 \to \frac{P^0_{jk\to\tilde{K}}}{s_{jk}}\ |\mathcal{M}^0_{m}(\cdots, p_i, p_{\tilde{K}}, p_l, \cdots)|^2$$

Double Unresolved Tree Factorizaion

At NNLO we have new singular limits

- Double soft
- Triple collinear
- Soft and collinear



Factorization holds $\longrightarrow$ new universal functions

$$S_{abcd} \quad S_{d,abc} \quad P_{ijk\to\tilde{K}} \quad \tilde{P}_{ijk\to\tilde{K}}$$

Details of factorization depend on colour ordering of partons

Subtraction Method

Construct a local counterterm, $d\sigma_{NNLO}^{RR,S}$,

$$\int_{d\Phi_{m+2}} d\sigma_{NNLO}^{RR} = \int_{d\Phi_{m+2}} \left[d\sigma_{NNLO}^{RR} - d\sigma_{NNLO}^{RR,S} \right] + \int_{d\Phi_{m+2}} d\sigma_{NNLO}^{RR,S}$$

- Absorbs all IR divergence
- New integrand is finite over the entire phase space
- Integrated numerically to finite result

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- Integrated numerically to finite result

Also construct a one loop counterterm, $d\sigma^{RV,S}_{NNLO}$,

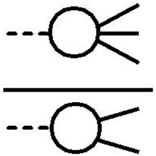
$$\int_{d\Phi_{m+1}} d\sigma^{RV}_{NNLO} = \int_{d\Phi_{m+1}} \left[d\sigma^{RV}_{NNLO} - d\sigma^{RV,S}_{NNLO} \right] + \int_{d\Phi_{m+1}} d\sigma^{RV,S}_{NNLO}$$

Why Antenna Functions?

- Built from physical matrix elements

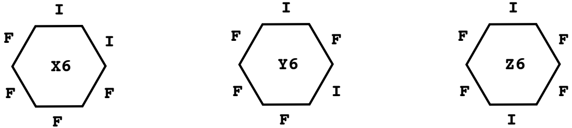
$$
\begin{aligned}
A,\ \tilde{A},\ B,\ C &\sim \gamma* \to q\ \bar{q}\ \ (+\ \text{partons}) \\
D,\ E,\ \tilde{E} &\sim \tilde{\chi} \to \tilde{g}\ \ (+\ \text{partons}) \\
F,\ G,\ \tilde{G},\ H &\sim H \to\ (\text{partons})
\end{aligned}
$$

- Contain multiple singular limits
  - Regulate multiple singularities
  - Spurious poles
- Analytically integrable
  - D dimensions $\longrightarrow$ ϵ expansion



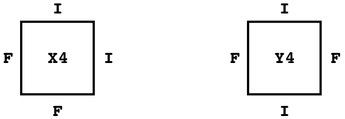
Matrix Element Structure

4 final + 2 initial state partons $\rightarrow$ 3 topologies $\mathcal{X}_6,\ \mathcal{Y}_6,\ \mathcal{Z}_6$



3 final + 2 initial state partons $\rightarrow$ 2 topologies $\mathcal{X}_5,\ \mathcal{Y}_5$

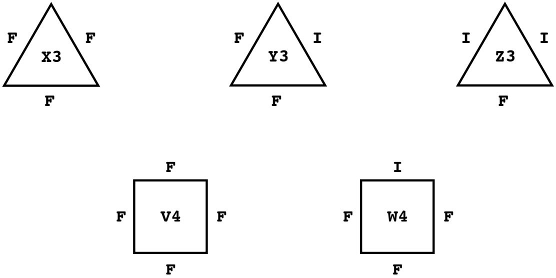
2 final + 2 initial state partons $\rightarrow$ 2 topologies $\mathcal{X}_4,\ \mathcal{Y}_4$



Constructing the Counterterm

$$d\sigma^{S}_{NNLO} = \sum_i \kappa_i \,(\text{Antenna})_i \otimes (\text{Reduced Matrix Element})_i$$

- Reduced matrix elements $\in \{\mathcal{X}_4, \mathcal{Y}_4\}$, $\{\mathcal{X}_5, \mathcal{Y}_5\}$
- Antenna functions $\in \{\mathcal{X}_3, \mathcal{Y}_3, \mathcal{Z}_3\}$, $\{\mathcal{V}_4, \mathcal{W}_4, \mathcal{X}_4, \mathcal{Y}_4\}$

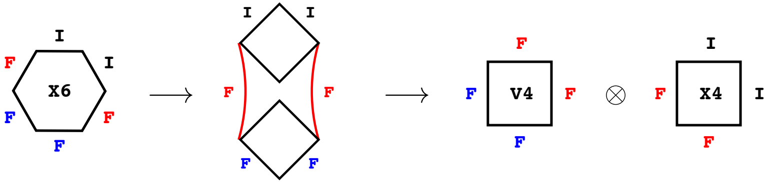


Constructing the Counterterm - Strategy

Define $\otimes$ operation diagrammatically

- Identify hard partons
- Stretch and pinch \ make insertions

e.g.



For each topology $\mathcal{T}$ remove singularities with counterterm $(\mathcal{A} \otimes \mathcal{R})$ such that,

$$(\mathcal{A} \otimes \mathcal{R}) \;\in\; \mathcal{T}$$

Constructing the Counterterm - Strategy

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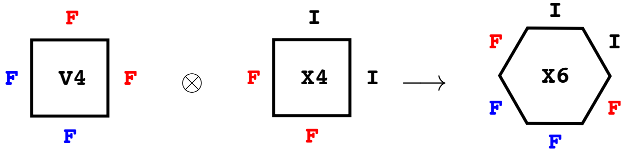

For each topology $\mathcal{T}$ remove singularities with counterterm $(\mathcal{A} \otimes \mathcal{R})$ such that,

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General Kinematic Structure

$$
\begin{aligned}
d\sigma_{NNLO}^{S,\mathcal{X}_6} &= (\mathcal{X}_3 \otimes \mathcal{X}_5) + (\mathcal{Y}_3 \otimes \mathcal{X}_5) \\
&+ (\mathcal{V}_4 \otimes \mathcal{X}_4) - (\mathcal{X}_3 \otimes \mathcal{X}_3) \otimes \mathcal{X}_4 \\
&+ (\mathcal{W}_4 \otimes \mathcal{X}_4) - (\mathcal{X}_3 \otimes \mathcal{Y}_3) \otimes \mathcal{X}_4 - (\mathcal{Y}_3 \otimes \mathcal{Y}_3) \otimes \mathcal{X}_4 + \cdots
\end{aligned}
$$

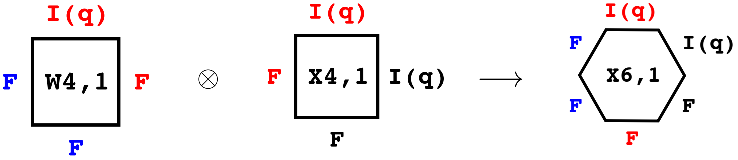
$$
\begin{aligned}
d\sigma_{NNLO}^{S,\mathcal{Y}_6} &= (\mathcal{X}_3 \otimes \mathcal{Y}_5) + (\mathcal{Y}_3 \otimes \mathcal{Y}_5) + (\mathcal{Z}_3 \otimes \mathcal{X}_5) \\
&+ (\mathcal{W}_4 \otimes \mathcal{Y}_4) - (\mathcal{X}_3 \otimes \mathcal{Y}_3) \otimes \mathcal{Y}_4 - (\mathcal{Y}_3 \otimes \mathcal{Y}_3) \otimes \mathcal{Y}_4 \\
&+ (\mathcal{Y}_4 \otimes \mathcal{X}_4) - (\mathcal{Z}_3 \otimes \mathcal{Z}_3) \otimes \mathcal{X}_4 + \cdots
\end{aligned}
$$

$$
\begin{aligned}
d\sigma_{NNLO}^{S,\mathcal{Z}_6} &= (\mathcal{Y}_3 \otimes \mathcal{Y}_5) \\
&+ (\mathcal{X}_4 \otimes \mathcal{X}_4) - (\mathcal{Y}_3 \otimes \mathcal{Z}_3) \otimes \mathcal{Z}_4 \\
&+ (\mathcal{Y}_4 \otimes \mathcal{Y}_4) - (\mathcal{Z}_3 \otimes \mathcal{Z}_3) \otimes \mathcal{Y}_4 + \cdots
\end{aligned}
$$

Introducing Quarks

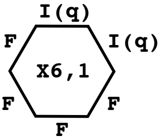
- Additional grading, fermion\boson
- Further classify topologies $\mathcal{X}_{6,1}, \mathcal{X}_{6,2}$, etc
- Follow same strategy

e.g.

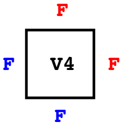


Application to $q\bar{q} \longrightarrow gggg$

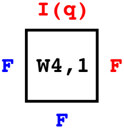
Only one topology to consider



Contributing 4 parton antennae:

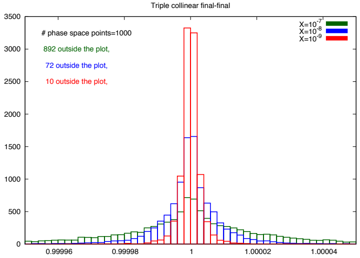
- 

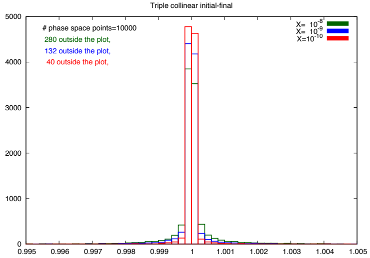
$\longrightarrow$ $F_4^0(g, g, g, g)$

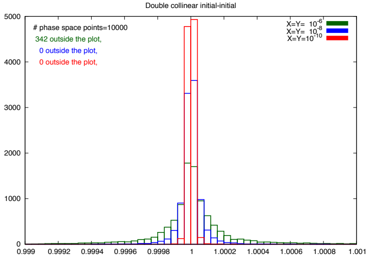
- 

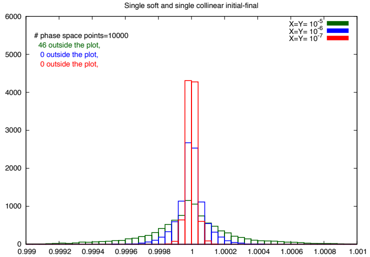
$\longrightarrow$ $D_4^0(\hat{q}, g, g, g)$

Numerical Testing - Results for $q\bar{q} \longrightarrow gggg$









Work in Progress

- Construction of the one loop counterterm $d\sigma_{NNLO}^{RV,S}$
- Numerical testing of $d\sigma_{NNLO}^{RV,S}$
- Mass factorization terms
- Integration of $d\sigma_{NNLO}^{RR,S} + d\sigma_{NNLO}^{RV,S}$ over antenna phase space
  - Need initial-initial integrated antennae
  - Combine with 2-loop singularities
  - Pole cancellation $\longrightarrow$ finite NNLO cross section
- Other channels, $qg \to qggg$, $gg \to q\bar{q}gg$, etc
- Repeat analysis for $4q + 2g$, and $6q$ scattering

To Take Away

- NNLO QCD highly desirable for LHC physics
- Antenna formalism general
  - NLO, NNLO, tree, 1-loop
  - Coloured initial states
- Double real counterterm structure understood
- "Close" to full subtraction for certain channels