Higher Order Corrections to the Drell-Yan Cross Section in the Mellin Space

Petra Kovačíková (DESY, Zeuthen)

petra.kovacikova@desy.de

HEPTOOLS final meeting, Granada

25^{th} November 2010

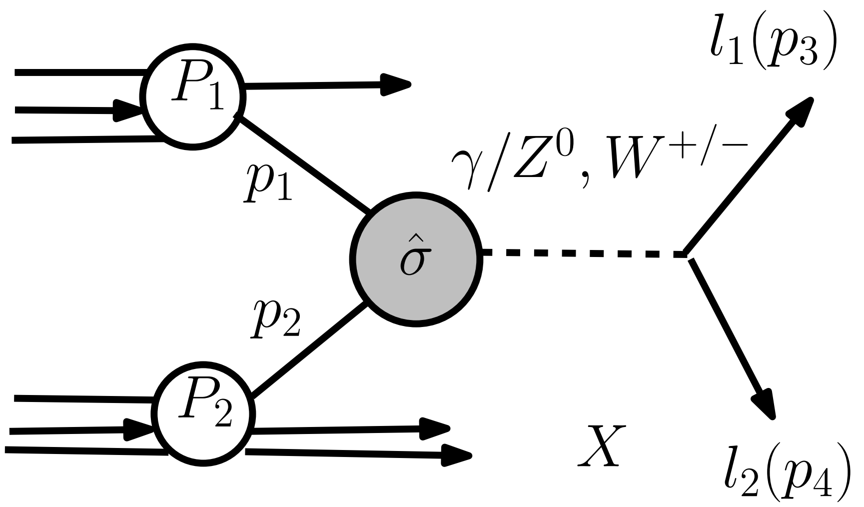
Outline

- The Drell-Yan mechanism
- The Mellin transform
- Calculation of the Drell-Yan structure functions in Mellin space up to NNLO
    - Coefficient functions
    - Parton distriubtion functions
- Partial results and checks
- Conclusions and outlook

The Drell-Yan Process

Massive lepton pair production in hadron-hadron collision, $M^2_{l_1 l_2} >> 1$ GeV^2

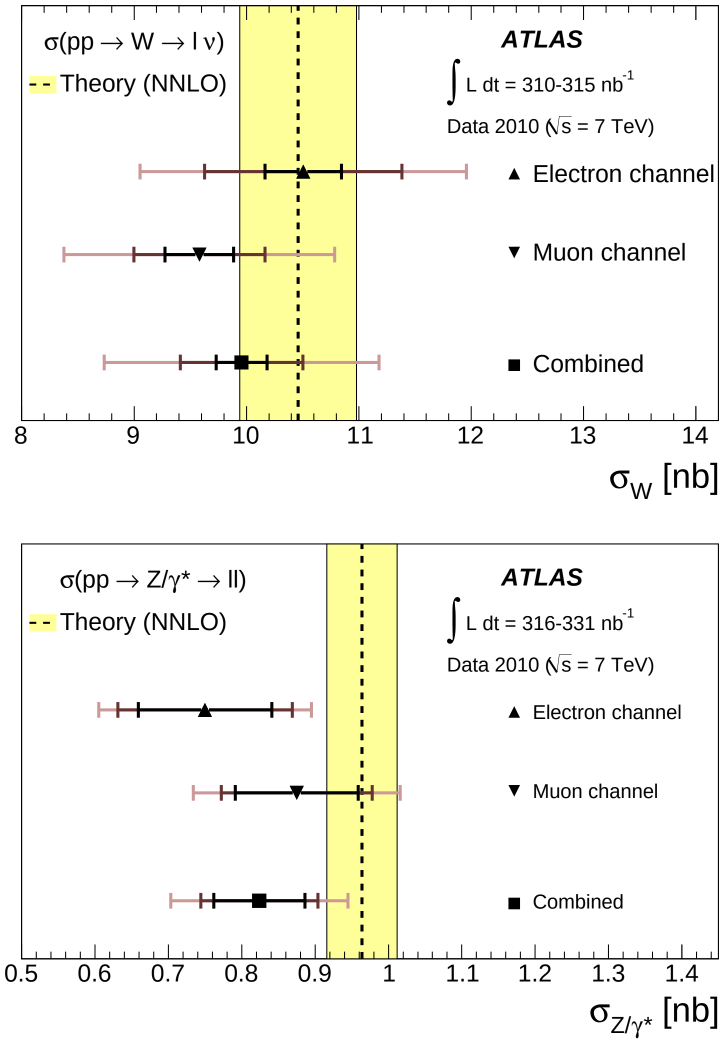
[Drell,Yan 1970]



- $M_{l_1 l_2} = Q^2 = (p_2 + p_3)^2$
- CM energy of hadrons
  $s = (P_1 + P_2)^2$
- $p_i = x_i P_i \qquad x_i \in (0, 1)$
- CM energy of partons
  $\hat{s} = (p_1 + p_2)^2 = s x_1 x_2$

- Neutral Current
  - $pp \to \gamma^* \to l\bar{l}X \qquad M_{l\bar{l}} \ll M_Z \qquad M_Z \sim 91.2$ GeV
  - $pp \to Z^0 \to l\bar{l}X \qquad M_{l\bar{l}} \sim M_Z$
- Charged Current
  - $pp \to W^\pm \to l\nu X \qquad M_{l\bar{l}} \sim M_W \qquad M_W \sim 80.4$ GeV

The Drell-Yan process at the LHC



- Large total cross sections for W and Z production
- Clear signal
- Background for new phyiscs measurements
- Test of of the Standard Model
- Constraints on parton distribution functions

The Drell-Yan Cross Section

$$\frac{\mathrm{d}\,\sigma^V_{DY}(Q^2)}{\mathrm{d}\,Q^2} = \sum_{a,b}\int_0^1 \mathrm{d}\,x_1 \int_0^1 \mathrm{d}\,x_2 \underbrace{f_a(x_1,Q^2) f_b(x_2,Q^2)}_{\text{PDFs}} \underbrace{\frac{\mathrm{d}\,\hat{\sigma}^V_{ab}(Q^2)}{\mathrm{d}\,Q^2}}_{\text{Hard Cross section}}$$

$$V = \gamma^*, Z^0, W^\pm \qquad a,b = q,\bar{q},g \qquad x_{1,2} \in (0,1)$$

- Higher order NLO QCD corrections [Altarelli, Ellis, Martinelli 1979]
- NNLO [Hamberg, Matsuura, van Neerven 1990; Harlander, Kilgore 2002]
- NNLO fully differential cross section [Anastasiou, Dixon, Melnikov, Petriello 2004; Catani, Ferrera, Grazzini 2010]
  $\Rightarrow$ new constraints on PDFs [Alekhin, Melnikov, Petriello 2006]
- Electroweak corrections up to NLO [Hollik, Wackeroth 1996; Vicini et.al. 2009]

The Drell-Yan Cross Section

$$\frac{d\,\sigma^V_{DY}(Q^2)}{d\,Q^2} = \sum_{a,b}\int_0^1 d\,x_1 \int_0^1 d\,x_2\, f_a(x_1,Q^2) f_b(x_2,Q^2) \frac{d\,\hat{\sigma}^V_{ab}(z,\alpha_s)}{d\,Q^2}$$

$$= xN^V \underbrace{\sum_{a,b} C^V_{ab}(f_a \otimes f_b \otimes \Delta_{ab})(x)}_{\text{Structure function } W_{DY}(x,Q^2)} \;; \qquad x = \frac{Q^2}{s} \qquad z = \frac{x}{x_1 x_2}$$

$$\Delta_{ab} = \sum_k \left(\frac{\alpha_s}{4\pi}\right)^k \Delta^{(k)}_{ab}$$

Coefficient functions

- Convolution

$$(f_1 \otimes f_2 \otimes \cdots \otimes f_k)(x)$$

$$= \int_0^1 d\,x_1 \int_0^1 d\,x_2 \ldots \int_0^1 d\,x_k\, \delta(x - x_1 x_2 \ldots x_k) f_1(x_1) f_2(x_2) \ldots f(x_k)$$

The Convolution and The Mellin Transform

- The Mellin transform - integral transform
  $f(x)$ ("x-space") $\to \tilde{f}(N)$ ("N-space")

$$\mathbf{M}[f(x)] \;=\; \int_0^1 \mathrm{d}\,x\,x^{N-1} f(x) = \tilde{f}(N)$$

- Mellin transform of the convolution is a product of functions in N-space

$$\mathbf{M}[(f_1 \otimes f_2 \otimes \cdots \otimes f_k)(x)](N) = f_1(N) f_2(N) \ldots f_k(N)$$

The Convolution and The Mellin Transform

- The structure function

$$W(x,Q^2) = (f_a \otimes f_b \otimes \Delta_{ab}^V)(x) \quad \rightarrow \quad f_a(N) f_b(N) \Delta_{ab}^V(N) = \tilde{W}(N,Q^2)$$

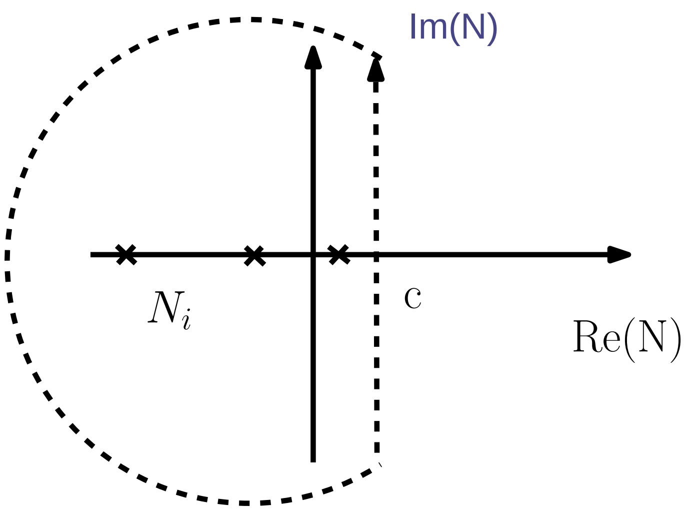
  - Two integrations $\rightarrow$ simple product
  - Need analytic expressions for all input functions in x-space for which the Mellin transforms exist [Vermaseren 1998]

    [Moch, Vermaseren, Vogt 2004; Blümlein, Kurth 1998 (2000); Blümlein, Ravindran 2005, . . .]
  - Original x-space recovered performing the inverse Mellin transform

The Inverse Mellin Transform

- The inverse Mellin Transform - integral over a complex plane

$$W(\tau) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \mathrm{d}\,N\,x^{-N}\tilde{W}(N)$$



- All poles of $\tilde{W}(N)$ lie to the left from the contour c
- In general inverse MT are performed numerically

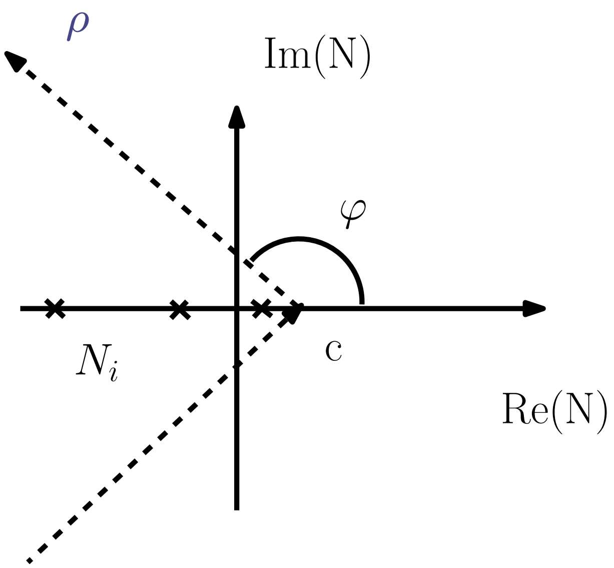
The Inverse MT - numerical evaluation

Rewriting the integral

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \mathrm{d}\,N\,x^{-N}\tilde{f}(N) \quad = \quad \frac{1}{\pi}\int_0^{\infty} \mathrm{d}\,\rho\,\mathrm{Im}[e^{i\varphi}x^{-c-\rho e^{i\varphi}}f(N)]$$

$\rightarrow$ integral over a real variable ρ
$N = c + \rho e^{i\phi}$ becomes complex variable

- Integration only up to ρ_{max} due to the factor $\exp[\rho\ln(1/x)\cos\varphi]$
- φ, c and number of N's can be tuned to get better accuracy
- Need analytic continuations of all N-space functions to the complex plane



Coefficient functions in N-space

- Starting from x-space expressions [Ravindran, unpublished]
  Mellin transforms obtained using `harmpol` package [Remiddi, Vermaseren 2000]
- N-space results (NNLO) have been published [Blümlein, Ravindran 2005]
- $\rightarrow N$-space results mostly in terms of harmonic sums $S_k(N)$
  with “most right” pole at N =1.
- Analytic continuations of harmonic sums in terms of polygamma functions
- Finite set of more complicated harmonic sums needs special treatment in order to get the analytic continuations, e.g. `ancont` [Blümlein, 2000]

Evolution of Parton Distribution Functions

- Scale dependence of $f_a(x, Q^2)$ described by evolution equations (DGLAP)

$$\frac{\partial f_a(x,Q^2)}{\partial \ln Q^2} = \sum_b \underbrace{P_{ab}(x,Q^2)}_{\text{Splitting functions}} \otimes f_b(x,Q^2)$$

- The solution

$$f_a(x,Q^2) = E\Big[P_{ab}(x), \alpha_s(Q^2), \underbrace{f_b(x,Q_0^2)}_{Q_0^2 \sim \text{few GeV}}\Big]$$

Parton Distribution Functions in N-space

- Evolution in N-space
  - Choose initial distribution $f(x, Q_0^2)$, typical parametrization

$$
\begin{aligned}
xf(x,\mu_0^2) &= ax^b(1-x)^c(1+dx^f+gx), \qquad a,\ldots,g \in \mathbb{R} \\
f(N,Q_0^2) &= a\Big[\beta(N+b-1,c+1)+\beta(N+b+f-1,c+1)+ \\
&\quad \beta(N+b,c+1)\Big]
\end{aligned}
$$

  - Splitting functions in N-space [Moch, Vermaseren, Vogt 2004]
  - Evolve PDFs in N-space [QCD-PEGASUS, Vogt 2004]

$$
f_a(N,Q^2) = E\Big[P_{ab}(N), \alpha_s(Q^2), f_b(N,Q_0^2)\Big]
$$

- Evolution in x-space (use LHAPDF) $\rightarrow$ find an interpolation function for $f(x, Q^2)$ and transform it to N-space.

The Calculation - Indirect Check

- Choose a toy initial parton distribution function, perform evolution in N-space with `QCD-PEGASUS` $\to \tilde{\sigma}_{\rm toy}(N, Q^2)$
- Choose the same initial distribution in x-space and do a standard convolution $\to \sigma_{\rm toy}(x, Q^2)$ [CUBA, Hahn 2005]
- Compare **M**$[\tilde{\sigma}_{\rm toy}(N, Q^2)] \Leftrightarrow \sigma_{\rm toy}(x, Q^2)$.
- Replace toy pdfs in x-space with real ones (LHAPDF) $\sigma_{\rm toy}(x, Q^2) \to \sigma_{\rm LHAPDF}(x, Q^2)$ and compare to publicly available codes
  - MCFM (NLO) [Campbell, Ellis]
  - ZWPROD [Hamberg, Matsuura, van Neerven, 2002]
  - DYNNLO [Catani, Cieri, Ferrera, de Florian, Grazzini 2009; Catani, Grazzini 2007]
  - FEWZ [Melnikov, Petriello]

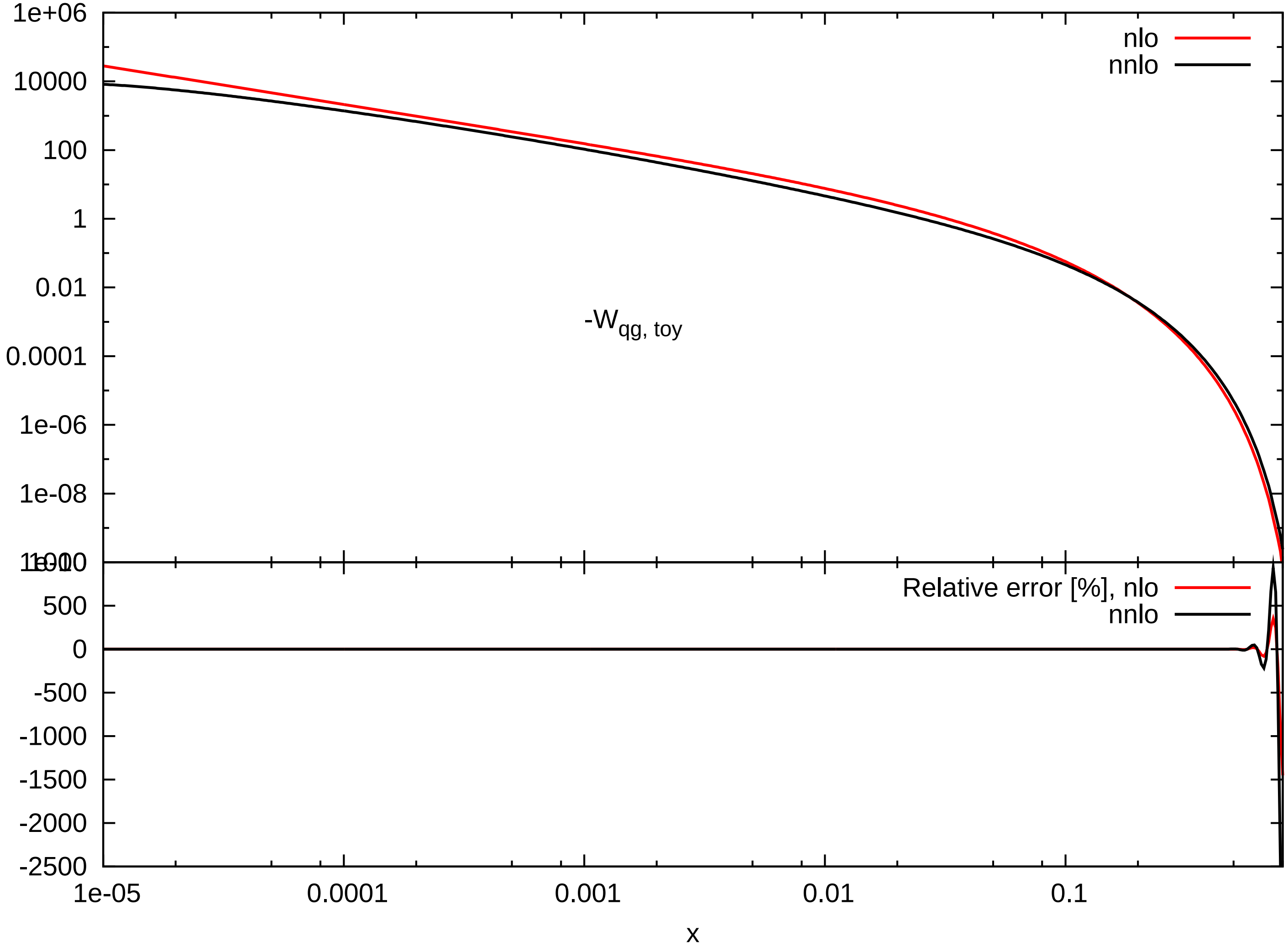
The Calculation - Direct Check

- Choose a real initial parton distribution function $\rightarrow$ Mellin transform, perform evolution in N-space with `QCD-PEGASUS` and compare **IM**$[\sigma(N, Q^2)]$ to cross section calculated by publicly available codes

- Needs to extend slightly `QCD-PEGASUS`
- Not possible for any distributions in LHAPDF - some initial parametrizations are not in form that Mellin transform on them can be performed

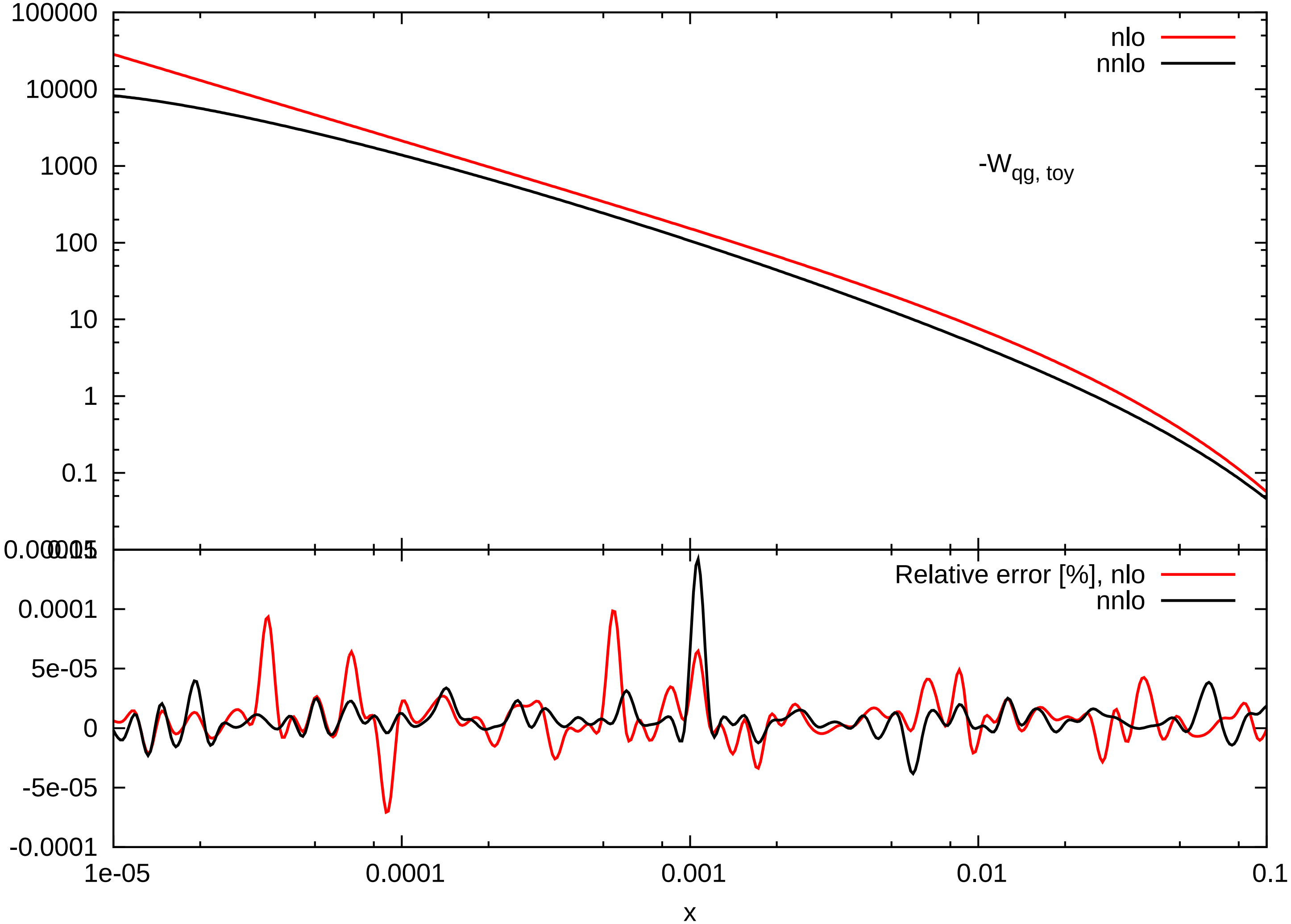
- Alternatively, access directly evolved PDFs from LHAPDF grids $\rightarrow$ interpolation $\rightarrow$ Mellin transform

QG contribution to the DY structure function



$$W^{\mathrm{n}^k\mathrm{lo}}(N)_{qg,toy} = \left(\frac{\alpha_s}{4\pi}\right)^k C_{qg}(N) f_{q,toy}(N) f_{g,toy}(N), \qquad W_{qg,toy} = \mathrm{IM}[W_{qg,toy}(N)]$$

QG contribution to the DY structure function



$$W^{\mathsf{n}^k\mathsf{lo}}(N)_{qg,toy} = \left(\frac{\alpha_s}{4\pi}\right)^k C_{qg}(N) f_{q,toy}(N) f_{g,toy}(N), \qquad W_{qg,toy} = \mathsf{IM}[W_{qg,toy}(N)]$$

Summary and Outlook

- Higher order corrections for Drell-Yan process are necessary for precise predictions
- Calculation in Mellin space is fast and accurate

- NNLO cross section
- Interface for PDFs accessing directly LHAPDF grids
- Implementation of DIS
- Release the code

Backup

- Analytic continuation of a Harmonic Sum

$$S_k(N) = \frac{(-1)^{k+1}}{(k-1)!}\psi^{k-1}(N+1) + \zeta(k)$$

- The Euler Beta function

$$\beta(a,b) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$