Nearest-Neighbour-Interactions and Abelian Discrete Flavour Symmetries

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In Collaboration with

- G. C. Simões (Work in progress)
- G. C. Branco and C. Simões, Phys. Lett. B690 (2010) 62
- C. Simões, Phys. Rev. D 79 (2009) 073006

HEPTOOLS Final Meeting in Granada 2010

26th November 2010

Motivation

- The simplest attempt for understanding the flavour structure encoded in the fermion mass matrices is by imposing some texture zeroes on the matrix elements which may reflect a flavour symmetry
- Many consistent texture zero structures can be found in the literature where some zeroes have or not Physical content
- NNI basis is non-Hermitian and has a maximum number o zeroes without physical implications [8]
- Fritzsch Ansatz: NNI and Hermiticity is ruled out $\Longrightarrow$ deviations from the hermiticiy in the quark sector
- Can the NNI arise from Phyics Beyond the Standard Model

Talk's Outline

- NNI structure and deviations from Fritzsch Ansatz
- Discrete Flavour Symmetries
- Flavour Symmetries in GUT Models
- Conclusions

Nearest-Neighbour-Interaction Basis

[Branco, Lavoura, Mota]

- Obtained through Weak Basis Transformations
- 8 Zeroes in Non-Hermitian Mass Matrices
- No further Physical Consequences, just a weak basis!
- Completely factorisable $V = O_u^\top \, K \, O_d$

$$M_u = \begin{pmatrix} 0 & A_u & 0 \\ A'_u & 0 & B_u \\ 0 & B'_u & C_u \end{pmatrix} \qquad M_d = \begin{pmatrix} 0 & A_d & 0 \\ A'_d & 0 & B_d \\ 0 & B'_d & C_d \end{pmatrix}$$

- or equivalently

$$(H_u)_{12} = 0 \qquad (H_d)_{12} = 0$$

- with $H_{u,d} = M_{u,d} M^\dagger_{u,d}$
- Can on have Hermiticity in addition?
  Fritzsch Ansatz [Physical Implications]

Fritzsch Ansatz

- NNI plus Hermiticity
- The orthogonal matrices $O^\top H O = \mathrm{diag}(m_1^2, m_2^2, m_3^2)$ are just function of mass ratios [exact]

$$O = \begin{pmatrix} \sqrt{\frac{m_2 m_3 (m_3+m_2)}{(m_2-m_1)(m_3+m_2+m_1)(m_3-m_1)}} & -\sqrt{\frac{m_1 m_3 (m_3+m_1)}{(m_1-m_2)(m_3+m_2+m_1)(m_3-m_2)}} & \sqrt{\frac{m_1 m_2 (m_1-m_2)}{(m_3-m_1)(m_3-m_2+m_1)(m_3-m_2)}} \\ \sqrt{\frac{m_1 (m_3+m_2)}{(m_1-m_2)(m_3-m_1)}} & \sqrt{\frac{m_2 (m_3+m_1)}{(m_2-m_1)(m_3-m_2)}} & \sqrt{\frac{m_3 (m_2+m_1)}{(m_3-m_1)(m_2-m_3)}} \\ -\sqrt{\frac{m_1 (m_2+m_1)(m_3+m_1)}{(m_2-m_1)(m_3+m_2+m_1)(m_3-m_1)}} & -\sqrt{\frac{m_2 (m_2+m_1)(m_3+m_2)}{(m_2-m_1)(m_3+m_2+m_1)(m_3-m_2)}} & \sqrt{\frac{m_3 (m_3+m_2)(m_3+m_1)}{(m_3-m_1)(m_3+m_2+m_1)(m_2-m_3)}} \end{pmatrix}$$

- If one takes into account the quark masss Hierarchy and neglet the up Sector contribution

$$V \simeq O_d \simeq \begin{pmatrix} 1 & -\sqrt{\frac{m_d}{m_s}} & \frac{m_s}{m_b}\sqrt{\frac{m_d}{m_b}} \\ \sqrt{\frac{m_d}{m_s}} & 1 & \sqrt{\frac{m_s}{m_b}} \\ -\sqrt{\frac{m_d}{m_b}} & -\sqrt{\frac{m_s}{m_b}} & 1 \end{pmatrix}$$

- Ruled out: m_t to heavy and V_{cb} inadequate
- Deviations of the Fritzsch Ansatz

Experimental Quark Masses and Mixings at Mz scale

[G. Rodrigo et al, Phys. Lett. B 313 (1993) 441
Y. Koide et al, Phys. Rev. D 57 (1998) 3986
Z. z. Xing at al, Phys. Rev. D 77 (2008) 113016]

- Runnig quark masses from PDG'10 to M_Z in the $\overline{MS}$ scheme using RGE for QCD @ 4 loops:

$$m_u = 1.4 \pm 0.5\,\text{MeV} \quad m_c = 0.62^{+0.06}_{-0.07}\,\text{GeV} \quad m_t = 170.2 \pm 1.0\,\text{GeV}$$

$$m_d = 2.9 \pm 0.5\,\text{MeV} \quad m_s = 58^{+16}_{-12}\,\text{MeV} \quad m_b = 2.86^{+0.16}_{-0.06}\,\text{GeV}$$

- CKM constructed from $|V_{us}|$, $|V_{ub}|$ and $|V_{cb}|$ and either $\sin 2\beta$ or γ constrained by J

$$\beta \equiv \arg(-V_{cd} V_{cb}^* V_{td}^* V_{tb}) \qquad \gamma = \arg(-V_{ud} V_{ub}^* V_{cd}^* V_{cb}) \qquad J \equiv \mathrm{Im}(V_{us} V_{ub}^* V_{cs}^* V_{cb})$$

$$|V_{us}| = 0.2253 \pm 0.0007 \quad |V_{ub}| = (3.47^{+0.16}_{-0.12}) \times 10^{-3} \quad |V_{cb}| = (41.0^{+0.11}_{-0.07}) \times 10^{-3}$$

$$\sin 2\beta = 0.673 \pm 0.023 \qquad \gamma = \left(73^{+22}_{-25}\right)^\circ \qquad J = \left(2.91^{+0.19}_{-0.11}\right) \times 10^{-5}$$

Minimal Deviations from Hermiticity

- Working with $H = MM^\dagger$

$$H = \begin{pmatrix} A^2 & 0 & A\,B' \\ 0 & {A'}^2 + B^2 & B\,C \\ A\,B' & B\,C & {B'}^2 + C^2 \end{pmatrix}$$

- Deviation from Hermiticity

$$A \equiv \overline{A}(1-\epsilon_a) \qquad A' \equiv \overline{A}(1+\epsilon_a)$$
$$B \equiv \overline{B}(1-\epsilon_b) \qquad B' \equiv \overline{B}(1+\epsilon_b)$$

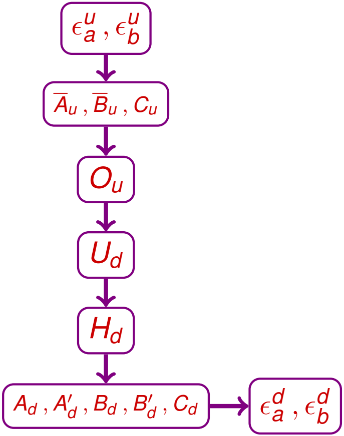
- $\epsilon_a\,,\epsilon_b << 1$ but not zero
- $\epsilon_a = 0 = \epsilon_b$ is the Fritzsch ansatz, ruled out by the data

$$O \simeq \begin{pmatrix} 1 & -\sqrt{\frac{m1}{m_2}}\left(1-\epsilon_a-\frac{m_2}{m_3}\epsilon_b\right) & \sqrt{\frac{m_1 m_2^2}{m_3^3}}\,(1+\epsilon_b-\epsilon_a) \\ \sqrt{\frac{m_1}{m_2}}\left(1-\epsilon_a-\frac{m_1}{m_3}\epsilon_b\right) & 1 & \sqrt{\frac{m_2}{m_3}}\,(1-\epsilon_b) \\ -\sqrt{\frac{m_1}{m_3}}\,(1-\epsilon_a-\epsilon_b) & -\sqrt{\frac{m_2}{m_3}}\left(1-\epsilon_b+\frac{m_1}{m_2}\epsilon_a\right) & 1 \end{pmatrix}$$

- Global deviation: $\varepsilon \equiv \sqrt{(\epsilon_a^u)^2+(\epsilon_b^u)^2+(\epsilon_a^d)^2+(\epsilon_b^d)^2}/2$

Numerical Procedure

- Reconstruction of M_u and M_d from quark masses and ϵ_a and ϵ_b



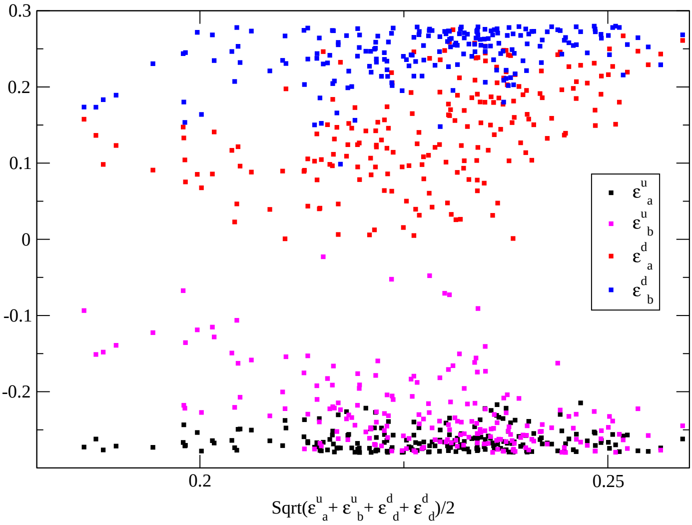
$$det(H_u) = m_u^2\, m_c^2\, m_t^2$$
$$Tr(H_u) = m_u^2 + m_c^2 + m_t^2$$
$$\chi(H_u) = m_u^2\, m_c^2 + m_u^2\, m_t^2 + m_c^2\, m_t^2$$

- $U_d = O_u\, K\, V$

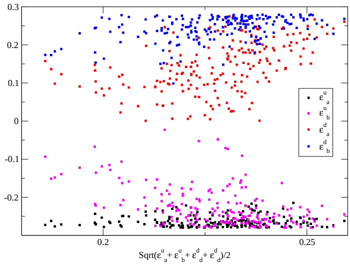
- K calculated solving $(H_d)_{12} = 0$

$$H_d = U_d\, diag(m_d, m_s, m_b)\, U_d^\dagger$$

Results for NNI at Mz



Results for NNI at Mz



Example:

$$\epsilon_a^u = -0.267982$$
$$\epsilon_b^u = -0.21014$$
$$\epsilon_a^d = 0.230599$$
$$\epsilon_b^d = 0.174908$$

Solution obtained for

$$m_u = 0.001910 \text{ MeV} \quad m_d = 0.003273 \text{ MeV}$$
$$m_c = 0.552049 \text{ GeV} \quad m_s = 0.045785 \text{ MeV}$$
$$m_t = 170.395835 \text{ GeV} \quad m_b = 2.850807 \text{ GeV}$$

and

$$V_{us} = 0.225732 \quad V_{ub} = 0.00345311 \quad V_{cb} = 0.0417001$$
$$\sin 2\beta = 0.667825 \quad J = 2.93573 \times 10^{-05}$$

- $\epsilon \gtrsim 0.188$

Starting from a symmetric Sector, large deviations in the other sector.

Nearest-Neighbour Interaction

- What is the minimal scenario for M_u and M_d be NNI form as a result of an Abelian family symmetry in a multi-Higgs extension in the context of the SM?

Discrete Flavour Symmetries

- NNI quark mass matrix implemented through the introduction of an Abelian family symmetry at the Lagrangian level.

@ least two Higgs doublets are needed: Φ_1, Φ_2

Under $\mathbf{Z_n}$ symmetry: $\Psi_j \longrightarrow \Psi'_j = e^{i\,\frac{2\pi}{n}\,\mathcal{Q}(\Psi_j)}\,\Psi_j$

$$\mathcal{Q}(\Phi_i) \equiv \phi_i \qquad \mathcal{Q}(Q_{Li}) \equiv q_i \qquad \mathcal{Q}(u_{Ri}) \equiv u_i \qquad \mathcal{Q}(d_{Ri}) \equiv d_i$$

The quark fields transformations:

$$(q_1, q_2) = (q_3 + \phi_1 - \phi_2, q_3 - \phi_1 + \phi_2)$$
$$(u_1,\, u_2,\, u_3) = (q_3 - \phi_1 + 2\phi_2, q_3 + \phi_1, q_3 + \phi_2)$$
$$(d_1,\, d_2,\, d_3) = (q_3 - 2\phi_1 + \phi_2, q_3 - \phi_2, q_3 - \phi_1)$$

Discrete Flavour Symmetries

Allowed bilineares:

$$\mathcal{Q}(\overline{Q}_{Li} u_{Rj}) = \begin{pmatrix} -2\phi_1 + 3\phi_2 & \phi_2 & -\phi_1 + 2\phi_2 \\ \phi_2 & 2\phi_1 - \phi_2 & \phi_1 \\ -\phi_1 + 2\phi_2 & \phi_1 & \phi_2 \end{pmatrix}$$

$$\mathcal{Q}(\overline{Q}_{Li} d_{Rj}) = \begin{pmatrix} -3\phi_1 + 2\phi_2 & -\phi_1 & -2\phi_1 + \phi_2 \\ -\phi_1 & \phi_1 - 2\phi_2 & -\phi_2 \\ -2\phi_1 + \phi_2 & -\phi_2 & -\phi_1 \end{pmatrix}$$

Discrete Flavour Symmetries

Allowed bilineares:

$$\mathcal{Q}(\overline{Q}_{Li}u_{Rj}) = \begin{pmatrix} -2\phi_1 + 3\phi_2 & \phi_2 & -\phi_1 + 2\phi_2 \\ \phi_2 & 2\phi_1 - \phi_2 & \phi_1 \\ -\phi_1 + 2\phi_2 & \phi_1 & \phi_2 \end{pmatrix}$$

$$\mathcal{Q}(\overline{Q}_{Li}d_{Rj}) = \begin{pmatrix} -3\phi_1 + 2\phi_2 & -\phi_1 & -2\phi_1 + \phi_2 \\ -\phi_1 & \phi_1 - 2\phi_2 & -\phi_2 \\ -2\phi_1 + \phi_2 & -\phi_2 & -\phi_1 \end{pmatrix}$$

- Neither Z_2 nor Z_3 works:

$$-2\phi_1 + 3\phi_2 = \phi_2 \pmod 2$$

$$-2\phi_1 + 3\phi_2 = \phi_1 \pmod 3$$

Minimal symmetry $\rightarrow$ Z_4

Discrete Flavour Symmetries

- The most general Yukawa couplings:

$$-\mathcal{L}_Y = \Gamma_u^1 \overline{Q}_L \widetilde{\Phi}_1 u_R + \Gamma_u^2 \overline{Q}_L \widetilde{\Phi}_2 u_R + \Gamma_d^1 \overline{Q}_L \Phi_1 d_R + \Gamma_d^2 \overline{Q}_L \Phi_2 d_R + \text{H.c.}\,,$$

$$\widetilde{\Phi}_j \equiv i\sigma_2 \Phi_j^*$$

Yukawa matrices $\Gamma_{u,d}^{1,2}$:

$$\Gamma_u^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_u \\ 0 & b'_u & 0 \end{pmatrix} \quad \Gamma_u^2 = \begin{pmatrix} 0 & a_u & 0 \\ a'_u & 0 & 0 \\ 0 & 0 & c_u \end{pmatrix}$$

$$\Gamma_d^1 = \begin{pmatrix} 0 & a_d & 0 \\ a'_d & 0 & 0 \\ 0 & 0 & c_d \end{pmatrix} \quad \Gamma_d^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_d \\ 0 & b'_d & 0 \end{pmatrix}$$

Discrete Flavour Symmetries

- The most general Yukawa couplings:

$$-\mathcal{L}_Y = \Gamma_u^1 \overline{Q}_L \widetilde{\Phi}_1 u_R + \Gamma_u^2 \overline{Q}_L \widetilde{\Phi}_2 u_R + \Gamma_d^1 \overline{Q}_L \Phi_1 d_R + \Gamma_d^2 \overline{Q}_L \Phi_2 d_R + \text{H.c.}\,,$$

$\widetilde{\Phi}_j \equiv i\sigma_2 \Phi_j^*$

- After spontaneous symmetry breaking, Higgs VEV's, $\langle \Phi_1 \rangle \equiv v_1$ and $\langle \Phi_2 \rangle \equiv v_2$ generate NNI mass matrices

$$M_u = \begin{pmatrix} 0 & v_2\, a_u & 0 \\ v_2\, a'_u & 0 & v_1\, b_u \\ 0 & v_1\, b'_u & v_2\, c_u \end{pmatrix} \qquad M_d = \begin{pmatrix} 0 & v_1\, a_d & 0 \\ v_1\, a'_d & 0 & v_2\, b_d \\ 0 & v_2\, b'_d & v_1\, c_d \end{pmatrix}$$

- Higher Dimensional Operators $\frac{\lambda}{\Lambda^2}\, \overline{Q}_{2\,L} \tilde{\Phi}_1 u_{2\,R} \Phi_1^\dagger \Phi_2\,,$

Accidental Global $U(1)$ symmetry

- Renormalisability implies that Z_4 has to be imposed on the full Lagrangian
- The most general renormalisable scalar potential consistent with Z_4 and gauge symmetry

$$\mathbf{V} = \mu_1 |\Phi_1|^2 + \mu_2 |\Phi_2|^2 + \lambda_1 |\Phi_1|^4 + \lambda_2 |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1 .$$

- Potential acquires a new accidental global symmetry which, upon spontaneous symmetry breaking leads to a massless neutral scalar (tree level)
- Soft-breaking term avoids the problem

$$\mathbf{V}' = \mu_{12} \, \Phi_1^\dagger \Phi_2 + \text{H.c.}$$

- Or by introducing a singlet Higgs field [Z_4 charged]

CP Violation in the Model

- Spontaneous CP Violation is not possible, essentially due to the absence of terms like

$$(\Phi_1^\dagger \Phi_2 \Phi_1^\dagger \Phi_2) + \text{H.c}$$

- scalar vacuum:

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} , \qquad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_2 \, e^{i\,\theta} \end{pmatrix} ,$$

- Two CP Conserving minima

$$\theta = 0 \qquad \text{for} \qquad \mu_{12} < 0$$

$$\theta = \pi \qquad \text{for} \qquad \mu_{12} > 0$$

- CP Violation via Kobayashi-Maskawa mechanism through Yukawa couplings

NNI in Grand Unification

- Minimal SM Fermion Content: SU(5) and SO(10)
- Quark and Leptons are together in higher dimension multiplets
- Flavour symmetry which give rise to NNI in the Quark Sector imposes restrictions on the Leptonic Sector
- SU(5): 5^*, 10 and Right-handed Neutrinos are singlets [minimal field content] and then free Z_4 charges

$$\mathcal{Q}(10) = \mathcal{Q}(q) = \mathcal{Q}(u^c) = \mathcal{Q}(e^c)$$
$$\mathcal{Q}(5^*) = \mathcal{Q}(\ell) = \mathcal{Q}(d^c)$$

- SO(10): all in a unique 16 spinorial representation

$$\mathcal{Q}(16) = \mathcal{Q}(q) = \mathcal{Q}(u^c) = \mathcal{Q}(e^c) = \mathcal{Q}(\ell) = \mathcal{Q}(d^c) = \mathcal{Q}(\nu^c)$$

SU(5) and Neutrinos

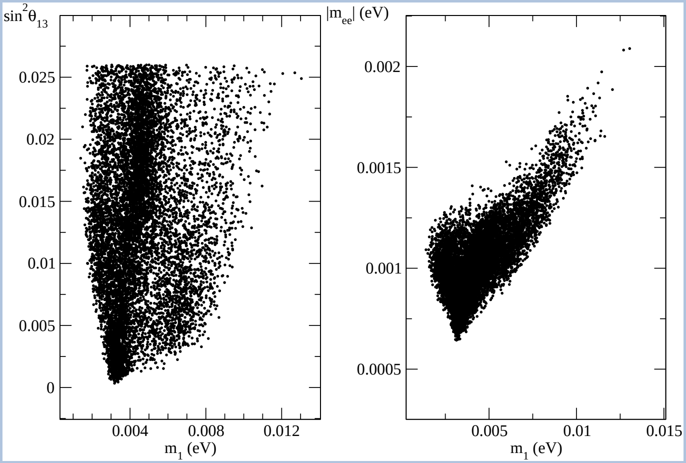
- Phenomenology makes further implications
- Effective Neutrino Mass Matrix for $\mathcal{Q}(\Phi_1) = 1$

Charges	$\nu_R = (0,1,3)$	$\nu_R = (1,2,3)$	$\nu_{R\,i} \in \{0,2\}$
$\phi_2{=}0$ $10{=}(1,3,0)$ $5^*{=}(2,0,1)$	$\mathrm{I}_{(123)}$ $\begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & 0 \end{pmatrix}$	$\mathrm{II}_{(12)}$ $\begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}$	$\mathrm{III}_{(12)}$ $\begin{pmatrix} * & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}$
$\phi_2{=}2$ $10{=}(2,0,3)$ $5^*{=}(1,3,2)$	II $\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$	$\mathrm{I}_{(13)}$ $\begin{pmatrix} * & * & 0 \\ * & 0 & * \\ 0 & * & 0 \end{pmatrix}$	III $\begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}$

- In the case of SO(10) does not work for Z_4
  [In the SUSY extension Works!!]

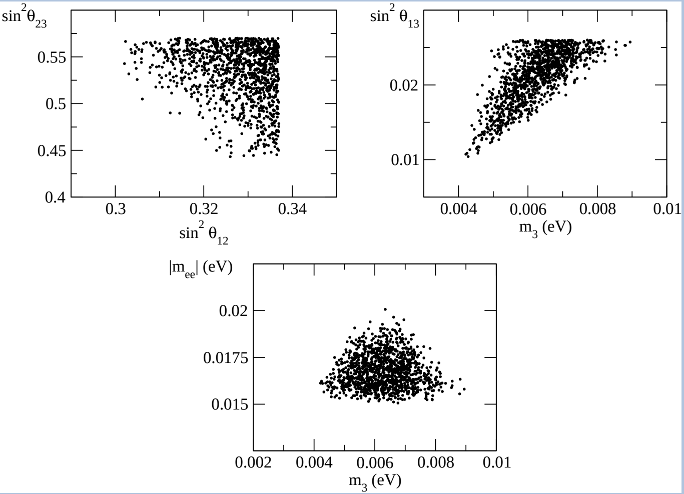
SU(5) and Neutrinos

- $\sin^2\theta_{13}$ (left) and $|m_{ee}|$ (right) as a function of m_1 for texture II in the case of normal hierarchy



SU(5) and Neutrinos

- $\sin^2\theta_{23}$ versus $\sin^2\theta_{12}$ (up left), $\sin^2\theta_{13}$ versus m_3 (up right) and $|m_{ee}|$ versus m_3 (down) for texture $II_{(12)}$ in the case of inverted hierarchy



Conclusions

- NNI textures are in aggreement with present experimental data on quark masses and mixings
- in both quark sectores allowed deviations of hermiticity around 20%

- The NNI textures for quark mass matrix obatined through Abelian flavour symmetry
- Minimal realisation in the context of two Higgs doublets is Z_4
- Accidental Global $U(1)$ symmetry
- CP Violation through KM mechanism

- Z_4 in SU(5) and SO(10)
- Special texture Zeroes in the Neutrino Sector

Thank You!