

$SU(5) \times SU(5)$ Unification Revisited

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- Unification of the three SM interactions in Λ -scale:
 - non-SUSY;
 - SUSY;
- Minimal groups (rank 4) can do that ($SU(5)$, $SO(10)$, E_6, \dots);
- Agreement with experimental data: $\sin^2\theta_W(M_Z)$ at Z-pole:
 - $\sin^2\theta_W(M_Z)$ at Z-pole;
 - Admit Neutrino masses;
- Avoid the usual problems in SUSY-GUTs:
 - The proton should live long enough;
 - R-parity must be conserved;
 - The doublet-triplet problem have to be alleviated;
- An possibility of extension to non-canonical gauge group;
- The $SU(5) \times SU(5)$ can accommodate the problems usual left unexplained;

- Briefly discuss minimal $SU(5)$ unification:
 - SM;
 - MSSM;
- The $SUSY SU(5) \times SU(5)$ model content;
 - Needs $(15, 1) + (1, \overline{15}) + h.c. + (24, 1) + (1, 24)$;
- Generalized seesaw mechanism for fermions;
- Running coupling constants @ two-loop level;
- Numerical analysis: with two and four Higgs doublets;
 - Unification scale Λ ;
 - $SU(5) \times SU(5)$ string constraint;
- Conclusions and outlook.

- $SU(5) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$

Fermion multiplets (chosen to the theory be anomaly-free):

$$\bar{\mathbf{5}} = \begin{pmatrix} d^c \\ \epsilon_2 \ell \end{pmatrix}_L = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}_L$$

$$\mathbf{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} \epsilon_3 u^c & -q \\ q & \epsilon_2 e^c \end{pmatrix}_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^c \\ d_1 & d_2 & d_3 & e^c & 0 \end{pmatrix}_L$$

with the following notation

$$SU(3) : d_L^c = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix}_L, \dots \quad SU(2) : \ell_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L, q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$$

For first approximation we consider the one-loop evolution of gauge coupling constants:

$$\alpha_i^{-1}(\mu) = \frac{1}{k_i} \left[\alpha_i^{-1}(M_z) - \frac{b_i}{2\pi} \ln \left(\frac{\mu}{M_z} \right) \right]$$

with the canonical group factors: $k_i = 5/3, 1, 1$

$$b_1 = \frac{20}{9}N_g + \frac{n_H}{6} \quad b_1^{\text{SUSY}} = \frac{10}{3}N_g + \frac{n_H}{2}$$

$$b_2 = \frac{4}{3}N_g + \frac{n_H}{6} - \frac{22}{3} \quad b_2^{\text{SUSY}} = 2N_g + \frac{n_H}{2} - 6$$

$$b_3 = \frac{4}{3}N_g - 11 \quad b_3^{\text{SUSY}} = 2N_g - 9$$

- Scalar multiplets:

For Standard model we include only a $H \sim \mathbf{5}$ and the usual $\mathbf{24}$ which breaks $\text{SU}(5)$. For the SUSY we need to include $H \sim \mathbf{5}$ and $\bar{H} \sim \bar{\mathbf{5}}$.

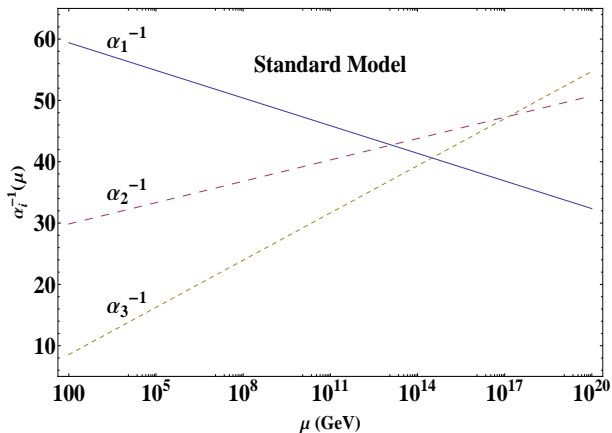


Figure: In this figure we show the non-convergence of the Minimal SU(5) at one-loop.

Standard

- The p^+ decay is mediated by GUT bosons (dim 6 operators) leptoquarks $(X, Y)\mathcal{O} \sim \frac{1}{M_X^2} QQQ_L$;
- It not exclude the model (since we consider $M_X \sim 10^{16} GeV$) in compatibility with the experimental bounds set by p^+ decay: $M_V \gtrsim 5 \times 10^{13} GeV$ (2×10^{15}) if we do (not) neglect fermion mixing ^a;
- However, the couplings do not unify, it cannot be a unification model;
- New scalar content must be include to avoid the relation $m_e(\Lambda) \neq m_d(\Lambda)$.

^aDorsner and Fileviez Perez, Phys. Lett. B **625**:88, 2005

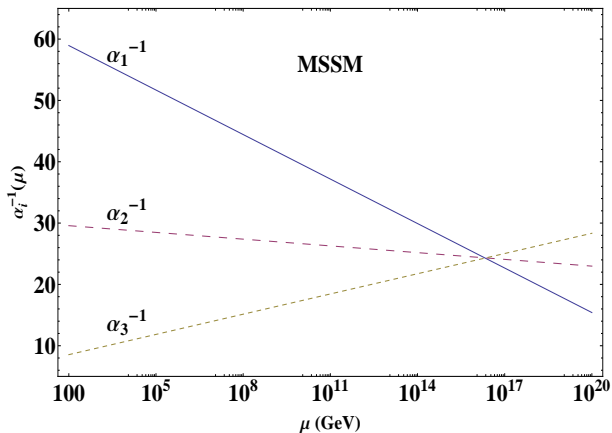


Figure: In this figure we show the convergence point for the Minimal SUSY SU(5) at one-loop.

SUSY

- Risk of weak-like proton decay (dim 4 operators), but it can be removed by appropriate R-symmetry;
- Dangerous process with triplet higgsino (dim 5 operators): the decay is due to interaction of higgsino bosons H^c ;
- Some specific choices of Yukawa couplings ($Y_d = Y_e$) dramatically induces the p^+ decay^a;
- However, different selections not exclude the model^b;
- Precise unification without the use of new particles at ISs.

^aFileviez Perez, Phys. Lett. **B595**:476, 2004; Roy, Phys. Rev. D **71**:035010, 2005; Boer and Sander, Phys. Lett. **B585**:276, 2004.

^bSee for example, Bajc, Fileviez Perez and Senjanovic Phys. Rev. D **66**, 075005 (2002); Emmanuel-Costa and Wiesenfeldt Nucl. Phys. B **661**, 62 (2003).

- It is in the branching of $E_8 \times E_8 \Rightarrow$ String theory.
- Hybrid unification of L-R-handed sectors;
- Implies the introduction of vector-like fermions;

Fermion multiplets

$$\psi_L = \begin{pmatrix} D^c \\ \epsilon_2 \ell \end{pmatrix}_L \sim (\bar{\mathbf{5}}, \mathbf{1}) \quad \chi_L = \frac{1}{\sqrt{2}} \begin{pmatrix} \epsilon_3 U^c & -q \\ q & \epsilon_2 E^c \end{pmatrix}_L \sim (\mathbf{10}, \mathbf{1})$$

$$\psi_R = \begin{pmatrix} D^c \\ \epsilon_2 \ell \end{pmatrix}_R \sim (\mathbf{1}, \bar{\mathbf{5}}) \quad \chi_R = \frac{1}{\sqrt{2}} \begin{pmatrix} \epsilon_3 U^c & -q \\ q & \epsilon_2 E^c \end{pmatrix}_R \sim (\mathbf{1}, \mathbf{10})$$

The uppercase letters (U , D and E) are the vector-like fermions and the lowercase still denoting the SM fermions.

The SUSY left-right $SU(5) \times SU(5)$ gauge group contains two copies per generation of the usual SUSY $SU(5)$ theory. In the left-handed picture, the $(\bar{5} + 10, 1)$ fermion representations are given by:

$$\psi = \begin{bmatrix} D_1^c \\ D_2^c \\ D_3^c \\ e \\ -\nu \end{bmatrix}, \quad \chi = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & U_3^c & -U_2^c & -u_1 & -d_1 \\ -U_3^c & 0 & U_1^c & -u_2 & -d_2 \\ U_2^c & -U_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -E^c \\ d_1 & d_2 & d_3 & E^c & 0 \end{bmatrix},$$

while the $(1, 5 + \bar{10})$ fields, represented by ψ^c and χ^c , are

$$\psi^c = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ e^c \\ -\nu^c \end{bmatrix}, \quad \chi^c = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & U_3 & -U_2 & -u_1^c & -d_1^c \\ -U_3 & 0 & U_1 & -u_2^c & -d_2^c \\ U_2 & -U_1 & 0 & -u_3^c & -d_3^c \\ u_1^c & u_2^c & u_3^c & 0 & -E \\ d_1^c & d_2^c & d_3^c & E & 0 \end{bmatrix}.$$

Extra fermions to SM: the vector-like (U, U^c, D, D^c, E, E^c) and the well-motivated right-handed neutrino, ν^c . There is no vector-like analog of the neutrino.

Among all possible patterns we choose the following:

$$\begin{aligned}
 & \text{SU}(5)_L \times \text{SU}(5)_R \\
 & \quad \downarrow \Lambda \\
 & \text{SU}(3)_L \times \text{SU}(2)_L \times \text{U}(1)_L \times \text{SU}(3)_R \times \text{SU}(2)_R \times \text{U}(1)_R \\
 & \quad \downarrow \Lambda_{LR} \\
 & \text{SU}(3)_C \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L} \\
 & \quad \downarrow v_R \\
 & \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \\
 & \quad \downarrow v_L \\
 & \text{SU}(3)_C \times \text{U}(1)_{em}.
 \end{aligned}$$

Identification:

- $\text{SU}(3)_C$ with the $\text{SU}(3)_{L+R}$ diagonal subgroup;
- $\text{U}(1)_{B-L}$ with $\text{U}(1)_{L+R}$.

The breaking energy scales Λ , Λ_{LR} and v_R are determined by the Higgs content of the model.

We need the adjoint representations of both $SU(5)$ subgroups;

- $\Phi_L \sim (24, 1)$ and $\Phi_R \sim (1, 24)$, which accomplish the first breaking of $SU(5)_L \times SU(5)_R @ \Lambda$ but preserve the **LR** symmetry;
- **LR** symmetry breaking @ Λ_{LR} , by the Higgs fields $\omega_1 \sim (5, \bar{5})$, $\omega_2 \sim (\bar{5}, 5)$, $\Omega_1 \sim (10, \bar{10})$ and $\Omega_2 \sim (\bar{10}, 10)$ ¹;
- The last two steps in the pattern are driven by the add. Higgs fields $\phi_R, \phi_R^c \sim (1, 5), (1, \bar{5})$ and $\phi_L, \phi_L^c \sim (5, 1), (\bar{5}, 1)$, respectively;
- Finally, as mentioned in the Introduction, the inclusion of the representations $T_L \sim (15, 1)$, $T_L^c \sim (\bar{15}, 1)$, $T_R \sim (1, \bar{15})$ and $T_R^c \sim (1, 15)$ turn out to be crucial for unification and are responsible for the Majorana masses of neutrinos.

¹Alternatively, one could break directly the left-right symmetry at the scale $\Lambda_{LR} = \Lambda$ without the need of the adjoint Higgs fields in the $(24, 1)$ and $(1, 24)$ representations.

- $SU(5) \times SU(5)$ attractive feature: generalized seesaw mechanism to give masses to all SM fermions through the heavy vector-like fermions².

The Yukawa contribution to the superpotential reads as

$$W_Y = \psi Y_1 \omega_1 \psi^c + \psi^c Y_1^T \omega_2 \psi + \chi Y_2 \Omega_1 \chi^c + \chi^c Y_2^T \Omega_2 \chi + \sqrt{2} \psi Y_3 \chi \phi_L^c + \sqrt{2} \psi^c Y_3^T \chi^c \phi_R^c + \frac{1}{4} \chi Y_4 \chi \phi_L + \frac{1}{4} \chi^c Y_4 \chi^c \phi_R,$$

where Y_i denotes the Yukawa coupling matrices. We choose the breaking directions as $\langle \omega_i \rangle_k^k = \langle \Omega \rangle_{12}^{12} = \langle \Omega \rangle_{23}^{23} = \langle \Omega \rangle_{31}^{31} = \langle \Omega \rangle_{45}^{45} = \Lambda_{LR}$, $i = 1, 2$, $k = 1, 2, 3$ and $\langle \phi_{L,R} \rangle = (0, 0, 0, 0, v_{uL,R})^T$, $\langle \phi_{L,R}^c \rangle = (0, 0, 0, 0, v_{dL,R})^T$, with $v_{L,R}^2 = v_{uL,R}^2 + v_{dL,R}^2$.

²A. Davidson and K. C. Wali, *Phys.Rev.Lett.* **59** (1987) 393; A. Davidson and K. C. Wali, *Phys.Rev.Lett.* **60** (1988) 1813; P. L. Cho, *Phys.Rev.* **D48** (1993) 5331–5341

The final mass contribution to all charged fermions can then be written as

$$\begin{aligned} \mathcal{L}_m = & (u \quad U) \begin{pmatrix} 0 & Y_4 v_{uL} \\ Y_4 v_{uR} & Y_2 \Lambda_{LR} \end{pmatrix} \begin{pmatrix} u^c \\ U^c \end{pmatrix} + \\ & (d \quad D) \begin{pmatrix} 0 & Y_3 v_{dL} \\ Y_3^T v_{dR} & Y_1 \Lambda_{LR} \end{pmatrix} \begin{pmatrix} d^c \\ D^c \end{pmatrix} + \\ & (e \quad E) \begin{pmatrix} 0 & Y_3 v_{dL} \\ Y_3^T v_{dR} & Y_2^T \Lambda_{LR} \end{pmatrix} \begin{pmatrix} e^c \\ E^c \end{pmatrix}. \end{aligned}$$

For the neutrino sector, the relevant terms in the superpotential are

$$W_N = \sqrt{2} Y_5 (\psi \psi T_L + \psi^c \psi^c T_R),$$

which leads to the effective neutrino mass Lagrangian³.

³In progress.

The general renormalization group equations (RGE) at two-loop for a gauge coupling constant, corresponding to a gauge group G_i are:

$$\frac{d}{dt}\alpha_i^{-1} = -\frac{b_i}{2\pi} - \frac{1}{8\pi^2} \sum_j b_{ij} \alpha_j - \frac{1}{32\pi^3} \sum_{f=u,d,e} C_{if} \text{Tr} \left(Y_f^\dagger Y_f \right),$$

where $i = 1, 2, 3$ for $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$, respectively; b_i are the usual one-loop beta coefficients and b_{ij} , C_{if} are the two-loop beta coefficients⁴. The quantities Y_f are the quark and lepton Yukawa coupling matrices.

⁴D. R. T. Jones, *Phys. Rev.* **D25** (1982) 581.

If one assumes that $\text{SU}(5) \times \text{SU}(5)$ breaks directly to the SM gauge group $\text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$ at the unification scale Λ , then the three SM gauge couplings g_a ($a = s, w, y$) meet together into a single value,

$$\alpha_U \equiv k_3 \alpha_s = k_2 \alpha_w = k_1 \alpha_y,$$

where $\alpha_a = g_a^2/(4\pi)$. The coefficients k_i are group factors, $k_i = (\text{Tr } T_i^2)/(\text{Tr } T^2)$, ($i = 1, 2, 3$), where T and T_i are generators of the GUT group properly normalized over the full group and its SM subgroup G_i , respectively. For $\text{SU}(5) \times \text{SU}(5)$ one obtains the non-canonical values $k_1 = 13/3$, $k_2 = 1$ and $k_3 = 2$.

Adopting the following experimental values at M_Z^5

$$\alpha^{-1} = 127.916 \pm 0.015,$$

$$\sin^2 \theta_W = 0.23116 \pm 0.00013,$$

$$\alpha_s = 0.1184 \pm 0.0007.$$

⁵K. Nakamura and et al., *J. Phys.* **G37** (2010) 075021.

Running Gauge Couplings One-Loop Example

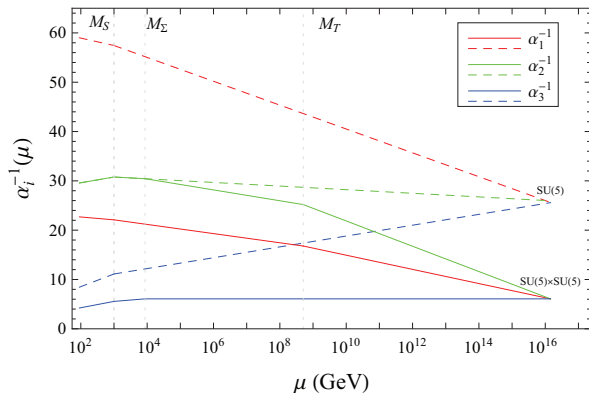


Figure: Comparison of RGEs between MSSM unifying in canonical SU(5)(dashed lines) with SU(5) \times SU(5) theory (solid lines) with the same unification scale, $\Lambda \sim 2 \times 10^{16}$ GeV, at 1-loop level. The SUSY scale is fixed at $M_S = 1$ TeV. Notice that for the non-canonical case we need Σ_3 and Σ_8 just below 10 TeV and two SU(2)_L triplets at high intermediate scales near to 10^9 GeV.

Mixing angle

$$\sin^2 \theta_W = \frac{\alpha_y}{\alpha_y + \alpha_w} = \frac{1}{1 + k_1/k_2} = \frac{3}{16}$$

It is commonly believed that this value cannot be reconciled with measurements at the electroweak scale ($\sin^2 \theta_W = 0.23116$): it is rather small and, in general, $\sin^2 \theta_W$ **decreases** from high to low energies.

- $(\overline{15}, 1) + (1, 15)$ and $(15, 1) + (1, \overline{15})$ representations: $\sin^2 \theta_W$ to the correct value;
- $SU(2)_L$ triplets in **15** and $\overline{15}$ representations strongly adjust α_w .

It is also remarkable that the above representations play a crucial role in implementing the seesaw mechanism for neutrino masses.

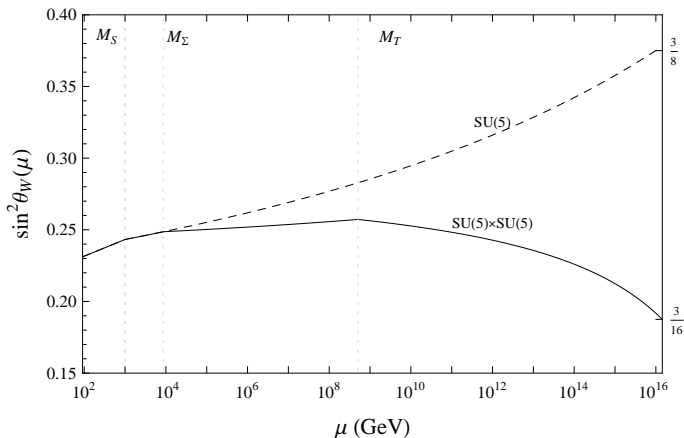


Figure: Comparison of evolution of $\sin^2 \theta_W$ between MSSM unifying in canonical SU(5) (dashed line) with SU(5) \times SU(5) theory (solid line) with the same unification scale, $\Lambda \sim 10^{16}$ GeV, at 1-loop level. The SUSY scale is fixed at $M_S = 1$ TeV while Σ_3 and Σ_8 are near to 10 TeV and the two SU(2)_L triplets are about $M_T \sim 10^9$ GeV.

Running Gauge Couplings Two-Loop Example

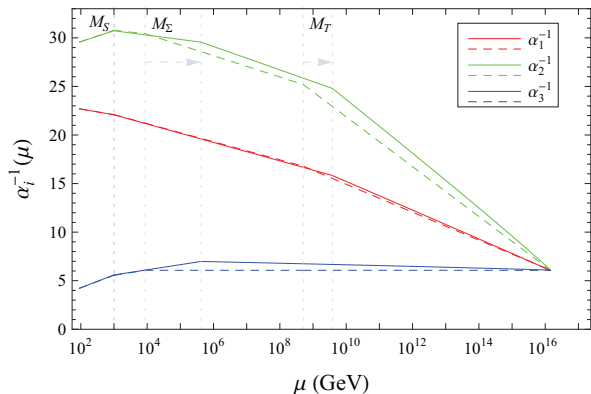


Figure: Comparison of $SU(5) \times SU(5)$ running of gauge couplings at 1-loop level (dashed lines) and 2-loops level (solid lines). The 2-loops effects increases both M_Σ and M_T scales with fixed M_S at 1 TeV and the same unification scale at $\Lambda \sim 2 \times 10^{16} GeV$.

To our study we need to define a evaluation function which defines the accuracy of unification we are choosing. We define this function ϵ as

$$\epsilon = \sqrt{(\alpha_{1\Lambda}^{-1} - \alpha_{2\Lambda}^{-1})^2 + (\alpha_{1\Lambda}^{-1} - \alpha_{3\Lambda}^{-1})^2 + (\alpha_{2\Lambda}^{-1} - \alpha_{3\Lambda}^{-1})^2}$$

Minimal values:

- $M_S = M_Z$
 - one-loop: $\epsilon_{M_Z}^{1l} \simeq 0.08$ @ $\Lambda_{M_Z}^{1l} \simeq 2.14 \times 10^{16} \text{GeV}$
 - two-loop: $\epsilon_{M_Z}^{2l} \simeq 0.63$ @ $\Lambda_{M_Z}^{2l} \simeq 2.17 \times 10^{16} \text{GeV}$

- $M_S = 1 \text{TeV}$
 - one-loop: $\epsilon_{1\text{TeV}}^{1l} \simeq 0.50$ @ $\Lambda_{1\text{TeV}}^{1l} \simeq 1.44 \times 10^{16} \text{GeV}$
 - two-loop: $\epsilon_{1\text{TeV}}^{2l} \simeq 0.18$ @ $\Lambda_{1\text{TeV}}^{2l} \simeq 1.38 \times 10^{16} \text{GeV}$

Our numerical analysis $\epsilon \leq 0.1$ and $M_S = 1 \text{TeV}$ @ two-loop level.

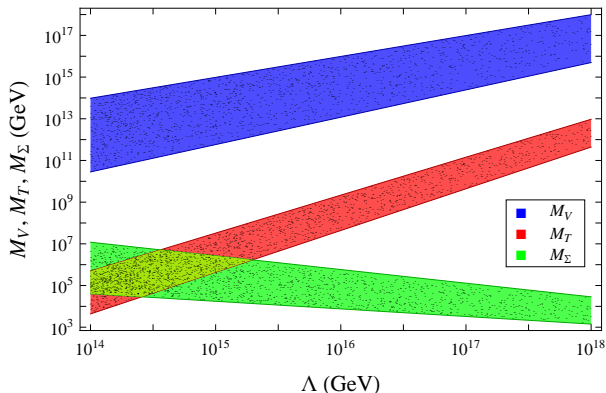


Figure: Numerical solutions to M_V , M_T and M_Σ by unification scale, Λ in $SU(5) \times SU(5)$ unification. The bands of solutions are due to not impose exact solution for the triangle formed by α_1^{-1} , α_2^{-1} and α_3^{-1} but within the possible threshold corrections which can be implemented in the same way as MSSM, but better than latter.

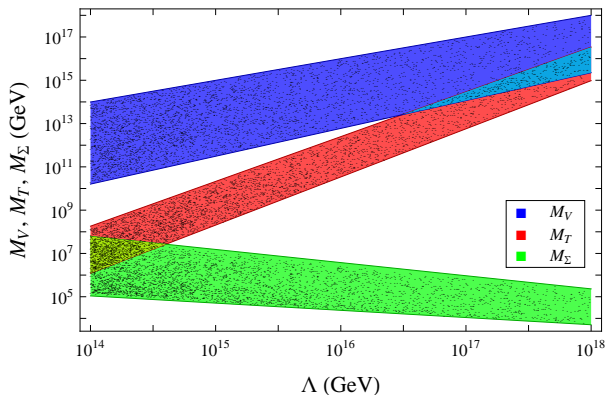


Figure: Numerical solutions to M_V , M_T and M_Σ by unification scale, Λ in $SU(5) \times SU(5)$ unification with two additional Higgs doublets. The bands of solutions are due to not impose exact solution for the triangle formed by α_1^{-1} , α_2^{-1} and α_3^{-1} . Notice the significant increase in the M_T scale which now reach the vector-like fermions scale.

Motivations:

- Superstring theory has emerged as the most promising candidate for a quantum theory of all known interactions;
- The phenomenology of $E_8 \times E_8$ heterotic string theory exhibits many of the attractive features of the low-energy known physics.

Now we wonder if the crucial test for the coupling constants could reach the minimal string unification value. In the heterotic scenario, the additional constraint on gauge couplings unification must be introduced,

$$\alpha_{\text{string}}^{-1} = 4\pi \left(\frac{\Lambda_S}{\Lambda} \right)^2,$$

where $\Lambda_S = 5.27 \times 10^{17}$ GeV. The lower bound on unification coupling constant is given by $\alpha_{\Lambda}^{-1} \geq 4\pi$ (for the perturbative region).

Lower bound on String Unification

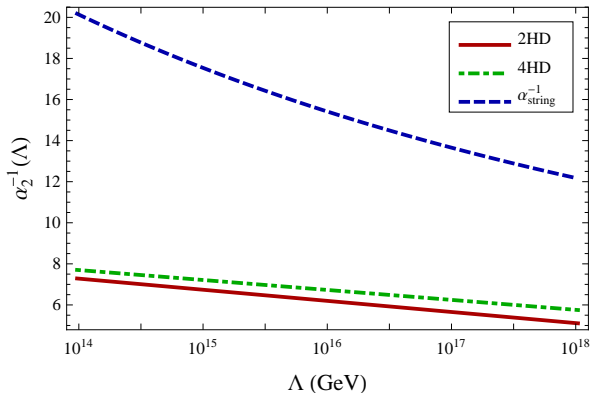


Figure: Upper bounds on $\alpha_2^{-1}(\Lambda)$ found in numerical analysis at 2-loops level by unification scale, Λ , in $SU(5) \times SU(5)$. 2H stands for the usual MSSM with two Higgs doublets while 4H stands for the MSSM plus two additional Higgs doublets at some scale between 10^3 – 10^{18} GeV.

- $(\bar{5}, 1)$ -scalar allows⁶: $q_{mL}^T y'_1 C H_m^c \ell + \epsilon_{mnp} U_{mL}^T y'_2 C H_n^c D_{pL}$, H_m^c
C-tpt: only interacts SM-dbts with vector-like quark sgts;
- Similarly R-parity-breaking in simple $SU(5)$ is due to interaction of $\bar{5}510$. In $SU(5) \times SU(5)$ model, R-parity-breaking is induced only by $q^T \ell D^c$ and $U^c D^c D^c$, which does not lead to R-parity violation involving light fermions.
- In this way, only a controlled amount of baryon and lepton asymmetries are produced given motivation to the study of baryogenesis and leptogenesis.

⁶R. Mohapatra, *Phys.Lett.* **B379** (1996) 115–120; R. Mohapatra, *Phys.Rev.* **D54** (1996) 5728–5733.

- The idea of unification still open for non-can. $SU(5) \times SU(5)$ group;
- $(15, 1) + (1, \overline{15}) + h.c. + (24, 1) + (1, 24)$
 - Left-decompositions: T_{15} and $\Sigma_{3,8}$;
 - Compatibility with EW data $\Rightarrow \sin^2\theta_w(\Lambda)$ agreement;
- $SU(5) \times SU(5)$ can accommodate all fermions masses due to gen. seesaw;
 - Vector-like fermions are naturally at high-energy scale;
- Proton is stable at tree-level;
- R-parity is conserved;
 - Only a controlled amount of CP-violation could be generated;
- The string cannot be incorporated in this minimal model;
 - Only a controlled amount of CP-violation could be generated;

Works in progress...questions to be answered...

- Is possible string unification with $SU(5) \times SU(5)$?

Thank you for your attention!



Ibituruna ao Luar...

Tespo - Francisco Padilha