# What can radiative decays tell us about the anomalous magnetic moment of the $\tau$?

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In collaboration with S. Eidelman, M. Fael, C. Ng and M. Passera.

# Outline

1. Introduction
2. Measurement of $a_\tau$
3. Radiative decays of the $\tau$

# Properties of elementary particles

How to we characterize SM particles?

The PDG tells us something about:

- Mass
- Spin
- Anomalous magnetic moment
- Electric dipole moment
- Decay channels and branching fractions
- ...

# Anomalous magnetic moment

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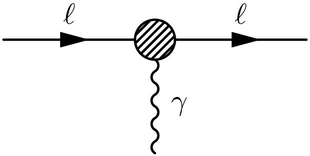


$$\Gamma^{\mu} \; = \; F_1\,\gamma^{\mu} + F_2\,\frac{i\sigma^{\mu\nu}q_{\nu}}{2m_{\ell}} + \ldots$$

$$a_{\ell} \; = \; F_2\Big|_{\text{on-shell}}$$

## Why is $a_\ell$ important?

- $a_e$: precise multi-loop calculations yield measurement of $\alpha$.

  [ D. Hanneke et al. '08, T. Kinoshita & collaborators ]

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- $a_\tau$: there is a SM prediction but what about measurements?

  [ S. Eidelman & M. Passera '07 ]

# Measurement of $a_\tau$

The limits on $a_\tau$ quoted by the PDG are:

$$-0.052 \leq a_\tau \leq 0.013$$

[ DELPHI Collaboration '04 ]

This is not even a test of the leading contribution!

$$a_\tau \;=\; \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2) \approx 0.00116$$

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*Is it possible to do better?*

Mean lifetime of the $\tau$ is $\sim 10^7$ times smaller than for the $\mu$.
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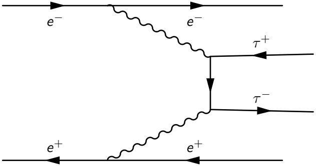
DELPHI Collaboration: $e^+e^- \to e^+e^-\tau^+\tau^-$




$$\sigma_{\text{exp}} = \sigma_{\text{tree}} + \sigma_a$$

[ F. Cornet & J. Illana '96 ]

$e^+e^- \to \tau^+\tau^-$ analysis of LEP1/2 & SLD data using effective operators:

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[ G. A. González-Sprinberg et al. '00 ]

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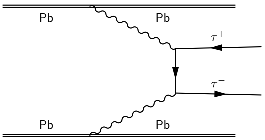
LHC: Pb Pb $\to$ Pb Pb $\tau^+\tau^-$



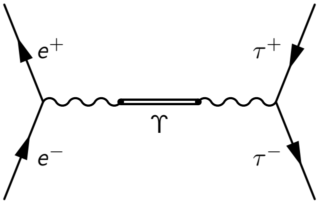

Advantage:
Pb Pb $\gamma\gamma \approx$ on-shell

$\Rightarrow\ q^2 \approx 0$

Expected sensitivity is $\sim\ 10^{-5}$

[ F. del Aguila et al. '91 ]

Where are $\tau$ mostly produced? $\Longrightarrow$ B-factories!



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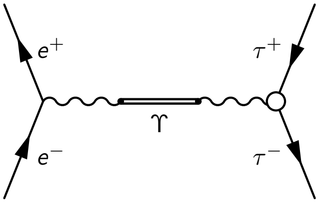


$\Longrightarrow$ bounds on $F_2(q^2 = M_\Upsilon^2)$

Remember: $a_\tau \doteq F_2(q^2 = 0)$

Expected sensitivity for $F_2(q^2 = M_\Upsilon^2)$ at Super B: $\sim \; 10^{-6}$

[ J. Bernabéu et al. '08 ]

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- probe $a_\tau$ at $\mathcal{O}(10^{-3})$ with minimal assumptions.
- provide an alternative determination of $a_\tau$.

No search for BSM physics!

Discrepancy of $a_\mu$ scaled as $\frac{m_\tau^2}{m_\mu^2}$: $\sim 10^{-6}$ needed.

Effective Lagrangian:

$$\mathcal{L}_{eff} = \mathcal{L}_{\text{QED}} + \mathcal{L}_{\text{Fermi}} + e\frac{\tilde{a}}{4\Lambda}\bar{\tau}\sigma_{\mu\nu}\tau F^{\mu\nu}$$

$$\Rightarrow a_\tau = a_\tau^{\text{QED}} + \tilde{a}\frac{m_\tau}{\Lambda}$$

From $\mathcal{L}_{eff}$ we calculate the decay rate of the tau:

$$d\Gamma(\tau^- \to \ell^- \nu_\tau \bar{\nu}_\ell \gamma) = d\Gamma_{\text{QED}} + \frac{\tilde{a}}{\Lambda} d\Gamma_a$$

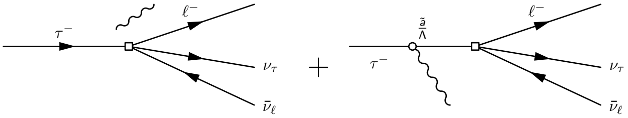
To probe $a_\tau$ at $\mathcal{O}(10^{-3})$, we need $d\Gamma_{\text{QED}}$ at $\mathcal{O}(\alpha^2)$.

[ A. Fischer et al. '94 ]

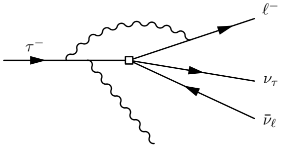
[ A. B. Arbuzov & E.S. Scherbakova '04 ]

Nevertheless, we are do the calculation again (in progress).

Tree-level:




One-loop:




$+ \ \ldots$

Additionally: double Bremsstrahlung.

What are the problems we encounter?

Phase space integration kills $d\Gamma_a$:

$d\Gamma_{\text{QED}}$ has phase space singularities ($E_\gamma$ and collinear photon)

$$\Gamma(\tau^- \to \ell^- \nu_\tau \bar{\nu}_\ell \gamma) = \int d\Pi \, d\Gamma_{\text{QED}} + \frac{\tilde{a}}{\Lambda} \int d\Pi \, d\Gamma_a$$

$\Rightarrow$ a full phase space analysis is needed!

# Summary and outlook

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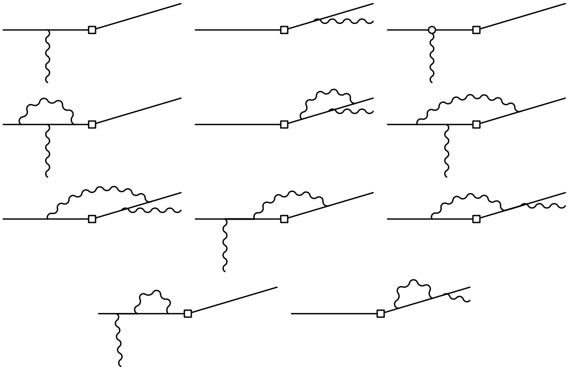
- A careful theoretical & experimental analysis has to be done to probe $a_\tau$ at $\mathcal{O}(10^{-3})$ (in progress).
- Outlook: polarizations, electric dipole moments, maybe even NP ....

# Diagrams to calculate



# PDG

**$\tau$**

$$J = \frac{1}{2}$$

Mass $m = 1776.82 \pm 0.16$ MeV
$(m_{\tau^+} - m_{\tau^-})/m_{\text{average}} < 2.8 \times 10^{-4}$, CL = 90%
Mean life $\tau = (290.6 \pm 1.0) \times 10^{-15}$ s
$c\tau = 87.11\ \mu$m
Magnetic moment anomaly $> -0.052$ and $< 0.013$, CL = 95%
$\text{Re}(d_\tau) = -0.22$ to $0.45 \times 10^{-16}$ $e$ cm, CL = 95%
$\text{Im}(d_\tau) = -0.25$ to $0.008 \times 10^{-16}$ $e$ cm, CL = 95%

[ K. Nakamura et al. (PDG) '10 ]

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## $\Gamma(\mu^-\overline{\nu}_\mu\nu_\tau\gamma)/\Gamma_{total}$ $\Gamma_4/\Gamma$

| VALUE (%) | EVTS | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|---|
| **0.361±0.016±0.035** | | [1] BERGFELD | 00 | CLEO | $E^{ee}_{cm}$= 10.6 GeV |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| 0.30 ±0.04 ±0.05 | 116 | [2] ALEXANDER | 96S | OPAL | 1991–1994 LEP runs |
| 0.23 ±0.10 | 10 | [3] WU | 90 | MRK2 | $E^{ee}_{cm}$= 29 GeV |

[1] BERGFELD 00 impose requirements on detected $\gamma$'s corresponding to a $\tau$-rest-frame energy cutoff $E^*_\gamma > 10$ MeV. For $E^*_\gamma > 20$ MeV, they quote $(3.04\pm0.14\pm0.30)\times10^{-3}$.

[2] ALEXANDER 96S impose requirements on detected $\gamma$'s corresponding to a $\tau$-rest-frame energy cutoff $E_\gamma$ >20 MeV.

[3] WU 90 reports $\Gamma(\mu^-\overline{\nu}_\mu\nu_\tau\gamma)/\Gamma(\mu^-\overline{\nu}_\mu\nu_\tau) = 0.013 \pm 0.006$, which is converted to $\Gamma(\mu^-\overline{\nu}_\mu\nu_\tau\gamma)/\Gamma_{total}$ using $\Gamma(\mu^-\overline{\nu}_\mu\nu_\tau\gamma)/\Gamma_{total} = 17.35\%$. Requirements on detected $\gamma$'s correspond to a $\tau$ rest frame energy cutoff $E_\gamma > 37$ MeV.

## $\Gamma(e^-\overline{\nu}_e\nu_\tau\gamma)/\Gamma_{total}$ $\Gamma_6/\Gamma$

| VALUE (%) | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|
| **1.75±0.06±0.17** | [1] BERGFELD | 00 | CLEO | $E^{ee}_{cm}$= 10.6 GeV |

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[ K. Nakamura et al. (PDG) '10 ]