Understanding quarkonium polarization

- Motivation of quarkonium polarization studies
- General considerations on the study of di-fermion decays of J = 1 states
  - The role of the choice of the reference frame
  - The interplay between production and decay kinematics
  - A frame-invariant formalism for polarization measurements
- "Messages" for polarization analyses (and calculations)

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CERN, May 3rd, 2010

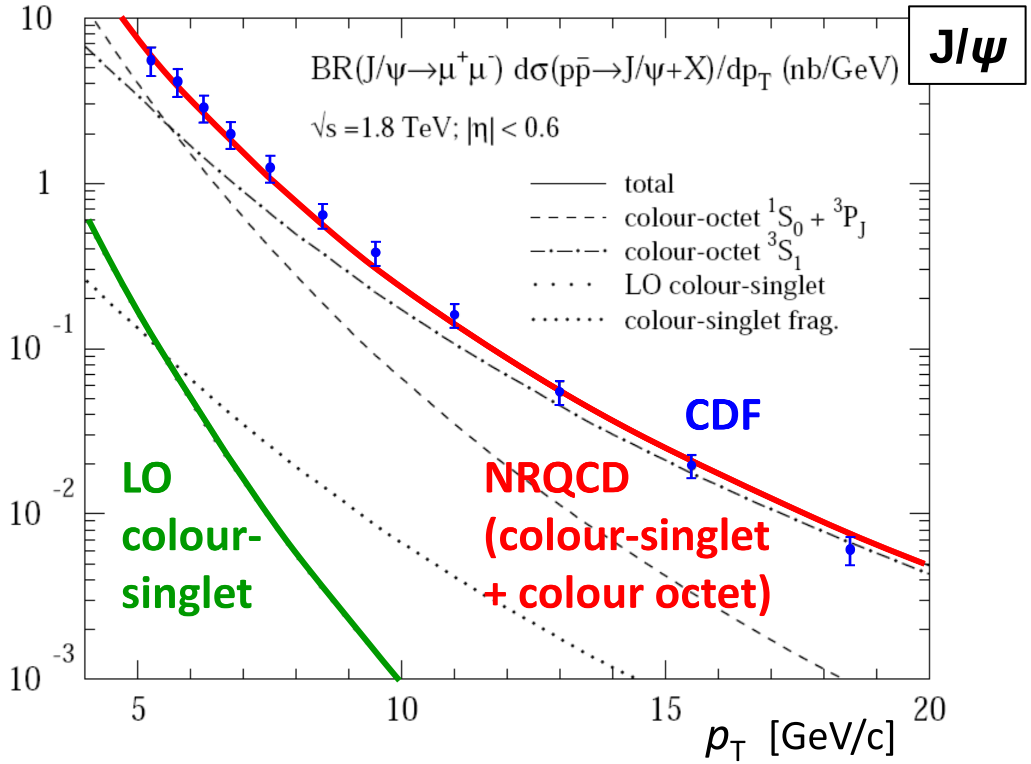
Importance of polarization for quarkonium studies

Experimental studies of the decay distributions of vector particles provide a uniquely detailed way of testing fundamental theories

In particular, quarkonium polarization measurements are expected to provide key information for the understanding of QCD

"Quarkonium represents for QCD what positronium is in QED"
But how well do we know the mechanisms of its production?

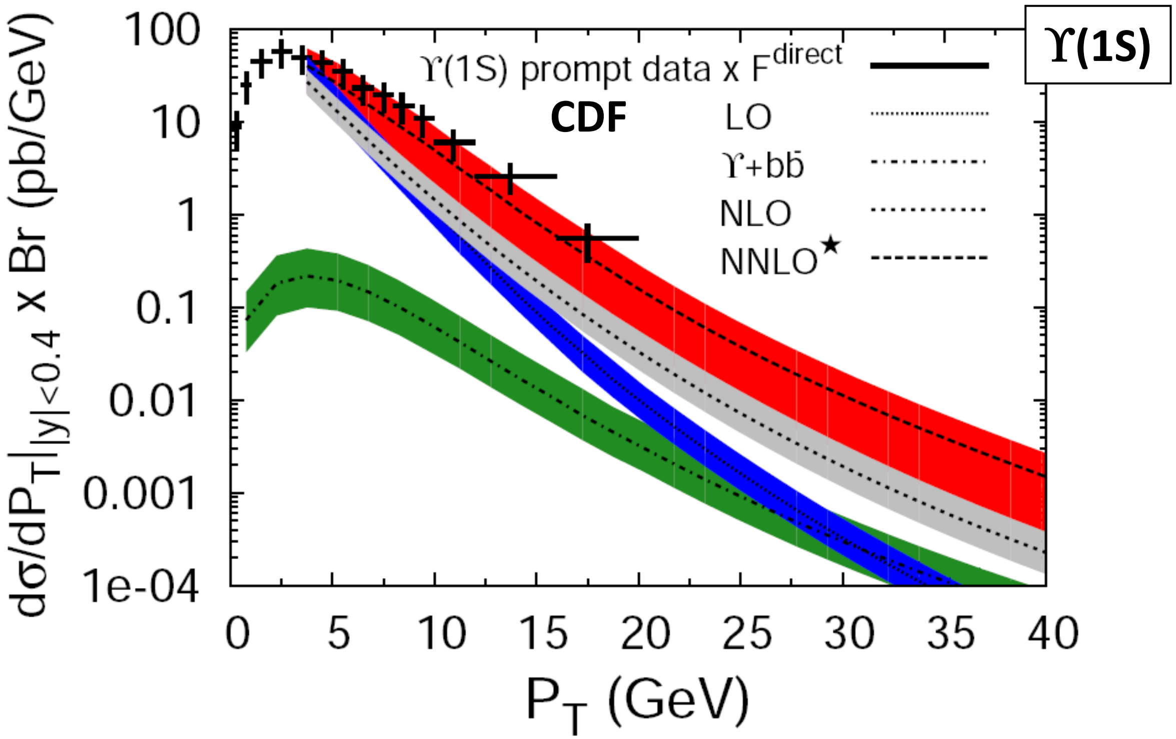
The seeming success of NRQCD



In 1995, **CDF** observed J/ψ and ψ' direct production cross sections ~50 times larger than existing calculations based on **leading-order colour-singlet production**

The **NRQCD framework** (where quarkonia are also produced as coloured quark pairs) apparently solved the problem… by freely *adjusting* long distance colour-octet matrix elements to describe the measurements

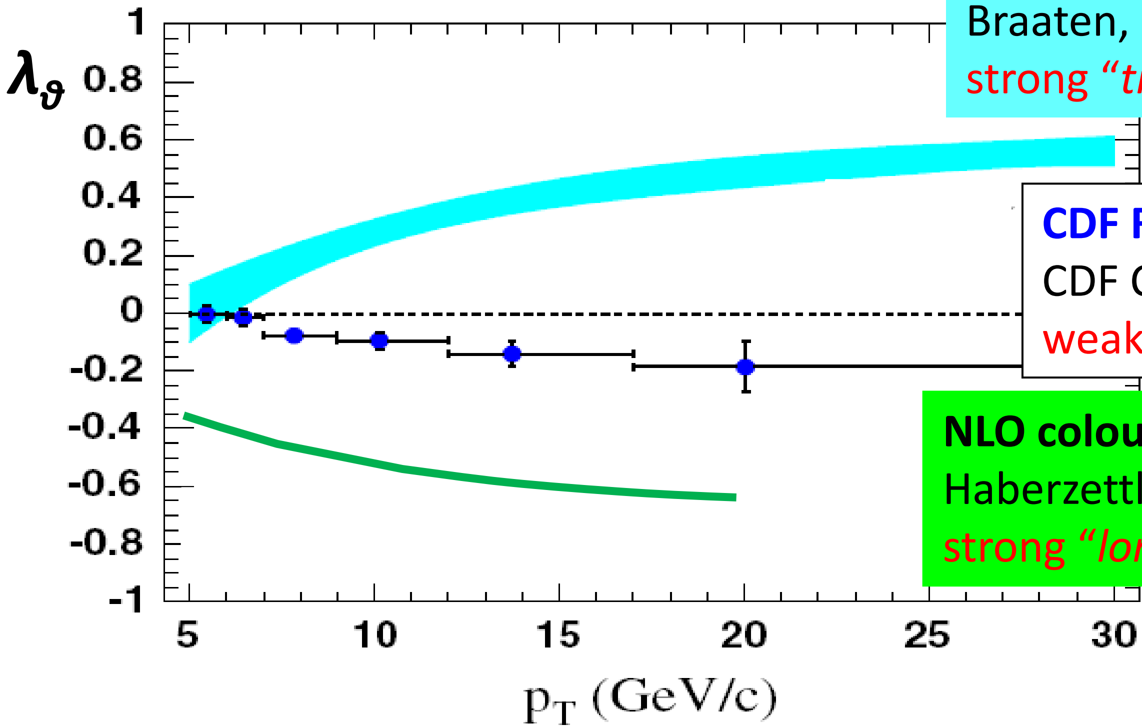
A rebirth of the colour-singlet model?



Recently calculated high-order corrections to the colour-singlet mechanism give flatter p_T spectra, leading to much larger high-p_T rates. The Tevatron data no longer require a large colour-octet component, especially in the Υ case…

→ Differential cross sections are insufficient information to ensure progress in our understanding of quarkonium production

The crucial test: polarization measurements



NRQCD factorization: prompt J/ψ
Braaten, Kniehl, Lee, PRD62, 094005 (2000)
strong "*transverse*" polarization

CDF Run II data: prompt J/ψ @1.96TeV
CDF Coll., PRL 99, 132001 (2007)
weak "*longitudinal*" polarization

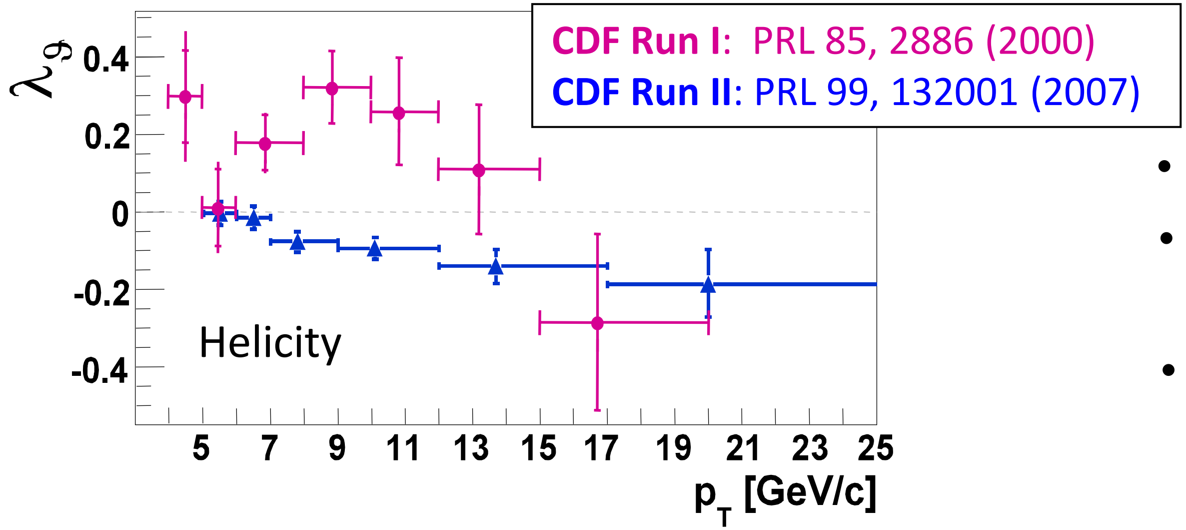
NLO colour-singlet: *direct* J/ψ
Haberzettl, Lansberg, PRL100, 032006 (2008)
strong "*longitudinal*" polarization

$$\frac{dN}{d\cos\vartheta} \propto 1 + \lambda_\vartheta (\cos\vartheta)^2$$

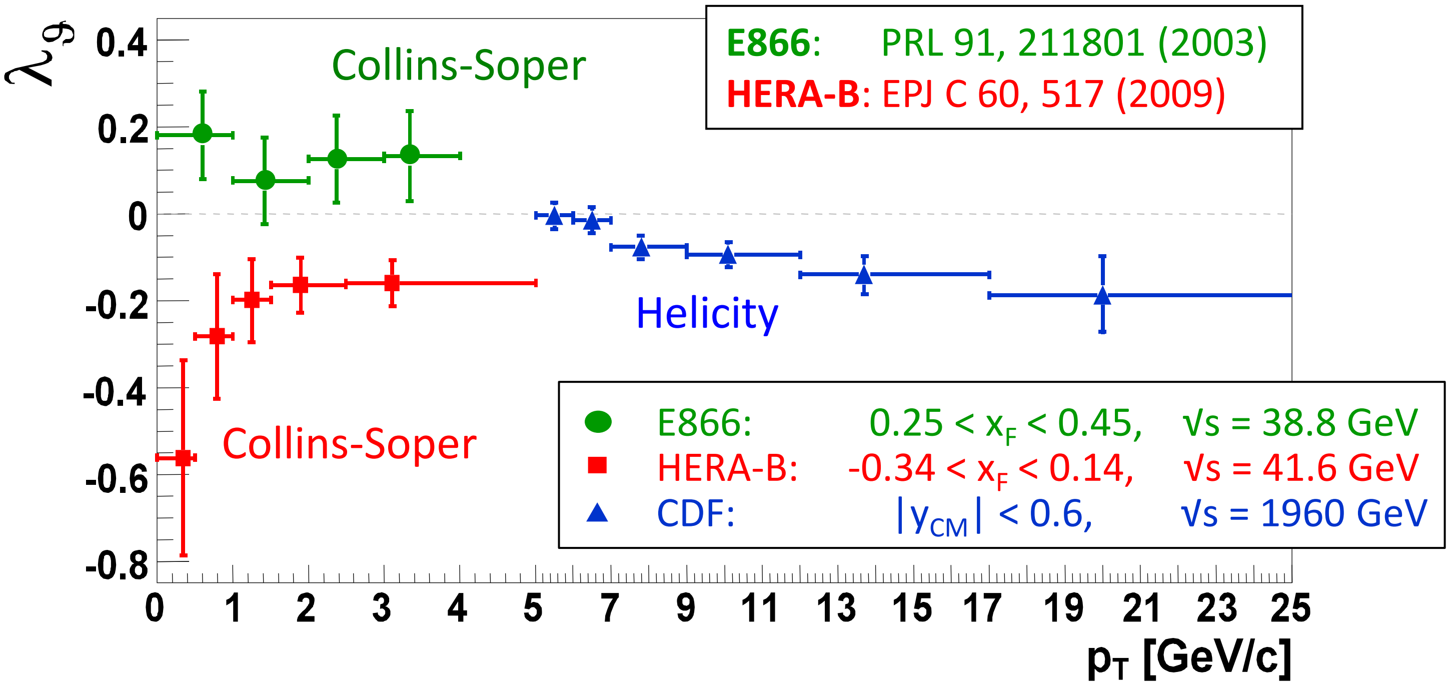
ϑ = angle between lepton direction (in the J/ψ rest frame) and J/ψ lab direction (helicity axis)

$\lambda_\vartheta > 0$: transverse (= photon-like)
$\lambda_\vartheta < 0$: longitudinal

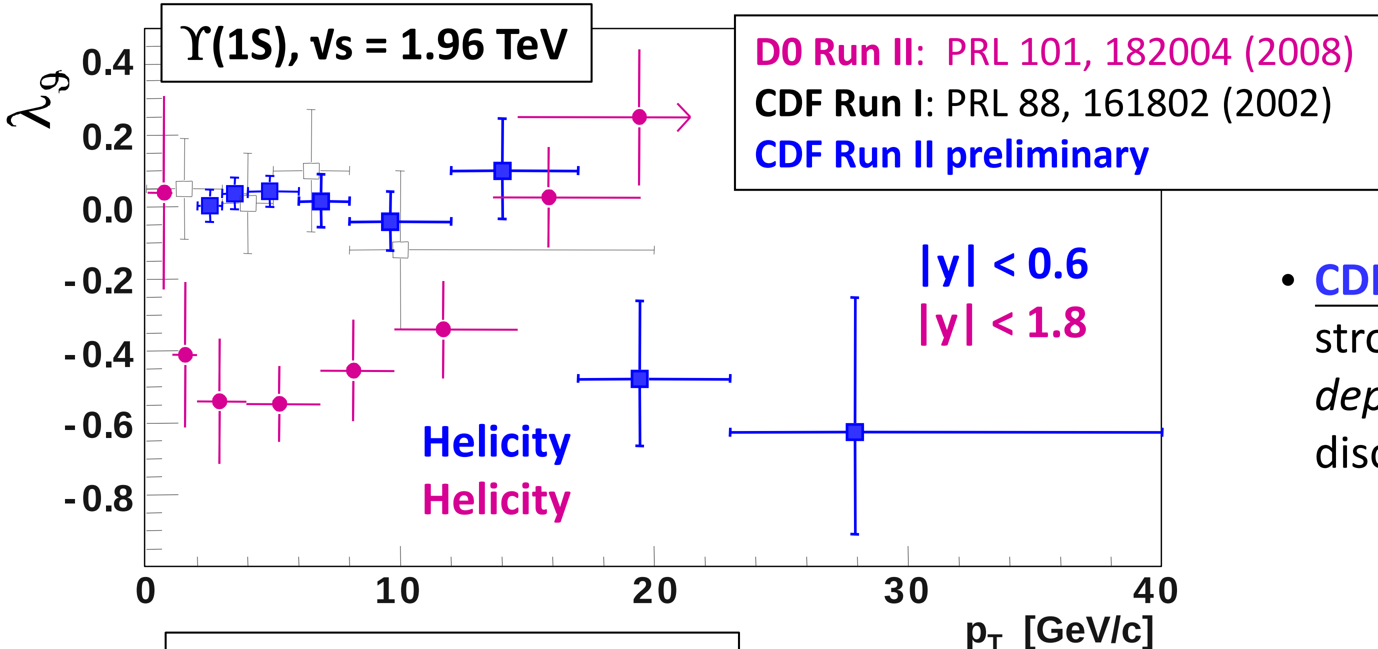
Experimental puzzles: J/ψ



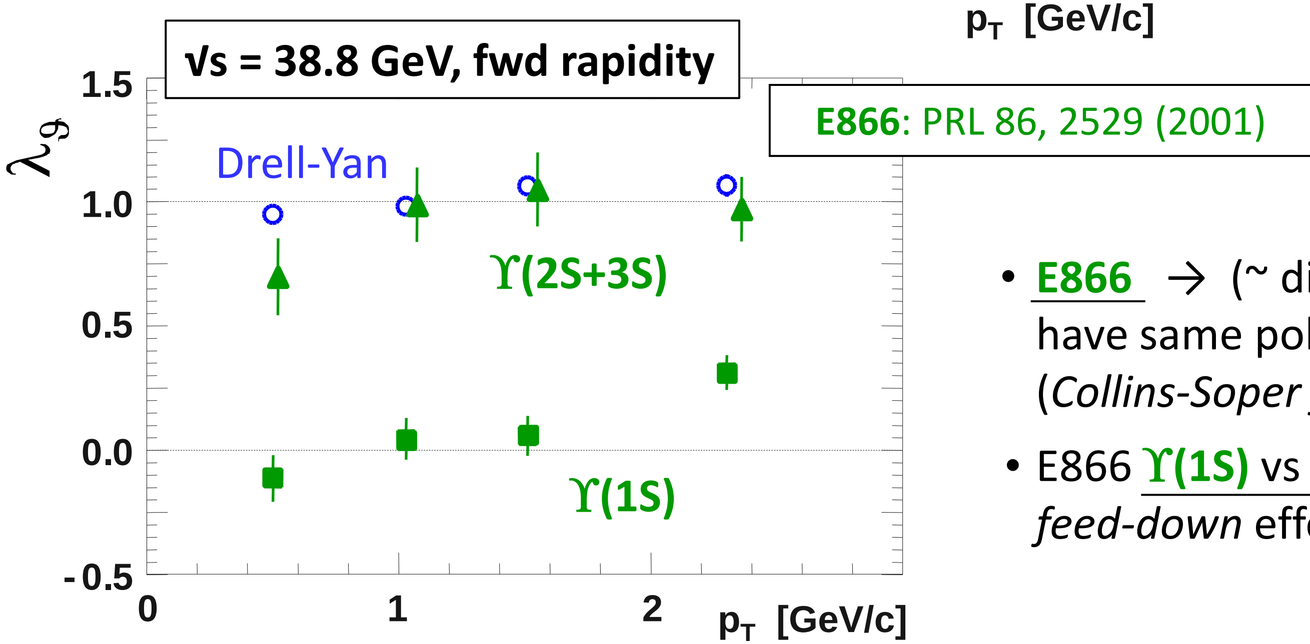
- CDF vs CDF
- E866 vs HERA-B → there must be a *strong* p_L *dependence*
- CDF vs low-p_T → how do the different *frame conventions* affect the comparison?



Experimental puzzles: ϒ



- **CDF** vs **D0** → may a strong *rapidity dependence* justify the discrepancy?



- **E866** → (~ directly produced) ϒ(2S+3S) have same polarization as Drell-Yan (*Collins-Soper frame*!)
- E866 **ϒ(1S)** vs **ϒ(2S+3S)** → dominant *feed-down* effects for ϒ(1S)?

Back to basics

- Production models involve complex calculations
- Measurements are experimentally challenging (need high statistics and perfect control of geometric/kinematic acceptance limitations).
- Today, theory is unsuccessful, while the experimental scenario is contradictory.

We believe that better experimental and theoretical results can be achieved by going back to the fundamentals.

What is a polarization measurement?

Given a chosen quantization axis *z*, a $\boldsymbol{J}$ **= 1** particle can be produced in one of three possible $\boldsymbol{J_z}$ **eigenstates (-1, 0, +1)** or in a certain mixture of the three

We **measure the (average) angular momentum state** in which the particle is produced **by studying its decay distribution**

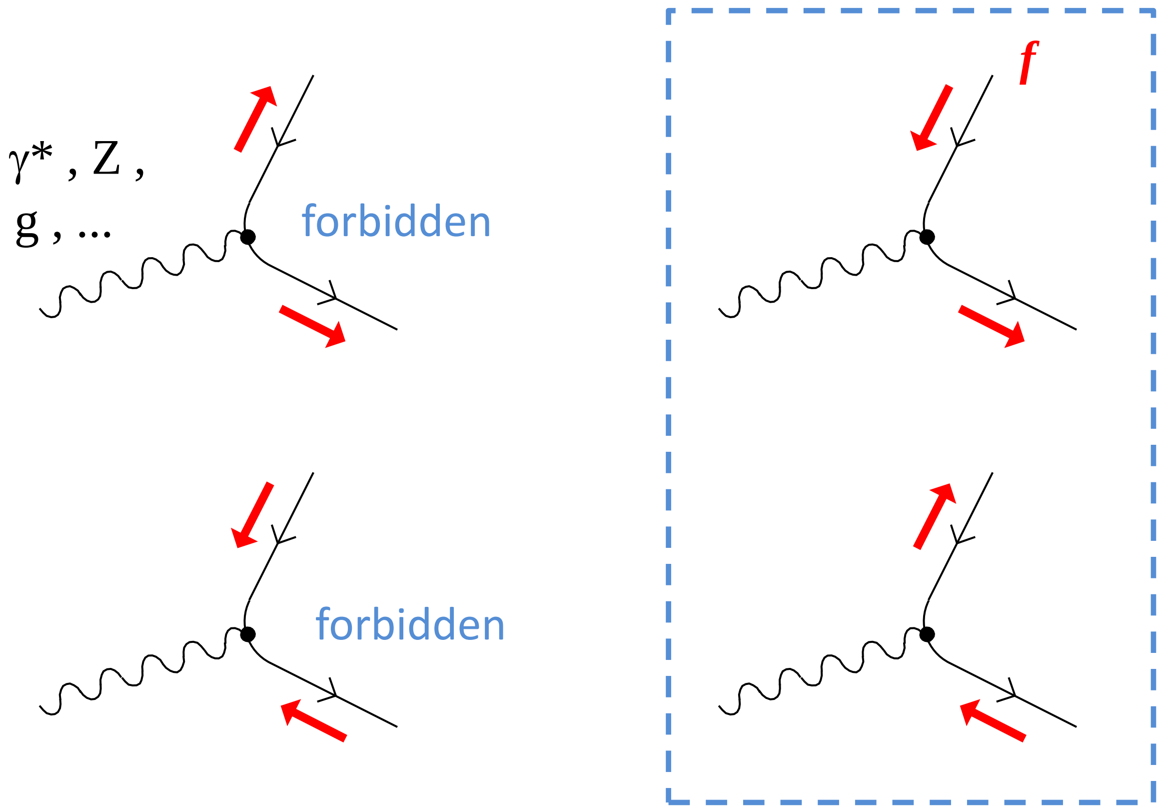
The decay **into fermion-antifermion pair** is an especially clean case to be studied

The shape of the observable angular distribution is determined by few basic principles:

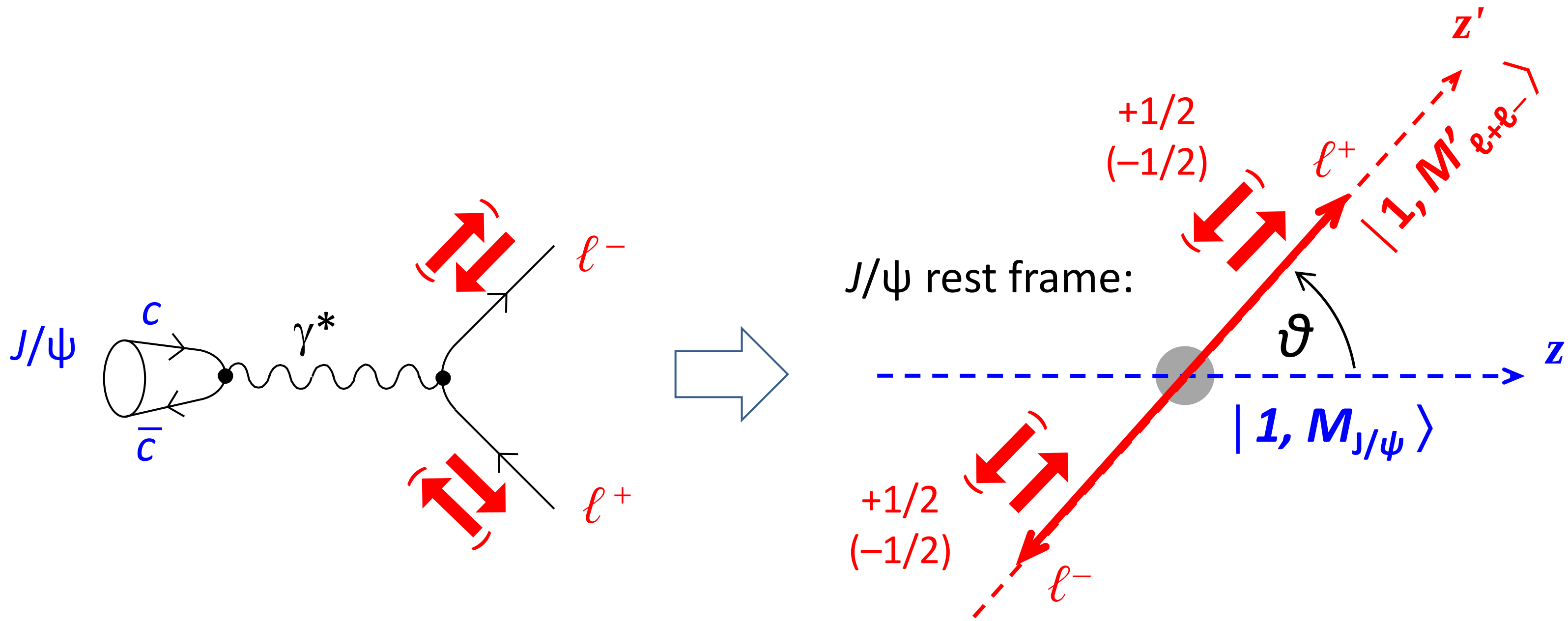
1) **"helicity conservation"** → fixes relative spin orientations of the two decay fermions
2) **rotational covariance** of angular momentum eigenstates
3) **parity conservation** (when relevant)

1) helicity conservation

EW and strong forces preserve the *chirality* (L/R) of fermions.
In the relativistic (massless) limit, *chirality* = *helicity* = spin-momentum alignment
→ the **fermion spin** never flips in the coupling to gauge bosons:



Example: leptonic decay of J/ψ



Whatever the J/ψ angular momentum component

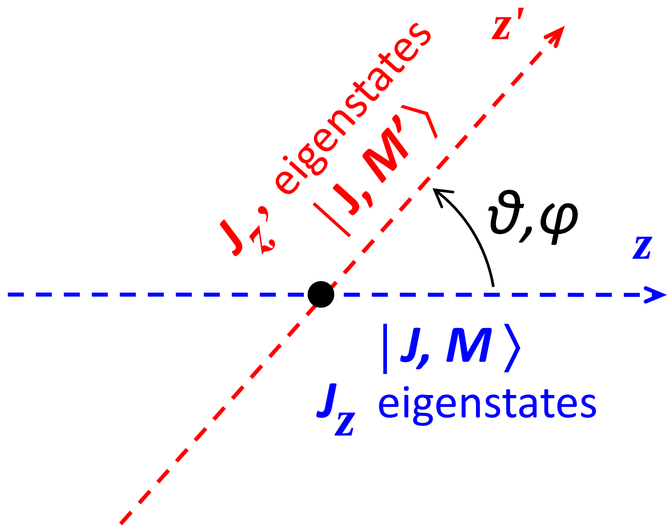
$M_{J/\psi}$ = -1, 0, +1 along the polarization axis z,

the **two leptons** can only have total angular momentum component

$M'_{\ell^+\ell^-}$ = -1 or +1 along their common direction z'

0 forbidden

2) rotation of angular momentum eigenstates



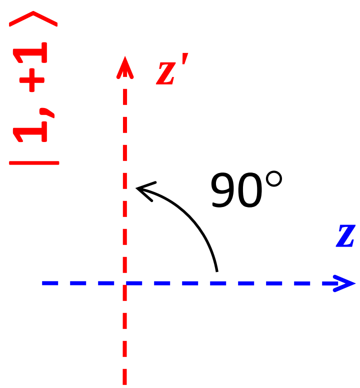
change of quantization frame:

$R(\vartheta,\varphi)$: $z \rightarrow z'$
$y \rightarrow y'$
$x \rightarrow x'$

$$|\,J, M'\rangle = \sum_{M=-J}^{+J} D^{J}_{MM'}(\vartheta,\varphi)\ |\,J, M\,\rangle$$

Wigner D-matrices
(in angular momentum textbooks)

Example:

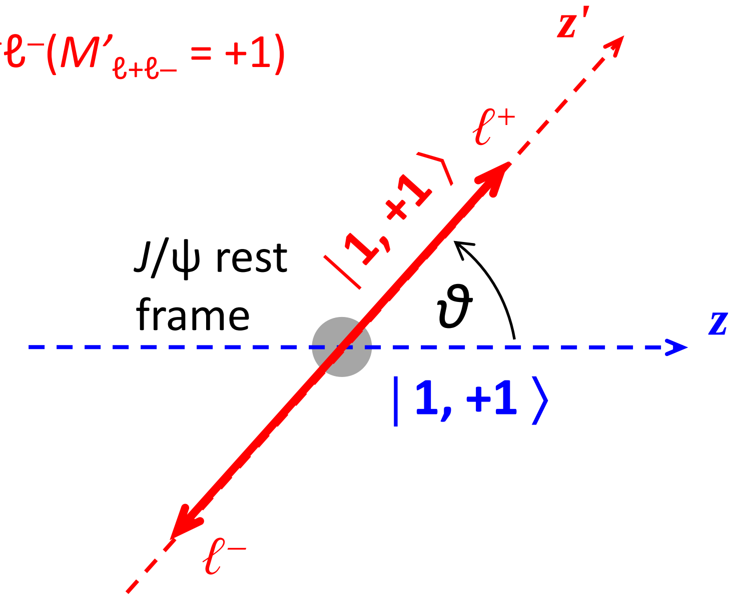


$$\frac{1}{2}|\,1, +1\,\rangle + \frac{1}{2}\,|\,1, -1\,\rangle - \frac{1}{\sqrt{2}}|\,1, 0\,\rangle$$

Classically, we would expect only $|\,1, 0\,\rangle$

Basic angular distribution

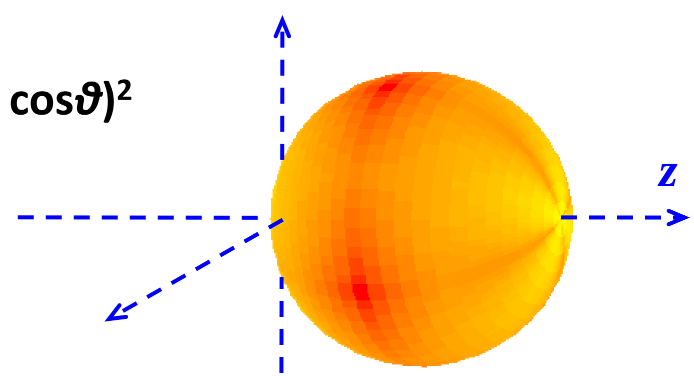
$J/\psi\ (M_{J/\psi} = +1) \rightarrow \ell^+\ell^-(M'_{\ell^+\ell^-} = +1)$



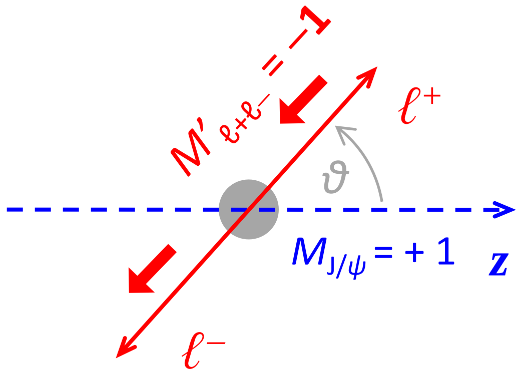
$$|1,+1\rangle = D^1_{-1,+1}(\vartheta,\varphi)\ |1,-1\rangle + D^1_{0,+1}(\vartheta,\varphi)\ |1,0\rangle + D^1_{+1,+1}(\vartheta,\varphi)\ |1,+1\rangle$$

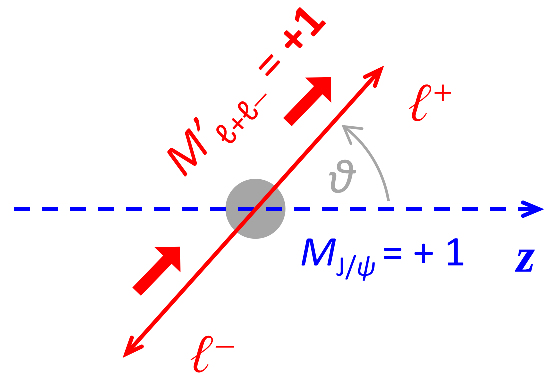
→ the $J_{z'}$ eigenstate $|1,+1\rangle$ "contains" the J_z eigenstate $|1,+1\rangle$ with component amplitude $D^1_{+1,+1}(\vartheta,\varphi)$

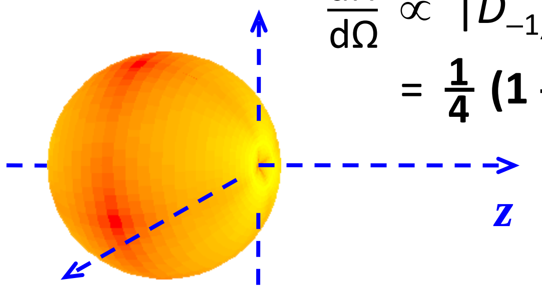
→ the decay distribution is $\propto |D_{+1,+1}(\vartheta,\varphi)|^2 = \frac{1}{4}(1 + \cos\vartheta)^2$



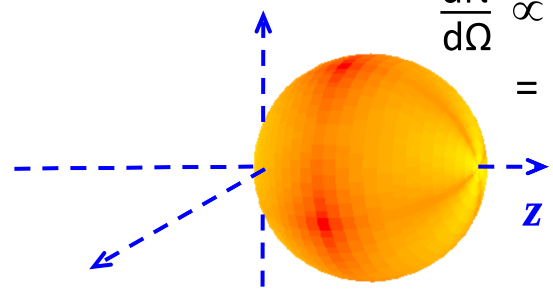
3) parity conservation





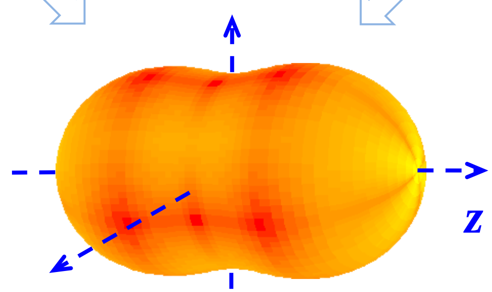


$$\frac{dN}{d\Omega} \propto |D_{-1,+1}(\vartheta,\varphi)|^2 = \frac{1}{4}(1-\cos\vartheta)^2$$



$$\frac{dN}{d\Omega} \propto |D_{+1,+1}(\vartheta,\varphi)|^2 = \frac{1}{4}(1+\cos\vartheta)^2$$

The two processes have identical probabilities



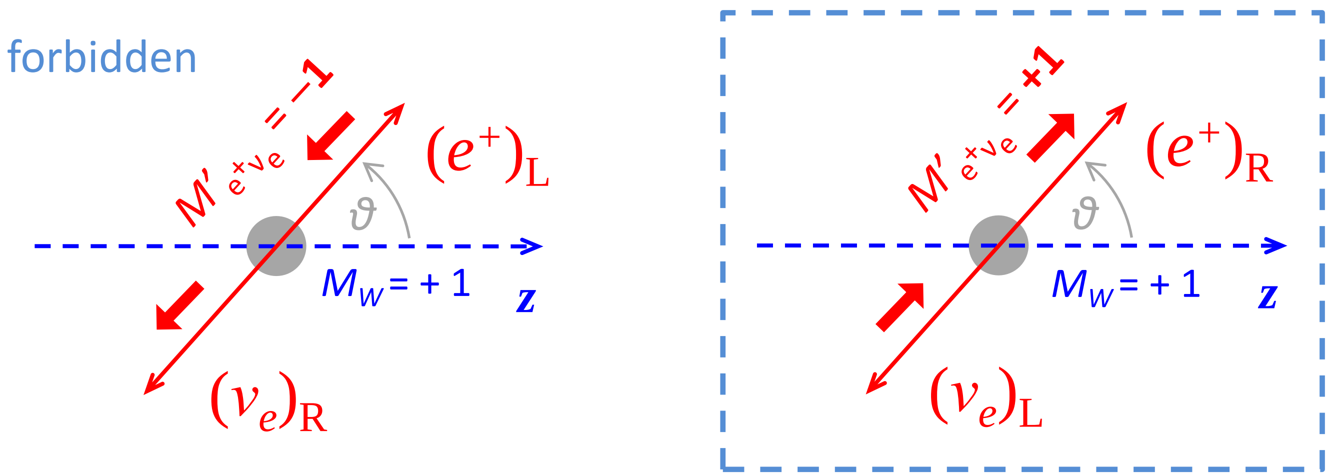
$$\frac{dN}{d\Omega} \propto |D_{-1,+1}(\vartheta,\varphi)|^2 + |D_{+1,+1}(\vartheta,\varphi)|^2$$

$$\propto 1 + \cos^2\vartheta$$

3^{bis}) parity violation

Example: *W*-boson decay **$W \rightarrow e^+\nu_e$**

W only couples to *left-handed fermions* (and *right-handed antifermions*):

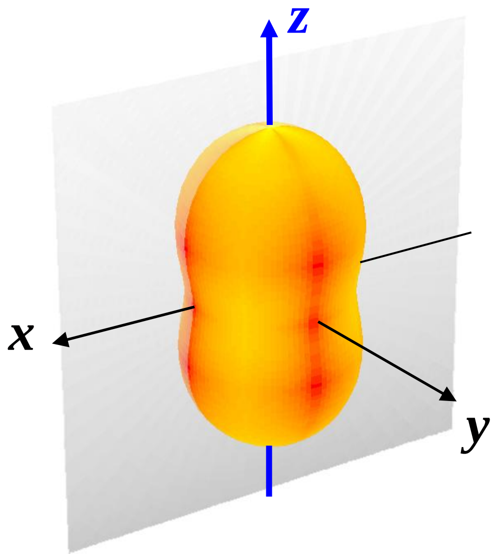


$$\rightarrow \quad \frac{dN}{d\Omega} \propto (1 - \cos\vartheta)^2$$

$$= \boxed{1 + \cos^2\vartheta - \mathbf{2\cos\vartheta}}$$

parity-violating term
("forward-backward" asymmetry)

"Transverse" and "longitudinal"

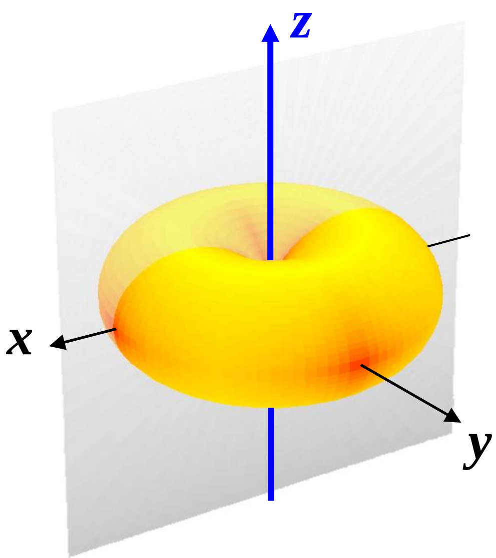


$|\,J/\psi\,\rangle = |\,1, \mathbf{+1}\,\rangle$

or $|\,1, \mathbf{-1}\,\rangle$

$$\frac{dN}{d\Omega} \propto 1 + \cos^2\vartheta$$

"Transverse" polarization, like for *real photons*.
The word refers to the alignment of the *field* vector, not to the *spin* alignment!



$|\,J/\psi\,\rangle = |\,1, \mathbf{0}\,\rangle$

$$\frac{dN}{d\Omega} \propto 1 - \cos^2\vartheta$$

"Longitudinal" polarization

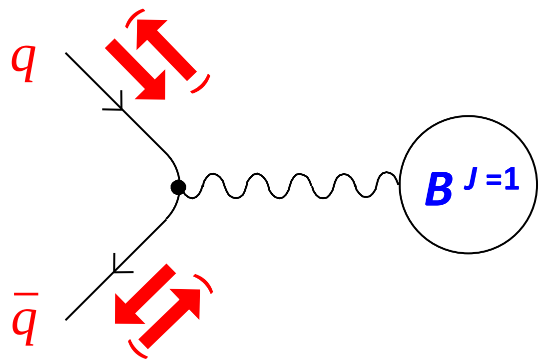
Transverse and longitudinal processes can be mixed together:

$$\frac{dN}{d\Omega} \propto 1 + \boldsymbol{\lambda_\vartheta} \cos^2\vartheta$$

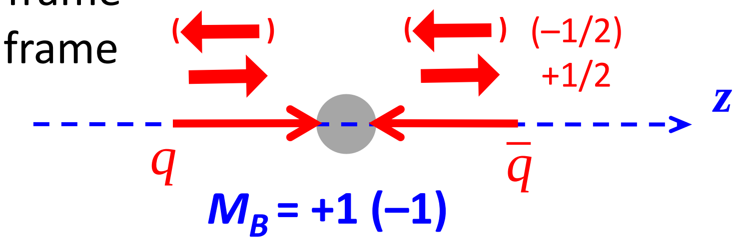
$-1 < \boldsymbol{\lambda_\vartheta} < +1$

Why “photon-like” polarizations are common

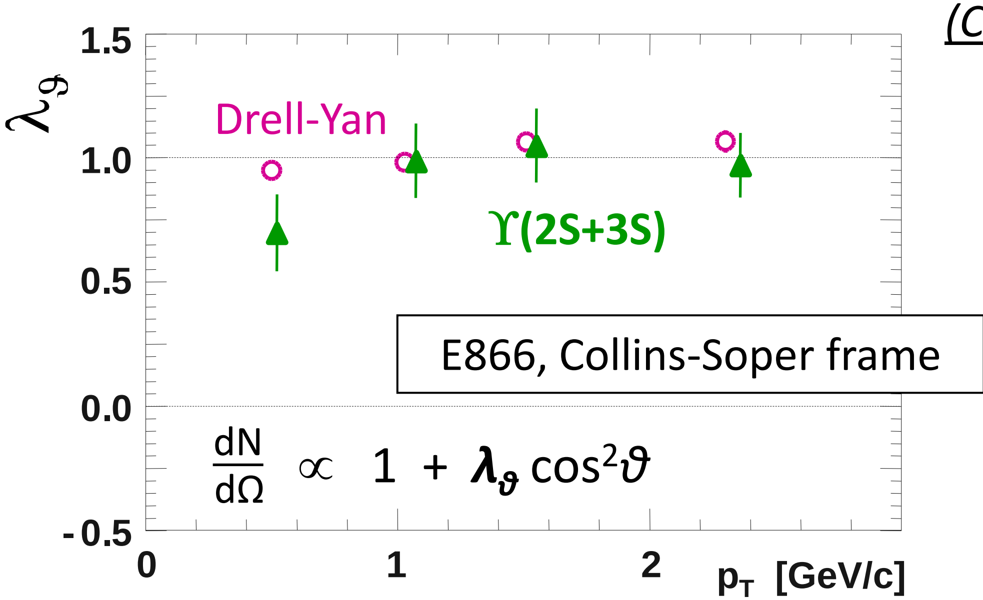
We can apply **helicity conservation at the *production* vertex** to predict that all $J=1$ states produced in *fermion-antifermion annihilations* (q-$\bar{q}$ or e^+e^-) at Born level have *transverse* polarization



q-$\bar{q}$ rest frame
= B rest frame



The “natural” polarization axis in this case is the *relative direction of the colliding fermions (Collins-Soper axis)*

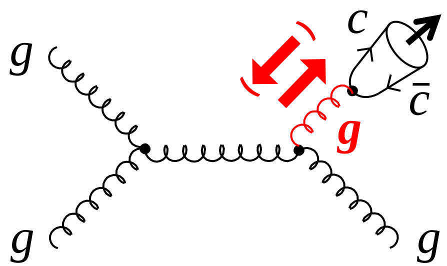


Drell-Yan is a paradigmatic case

But not the only one

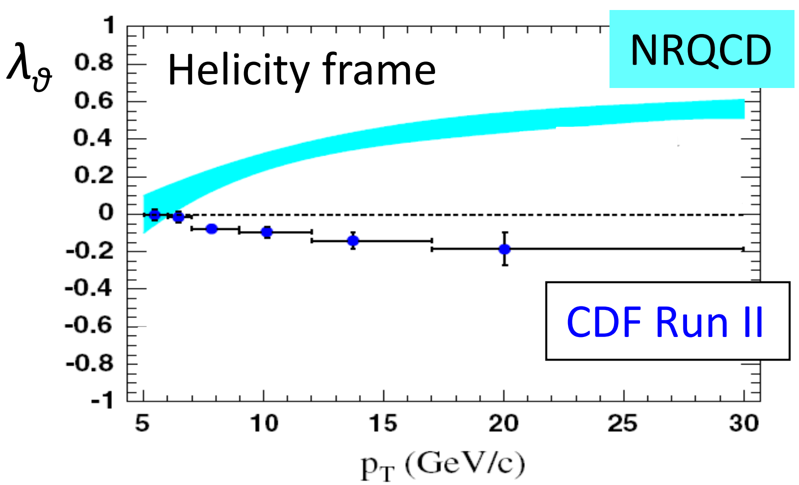
What polarization axis?

We have seen examples of polarizations naturally defined along the direction of the collision (Collins-Soper axis). Today, the high-energy quarkonium community is rather focussing on another axis definition.



NRQCD predicts that, at *very large* p_T, quarkonium should be produced from the fragmentation of a quasi-real gluon, inheriting its natural spin alignment.

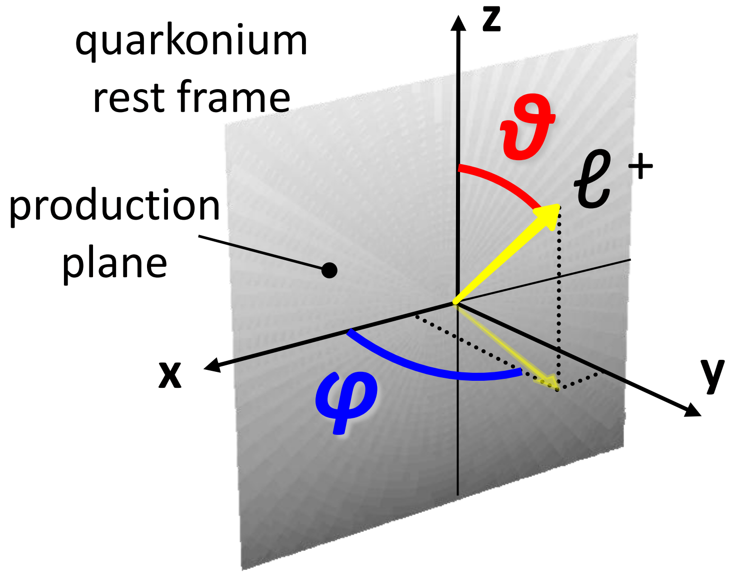
A large, transverse polarization should therefore be observed *along the J/ψ (=gluon) momentum* (*helicity axis*).



Existing high-energy measurements, essentially driven by the NRQCD hypothesis, chose the helicity axis and made no further investigations. In the kinematic regime probed today, the model fails.

What would CDF find, e.g., in the Collins-Soper frame?
How well can the current measurement constrain other (non-NRQCD) hypotheses?

Angles and frames



direction of one of the two decay fermions (e.g.: positive lepton):

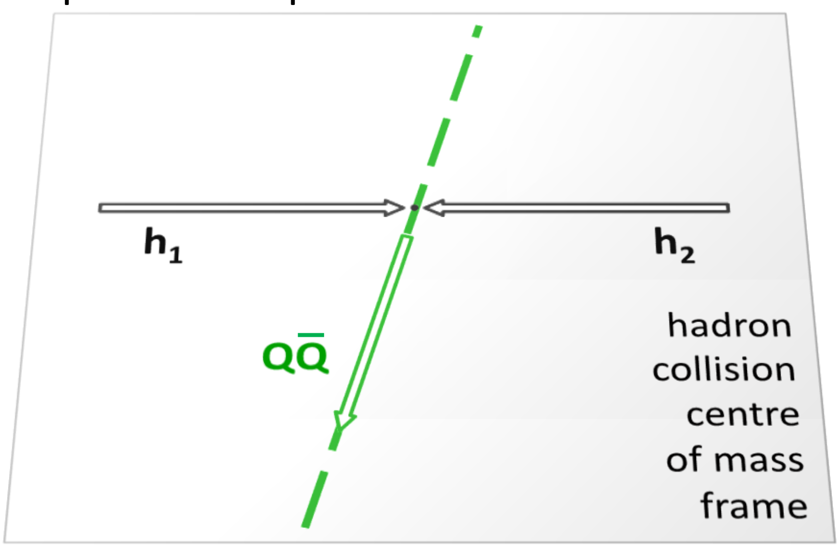
ϑ wrt a chosen polarization axis (*z*)

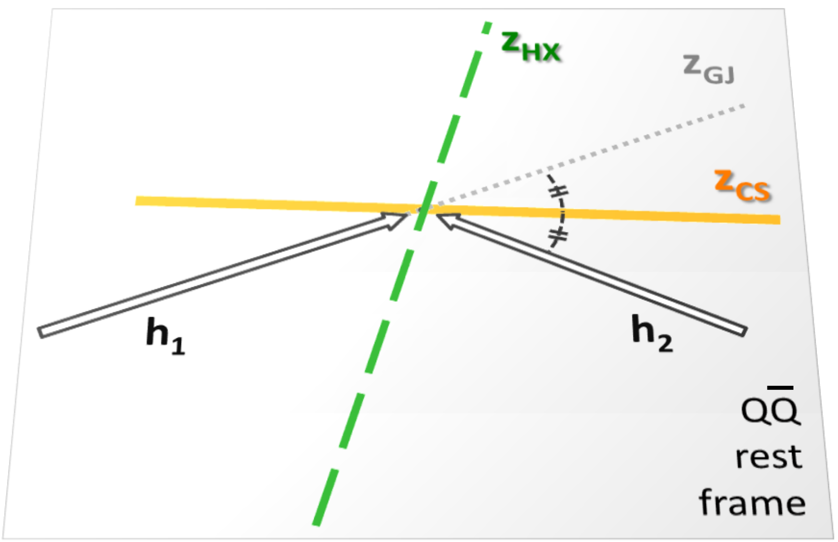
φ wrt the production plane (*xz*)

Inclusive production studies:

- **Helicity axis (HX)** = quarkonium momentum dir.
- **Collins-Soper axis (CS)** = beam line

production plane





The most general distribution

We have considered up to now *pure* angular momentum eigenstates.

The most general $J = 1$ state that can be produced in *one elementary subprocess* can be represented (wrt the chosen z axis) as a superposition of the three J_z eigenstates:

$$|\psi\rangle = a_{-1}\,|1,\mathbf{-1}\rangle + a_0\,|1,\mathbf{0}\rangle + a_{+1}\,|1,\mathbf{+1}\rangle$$

The general angular distribution of its decay into two fermions is:

$$\frac{dN}{d\Omega} \propto 1 + \lambda_\theta \cos^2\theta + \lambda_\varphi \sin^2\theta \cos 2\varphi + \lambda_{\theta\varphi} \sin 2\theta \cos\varphi$$
$$+ \lambda_\varphi^\perp \sin^2\theta \sin 2\varphi + \lambda_{\theta\varphi}^\perp \sin 2\theta \sin\varphi$$
$$+ 2A_\theta \cos\theta + 2A_\varphi \sin\theta \cos\varphi + 2A_\varphi^\perp \sin\theta \sin\varphi$$

violate parity

asymmetric by reflection about the production plane

- vanish (in the event average) in the parity-conserving case
- small in the parity-violating case

"Unpolarized" J/ψ does not exist

For a ***single elementary subprocess***, for simplicity in the parity-conserving case:

$$\frac{dN}{d\Omega} \propto 1 + \lambda_\theta \cos^2\theta + \lambda_\varphi \sin^2\theta \cos 2\varphi + \lambda_{\theta\varphi} \sin 2\theta \cos\varphi + \ldots$$

$$\lambda_\theta = \frac{1 - 3|a_0|^2}{1 + |a_0|^2} \qquad \lambda_\varphi = \frac{2\,\mathrm{Re}\, a_{+1}^* a_{-1}}{1 + |a_0|^2} \qquad \lambda_{\theta\varphi} = \frac{\sqrt{2}\,\mathrm{Re}[a_0^*(a_{+1} - a_{-1})]}{1 + |a_0|^2}$$

There is no combination of a_0, a_{+1} and a_{-1} such that $\lambda_\vartheta = \lambda_\varphi = \lambda_{\vartheta\varphi} = 0$

The angular distribution is never intrinsically *isotropic*

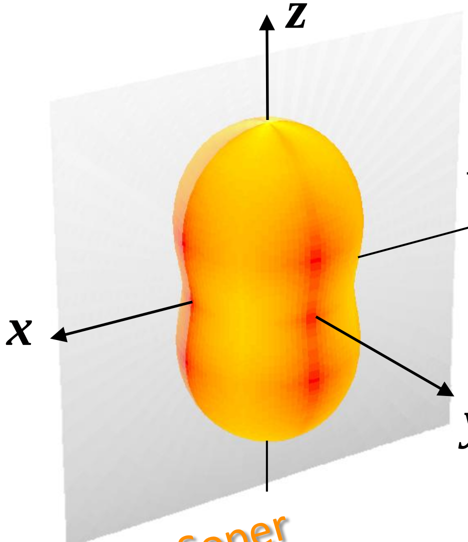
Only a fortunate ***mixture of subprocesses*** (or randomization effects) can lead to a cancellation of all three *measured* anisotropy parameters

→ Polarization is a "necessary" property of $J = 1$ states
Measuring and understanding it is crucial

... also from an "experimental" point of view: quarkonium acceptances depend strongly on the dilepton decay kinematics. Quarkonium is by default unpolarized in MC generators...

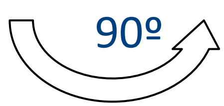
The observed polarization depends on the frame

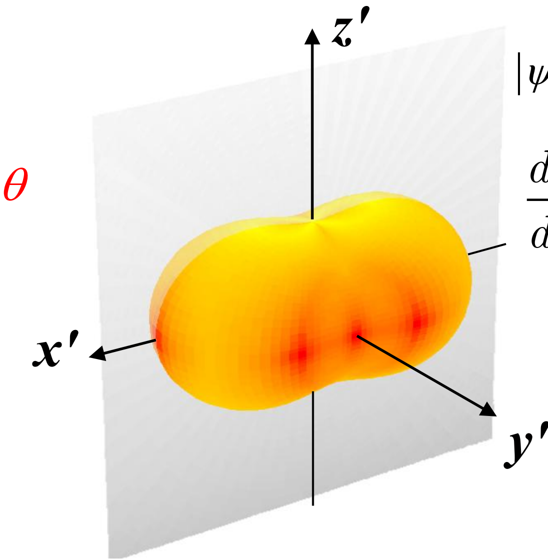
For $|p_L| << p_T$ the CS and HX frames differ by a rotation of 90º



$$|\psi\rangle = |\pm 1\rangle$$

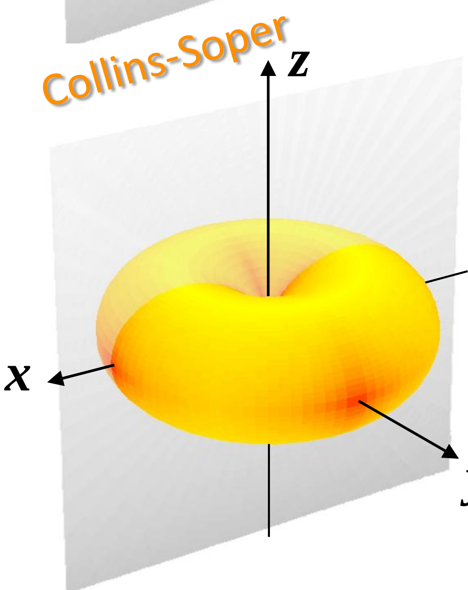
$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta$$





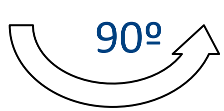
$$|\psi\rangle = \frac{1}{2}|+1\rangle + \frac{1}{2}|-1\rangle \mp \frac{1}{\sqrt{2}}|0\rangle$$

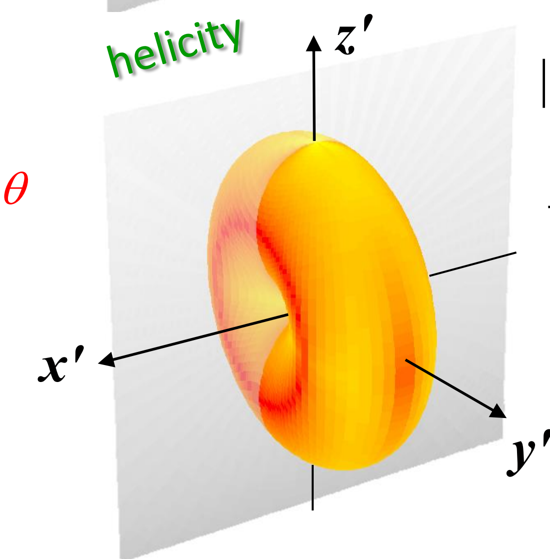
$$\frac{dN}{d\Omega} \propto 1 - \tfrac{1}{3}\cos^2\theta + \tfrac{1}{3}\sin^2\theta\cos 2\varphi$$



$$|\psi\rangle = |0\rangle$$

$$\frac{dN}{d\Omega} \propto 1 - \cos^2\theta$$



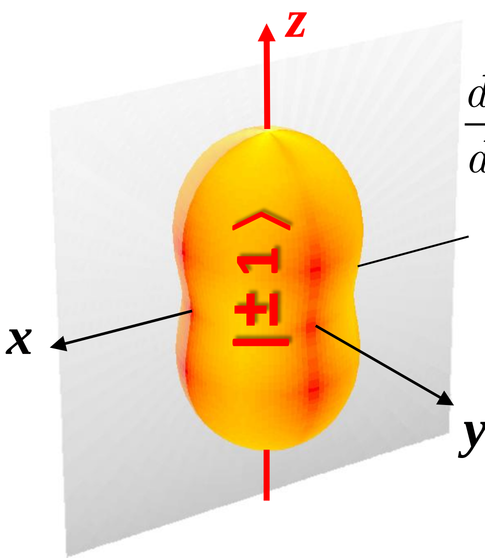


$$|\psi\rangle = \frac{1}{\sqrt{2}}|+1\rangle - \frac{1}{\sqrt{2}}|-1\rangle$$

$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta - \sin^2\theta\cos 2\varphi$$

The azimuthal anisotropy is not a detail

Case 1: natural transverse polarization

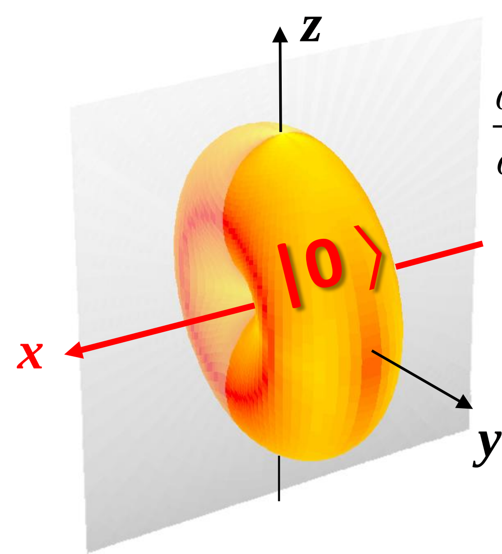


$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta$$

$\lambda_\vartheta = +1$

$\lambda_\varphi = 0$

Case 2: natural longitudinal polarization, observation frame ⊥ to the natural one



$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta - \sin^2\theta \cos 2\varphi$$

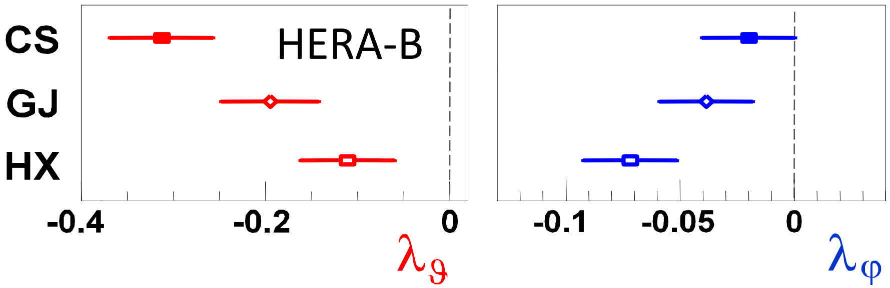
$\lambda_\vartheta = +1$

$\lambda_\varphi = -1$

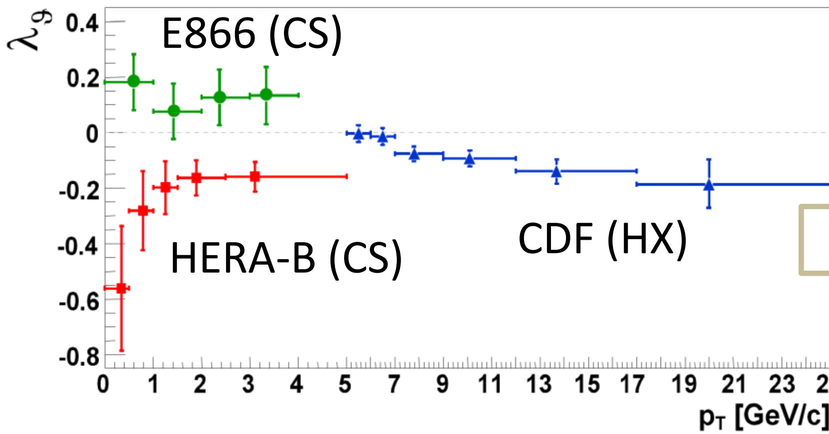
These two decay distributions are indistinguishable when the azimuthal dependence is integrated out. But they correspond to opposite *natural* polarizations, which can only be originated by completely different production mechanisms.

In general, measurements not reporting the azimuthal anisotropy provide an incomplete physical result. Their fundamental interpretation is impossible (relies on arbitrary assumptions).

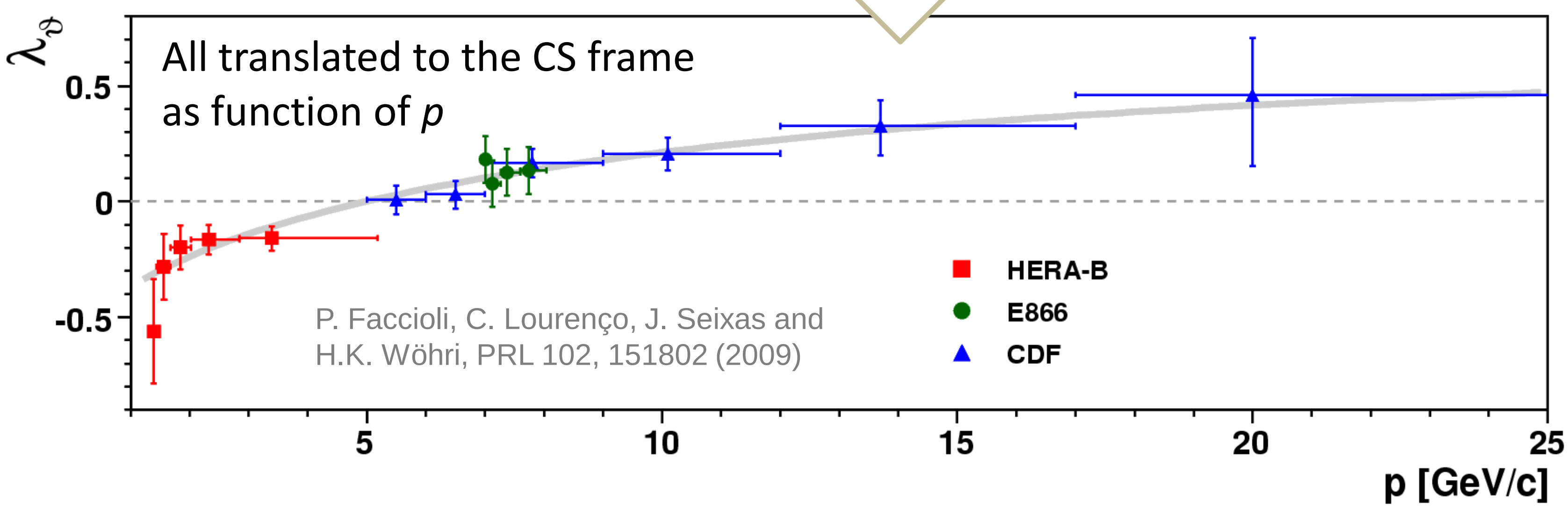
A possible hypothesis about CDF's J/ψ



HERA-B has shown that low-p_T J/ψ's (at fixed-target energies) are **naturally polarized in the Collins-Soper frame** (**most significant λ_ϑ** and **purely polar anisotropy, λ_φ = 0**).



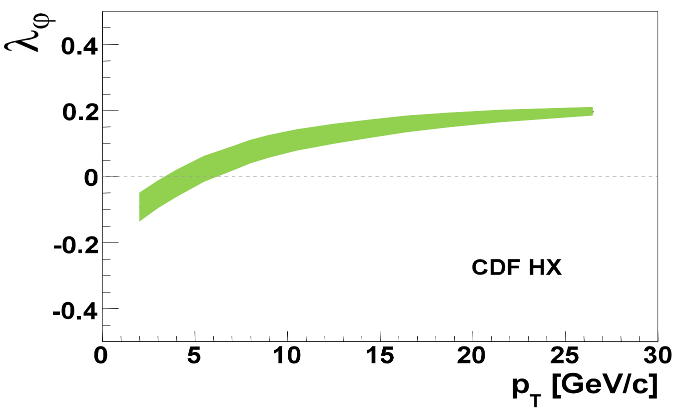
If we assume that this continues to be valid up to collider energies, we can translate the CDF points from the helicity frame to the Collins-Soper frame and recognize a smoothly varying polarization from low to high quarkonium *momentum*.



Message nº1

Today, we are allowed to make the speculation in the previous slide because CDF has not reported the azimuthal anisotropy.

We have assumed that $\lambda_\varphi = 0$ in the CS frame, automatically implying that a significant value of λ_φ should be measured in the HX frame:



By measuring also λ_φ CDF will remove this ambiguity of interpretation.

Measure the *full* angular decay distribution, not only the polar anisotropy.

Reference frames are not all equally good

How the anisotropy parameters transform from one frame to another depends *explicitly* on the production kinematics. In fact, the angle δ between helicity and Collins-Soper axes is given by

$$\cos\delta = \frac{m \, p_L}{m_T \, p}$$

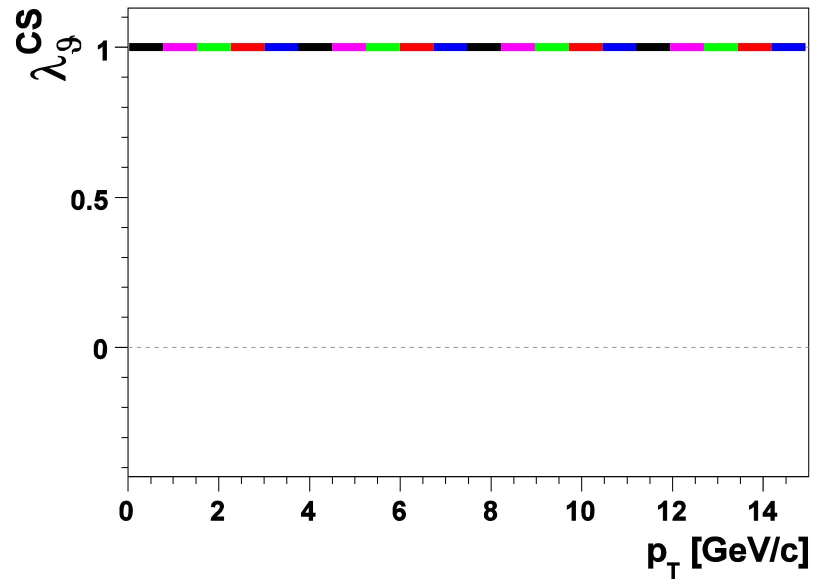
Example: **how would different experiments observe a Drell-Yan-like decay distribution** [“naturally” of the kind **$1 + \cos^2\vartheta$** in the Collins-Soper frame – see e.g. E866’s Υ result] **with an arbitrary choice of the reference frame?**

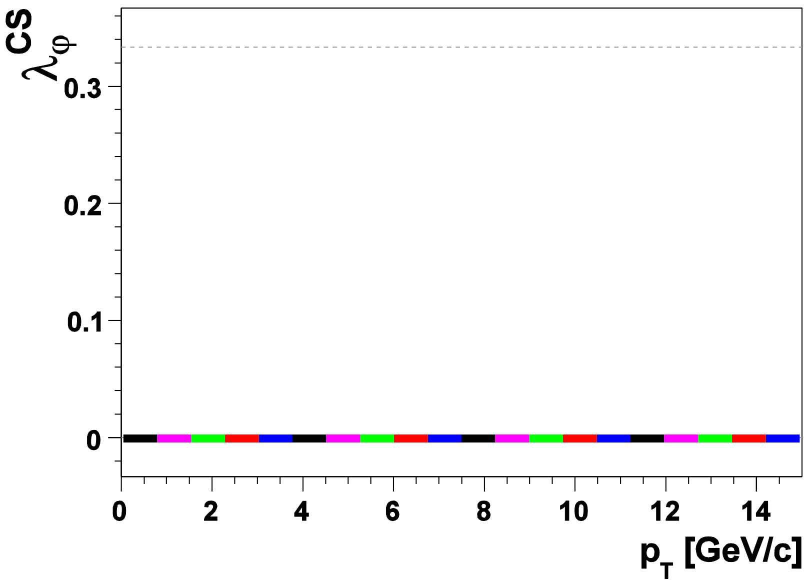
We consider Υ decay. For simplicity of illustration we assume that each experiment has a flat acceptance in its nominal rapidity range:

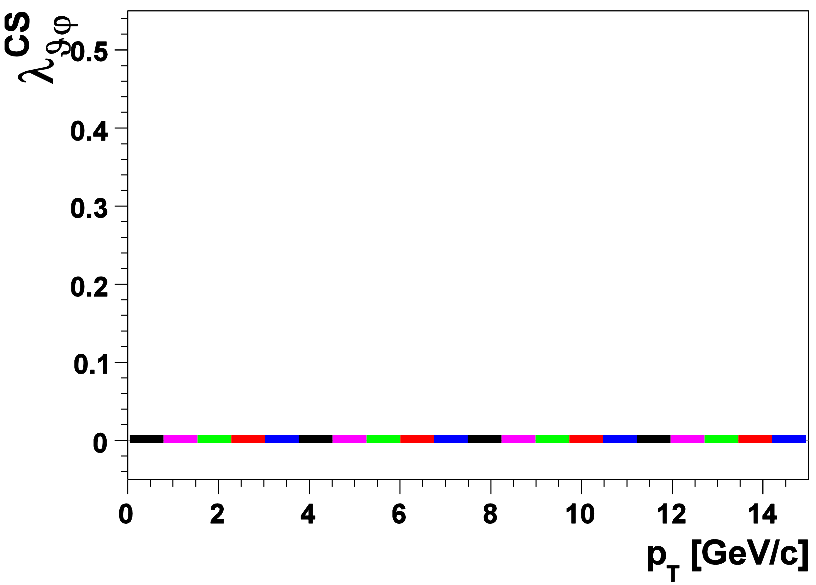
CDF	$\|y\| < 0.6$
D0	$\|y\| < 1.8$
ATLAS & CMS	$\|y\| < 2.5$
ALICE e^+e^-	$\|y\| < 0.9$
ALICE $\mu^+\mu^-$	$2.5 < \|y\| < 4$
LHCb	$2 < \|y\| < 5$

The lucky frame choice

(CS in this case)







CDF

D0

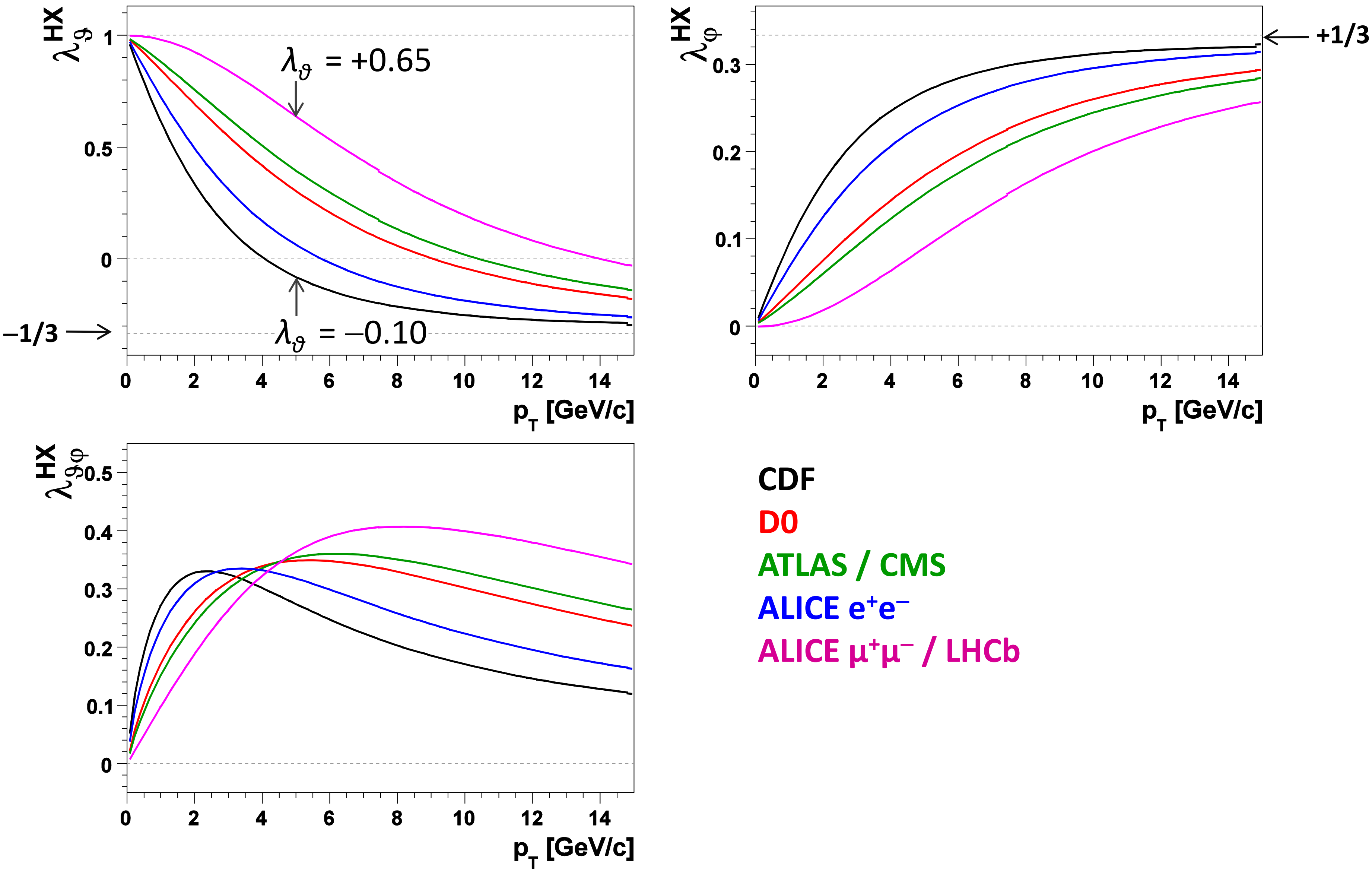
ATLAS / CMS

ALICE e^+e^-

ALICE $\mu^+\mu^-$ / LHCb

Less lucky choice

(HX in this case)



CDF

D0

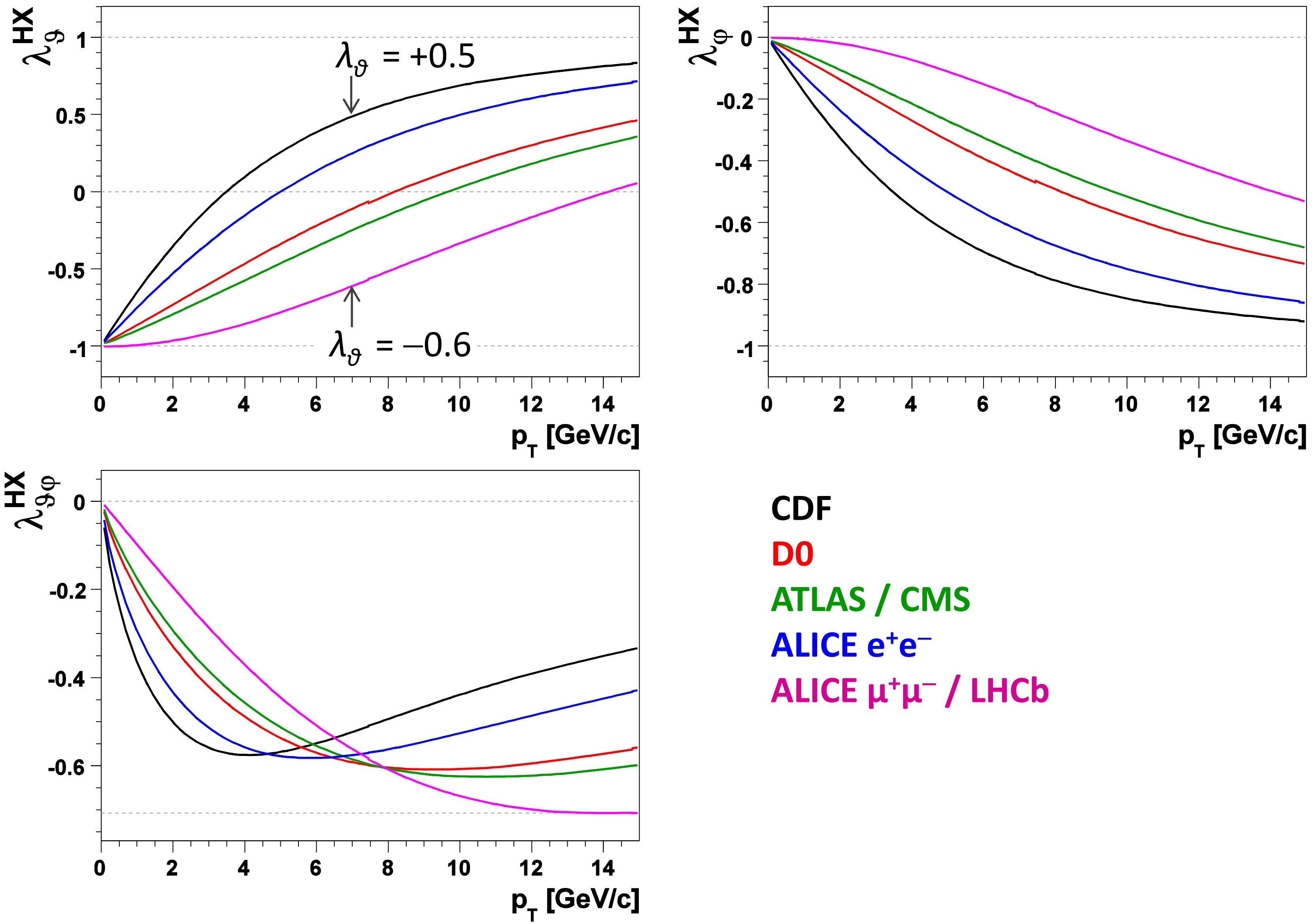
ATLAS / CMS

ALICE e^+e^-

ALICE $\mu^+\mu^-$ / LHCb

One more example

“natural” polarization $\boldsymbol{\lambda_\vartheta = -1}$ in the CS frame, as seen in the HX frame



Message nº2

When observed in an arbitrarily chosen frame, the simplest possible pattern of a constant natural polarization may be seen as a complex decay distribution rapidly changing with p_T and rapidity. This is not wrong, but gives a misleading view of the phenomenon, even inducing an artificial dependence of the measurement on the specific kinematic window of the experiment.

Measure in more than one frame.

Message nº3

In general, the polarization depends on the kinematics.

What is measured by an experiment is the *average* polarization in a certain kinematic range.

This average depends on the *effective population of collected events*, as accepted by detector, trigger and analysis cuts.

Two experiments may find different average polarizations even in the *same* kinematic interval, if they have very different *kinematic acceptance shapes*.

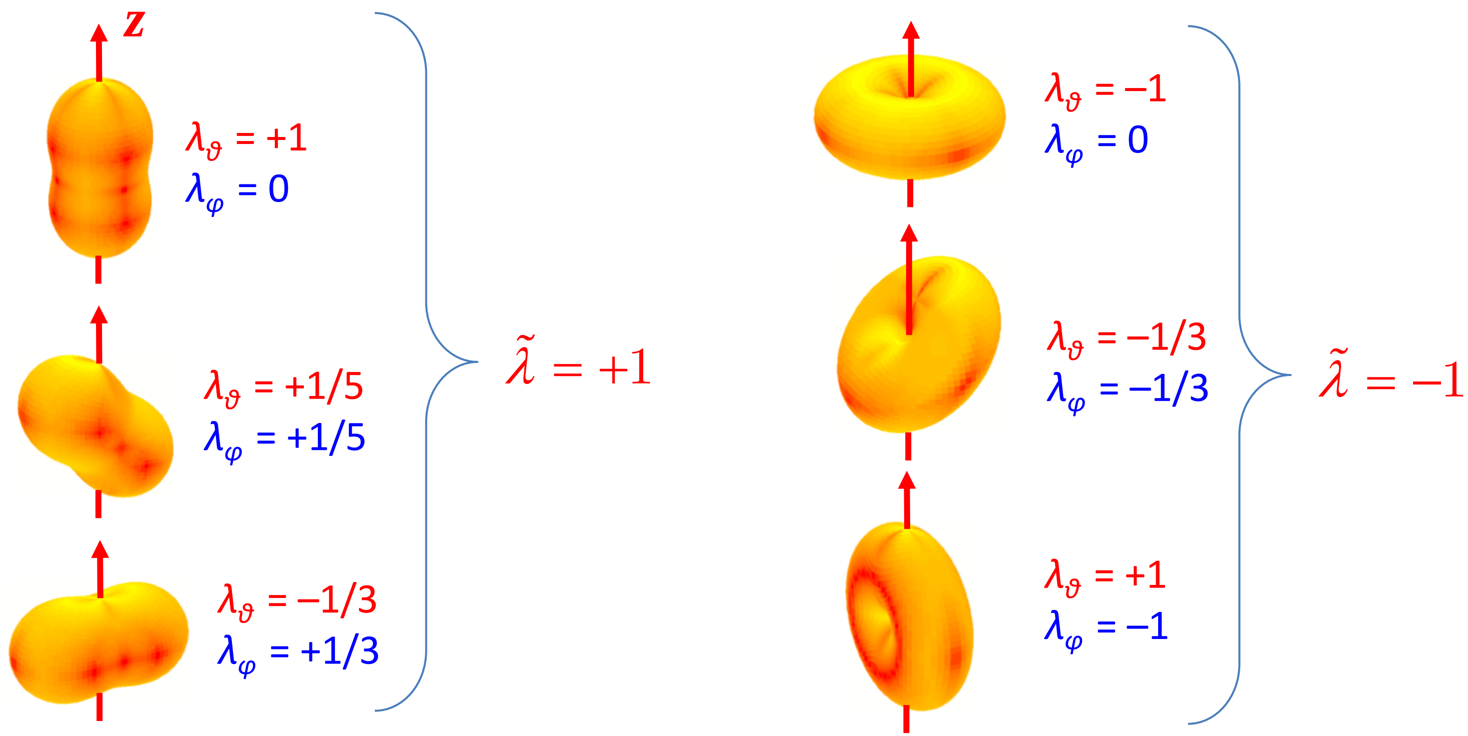
The problem can be solved by measuring in *small* kinematic cells.

Also *theoretical calculations* should take into account how the momentum distribution is distorted by the acceptance of the specific experiment, or provide *event-level* predictions.

Avoid (as much as possible) kinematic averages.

Frame-independent polarization

The *shape* of the distribution is obviously frame-invariant.
→ there exists a family of frame-independent quantities, e.g. $\tilde{\lambda} = \dfrac{\lambda_\vartheta + 3\lambda_\varphi}{1 - \lambda_\varphi}$ (and any function of it)



Measuring frame-invariant quantities is useful for

- a self-consistency check of the analysis (is $\tilde{\lambda}$ really the same in two frames?)
- a clearer representation of the results, removing frame-induced kinematic dependencies

Advantages

Invariant quantities provide an easier representation of polarization results.

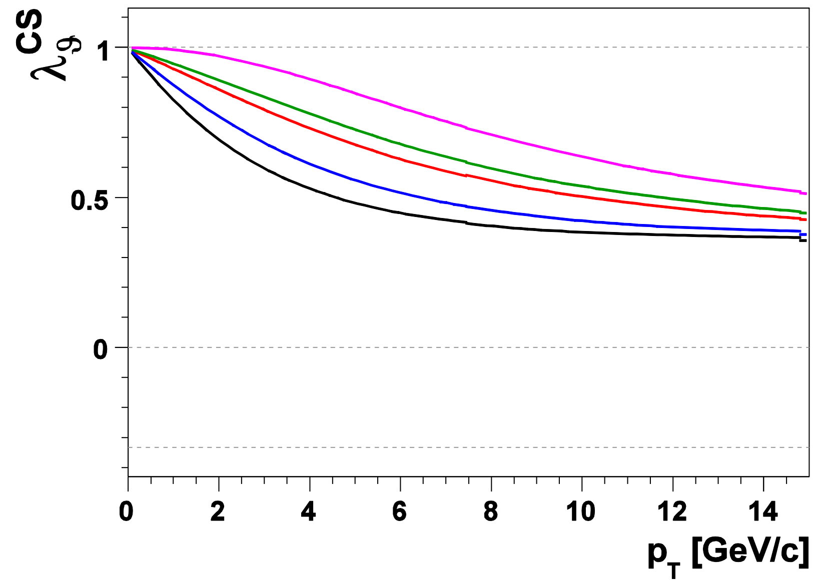
Let us consider, for illustrative purposes, the following (purely hypothetic) mixture of subprocesses for Υ production:

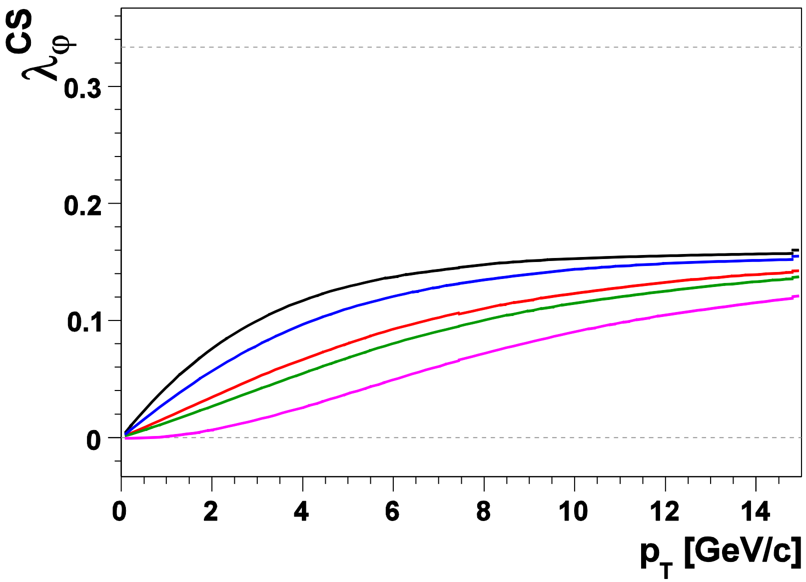
1) $\boldsymbol{f^{(1)}}$ **= 60%** of the events have a natural **transverse** polarization in the **CS** frame

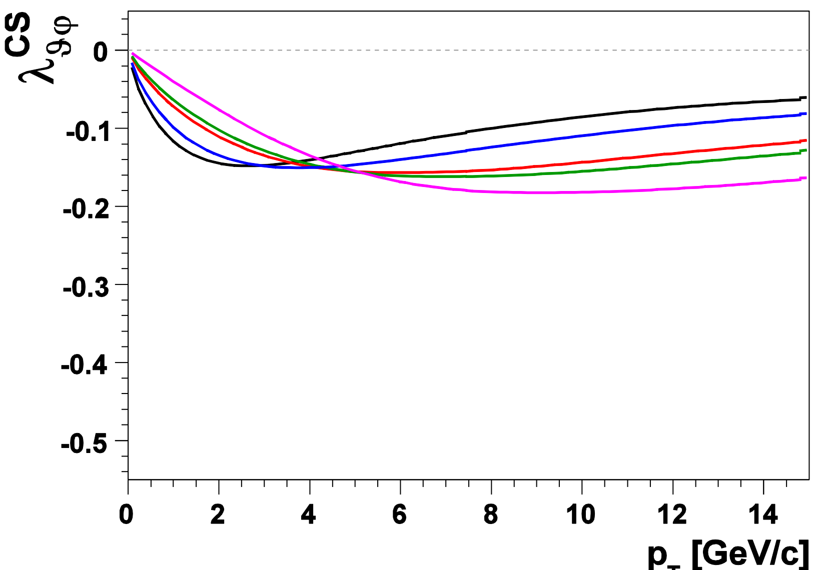
2) $\boldsymbol{f^{(2)}}$ **= 40%** of the events have a natural **transverse** polarization in the **HX** frame

Frame choice 1

All experiments choose the CS frame







CDF

D0

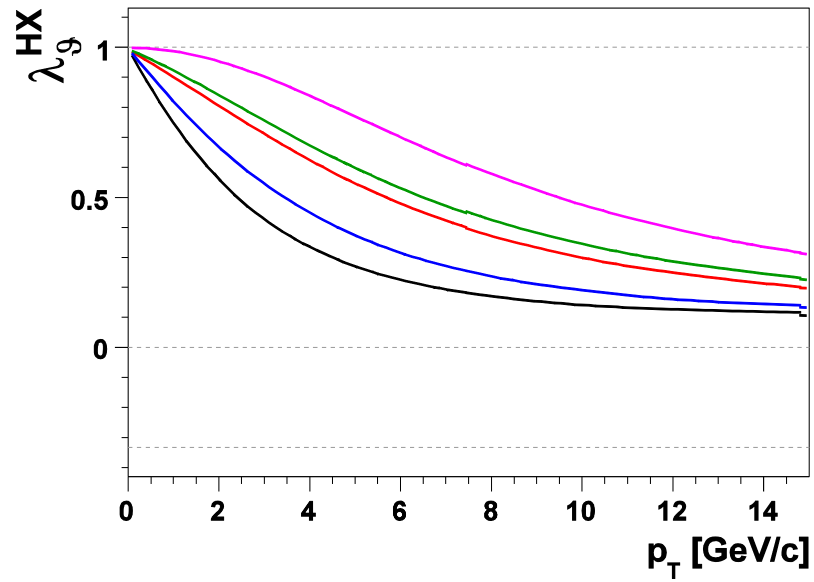
ATLAS / CMS

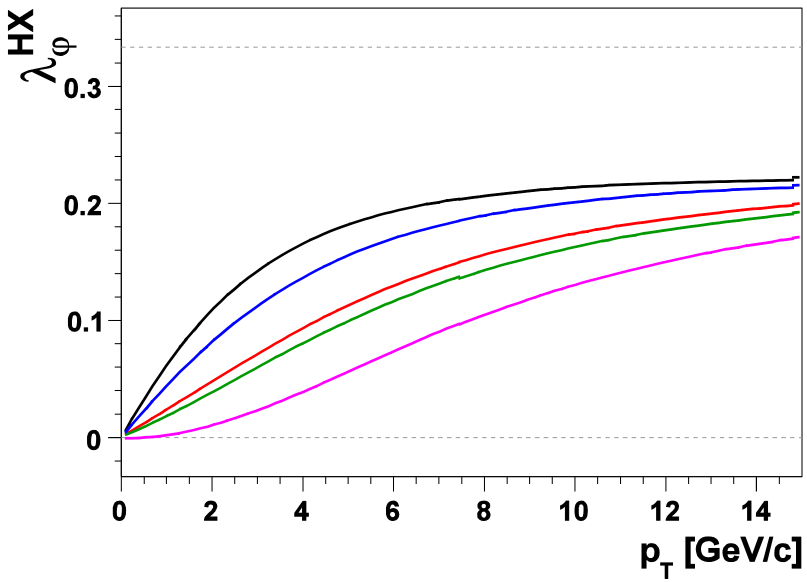
ALICE e^+e^-

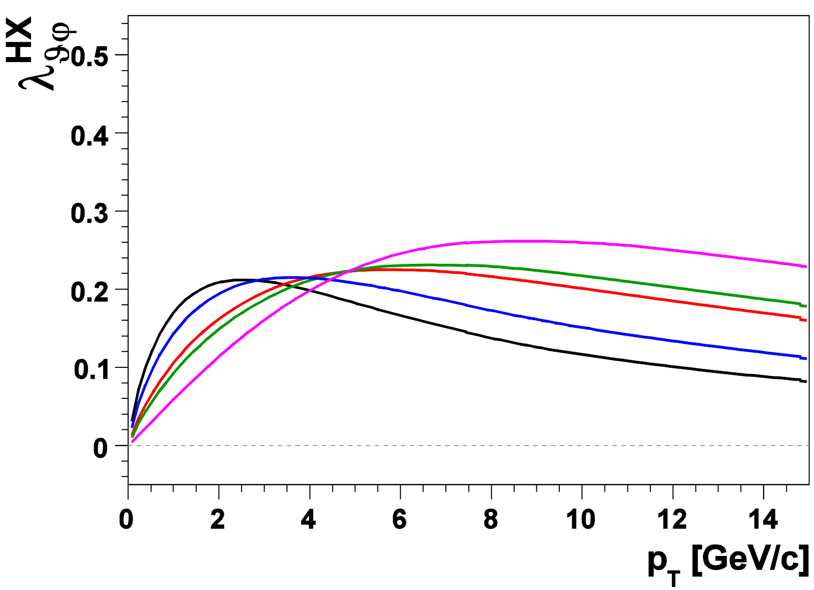
ALICE $\mu^+\mu^-$ / LHCb

Frame choice 2

All experiments choose the HX frame







CDF

D0

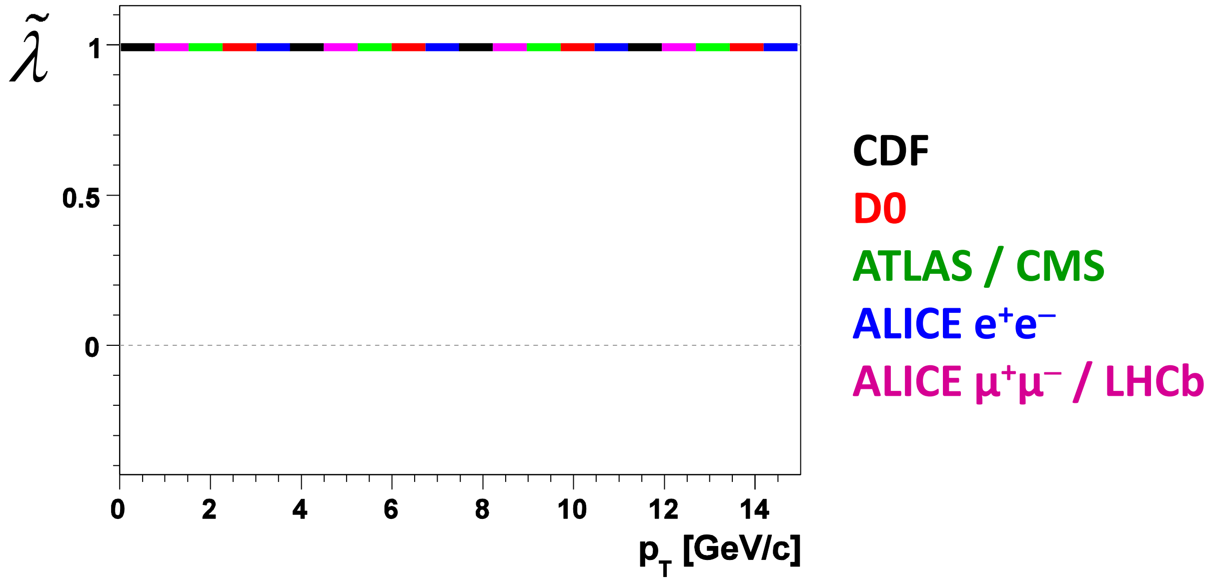
ATLAS / CMS

ALICE e^+e^-

ALICE $\mu^+\mu^-$ / LHCb

Any frame choice

The experiments measure an invariant quantity, for example $\tilde{\lambda} = \dfrac{\lambda_\vartheta + 3\,\lambda_\varphi}{1 - \lambda_\varphi}$



$\tilde{\lambda}$ is an "average of the natural polarizations",
irrespective of the directions of the respective axes:

$f^{(i)}$ = statistical weight of the *i*-th process

$\lambda_\vartheta^{*(i)}$ = *i*-th "natural" polarization

$$\tilde{\lambda} = \frac{\sum_{i=1}^{n} \frac{f^{(i)}}{3 + \lambda_\vartheta^{*(i)}} \lambda_\vartheta^{*(i)}}{\sum_{i=1}^{n} \frac{f^{(i)}}{3 + \lambda_\vartheta^{*(i)}}}$$

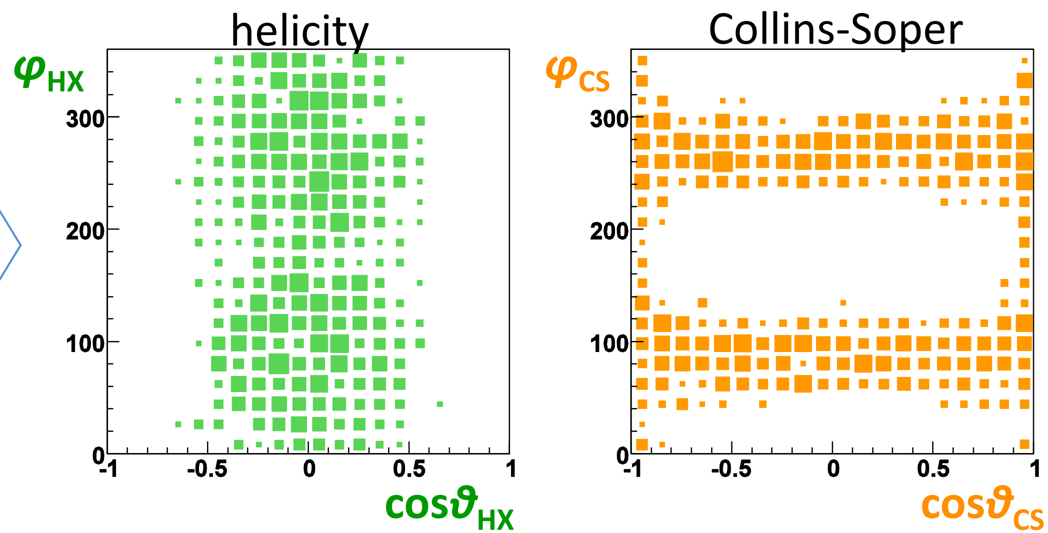
Message nº4

Frame-invariant quantities are immune to “extrinsic” kinematic dependencies induced by the observation perspective.
They minimize the acceptance-dependence of the measurement.

Use invariant relations to facilitate comparisons.

Experimental biases are not frame-invariant

Minimum detector sensitivity to muon momenta + trigger cuts



Reconstructed *unpolarized* Υ(1S)

CMS-like MC with $p_T(\mu)$ > 3 GeV/c (both muons)

$p_T(\Upsilon)$ > 10 GeV/c, $|y(\Upsilon)|$ < 1,

These spurious anisotropies must be accurately corrected.

The "detector polarization frame" is naturally defined in the LAB frame.

The physical polarization frame is the particle rest frame.

There is no "rotation" correlating the two.

→ unaccounted detector effects due to acceptance limitations will *violate the physical frame-invariant relations* between decay angular parameters.

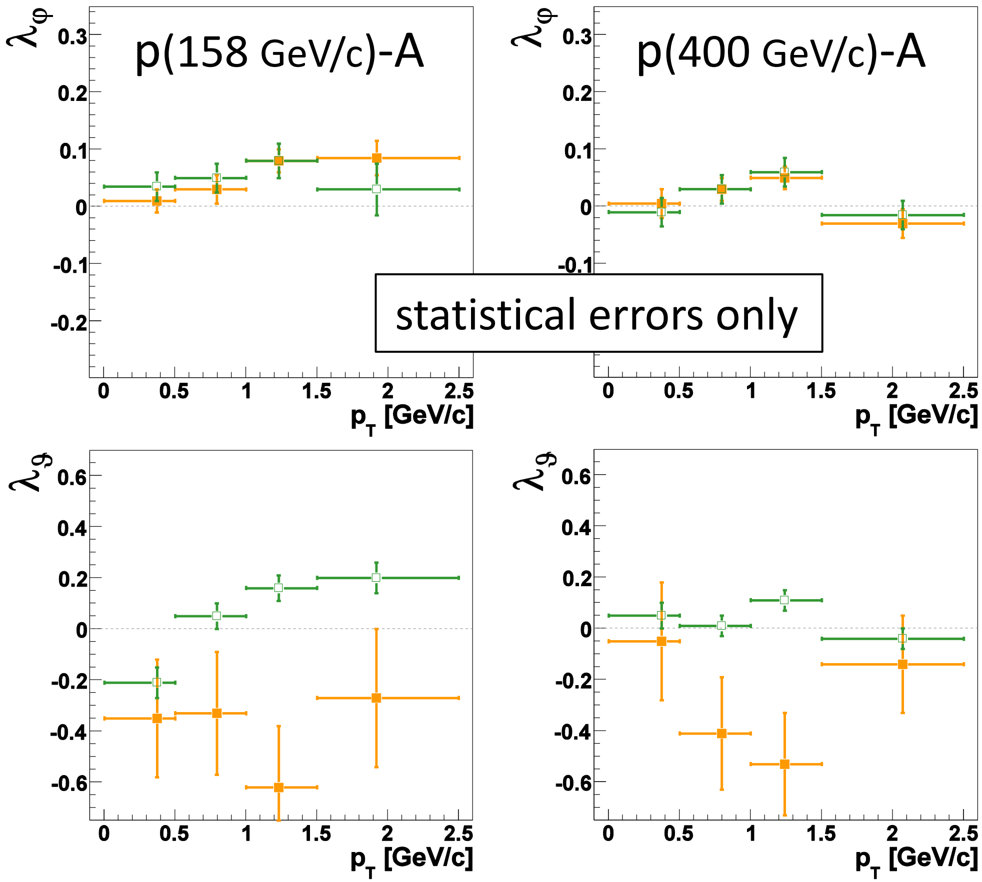
→ checking whether the same value of an invariant quantity is obtained (within systematic errors) *in two distinct polarization frames* is a non-trivial test.

Example

Given two frames *A* and *B*,

$$\lambda_\vartheta^B = \lambda_\vartheta^A \Leftrightarrow \lambda_\varphi^B = \lambda_\varphi^A \Leftrightarrow \begin{cases} B = A \\ \text{or } \lambda_\vartheta = \lambda_\varphi = 0 \end{cases}$$

NA60 J/ψ prelim. (QM09) **HX** / **CS**



At first glance: $\lambda_\varphi(\text{CS}) \approx \lambda_\varphi(\text{HX})$
while $\lambda_\vartheta(\text{CS}) < \lambda_\vartheta(\text{HX})$

→ check quantitatively by calculating the average "polarization" constant

$$\tilde{\lambda} = \frac{\lambda_\vartheta + 3\lambda_\varphi}{1 - \lambda_\varphi}$$

$$\tilde{\lambda}(\text{HX}) - \tilde{\lambda}(\text{CS}) = \begin{cases} 0.49 \; [\pm 0.13] & 158 \text{ GeV/c} \\ 0.28 \; [\pm 0.12] & 400 \text{ GeV/c} \end{cases}$$

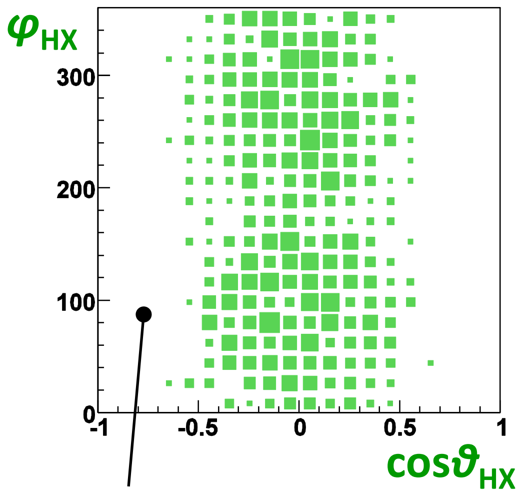
(errors not so relevant: CS and HX data are statistically correlated)

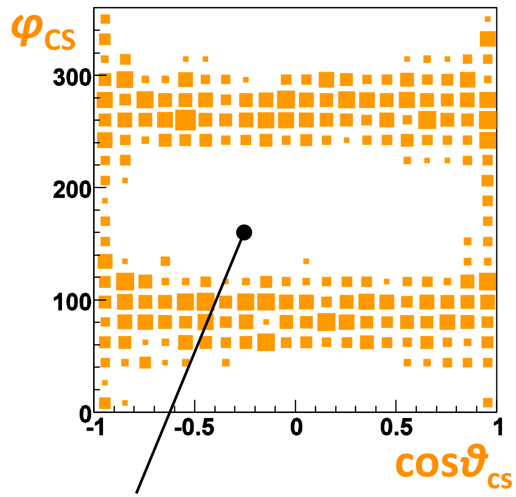
order of magnitude of the expected systematic error on the anisotropy parameters

Message nº5

Use invariant relations
for a better control over systematic effects.

Polarization dependence of the dilepton acceptance





The efficiency determination in the zero-acceptance domains is 100% dependent on the polarization information fed into the Monte Carlo simulation

The acceptance *depends on both polar and azimuthal anisotropies, differently in different frames*.
E.g., high p_T: depends mostly on λ_ϑ(HX) and on λ_φ(CS)

Basic meaning of the frame-invariant quantities

Let us suppose that, in the collected events, *n* different elementary subprocesses yield angular momentum states of the kind

$$\left|\psi^{(i)}\right\rangle = a_{-1}^{(i)}\,|1,-1\rangle + a_{0}^{(i)}\,|1,0\rangle + a_{+1}^{(i)}\,|1,+1\rangle, \qquad i = 1,2,\ldots n$$

(wrt a given quantization axis), each one with probability $f^{(i)}$ $(\sum f^{(i)} = 1)$.

The **rotational properties of angular momentum eigenstates** imply that

the combinations $a_{+1}^{(i)} + a_{-1}^{(i)}$ *are independent of the choice of the quantization axis*

The quantity

$$F = \sum_{i=1}^{n} f^{(i)} F^{(i)} = \frac{1}{2}\sum_{i=1}^{n} f^{(i)} \left|a_{+1}^{(i)} + a_{-1}^{(i)}\right|^2 \qquad (0 \le F \le 1)$$

is therefore frame-independent. It can be shown to be equal to

$$F = \frac{1 + \lambda_\vartheta + 2\lambda_\varphi}{3 + \lambda_\vartheta}$$

In other words, there always exists *a calculable frame-invariant relation* of the form

$$\boxed{(1-F)\lambda_\vartheta + 2\lambda_\varphi = 3F - 1}$$

The Lam-Tung limit

Another consequence of rotational properties of angular momentum eigenstates:

for each state $\left|\psi^{(i)}\right\rangle = a_0^{(i)}\,|0\rangle + a_{+1}^{(i)}\,|+1\rangle + a_{-1}^{(i)}\,|-1\rangle$

there exists a quantization axis z' wrt which $a_0^{(i)\prime} = 0$

→ quarkonium produced in each single elementary subprocess has a dilepton decay distribution of the type

$$\lambda_\vartheta^{(i)\prime} = +1, \quad \lambda_\varphi^{(i)\prime} = 2F^{(i)} - 1, \quad \lambda_{\vartheta\varphi}^{(i)\prime} = 0 \qquad \left(F^{(i)} = \tfrac{1}{2}\left|a_{+1}^{(i)} + a_{-1}^{(i)}\right|^2\right)$$

wrt its specific "$a_0^{(i)\prime} = 0$" axis.

Case $F^{(i)} = \tfrac{1}{2}$: each subprocess is characterized by a *fully transverse* polarization

$$\lambda_\vartheta^{(i)\prime} = +1, \qquad \lambda_\varphi^{(i)\prime} = 0, \qquad \lambda_{\vartheta\varphi}^{(i)\prime} = 0$$

wrt a certain "natural" axis (which may be different from subprocess to subprocess).

$$\rightarrow \quad F = \sum f^{(i)} F^{(i)} = \frac{1}{2} = \frac{1 + \lambda_\vartheta + 2\lambda_\varphi}{3 + \lambda_\vartheta}$$

$$\rightarrow \quad \lambda_\vartheta + 4\lambda_\varphi = 1$$

Lam-Tung identity
(Drell-Yan up to NLO QCD corrections)

Simple interpretation of the LT relation

1. The *existence* (*and frame-independence*) of the LT relation is the *kinematic* consequence of the rotational properties of J = 1 angular momentum eigenstates
2. Its *form* derives from the *dynamical* input that all contributing processes produce the dilepton via one *transversely* polarized photon

More generally:

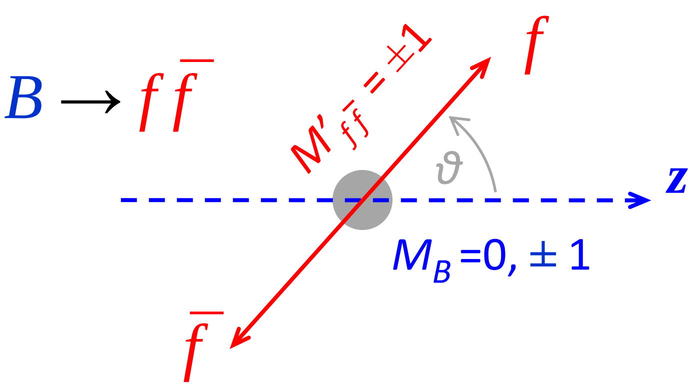
- Corrections to the Lam-Tung relation (parton-k_T, higher-twist effects) should continue to yield *invariant* relations.
  In the literature, deviations are searched in the form
  $$\lambda_\vartheta + 4\lambda_\varphi = 1 - \Delta$$
  But this is not a frame-invariant relation!
- For *any* superposition of processes, concerning *any* J = 1 particle (even in parity-violating cases: W, Z), we can always calculate a *frame-invariant* relation analogous to the LT relation.

Rotation-invariant parity asymmetry

$$\frac{dN}{d\Omega} \propto 1 + \ldots + 2\boldsymbol{A_\theta} \cos\theta + 2\boldsymbol{A_\varphi} \sin\theta\cos\varphi + 2\boldsymbol{A_\varphi^\perp} \sin\theta\sin\varphi$$

$$\tilde{\mathcal{A}} = \frac{4}{3+\lambda_\vartheta}\sqrt{A_\theta^2 + A_\varphi^2 + A_\varphi^{\perp 2}}$$ is rotationally invariant

It represents the magnitude of the *maximum observable parity asymmetry*, i.e. of the *net* asymmetry as it can be measured along the polarization axis that maximizes it



$$\tilde{\mathcal{A}} = \max_{z} \frac{P(\pm 1, \pm 1) - P(\pm 1, \mp 1)}{P(\pm 1, \pm 1) + P(\pm 1, \mp 1)}$$

Appendix

Rotation-invariant "forward-backward" asymmetry

It can also be written as

$$\tilde{\mathcal{A}} = \frac{4}{3}\sqrt{\mathcal{A}^2_{\cos\theta} + \mathcal{A}^2_{\cos\varphi} + \mathcal{A}^{\perp 2}_{\sin\varphi}}$$

$$\mathcal{A}_{\cos\theta} = \frac{N(\cos\theta > 0) - N(\cos\theta < 0)}{N_{\text{tot}}}$$

$$\mathcal{A}_{\cos\varphi} = \frac{N(\cos\varphi > 0) - N(\cos\varphi < 0)}{N_{\text{tot}}}$$

$$\mathcal{A}_{\sin\varphi} = \frac{N(\sin\varphi > 0) - N(\sin\varphi < 0)}{N_{\text{tot}}}$$

"forward-backward asymmetry"

experiments usually measure this (in the Collins-Soper frame)

$\tilde{\mathcal{A}}$ can provide a better measurement of parity violation:

- it is not reduced by a not-optimal frame choice
- it can be checked in two "orthogonal" frames

Summary

- Even if experimentally challenging, polarization measurements are textbook exercises of basic quantum mechanics. By keeping in mind fundamental notions we will perform better polarization measurements
- The observable angular distribution reflects the rotational-covariance properties of angular momentum
  - it depends (strongly) on the reference frame according to definite rules
  - its parameters satisfy a frame-independent identity, a special case of which is the Lam-Tung relation
- In the quarkonium analyses of CMS, we will
  - determine the *full* angular decay distribution, not only the polar anisotropy
  - provide results in two polarization frames
  - avoid averages over large kinematic intervals, using (p_T ,y) cells
  - exploit the existence of frame-independent relations
    - to detect residual systematic effects
    - to facilitate the comparison with theoretical calculations and other results