Introduction to Lattice QCD

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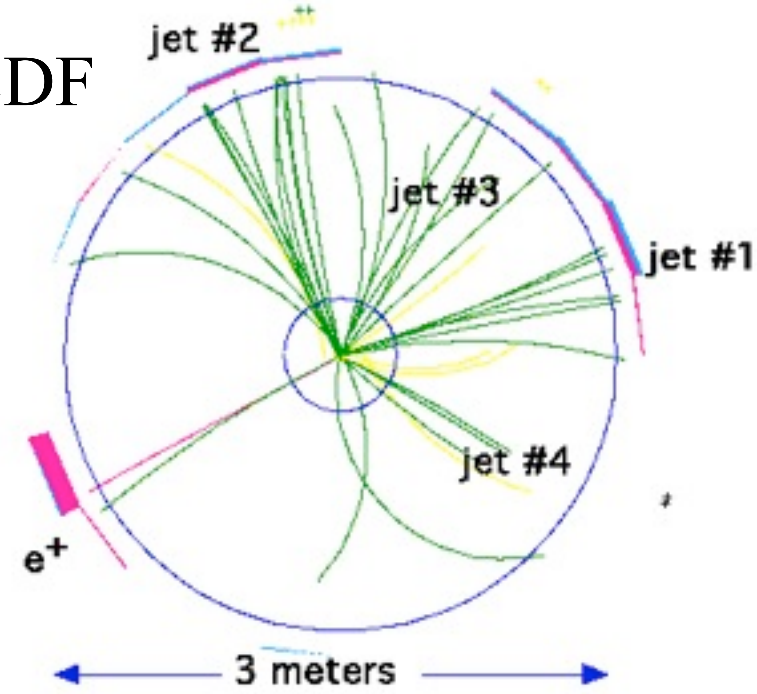
Lattice QCD meets experiment in flavour physics
Glasgow, June 2010

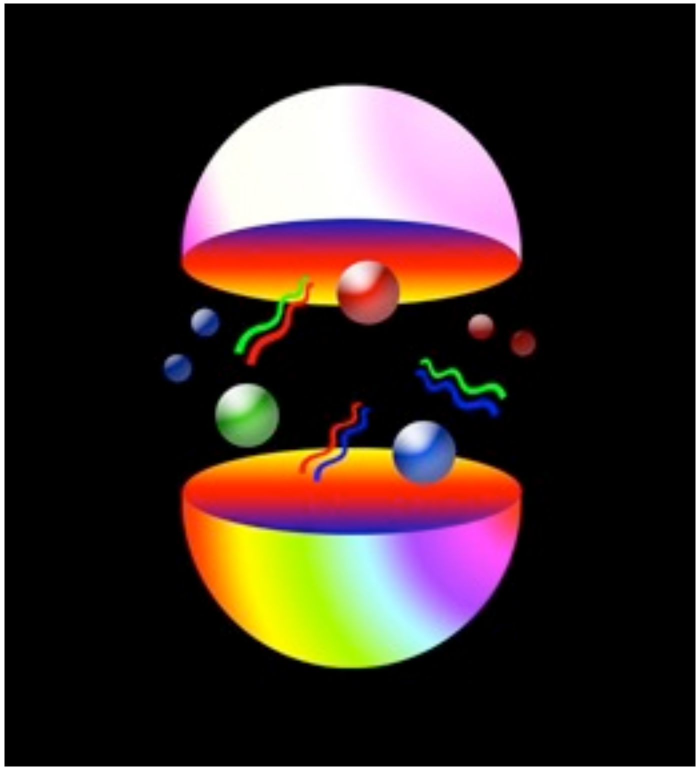
QCD is a key part of the Standard Model but quark confinement complicates things.

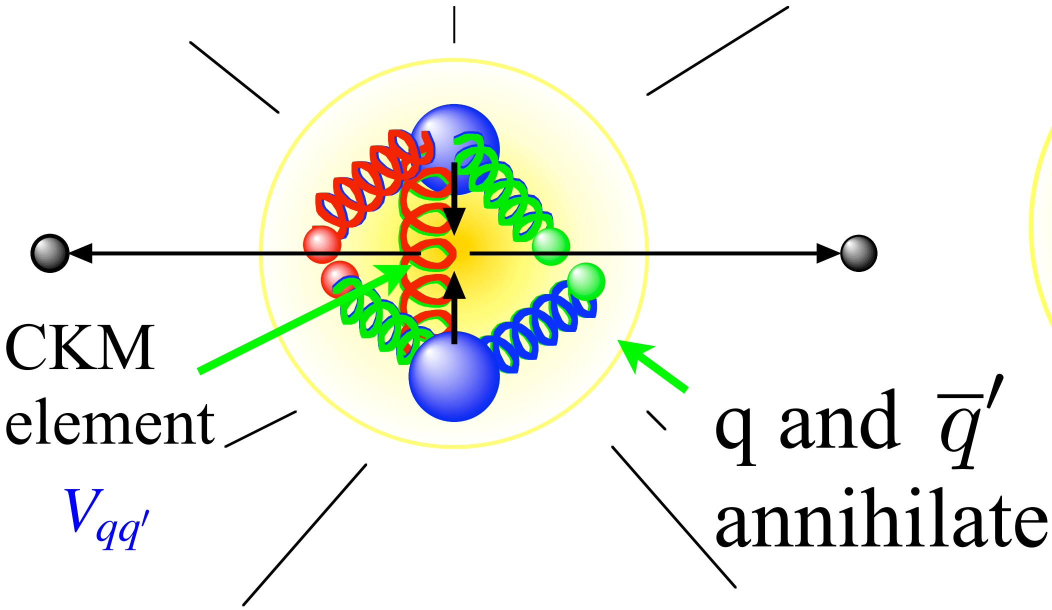
CDF

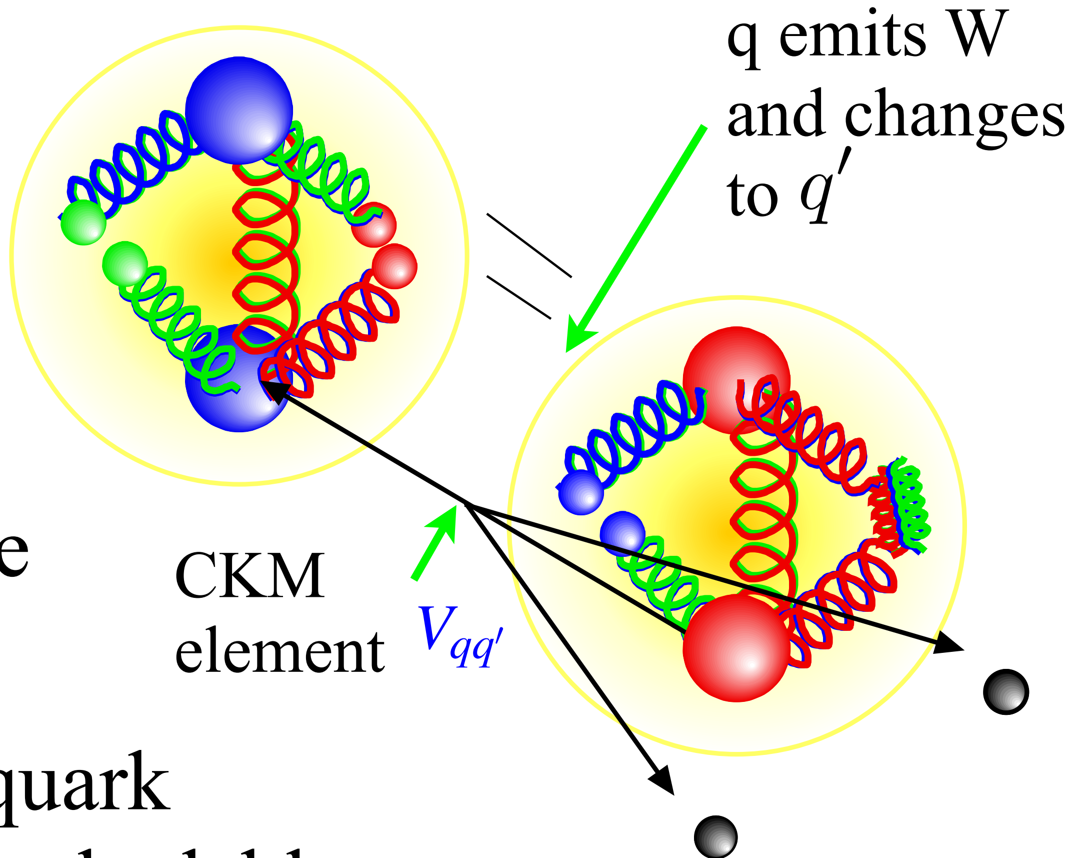


QCD only tested to 5-10% level at high energies from comparison of e.g. jet phenomena to pert.th.

But properties of hadrons calculable from QCD if fully nonperturbative calc. is done - can test QCD and determine parameters very accurately (1%).



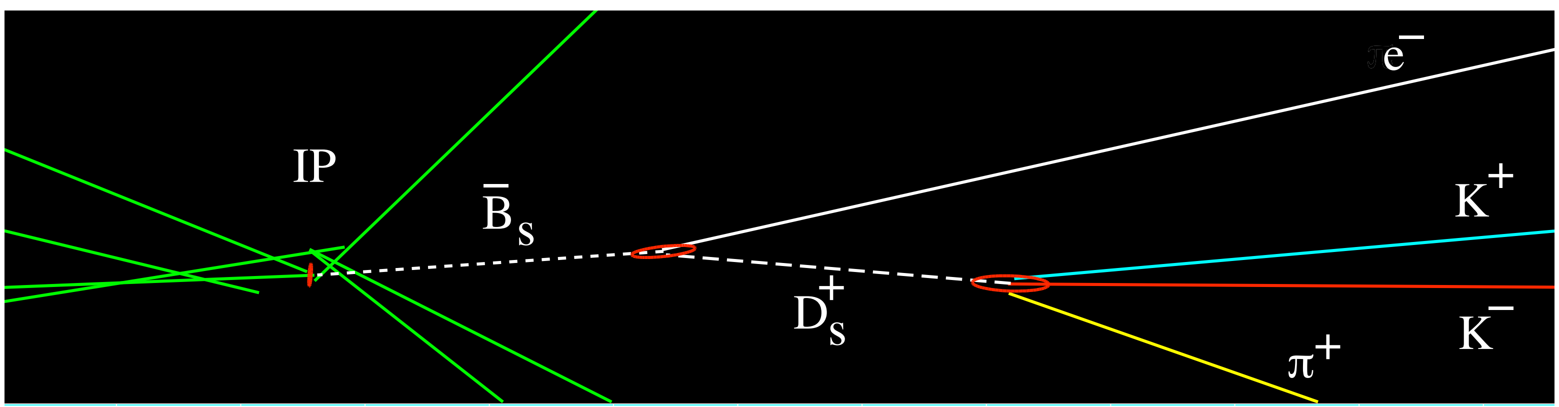




Rates for simple weak or em quark processes inside hadrons also calculable, but *not* multi-hadron final states.

$\overline{B}_s \longrightarrow D_s e^- \nu$

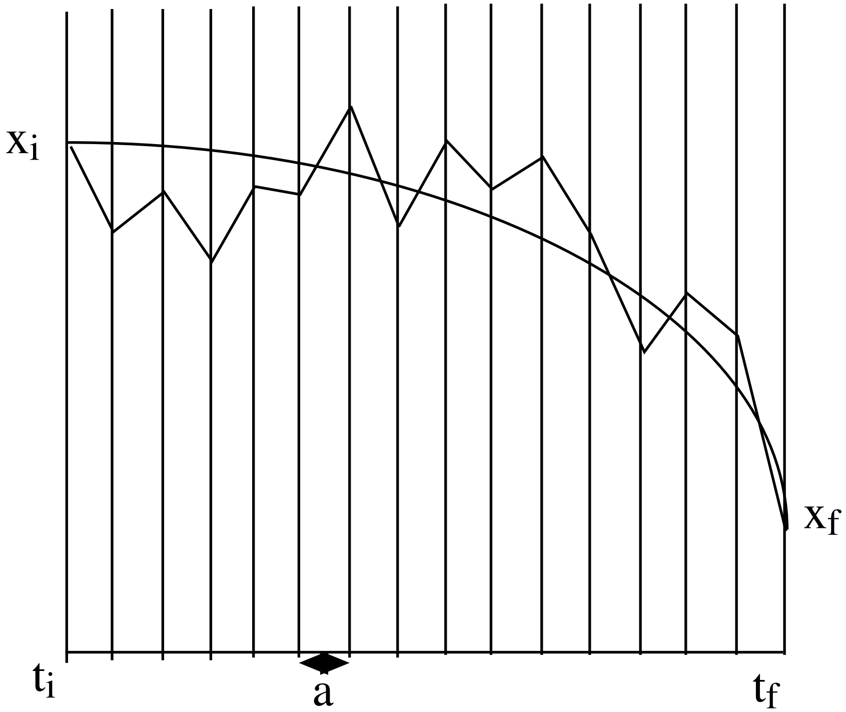
ALEPH $(D_S \longrightarrow K^+ K^- \pi^+)$



Compare to exptl rate gives $V_{qq'}$ accurately

Solving a path integral: quantum mechanical case

Solve Schrödinger's eq. for eigenvalues/fns of H or:



discretise time and integrate over all paths possible weighted by e^{iS}

$$S = \int dt \mathcal{L}; \quad \mathcal{L} = \frac{1}{2} m \dot{x}^2 - V(x)$$

classical path is $m\ddot{x} = V'$
qm path fluctuates about this.

In Euclidean time solve numerically, by making sets of $x(t_i)$

$$< x(t_2)x(t_1) >= \frac{\int \mathcal{D}x \, x(t_2)x(t_1)e^{-S}}{\int \mathcal{D}x e^{-S}} = \sum_n A_n e^{-(E_n - E_0)(t_2 - t_1)}$$

average over 'ensemble' of paths - paths chosen with prob. e^{-S}

fit as fn of time can extract excitation energies

further reading: G.P.Lepage, hep-lat/0506036

Solving a path integral: QCD

Now path integral over gluon and quark fields on a 4-d space-time lattice - quarks anticommute so do by hand.

$$\mathcal{L}_{QCD} = \frac{1}{2} Tr F^2_{\mu\nu} + \overline{\psi}(\gamma \cdot D + m)\psi$$

= a huge matrix, M

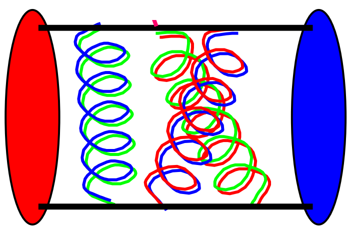
$$\int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\overline{\psi} O(\psi,\overline{\psi}) e^{-S_{QCD}} \rightarrow$$

$$\int \mathcal{D}U O(M^{-1}) e^{-(S_g - \ln(\det M))}$$

Integral over gluon fields only

valence quarks inc. in operator

complicated prob, distn for gluons - inc. effects of sea quarks

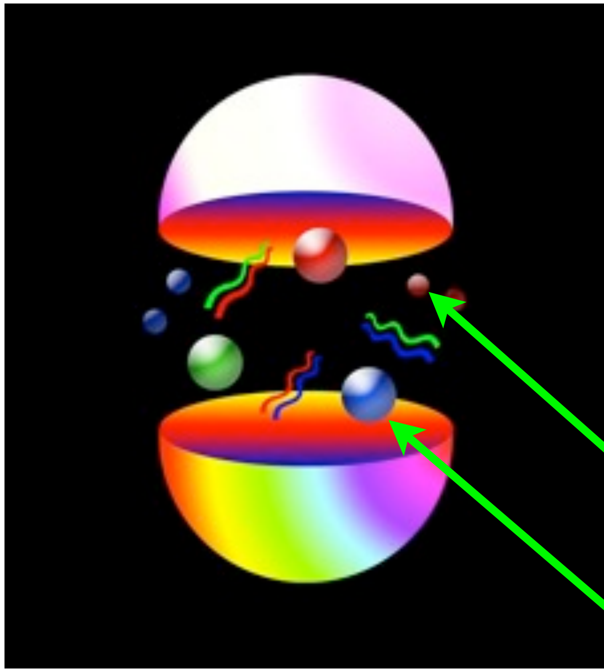


$$< O > = < H(t)H^\dagger(0) > = \sum_n A_n e^{-E_n t}$$

ensemble average

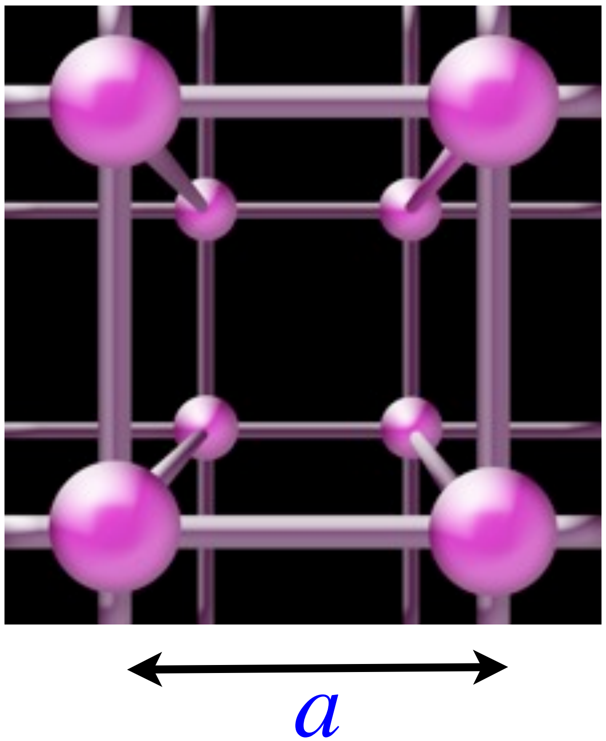
Fit as fn of t to get hadron mass

Lattice QCD = fully nonperturbative QCD calculation

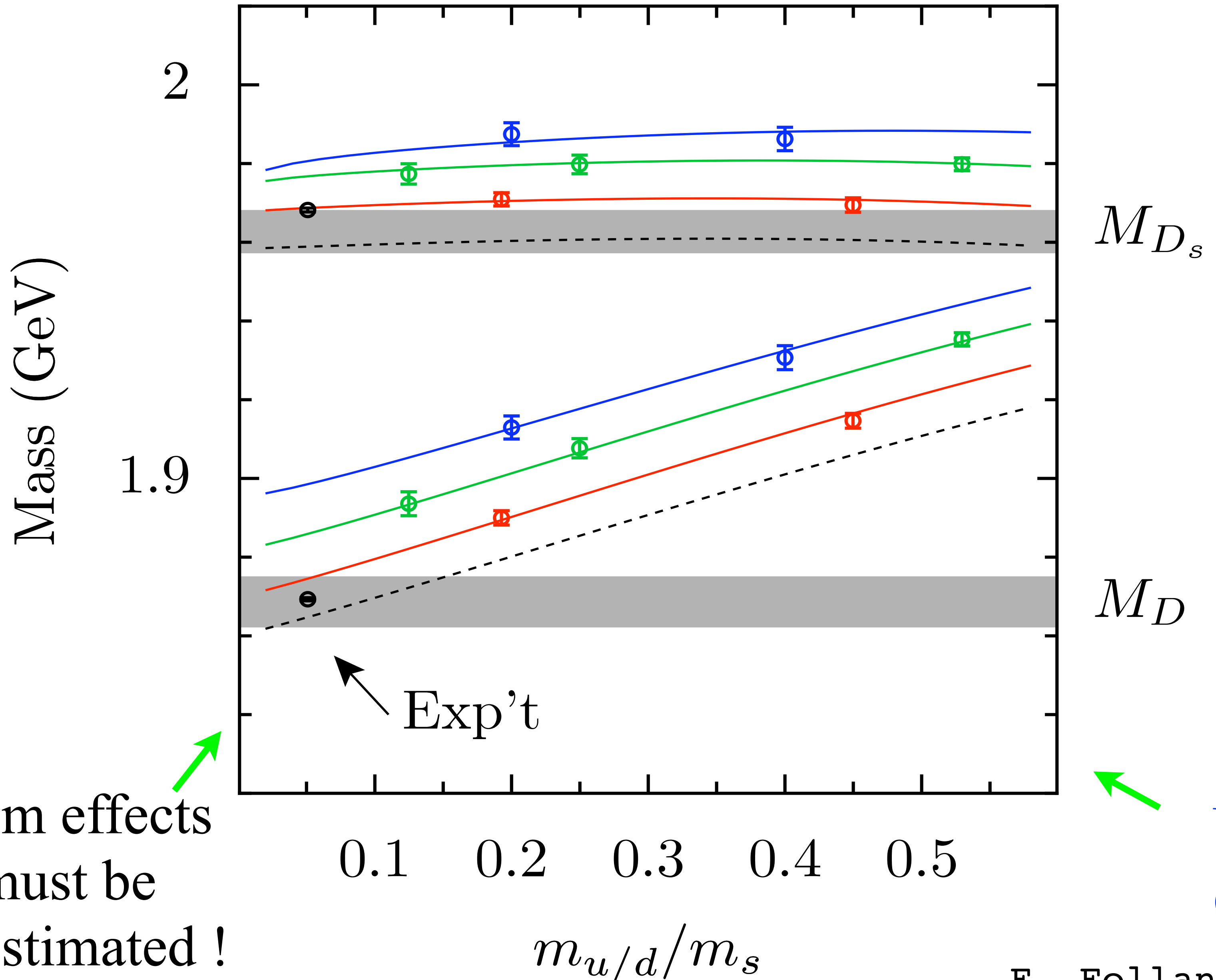


RECIPE

- Generate sets of gluon fields for Monte Carlo integrn of Path Integral (inc effect of u, d and s sea quarks)
- Calculate averaged “hadron correlators” from valence q props.
- Fit for masses and simple matrix elements
- Fix m_q and determine a to get results in physical units.
- extrapolate to $a = 0, m_{u,d} = phys$ for real world



Lattice results need to be extrapolated to the real world where a=0 and $m_{u/d}$ = small.



To do this well needs:

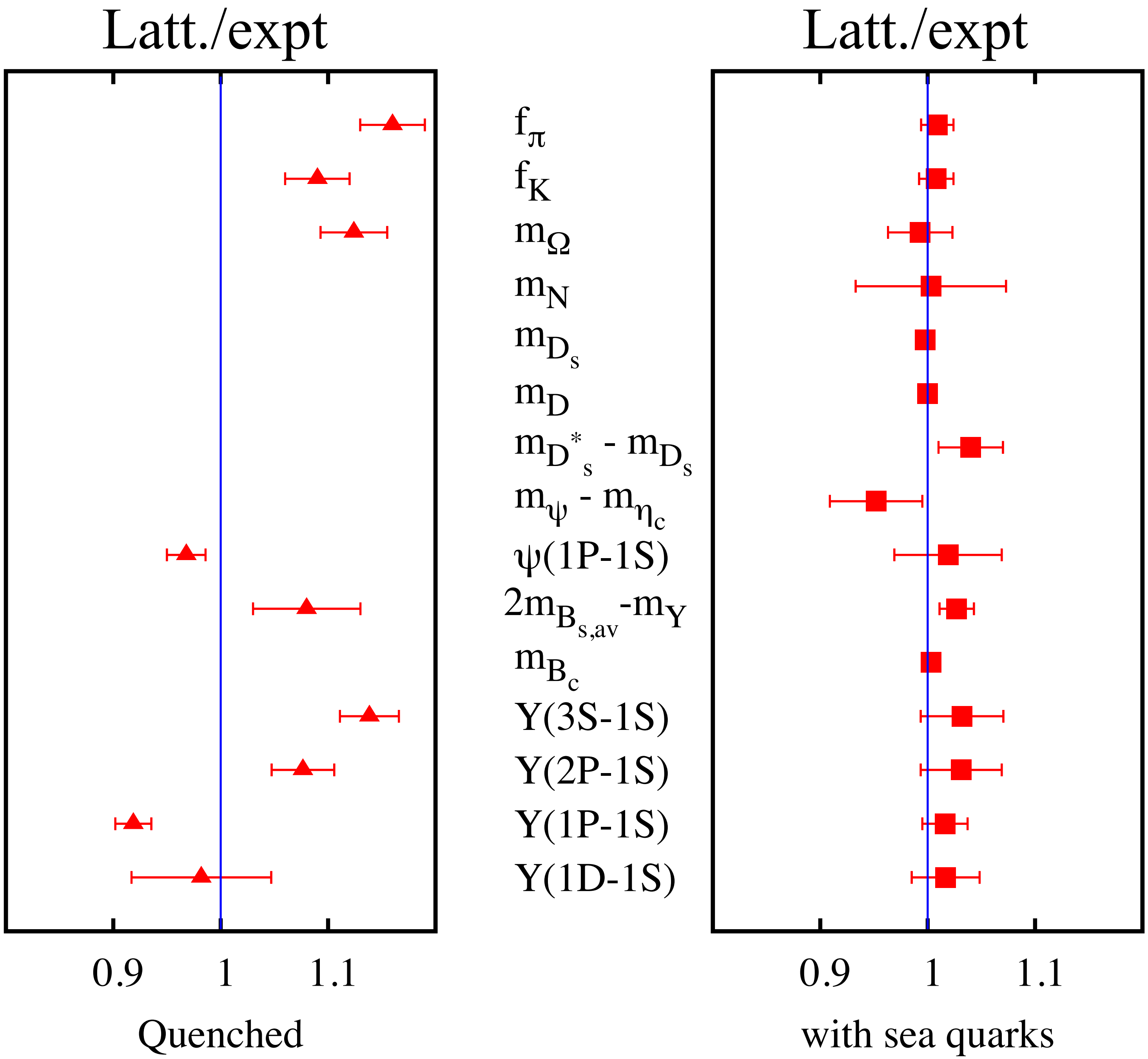
- statistical precision
- small disc. errors and several values of a
- small $m_{u/d}$

em effects must be estimated !

using HISQ charm quarks

E. Follana et al, 0706.1726

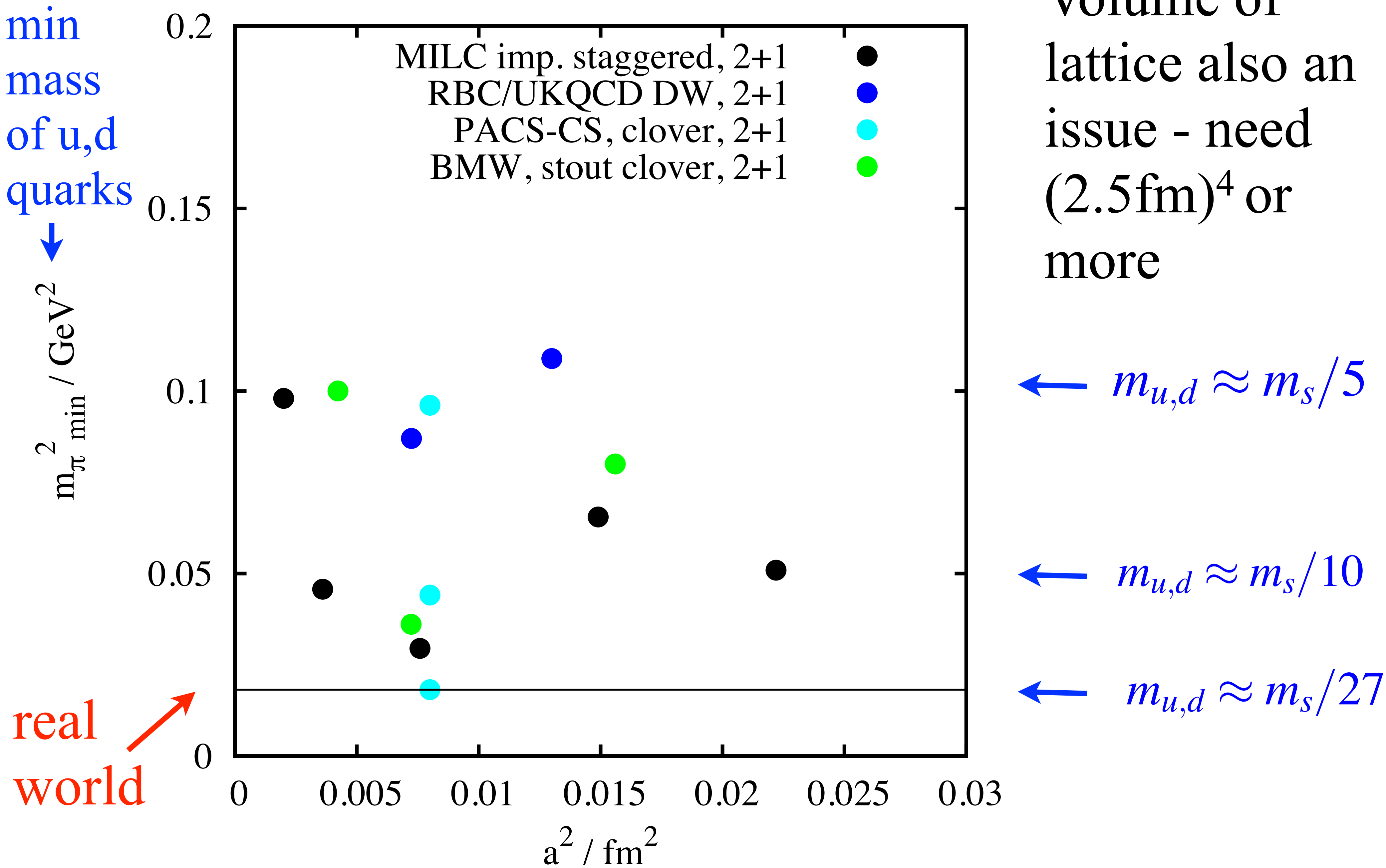
Including u, d and s sea quarks is critical for accurate results, but numerically expensive - particularly light $m_{u,d}$.



HPQCD/MILC 2008 "ratio plot".

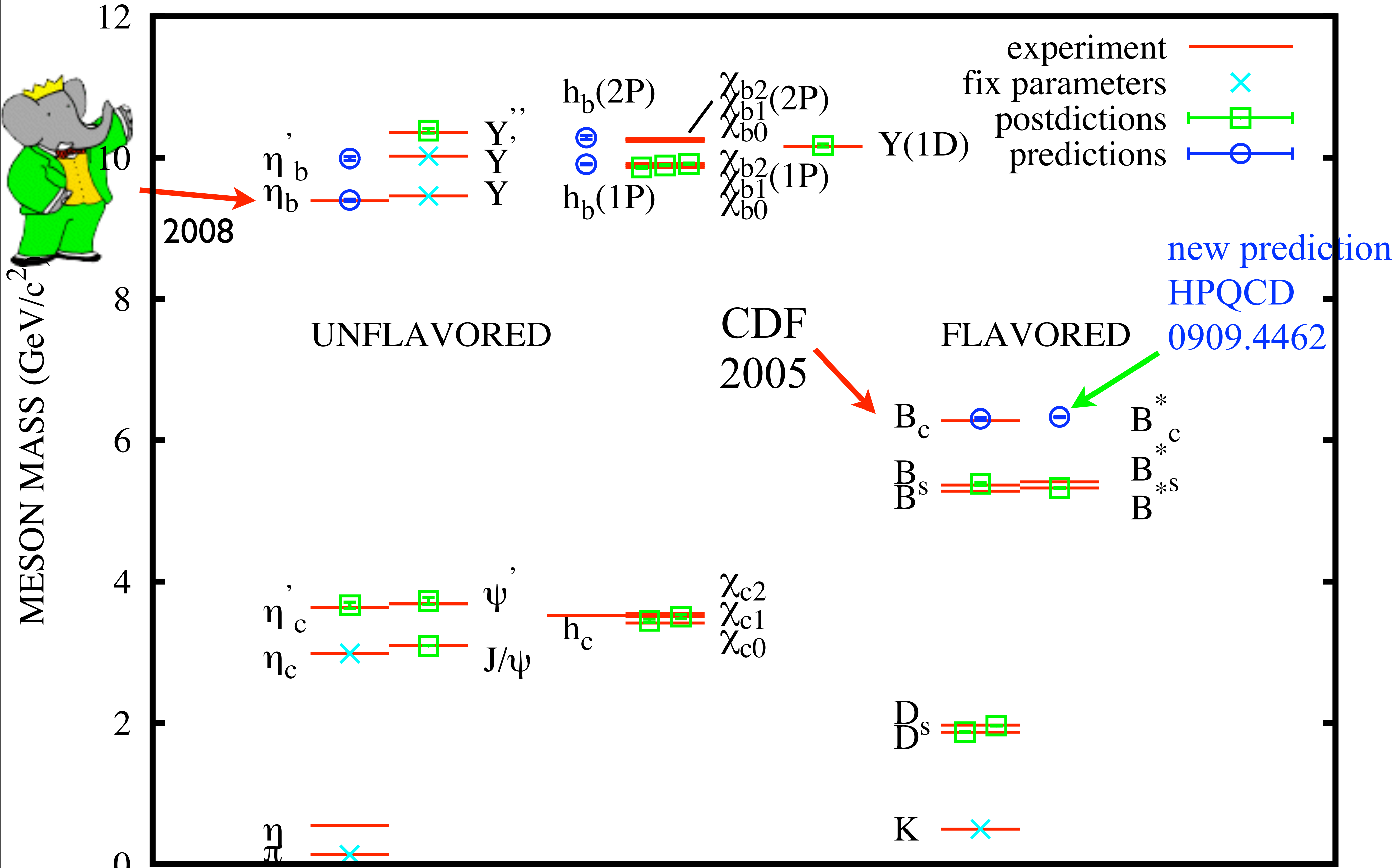
Multiple values of a, and of $m_{u,d}$. Extrapolate to physical point.

Example parameters for calculations now being done.
Lots of different formalisms for handling quarks.



Volume of lattice also an issue - need $(2.5\text{fm})^4$ or more

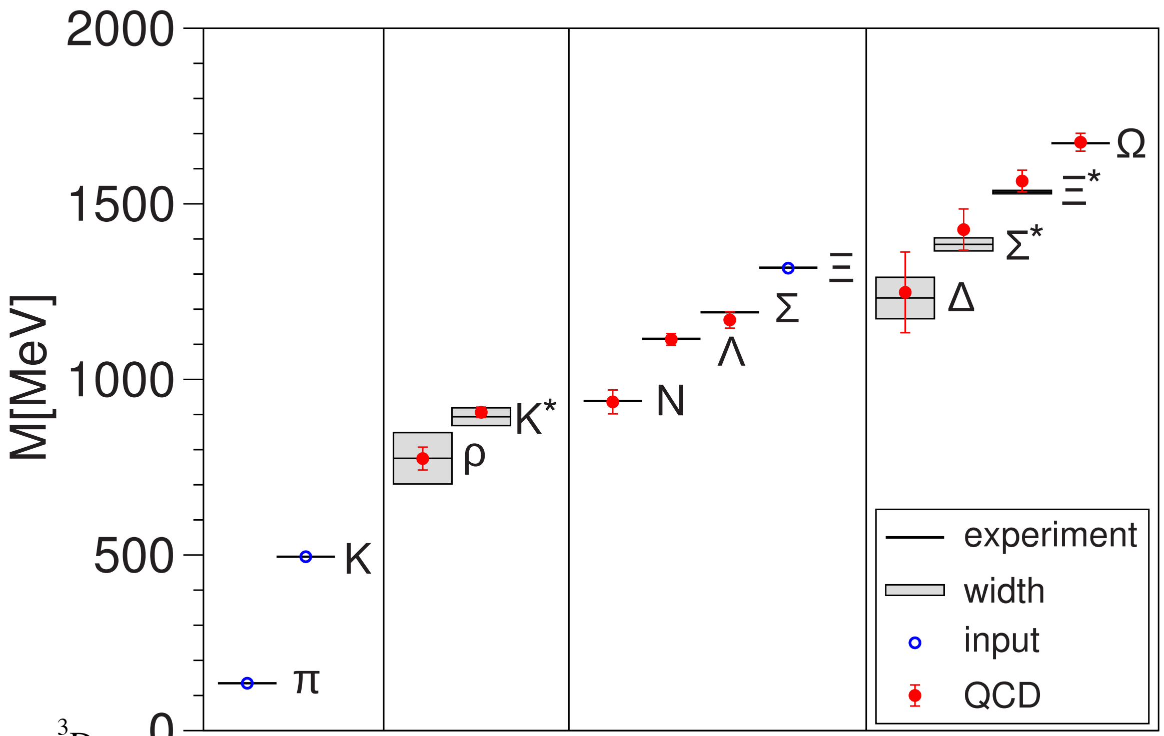
The gold-plated meson spectrum - HPQCD 2009



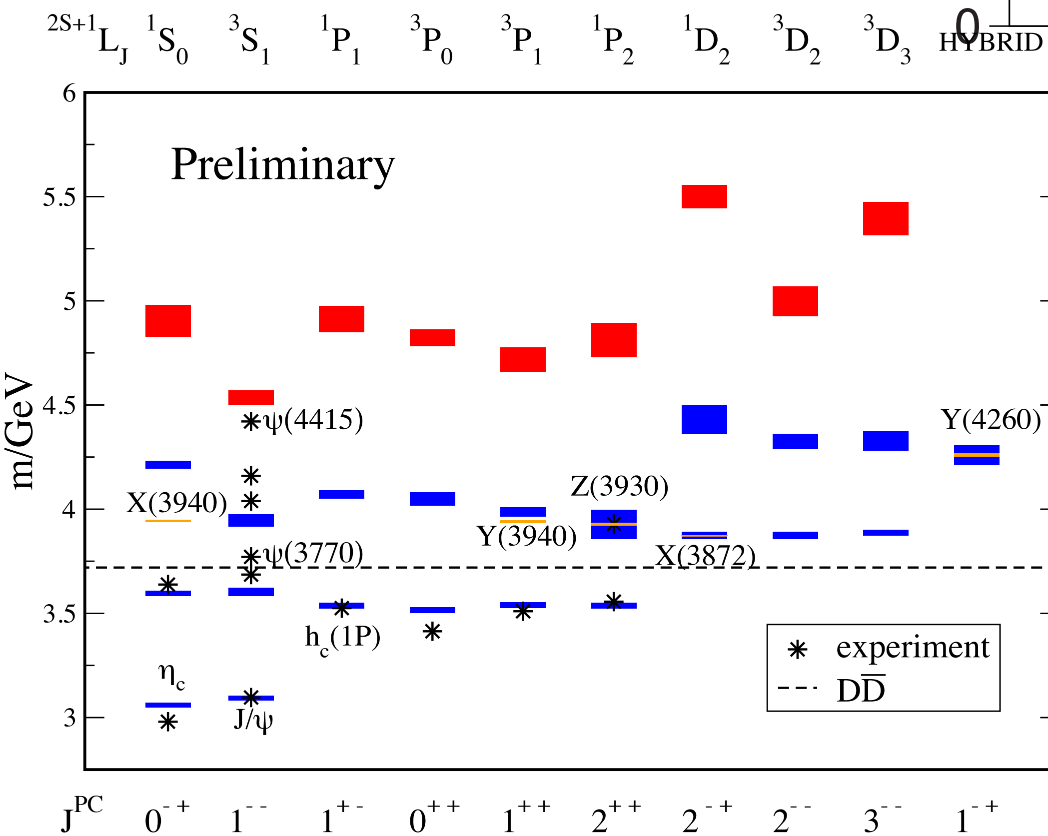
I. Allison et al, hep-lat/0411027, A. Gray et al, hep-lat/0507013

Harder states to do

Light hadron spectrum
inc. baryons with
'fat clover' quarks



BMW collaboration 2008



Even harder ....

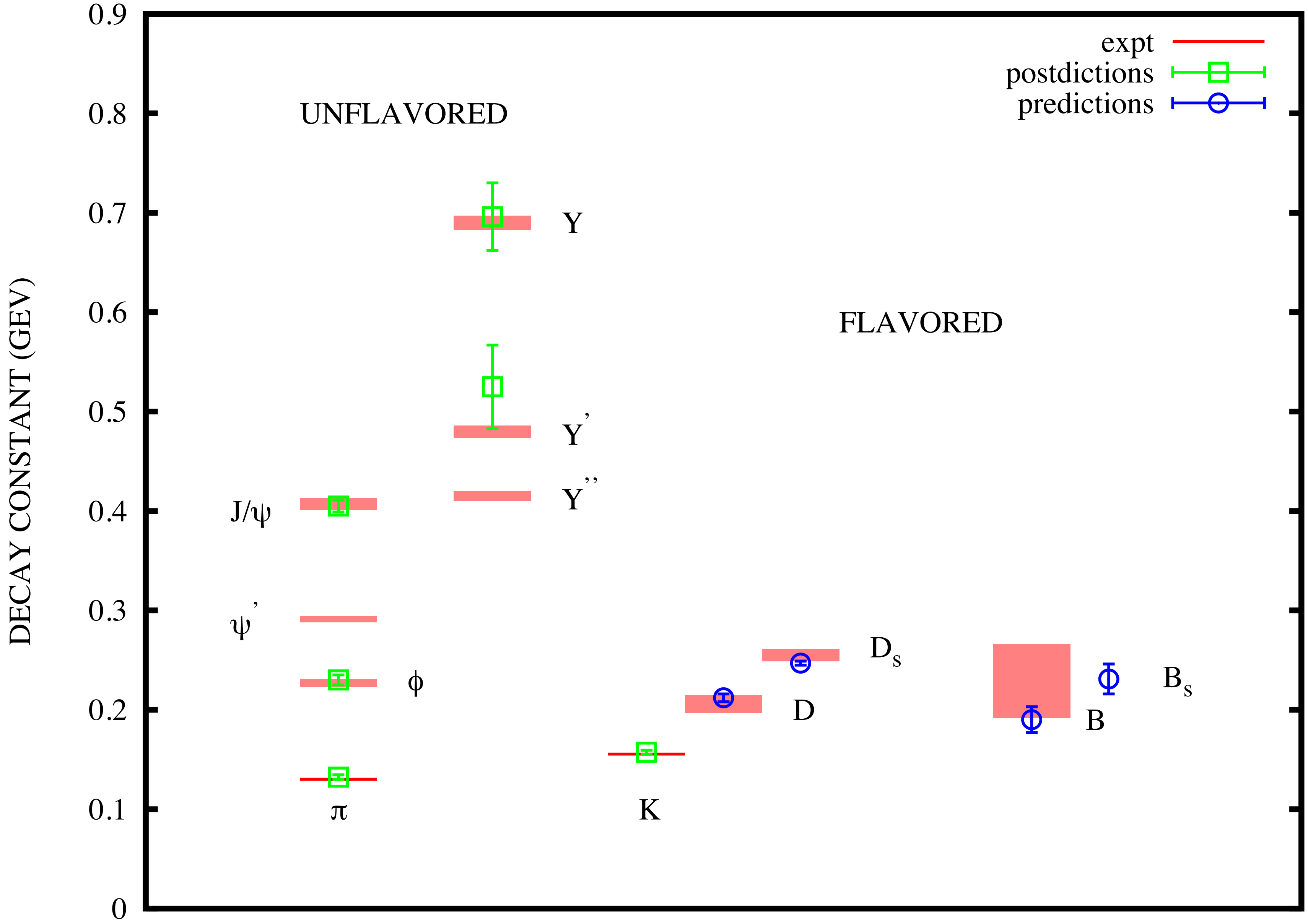
v. preliminary excited charmonium spectrum using anisotropic lattices.

Trinlat collaboration - 0801.0973

Determine em and weak decay constants from amplitudes

→ CKM elements - see other talks

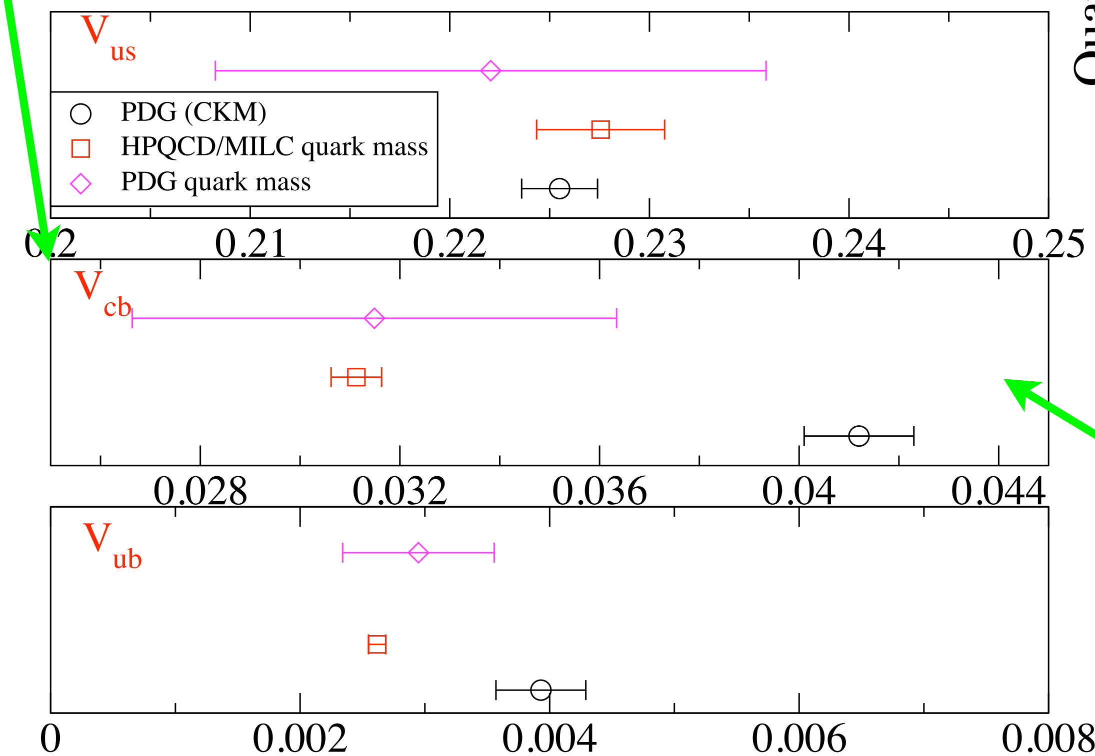
HPQCD 2009

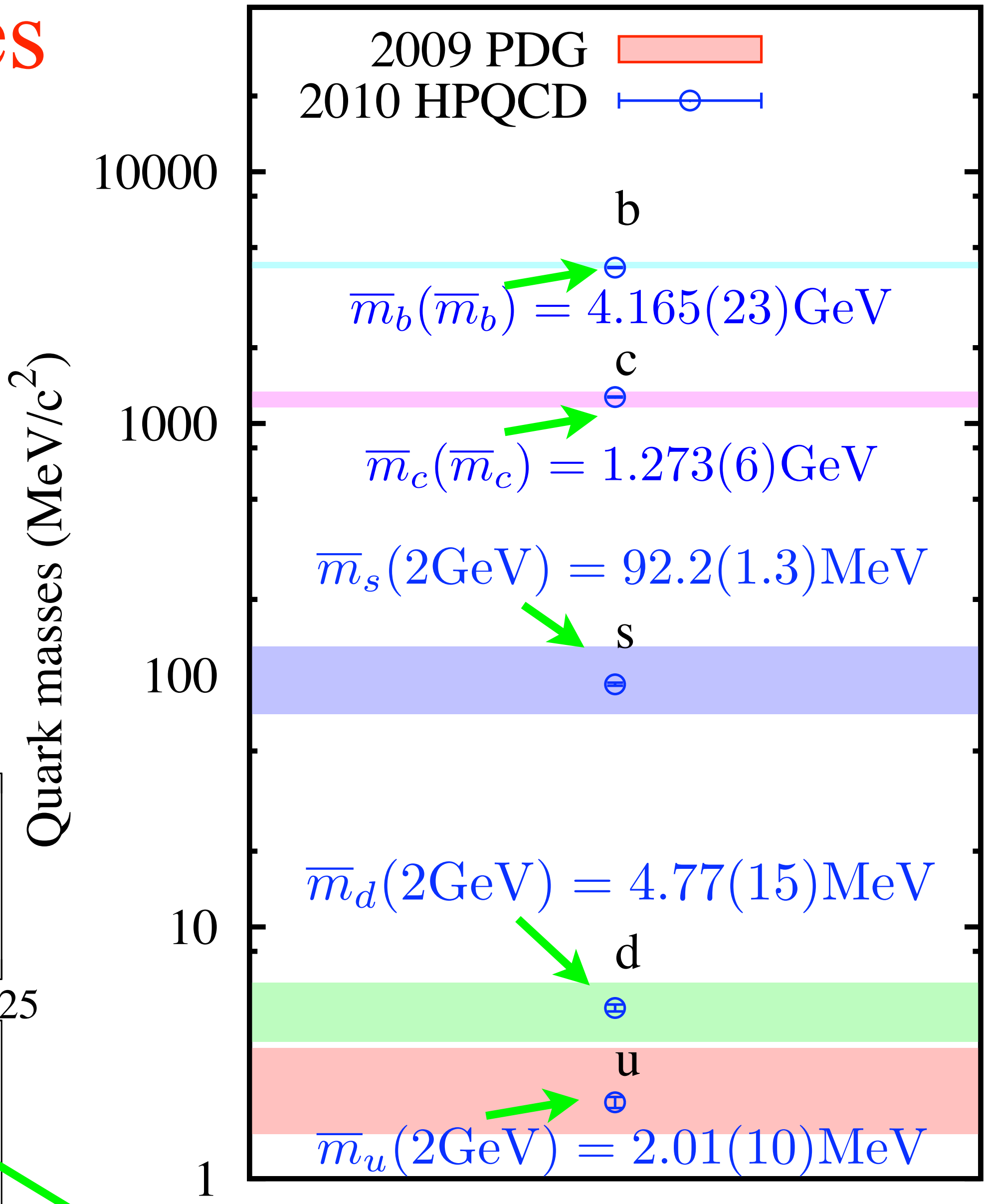


Determining quark masses

Lattice QCD has direct access to parameters in Lagrangian for accurate tuning
- issue is converting to contnm schemes such as $\overline{MS}$

Can now rule out some quark mass matrix models





HPQCD, 1004.4285

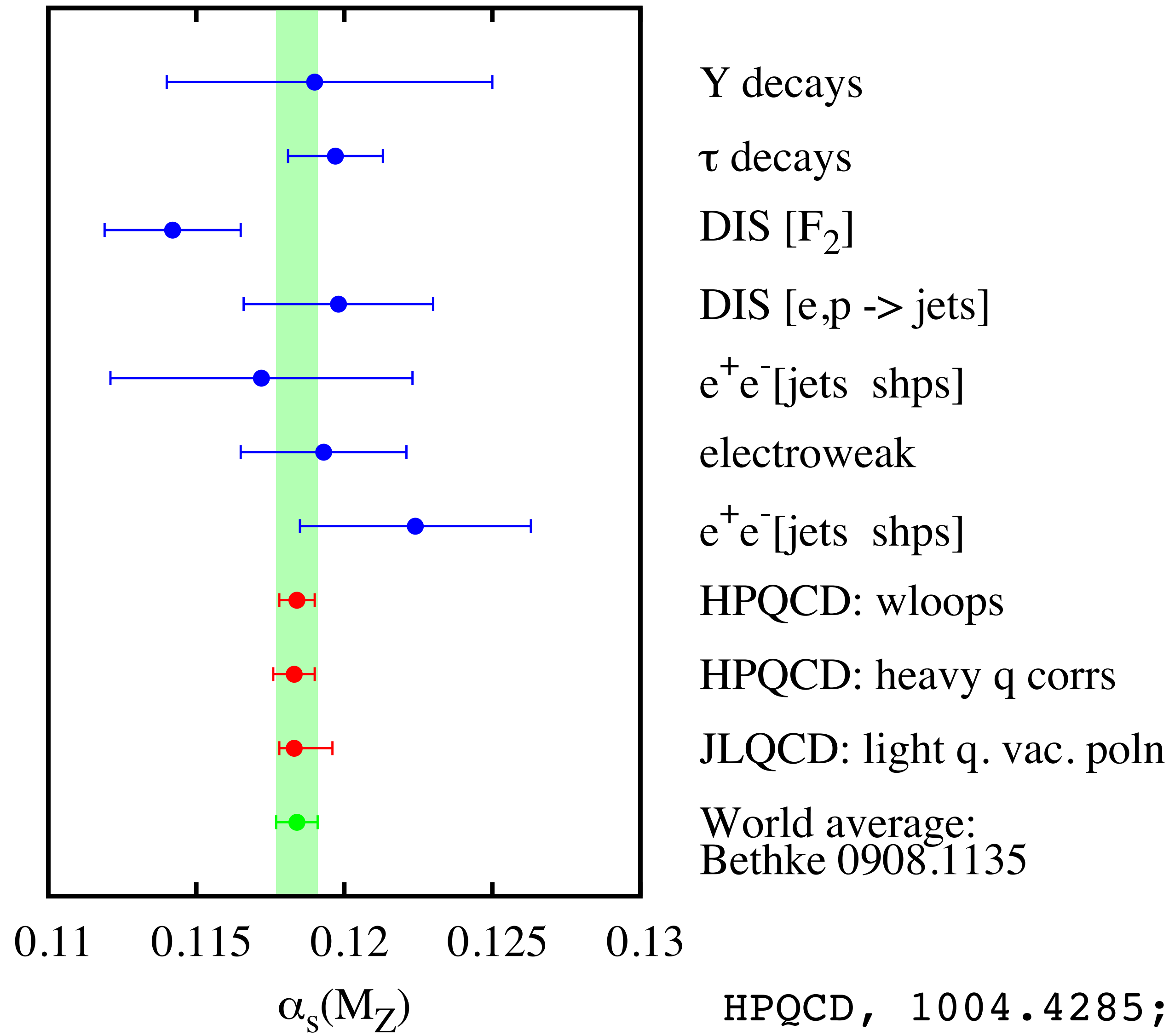
$V_{cb} = \sqrt{m_d/m_b}$ vs PDG

C. McNeile, 1004.4985, model from Chkareuli+Froggatt, hep-ph/9812499

Determining α_s

Lattice QCD now has several determns of α_s to 1%.

Key points:
- high statistical precision
- high order pert. th. exists and can estimate higher orders
- higher twist not a significant issue
- approaches very different - good test



HPQCD, 1004.4285; JLQCD,1002.0371.

Full error budgets now available for lattice calcs

stats
tuning
chiral
continuum

$$\Delta_q = 2m_{Dq} - m_{\eta c}$$

	f_K/f_π	f_K	f_π	f_{D_s}/f_D	f_{D_s}	f_D	Δ_s/Δ_d
r_1 uncerty.	0.3	1.1	1.4	0.4	1.0	1.4	0.7
a^2 extrap.	0.2	0.2	0.2	0.4	0.5	0.6	0.5
Finite vol.	0.4	0.4	0.8	0.3	0.1	0.3	0.1
$m_{u/d}$ extrap.	0.2	0.3	0.4	0.2	0.3	0.4	0.2
Stat. errors	0.2	0.4	0.5	0.5	0.6	0.7	0.6
m_s evoln.	0.1	0.1	0.1	0.3	0.3	0.3	0.5
m_d, QED, etc.	0.0	0.0	0.0	0.1	0.0	0.1	0.5
Total %	0.6	1.3	1.7	0.9	1.3	1.8	1.2

→ will tell you what is possible in future
e.g. is error from disc. errors, $m_{u,d}$ extrapoln, stats ...

Conclusion

• very accurate results are available now from lattice QCD for QCD parameters and for simple hadron masses and decay matrix elements important for flavour physics.

Future

• sets of 'next generation' gluon configs will have
$m_{u,d}$ at physical value (so no extrapoln) *or*
a down to 0.03fm (so b quarks are 'light') *or*
much higher statistics (for harder hadrons)
also can include charm in the sea now.

• Pushing errors down to 1% level will mean em corrections and $m_u \neq m_d$ must be understood.

• some harder calculations (flavor singlet, excited states, nuclear physics) will also become possible