Lattice calculations of charm and bottom physics

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Lattice meets experiment
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Introduction

Lattice now delivers unquenched (2+1 flavor) determinations for many quantities of interest.

Focus on heavy-light quantities needed for CKM fits.

Heavy quarks on the lattice

The lattice cut-off is smaller than the heavy quark masses for realistic lattices. The solution(s): heavy quark effective theory(HQET) or nonrelativistic QCD

Fermilab Method:

Continuum QCD $\longrightarrow$ Lattice gauge theory
(using HQET)

nonrelativistic QCD method:

Continuum QCD $\longrightarrow$ Nonrelativistic QCD $\longrightarrow$ Lattice gauge theory

- Both methods require tuning parameters of the lattice action
- The currents and 4-quark operators must be matched as well. Typically this is done in lattice perturbation theory.

Other Approaches

The extrapolation method (Becirevic, et al, hep-lat/0002025; QCDSF, hep-lat/0701015):

In this case one simulates at masses around the charm quark (or heavier) and extrapolates to bottom with fit functions motivated by HQET.

The step-scaling method (Guazzini, Sommer, and Tantalo, hep-lat/0609065):

One starts with a small volume where the b quark can be computed directly, where the finite size effects can be eliminated through step scaling functions which give the change of the observables when L is changed to $2L$.

Matching Errors

One must estimate errors due to inexact matching of the lattice to the continuum.

In the Fermilab method, all errors associated with discretizing the action are combined. These errors are then estimated using knowledge of HQET power counting.

In the nonrelativistic QCD method, there are “relativistic errors” associated with using NRQCD [$O(\alpha_s \Lambda_{QCD}/m_Q)$, $O(\Lambda_{QCD}^2/m_Q^2)$], and “perturbation theory errors” associated with matching NRQCD to the lattice [$O(\alpha_s^2)$].

Lattice averages

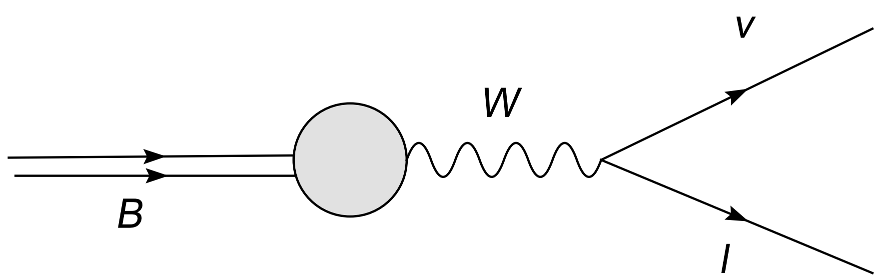
Results from at least two groups for most quantities. Need to average. See www.latticeaverages.org, page maintained by JL, E Lunghi, and R S Van de Water.

We don't have a complete correlation matrix between various lattice calculations. We combine lattice errors with the following assumptions:

- Only average 2+1 flavor results with complete, itemized error budget.
- Whenever a source of error is at all correlated between two lattice calculations, we assign the degree-of-correlation a value of 100%.
- We adopt the PDG prescription to combine several measurements whose spread is wider than what is expected from the quoted errors. The error on the average is rescaled by the square root of the minimum of the chi-square per degree of freedom:

$$\sqrt{\sum (x_i - x_{\rm avg})(C^{-1})_{ij}(x_j - x_{\rm avg})/(n-1)} \tag{1}$$

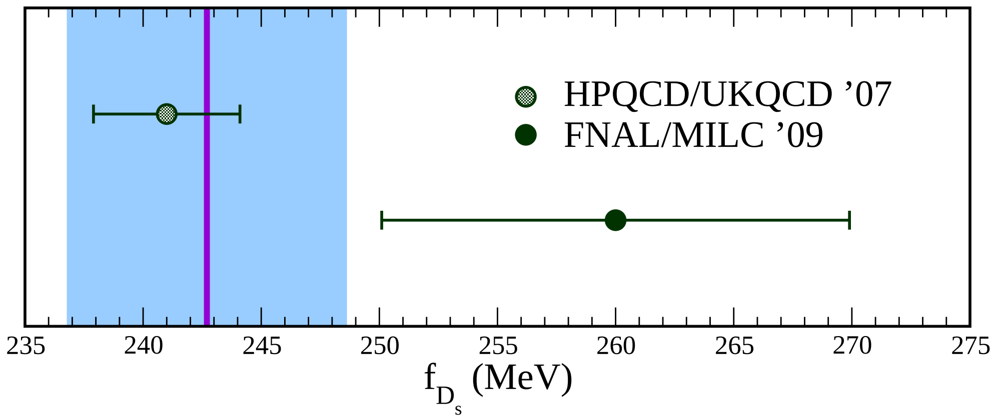
Leptonic decay constants

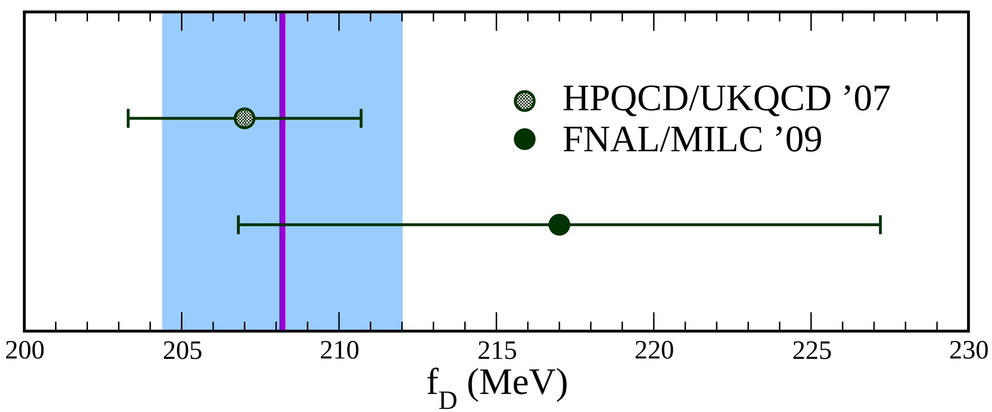


$$\mathcal{B}(B \to \tau \overline{\nu}_\tau) = (\text{known factor})\,(\text{CKM factor})(\text{QCD factor}) \tag{2}$$

$$\mathcal{B}(B \to \tau \overline{\nu}_\tau) = \frac{G_F^2 m_B m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B \tag{3}$$

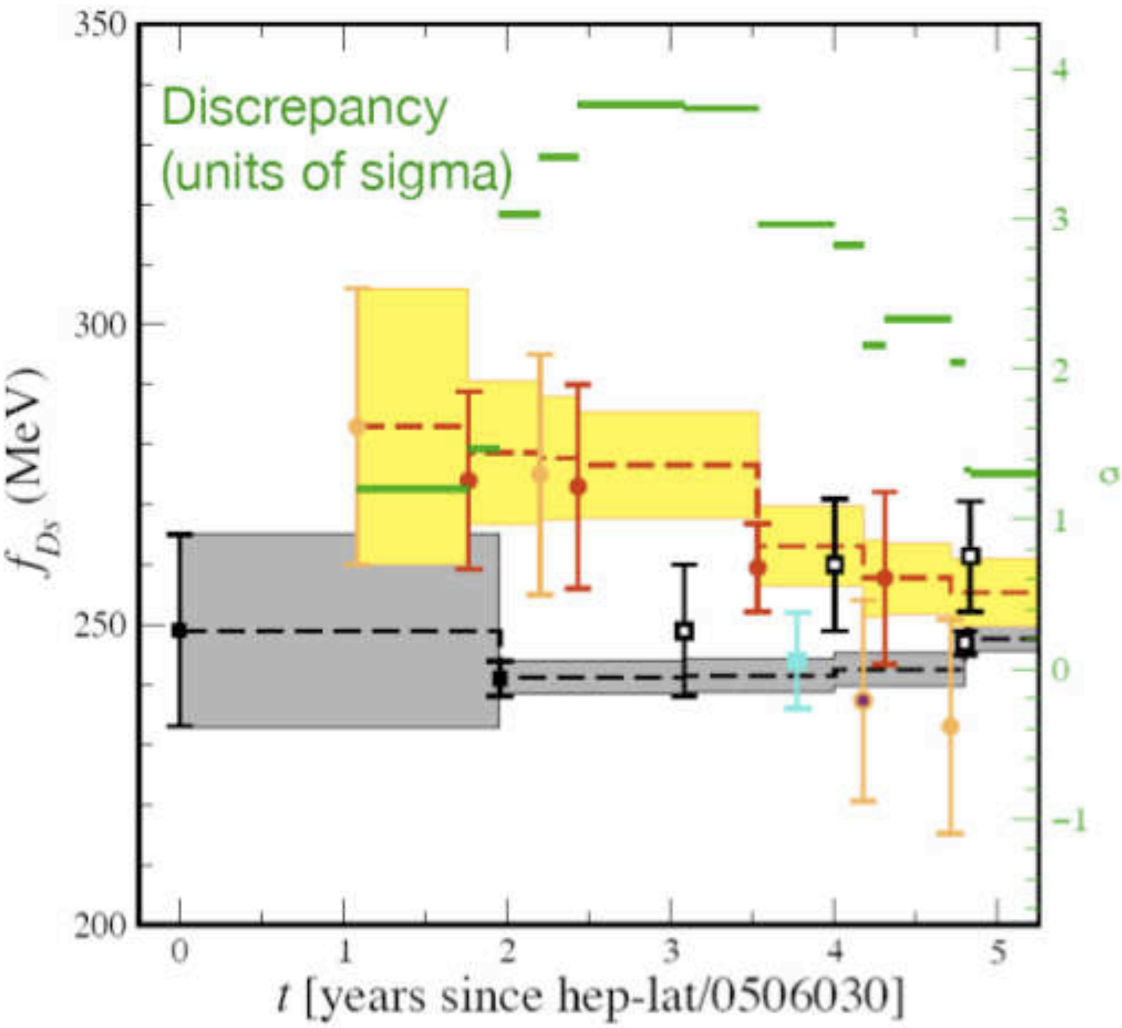
f_{D^+}, f_{D_s}



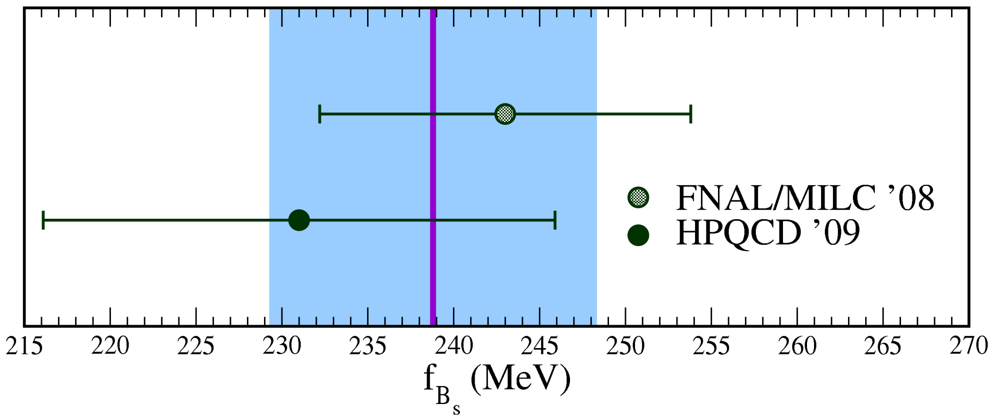


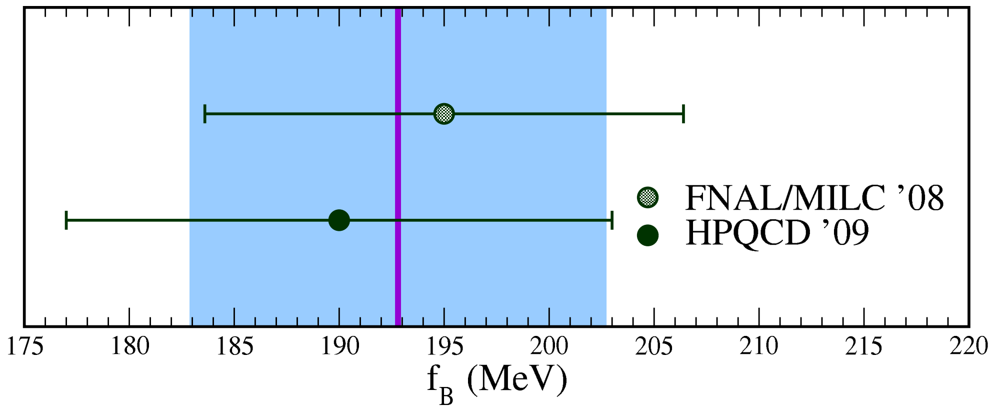
$f_{D_s} = 242.8 \pm 6.0$ MeV, $\quad f_D = 208.1 \pm 3.7$ MeV.

Saga of f_{D_s} from Andreas Kronfeld



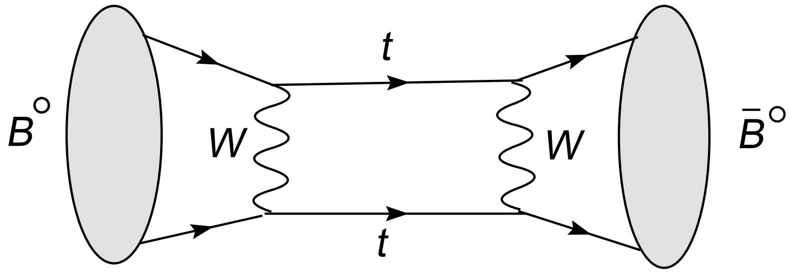
f_B, f_{B_s}





$f_{B_s} = 238.8 \pm 9.5, \quad f_B = 192.8 \pm 9.9$ MeV.

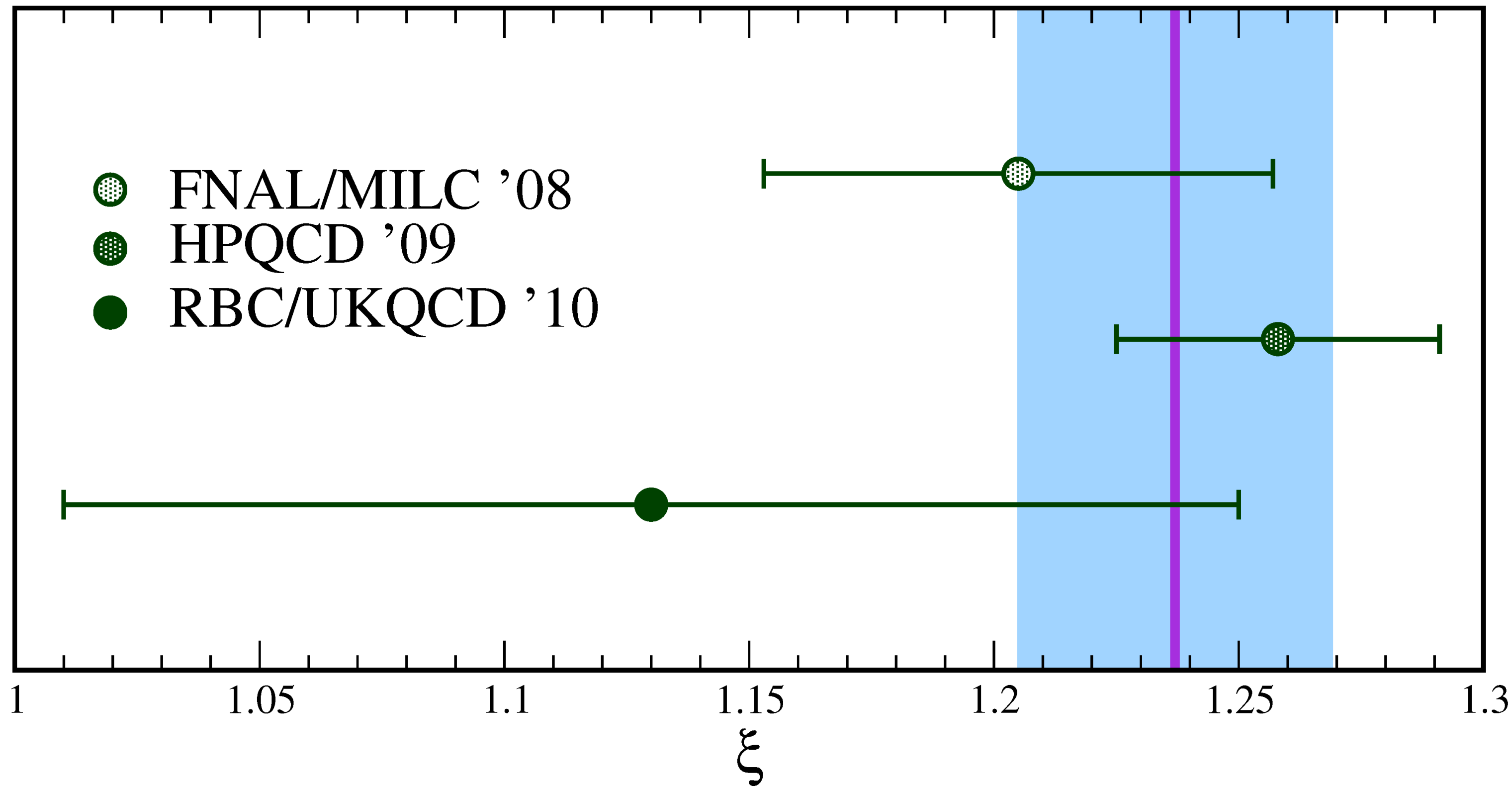
B-$\overline{B}$ Mixing



$$\langle \overline{B}^0 | (\bar{b}d)_{V-A} (\bar{b}d)_{V-A} | B^0 \rangle \equiv \frac{8}{3} m_B^2 f_B^2 B_B, \tag{4}$$

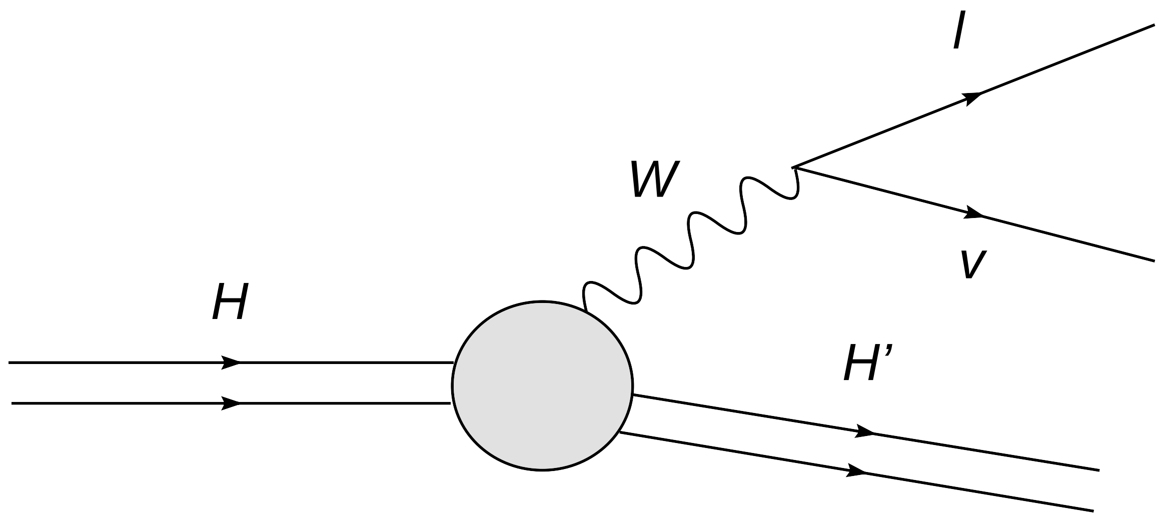
$$\Delta M_s = \frac{G_F^2 M_W^2}{6\pi^2} |V_{ts}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_s} f_{B_s}^2 \widehat{B}_{B_s} \tag{5}$$

ξ



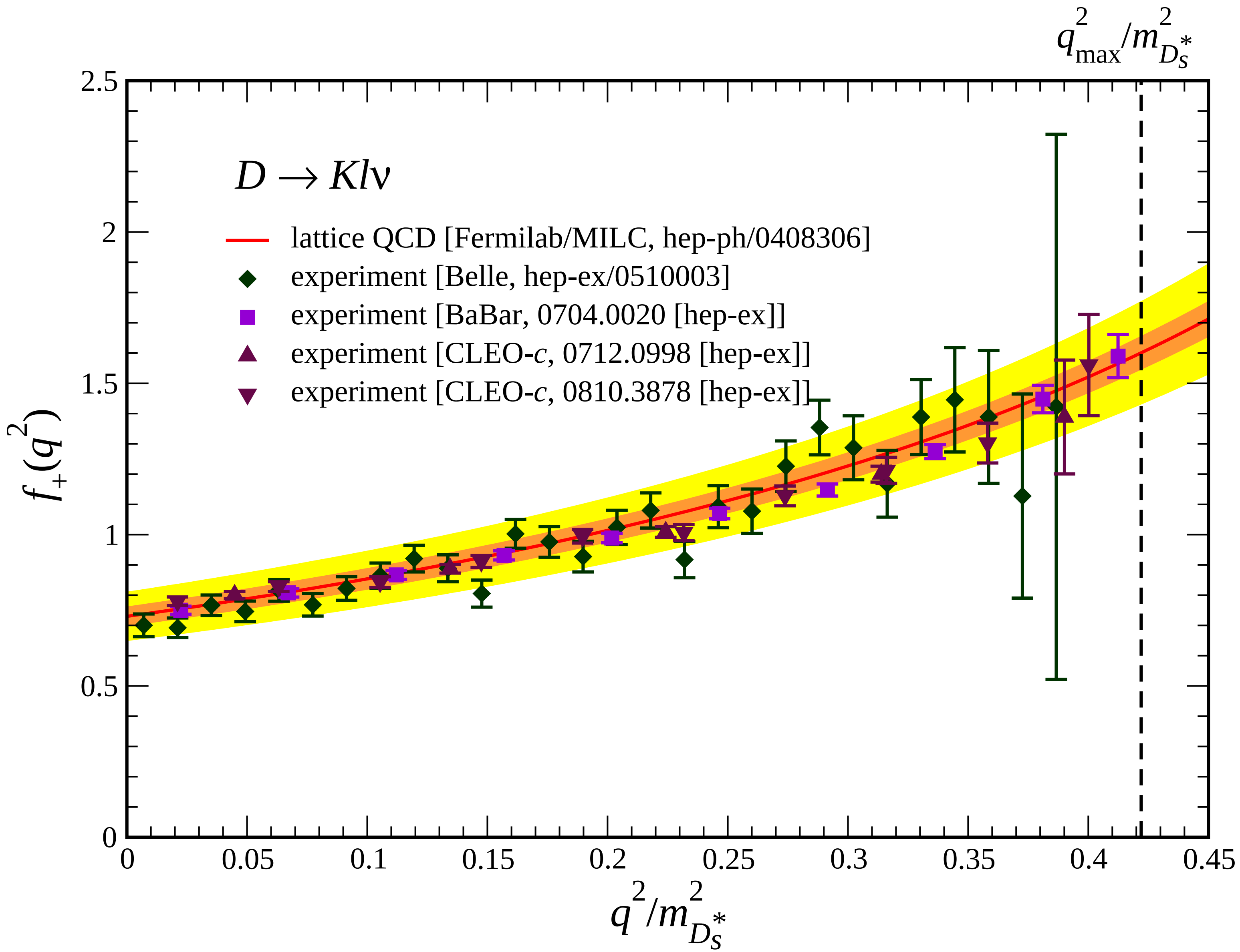
$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}} = 1.237 \pm 0.032.$$

Semileptonic decays

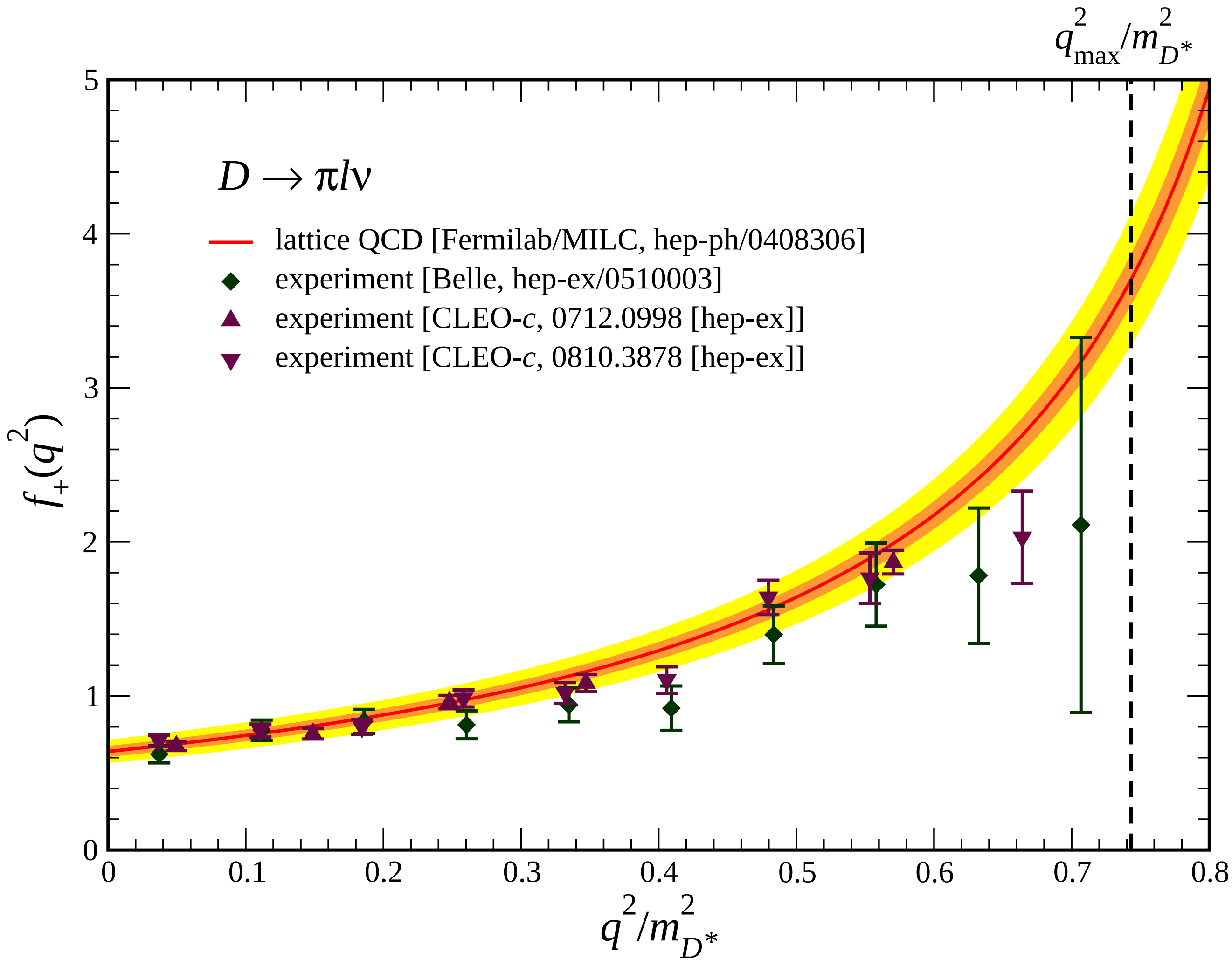


Vertex proportional to $|V_{qq'}|$. In order to extract it, nonperturbative input is needed.

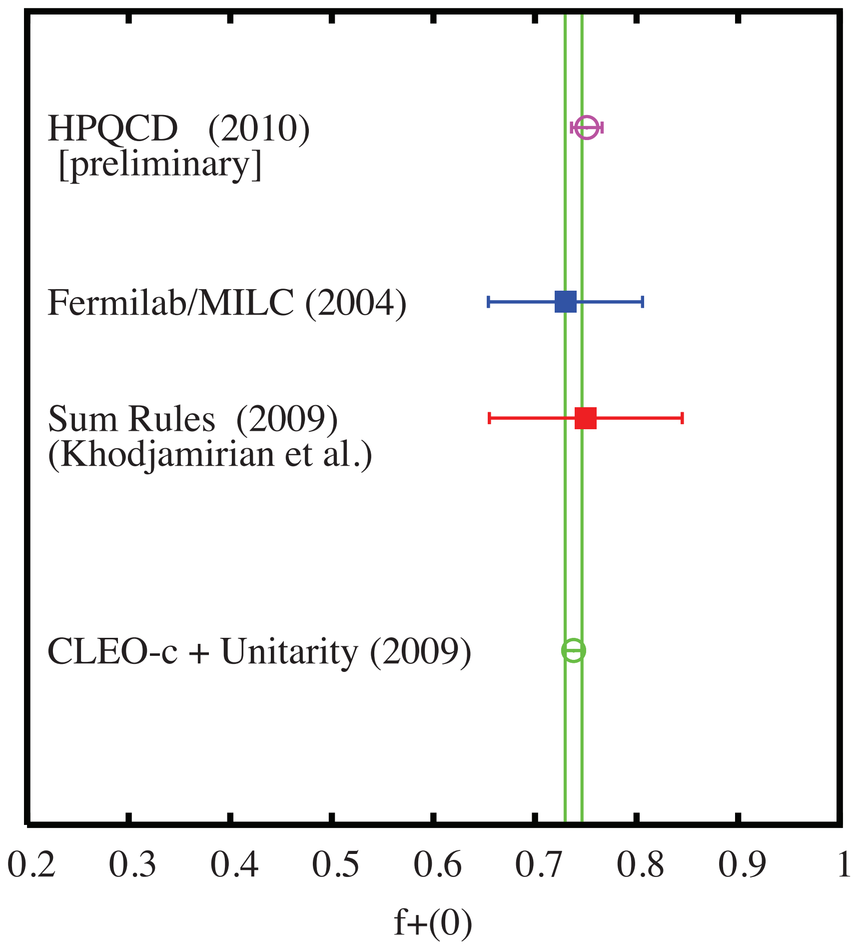
Prediction of form factor shape



Prediction of form factor shape

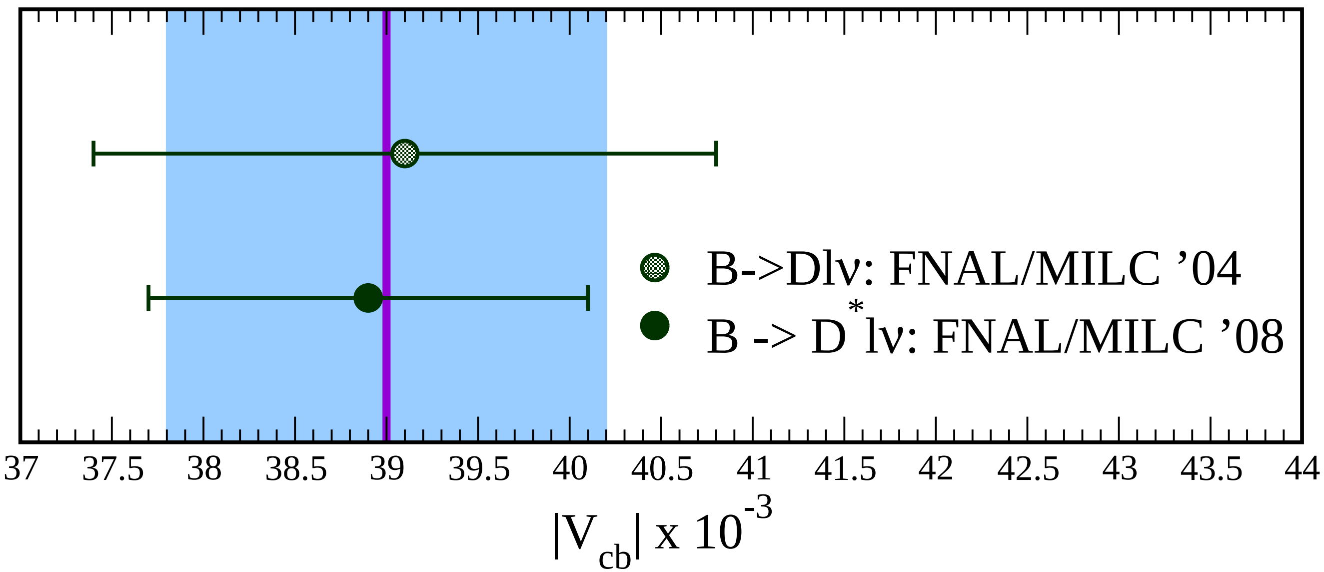


Preliminary HPQCD result



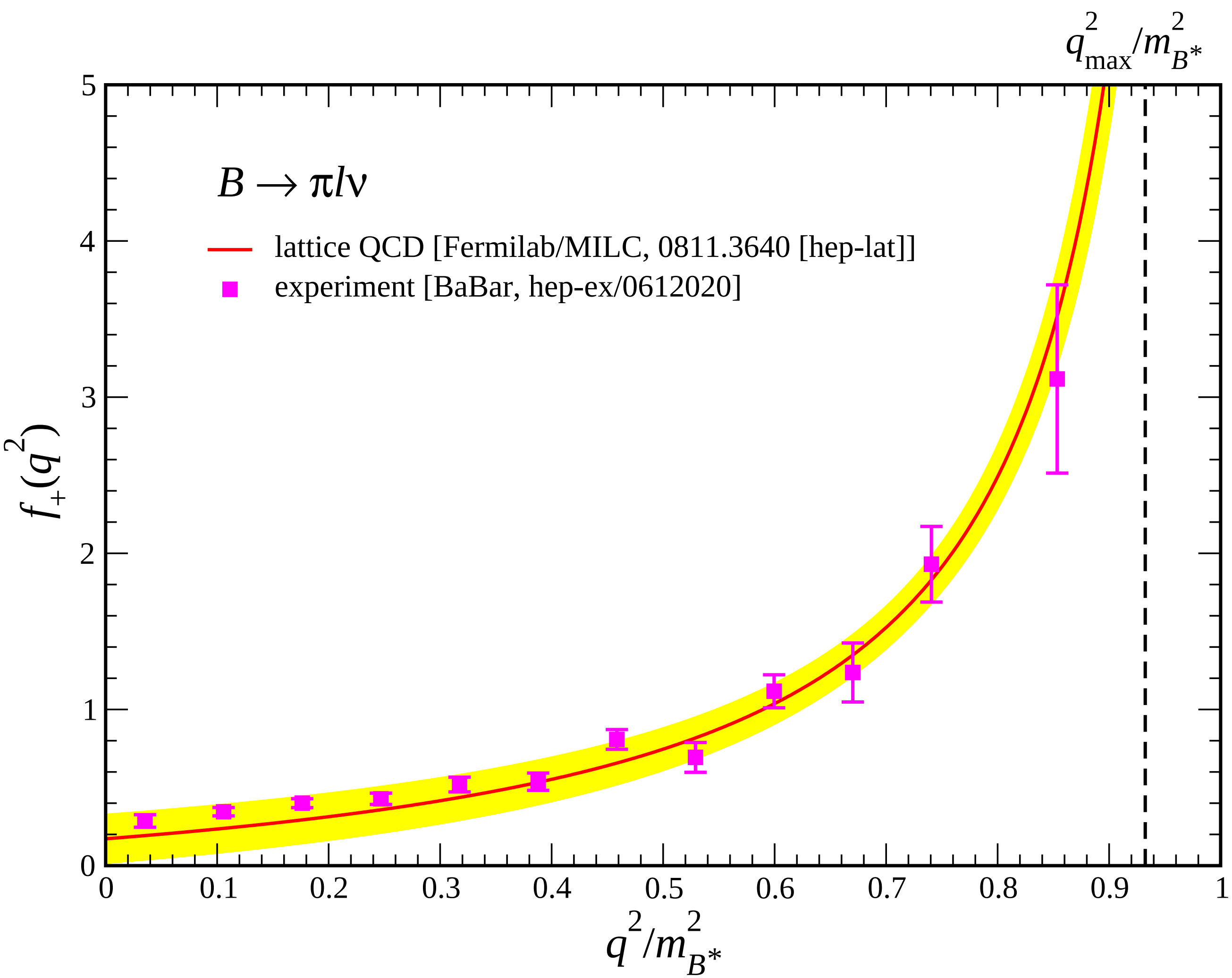
$D \to K \ell \nu$

$|V_{cb}|$

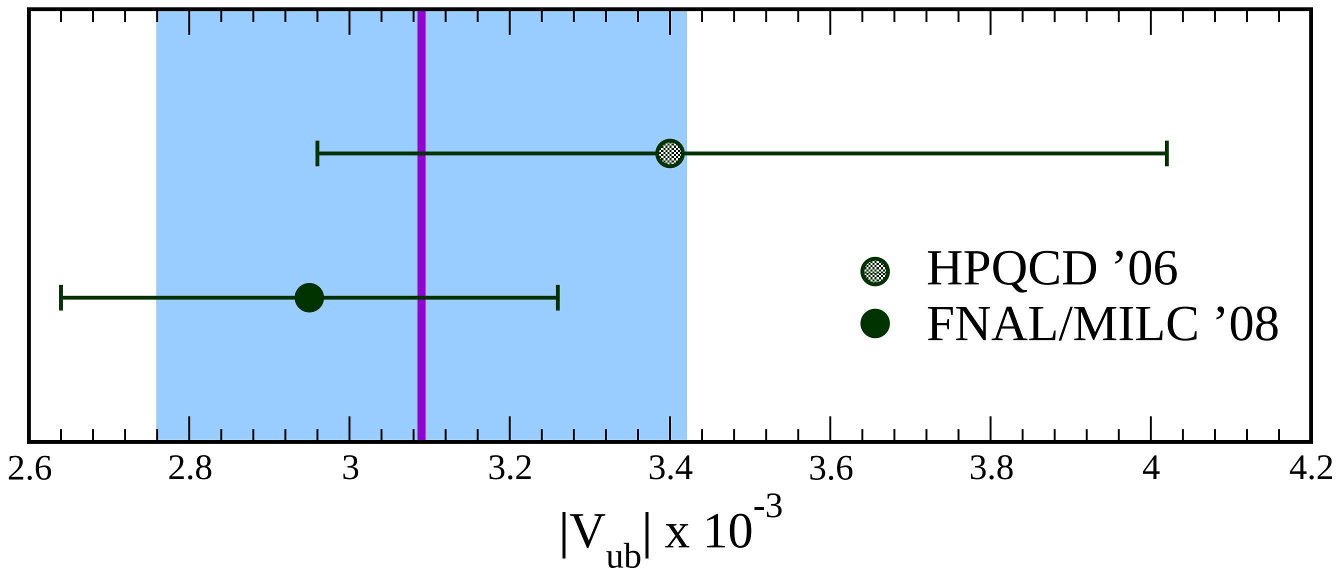


$$|V_{cb}| = (39.0 \pm 1.2) \times 10^{-3}$$

Prediction of form factor shape



$|V_{ub}|$

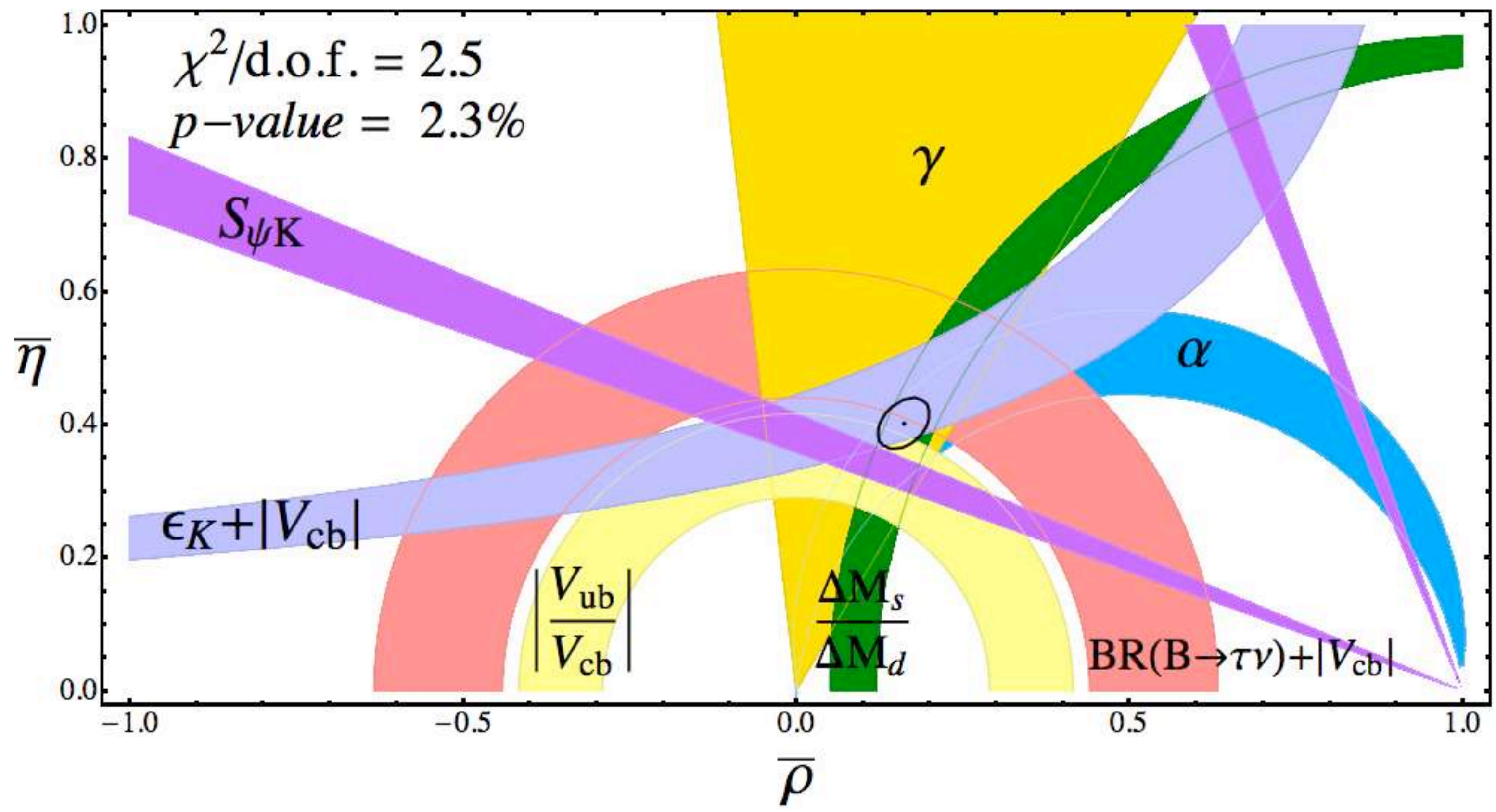


$$|V_{ub}| = (3.09 \pm 0.33) \times 10^{-3}$$

Future Error Budgets

Quantity	Current lattice error	Current exp error	2 year lattice error
$D_{+,s} \to \ell\nu$	4%	2%	$\sim 1\%$
$B \to \ell\nu$	5%	20%	$\sim 2\%$
$B \to D^*\ell\nu$	3%	2%	$\sim 2\%$
$B \to D\ell\nu$	2%	5%	$\sim 2\%$
$B \to \pi\ell\nu$	11%	4%	$\sim 5\%$
$B_{s,d}$	5%	0.5%	$\sim 3\%$
ξ	3%	0.5%	$\sim 1\%$

Conclusion



For latest averages, see www.latticeaverages.org