Kaon physics lattice calculations

Lattice QCD meets experiment

Glasgow, 03.05.2010

Andreas Jüttner
CERN Theory Division

Lattice QCD results relevant for flavour physics

- quality of lattice results
- status: f_K/f_π, $f_+^{K\to\pi}(0)$, $\hat{B}_K$
- phenomenological implications
- will there be improvements?
- $K \to \pi\pi$ status
- outlook

Quality of lattice results

- 9 good quality results for f_K/f_π
- 5 good quality results for $\hat{B}_K$
- 2 good quality results for $f_+^{K\pi}(0)$
- 0 good quality results for $K \to \pi\pi$

Quality of lattice results

- 9 good quality results for f_K/f_π
- 5 good quality results for $\hat{B}_K$
- 2 good quality results for $f_+^{K\pi}(0)$
- 0 good quality results for $K \to \pi\pi$

- all results have been determined using different lattice techniques and different sets of simulation parameters
- in most cases no correlation between *bare* results
- data analysis affected by very similar systematics (L, a, m_π, renormalization, running)

Quality of lattice results - FLAG

Flavia Net Lattice Averaging Group (**FLAG**) was founded to allow also to an outsider to judge the quality and 'state-of-the-art'-fulnes of lattice results relevant to flavor physics

- G. Colangelo, S. Dürr, A. J., L. Lellouch, H. Leutwyler, V. Lubicz, S. Necco, C. Sachrajda, S. Simula, A. Vladikas, U. Wenger, H. Wittig

Quality of lattice results - FLAG

Flavia Net Lattice Averaging Group (**FLAG**) was founded to allow also to an outsider to judge the quality and 'state-of-the-art'-fulnes of lattice results relevant to flavor physics

- G. Colangelo, S. Dürr, A. J., L. Lellouch, H. Leutwyler, V. Lubicz, S. Necco, C. Sachrajda, S. Simula, A. Vladikas, U. Wenger, H. Wittig
- currently: f_K/f_π, $f_+^{K\pi}(0)$, $\hat{B}_K$, LEC's, $m_{u,d,s}$
- citeria:
  - chiral extrapolation
  - continuum extrapolation
  - finite volume effects
  - renormalization
  - running

Quality of lattice results - FLAG

Flavia Net Lattice Averaging Group (**FLAG**) was founded to allow also to an outsider to judge the quality and 'state-of-the-art'-fulnes of lattice results relevant to flavor physics

- G. Colangelo, S. Dürr, A. J., L. Lellouch, H. Leutwyler, V. Lubicz, S. Necco, C. Sachrajda, S. Simula, A. Vladikas, U. Wenger, H. Wittig
- currently: f_K/f_π, $f_+^{K\pi}(0)$, $\hat{B}_K$, LEC's, $m_{u,d,s}$
- citeria:
  - chiral extrapolation
  - continuum extrapolation
  - finite volume effects
  - renormalization
  - running
- color code ★, ●, ■
- criteria will change with time
- averages/best values: no red (if possible)
- dead-line (skip all later, freeze publication status)

Quality of lattice results - FLAG

Flavia Net Lattice Averaging Group (**FLAG**) was founded to allow also to an outsider to judge the quality and 'state-of-the-art'-fulnes of lattice results relevant to flavor physics

- G. Colangelo, S. Dürr, A. J., L. Lellouch, H. Leutwyler, V. Lubicz, S. Necco, C. Sachrajda, S. Simula, A. Vladikas, U. Wenger, H. Wittig
- currently: f_K/f_π, $f_+^{K\pi}(0)$, $\hat{B}_K$, LEC's, $m_{u,d,s}$
- citeria:
  - chiral extrapolation
  - continuum extrapolation
  - finite volume effects
  - renormalization
  - running
- color code ★, ●, ■
- criteria will change with time
- averages/best values: no red (if possible)
- dead-line (skip all later, freeze publication status)
- FLAG averages/best values soon on arXiv
- also: Lattice results relevant for CKM triangle analysis
  Laiho, Lunghi, van de Water PRD, 81, 034503 (2008)

Lattice QCD and experiment in flavour physics

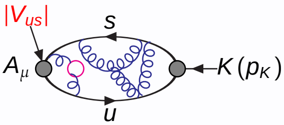
in practice:

- measure decay rates $\Gamma(i \to j)$
- compute process in SM (FF, RC)
- $\Gamma(i \to j) = const. \times G_F^2 |V_{ij}|^2 \times FF \times RC$

Lattice QCD and experiment in flavour physics

in practice:

- measure decay rates $\Gamma(i \to j)$
- compute process in SM (FF, RC)
- $\Gamma(i \to j) = const. \times G_F^2 |V_{ij}|^2 \times FF \times RC$

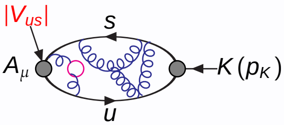


$\langle 0|A_\mu(0)|K(p_K)\rangle|_{\rm QCD}$

Lattice QCD and experiment in flavour physics

in practice:

- measure decay rates $\Gamma(i \to j)$
- compute process in SM (FF, RC)
- $\Gamma(i \to j) = const. \times G_F^2 |V_{ij}|^2 \times FF \times RC$



$\langle 0|A_\mu(0)|K(p_K)\rangle|_{\mathrm{QCD}}$

In 2004 Marciano first used the lattice determination of f_K/f_π to determine $|V_{us}|$:
(Marciano, Phys.Rev.Lett. 2004)

$$\frac{\Gamma(K \to \mu\bar{\nu}_\mu(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}_\mu(\gamma))} = \frac{|V_{us}|^2}{|V_{ud}|^2} \left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K(1 - m_\mu^2/m_K^2)}{m_\pi(1 - m_\mu^2/m_\pi^2)} \times 0.9930(35)$$

Results for f_K/f_π

Collaboration	N_f	publication status	chiral extrapolation	finite volume errors	continuum extrapolation	f_K/f_π
MILC 09A	2+1	C	★	★	★	$1.198(2)(^{+6}_{-8})$
MILC 09	2+1	P	★	★	★	$1.197(3)(^{+6}_{-13})$
ALVdW 08	2+1	C	★	●	●	1.191(16)(17)
PACS-CS 08, 08B	2+1	A	★	■	■	1.189(20)
BMW 08	2+1	C	★	★	★	1.18(1)(1)
HPQCD/UKQCD 08	2+1	A	★	●	★	1.189(2)(7)
RBC/UKQCD 08	2+1	A	●	★	■	1.205(18)(62)
NPLQCD 06	2+1	A	●	■	■	$1.218(2)(^{+11}_{-24})$
ETM 09	2	A	●	●	★	1.210(6)(15)(9)
QCDSF/UKQCD 07	2	C	●	★	●	1.21(3)

precision of $\approx 0.6 - 0.8\%$ possible

Results for f_K/f_π

BMW Phys.Rev.D81:054507,2010

Source uncertainty/error	uncertainty/error on f_K/f_π
statistics	0.6%
chiral extrapolation	
- functional form	0.3%
- pion mass range	0.3%
continuum extrapolation	0.3%
exited states	0.2%
scale setting	0.1%
finite volume	0.1%
total syst	0.5%
total	0.8%

- “dominant” uncertainties: chiral and continuum extrapolation (other collabs reach much smaller stat. error than BMW)

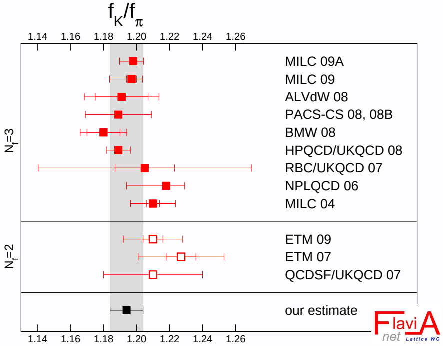
Results for f_K/f_π

BMW Phys.Rev.D81:054507,2010

Source uncertainty/error	uncertainty/error on f_K/f_π	
statistics	0.6%	
chiral extrapolation		
- functional form	0.3%	
- pion mass range	0.3%	$\gtrsim$ 190MeV
continuum extrapolation	0.3%	$\gtrsim$ 0.064fm
exited states	0.2%	
scale setting	0.1%	
finite volume	0.1%	
total syst	0.5%	
total	0.8%	

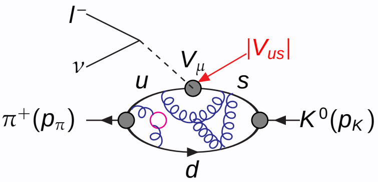
- “dominant” uncertainties: chiral and continuum extrapolation (other collabs reach much smaller stat. error than BMW)

Results for f_K/f_π



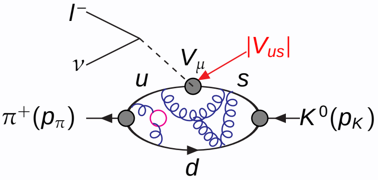
- very good agreement
- two (three) very advanced $N_f = 2+1$-results:
  MILC 09, HPQCD/UKQCD 08, (BMW)
  one very advanced $N_f = 2$-result: ETM 09

Results for $f_+^{K\pi}(0)$



$$\langle\pi(p_\pi)|V_\mu(0)|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K-p_\pi)_\mu$$

Results for $f_+^{K\pi}(0)$



$$\langle\pi(p_\pi)|V_\mu(0)|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K-p_\pi)_\mu$$

$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} I\, S_{\rm EW}[1+\Delta_{SU(2)}+\Delta_{\rm EM}] \times |V_{us}|^2 |f_+^{K\pi}(0)|^2$$

- I phase space integral (via FF shape from experiment)
- $S_{\rm EW}$ short distance EW corrections
- $\Delta_{SU(2)}$ Iso-spin breaking corrections
- $\Delta_{\rm EM}$ long distance EM corrections

Antonelli et al., arXiv:0801.1817 (KLOE, KTeV, ISTRA+, NA48): $|V_{us} f_+^{K\pi}(0)| = 0.21661(47)$

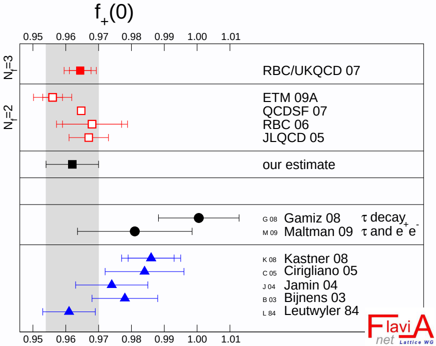
$\rightarrow$ sub-1%-precision for $f_+^{K\pi}(0)$ required

Results for $f_+^{K\pi}(0)$

Collaboration	N_f	publication status	chiral extrapolation	finite volume errors	continuum extrapolation	$f_+(0)$
RBC/UKQCD 07	2+1	A	●	★	■	0.9644(33)(34)(14)
ETM 09A	2	A	●	●	●	0.9560(57)(62)
QCDSF 07	2	C	■	★	■	$0.9647(15)_{stat}$
RBC 06	2	A	■	★	■	0.968(9)(6)
JLQCD 05	2	C	■	★	■	0.967(6)

precision $\approx$ 0.5% possible

Results for $f_+^{K\pi}(0)$



Comments:

- only two state-of-the art calculations:
  $N_f = 2$ ETM 09A *PRD80:111502,2009* and
  $N_f = 2+1$ RBC+UKQCD 07 *PRL100:141601,2008, arXiv:1004.0886*
- the former ($N_f = 2$) result being technically advanced:
  lighter pion masses, 3 lattice spacings
- no *s*-quark effects - but still two different theories, so wait for higher precision and don't average!

Results for $f_+^{K\pi}(0)$

RBC+UKQCD Phys.Rev.Lett.100:141601,2008, arXiv:1004.0886

Source uncertainty/error	uncertainty/error on $f_+^{K\pi}(0)$
statistical	0.3%
chiral extrapolation	0.4%
continuum extrapolation	0.1%
total systematic	0.4%
total	0.5%

- dominant uncertainty from chiral extrapolation

Results for $f_+^{K\pi}(0)$

RBC+UKQCD Phys.Rev.Lett.100:141601,2008, arXiv:1004.0886

Source uncertainty/error	uncertainty/error on $f_+^{K\pi}(0)$	
statistical	0.3%	
chiral extrapolation	0.4%	$\gtrsim$ 330MeV
continuum extrapolation	0.1%	
total systematic	0.4%	
total	0.5%	

- dominant uncertainty from chiral extrapolation

Standard Model correlations

$$|V_{us}|f_+^{K\pi}(0) = 0.21661(47) \quad \text{Antonelli et al., arXiv:0801.1817} \tag{1}$$

$$\left|\frac{V_{us}f_K}{V_{ud}f_\pi}\right| = 0.27599(59) \quad \text{Marciano, Phys.Rev.Lett. 2004} \tag{2}$$

$$|V_u| \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1+? \tag{3}$$

$$|V_{ud}| = 0.97425(22) \quad \text{Towner and Hardy Phys.Rev.C 2008} \tag{4}$$

Standard Model correlations

$$|V_{us}|f_+^{K\pi}(0) = 0.21661(47) \quad \text{Antonelli et al., arXiv:0801.1817} \qquad (1)$$

$$\left|\frac{V_{us}f_K}{V_{ud}f_\pi}\right| = 0.27599(59) \quad \text{Marciano, Phys.Rev.Lett. 2004} \qquad (2)$$

$$|V_u| \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1+? \qquad (3)$$

$$|V_{ud}| = 0.97425(22) \quad \text{Towner and Hardy Phys.Rev.C 2008} \qquad (4)$$

types of analysis

- information gathered without lattice
  - **use (1-4), no lattice - "experimental values"**
    $|V_{us}| = 0.22544(95)$, $f_+^{K\pi}(0) = 0.9608(46)$, $f_K/f_\pi = 1.1927(59)$

Standard Model correlations

$$|V_{us}|f_+^{K\pi}(0) = 0.21661(47) \quad \text{Antonelli et al., arXiv:0801.1817} \qquad (1)$$

$$\left|\frac{V_{us}f_K}{V_{ud}f_\pi}\right| = 0.27599(59) \quad \text{Marciano, Phys.Rev.Lett. 2004} \qquad (2)$$

$$|V_u| \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1+? \qquad (3)$$

$$|V_{ud}| = 0.97425(22) \quad \text{Towner and Hardy Phys.Rev.C 2008} \qquad (4)$$

types of analysis

- information gathered without lattice
  - **use (1-4), no lattice - "experimental values"**
    $|V_{us}| = 0.22544(95)$, $f_+^{K\pi}(0) = 0.9608(46)$, $f_K/f_\pi = 1.1927(59)$
- information gathered with lattice
  - **use lattice and (1-2) (and (4)) - testing the SM**
    i.e. use lattice and kaon/pion BRs (and $|V_{ud}|$) $\rightarrow$ unitarity test
    is Fermi coupling universal?
  - **use lattice and (1)-(3) - analysis within the SM**
    3 equations and four unknowns: $|V_{ud}|$, $|V_{us}|$, $f_+^{K\pi}(0)$, f_K/f_π; let lattice provide one of them and predict the other ones
    are all results consistent?

Standard Model correlations

in the following: $N_f = 2 + 1$, only with *good* lattice results

$f_+^{K\pi}(0)$ by (RBC/UKQCD) and
f_K/f_π by (MILC+HPQCD)

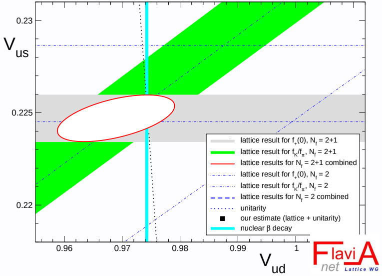
- using SM correlations only:
  $|V_{us}| = 0.2247(13)$ from $f_+^{K\pi}(0)$ and $|V_{us}/V_{ud}| = 0.2319(20)$ from f_K/f_π

Standard Model correlations

in the following: $N_f = 2 + 1$, only with *good* lattice results

$f_+^{K\pi}(0)$ by (RBC/UKQCD) and
f_K/f_π by (MILC+HPQCD)

- using SM correlations only:
  $|V_{us}| = 0.2247(13)$ from $f_+^{K\pi}(0)$ and $|V_{us}/V_{ud}| = 0.2319(20)$ from f_K/f_π

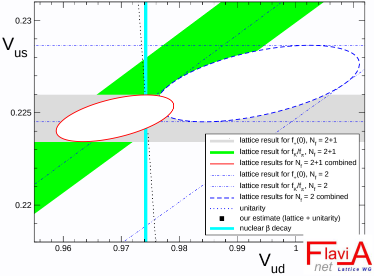


Standard Model correlations

in the following: $N_f = 2+1$, only with *good* lattice results

$$f_+^{K\pi}(0) \text{ by (RBC/UKQCD) and}$$
$$f_K/f_\pi \text{ by (MILC+HPQCD)}$$

- using SM correlations only:
  $|V_{us}| = 0.2247(13)$ from $f_+^{K\pi}(0)$ and $|V_{us}/V_{ud}| = 0.2319(20)$ from f_K/f_π

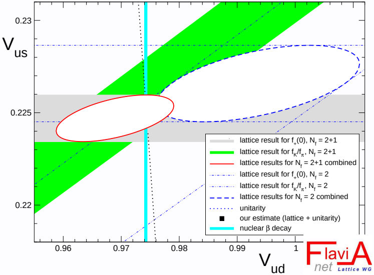


Standard Model correlations

in the following: $N_f = 2+1$, only with *good* lattice results

$f_+^{K\pi}(0)$ by (RBC/UKQCD) and
f_K/f_π by (MILC+HPQCD)

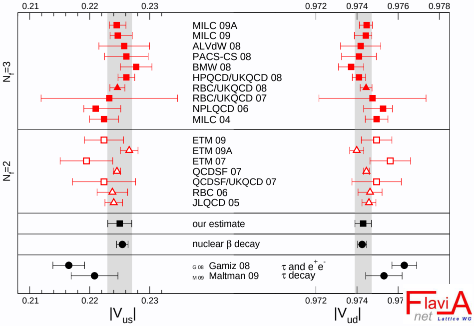
- using SM correlations only:
  $|V_{us}| = 0.2247(13)$ from $f_+^{K\pi}(0)$ and $|V_{us}/V_{ud}| = 0.2319(20)$ from f_K/f_π



- testing SM unitarity:
  - using lattice + BRs: $|V_u| = 0.989(20)$
  - using lattice + BRs + $|V_{ud}|$: $|V_u| = 1.0002(10)$
  - can we rely on $|V_{ud}|$?

First row unitairty

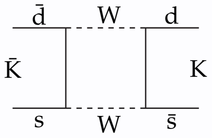
- analysis within the SM: using lattice + BRs + unitarity



- squares (input f_K/f_π) and triangles (input $f_+^{K\pi}(0)$) agree very well for determination of $|V_{us}|$ and $|V_{ud}|$
- $N_f = 2$ and $N_f = 2+1$ results nearly the same
- remarkable precision and agreement with results for $|V_{ud}|$ from nuclear β-decay
  (precision intuitively clear from plot)

Results for B_K

$$\epsilon_K = \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})}$$



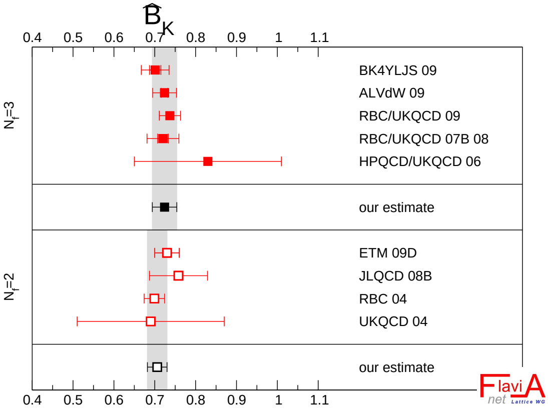
$$|\epsilon_K| = \kappa_\epsilon C \hat{B}_K |V_{cb}|^2 |V_{us}|^2 \left(\frac{1}{2}|V_{cb}|^2 R_t^2 \sin 2\beta\, \eta_{tt} S_0(x_t) + R_t \sin\beta\, (\eta_{ct} S_0(x_c, x_t) - \eta_{cc} x_c)\right)$$

$$\hat{B}_K = b(\mu) B_K(\mu) = b(\mu) \frac{\left\langle \bar{K}^0 \left| Q_R^{\Delta S=2}(\mu) \right| K^0 \right\rangle}{\frac{8}{3} f_K^2 m_K^2}$$

Standard model test

- direct computation of $\hat{B}_K$ on the lattice
- $\hat{B}_K$ from fit

Results for B_K



This is interesting: There is now a tension with respect to $\hat{B}_K$ from UT-fit using $S_{\psi K_S}$, $\Delta M_{B_s}/\Delta_{M_{B_d}}$ and ϵ_K: $\hat{B}_K = 1.09(12)_{V_{cb}\ excl}$, $\hat{B}_K = 0.903(86)_{V_{cb}\ incl}$

Laiho, Lunghi, van de Water PRD, 81, 034503 (2008)
also: Lunghi, Soni, PLB 666, 162(2008), Buras, Guadagnoli, PRD79,053010(2009)

Results for B_K

Aubin, Laiho, van de Water Phys.Rev.D81:014507,2010

Source uncertainty/error	uncertainty/error on B_K
statistics	1.2%
chiral & continuum extrapolation	1.9%
scale and quark-mass uncertainties	0.8%
finite volume errors	0.6%
renormalization factor	3.4%
total systematic	4.0%
total	4.2%

Results for B_K

Aubin, Laiho, van de Water Phys.Rev.D81:014507,2010

Source uncertainty/error	uncertainty/error on B_K	
statistics	1.2%	
chiral & continuum extrapolation	1.9%	$a \gtrsim 0.09$fm $m_\pi \gtrsim 240$MeV
scale and quark-mass uncertainties	0.8%	
finite volume errors	0.6%	
renormalization factor	3.4%	$\leftarrow$ mainly NLO running
total systematic	4.0%	
total	4.2%	

	f_K/f_π	a and m_π
summary:	B_K	renormalisation scale running
	$f_+^{K\pi}(0)$	statistics and m_π

	f_K/f_π	a and m_π
summary:	B_K	renormalisation scale running
	$f_+^{K\pi}(0)$	statistics and m_π

Outlook

- improving statistics = money

Outlook

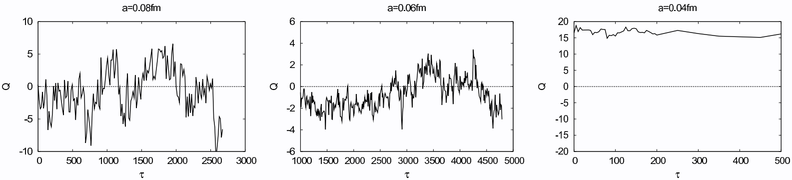
- improving statistics = money - not quite

Outlook

- improving statistics = money - not quite
- reducing *a* beyond $\approx$ 0.06fm turns out to be problematic
  - there are indications that for some observables auto-correlation times are much longer than accessible MC-chain lenghts
    Schäfer et al. arXiv:0910.1465, Lüscher Commun.Math.Phys.293:899-919,2010

Outlook

- improving statistics = money - not quite
- reducing a beyond $\approx$ 0.06fm turns out to be problematic
  - there are indications that for some observables auto-correlation times are much longer than accessible MC-chain lenghts
    Schäfer et al. arXiv:0910.1465, Lüscher Commun.Math.Phys.293:899-919,2010
  - the problem gets worse as the lattice spacing is reduced as seen for example by CLS and MILC for the topological charge:



  - there is currently no cure and it seems that all simulations with a much smaller than 0.1fm are affected, i.e., have modes with very long correlation times
  - there is no theoretical understanding of which observables couple to slow modes and which ones don't
  - estimation of statistical error is a delicate issue

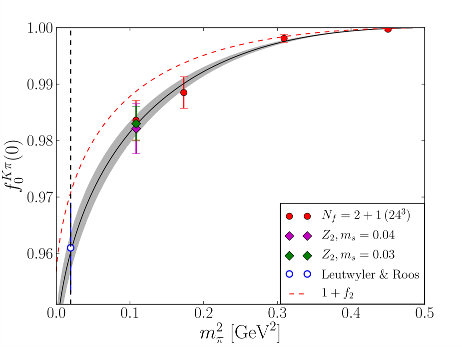
Outlook

- reducing m_π: improved algorithms and/or more FLOP/s

  example $f_0^{K\pi}(0)$

$$f_0^{K\pi}(0) = 1 + \underbrace{f_2(f_\pi, m_\pi, m_K)}_{\text{Gasser \& Leutwyler}} + \underbrace{\Delta f}_{\text{lattice}}$$

Gasser & Leutwyler
Nucl. Phys. B250 (1985) 517538



RBC+UKQCD arXiv:1004.0886

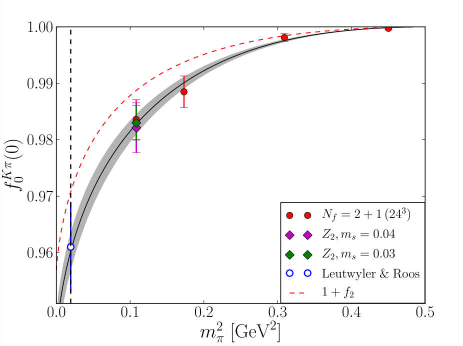
Outlook

- reducing m_π: improved algorithms and/or more FLOP/s

  example $f_0^{K\pi}(0)$

$$f_0^{K\pi}(0) = 1 + \underbrace{f_2(f_\pi, m_\pi, m_K)} + \underbrace{\Delta f}$$

Gasser & Leutwyler Nucl. Phys. B250 (1985) 517538 — lattice

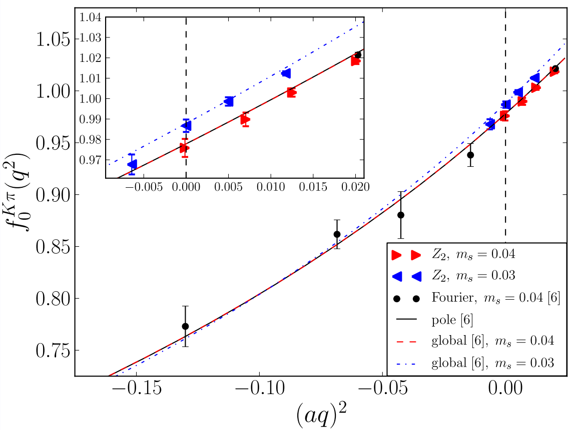


RBC+UKQCD arXiv:1004.0886

- systematics dominated by limited control of chiral extrapolation
- $f_0^{K\pi}(0)$ in NNLO χPT???

Outlook

interpolation in q^2 - systematic due to interpolation in q^2 entirely removed through using partially twisted boundary conditions



RBC+UKQCD arXiv:1004.0886

$K \to \pi\pi$

$$\langle \pi\pi(I)|\mathcal{H}^{\text{eff}}|K\rangle$$

Maiani Testa: standard lattice approach will project on unphysical 2-pion state

$K \to \pi\pi$

$$\langle \pi\pi(I)|\mathcal{H}^{\mathrm{eff}}|K\rangle$$

Maiani Testa: standard lattice approach will project on unphysical 2-pion state

Current approaches

- indirectly via χPT *RBC PRD68, (2003), CP-PACS PRD68, (2003)*
  depends crucially on SU(3) chiral PT which turns out to converge badly
  repeat efforts with unphysically light strange quarks?

$K \to \pi\pi$

$$\langle \pi\pi(I)|\mathcal{H}^{\rm eff}|K\rangle$$

Maiani Testa: standard lattice approach will project on unphysical 2-pion state

Current approaches

- indirectly via χPT *RBC PRD68, (2003), CP-PACS PRD68, (2003)*
  depends crucially on SU(3) chiral PT which turns out to converge badly
  repeat efforts with unphysically light strange quarks?
- directly, using (partially twisted, G-parity) boundary condition tricks to project on the desired state *arXiv:0912.2917*
  - very feasible in $\Delta I = 3/2$ channel
  - very cost-intensive in $\Delta I = 1/2$ channel

  N. Christ in his Kaon 2009 write-up *arXiv:0912.2917*: “$\Delta I = 1/2\,,3/2$ amplitudes A_0 and A_2 with 10-20% precision within the next 2-3 years” (RBC+UKQCD)

Summary

f_K/f_π

- reduction of m_π will reduce systematic in chiral extrapolation
- a is already in critical range in best computations, further reduction may be hazardous
- increase statistics $\sigma \propto 1/\sqrt{N}$

$f_+^{K\pi}(0)$

- most important: reduce pion mass in simulations and motivate other collaborations to compute it
- increase statistics $\sigma \propto 1/\sqrt{N}$
- luckily a is sub-dominant here

B_K

- lattice doing very well with $\approx 4\%$ uncertainty
- reduction of m_π and a not primary goal
- dominant systematic in renormalization $\rightarrow$ non-perturbative running
- I have heard that short distance corrections η are currently being computed beyond NNLO

$K \rightarrow \pi\pi$ "10-20% error on A_0 and A_2 in 2-3 years"