Anomalous gauge boson couplings in $e^- e^+ \to W^- W^+$ and $\gamma\gamma \to W^- W^+$ at an ILC

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Work by

M. Diehl
F. Nagel
M. Pospischil
A. Utermann
M. Maniatis
A. v. Manteuffel
O. N.

1 Introduction, Motivation

There is an ongoing workshop where the physics at a e^+e^- linear collider at c.m. energies 0.5 to 1.0 TeV, the **ILC**, is discussed.

One topic there is the study of the couplings of the gauge bosons γ, Z, $W^\pm$.

In the SM these couplings are in essence fixed by the requirement of renormalisability.

Deviations are, therefore very interesting and signal new physics.

We studied the accuracies achievable for anomalous gauge-boson couplings in various modes of operation / reactions at an ILC:

- e^+e^- annihilation

$$e^- + e^+ \longrightarrow W^- + W^+$$

with one W decaying into $e\nu$ or $\mu\nu$, the other W decaying to hadrons

- Giga Z

$$e^- + e^+ \longrightarrow Z$$

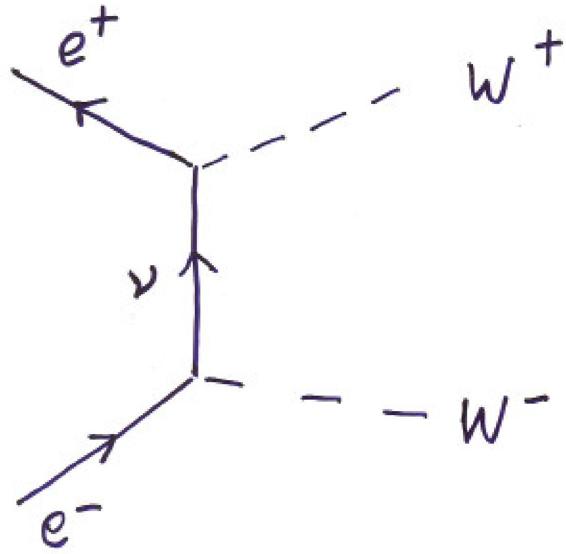
100 times LEP statistics

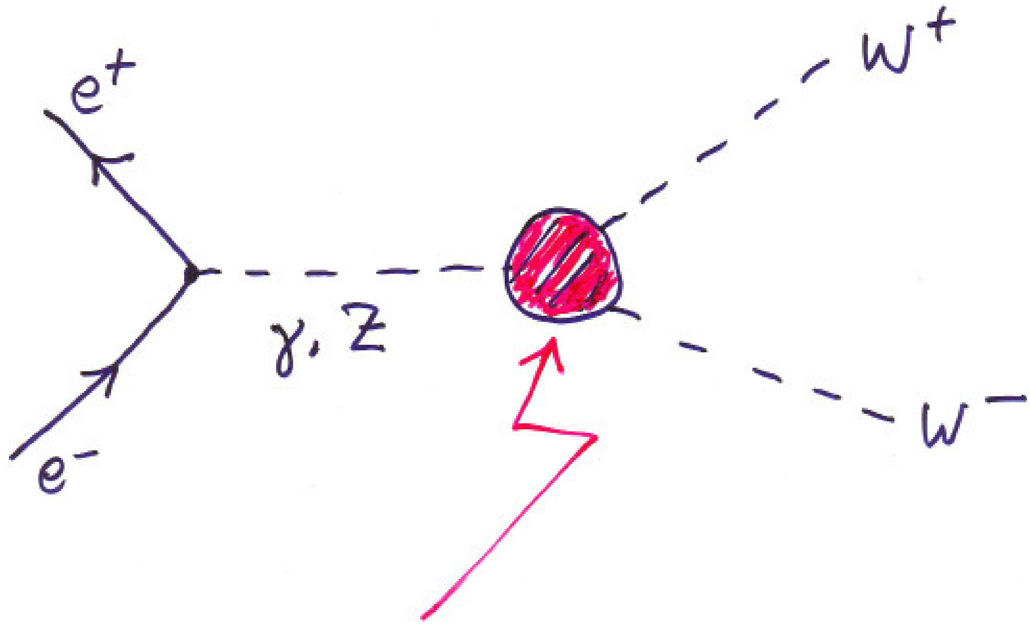
- $\gamma\gamma$ annihilation

$$\gamma + \gamma \longrightarrow W^- + W^+$$

2 Form factor approach to $e^- e^+ \to W^- W^+$

Diagrams:





14 complex form factors

Hagiwara et al: NP B282, 253 (1987):

$$\boxed{\begin{aligned}\mathscr{L}_{\mathrm{WWV}}/g_{\mathrm{WWV}} = {} & ig_1^{\mathrm{V}}\left(W_{\mu\nu}^{\dagger}W^{\mu}V^{\nu} - W_{\mu}^{\dagger}V_{\nu}W^{\mu\nu}\right) + i\kappa_{\mathrm{V}}W_{\mu}^{\dagger}W_{\nu}V^{\mu\nu} \\ & + \frac{i\lambda_{\mathrm{V}}}{m_{\mathrm{W}}^2}W_{\lambda\mu}^{\dagger}W^{\mu}{}_{\nu}V^{\nu\lambda} - g_4^{\mathrm{V}}W_{\mu}^{\dagger}W_{\nu}\left(\partial^{\mu}V^{\nu} + \partial^{\nu}V^{\mu}\right) \\ & + g_5^{\mathrm{V}}\varepsilon^{\mu\nu\rho\sigma}\left(W_{\mu}^{\dagger}\overleftrightarrow{\partial}_{\rho}W_{\nu}\right)V_{\sigma} + i\tilde{\kappa}_{\mathrm{V}}W_{\mu}^{\dagger}W_{\nu}\tilde{V}^{\mu\nu} \\ & + \frac{i\tilde{\lambda}_{\mathrm{V}}}{m_{\mathrm{W}}^2}W_{\lambda\mu}^{\dagger}W^{\mu}{}_{\nu}\tilde{V}^{\nu\lambda}.\end{aligned}}$$

$V = \gamma, Z$

$g_{WW\gamma} = -e$, $g_{WWZ} = -e\cot\vartheta_W$

SM values

$g_1^V = \varkappa_V = 1$,

all others 0.

$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$

etc.

SM couplings plus constant form factors $\Longrightarrow$ 28 real anomalous coupling terms.

Best is to use L and R couplings:

$$g_1^L = 4\sin^2\vartheta_W\, g_1^\gamma + (2-4\sin^2\vartheta_W)\,\xi\, g_1^Z$$

$$g_1^R = 4\sin^2\vartheta_W\, g_1^\gamma - 4\sin^2\vartheta_W\,\xi\, g_1^Z$$

etc.

$$\xi = \frac{s}{s-m_Z^2}$$

For purely left-handed initial e^- beam there is only sensitivity to the L couplings, for right-handed e^- beam to the R couplings.

Method: use optimal observables, that is, integrated observables giving optimal statistical accuracy in measuring anomalous terms (Atwood & Soni, Davier et al., Diehl & O.N.).

Exploit discrete symmetry properties under CP and $CP\tilde{T}$ of the couplings.

class	CP	$CP\tilde{T}$
(a)	+	+
(b)	+	−
(c)	−	+
(d)	−	−

$CP\tilde{T}$: Like CPT but without reversal of initial and final states

Optimal observables:

Consider the cross section; ϕ phase space var.

h_i : anomalous couplings

$$S(\phi) = S_0(\phi) + \sum_i S_{1i}(\phi)\, h_i + \sum_{i,j} S_{2\,ij}(\phi)\, h_i h_j$$

best observables for estimating (small) h_i:

$$O_i(\phi) = \frac{S_{1i}(\phi)}{S_0(\phi)}$$

Define

$$c_{ij} = \frac{1}{\sigma_0}\int d\phi\, O_i\, S_{1j} - \frac{\sigma_{1j}}{\sigma_0^2}\int d\phi\, S_0\, O_i$$

$$\sigma_0 = \int d\phi\, S_0 \quad , \sigma_{1j} = \int d\phi\, S_{1j}$$

Then the estimate for h is

$$h_i = \sum_j (c^{-1})_{ij}\, \left(E(O_i) - E_0(O_i)\right)$$

Cov. Matrix $\quad V(h) = \frac{1}{N} c^{-1}$

Results for $e^- e^+ \longrightarrow W^- W^+$

$\sqrt{s} = 500$ GeV ; $\int \mathcal{L}\, dt = 500\ fb^{-1}$

for unpolarised and long. pol. e^-, e^+

Polarisation (%)		number of semi lept. WW events
$P(e^-)$	$P(e^+)$	
-80	$+60$	3.3×10^6
-80	0	2.1×10^6
0	0	1.1×10^6
$+80$	0	0.2×10^6
$+80$	-60	0.1×10^6

To Neutrino exchange which gives a large rate only the left handed e^- and right handed e^+ contribute.

Table 12. Errors $\delta h \times 10^3$ on the couplings of symmetry (a) at 500 GeV for different initial beam polarisations

P^-	P^+	$\mathrm{Re}\,\Delta g_1^{\mathrm{L}}$	$\mathrm{Re}\,\Delta\kappa_{\mathrm{L}}$	$\mathrm{Re}\,\lambda_{\mathrm{L}}$	$\mathrm{Re}\,g_5^{\mathrm{L}}$	$\mathrm{Re}\,\Delta g_1^{\mathrm{R}}$	$\mathrm{Re}\,\Delta\kappa_{\mathrm{R}}$	$\mathrm{Re}\,\lambda_{\mathrm{R}}$	$\mathrm{Re}\,g_5^{\mathrm{R}}$
−80%	+60%	1.5	0.47	0.34	1.1	169	40	57	112
−80%	0	1.9	0.60	0.43	1.5	62	14	21	41
0	0	2.6	0.85	0.59	2.0	10	2.4	3.6	6.7
+80%	0	6.9	2.3	1.5	5.3	3.5	0.83	1.2	2.3
+80%	−60%	13	4.5	2.8	10	2.0	0.47	0.67	1.3

- Here combined fits to all parameters are done. Single parameter LEP 2 constraints are typically worse by factor 10 − 100.
- Similar results apply to symmetry classes (b), (c), (d).
- One coupling paramter,

$$\frac{1}{\sqrt{2}}\,\mathrm{Im}\left(g_1^R + \kappa_R\right)$$

cannot be measured with unpolarised or long. pol. e^+e^- beams. It can be measured with transversely polarised e^+e^- beams.

3 Effective Lagrangian approach

The form factor approach is general but not very suitable to compare anomalous couplings in different reactions. To compare $e^- e^+ \to W^- W^+$ and $\gamma\gamma \to W^- W^+$ we use an effective Lagrangian approach.

Add to SM Lagrangian, before spontaneous symmetry breaking, coupling terms of dimension 6, respecting $SU(2)_L \times U(1)_Y$ invariance.

There are 80 terms (Buchmüller & Wyler 1986).

We restrict ourselves to terms containing only gauge bosons or in addition the Higgs field.

$$
\begin{aligned}
O_W &= \epsilon_{ijk}\, W_\mu^{i\,\nu} W_\nu^{j\,\lambda} W_\lambda^{k\,\mu}, & O_{\tilde W} &= \epsilon_{ijk}\, \tilde W_\mu^{i\,\nu} W_\nu^{j\,\lambda} W_\lambda^{k\,\mu}, \\
O_{\varphi W} &= \frac{1}{2}\left(\varphi^\dagger\varphi\right)\, W^i_{\mu\nu} W^{i\,\mu\nu}, & O_{\varphi \tilde W} &= \left(\varphi^\dagger\varphi\right)\, \tilde W^i_{\mu\nu} W^{i\,\mu\nu}, \\
O_{\varphi B} &= \frac{1}{2}\left(\varphi^\dagger\varphi\right)\, B_{\mu\nu} B^{\mu\nu}, & O_{\varphi \tilde B} &= \left(\varphi^\dagger\varphi\right)\, \tilde B_{\mu\nu} B^{\mu\nu}, \\
O_{WB} &= \left(\varphi^\dagger\tau^i\varphi\right)\, W^i_{\mu\nu} B^{\mu\nu}, & O_{\tilde W B} &= \left(\varphi^\dagger\tau^i\varphi\right)\, \tilde W^i_{\mu\nu} B^{\mu\nu}, \\
O_\varphi^{(1)} &= \left(\varphi^\dagger\varphi\right)\left(\mathcal{D}_\mu\varphi\right)^\dagger\left(\mathcal{D}^\mu\varphi\right), & O_\varphi^{(3)} &= \left(\varphi^\dagger\mathcal{D}_\mu\varphi\right)^\dagger\left(\varphi^\dagger\mathcal{D}^\mu\varphi\right).
\end{aligned}
$$

Here the dual field strengths are defined as

$$
\tilde W^i_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} W^{i\,\rho\sigma}, \qquad \tilde B_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} B^{\rho\sigma}.
$$

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \mathcal{L}_2$$

$$\mathcal{L}_2 = \sum_{i=1}^{10} \frac{h_i}{v^2} O_i$$

$v = 246$ GeV = standard Higgs vev.

$$h_i = \mathcal{O}\left(\frac{v^2}{\Lambda^2}\right)$$

$\Lambda \approx$ scale of new physics

Spontanous Symmetry Breaking:

Higgs field obtains vacuum exp. value.
Anomalous terms contribute now also to the kinetic and mass terms of the gauge bosons.

kinetic term: $h_{\varphi W}$, $h_{\varphi B}$, h_{WB}

mass term: $h_\varphi^{(1)}$, $h_\varphi^{(3)}$

This necessitates redefinition of $W^\pm$ and γ, Z fields:

Renormalisation of $W^\pm$;
simultanous diagonalisation of kinetic and mass terms in the $\gamma - Z$ sector.

$\Rightarrow$ Z decay properties are changed.
Z decays are also sensitive to anomalous couplings in this approach.

Choice of parameter scheme for the electro weak parameters:

$\mathcal{P}_{\mathcal{L}}$ $\quad g,\ g',\ v,\ h_i$

$\mathcal{P}_{Z}$ $\quad \alpha(m_Z),\ G_F,\ m_Z,\ h_i$

$\mathcal{P}_{W}$ $\quad \alpha(m_Z),\ G_F,\ m_W,\ h_i$

Bounds from existing data

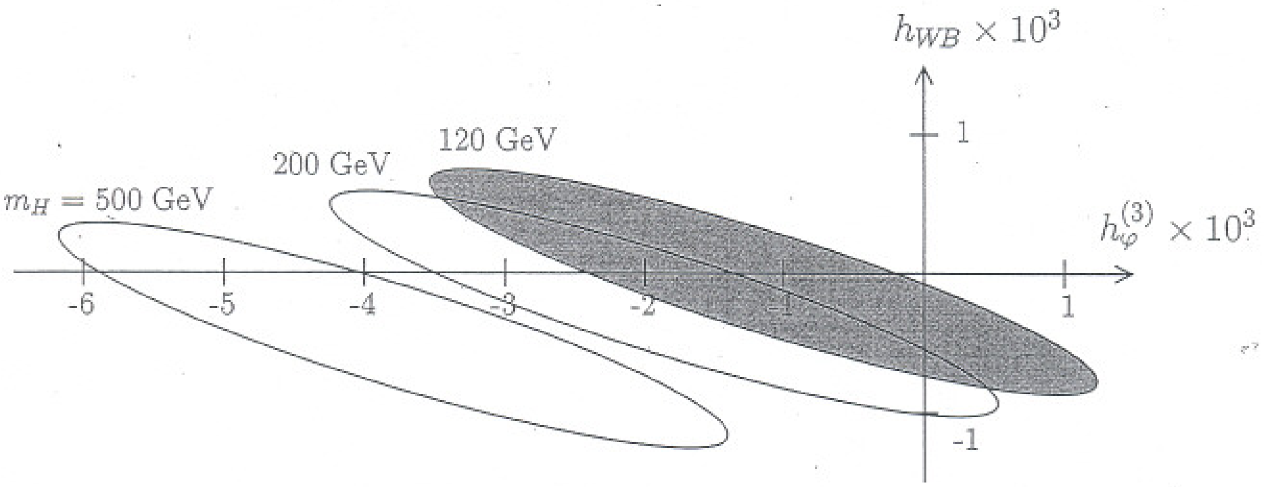


Fig. 1. Error ellipses of h_{WB} and $h_\varphi^{(3)}$ for different Higgs masses

Table 8. Final results from already existing data for CP conserving couplings in units of 10^{-3} for a Higgs mass of 120 GeV, 200 GeV and 500 GeV. The anomalous couplings are extracted from the observables listed in the first row using (5.25). The errors δh and the correlations of the errors are independent of the Higgs mass with the accuracy given here. The correlation matrix is given on the right

		s^2_{eff}, Γ_Z, σ^0_{had}, R^0_ℓ, m_W, Γ_W, TGCs						
m_H		120 GeV	200 GeV	500 GeV	$\delta h \times 10^3$			
h_W	$\times 10^3$	-62.4	-62.5	-62.8	36.3	1	-0.007	0.008
h_{WB}	$\times 10^3$	-0.06	-0.22	-0.45	0.79		1	-0.88
$h_\varphi^{(3)}$	$\times 10^3$	-1.15	-1.86	-3.79	2.39			1

Data : LEP 1, 2 ; SLD ; Γ_W , m_W

Scheme P_Z

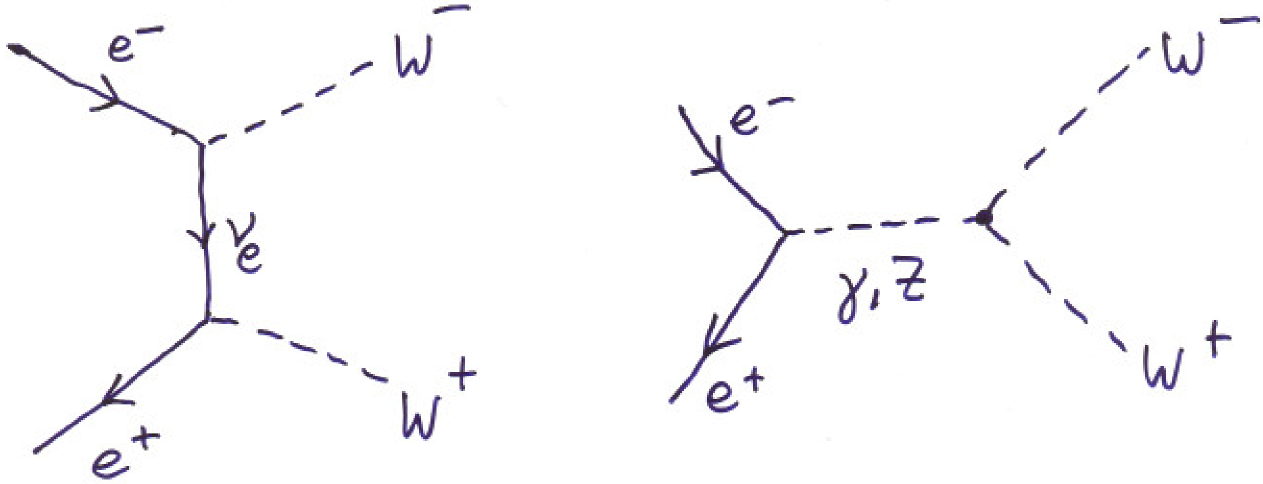
Note the correlation of

m_{Higgs} and $h_\varphi^{(3)}$.

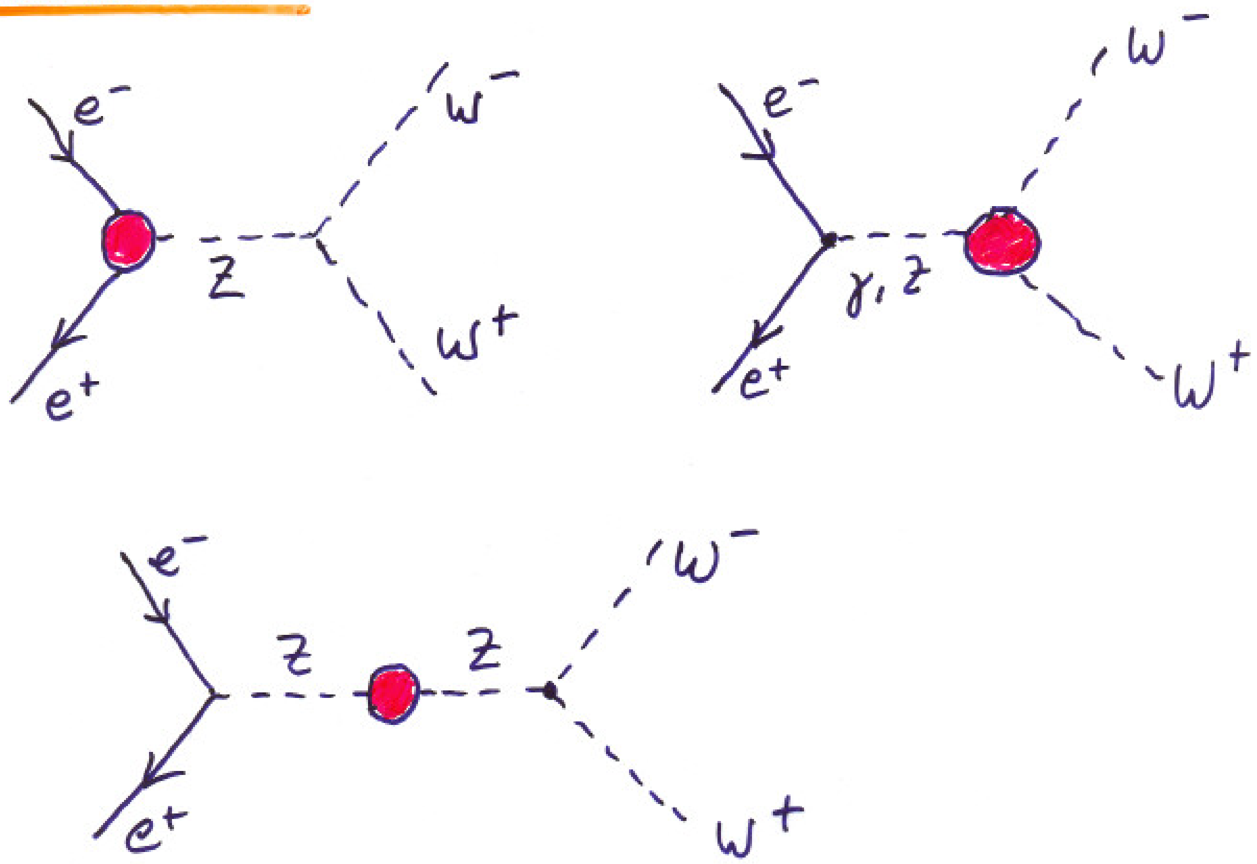
Effective Lagrangian approach,

P_W scheme. Diagrams for $e^- e^+ \to W^- W^+$

SM

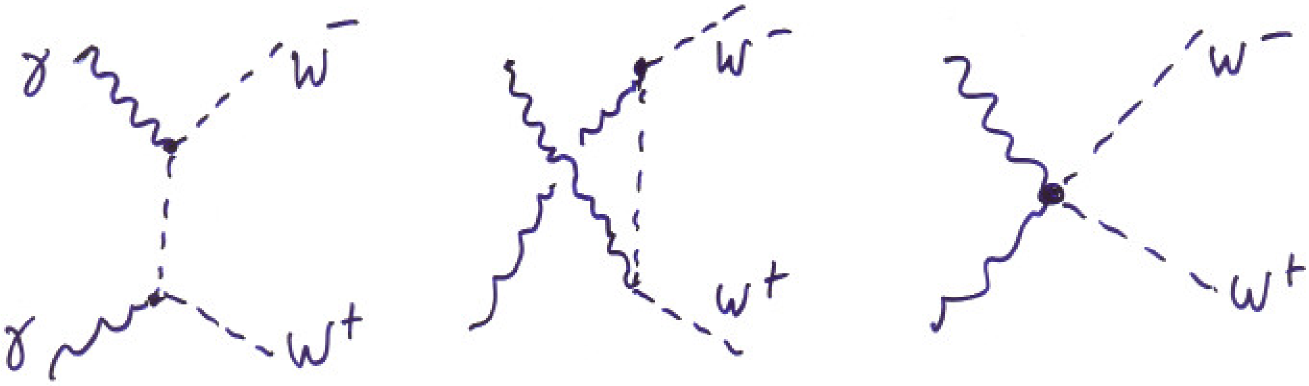


Anomalous

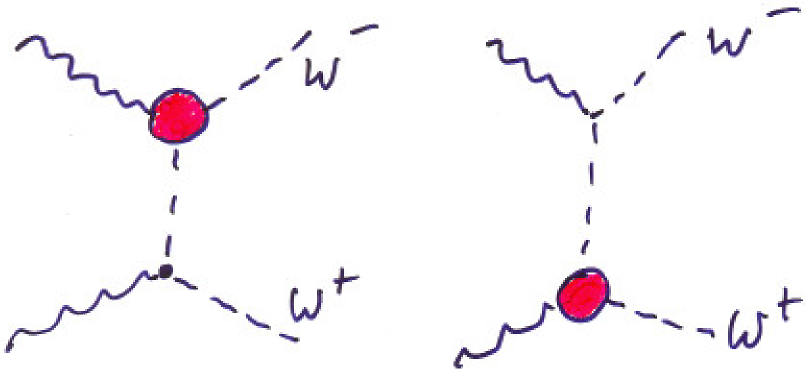


Diagrams for $\gamma\gamma \to W^- W^+$

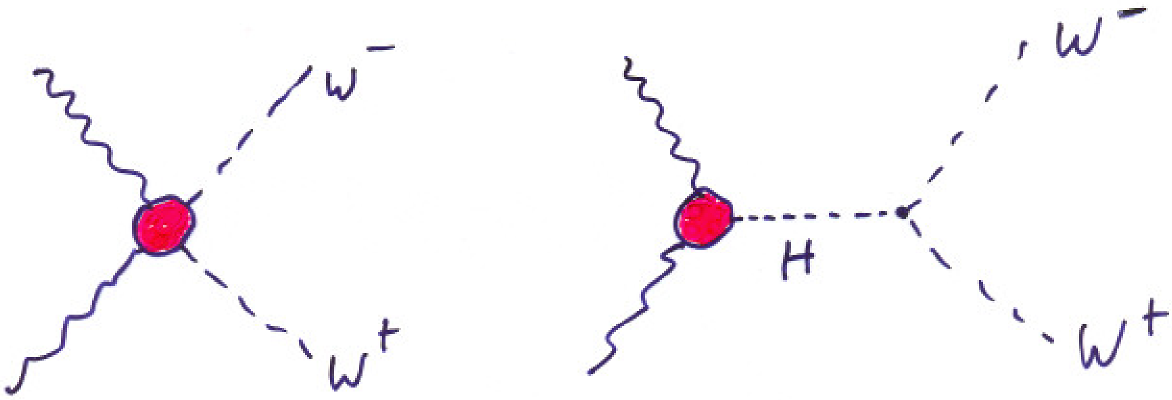
SM



Anomalous



+ crossed



Measurable couplings from normalised event distributions:

$$h_W\,,\ h_{WB}\,,\ h_{\tilde{W}}\,,\ h_{\tilde{W}\tilde{B}}\,,\ h_{\varphi}^{(3)}\,,$$

$$h_{\varphi WB} = \sin^2\vartheta_W\, h_{\varphi W} + \cos^2\vartheta_W\, h_{\varphi B}\,,$$

$$h_{\varphi \tilde{W}\tilde{B}} = \sin^2\vartheta_W\, h_{\varphi \tilde{W}} + \cos^2\vartheta_W\, h_{\varphi \tilde{B}}\,.$$

Unmeasurable couplings

$$h_{\varphi}^{(1)}$$

$$h'_{\varphi WB} = \cos^2\vartheta_W\, h_{\varphi W} - \sin^2\vartheta_W\, h_{\varphi B}\,,$$

$$h'_{\varphi \tilde{W}\tilde{B}} = \cos^2\vartheta_W\, h_{\varphi \tilde{W}} - \sin^2\vartheta_W\, h_{\varphi \tilde{B}}\,.$$

Results for $\sqrt{s} = 500$ GeV,

integrated luminosity:

$$L_{e^-e^+} = L_{e^-e^-} = 500\ fb^{-1}$$

Number of produced $W^- W^+$ decaying semileptonically

$e^- e^+$ mode	1.1×10^6
$\gamma\gamma$ mode, Compton spectrum	4.4×10^6

Assumption:

$$m_{Higgs} = 120 \text{ GeV}$$

	LEP + SLD	ILC e^+e^-	ILC $\gamma\gamma$
	$h_i \times 10^3$	$\delta h_i \times 10^3$	$\delta h_i \times 10^3$
h_W	-69 ± 39	0.28	0.36
h_{WB}	-0.06 ± 0.79	0.32	1.08
$h_{\varphi WB}$	no contr.	no contr.	1.17
$h_{\varphi}^{(3)}$	-1.15 ± 2.39	36.4	no contr.
$h_{\tilde{W}}$	68 ± 81	0.28	0.46
$h_{\tilde{W}B}$	33 ± 84	2.2	3.17
$h_{\varphi \tilde{W} \tilde{B}}$	no contr.	no contr.	1.01

4 Conclusions

Effective Lagrangian approach:

- Out of the original 10 couplings only 5 are measurable in $e^-e^+ \to W^-W^+$, $e^-e^+ \to Z$,

  2 more lin. combinations can be measured with $\gamma\gamma \to W^-W^+$.

- The coupling $h_\varphi^{(3)}$ is best measured with GigaZ.

- All options considered for an ILC are needed to get the complete picture.

- The approach followed, form factor or $\mathcal{L}_{eff}$, must be specified. The relation of approaches is quite non trivial.
- The parameter scheme for the electroweak interactions must be specified carefully.
- Setting $\delta h = v^2/\Lambda^2$ we find that at a 500 GeV ILC one is sensitive to a "new physics scale" up to $\Lambda \approx 10$ TeV.

For the details see

Z. Phys. **C62**, 397 (1994)

EPJ **C1**, 177 (1998)

C27, 375 (2003)

C32, 17 (2003)

C40, 497 (2005)

C42, 139 (2005)

C45, 679 (2006)

C46, 93 (2006)

Work in progress on $\gamma\gamma \to W^- W^+$
with polarised γ's.