UPC lepton pair production to all orders in $Z\alpha$

ECT*, Trento, January 19, 2007

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Topics:

- **BOUND-ELECTRON POSITRON PAIRS**
- **CONTINUUM LEPTON PAIRS**
- **IMPACT PARAMETER DEPENDENCE**
- **LHC POSSIBILITIES**

Forward e^+e^- Pairs

$\mu^+\mu^-$ Pairs

Deviations from Z^4 Scaling

- **CONCLUSIONS ETC.**

ULTRARELATIVISTIC δ FUNCTION POTENTIAL

- Begin with the Liénhard-Wiechart potential

$$V(\rho,z,t) = \frac{\alpha Z(1 - v\alpha_z)}{\sqrt{[(\mathbf{b}-\rho)/\gamma]^2 + (z-vt)^2}}$$

- If one makes the gauge transformation on the wave function

$$\psi = e^{-i\chi(\mathbf{r},t)}\psi'; \qquad \chi(\mathbf{r},t) = \frac{\alpha Z}{v}\ln[\gamma(z-vt) + \sqrt{b^2 + \gamma^2(z-vt)^2}]$$

$$V(\rho,z,t) = \frac{\alpha Z(1 - v\alpha_z)}{\sqrt{[(\mathbf{b}-\rho)/\gamma]^2 + (z-vt)^2}} - \frac{\alpha Z(1-(1/v)\alpha_z)}{\sqrt{b^2/\gamma^2 + (z-vt)^2}}$$

A.J.B., M J. Rhoades-Brown, and J. Weneser, Phys. Rev. A 44, 5568 (1991)

- In the ultrarelativistic limit (ignoring correction terms in $[(\mathbf{b}-\rho)/\gamma]^2$)

$$V(\rho,z,t) = -\delta(z-t)\alpha Z_P(1-\alpha_z)\ln(\mathbf{b}-\rho)^2$$

A.J.B., Phys. Rev. A 52, 4970 (1995)

BOUND-ELECTRON POSITRON PAIRS

A.J.B., Phys. Rev. Lett. 78, 1231 (1997)

- The δ function potential allowed the closed form solution of the Dirac equation for the bound-electron positron problem.
- The full solution of the problem is in perturbation theory form, but with an eikonalized interaction in the transverse direction

$$V(\rho, z, t) = -i\delta(z - t)(1 - \alpha_z)(\exp[-i\alpha Z_P \ln(\mathbf{b} - \rho)^2] - 1).$$

- Calculation is in the frame of the ion that receives the bound electron. Matrix element wave functions include rest ion's static field $\alpha Z_T/r$, thus including higher order effects from the rest ion (G. Baur, this workshop).
- Recall that this exact semiclassical solution produced a reduction of a little less than 10% in the predicted cross section for Au + Au at RHIC.
- One can identify this reduction as an additional Coulomb correction from the moving ion to bound-electron positron pair production.

CONTINUUM PAIRS

- Two center light cone calculation of continuum pairs by solving the semi-classical Dirac equation for colliding δ function potentials

$$V(\rho, z, t) = \delta(z-t)(1-\alpha_z)\Lambda^-(\rho) + \delta(z+t)(1+\alpha_z)\Lambda^+(\rho)$$

in collider center of mass (lab) frame, and

$$\Lambda^\pm(\rho) = -Z\alpha \ln \frac{(\rho \pm \mathbf{b}/2)^2}{(b/2)^2}$$

B. Segev and J. C. Wells, Phys. Rev. A 57, 1849 (1998)

A.J.B., Larry McLerran, Phys. Rev. C 58, 1679 (1998)

U. Eichmann, J. Reinhardt, S. Schramm, and W. Greiner, Phys. Rev. A 59, 1223 (1999)

- A.J.B and McLerran note the agreement with perturbation theory; Segev and Wells also note the scaling with $Z_1^2 Z_2^2$ seen in SPS data.

B. Segev and J. C. Wells, Phys. Rev. C 59, 2753 (1999)

- CERN SPS data

  160 GeV/c Pb ions on C, Al, Pa, Au; 200 Gev/c S ions on C, Al, Pa, Au:

  "Cross sections scale as the product of the squares of the projectile and target nuclear charges"

  C. R. Vane, S. Datz, E. F. Deveney, P. F. Dittner, H. F. Krause, R. Schuch, H. Gao, and R. Hutton, Phys. Rev. A 56, 3682 (1997)

- On the other hand, photoproduction on a heavy target shows a negative correction proportional to Z^2.

  H. A. Bethe and L. C. Maximon, Phys. Rev. 93, 768 (1954); Handel Davies, H. A. Bethe and L. C. Maximon, Phys. Rev. 93, 788 (1954)

- Several authors have argued that a correct regularization of the exact Dirac equation amplitude should lead to Coulomb corrections.

  D. Yu. Ivanov, A. Schiller, and V. G. Serbo, Phys. Lett. B 454, 155 (1999)

  R. N. Lee and A. I. Milstein, Phys. Rev. A 61, 032103; 64, 032106

EXACT CROSS SECTION FOR CONTINUUM PAIRS

The amplitude takes the form

$$M(p,q) = \int \frac{d^2k}{(2\pi)^2} \exp[i\mathbf{k}\cdot\mathbf{b}]\mathcal{M}(\mathbf{k})F_B(\mathbf{k})F_A(\mathbf{q}_\perp + \mathbf{p}_\perp - \mathbf{k})$$

p and q are the momenta of the produced electron and positron,

$$\begin{aligned}\mathcal{M}(\mathbf{k}) &= \bar{u}(p)\frac{\alpha(\mathbf{k}-\mathbf{p}_\perp)+\gamma_0 m}{-p_+q_- - (\mathbf{k}-\mathbf{p}_\perp)^2 - m^2 + i\varepsilon}\gamma_- u(-q) \\ &\quad +\bar{u}(p)\frac{-\alpha(\mathbf{k}-\mathbf{q}_\perp)+\gamma_0 m}{-p_-q_+ - (\mathbf{k}-\mathbf{q}_\perp)^2 - m^2 + i\varepsilon}\gamma_+ u(-q),\end{aligned}$$

and the transverse integrals F_B and F_A originally took the eikonalized form

$$F(\mathbf{k}) = 2\pi \int_0^\infty \rho\, d\rho J_0(k\rho)\{\exp[-i2Z\alpha \ln\rho] - 1\}.$$

$F(\mathbf{k})$ has to be regularized at large ρ.

How it is regularized is the key to understanding Coulomb corrections.

σ_T is the number weighted inclusive cross section

$$\sigma_T = \int d^2b < N >= \int d^2b \sum_{n=1}^{\infty} nP_n(b),$$

$$\sigma_T = \int \frac{m^2 d^3p\, d^3q}{(2\pi)^6 \varepsilon_p \varepsilon_q} \int \frac{d^2k}{(2\pi)^2} |\mathcal{M}(\mathbf{k})|^2 |F_A(\mathbf{q}_\perp + \mathbf{p}_\perp - \mathbf{k})|^2 |F_B(\mathbf{k})|^2$$

$$|\mathcal{M}(\mathbf{k})|^2 = \frac{p_+q_-[(\mathbf{k}-\mathbf{p}_\perp)^2+m^2]}{[p_+q_- + (\mathbf{k}-\mathbf{p}_\perp)^2+m^2]^2} + \frac{p_-q_+[(\mathbf{k}-\mathbf{q}_\perp)^2+m^2]}{[p_-q_+ + (\mathbf{k}-\mathbf{q}_\perp)^2+m^2]^2}$$
$$+\frac{2[\mathbf{k}\cdot\mathbf{p}_\perp q_+q_- + \mathbf{k}\cdot\mathbf{q}_\perp p_+p_- - 2\mathbf{k}\cdot\mathbf{p}_\perp \mathbf{k}\cdot\mathbf{q}_\perp + k^2(\mathbf{p}_\perp\cdot\mathbf{q}_\perp - m^2) - p_+p_-q_+q_-]}{[p_+q_- + (\mathbf{k}-\mathbf{p}_\perp)^2+m^2][p_-q_+ + (\mathbf{k}-\mathbf{q}_\perp)^2+m^2]}.$$

If one merely regularizes the eikonal integrals $F_{A,B}$ at large ρ one obtains apart from a trivial phase

$$F(\mathbf{k}) = \frac{4\pi\alpha Z}{k^{2-2i\alpha Z}}.$$

If one puts in a cutoff by hand by replacing k^2 with $k^2+\omega^2/\gamma^2$

$$F(\mathbf{k}) = \frac{4\pi\alpha Z}{(k^2+\omega^2/\gamma^2)^{1-i\alpha Z}}.$$

the cross section formula goes to the known perturbation theory limit.

All the higher order $Z\alpha$ effects in $M(p,q)$ are contained only in the phase of the denominator, which falls out of the expression when squared.

A PHYSICAL REGULARIZATION

Alternatively, a physically motivated cutoff of the transverse potential leads to an "exact" cross section expression:

- Coulomb corrections consistent with the Lee and Milstein result,
- Cross section consistent with perturbation theory in that limit.

A.J.B., Phys. Rev. C 68, 034906 (2003)

- "Exact" Dirac equation cross sections evaluated on a computer.

A.J.B., Phys. Rev. C 71, 024901 (2005)

- In a Weizsacker-Williams equivalent photon treatment the potential is cut off at impact parameter $b \simeq \gamma/\omega$, where γ is the relativistic boost of the ion producing the photon and ω is the energy of the photon. If

$$\chi(\rho) = \int_{-\infty}^{\infty} dz V(\sqrt{z^2+\rho^2})$$

and $V(r)$ is cut off in a physically motivated way, such as an equivalent photon cutoff, then

$$V(r) = \frac{-Z\alpha \exp[-r\omega_{A,B}/\gamma]}{r}$$

where

$$\omega_A = \frac{p_+ + q_+}{2};\ \omega_B = \frac{p_- + q_-}{2}$$

- ω_A the energy of the virtual photon from ion A moving in the positive z direction and ω_B the energy of the virtual photon from ion B moving in the negative z direction.

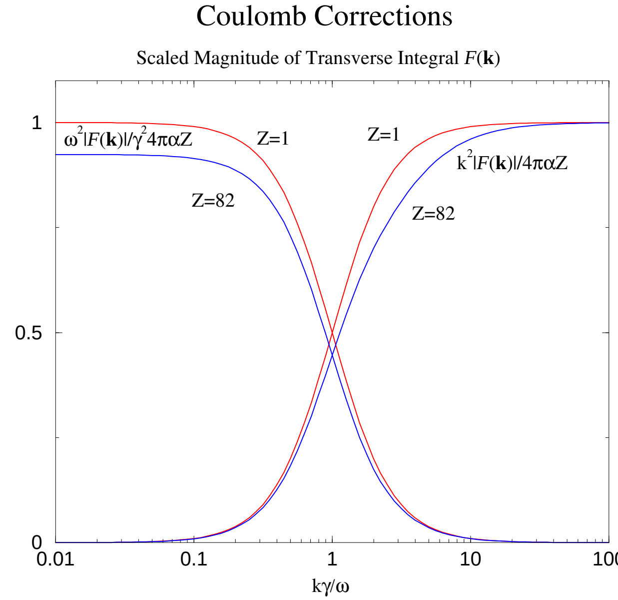
- All beam energy dependence (γ) and some of the electron and positron energy dependence ($\omega_{A,B}$) is contained in this cutoff. The integral

$$\chi(\rho) = \int_{-\infty}^{\infty} dz V(\sqrt{z^2 + \rho^2})$$

can be carried out to obtain

$$\chi(\rho) = -2Z\alpha K_0(\rho\omega_{A,B}/\gamma),$$

and



$$F_{A,B}(\mathbf{k}) = 2\pi \int d\rho\rho J_0(k\rho)\{\exp[2iZ_{A,B}\alpha K_0(\rho\omega_{A,B}/\gamma)] - 1\}.$$

- $F_A(\mathbf{k})$ and $F_B(\mathbf{k})$ are functions of virtual photon ω_A and ω_B respectively.
- $K_0(\rho\omega/\gamma) = -\ln(\rho\omega/2\gamma) - \gamma_{Euler}$ for small ρ and cuts off exponentially at $\rho \sim \gamma/\omega$. This is the physical cutoff to the transverse potential $\ln(\rho)$.

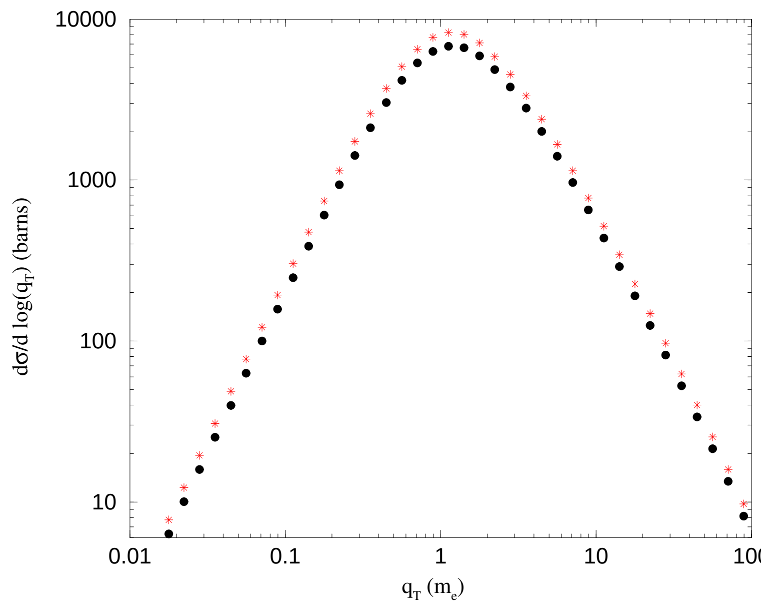
CALCULATIONS: NUMERICAL TECHNIQUES AND RESULTS

- The expression for the total cross section involves an eight dimensional integral over the positron and electron momenta as well as the virtual photon transverse momentum.
- Reduces to seven dimensions in the usual way by symmetry, e. g. let the positron transverse momentum define the x-axis.
- The usual method of evaluation e. g. in perturbation theory is via Monte Carlo.
- I have chosen to do the seven dimensional integral directly on meshes uniform on a logarithmic scale in each momentum dimension.
- It was possible to carry the calculation out without using Monte Carlo because the integrand is very smooth and smoothly goes to zero at both high end and low end of the momentum ranges.
- No cutoffs were applied except the cutoffs implicit in the $\omega_{A,B}/\gamma$ of the of the virtual photon sources $F_{A,B}(\mathbf{k})$.

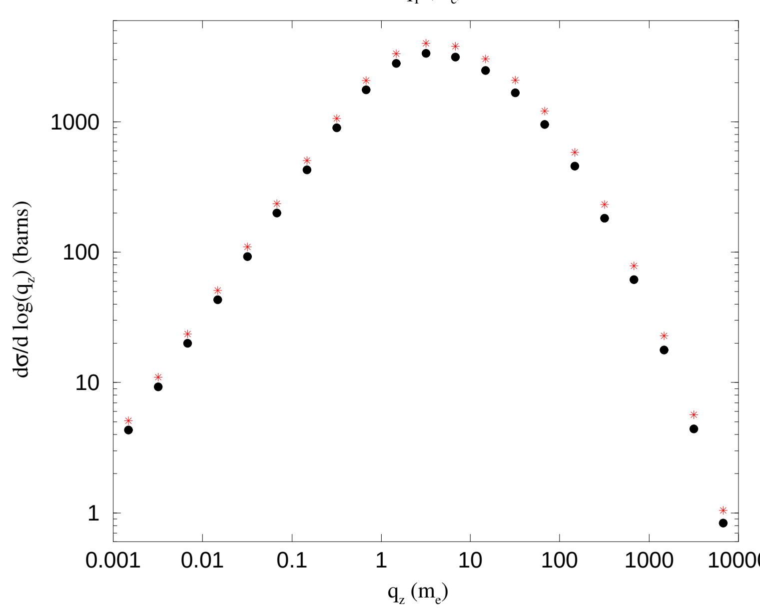
	RESULTS IN BARNS	Exact	Perturb.	Difference
Pb + Au	Computer Evaluation	2670	3720	-1050
$\gamma = 9.2$	Racah Formula		3480	
SPS	Lee-Milstein	3050	5120	-2070
Au + Au	Computer Evaluation	28,600	34,600	-6,000
$\gamma = 100$	Racah Formula		34,200	
RHIC	Lee-Milstein	34,100	42,500	-8,400
	Hencken, Trautman, Baur[*]		34,000	
Pb + Pb	Computer Evaluation	199,000	224,000	-25,000
$\gamma = 2960$	Racah Formula		226,000	
LHC	Lee-Milstein	226,000	258,000	-32,000

[*]Kai Hencken, Dirk Trautmann, and Gerhard Baur, Phys. Rev. C 59, 841 (1999)

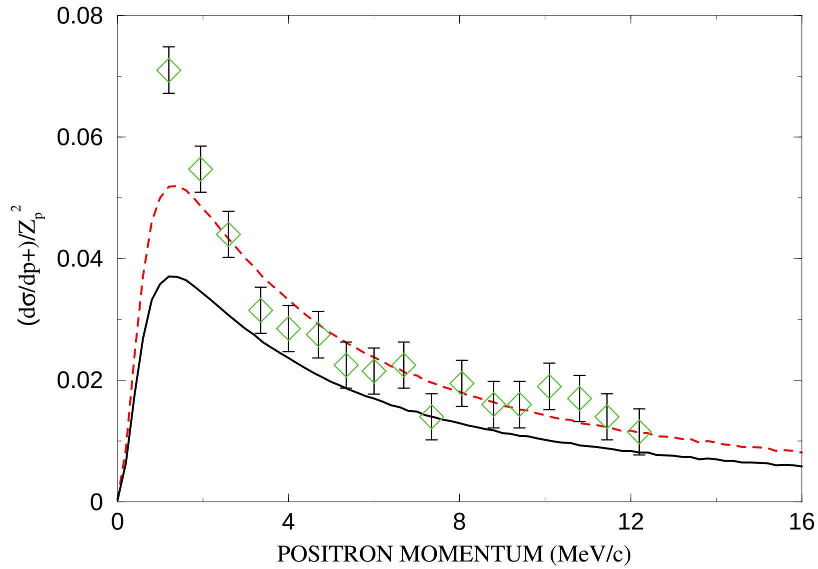
- Positron transverse momentum spectra for Au + Au at RHIC with $\gamma = 100$. The exact calculation (filled circles) is below perturbation theory (stars) for the entire range of $\mathbf{q}_\perp$.



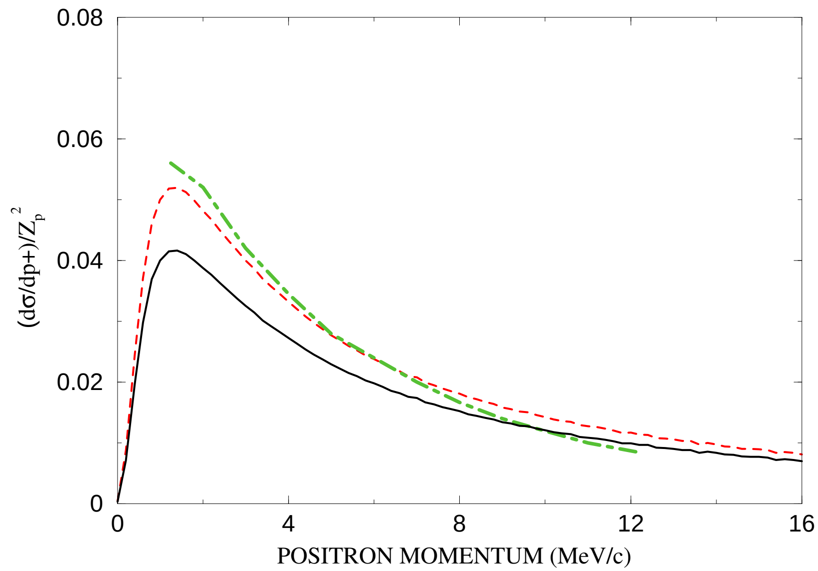
- Positron longitudinal momentum spectra for Au + Au at RHIC with $\gamma = 100$. The exact calculations (filled circles) are below perturbation theory (stars) for the entire range of q_z.



- Calculated positron momentum spectra compared with CERN SPS Pb+Au data[*] at $\gamma = 9.2$ c.m. The solid line is the exact calculation and the dashed line perturbation theory. The closer agreement of data to perturbation theory is a puzzle.



- As above for SPS S+Au data[*]. The dot-dashed line follows the experimental authors' repesentation of their data.



[*] C. R. Vane, S. Datz, E. F. Deveney, P. F. Dittner, H. F. Krause, R. Schuch, H. Gao, and R. Hutton, Phys. Rev. A 56, 3682 (1997)

RHIC STAR DATA

- e^+e^- pairs accompanied by nuclear dissociation have been measured by STAR[*]. Comparison with perturbative QED calculations allowed a limit to be set "on higher-order corrections to the cross section,

$$-0.5\sigma_{QED} < \Delta\sigma < 0.2\sigma_{QED}$$

at a 90% confidence level.

[*]STAR Collaboration, J. Adams *et al.*, Phys. Rev. C **70**, 031902(R) (2004)

- Calculations in the STAR acceptance without dissociation provide an indication of the relative difference between perturbation theory and the exact result. In the STAR acceptance the exact result is calculated to be 17% lower than perturbation theory. This rough estimate,

$$\Delta\sigma = -0.17\sigma_{QED}$$

is not excluded by STAR.

IMPACT PARAMETER DEPENDENCE OF HEAVY ION e^+e^- PAIR PRODUCTION TO ALL ORDERS IN $Z\alpha$

A.J.B., Phys. Rev. C 74, 054903 (2006)

- ZDC triggering (e.g. for the STAR pair production) weights smaller parameter contributions: the probability at each impact parameter goes as the product of the dissociation probabilty (ZDC) and the pair production probability.
- Pair production probability as a function of impact parameter is needed to describe ZDC triggered events as was done for ρ production at STAR.[*]
- We calculate the number weighted probability P_T (or number operator) for producing e^+e^- pairs at some impact parameter b

$$P_T = \sum_{n=1}^{\infty} nP_n(b) = \int \frac{m^2 d^3p\, d^3q}{(2\pi)^6 \epsilon_p \epsilon_q} |M(p,q)|^2$$

[*]A. J. B., S. R. Klein, and J. Nystrand, Phys.Rev.Lett. 89, 012301 (2002)

IMPACT PARMETER DEPENDENT AMPLITUDE

- Express $\mathbf{k}$ in Cartesian co-ordinates:

$$\mathcal{M}(\mathbf{k}) = \bar{u}(p)\frac{\alpha_x k_x + \alpha_y k_y - \alpha\cdot\mathbf{p}_\perp + \gamma_0 m}{-p_+q_- - (\mathbf{k}-\mathbf{p}_\perp)^2 - m^2 + i\varepsilon}\gamma_- u(-q) + \bar{u}(p)\frac{-\alpha_x k_x - \alpha_y k_y + \alpha\cdot\mathbf{q}_\perp + \gamma_0 m}{-p_-q_+ - (\mathbf{k}-\mathbf{q}_\perp)^2 - m^2 + i\varepsilon}\gamma_+ u(-q).$$

- The expression for the amplitude $M(p,q)$ then becomes

$$M(p,q) = \bar{u}(p)[I_{px}\alpha_x + I_{py}\alpha_y + (-\alpha\cdot\mathbf{p}_\perp + \gamma_0 m)J_p]\gamma_- u(-q) + \bar{u}(p)[-I_{qx}\alpha_x - I_{qy}\alpha_y + (\alpha\cdot\mathbf{q}_\perp + \gamma_0 m)J_q]\gamma_+ u(-q),$$

where letting $\mathbf{b}$ define the x-axis,

$$I_{px} = \frac{1}{(2\pi)^2}\int \exp[i\,k_x\,b]\,dk_x \int \frac{F_B(\mathbf{k})\,F_A(\mathbf{q}_\perp + \mathbf{p}_\perp - \mathbf{k})\;k_x\,dk_y}{-p_+q_- - (\mathbf{k}-\mathbf{p}_\perp)^2 - m^2}$$

$$I_{py} = \frac{1}{(2\pi)^2}\int \exp[i\,k_x\,b]\,dk_x \int \frac{F_B(\mathbf{k})\,F_A(\mathbf{q}_\perp + \mathbf{p}_\perp - \mathbf{k})\;k_y\,dk_y}{-p_+q_- - (\mathbf{k}-\mathbf{p}_\perp)^2 - m^2}$$

$$I_{qx} = \frac{1}{(2\pi)^2} \int \exp[i\,k_x\,b]\,dk_x \int \frac{F_B(\mathbf{k})\,F_A(\mathbf{q}_\perp + \mathbf{p}_\perp - \mathbf{k})\;k_x\,dk_y}{-p_- q_+ - (\mathbf{k} - \mathbf{q}_\perp)^2 - m^2}$$

$$I_{qy} = \frac{1}{(2\pi)^2} \int \exp[i\,k_x\,b]\,dk_x \int \frac{F_B(\mathbf{k})\,F_A(\mathbf{q}_\perp + \mathbf{p}_\perp - \mathbf{k})\;k_y\,dk_y}{-p_- q_+ - (\mathbf{k} - \mathbf{q}_\perp)^2 - m^2}$$

$$J_p = \frac{1}{(2\pi)^2} \int \exp[i\,k_x\,b]\,dk_x \int \frac{F_B(\mathbf{k})\,F_A(\mathbf{q}_\perp + \mathbf{p}_\perp - \mathbf{k})\;dk_y}{-p_+ q_- - (\mathbf{k} - \mathbf{p}_\perp)^2 - m^2}$$

$$J_q = \frac{1}{(2\pi)^2} \int \exp[i\,k_x\,b]\,dk_x \int \frac{F_B(\mathbf{k})\,F_A(\mathbf{q}_\perp + \mathbf{p}_\perp - \mathbf{k})\;dk_y}{-p_- q_+ - (\mathbf{k} - \mathbf{q}_\perp)^2 - m^2}.$$

- There is an apparent numerical difficulty in evaluating the integrals over k_x due to the oscillating factor $\exp[i\,k_x\,b]$.
- In the $\mathbf{b} = 0$ limit this factor is absent:
  we will first investigate this numerically more tractable case.
- The general case of non-zero **b** is addressed by a technique involving piecewise analytical integration.

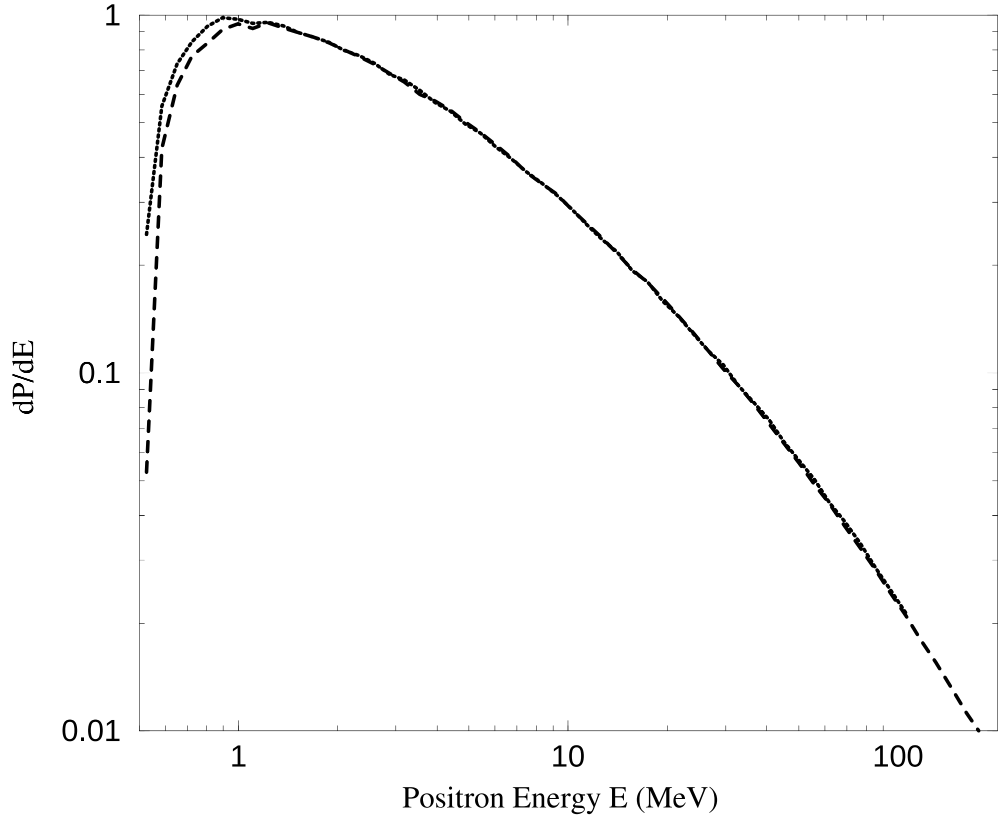
After squaring, summing over spin states, and taking traces with the aid of the computer program FORM[*]

$$
\begin{aligned}
|M(p,q)|^2 \quad = \quad & p_+q_-[(m^2+p_\perp^2)|J_p|^2+|I_{px}|^2+|I_{py}|^2 \\
& -2p_x Re(J_p I_{px}^*) - 2p_y Re(J_p I_{py}^*)] \\
+ \quad & p_-q_+[(m^2+q_\perp^2)|J_q|^2+|I_{qx}|^2+|I_{qy}|^2 \\
& -2q_x Re(J_q I_{qx}^*) - 2q_y Re(J_q I_{qy}^*)] \\
+ \quad & 2[(m^2+p_\perp^2)(q_x Re(J_p I_{qx}^*) + q_y Re(J_p I_{qy}^*)) \\
& +(m^2+q_\perp^2)(p_x Re(J_q I_{px}^*) + p_y Re(J_q I_{py}^*)) \\
& +(\mathbf{p}_\perp \cdot \mathbf{q}_\perp - m^2)(Re(I_{px} I_{qx}^*) + Re(I_{py} I_{qy}^*)) \\
& -(m^2+p_\perp^2)(m^2+q_\perp^2) Re(J_p J_q^*) \\
& -(p_x q_y + p_y q_x)(Re(I_{px} I_{qy}^*) + Re(I_{py} I_{qx}^*)) \\
& -2p_x q_x Re(I_{px} I_{qx}^*) - 2p_y q_y Re(I_{py} I_{qy}^*)].
\end{aligned}
$$

[*]J. A. M. Vermaseren, arXiv:math-ph/0010025 (2000)

THE b = 0 LIMIT

- Present calculation: Dashed line, perturbation theory.
- Comparison with previous calculation[*]:
  Dotted line, perturbation theory.



[*]Kai Hencken, Dirk Trautmann, and Gerhard Baur, Phys. Rev. **A 49**, 1584 (1994)

- Dashes: perturbation theory

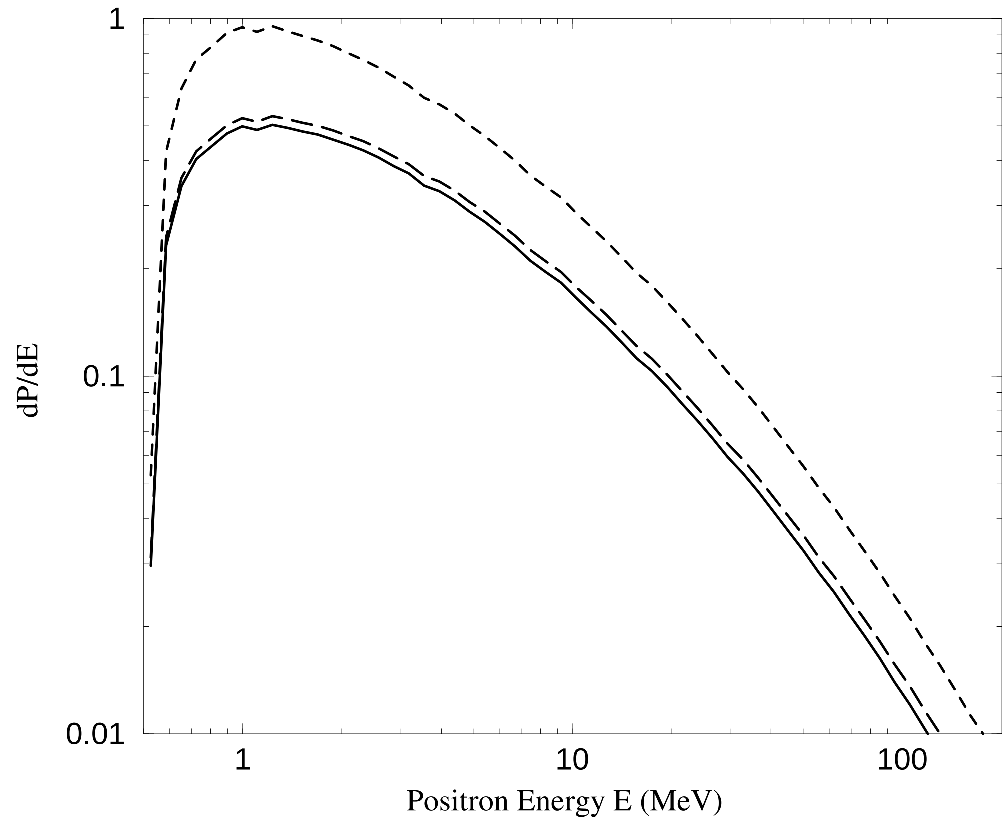
$$P^0(0) = 1.64$$

- Solid line: exact

$$P(0) = .94 = .57P^0(0)$$

- Long dashed line: eikonal

$$P^e(0) = 1.03 = .63P^0(0)$$



- Pb + Pb at LHC

$$P^0(0) = 4.07; \qquad P(0) = 2.39 = .59P^0(0)$$

EIKONAL, EXACT, AND PERTURBATIVE CASES

Exact:

$$F_{A,B}(\mathbf{k}) = 2\pi \int d\rho\rho J_0(k\rho)\{\exp[2iZ_{A,B}\alpha K_0(\rho\omega_{A,B}/\gamma)] - 1\},$$

or

$$F_{A,B}(\mathbf{k}) = \frac{4\pi i Z_{A,B}\alpha}{k^2} I_{A,B}(\gamma k/\omega)$$

where

$$I_{A,B}(\gamma k/\omega) = \frac{1}{2iZ_{A,B}\alpha}\int d\xi\xi J_0(\xi)\{\exp[2iZ_{A,B}\alpha K_0(\xi\omega/\gamma k)] - 1\}.$$

eikonal,

$$I^E_{A,B}(\gamma k/\omega) = -i(\frac{\exp[\gamma_e]\omega}{\gamma k})^{-2i\alpha Z}\frac{\Gamma(-i\alpha Z)}{\Gamma(i\alpha Z)}\frac{1}{(1+\omega^2/k^2\gamma^2)^{1-i\alpha Z}}.$$

and perturbative

$$I^0_{A,B}(\gamma k/\omega) = \frac{-i}{1+\omega^2/k^2\gamma^2},$$

Top:

Exact, solid;

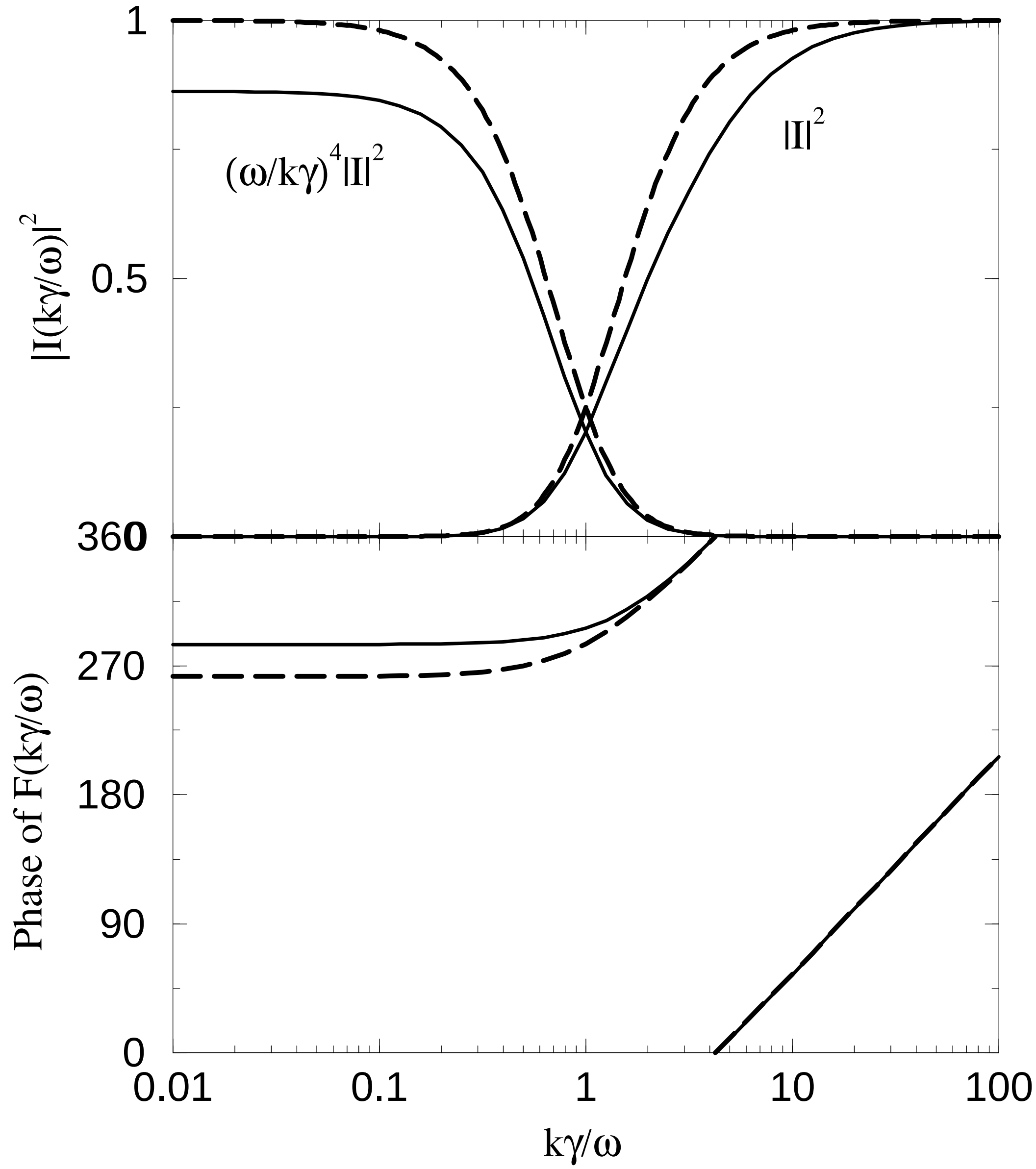
Eikonal or perturbative, dashes.

Bottom:

Exact, solid;

Eikonal, dashes ;

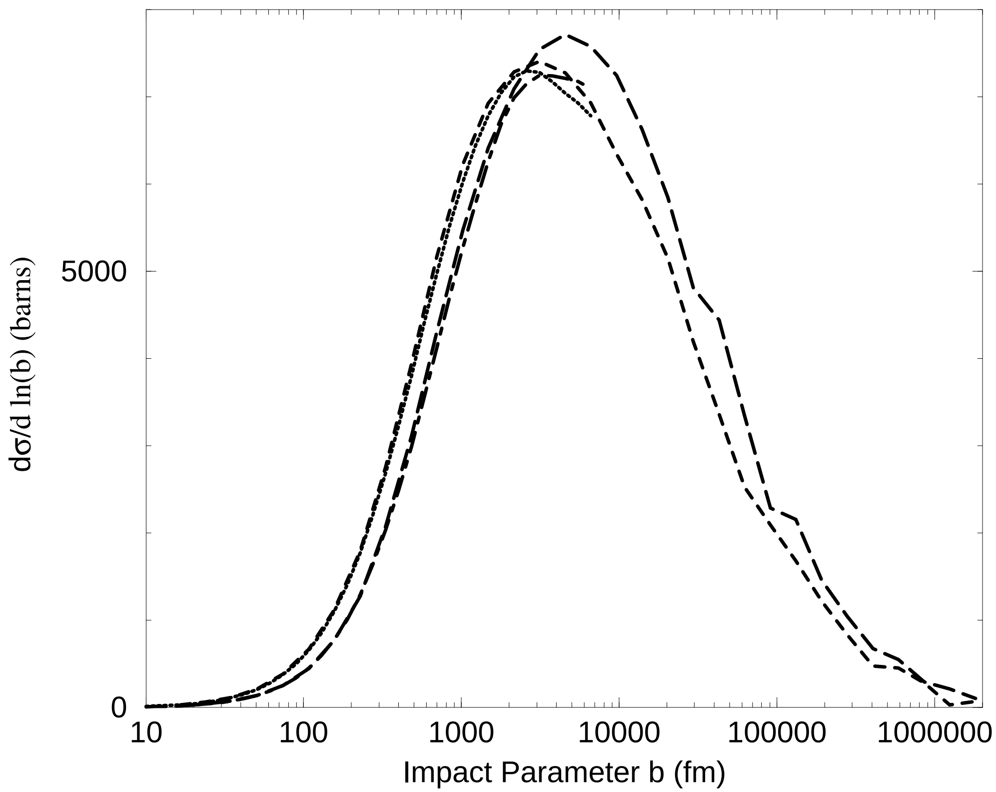
Perturbative (not shown) does not vary.



b DEPENDENT PROBABILITIES

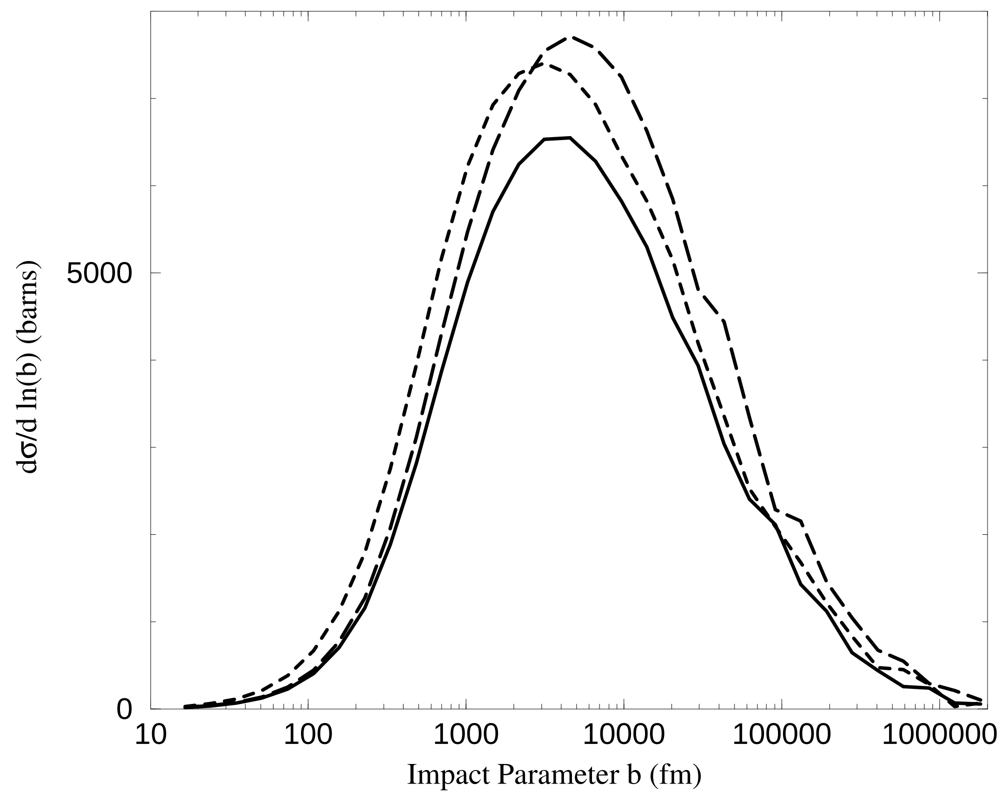
RHIC case of Au + Au at $\gamma = 100$

- Present calculations:
  Dashed line, perturbation theory; long dashed line eikonal.
- Comparison with previous calculations [*]:
  Dotted line, perturbation theory; Dot dashed line, eikonal.



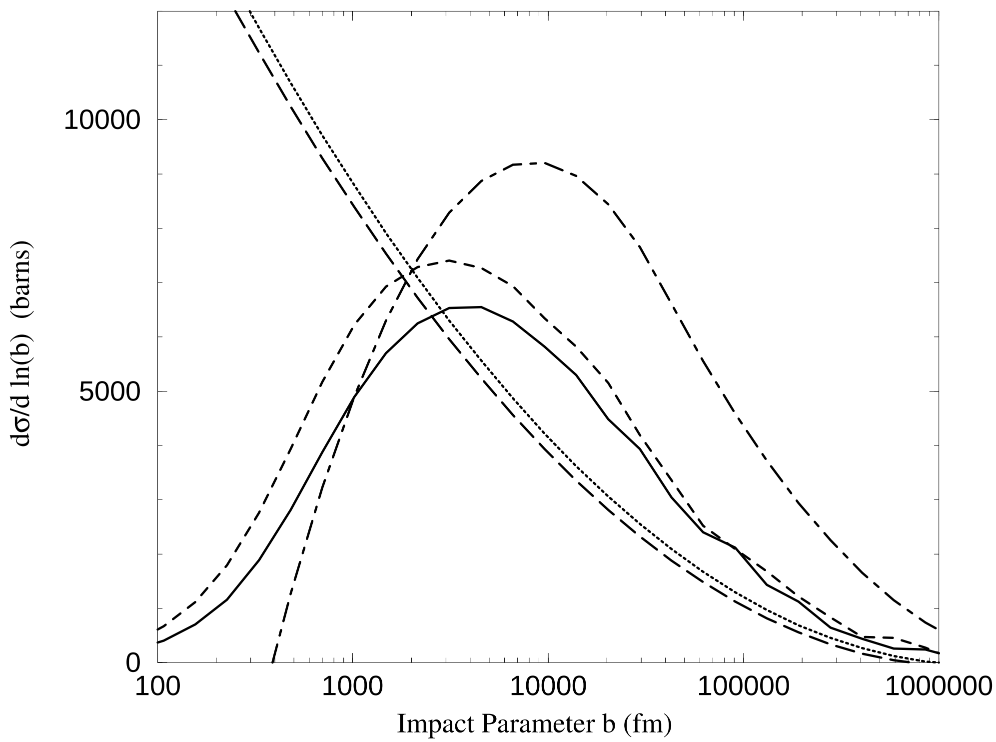
[*]Kai Hencken, Dirk Trautmann, and Gerhard Baur, Phys. Rev. **C 59**, 841 (1999)

- Dashed line, perturbation theory.
- Solid line, exact.
- Long dashed line eikonal.



RESULTS IN BARNS	Perturb.	Exact	Eikonal
Integration over b	34,600	29,400	35,500
Previous no b	34,600	28,600	34,600

- Present calculations: Dashed line, perturbation theory; solid line exact.
- Bertulani and Baur[*]:Dotted line pertubation theory; long dashed line, Coulomb corrected.
- Lee, Milstein, and Serbo[†]: Dot-dashed line, perturb.



[*]Carlos A. Bertulani and Gerhard Baur, Physics Reports **163**, 299 (1988)

[†]R. N. Lee, A. I. Milstein, and V. G. Serbo, Phys. Rev. **A 65**, 022102 (2002)

FORWARD PAIRS AT LHC

- A sample numerical calculation has been performed using the same method for e^+e^- production by Pb + Pb ions with cuts from a possible detector setup suggested by Bocian and Piotrzkowski[*] at the LHC. With electron and positron energy E and angle θ in the range,

$$3 \text{ Gev} < \text{E} < 20 \text{ GeV}$$

and

$$.00223 \text{ radians} < \theta < .00817 \text{ radians},$$

the no form factor perturbation theory cross section of 2.88 b is reduced by 18% to 2.36 b in an exact numerical calculation.

- If forward e^+e^- pairs are to be employed for luminosity measurements at LHC, then it seems necessary to consider the Coulomb corrections to the predicted cross sections.

[*]D. Bocian and K. Piotrzkowski, Acta Phys. Polon. B 35, 2417 (2004)

$\mu^+\mu^-$ PAIRS

For point charge heavy ions (no form factor) if length is expressed in terms of $1/m_l$ and energy in terms of m_l then the total cross section $\sigma(\mu^+\mu^-)$ is identical to $\sigma(e^+e^-)$

Assume simple form factor

$$F(k) = \frac{1}{1 + k^2/\Lambda^2}$$

where for Au or Pb

$$\Lambda \simeq 80\text{MeV} = 160 m_e = .75 m_\mu$$

The form factor is relatively insignificant for $\sigma(e^+e^-)$,
but for $\sigma(\mu^+\mu^-)$ contributions at large k are cut off more rapidly.

Without a form factor

$$\frac{\sigma(\mu^+\mu^-)}{\sigma(e^+e^-)} = \left(\frac{m_e}{m_\mu}\right)^2 = 23.4 \times 10^{-6}$$

But with a form factor the perturbation theory result

$$\frac{\sigma(\mu^+\mu^-)}{\sigma(e^+e^-)} = 6.1 \times 10^{-6} = .26 \times (\frac{m_e}{m_\mu})^2 \quad \text{RHIC}$$

and

$$\frac{\sigma(\mu^+\mu^-)}{\sigma(e^+e^-)} = 11.6 \times 10^{-6} = .50 \times (\frac{m_e}{m_\mu})^2 \quad \text{LHC}$$

As previously noted there is a 17% (RHIC) and 11% (LHC) reduction in the exact $\sigma(e^+e^-)$ from the perturbation theory result.

For $\sigma(\mu^+\mu^-)$ the reduction from perturbation theory is even greater, 22% (RHIC) and 14% (LHC).

Present perturbative $\sigma(\mu^+\mu^-)$ calculations are in good agreement with recent calculations of Hencken, Kuraev, and Serbo[*], but the present exact cross section calculations are in disagreement with their arguement that Coulomb corrections are relatively insignificant for mu pairs.

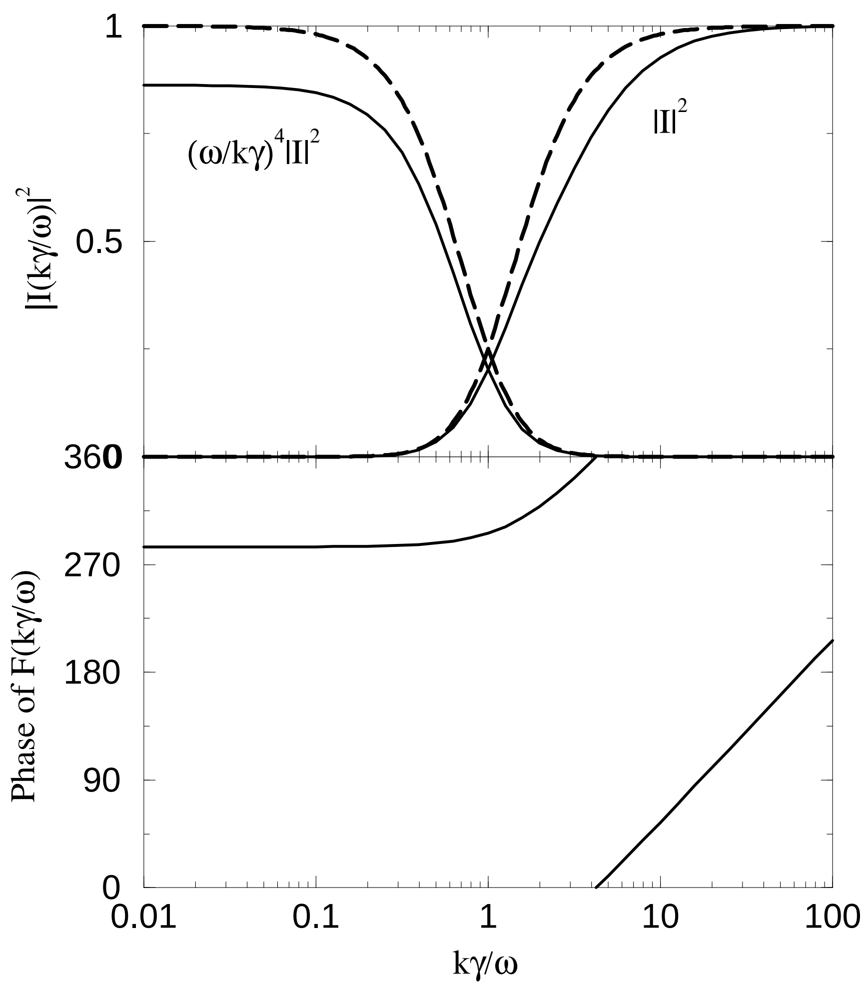
[*]K. Hencken, E. A. Kuraev, and V. G. Serbo, Acta Phys. Polon. B37, 969 (2006)

- The difference between the perturbative and exact cross sections arises from:

  (1) differences between the perturbative (long dashed) and exact (solid) values of the magnitude (top) of

$$F_{A,B}(\mathbf{k}) = \frac{4\pi i Z_{A,B}\alpha}{k^2} I_{A,B}(\gamma k/\omega);$$

  (2)the change in the phase of the exact value of $F_{A,B}(\mathbf{k})$ (bottom). (The perturbative expression for $F_{A,B}(\mathbf{k})$ does not change in phase.)



- At the highest values of $\gamma k/\omega$ (corresponding to the region where μ pairs are suppressed by the form factor) the exact magnitude is not reduced.

DEVIATIONS FROM Z^4 SCALING

- No statistically significant deviation of continuum e^+e^- pair production rates from $Z_A^2 Z_B^2$ scaling was observed in the SPS data of

  160 GeV/c Pb ions on C, Al, Pa, Au and

  200 Gev/c S ions on C, Al, Pa, Au

- The statistics of the published Au + Au STAR data are insuffient to rule out Coulomb correction deviations from perturbation theory calculations.

- Observe lepton pairs at LHC from Ca + Ca and Pb + Pb

  at the same relativistic beam γ

  and with the same detector acceptance

  A statistically significant lack of deviations from Z^4 scaling would provide a severe challenge to our present understanding of QED with UPCs.

CONCLUSIONS: THEORY

- A full numerical evaluation of the “exact” total cross section for e^+e^- production with gold or lead ions shows reductions from perturbation theory of 28% (SPS), 17% (RHIC), and 11%(LHC).
- For large Z no final momentum region was found in which there was no reduction or an insignificant reduction of the exact cross section.
- Reductions in the exact total probability of e^+e^- production from perturbation theory were seen at all impact parameters.
- The reduction of the $\mu^+\mu^-$ cross section from perturbation theory is even larger than in the e^+e^- case.

FOR THE FUTURE: THEORY

- Still to be done is an accurate exact calculation of the high transverse momentum slice of data seen by STAR to be combined with a Coulomb dissociation calculation for the zero degree calorimeter acceptance.
- The present approach is strictly speaking valid only when either the positrons or electons have been integrated over; in the STAR case both electron and positron are constrained to be in the high momentum slice. At present the best one can do is observe that the present method is valid for both uncorrelated positrons and electrons of all momenta, and ignore the correlations. The effect of correlations averages to zero, but some estimate of individual magnitudes would be useful.