On the Dipole picture in photon induced reactions

1 Introduction, Motivation

2 Rigorous Results

3 High Energy Limit

4 Tests for the standard dipole picture in DIS

5 Summary and Conclusions

C. Ewerz — ECT

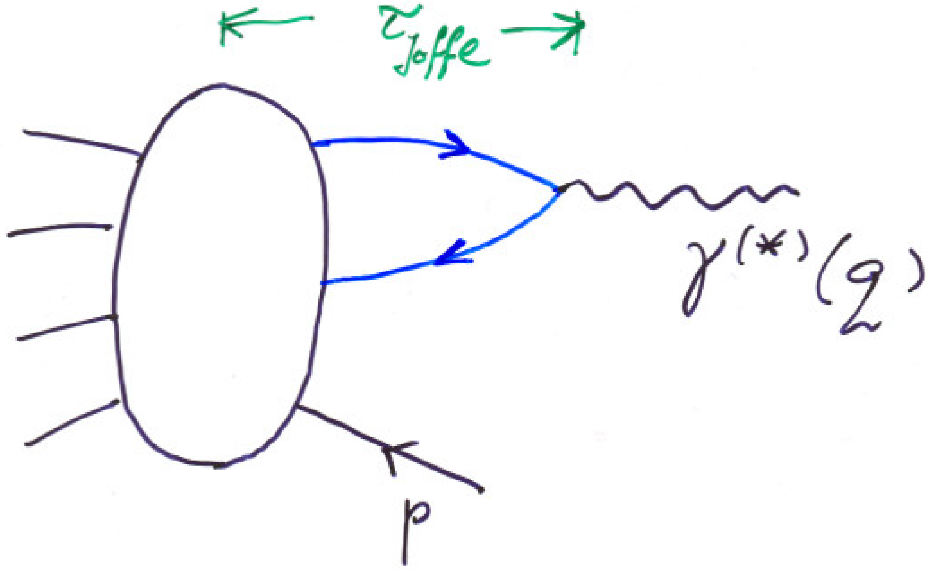
O. Nachtmann — Univ. Heidelberg

1 Introduction, Motivation

Consider a photon induced hadronic reaction at high energies, for instance

$$\gamma^{(*)}(q) + p(p) \rightarrow X$$

Restframe of target p:
$\gamma^{(*)}$ fluctuates into a virtual quark - antiquark pair long before hitting the proton.
Interaction of $\gamma^{(*)}$ is equivalent to the interaction of the colour dipole $q\bar{q}$ pair.
This is similar to the picture developed by Joffe (1969) of γ^* fluctuating to hadrons.



$|\vec{q}| \to \infty$

Ioffe time $\tau_{Ioffe} \approx q^0/m^2 \to \infty$

Revival of interest in this approach

for DIS :

Nikolaev & Zakharov , 1991, 1992

Mueller , 1994, 1995

....

Dipole approach for hadrons in soft hadronic high energy scattering

Dosch, Krämer, Ferreira, O.N. , 1990...

Successful applications to DIS:

Golec-Biernat & Wüsthoff, 1999, ...

Forshaw, Kerley, Shaw, 1999, ...

Donnachie, Dosch,

McDermott, Buchmüller, Hebecker,

J. Bartels et al.,

....

Dipole picture is frequently (mis-?) used to motivate saturation in ~~pho~~ proton-induced reactions, e.g. to claim a Froissart-limit for $\gamma^{(*)}p$ total cross sections:

$$\sigma_{tot}(\gamma^{(*)}p) \leq \text{const.}\left(\ln\left(s/s_0\right)\right)^2$$

for $s \to \infty$

Our motivation (C. Ewerz, O. N.):

- How far can we come to give a rigorous foundation for something like the dipole picture of photon-induced reactions?
- Such a foundation should be a prerequisite for deriving reliable results for saturation.
- We do not rely on perturbation theory but use functional-integral techniques (see O. N., Ann. Phys. 209, 436 (91)).
- Can we give a definition of the "photon wave function" using Lagrangian functional integral techniques?

2 Rigorous Results

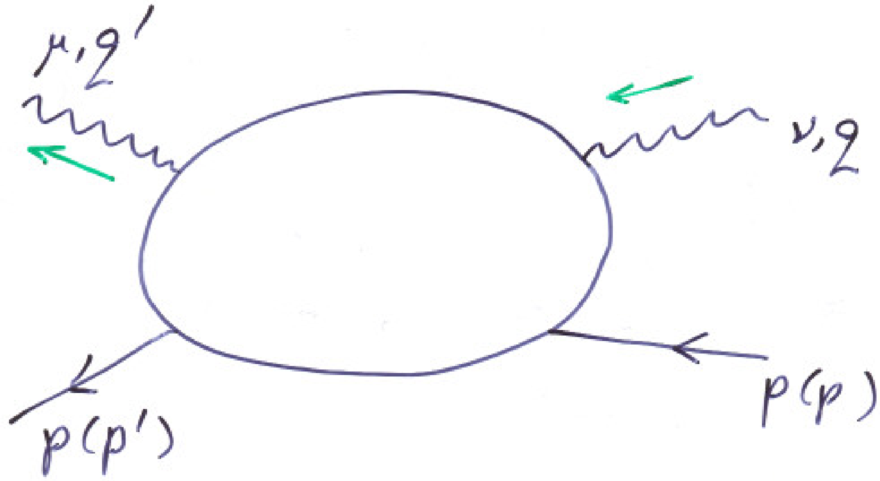
Consider as an example real or virtual Compton scattering on a proton

$$\gamma^{(*)}(q) + p(p) \longrightarrow \gamma^{(*)}(q') + p(p')$$

$$q^2 = -Q^2 \leq 0 ,$$

$$q'^2 = -Q'^2 \leq 0 .$$

$$\gamma^{(*)}(q) + p(p) \longrightarrow \gamma^{(*)}(q') + p(p')$$



$$\mathcal{M}^{\mu\nu}_{s's}(p',p,q) = \frac{i}{2\pi m_p} \int d^4x \, e^{-iqx}$$

$$\langle p(p',s') | T^* J^\mu(0) J^\nu(x) | p(p,s) \rangle$$

Now we use the LSZ reduction formula and express Green's functions as functional integrals.

- LSZ reduction formula:

$$\mathcal{M}^{\mu\nu}_{s's} = \text{Integral over Green's function}$$

- Express Green's function as functional integral

Green's function =

$$\int \mathcal{D}(G, q, \bar{q}) \exp\left[i \int d^4x\, \mathcal{L}_{QCD}(x)\right] \cdots\cdots$$

- Integrate out the quarks using the fact that $\mathcal{L}_{QCD}$ is bilinear in the quark degrees of freedom

- Result: 7 contributions to the Compton amplitude

$$\mathcal{M} = \mathcal{M}^{(a)} + \mathcal{M}^{(b)} + \ldots + \mathcal{M}^{(g)}$$

Diagrammatic representation:

(i) Calculate diagrams for fixed gluon potential, where

means full quark propagator in this potential.

(ii) Integrate over all gluon potentials with a measure given by the functional integral. This includes the fermion determinant, gauge fixing and Fadeev - Popov terms.

leading at high energy:

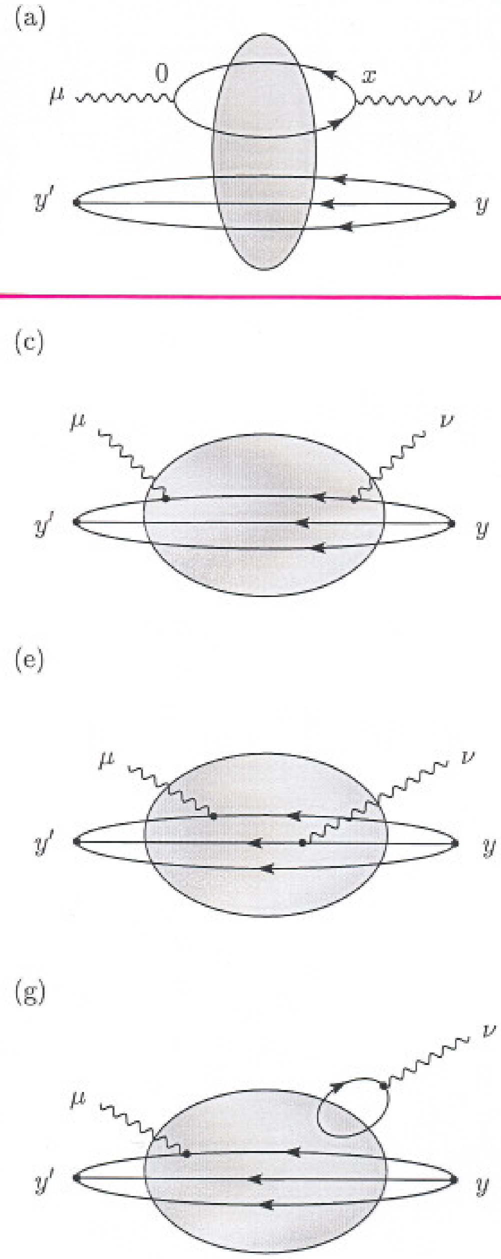


Figure 1: The diagrams resulting from the functional integral corresponding to $\mathcal{J}^{(a)}, \ldots, \mathcal{J}^{(g)}$ and to the amplitudes $\mathcal{M}^{(a)}, \ldots, \mathcal{M}^{(g)}$

$$\mathcal{M}^{(a)\mu\nu}_{s's}(p',p,q) =$$

$$\sum_q Q_q^2 \left\langle \mathcal{C}_{s's}(p',p)\, A^{(q)\mu\nu}(q) \right\rangle_G$$

$$\mathcal{C}_{s's}(p',p) = -\frac{i}{2\pi m_p Z_p} \int d^4y'\, d^4y$$

$$e^{-ip'y'}\, \bar{u}_{s'}(p')\, (-i\overrightarrow{\not\partial}_{y'} + m_p)$$

$$\overbrace{\psi_p(y')\ \bar{\psi}_p}(y)$$

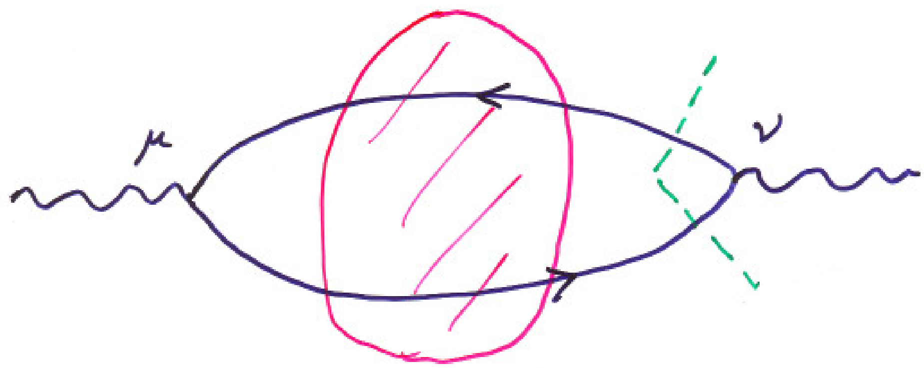
$$(i\overleftarrow{\not\partial}_y + m_p)\, u_s(p)\, e^{-ipy}$$

$$A^{(q)\mu\nu}(q) = \int d^4x$$

$$\mathrm{Tr}\left[\gamma^\mu S_F^{(q)}(0,x;G)\, e^{-iqx} \gamma^\nu S_F^{(q)}(x,0;G)\right]$$

Study of quark loop in fixed gluon potential:

$$A^{(q)\mu\nu}(q) =$$



- Cut the quark lines next to vertex ν
- Insert suitable factors 1

$$(-i\partial\!\!\!/_z + m_q)\, S_F^{(q,0)}(z,x) = \delta^{(4)}(z-x)$$

↑ free propagator

- Represent free propagators with spin sums of quark and antiquark spinors

$$S_F^{(q,0)}(x,y) = \theta(x^0-y^0)\langle 0|q(x)\bar{q}(y)|0\rangle - \theta(y^0-x^0)\langle 0|\bar{q}(y)q(x)|0\rangle$$

$$S_F^{(q,0)}(x,y) = \theta(x^0-y^0)$$

$$\frac{i}{(2\pi)^3}\int\frac{d^3k}{2k^0}\, e^{-ik(x-y)}\,(\not{k}+m_q)$$

$$+\ \theta(y^0-x^0)$$

$$\frac{i}{(2\pi)^3}\int\frac{d^3k}{2k^0}\, e^{ik(x-y)}\,(-\not{k}+m_q)$$

$$= -\frac{1}{2\pi}\int\frac{d\omega}{\omega+i\varepsilon}\int\frac{d^3k}{(2\pi)^3\,2k^0}\sum_r$$

$$\Big\{ e^{-ik_\omega x}\, u_r(k)\ \bar{u}_r(k)\ e^{ik_\omega y}$$

$$-\ e^{ik_\omega x}\ v_r(k)\ \bar{v}_r(k)\ e^{-ik_\omega y}\Big\}$$

$$k = \begin{pmatrix} k^0 \\ \vec{k} \end{pmatrix} = \begin{pmatrix} \sqrt{\vec{k}^{\,2}+m_q^2} \\ \vec{k} \end{pmatrix},\quad k_\omega = \begin{pmatrix} k^0+\omega \\ \vec{k} \end{pmatrix}$$

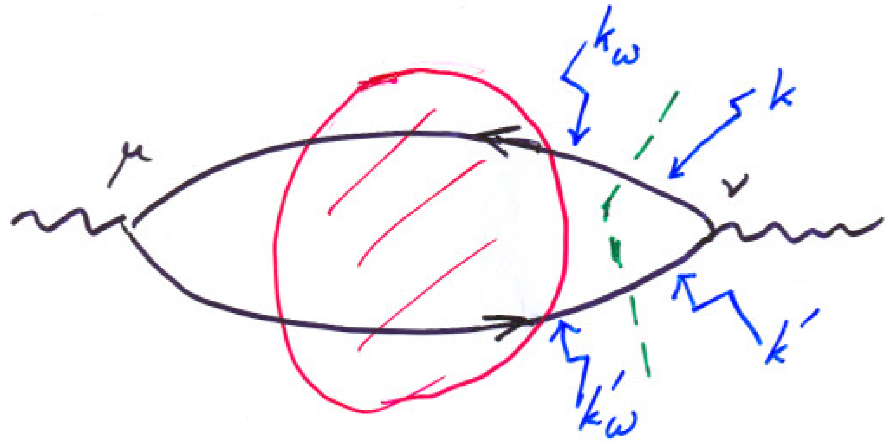
Putting everything together we find the following result:

- The (a) term of the Compton amplitude can be written as sum of 4 terms. The at high energy leading term has the structure

  photon splitting into $q\bar{q}$ times an amplitude for $q\bar{q}$, but off energy shell, scattering on the proton.

- According to LSZ this amplitude involves the quark wave function renormalisation constant Z_q. This must then also multiply the photon $\rightarrow q\bar{q}$ splitting factor.

- An integration over the q and $\bar{q}$ off shell energy is to be performed.



$$A^{(q)\mu\nu}(q) = \sum_{j=1}^{4} A^{(q,j)\mu\nu}(q)$$

$$A^{(q,1)\mu\nu}(q) = -\int \frac{d^3k}{(2\pi)^3 2k^0} \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega$$

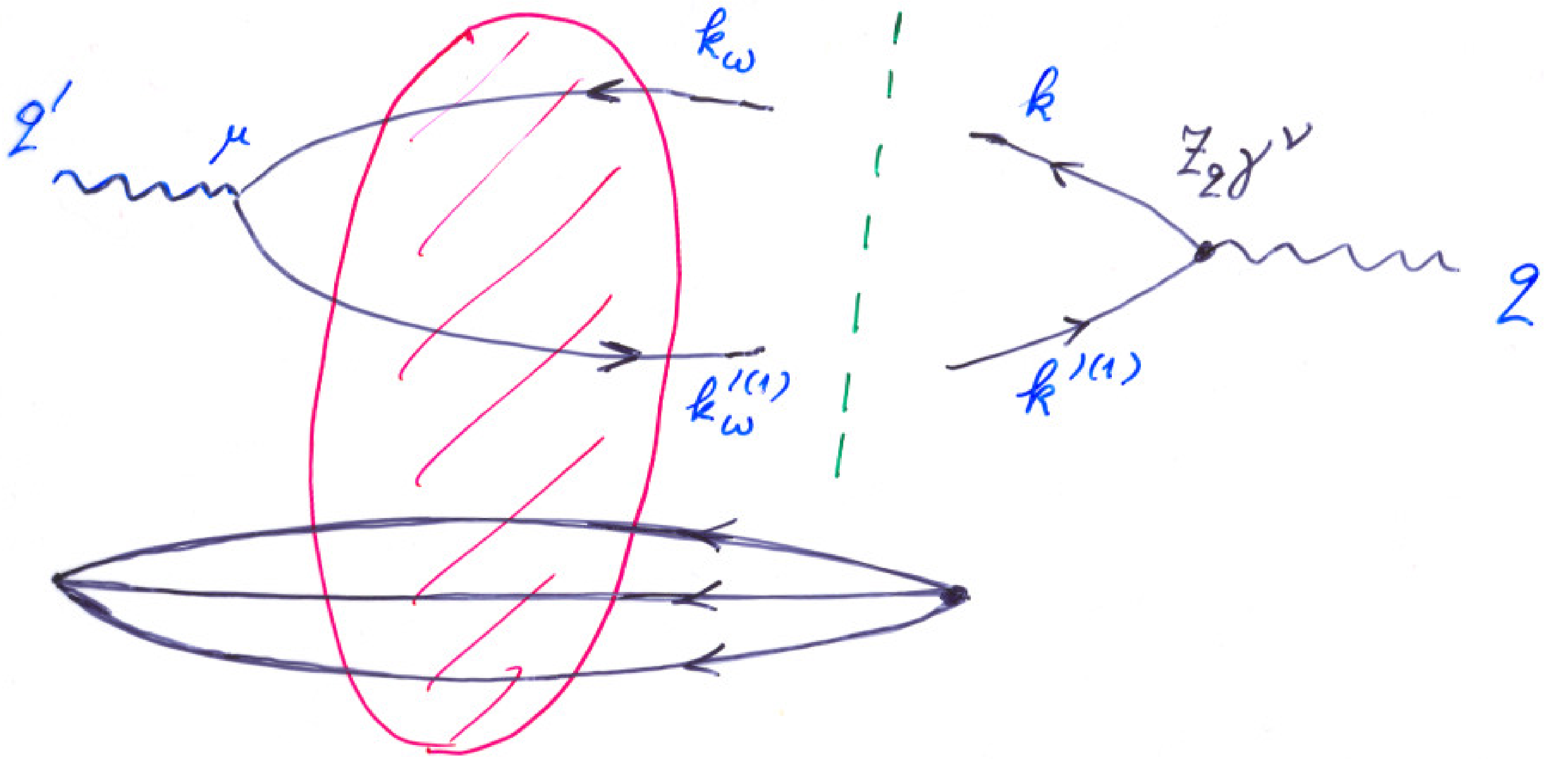
$$\frac{1}{(\omega + i\varepsilon)(q^0 - k^0 - k'^0 - \omega + i\varepsilon)\, 2k'^0}$$

$$\Big\{ \int d^4z\, d^4z'\, e^{-ik'_\omega z'}\, \bar{v}_{r'}(k')\,(-i\overrightarrow{\partial}_{z'} + m_q)$$

$$S_F^{(q)}(z', 0; G)\, \gamma^\mu\, S_F^{(q)}(0, z; G)\,(i\overleftarrow{\partial}_z + m_q)$$

$$u_r(k)\, e^{-ik_\omega z} \Big\}$$

$$\bar{u}_r(k)\, \gamma^\nu\, v_{r'}(k')$$

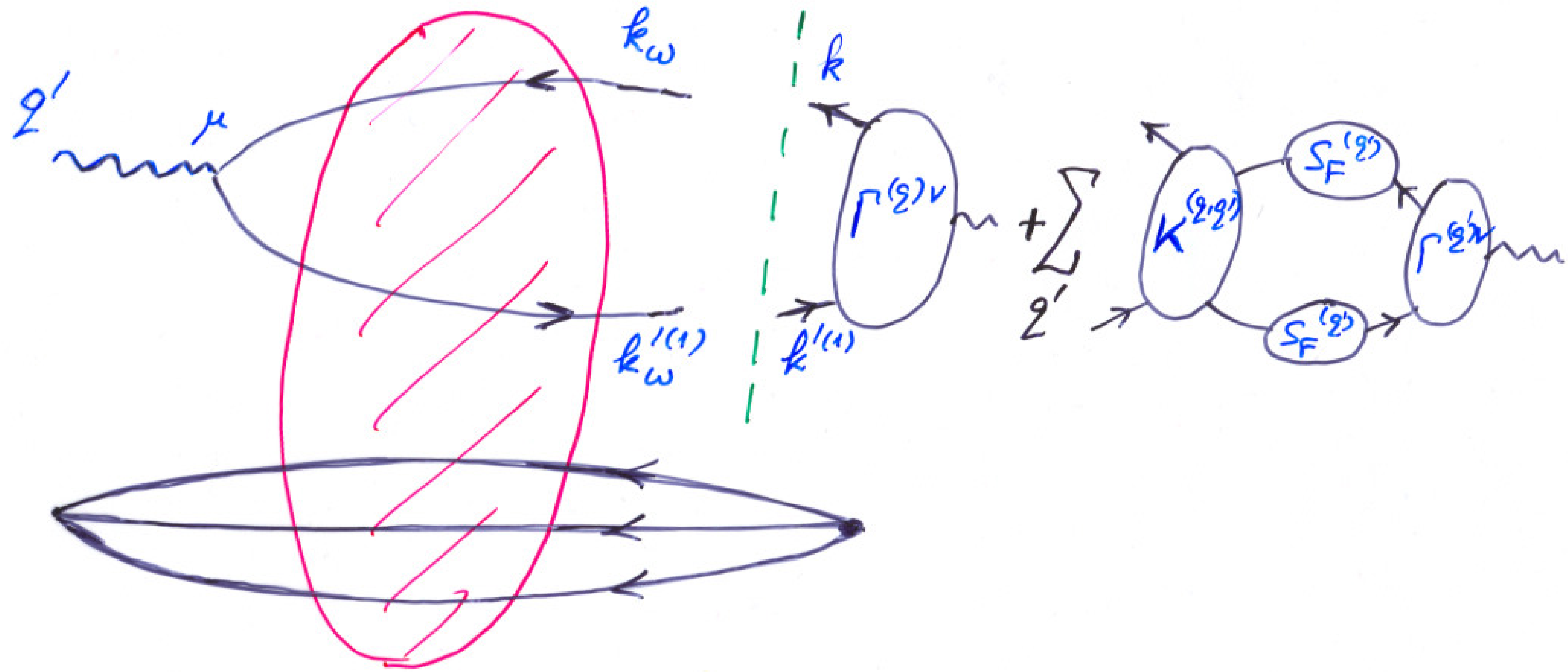


Integration over off shell energy ω!

- To get rid of Z_q we use the Schwinger - Dyson eq. for the photon - quark - antiquark vertex function.

  This procedure is completely analogous to the renormalisation of the vacuum polarisation function in QED where overlapping divergences occur.

- The same steps as for $\mathcal{M}^{(a)}$ can be done for $\mathcal{M}^{(b)}$



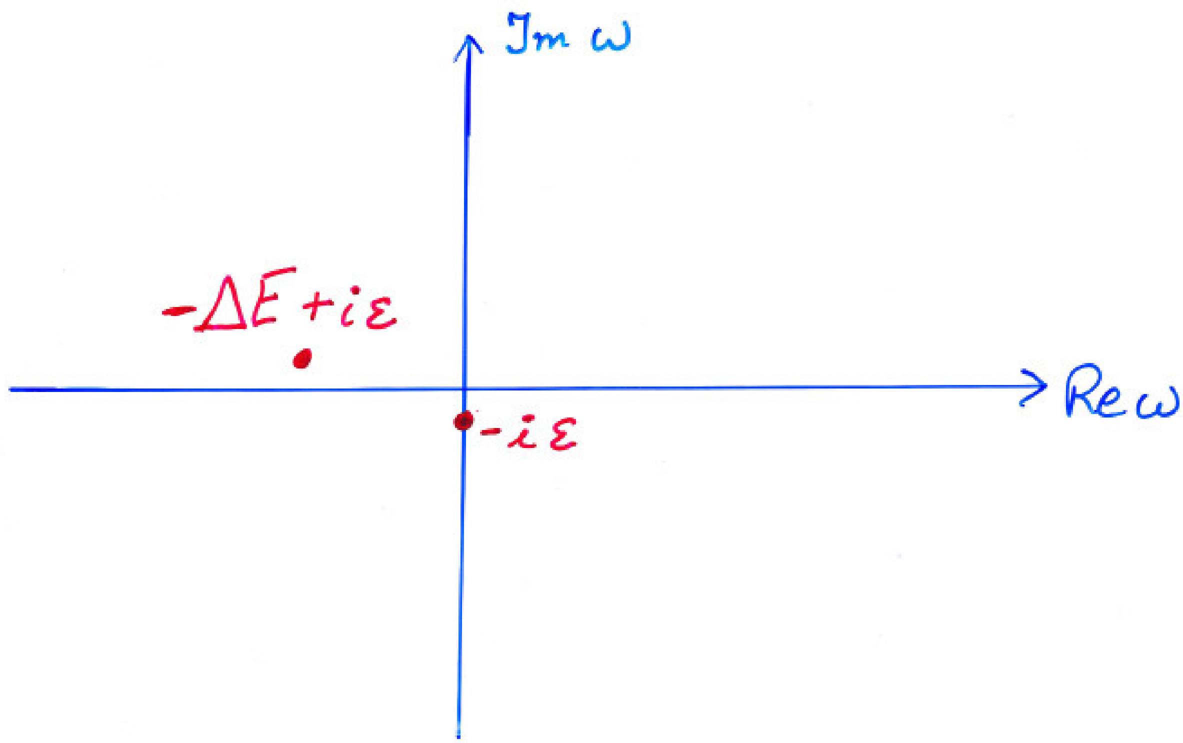
Integration over off shell energy ω!

3 High energy limit, $|\vec{q}| \to \infty$

Assume that quarks have a mass shell.

Choose m_q = ren. quark mass.

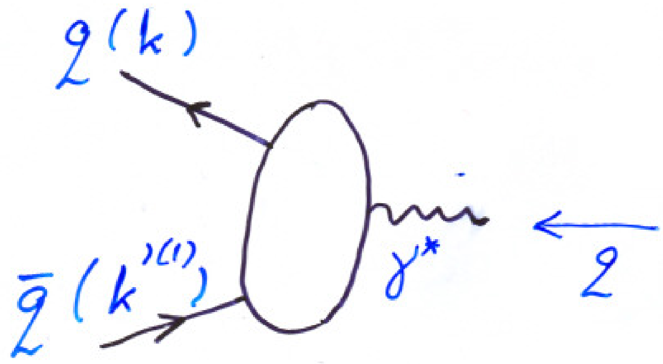
Suppose : $|\vec{q}| \to \infty$.



Energy difference quark + antiquark − $\gamma^{(*)}$:

$$\Delta E = k^0 + k'^{(1)0} - q^0$$

- pinch for $\Delta E \to 0$ in the ω integral!



$$\vec{k} = \alpha \vec{q} + \vec{k}_T$$

$$\vec{k}'^{(1)} = (1-\alpha)\vec{q} - \vec{k}_T$$

$0 \leq \alpha \leq 1$ required for pinch!

$$\Delta E = k^0 + k'^{(1)0} - q^0$$

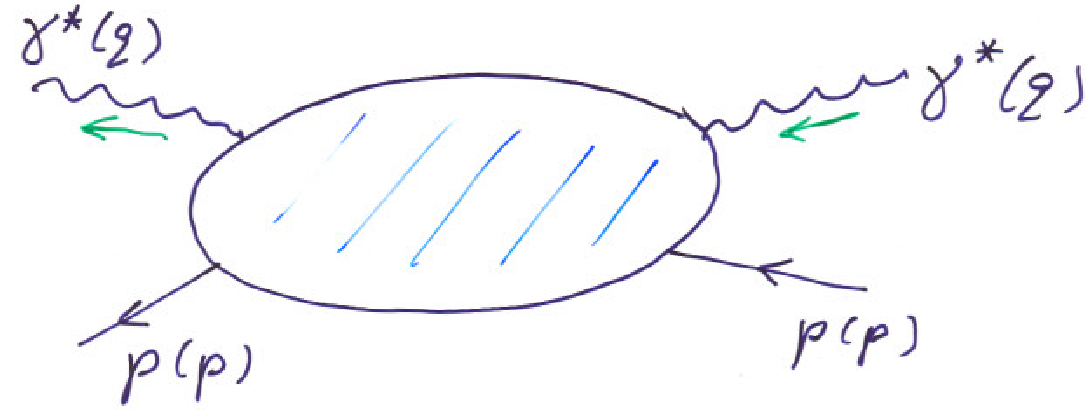
$$= \frac{Q^2 + \tilde{m}^2}{q^0 + k^0 + k'^{(1)0}}$$

$$\tilde{m}^2 \approx \frac{\vec{k}_T^{\,2} + m_q^2}{\alpha(1-\alpha)}$$

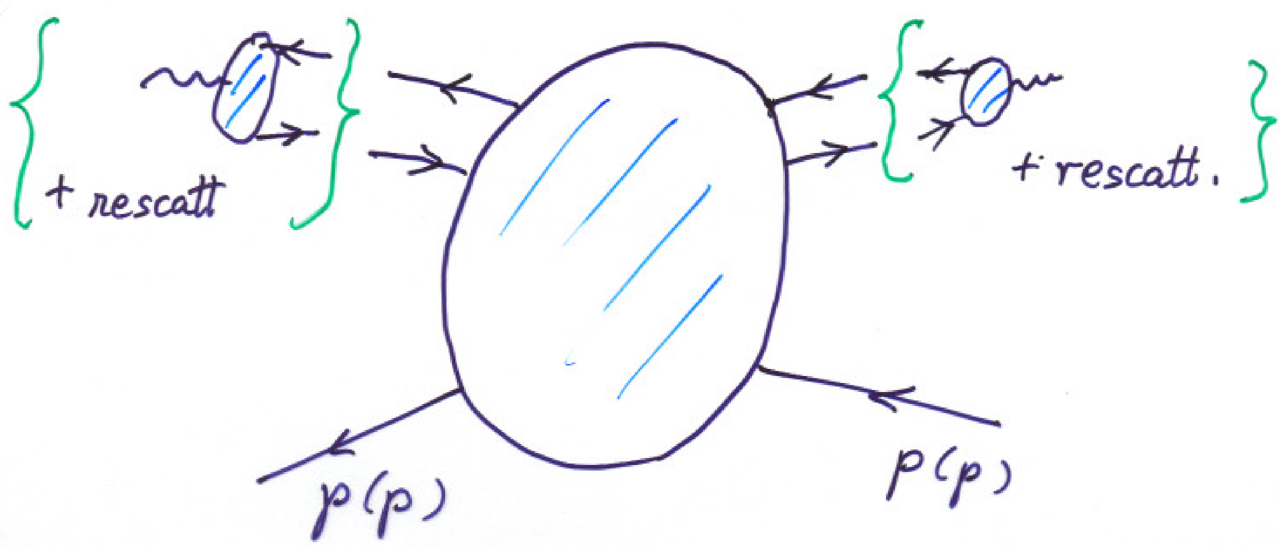
- Pinch, that is $\Delta E \to 0$, occurs for quark and antiquark moving in the same long. direction as the original γ^*.
- Pinch contribution: q and $\bar{q}$ on shell.
- Define $q\bar{q}$ dipole states, transverse separation $\vec{R}_T$.

Application to DIS

$$\gamma^*(q) + p(p) \longrightarrow \gamma^*(q) + p(p)$$



$$\Downarrow \; q^0 \longrightarrow \infty$$



Assumptions needed to arrive at the standard dipole picture:

(i) quarks have a mass shell. Only the pinch contribution is kept in the ω integral.

(ii) the photon vertex function is taken as γ^{ν}, the rescattering term is dropped.

(iii) Only $\mathcal{M}^{(a)}$ is kept, $\mathcal{M}^{(b)}$ is dropped.

(iv) The scattering of the dipole on the proton is diagonal in quark flavour, transverse size $\vec{R}_T$ and longitudinal momentum α.

(v) The dipole-proton cross section depends only on $\vec{R}_T^{\,2}$ and the (c.m. energy)^2 = W^2.

Result:

$$\sigma_T(W^2, Q^2) = \sum_q \int d^2R_T \, w_T^{(q)}(R_T, Q^2) \, \hat{\sigma}^{(q)}(R_T, W^2)$$

$$\sigma_L(W^2, Q^2) = \sum_q \int d^2R_T \, w_L^{(q)}(R_T, Q^2) \, \hat{\sigma}^{(q)}(R_T, W^2)$$

$\hat{\sigma}^{(q)}(R_T, W^2) \geq 0$, dipole-proton cross section

$w_{T,L}^{(q)}(R_T, Q^2) \propto$

|photon wave function$|^2$

$$w_{T,L}^{(q)}(R_T, Q^2) = \int_0^1 d\alpha \; v_{T,L}^{(q)}(\alpha, R_T, Q^2)$$

$$v_T^{(q)}(\alpha, R_T, Q^2) = \frac{3}{2\pi^2} \alpha_{em} Q_q^2$$

$$\left\{ \left[\alpha^2 + (1-\alpha)^2\right] \varepsilon_q^2 \left(K_1(\varepsilon_q R_T)\right)^2 + m_q^2 \left(K_0(\varepsilon_q R_T)\right)^2 \right\},$$

$$v_L^{(q)}(\alpha, R_T, Q^2) = \frac{6}{\pi^2} \alpha_{em} Q_q^2 Q^2$$

$$\left[\alpha(1-\alpha)\right]^2 \left(K_0(\varepsilon_q R_T)\right)^2,$$

$$\varepsilon_q = \sqrt{\alpha(1-\alpha)Q^2 + m_q^2}\,.$$

- There are non trivial problems/subtleties in choosing the polarisation vector for γ_L^* correctly.

$$q = \begin{pmatrix} q^0 \\ 0 \\ 0 \\ |\vec{q}| \end{pmatrix}$$

Bad choice of long. pol. vector

$$\varepsilon_L = \frac{1}{Q} \begin{pmatrix} |\vec{q}| \\ 0 \\ 0 \\ q^0 \end{pmatrix} \longrightarrow \infty$$

for $|\vec{q}| \to \infty$

Good choice:

$$\varepsilon_L' = \varepsilon_L - \frac{1}{Q} q = \frac{Q}{|\vec{q}| + q^0} \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

$\longrightarrow 0$ for $|\vec{q}| \to \infty$

4 Tests for the standard dipole picture in DIS

Bounds on ratios of cross sections and structure functions following from

$$w_T^{(q)}(R_T, Q^2) \geq 0,$$

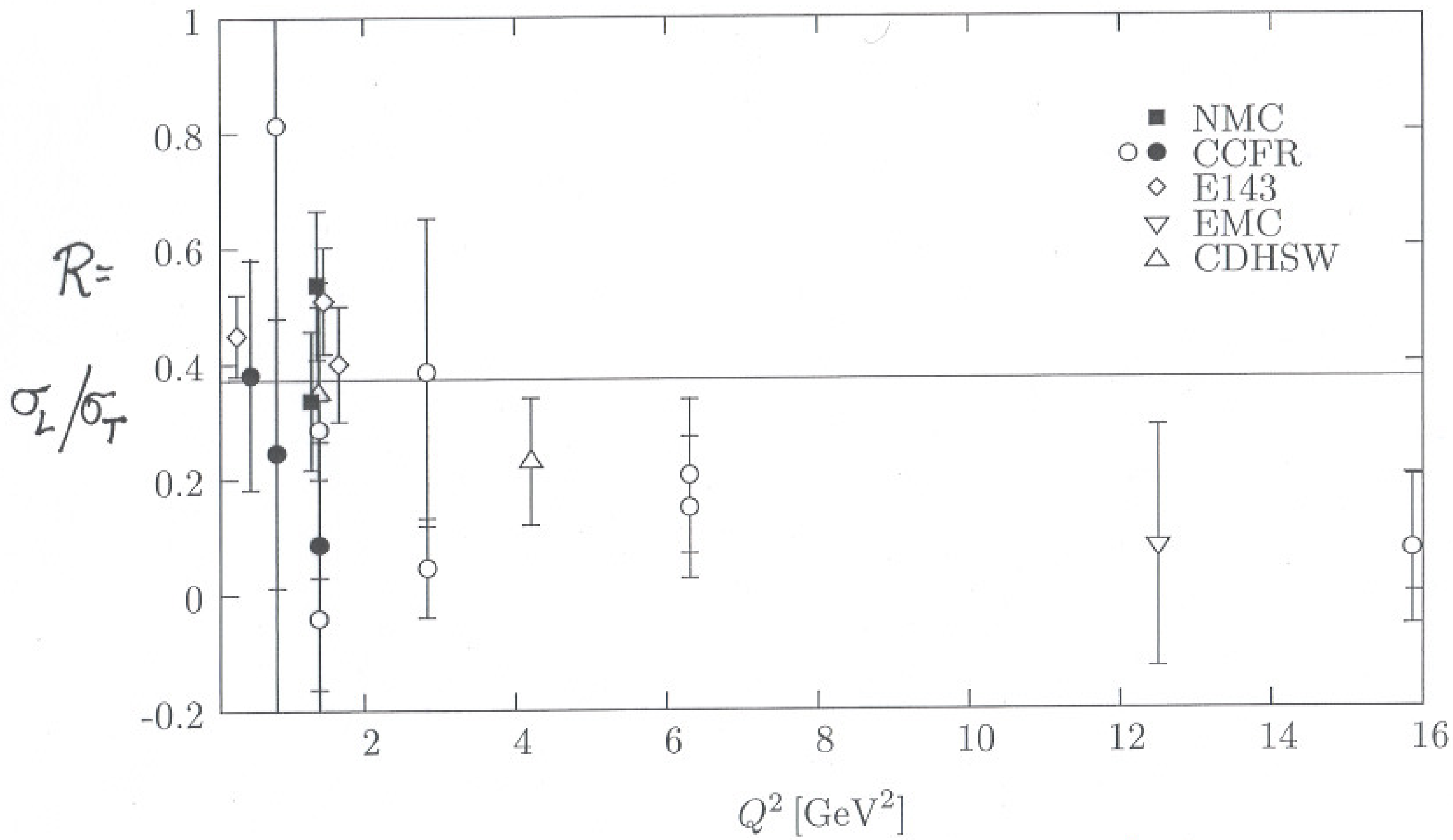
$$w_L^{(q)}(R_T, Q^2) \geq 0,$$

$$\hat{\sigma}^{(q)}(R_T, W^2) \geq 0,$$

and the explicit form of $w_{T,L}^{(q)}$.

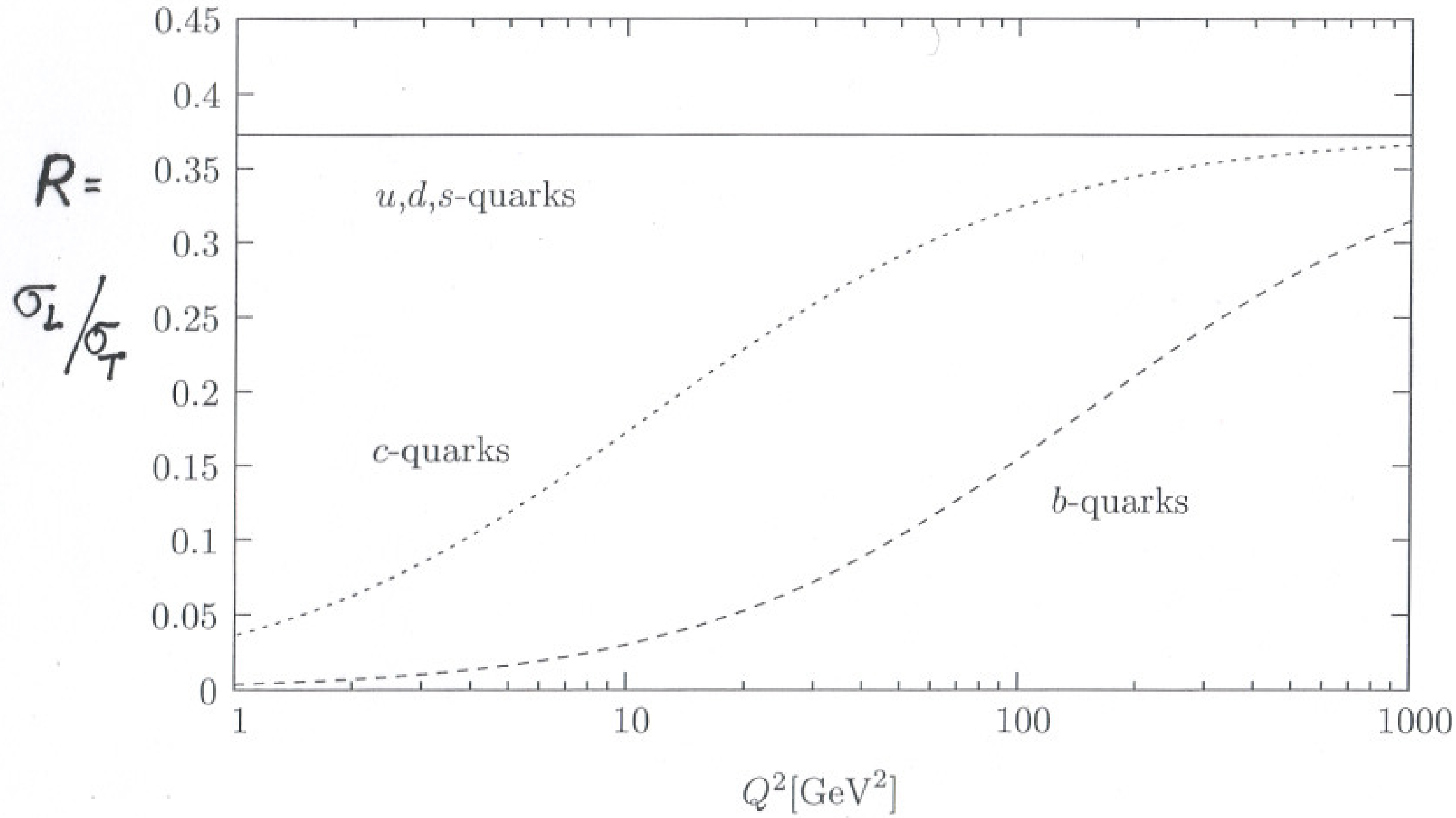
(1) A bound for R :

$$R(W^2, Q^2) = \frac{\sigma_L(W^2, Q^2)}{\sigma_T(W^2, Q^2)} \leq 0.37248$$



■ ● data with $x < 0.01$

○ ◇ ▽ △ data with $0.01 < x < 0.05$



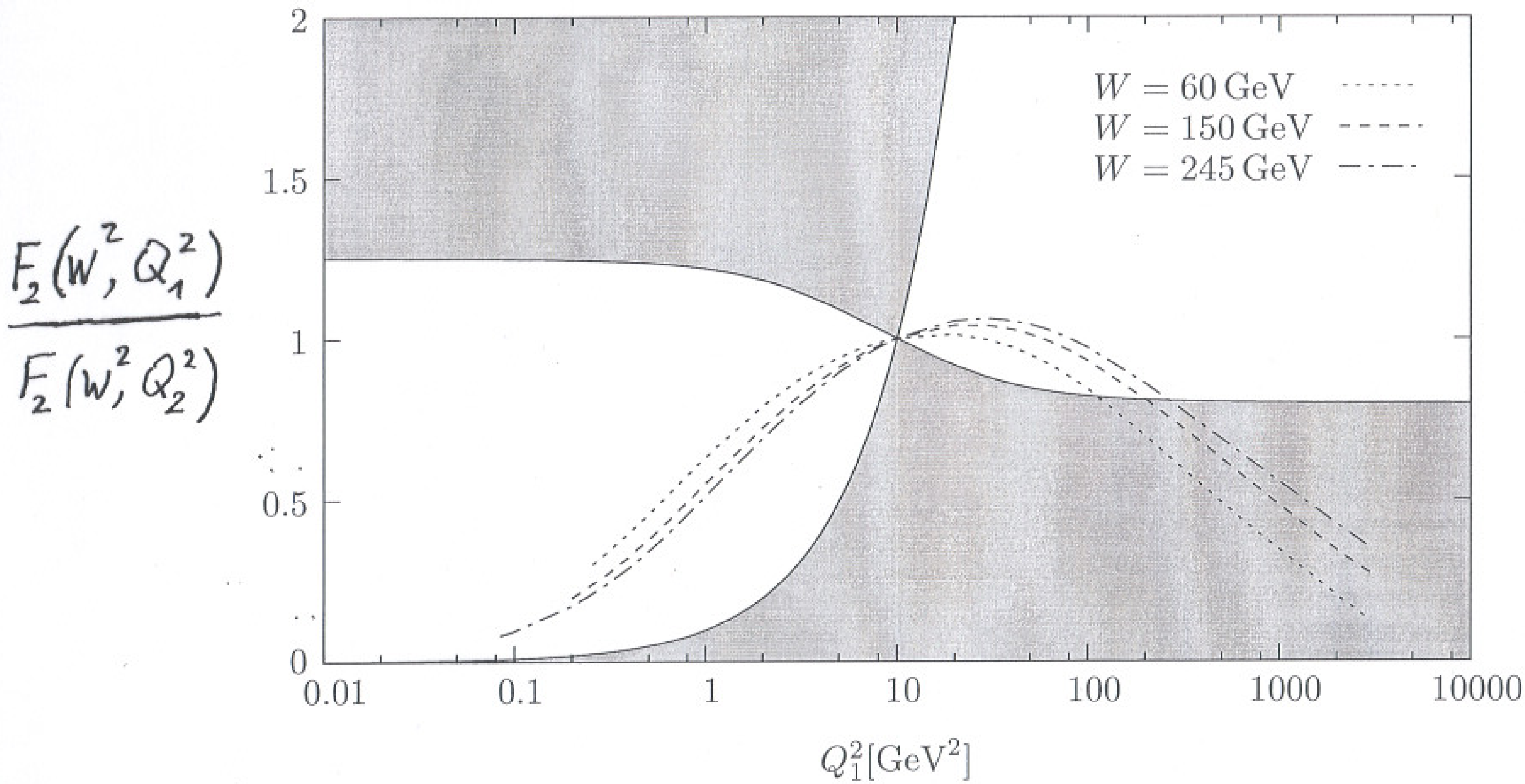
R=
σL/σT
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
u,d,s-quarks
c-quarks
b-quarks
1
10
100
1000
Q2[GeV2]

(2) Bounds for the ratio of structure functions at the same W^2 but different Q^2:

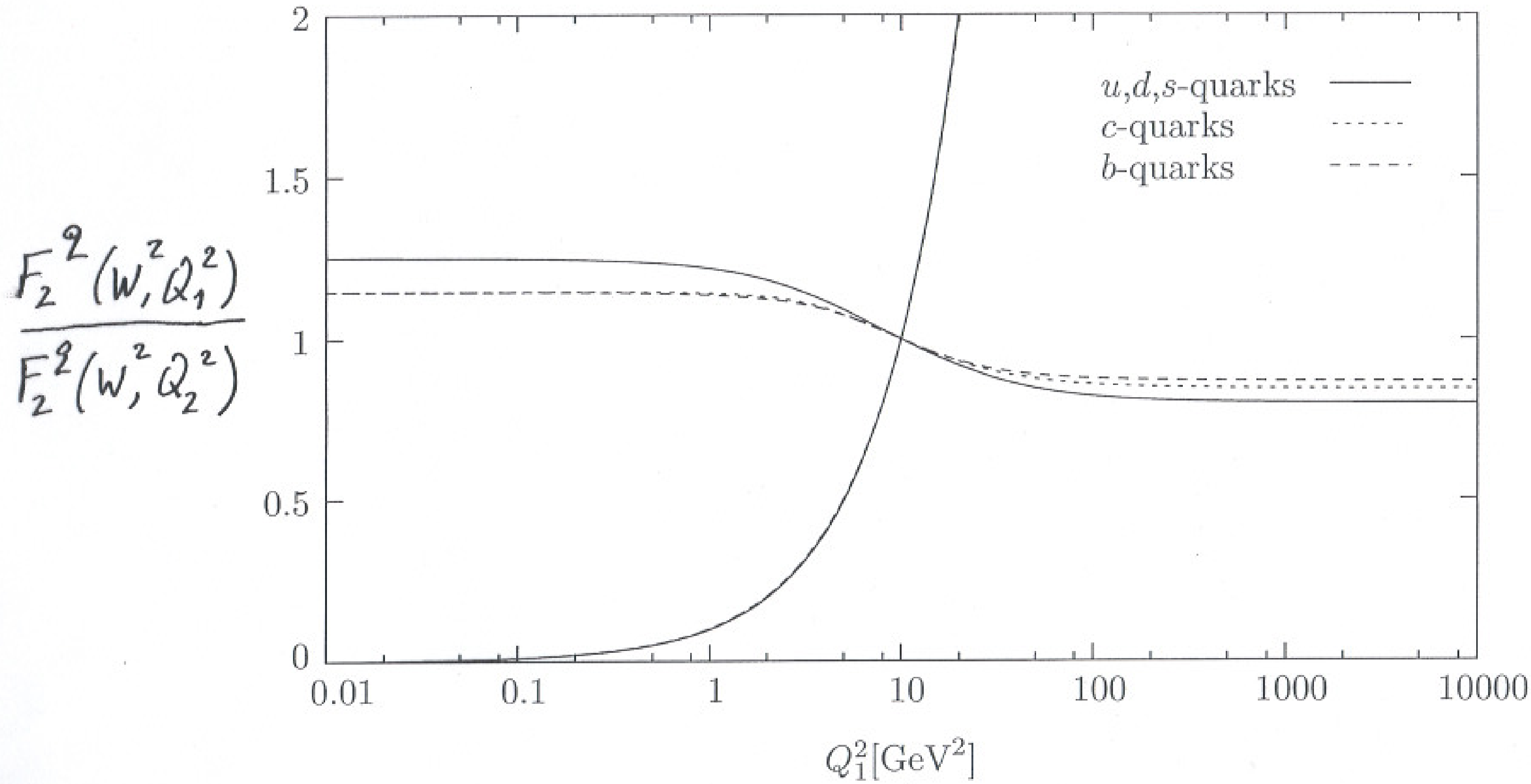
$$\frac{F_2(W^2; Q_1^2)}{F_2(W^2; Q_2^2)}$$

Example: set $Q_2^2 = 10\ \mathrm{GeV}^2$.

The data is <u>outside of the bounds</u> for $Q_1^2 \cong 100 - 200\ \mathrm{GeV}^2$.



$Q_2^2 = 10\ \mathrm{GeV}^2$



$Q_2^2 = 10\ \mathrm{GeV}^2$

5 Summary and Conclusions

- We have developed a non perturbative framework to study high energy (virtual) photon hadron scattering. Renormalisation is taken care of.
- The necessary assumptions leading to the standard dipole picture have been spelled out.
- Stringent tests for this dipole picture have been given. They suggest, that for HERA energies, $W = 60$ to $250\ \mathrm{GeV}$, the dipole picture may at best be valid in the range

$$2\ \mathrm{GeV}^2 \lesssim Q^2 \lesssim 100 \text{ to } 200\ \mathrm{GeV}^2.$$

- For more detailed derivations and many more results see

C. Ewerz & O.N.

hep-ph/ 04 04 254

06 04 087

06 11 076

Further work is in progress together with A.v. Manteuffel