Hard Photoproduction at HERA
- Theory -

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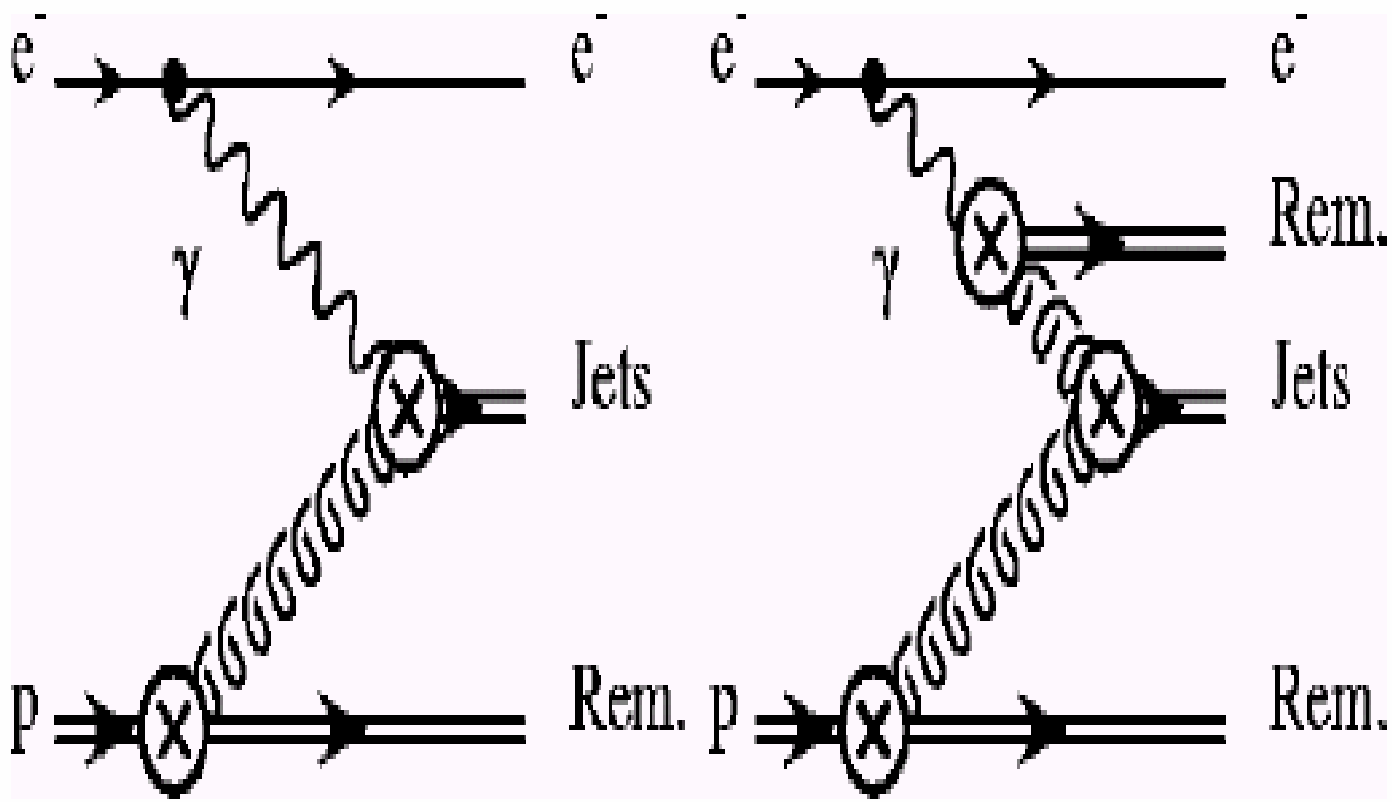
Outline

- QCD Factorization Theorem
  - Photon Spectra
  - Photon Structure
- Jet Production
  - Cone and cluster algorithms
  - Single jets, dijets, three jets, and diffraction
- Hadron Production
  - Fragmentation
  - Light hadrons, heavy hadrons, and quarkonia
- Prompt-Photon Production
  - Fragmentation and isolation
  - Inclusive photons and photons with jets
- Related Topics
  - Virtual photons
  - Polarized photons

References

- Topical review:
  - Klasen, Rev. Mod. Phys. 74 (2002) 1221
- Diffractive jet production:
  - Kaidalov, Khoze, Martin, Ryskin, EPJC 21 (2001) 521
  - Klasen, Kramer, EPJC 38 (2004) 93; PRL 93 (2004) 232002; JPG 31 (2005) 1391; hep-ph/0608235 (EPJC, in press)
- Heavy-quark production:
  - Cacciari, Frixione, Nason, JHEP 0103 (2001) 6
  - Heinrich, Kniehl, PRD 70 (2004) 094035
  - Kramer, Spiesberger, EPJC 38 (2004) 309
  - Kniehl, Kramer, Schienbein, Spiesberger, PRD 71 (2005) 014018
  - Banfi, Salam, Zanderighi, EPJC 47 (2006) 113

QCD Factorization Theorem



Direct Photons   Resolved Photons

Photon Spectra (1)

- Bremsstrahlung:
  - Weizsäcker-Williams (1934) approximation:

$$f_{\gamma/l}^{\mathrm{brems}}(x)=\frac{\alpha}{2\pi}\left[\frac{1+(1-x)^2}{x}\ln\frac{Q_{\max}^2(1-x)}{m_l^2x^2}+2m_l^2x\left(\frac{1}{Q_{\max}^2}-\frac{1-x}{m_l^2x^2}\right)\right]$$

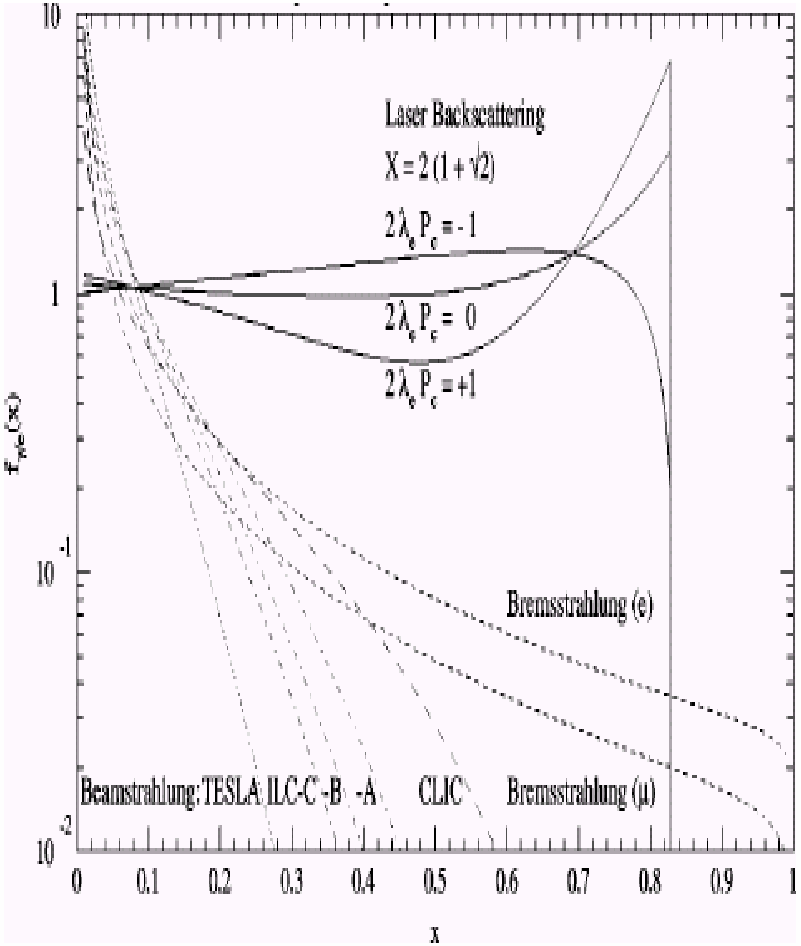
  - Subleading non-logarithmic terms (Kessler, 1975): 5%
- Beamstrahlung (very dense bunches at LC):
  - Transverse acceleration in EM-field ($\Upsilon=\frac{5r_e^2E_eN}{6\alpha\sigma_z(\sigma_x+\sigma_y)m_e}$) of bunch:

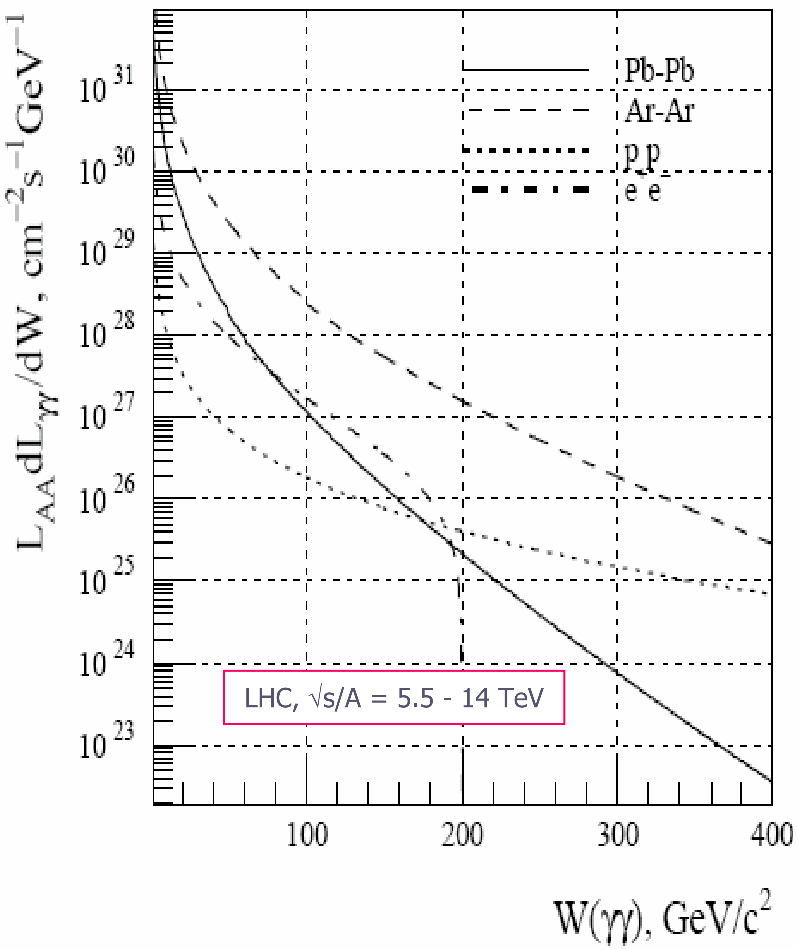
$$f_{\gamma/e}^{\mathrm{beam}}(x)=\frac{1}{\Gamma\left(\frac{1}{3}\right)}\left(\frac{2}{3\Upsilon}\right)^{1/3}x^{-2/3}(1-x)^{-1/3}e^{-2x/[3\Upsilon(1-x)]}\left\{\frac{1-\sqrt{\frac{\Upsilon}{24}}}{g(x)}\left[1-\frac{1}{g(x)N_\gamma}(1-e^{-g(x)N_\gamma})\right]+\sqrt{\frac{\Upsilon}{24}}\left[1-\frac{1}{N_\gamma}(1-e^{-N_\gamma})\right]\right\}$$

- Laser backscattering:
  - Compton effect (Ginzburg et al., 1984):

$$f_{\gamma/e}^{\mathrm{laser}}(x)=\frac{1}{N_c+2\lambda_eP_cN_c'}\left[1-x+\frac{1}{1-x}-\frac{4x}{X(1-x)}+\frac{4x^2-2\lambda_eP_cx(2-x)X[x(X+2)-X]}{X^2(1-x)^2}\right]$$

Photon Spectra (2)





Photon Structure (1)

- Structure functions:

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\{[1+(1-y)^2]F_2^\gamma(x,Q^2) - y^2 F_L^\gamma(x,Q^2)\}$$

- Evolution equations:

$$\frac{df_{q/\gamma}(Q^2)}{d\ln Q^2} = \boxed{\frac{\alpha}{2\pi}P_{q\leftarrow\gamma}\otimes f_{\gamma/\gamma}(Q^2)} + \frac{\alpha_s(Q^2)}{2\pi}[P_{q\leftarrow q}\otimes f_{q/\gamma}(Q^2) + P_{q\leftarrow g}\otimes f_{g/\gamma}(Q^2)]$$

- Factorization schemes:

$$P_{q\leftarrow\gamma}^{\mathrm{DIS}_\gamma} = P_{q\leftarrow\gamma}^{\overline{\mathrm{MS}}} - e_q^2 P_{q\leftarrow q}\otimes C_\gamma, \quad C_\gamma(x) = 3\left([x^2+(1-x)^2]\ln\frac{1-x}{x} + 8x(1-x) - 1\right)$$

- Hadronic solutions:

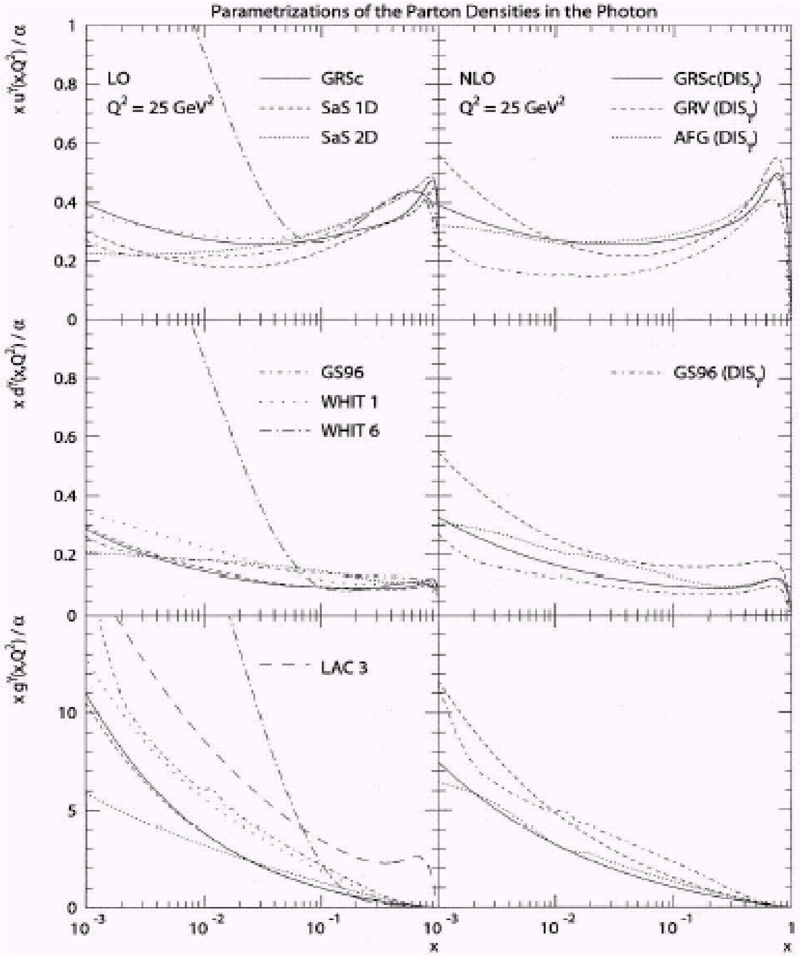
$$|\gamma\rangle = \sum_{V=\rho,\omega,\phi}\frac{e}{f_V}|V\rangle = \sqrt{\frac{e^2}{f_\rho^2}+\frac{e^2}{f_\omega^2}}(e_u^2+e_d^2)^{-1/2}(e_u|u\bar{u}\rangle + e_d|d\bar{d}\rangle) + \frac{e}{f_\phi}|s\bar{s}\rangle$$

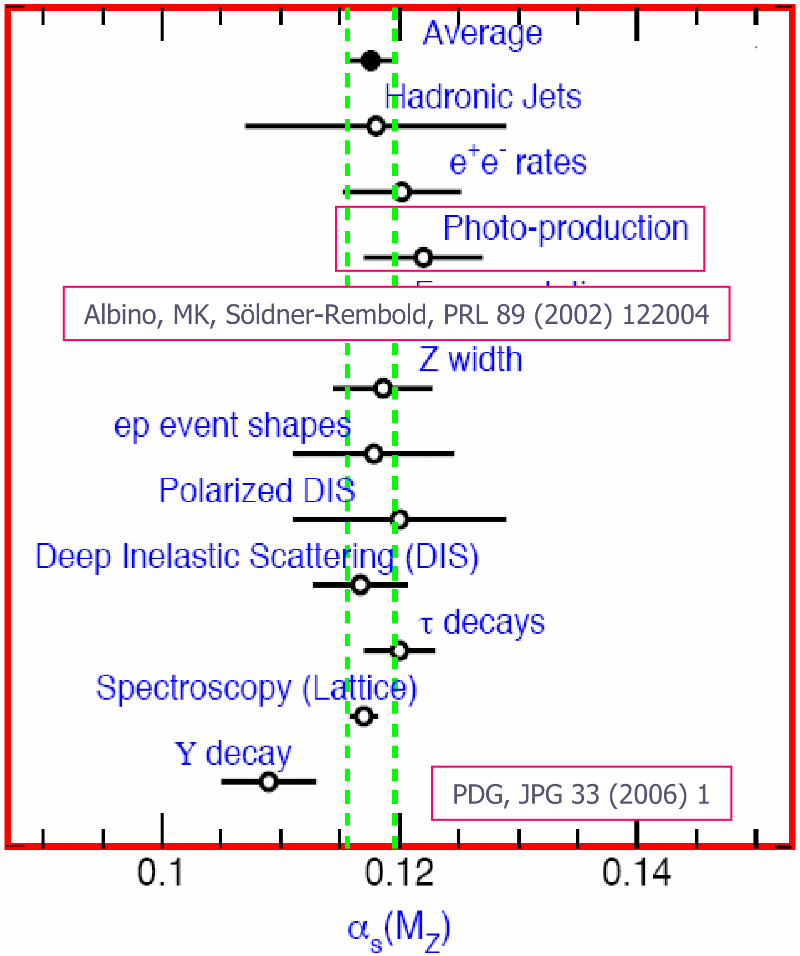
- Strong coupling constant:

$$F_2^\gamma(Q^2) = \sum_q 2xe_q^2\left\{f_{q/\gamma}(Q^2) + \frac{\boxed{\alpha_s(Q^2)}}{2\pi}[C_q\otimes f_{q/\gamma}(Q^2) + C_g\otimes f_{g/\gamma}(Q^2)] + \frac{\alpha}{2\pi}e_q^2 C_\gamma\right\}$$

[Albino, MK, Söldner-Rembold, PRL 89 (2002) 122004]

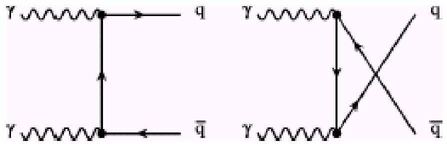
Photon Structure (2)



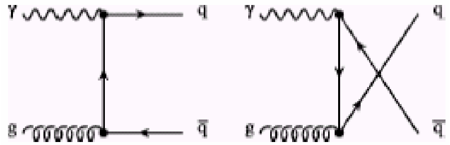


Jet Production (1)

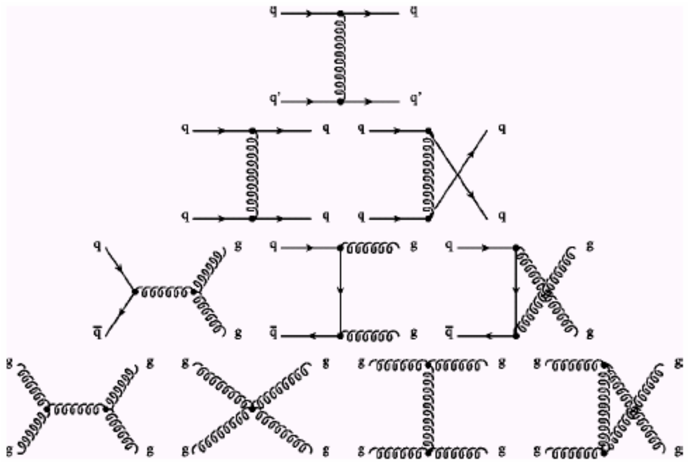
- Direct-direct ($\rightarrow$ LEP):



- Direct-resolved:



- Resolved-resolved:



- Hadronic cross-section:

$$\frac{d^3\sigma}{dE_T^2 d\eta_1 d\eta_2} = \sum_{a,b} x_a f_{a/A}(x_a, M_a^2) x_b f_{b/B}(x_b, M_b^2) \frac{d\sigma}{dt}$$

- Partonic cross-section:

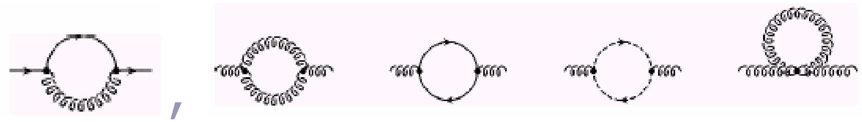
$$\frac{d\sigma^B}{dt} = \frac{1}{2s} \frac{1}{\Gamma(1-\varepsilon)} \left(\frac{4\pi s}{tu}\right)^{\varepsilon} \frac{1}{8\pi s} \frac{g_{a,b}^2}{S_a S_b C_a C_b} |\mathcal{M}^B|^2$$

- Matrix elements:

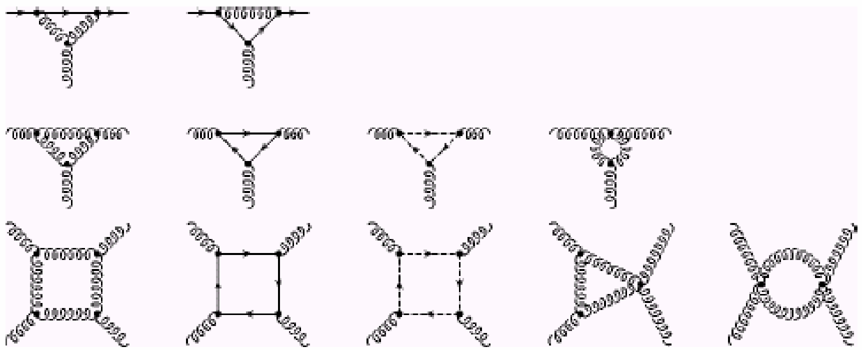
Process	LO matrix element squared $\lvert\mathcal{M}^B\rvert^2$
$\gamma\gamma \to q\bar{q}$	$8N_C(1-\varepsilon)\left[(1-\varepsilon)\left(\frac{u}{t}+\frac{t}{u}\right)-2\varepsilon\right]$
$\gamma g \to q\bar{q}$	$C_F\lvert\mathcal{M}^B\rvert^2_{\gamma\gamma\to q\bar{q}}(s,t,u)$
$qq' \to qq'$	$4N_C C_F\left(\frac{s^2+u^2}{t^2}-\varepsilon\right)$
$qq \to qq$	$\left[\lvert\mathcal{M}^B\rvert^2_{qq'\to qq'}(s,t,u)+\lvert\mathcal{M}^B\rvert^2_{qq'\to qq'}(s,u,t)\right.$
	$\left.-8C_F(1-\varepsilon)\left(\frac{s^2}{ut}+\varepsilon\right)\right] \Big/ 2!$
$q\bar{q} \to gg$	$\left[4C_F(1-\varepsilon)\left(\frac{2N_C C_F}{ut}-\frac{2N_C^2}{s^2}\right)(t^2+u^2-\varepsilon s^2)\right] \Big/ 2!$
$gg \to gg$	$\left[32N_C^3 C_F(1-\varepsilon)^2\left(3-\frac{ut}{s^2}-\frac{us}{t^2}-\frac{st}{u^2}\right)\right] \Big/ 2!$

Jet Production (2)

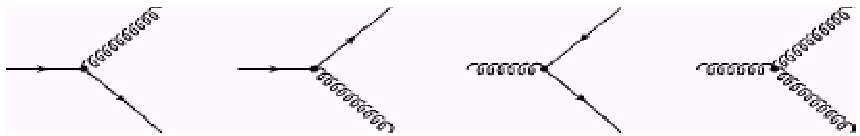
- Virtual loop corrections:
  - Self-energies:



  - Vertex corrections:

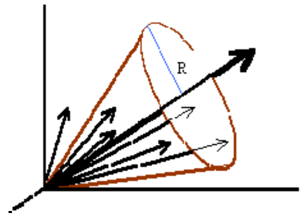


- Real emission corrections:



Process	Color factor	Correction	Singular parts of $\lvert\mathcal{M}^{V,F,I}\rvert^2/\lvert\mathcal{M}^{B}\rvert^2$
$\gamma q \to g q$	C_F	Virtual	$\left[-\frac{2}{\varepsilon^2}-\frac{1}{\varepsilon}[3-2l(t)]\right]$
		Final	$\left[+\frac{1}{\varepsilon^2}+\frac{1}{2\varepsilon}[3-2l(t)]\right]$
		Initial	$\left[+\frac{1}{\varepsilon^2}+\frac{1}{2\varepsilon}[3-2l(t)]\right]$
	N_C	Virtual	$\left[-\frac{1}{\varepsilon^2}-\frac{1}{2\varepsilon}[\frac{11}{3}-2l(s/t)-2l(u)]\right]$
		Final	$\left[+\frac{1}{\varepsilon^2}+\frac{1}{2\varepsilon}[\frac{11}{3}-l(s/t)-l(u)]\right]$
		Initial	$\left[+\frac{1}{2\varepsilon}[-l(s/t)-l(u)]\right]$
	N_f	Virtual	$+\frac{1}{3\varepsilon}$
		Final	$-\frac{1}{3\varepsilon}$

Cone and Cluster Algorithms



- JADE clustering algorithm (e^+e^--colliders):

  $$(p_i+p_j)^2 = 2E_iE_j(1-\cos\theta_{ij}) < yS$$

- Snowmass cone algorithm (pp-colliders):

  $$R_i = \sqrt{(\eta_i-\eta_J)^2+(\phi_i-\phi_J)^2} < R$$ with $$E_{T_J} = \sum_{R_i<R} E_{T_i},\ \eta_J = \frac{1}{E_{T_J}}\sum_{R_i<R} E_{T_i}\eta_i,\ \phi_J = \frac{1}{E_{T_J}}\sum_{R_i<R} E_{T_i}\phi_i$$

  → Broad, seedless jets → $R_{ij} = \sqrt{(\eta_i-\eta_j)^2+(\phi_i-\phi_j)^2} < \min\left[\frac{E_{T_i}+E_{T_j}}{\max(E_{T_i},E_{T_j})}R, R_{sep}\right]$

  → Midpoint: start with > 1 GeV seeds, add > 100 MeV clusters

- Durham k_T-algorithm:

  $$d_{ij} = \min(k_{ti}^2, k_{tj}^2)R_{ij}^2$$ with $$R_{ij}^2 = (\eta_i-\eta_j)^2+(\phi_i-\phi_j)^2$$

  → IR-safe, no R_{sep}, applies to e^+e^-- and pp-colliders

  → N^3 computing time (N searches of N^2 table for d_{min})

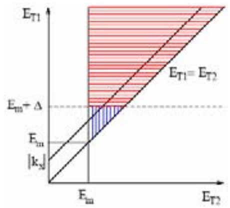
  → Take geometrical nearest neighbours, use Voronoi diagrams

  → N ln N computing time [Cacciari, Salam, PLB 641 (2006) 57]

Single Jets and Dijets

- Single jets:
  - High statistics, single scale E_T
  - $$\frac{d^2\sigma}{dE_T\, d\eta} = \sum_{a,b} \int_{x_{a,\min}}^{1} dx_a\, x_a f_{a/A}(x_a, M_a^2)\, x_b f_{b/B}(x_b, M_b^2) \times \frac{4E_A E_T}{2x_a E_A - E_T e^{\eta}} \frac{d\sigma}{dt},$$

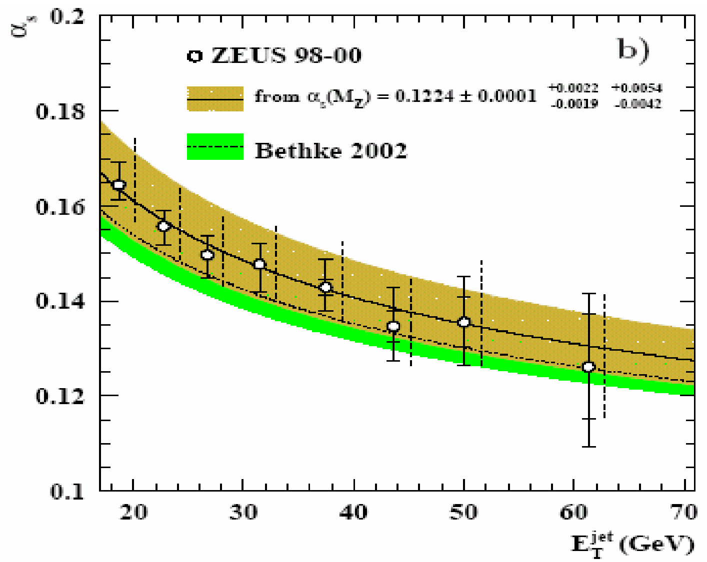
- Dijets: $\bar{E}_T = (E_{T_1} + E_{T_2})/2$



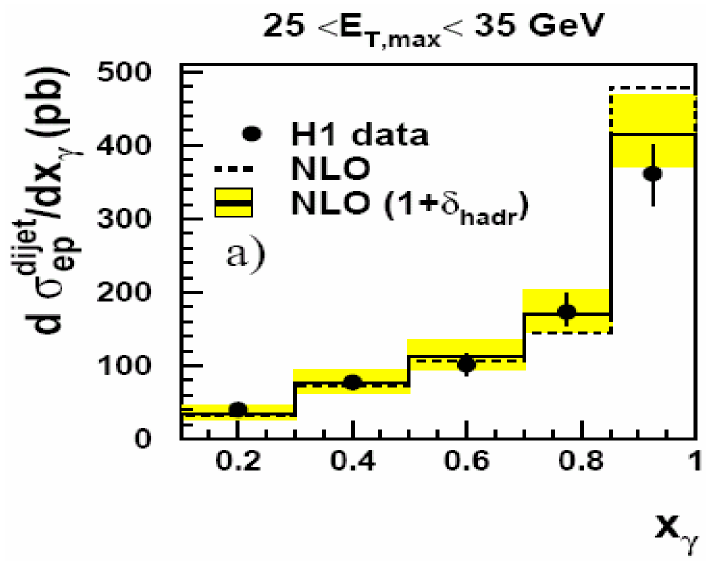
$$x_{a,b}^{\rm obs} = \sum_{i=1}^{2} E_{T_i} e^{\pm \eta_i} / (2E_{A,B})$$

$$\frac{d^3\sigma}{dE_T^2\, d\eta_1\, d\eta_2} = \sum_{a,b} x_a f_{a/A}(x_a, M_a^2)\, x_b f_{b/B}(x_b, M_b^2) \frac{d\sigma}{dt}$$

- α_s from scaling violations:

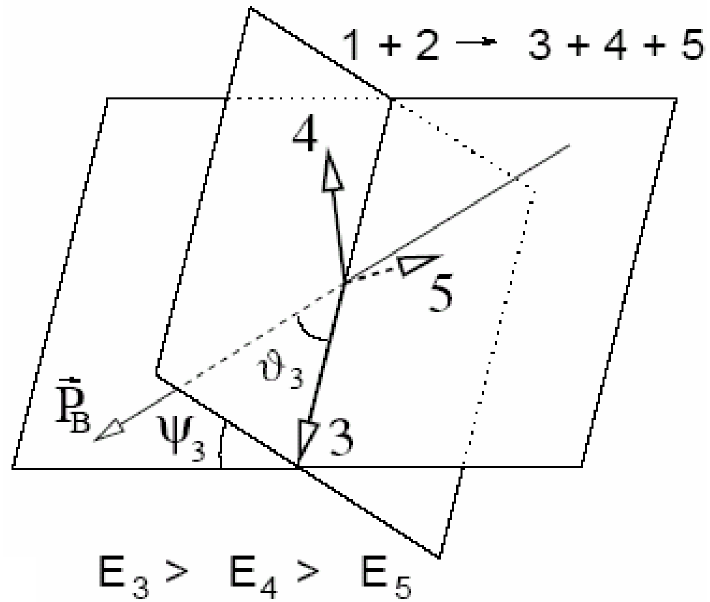


- Photon PDFs from x_γ^{obs}:



Three Jets

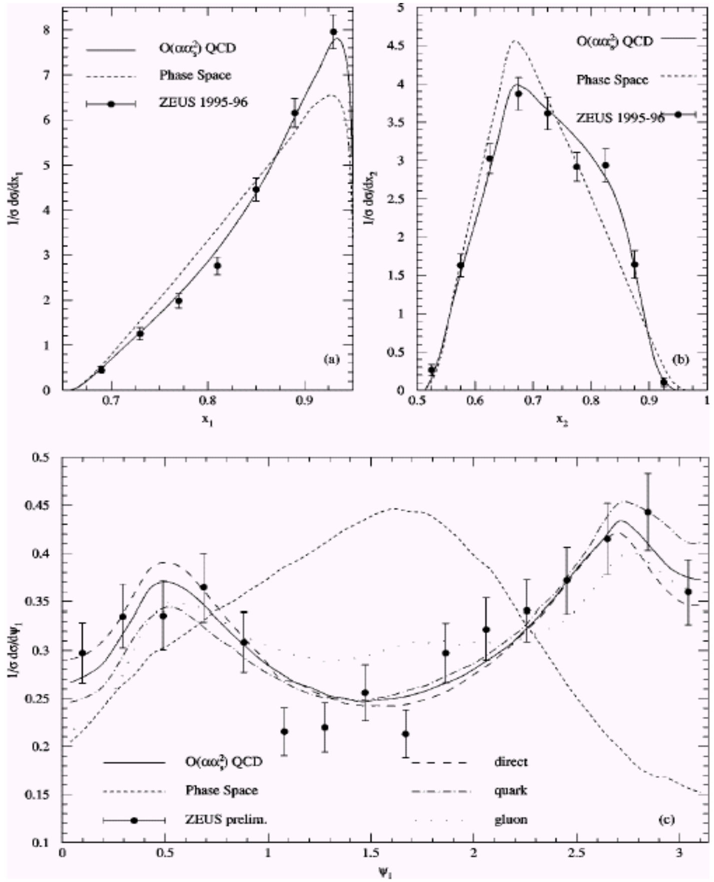
- Three-body rest frame:



- Three-jet cross section:

$$\frac{d^4\sigma}{dx_1 dx_2 d\cos\theta_1 d\psi_1} = \sum_{a,b} x_a f_{a/A}(x_a, M_a^2) x_b f_{b/B}(x_b, M_b^2) \frac{\overline{|\mathcal{M}|}^2_{ab\to 123}}{1024\pi^4}$$

- Energy/angular distributions:



Diffractive Dijet Production

Hard diffraction:

→ Does factorization hold?

Deep inelastic scattering: Yes

→ Direct photoproduction

Hadroproduction: No

→ Resolved photoproduction

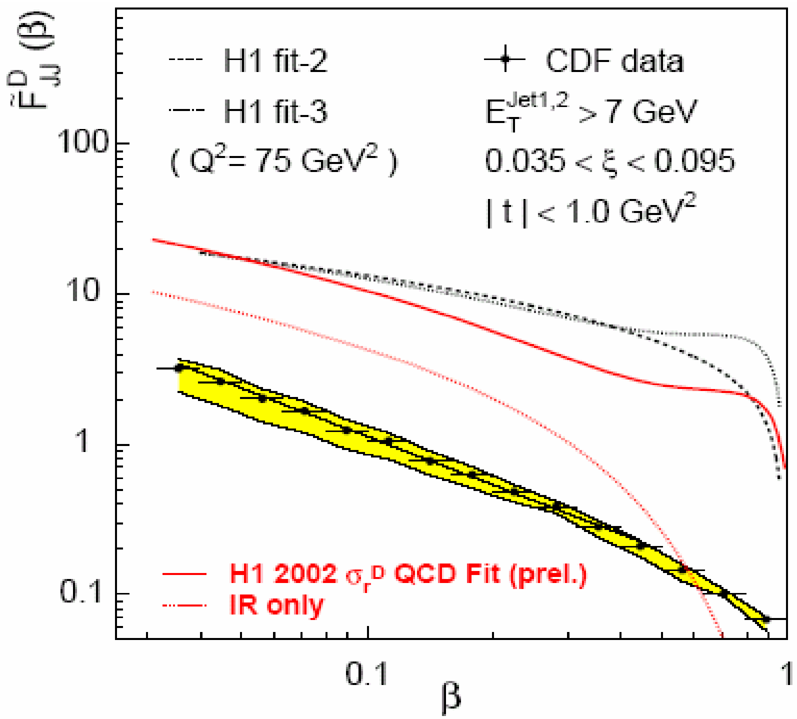
Why next-to-leading order?

→ $\sigma_{tot} = \sigma_{dir}(x_\gamma, M_\gamma) + \sigma_{res}(x_\gamma, M_\gamma)$

→ At LO $x_\gamma = 1$, but at NLO $x_\gamma \leq 1$

→ $\log(M_\gamma)$-dependence cancels

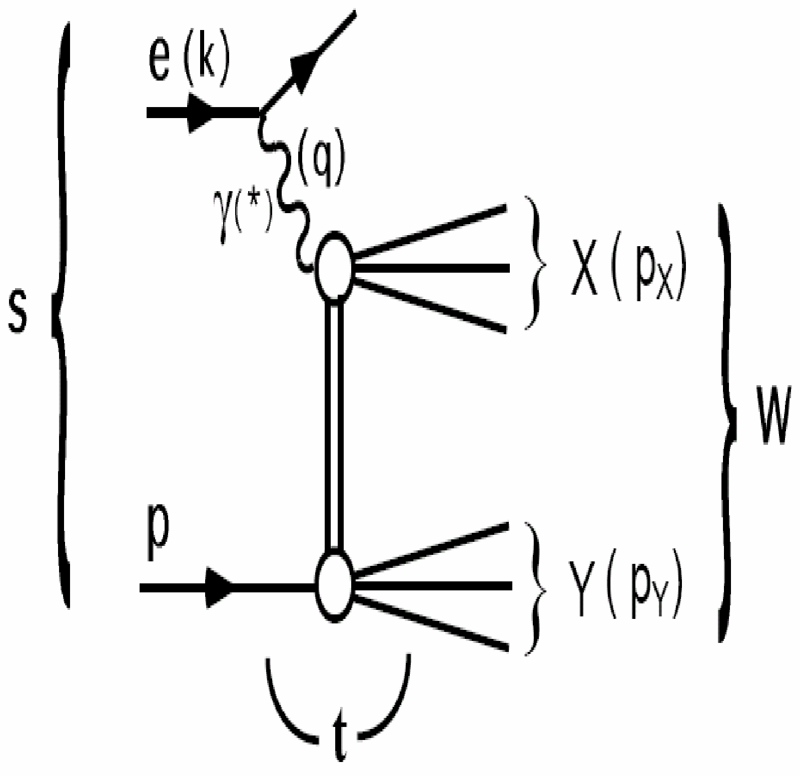
Diffr. hadroproduction of dijets:



CDF Coll., PRL 84 (2000) 5043

Kinematics

Diffractive processes at HERA:



H1 Coll., EPS 2003 and DIS 2004

Inclusive DIS:

$$s = (k+p)^2, \; Q^2 = -q^2, \text{ and } y = \frac{qp}{kp}$$

Diffractive DIS:

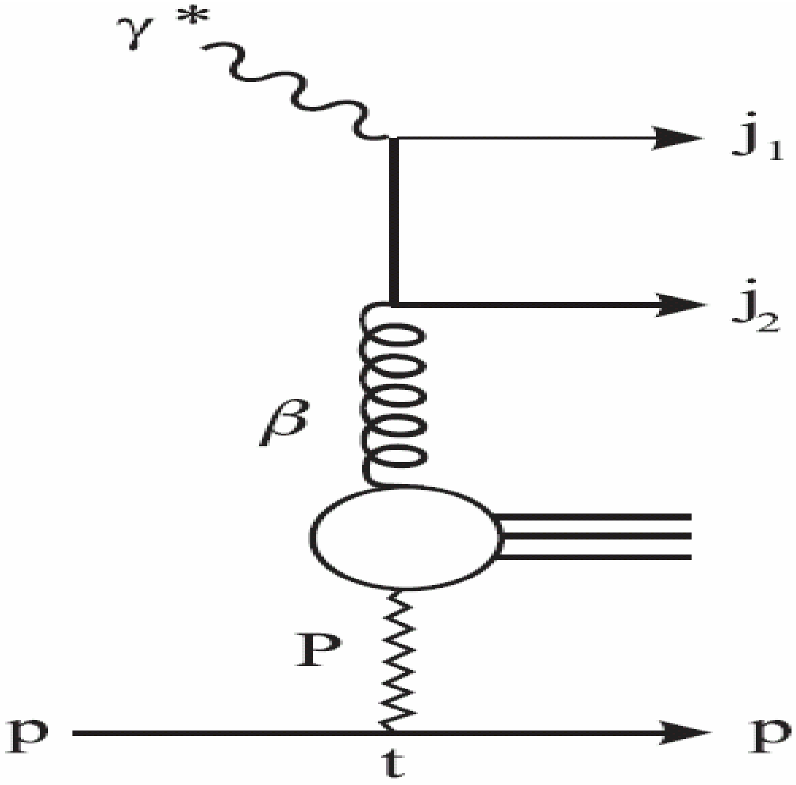
$$M_X^2 = p_X^2 \quad \text{and} \quad t = (p - p_Y)^2,$$
$$M_Y^2 = p_Y^2 \quad \text{and} \quad x_{I\!P} = \frac{q(p - p_Y)}{qp}$$

Experimental cuts:

0.3	$<$	y	$<$	0.65
		Q^2	$<$	0.01 GeV^2
		E_T^{jet1}	$>$	5 GeV
		E_T^{jet2}	$>$	4 GeV
-1	$<$	$\eta_{\text{lab}}^{\text{jet1,2}}$	$<$	2
		$x_{I\!P}$	$<$	0.03
		M_Y	$<$	1.6 GeV
		$-t$	$<$	1 GeV^2

Diffractive Parton Distributions

Double factorization:



G. Ingelman, P. Schlein, PLB 152 (1985) 256

Hard QCD factorization:

$$\frac{d^2\sigma}{dx_{I\!P}dt} = \sum_a \int_x^{x_{I\!P}} d\xi \sigma_a^{\gamma^*}(x, Q^2, \xi) f_a^D(\xi, Q^2; x_{I\!P}, t)$$

Regge factorization:

$$f_a^D(x, Q^2; x_{I\!P}, t) = f_{I\!P/p}(x_{I\!P}, t) f_{a/I\!P}(\beta = x/x_{I\!P}, Q^2)$$

Pomeron flux factor:

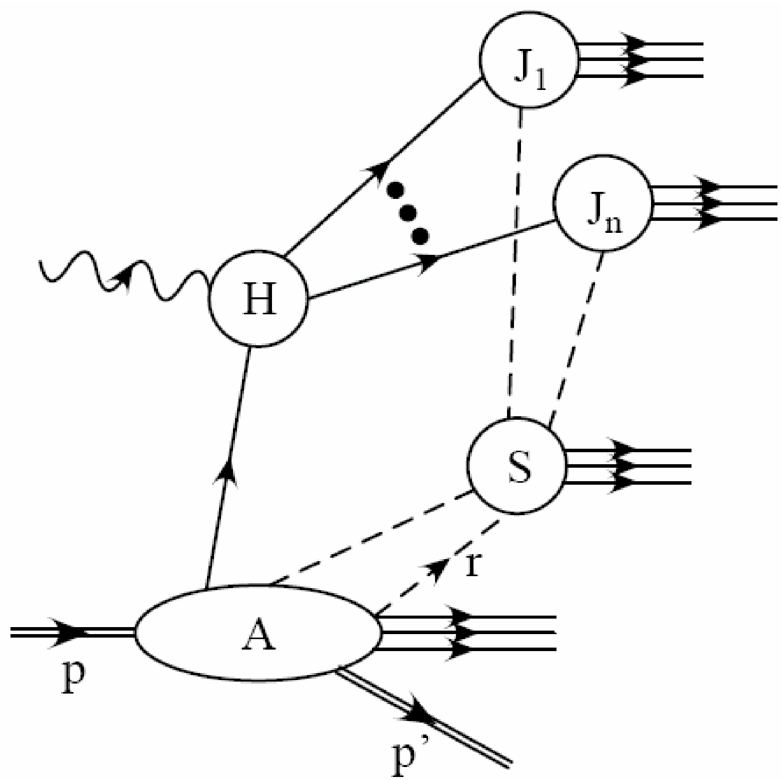
$$f_{I\!P/p}(x_{I\!P}, t) = x_{I\!P}^{1-2\alpha_{I\!P}(t)} \exp(B_{I\!P} t)$$

Pomeron tracectory:

$$\alpha_{I\!P}(t) = \alpha_{I\!P}(0) + \alpha'_{I\!P} t$$

Proof of Hard Factorization

Diffractive deep inelastic scattering:



J.C. Collins, PRD 57 (1998) 3051

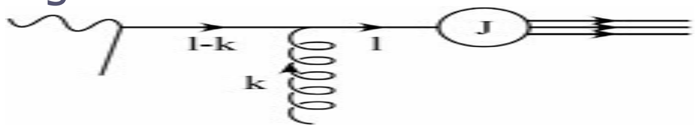
Light cone coordinates:

- $q^\mu = (q^+, q^-, \mathbf{q}_T)$

Leading regions:

- H: $q^\mu \approx O(Q)$
- J: $l^\mu \approx (0, Q/\sqrt{2}, \mathbf{0}_T)$
- A: $|k^\mu| \ll O(Q)$

Soft gluon attachments:

- 

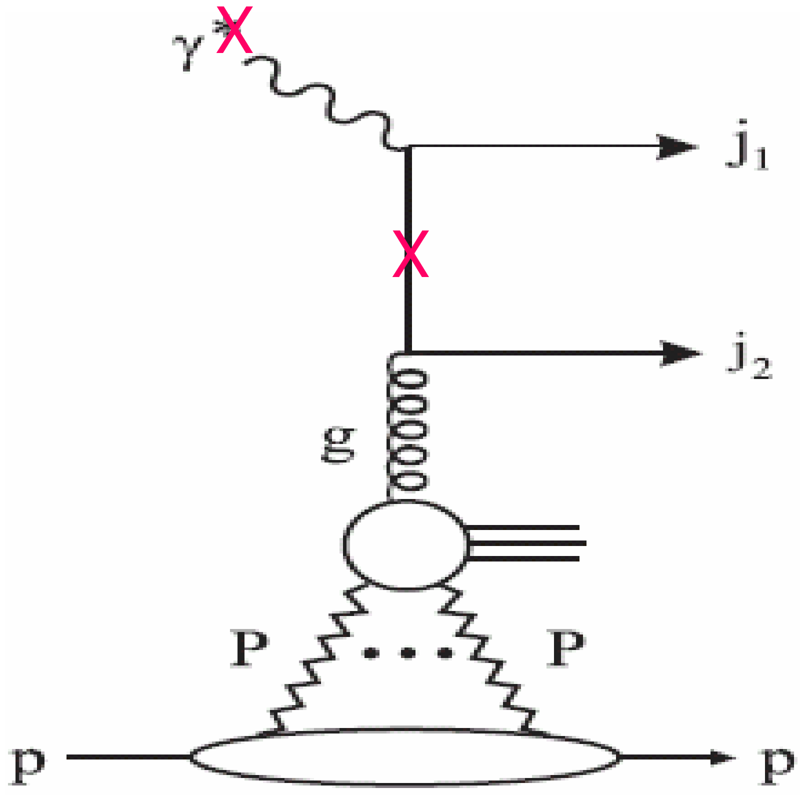
- $$\frac{1}{(l-k)^2 - m^2 + i\epsilon} \simeq \frac{1}{-2l^- k^+ + \text{transverse} + i\epsilon}$$

Poles in k^+-plane:

- Final state: Upper half-plane
- Initial state: Lower half-plane

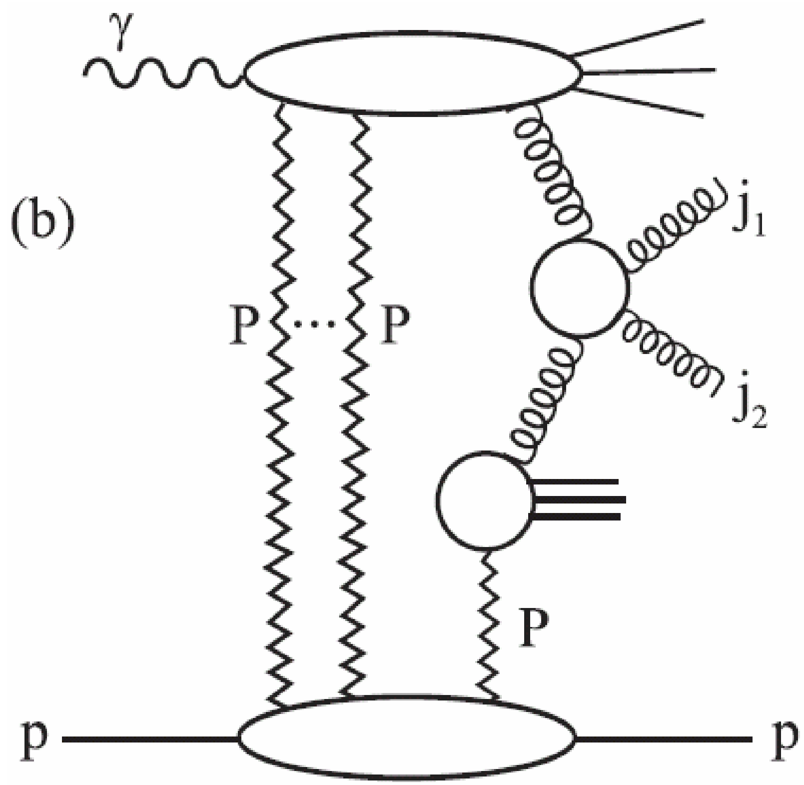
Multipomeron Exchanges

Direct photoproduction:



→ Modify the Regge trajectory

Resolved photoproduction:



→ Factorization breaking

Diffractive Photoproduction of Dijets

Cross section:

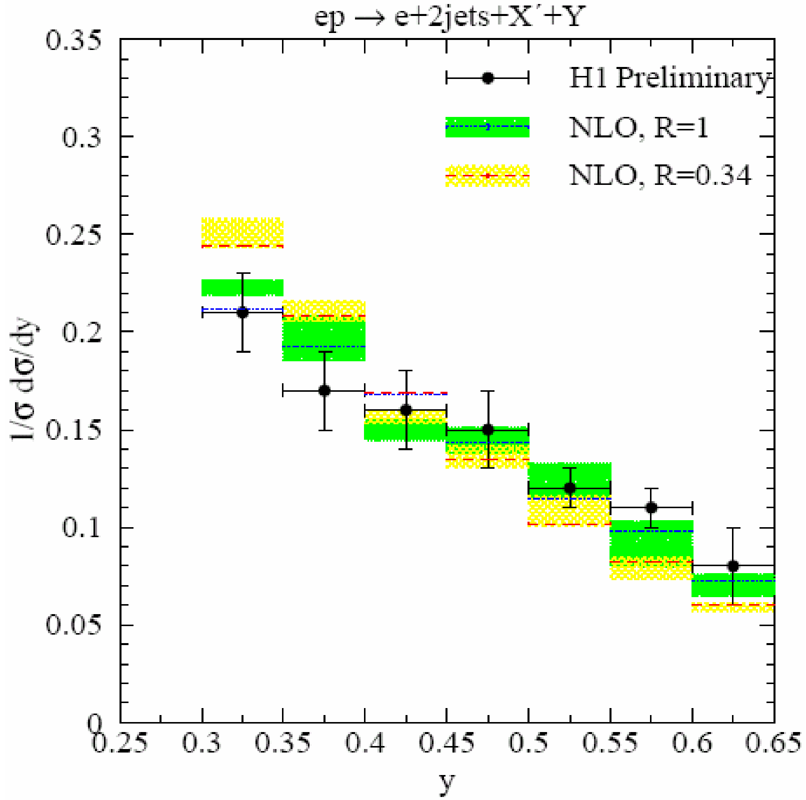
$$d\sigma^D(ep \to e + 2\text{ jets} + X' + Y) =$$
$$\sum_{a,b} \int_{t_{\rm cut}}^{t_{\rm min}} dt \int_{x_{I\!P}^{\rm min}}^{x_{I\!P}^{\rm max}} dx_{I\!P} \int_0^1 dz_{I\!P} \int_{y_{\rm min}}^{y_{\rm max}} dy \int_0^1 dx_\gamma$$
$$f_{\gamma/e}(y) f_{a/\gamma}(x_\gamma, M_\gamma^2) f_{I\!P/p}(x_{I\!P}, t) f_{b/I\!P}(z_{I\!P}, M_{I\!P}^2)$$
$$d\sigma^{(n)}(ab \to \text{jets}).$$

Photon flux: Weizsäcker-Williams approximation

$$f_{\gamma/e}(y) = \frac{\alpha}{2\pi}\left[\frac{1+(1-y)^2}{y} \ln \frac{Q^2_{\rm max}(1-y)}{m_e^2 y^2} + 2m_e^2 y \left(\frac{1-y}{m_e^2 y^2} - \frac{1}{Q^2_{\rm max}}\right)\right]$$

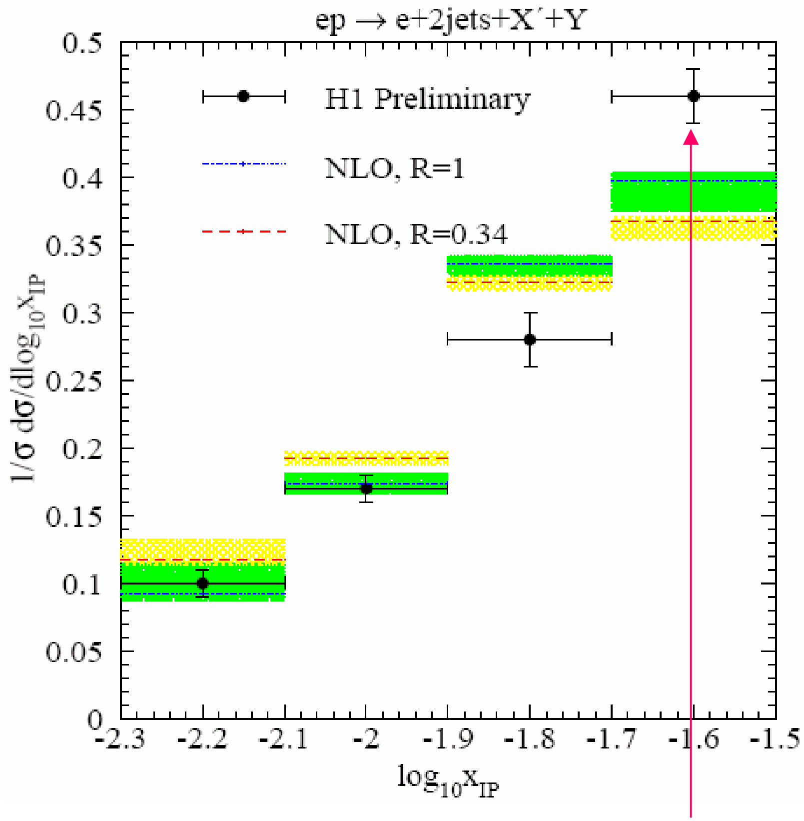
Factorizable Multipomeron Exchanges

y-dependence: Photon flux



→ Small correlations due to exp. cuts

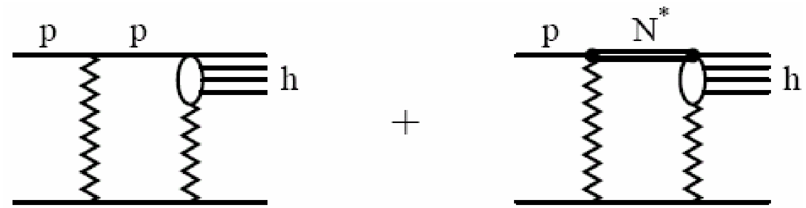
x_{IP}-dependence: Pomeron flux



→ Subleading Reggeon contribution

Two-Channel Eikonal Model

Hadronic collisions:



Survival probability:

$$|S|^2 = \frac{\int d^2b \left(|\mathcal{M}_{\rm v}|^2 \, e^{-\Omega_{\rm v}(s,b)} + |\mathcal{M}_{\rm sea}|^2 \, e^{-\Omega_{\rm sea}(s,b)}\right)}{\int d^2b \, (|\mathcal{M}_{\rm v}|^2 + |\mathcal{M}_{\rm sea}|^2)}$$

Opacity / optical density: $K_i = 1 \pm \gamma$

$$\Omega_i = K_i \frac{(g_{pp}^{I\!P})^2 \, (s/s_0)^{\Delta}}{4\pi B} e^{-b^2/4B}$$

Kaidalov et al., EPJC 21 (2001) 521

Photoproduction:

Generalized vector meson dominance:

- $J^{PC} = 1^{--}$: $\gamma \to \rho, \omega, \ldots$

Fitted parameters (W = 200 GeV):

- Total cross section: $\sigma^{tot}(\rho p) = 34$ mb
- Pomeron slope: B = 11.3 GeV^{-2}
- Transition probability: $\gamma = 0.6$

→ ZEUS Coll., EPJ C2 (1998) 247
→ H1 Coll., EPJ C13 (2000) 371

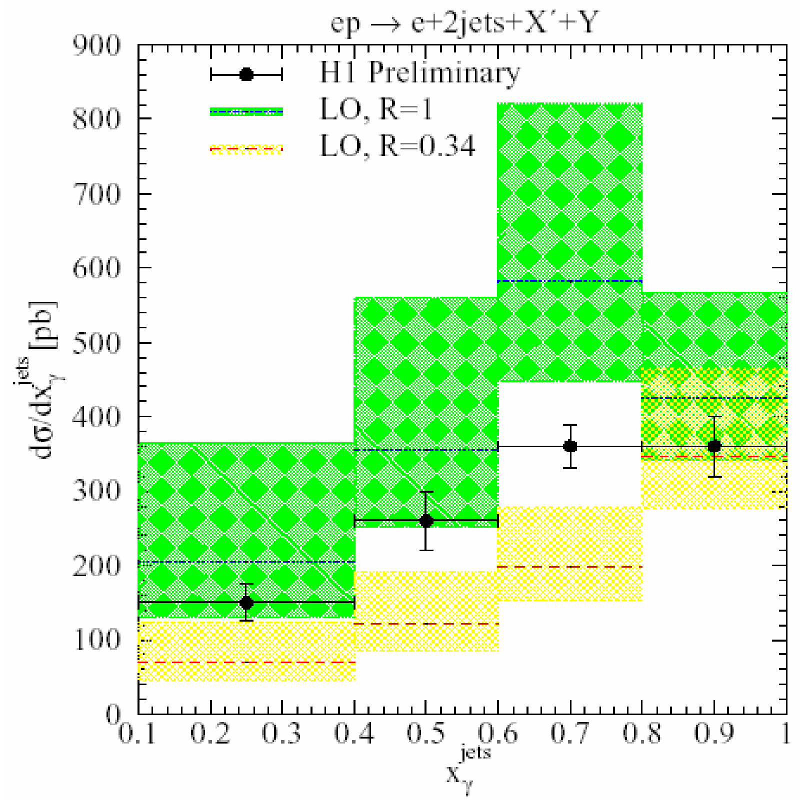
Survival probability:

- $R \equiv |S|^2 \approx 0.34$

Kaidalov et al., PLB 567 (2003) 61

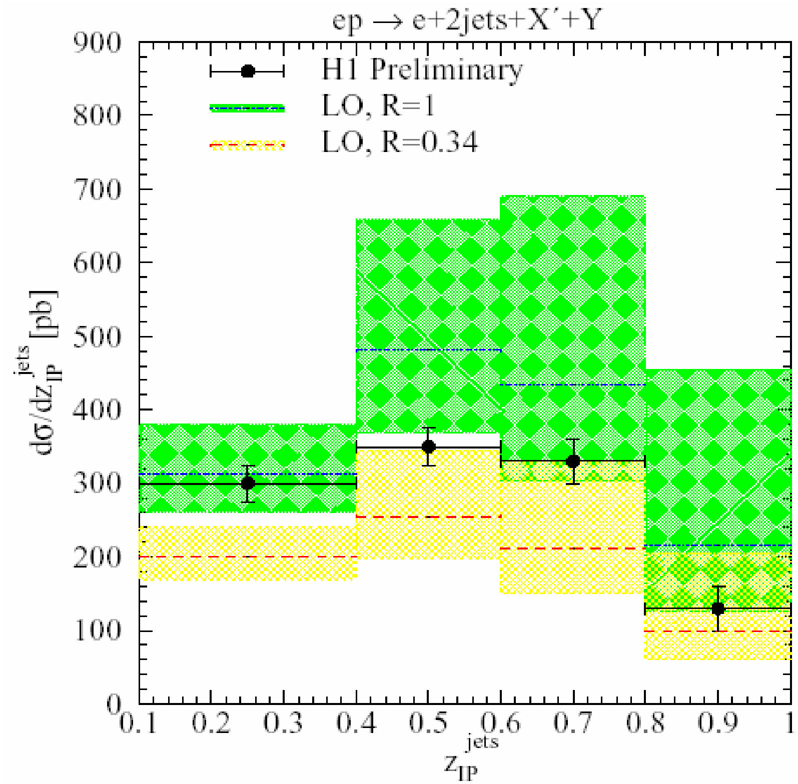
No Sign of Factorization Breaking at LO

x_γ-dependence: Direct/resolved photons



→ At LO, R = 1 agrees better with data!

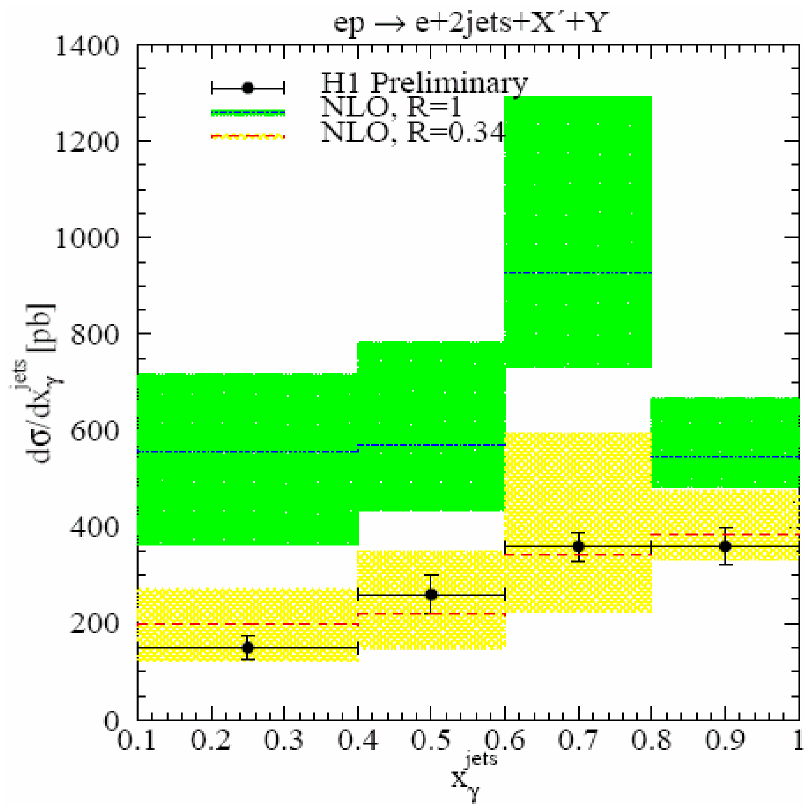
z_{IP}-dependence: Gluon density in pom.



→ Smaller uncertainties in $1/\sigma \, d\sigma/dz_{IP}$

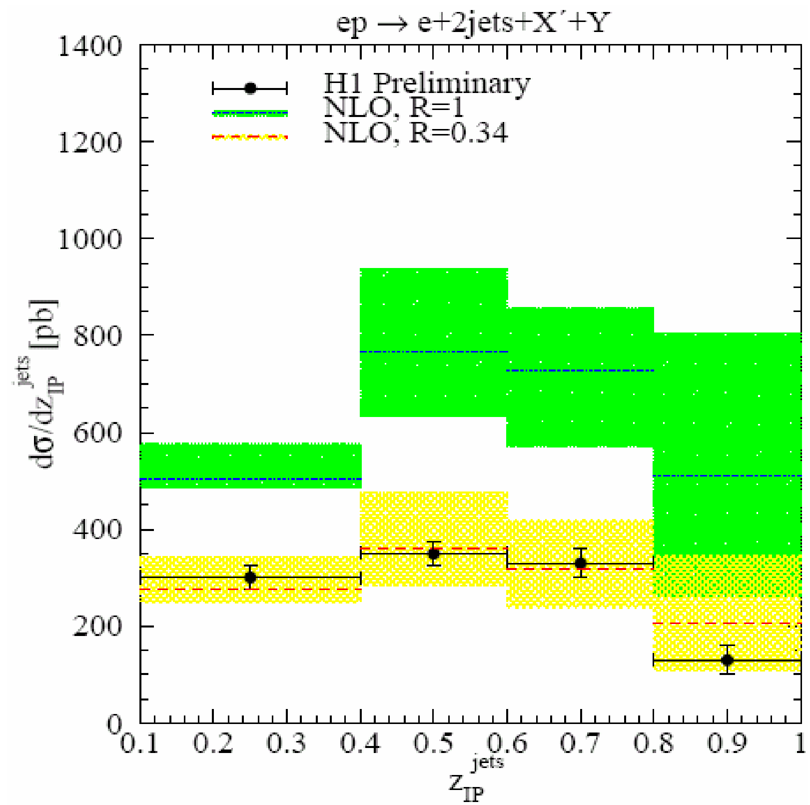
Non-Factorizable Multipomeron Exchanges

x_γ-dependence: Direct/resolved photons



→ Large K-Factor ↔ Survival probability

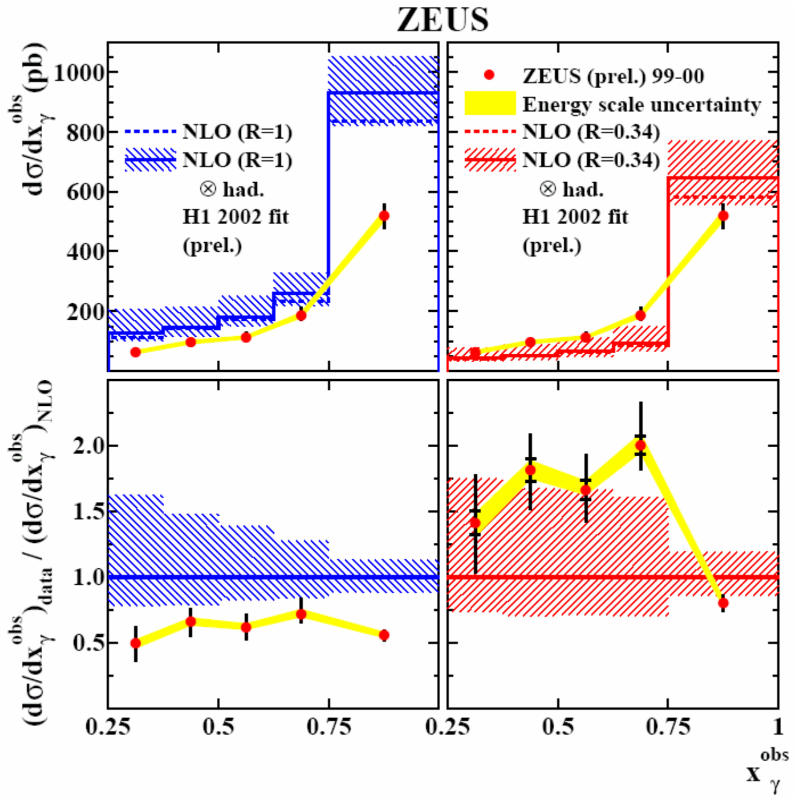
z_{IP}-dependence: Gluon density in pom.



→ Reduced scale uncertainties at NLO

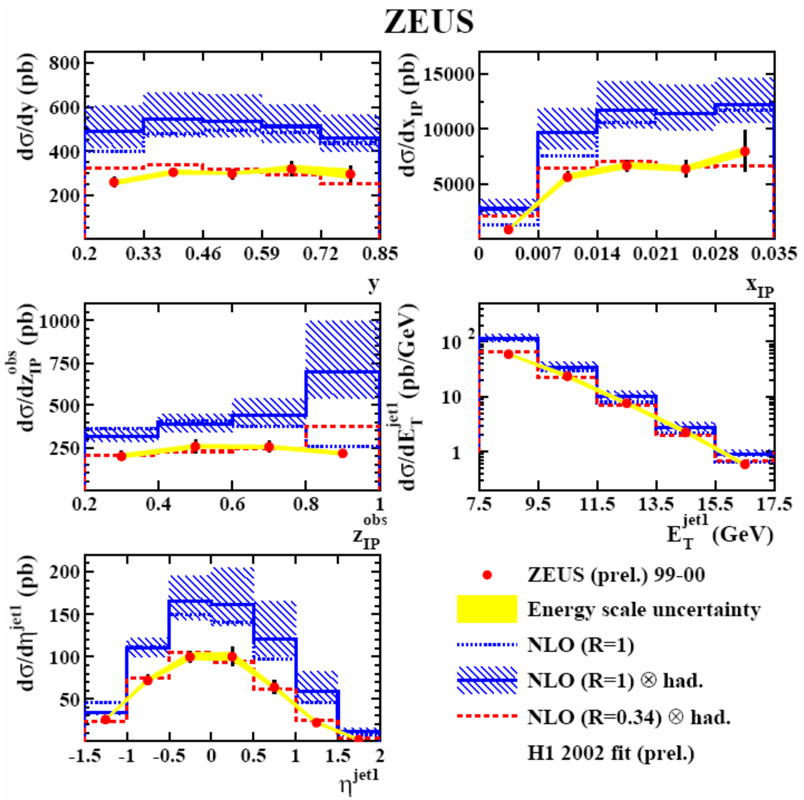
ZEUS Analysis

x_γ-dependence: Direct/resolved photons



→ At LO, R = 1 agrees better with data!

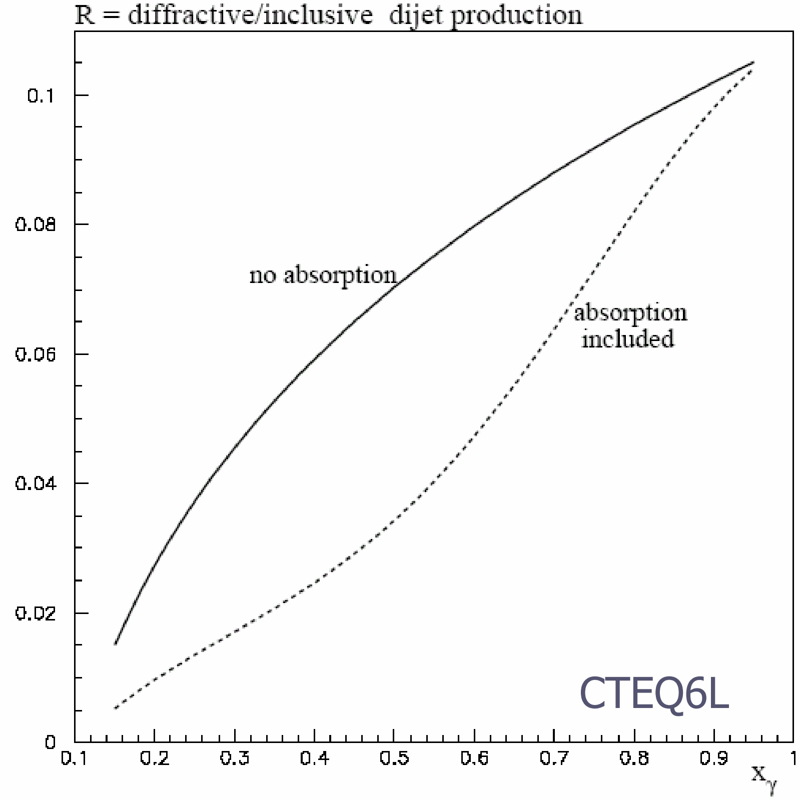
Other observables:



→ Excellent agreement for absolute σ's!

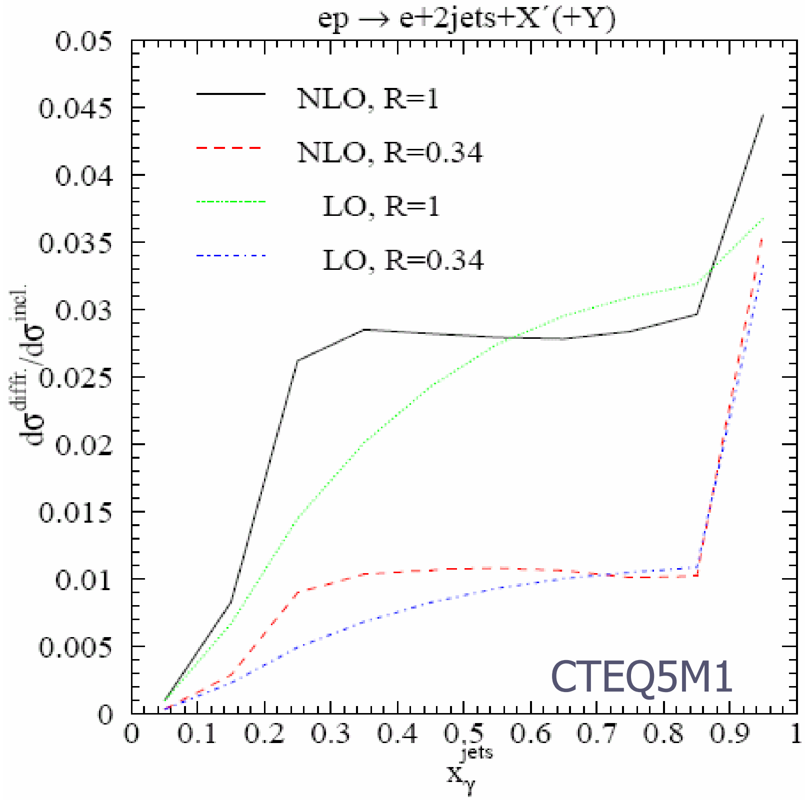
Diffractive / Inclusive Production

$R = f_{g/IP} \otimes f_{IP/p} / f_{g/p}$ with $M_{12} = x_\gamma z_{IP} x_{IP} W$:



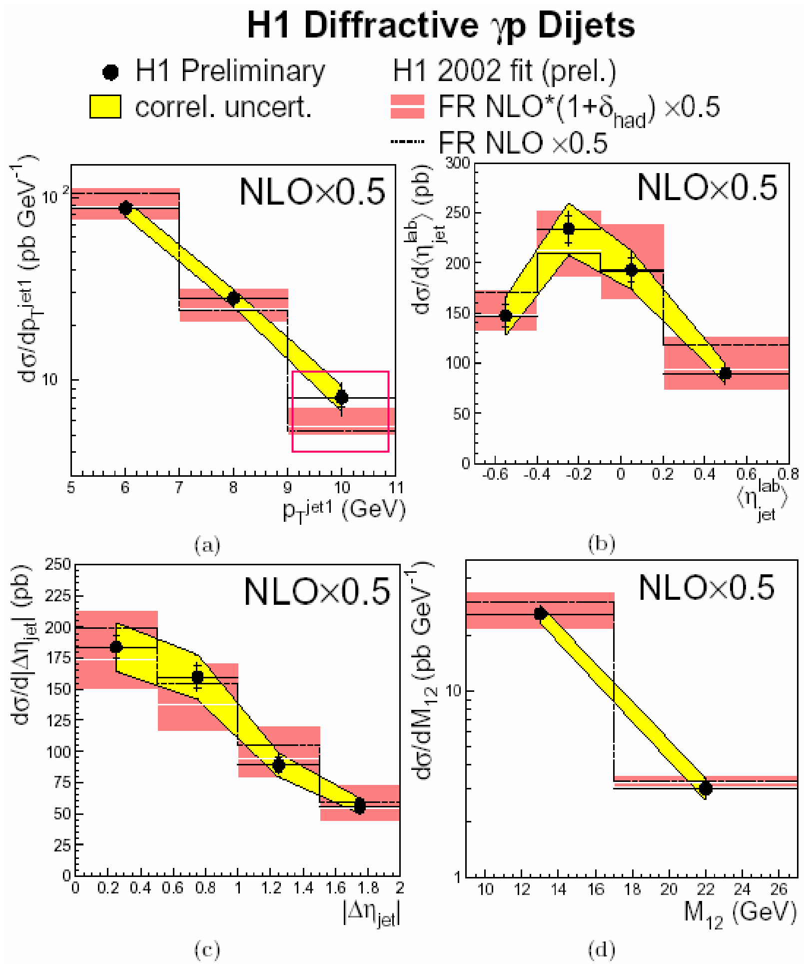
A. Kaidalov et al., PLB 567 (2003) 61

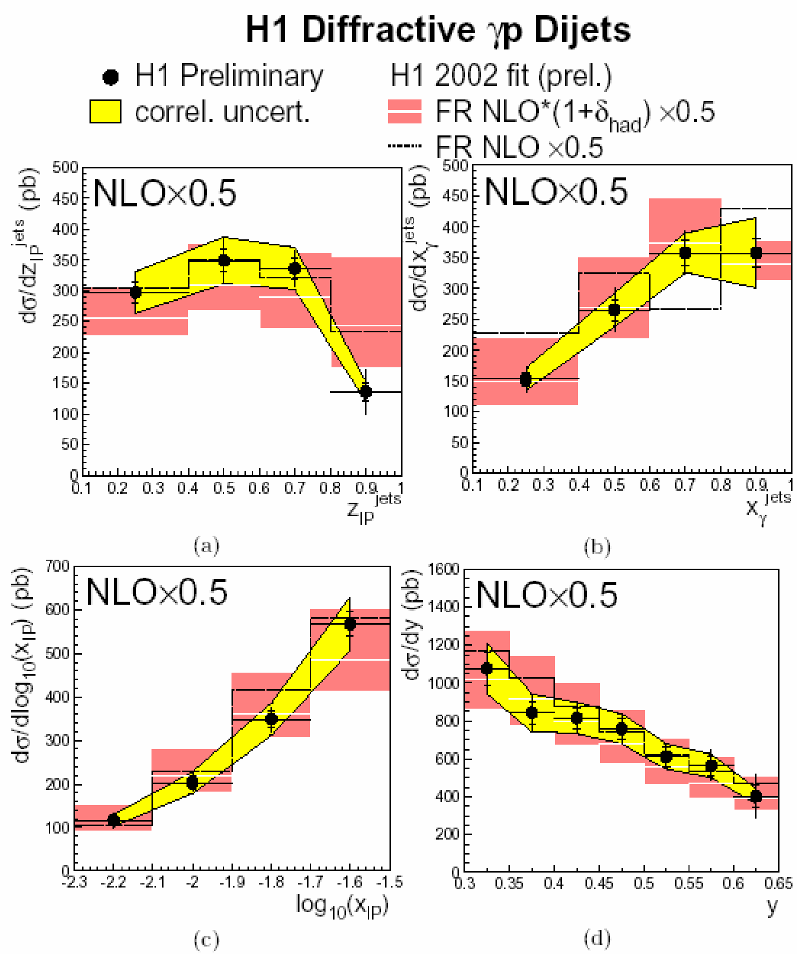
$R = \sigma^{diffr.} / \sigma^{incl.}$ with full kinematics:



MK, G. Kramer, EPJC 38 (2004) 39

But: Data also support direct suppression!

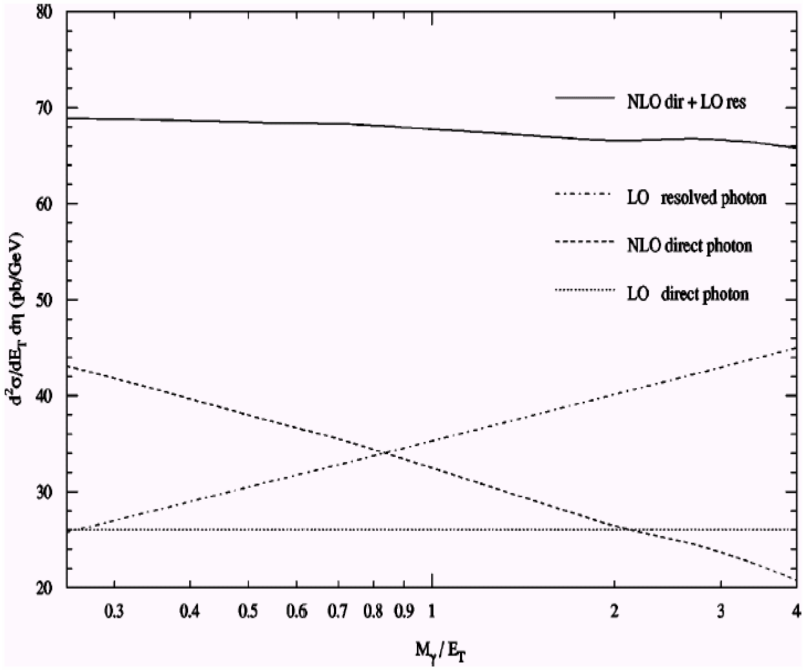




Factorization Scale Dependence

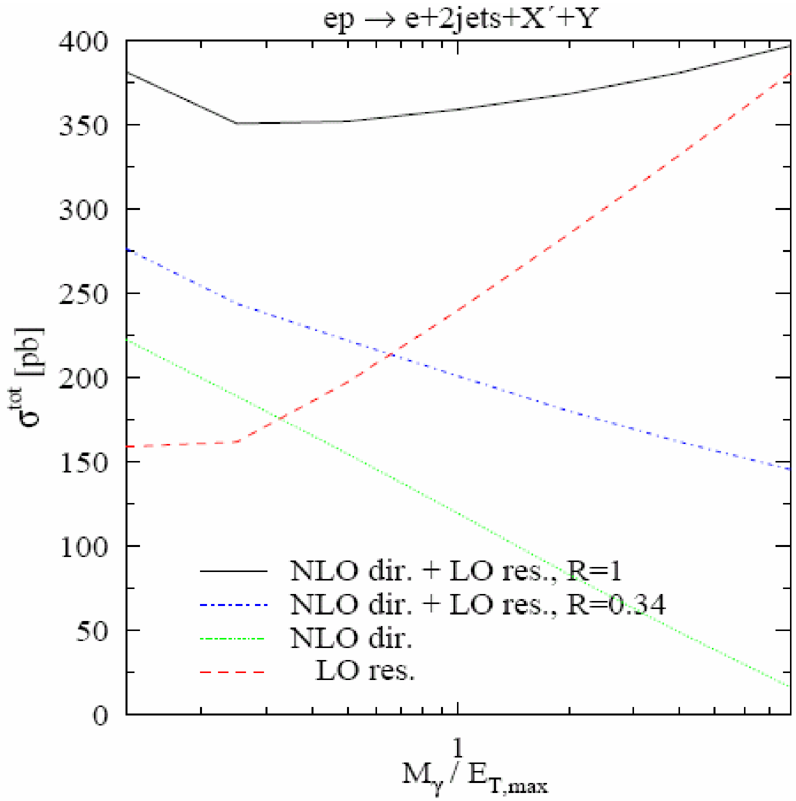
Inclusive photoproduction:

$$|\mathcal{M}^I|^2_{ab\to 123} = \ln\left(\frac{M^2}{Q^2}\right)|\mathcal{M}^B|^2_{cb\to 12} P_{c\leftarrow a}(x) + \cdots$$



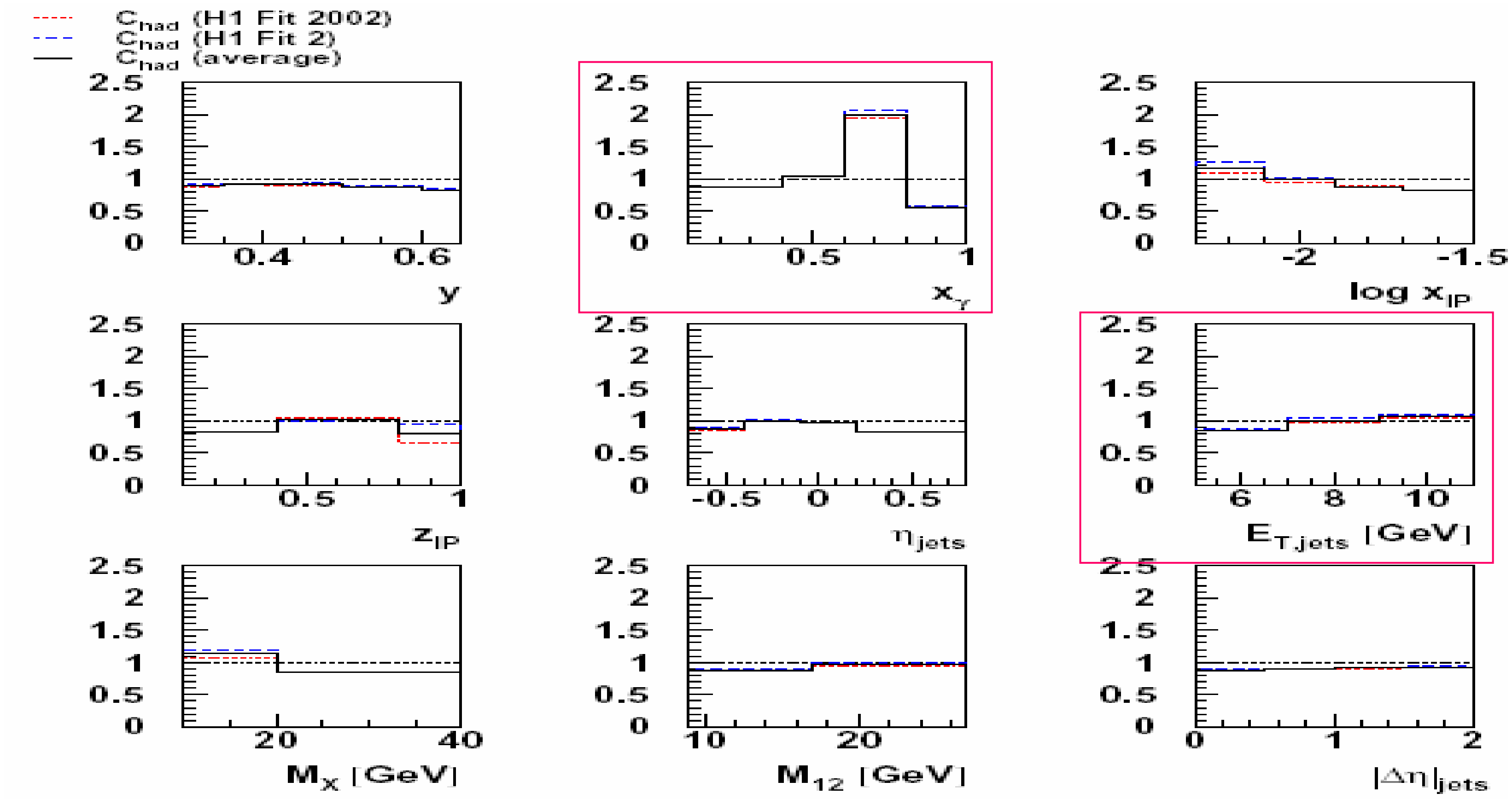
MK, Rev. Mod. Phys. 74 (2002) 1221

Diffractive photoproduction:



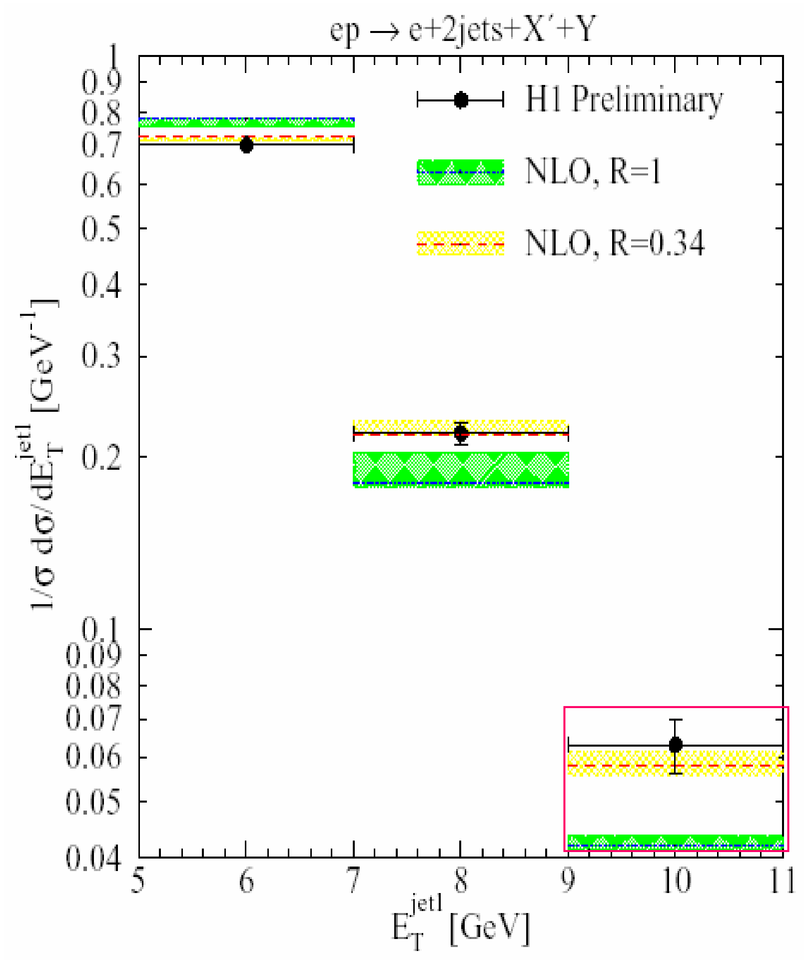
MK, G. Kramer, EPJC 38 (2004) 39

Hadronization Corrections



→ Observable and model dependent!

E_T-Distribution



Importance of large E_T:

- Direct process dominates
- IS singularity less important
- Hadronization corrections small
- Experimentally directly accessible
- Less sensitive than x_γ

Result:

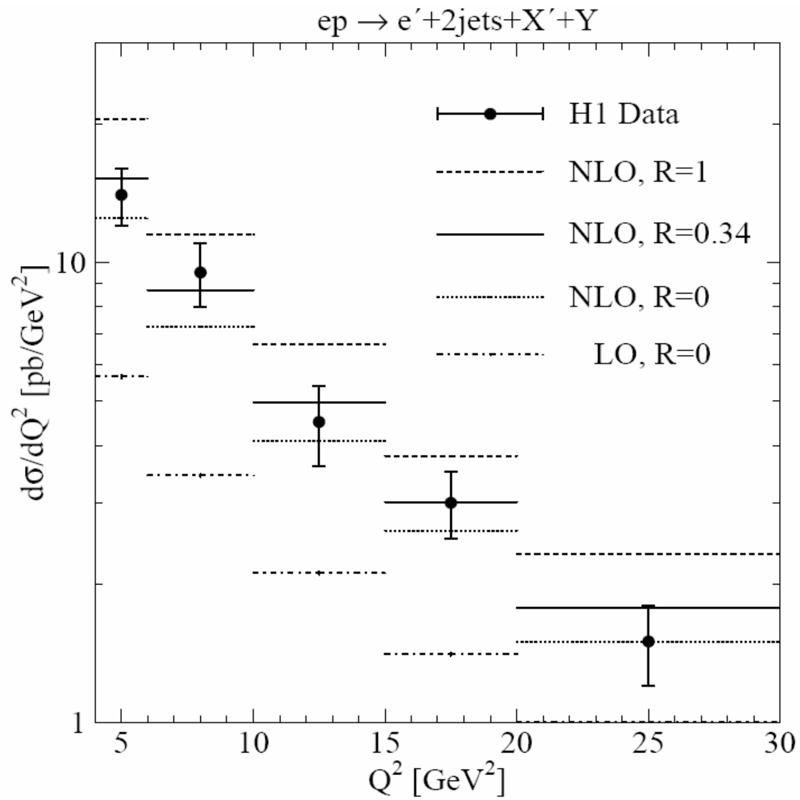
- Suppressed result agrees
- Unsuppressed 50% too low

How can we learn more?

- Critical role of IS singularity
- Transition from γp to DIS

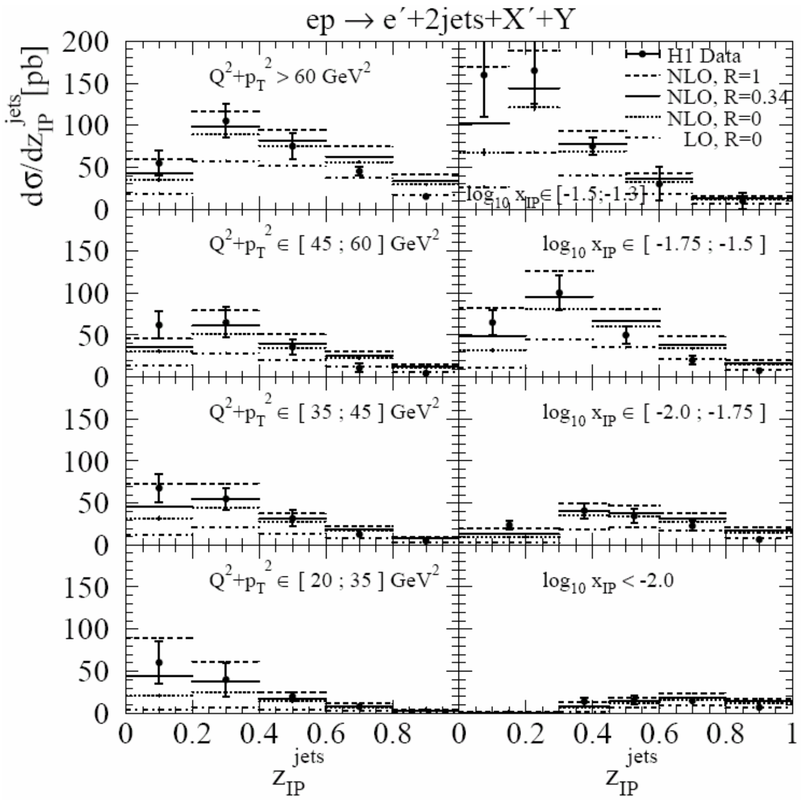
High- to Low-Q^2 Transition in DIS

Q^2-dependence:



MK, G. Kramer, PRL 93 (2004) 232002

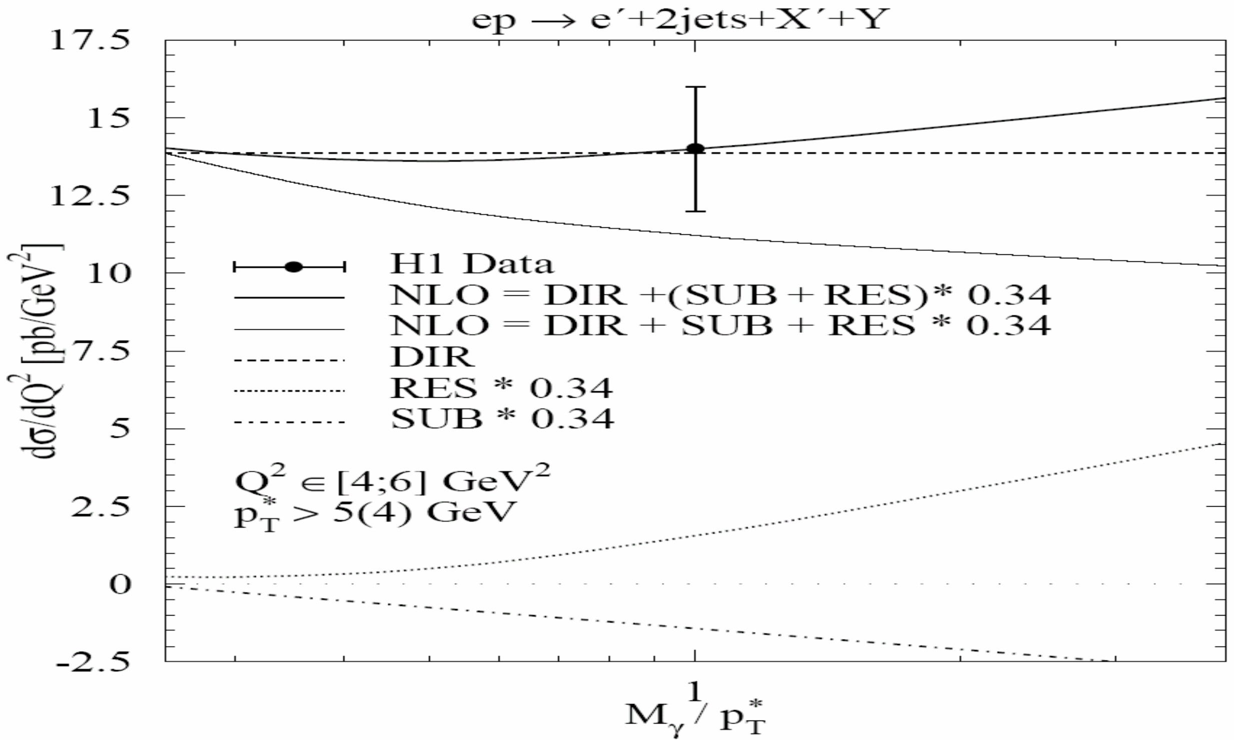
z_{IP}-dependence:



MK, G. Kramer, PRL 93 (2004) 232002

Factorization Scale Dependence (1)

$$M(P^2)_{\overline{MS}} = -\frac{1}{2N_c} P_{q_i \leftarrow \gamma}(z_a) \ln\left(\frac{M_\gamma^2 z_a}{(z_a P^2 + y_s s)(1 - z_a)}\right) + \frac{Q_i^2}{2}$$

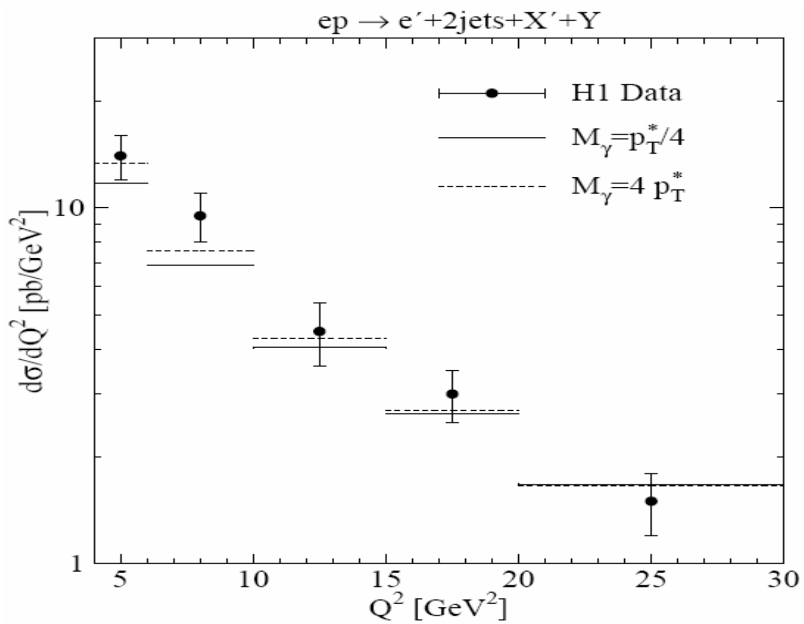


MK, G. Kramer, JPG 31 (2005) 1391

Factorization Scale Dependence (2)

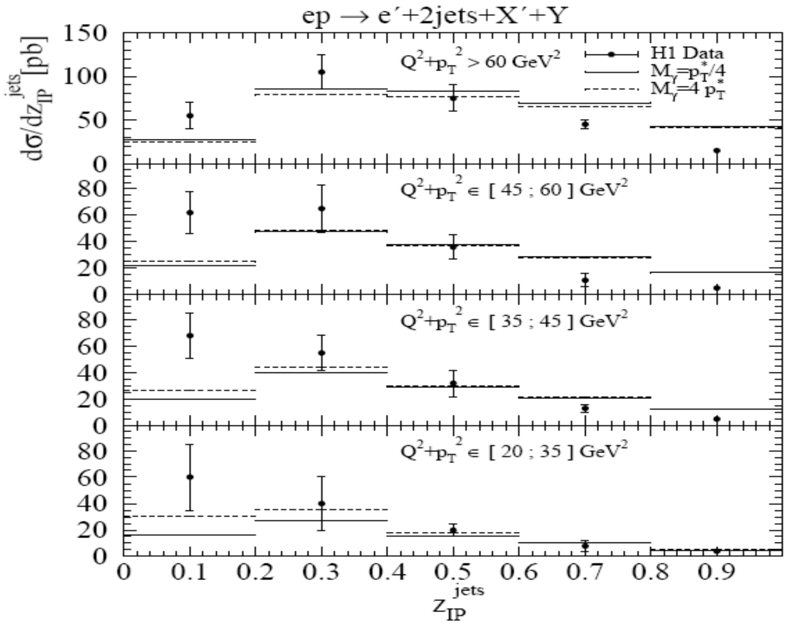
$$M(P^2)_{\overline{MS}} = -\frac{1}{2N_c} P_{q_i \leftarrow \gamma}(z_a) \ln \left(\frac{M_\gamma^2 z_a}{(z_a P^2 + y_s s)(1 - z_a)} \right) + \frac{Q_i^2}{2}$$

Q^2-dependence:



MK, G. Kramer, JPG 31 (2005) 1391

z_{IP}-dependence:



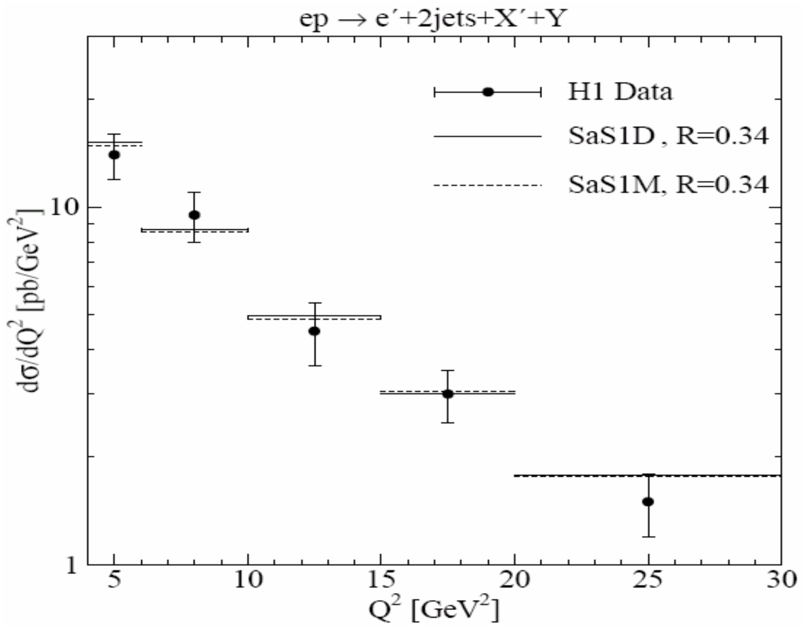
MK, G. Kramer, JPG 31 (2005) 1391

Factorization Scheme Dependence

$$F_2^\gamma(Q^2) = \sum_q 2xe_q^2 \left\{ f_{q/\gamma}(Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \left[C_q \otimes f_{q/\gamma}(Q^2) + C_g \otimes f_{g/\gamma}(Q^2) \right] + \frac{\alpha}{2\pi} e_q^2 C_\gamma \right\}.$$

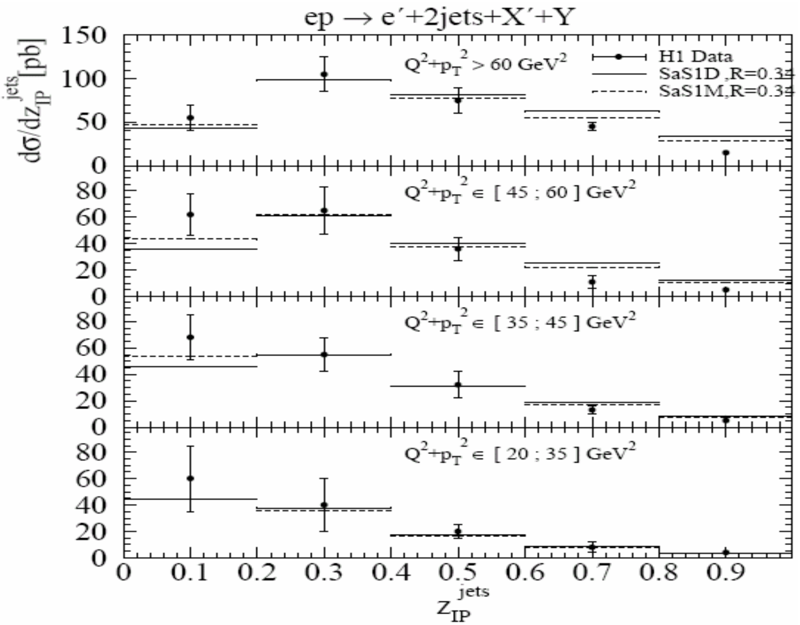
$$C_\gamma(x) = 2N_C\, C_g(x) = 3 \left[\left(x^2 + (1-x)^2 \right) \ln \frac{1-x}{x} + 8x(1-x) - 1 \right],$$

Q^2-dependence:



MK, G. Kramer, JPG 31 (2005) 1391

z_{IP}-dependence:



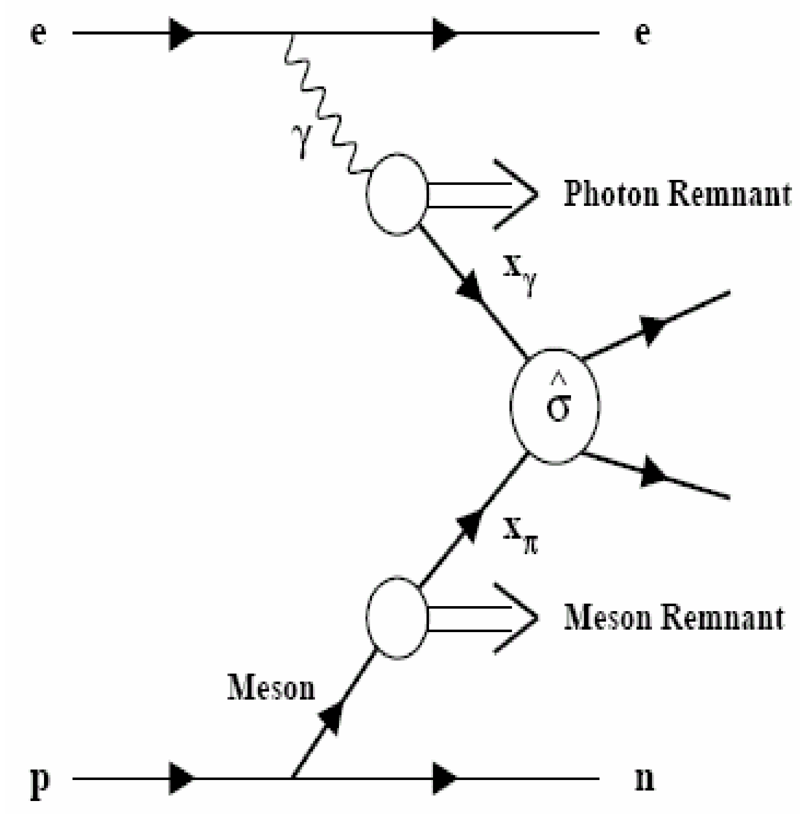
MK, G. Kramer, JPG 31 (2005) 1391

Components of Parton Densities in Photon

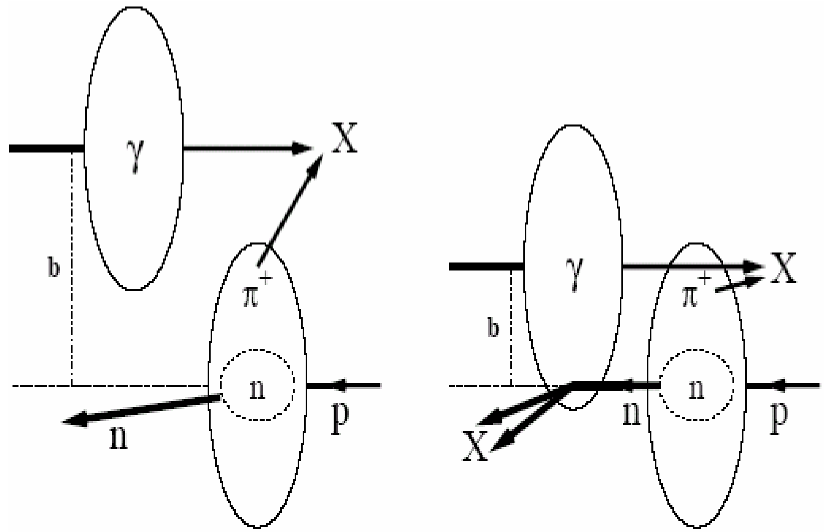
- SaS parameterizations allow for separation:
  - Anomalous component: Resummation of IS singularity
  - Hadronic component: Vector meson dominance model
- VMD component suppressed:
  - 10^{-4} for $Q^2 \approx 70$ GeV^2
  - 10^{-2} for $Q^2 \approx 5$ GeV^2
- Anomalous component dominates:
  - Direct higher order contributions
- Known from inclusive low-Q^2 production

Leading Neutron Production (1)

Scattering process:



Meson cloud model:

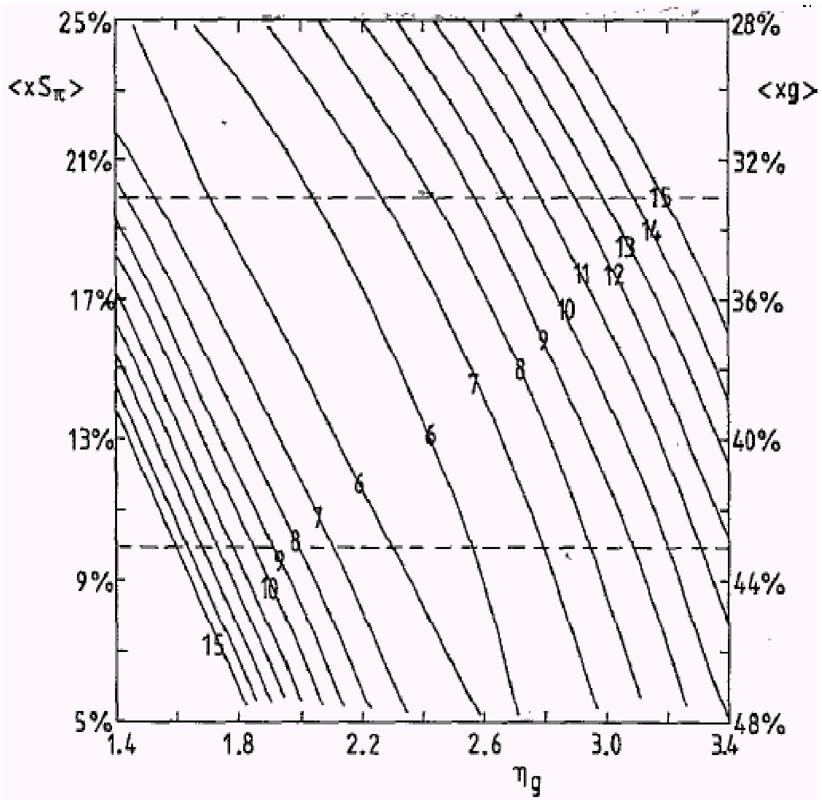


Pion flux factor:

$$f_{\pi/p}(1-x_n,t') = \int d^2 p_T |\phi_{\pi/p}(x_n, p_T)|^2 \delta(t' - f(p_T))$$
$$= \frac{3C_n}{4\pi}\frac{g^2_{n\pi p}}{4\pi}\frac{-t'}{(m_\pi^2 - t')^2}(1-x_n)^{1-2\alpha'_\pi(t'-m_\pi^2)}[F(x_n,t')]^2$$

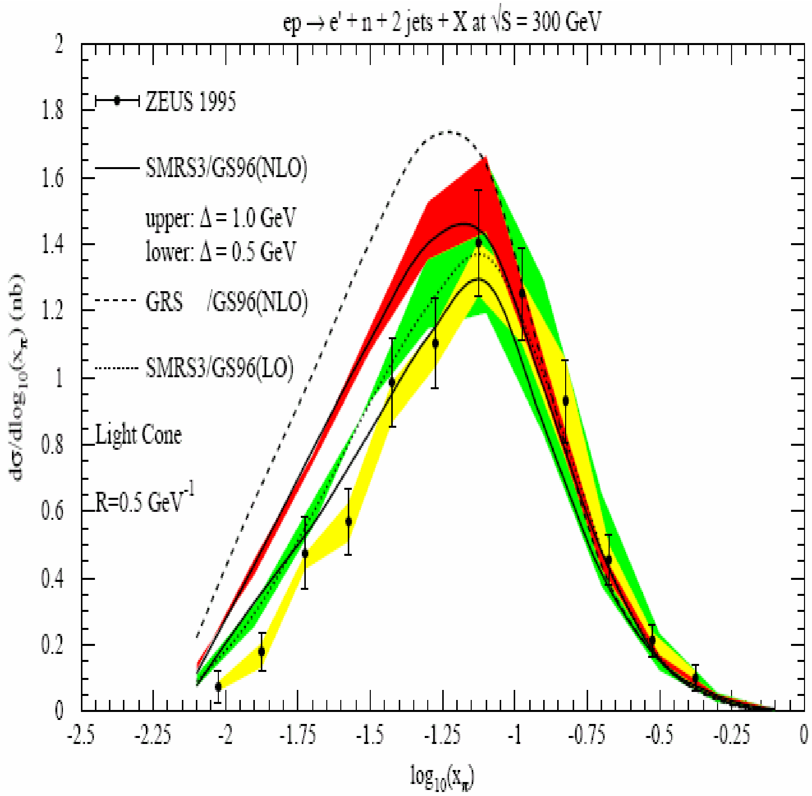
Leading Neutron Production (2)

Sea-quark and gluon density in pions:



Sutton, Martin, Roberts, Stirling, PRD 45

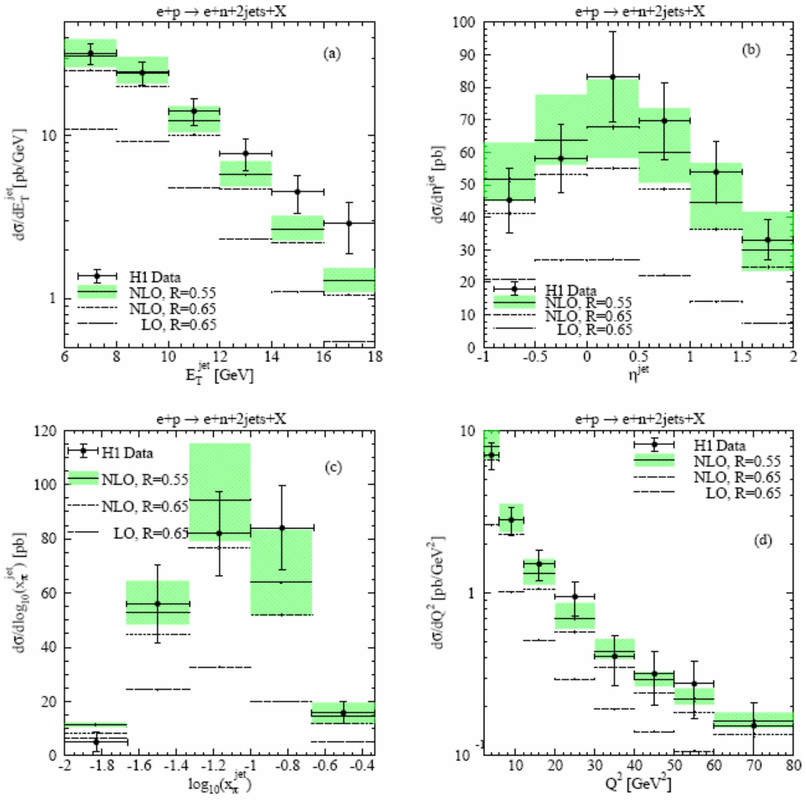
Dijet production with a leading neutron:

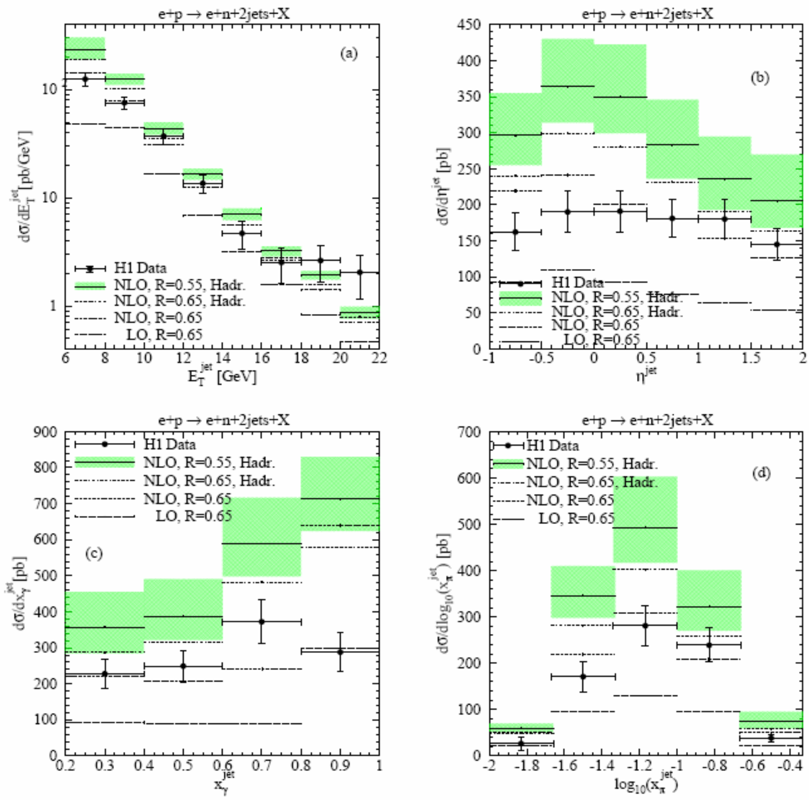


MK, G. Kramer, PLB 508 (2001) 259

Leading Neutron Production (3)

- DIS: Fix R in $F(x_L,t) = \exp[R^2(t - m_\pi^2)/(1 - x_L)]$
- γp: Find S = 0.48 (0.64)

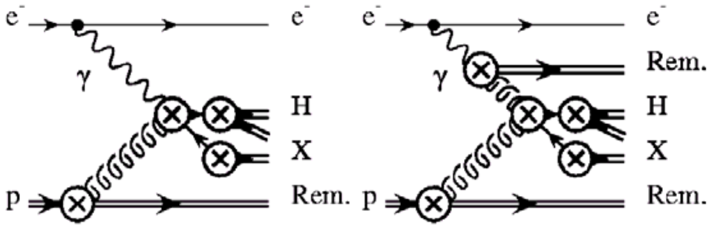




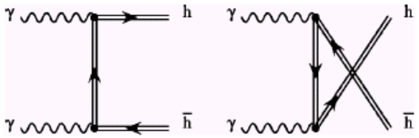
[MK, G. Kramer, hep-ph / 0608235 (EPJC in press)]

Hadron Production

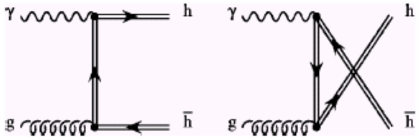
- QCD factorization theorem:



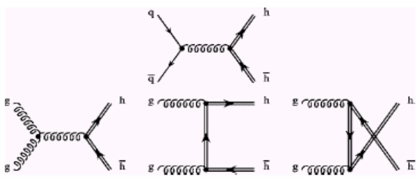
- Direct-direct (→ LEP):



- Direct-resolved:



- Resolved-resolved:



- Hadronic cross section:

$$\frac{d^2\sigma}{dp_T^2dy}=\sum_{a,b,c}\int dx_a dx_b \frac{dz}{z^2} f_{a/A}(x_a,M_a^2) f_{b/B}(x_b,M_b^2) \times D_{H/c}(z,M_c^2)\frac{d\sigma}{dt},$$

- Partonic cross-section:

$$\frac{d\sigma^B}{dt}=\frac{1}{2s}\frac{1}{\Gamma(1-\varepsilon)}\left(\frac{4\pi s}{(t-p_2^2)(u-p_2^2)-p_2^2 s}\right)^{\varepsilon}\frac{1}{8\pi s} \times \frac{g_{a,b}^2}{S_aS_bC_aC_b}|\mathcal{M}^B|^2.$$

- Matrix elements:

Process	LO matrix element squared $\lvert\mathcal{M}^B\rvert^2$
$\gamma\gamma\to h\bar{h}$	$8N_C\left[\frac{t_2}{u_2}+\frac{u_2}{t_2}+\frac{4p_2^2 s}{t_2u_2}\left(1-\frac{p_2^2 s}{t_2u_2}\right)+\varepsilon\left(-1+\frac{s^2}{t_2u_2}\right)+\varepsilon^2\frac{s^2}{4t_2u_2}\right]$
$\gamma g\to h\bar{h}$	$C_F\lvert\mathcal{M}^B\rvert^2_{\gamma\gamma\to h\bar{h}}(s,t,u)$
$q\bar{q}\to h\bar{h}$	$4N_CC_F\left[\frac{t_2^2+u_2^2}{s^2}+\frac{2p_2^2}{s}+\frac{\varepsilon}{2}\right]$
$gg\to h\bar{h}$	$\left[\frac{N_CC_F}{2}\left(1-2\frac{t_2u_2}{s^2}\right)-\frac{C_F}{2N_C}\right]\lvert\mathcal{M}^B\rvert^2_{\gamma\gamma\to h\bar{h}}(s,t,u)$

Fragmentation (1)

- Transition functions:

$$\overline{\Gamma}_{j\leftarrow i}(x,M_f^2)=\delta_{ij}\delta(1-x)-\frac{1}{\varepsilon}\frac{\alpha_s(\mu^2)}{2\pi}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{4\pi\mu^2}{M_f^2}\right)^{\varepsilon}P_{j\leftarrow i}(x)+\mathcal{O}(\varepsilon,\alpha_s^2)$$

- Evolution equations:

$$\frac{dD_{H/q}(Q^2)}{d\ln Q^2}=\frac{\alpha_s(Q^2)}{2\pi}[P_{q\leftarrow q}\otimes D_{H/q}(Q^2)+P_{g\leftarrow q}\otimes D_{H/g}(Q^2)]$$

- Fit to e^+e^- cross sections:

$$\frac{1}{\sigma_{\text{tot}}}\frac{d\sigma(Q^2)}{dx}=\sum_q 2\left(D_{H/q}(Q^2)+\sum_{i=T,L}\frac{\alpha_s(Q^2)}{2\pi}\times[C_q^i\otimes D_{H/q}(Q^2)+C_g^i\otimes D_{H/g}(Q^2)]\right)$$

- Peterson (1983):

$$D_{H/h}(x)=\frac{N}{x[1-1/x-\varepsilon_h/(1-x)]^2} \qquad \varepsilon_h\propto m_q^2/m_h^2$$

- Mele and Nason (1991):

$$D_{H/h}(x,Q^2)=\delta(1-x)+\frac{\alpha_s(Q^2)C_F}{2\pi}\left[\frac{1+x^2}{1-x}\left(\ln\frac{Q^2}{m_h^2}-2\ln(1-x)-1\right)\right]_+$$

Fragmentation (2)

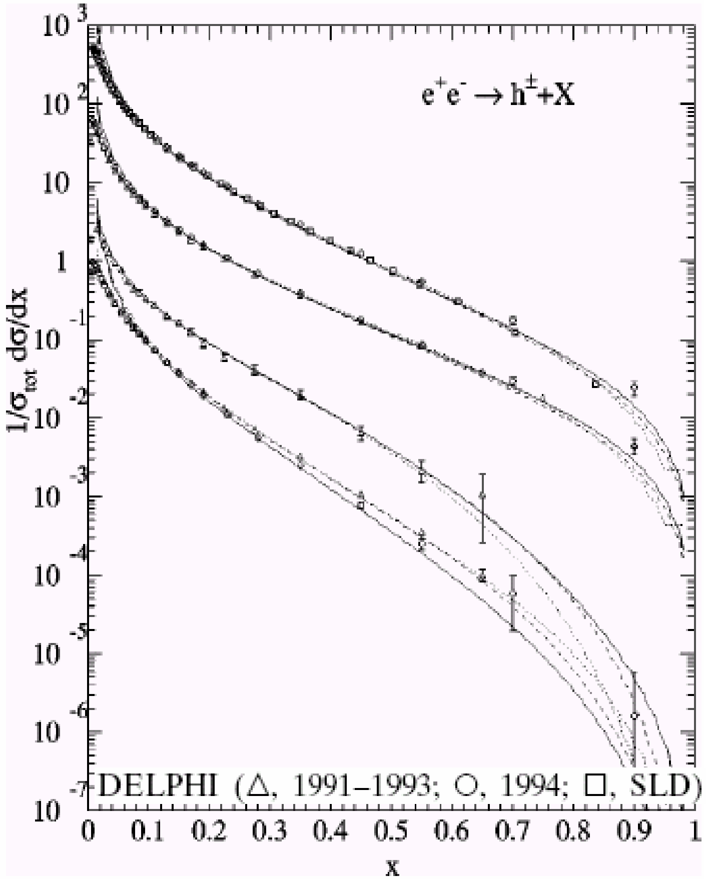
- Available parameterizations:

Group	Year	Hadron	Q_0^2 (GeV^2)	Factor. scheme	N_f	$\Lambda_{\overline{MS}}^{N_f=4}$ (MeV)
BEP	1979	$\pi^{\pm,0},K^{\pm},p$	25.0	LO	3	450/600
AKL	1983	$\pi^{\pm},K^{\pm}$	25.0	LO	3	300/400
GR+	1993/5	$\pi^{\pm,0},\eta,K^{\pm,0}$	900	$\overline{MS}$	5	269/319
NW	1994	$h^{\pm}$	m_Z^2	$\overline{MS}$	5	344
BKK	1995	$\pi^{\pm},K^{\pm}$	2.00	LO	5	190
				$\overline{MS}$		190
	1995/8	$\pi^{\pm},K^{\pm,0}$,	2.00	LO	5	146 (fit)
		$D^{*\pm},B^{+,0}$	$4m_{c,b}^2$	$\overline{MS}$		317 (fit)
CGRT	1997	D^0,D^{*0}	m_c^2	$\overline{MS}$	5	151
KKP	2000	$\pi^{\pm},K^{\pm},p$	2.00	LO	5	121 (fit)
				$\overline{MS}$		299 (fit)
Kretzer	2000	$\pi^{\pm},K^{\pm},h^{\pm}$	0.26	LO	5	175
			0.40	$\overline{MS}$		246
BFGW	2001	$h^{\pm}$	2.00	$\overline{MS}$	5	300

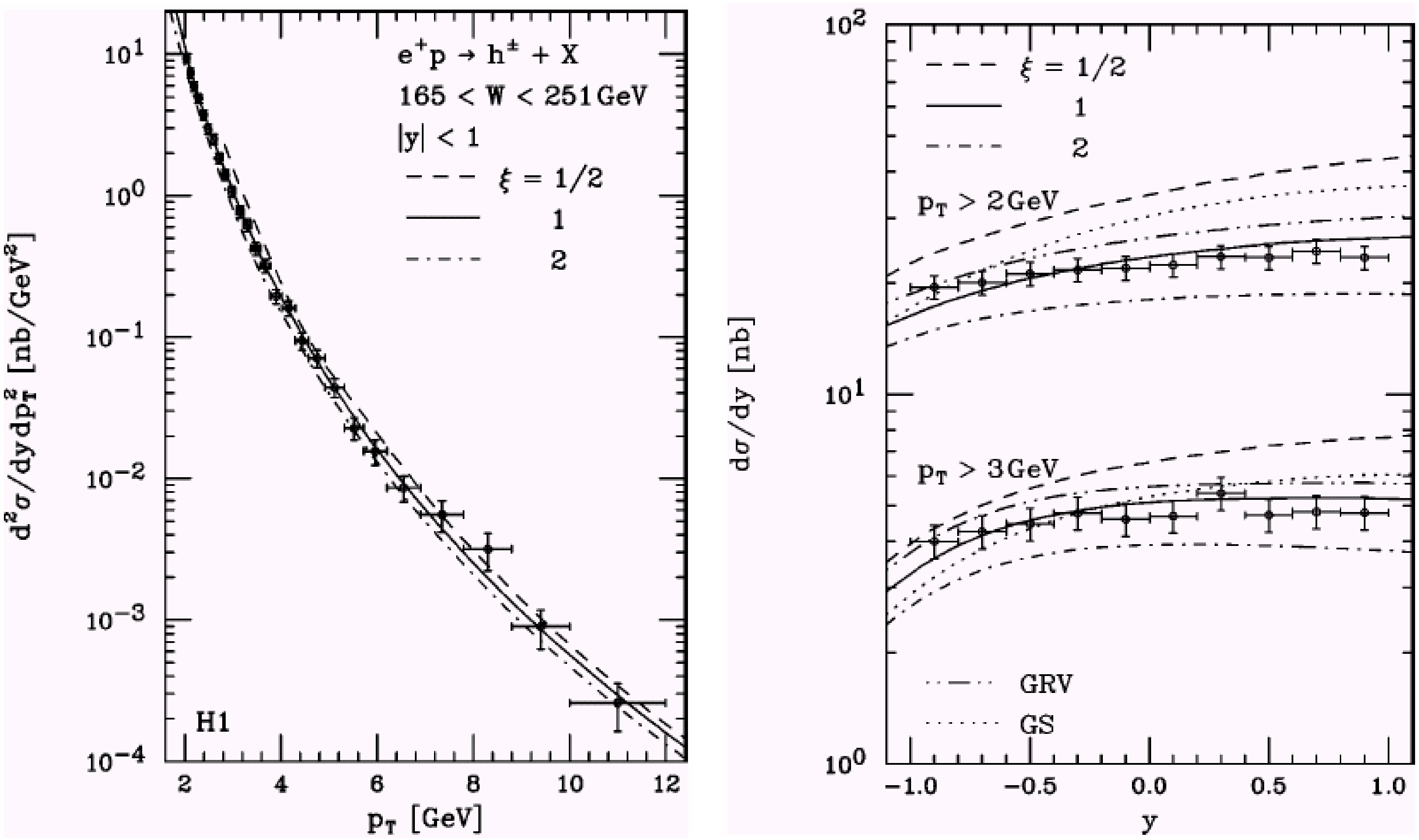
- Quality of the fits:

Energy (GeV)	Flavor	Experiment	FF set: KKP	FF set: Kretzer	FF set: BFGW	No. of points
29	*uds*	TPC	0.178*	0.159	0.167*	7
	c		0.876*	0.911	0.923*	7
	b		2.23*	1.21	1.14*	7
91.2	all	DELPHI 94	1.28	1.51*	1.49	12
		SLD	1.32	0.370	0.421	21
	uds	DELPHI 91-3	3.17*	0.990*	1.95	13
		DELPHI 94	0.201	0.588*	1.00*	12
	c	DELPHI 91-3	0.473*	0.388*	0.401	11
	b	DELPHI 91-3	28.9*	0.887*	1.03	12
		DELPHI 94	0.433	9.14*	8.74	12
189	all	OPAL	0.568*	0.250*	0.414*	11

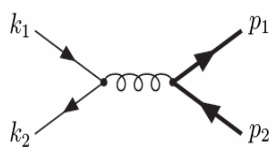
- Comparison to LEP/SLC data:

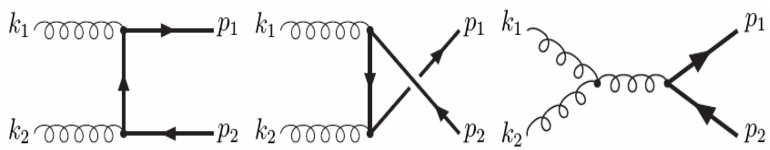


Light Hadrons



Heavy Hadrons (1)

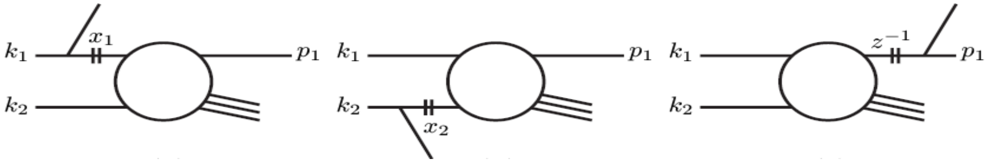
- Fixed flavor number scheme:
  - Heavy flavors: $m_q > \Lambda \to q = c,b,t$
  - Active partons in PDFs and FFs: $q = u,d,s$ and g
  - Applicability: σ_{tot}, $p_T \approx m_q$
- Partonic processes:
  - qq: 

gg: 

- Fixed-order next-to-leading-logarithm scheme:
  - Cacciari, Greco, Nason, JHEP 05 (1998) 007
  - Subtraction of $\ln(m_q^2/p_T^2) \neq$ massless calculation
  - Finite $O(m_q^2/p_T^2)$ terms $\to$ initial condition of FFs

Heavy Hadrons (2)

- Variable flavor number scheme:
  - Kniehl, Kramer, Schienbein, Spiesberger, EPJC 41 (2005) 199
  - Kinematics of mass factorization:



  - Factorized partonic cross section:

$$\mathrm{d}\hat{\sigma}^{(1)}(a+b\to Q+X) = \mathrm{d}\tilde{\sigma}^{(1)}(a+b\to Q+X)$$
$$-f^{(1)}_{a\to i}(x_1)\otimes \mathrm{d}\sigma^{(0)}(i+b\to Q+X)$$
$$-f^{(1)}_{b\to j}(x_2)\otimes \mathrm{d}\sigma^{(0)}(a+j\to Q+X)$$
$$-\mathrm{d}\sigma^{(0)}(a+b\to k+X)\otimes d^{(1)}_{k\to Q}(z)\,.$$

  - Partonic splitting functions:

$$d^{(1)}_{Q\to Q}(z,\mu_{\mathrm{R}},\mu'_{\mathrm{F}})$$
$$= \frac{\alpha_s(\mu_{\mathrm{R}})}{2\pi}C_{\mathrm{F}}\left[\frac{1+z^2}{1-z}\left(\ln\frac{\mu'^2_{\mathrm{F}}}{m^2}-2\ln(1-z)-1\right)\right]_+$$

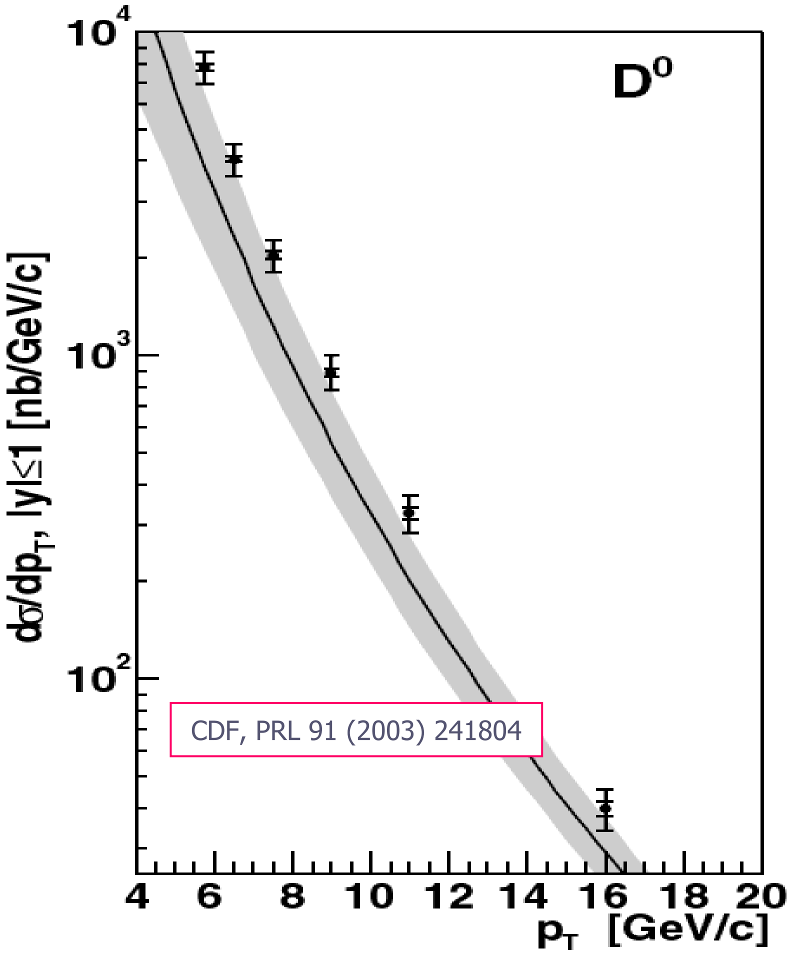
$$\boxed{d^{(1)}_{g\to Q}(z,\mu_{\mathrm{R}},\mu'_{\mathrm{F}}) = \frac{\alpha_s(\mu_{\mathrm{R}})}{2\pi}P^{(0)}_{g\to q}(z)\ln\frac{\mu'^2_{\mathrm{F}}}{m^2}}$$

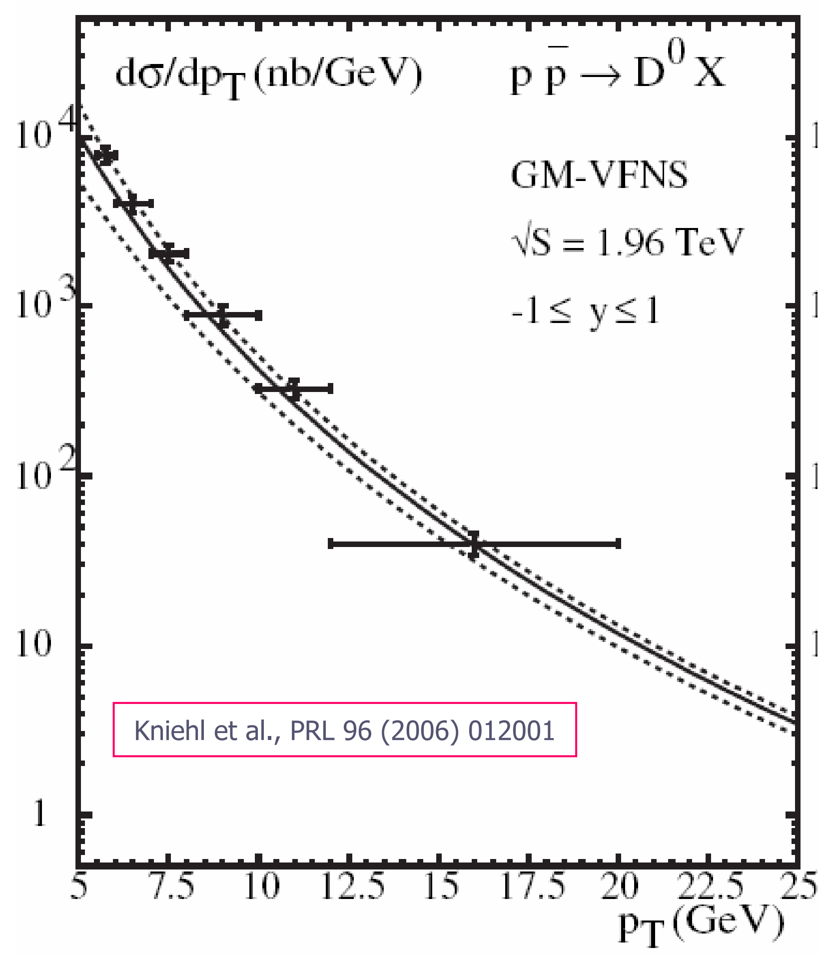
$$f^{(1)}_{g\to Q}(x,\mu_{\mathrm{R}},\mu_{\mathrm{F}}) = \frac{\alpha_s(\mu_{\mathrm{R}})}{2\pi}P^{(0)}_{g\to q}(x)\ln\frac{\mu^2_{\mathrm{F}}}{m^2}\,,$$

$$f^{(1)}_{Q\to Q}(x,\mu_{\mathrm{R}},\mu_{\mathrm{F}})$$
$$= \frac{\alpha_s(\mu_{\mathrm{R}})}{2\pi}C_{\mathrm{F}}\left[\frac{1+x^2}{1-x}\left(\ln\frac{\mu^2_{\mathrm{F}}}{m^2}-2\ln(1-x)-1\right)\right]_+$$

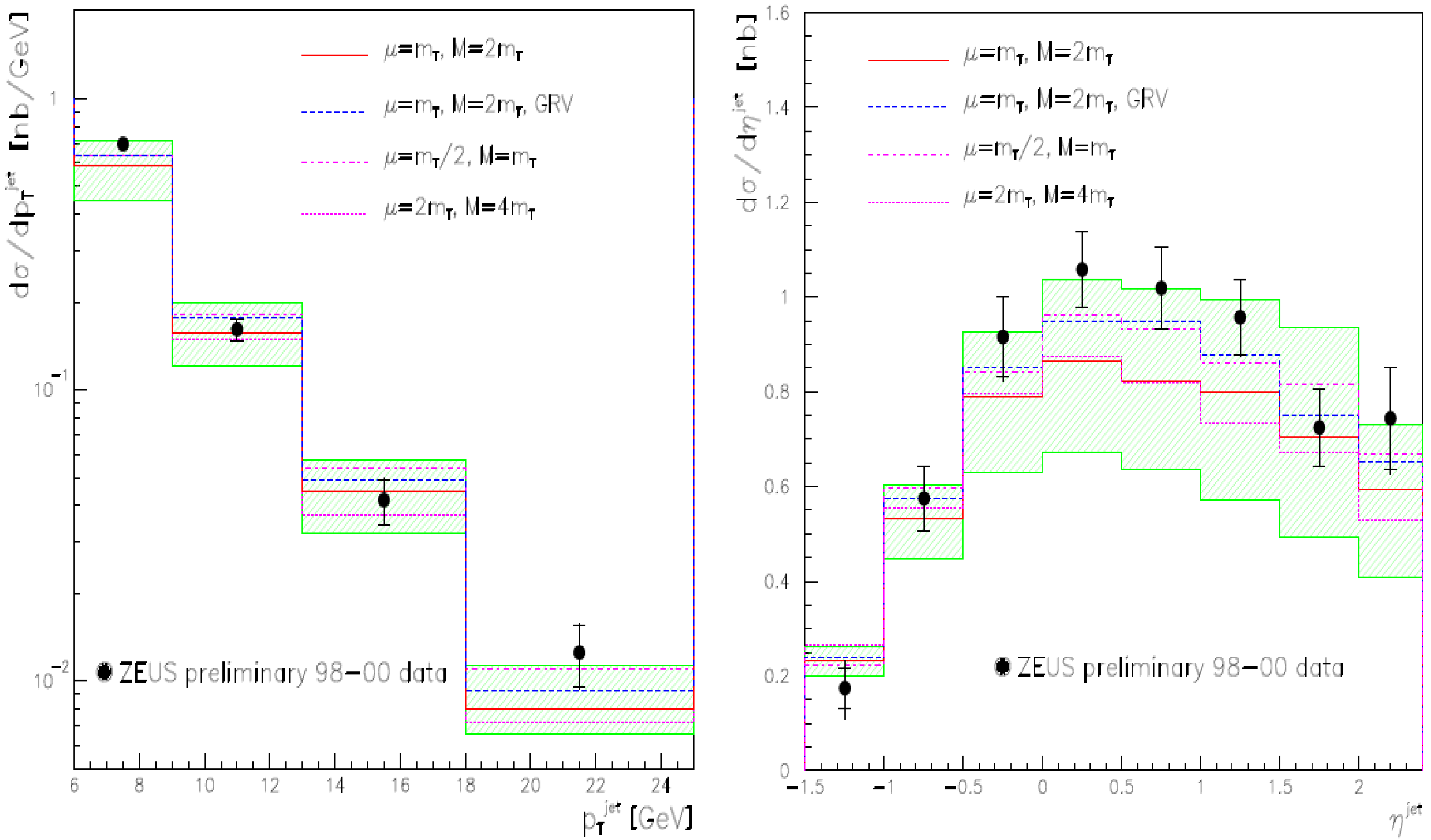
$$f^{(1)}_{g\to g}(x,\mu_{\mathrm{R}},\mu_{\mathrm{F}}) = -\frac{\alpha_s(\mu_{\mathrm{R}})}{2\pi}\frac{2}{3}T_f\ln\frac{\mu^2_{\mathrm{F}}}{m^2}\delta(1-x)\,,$$

Open Charm at the Tevatron





D*+jet in Photoproduction at HERA



Quarkonia

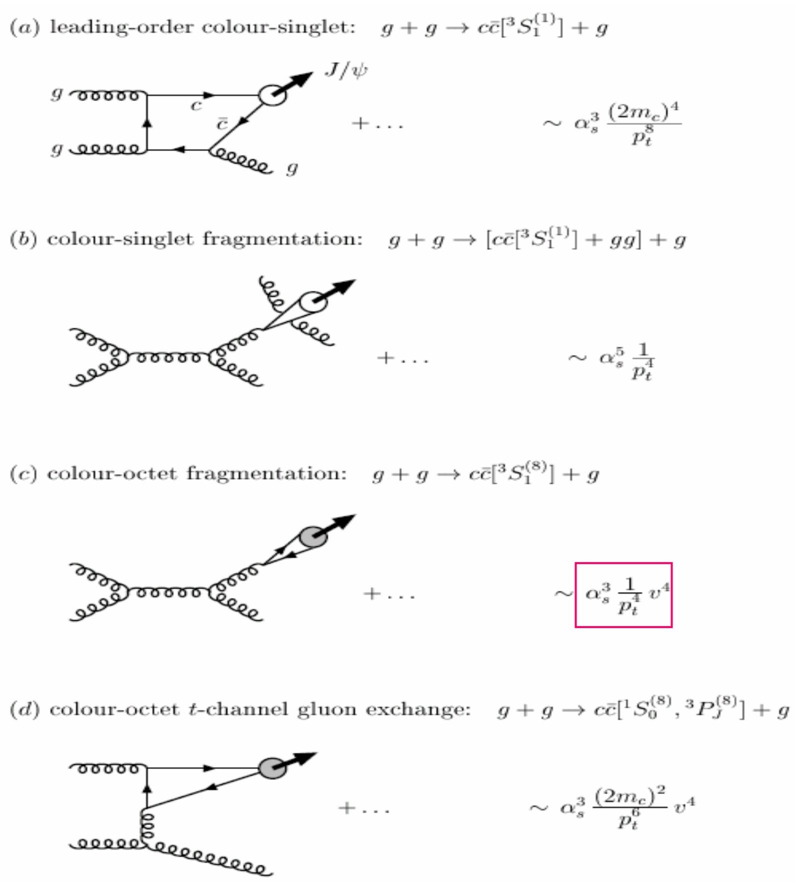
- The two sides of QCD:
  - Quarks and gluons: Asymptotically free, carry color
  - Hadrons: Color-singlets, non-perturbative, confinement
- Simplest laboratory: Quarkonia $\rightarrow$ Rich spectroscopy
  - Charmonia: $\eta_c'(2S)$, X(3872) (?)
  - Bottomonia: $\Upsilon(1D)$
- Experimental data:
  - C-/B-factories: BES, CLEO, Babar, Belle
  - Fixed target: HERA-B, E866
  - Colliders: LEP, HERA, Tevatron, LHC-B
- Review:
  - N. Brambilla et al.: CERN Yellow Report, hep-ph/0412158

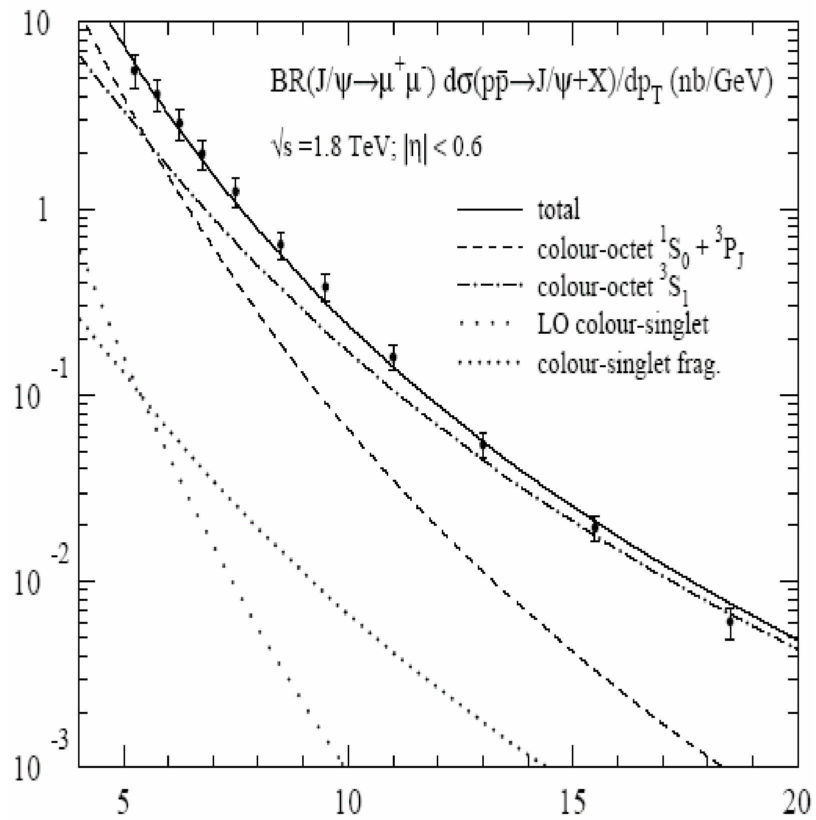
NRQCD

- Color singlet state:
  - Created by soft gluon exchange with underlying event
- Separation of mass scales: ($v_c^2 = 0.3$, $v_b^2 = 0.1$)
  - m_c = 1.5 GeV (annihilation and production threshold)
  - $m_c v_c$ = 0.9 GeV (size of bound state)
  - $m_c v_c^2$ = 0.5 GeV (splitting of resonances)
  - Double expansion in α_s and v (pertur. and relat. corrections)
- Lagrangian: $\mathcal{L}_{\rm NRQCD} = \psi^\dagger \left(iD_0 + \frac{\vec{D}^2}{2m} \right) \psi + \chi^\dagger \left(iD_0 - \frac{\vec{D}^2}{2m} \right) \chi + \mathcal{L}_{\rm light} + \delta\mathcal{L}$
  - ψ and χ are two-component heavy (anti-)quark spinors
  - δL reproduces L_{QCD}: $\delta\mathcal{L}_{\rm bilinear}$, $\delta\mathcal{L}_{4-{\rm fermion}} = \sum \frac{d_i}{m^2} (\psi^\dagger \mathcal{K}_i \chi)(\chi^\dagger \mathcal{K}'_i \psi)$
  - Gauge completion: $\mathcal{O}_n^H(0) \to \chi^\dagger \mathcal{K}_{n,c} \psi(0)\, \Phi_l^\dagger[0,A]_{cb} \left(a_H^\dagger a_H \right) \Phi_l[0,A]_{ba}\, \chi^\dagger \mathcal{K}'_{n,a} \psi(0)$

  [Nayak, Qiu and Sterman, PLB 613 (2005) 45]

J/Ψ-Cross Section at the Tevatron

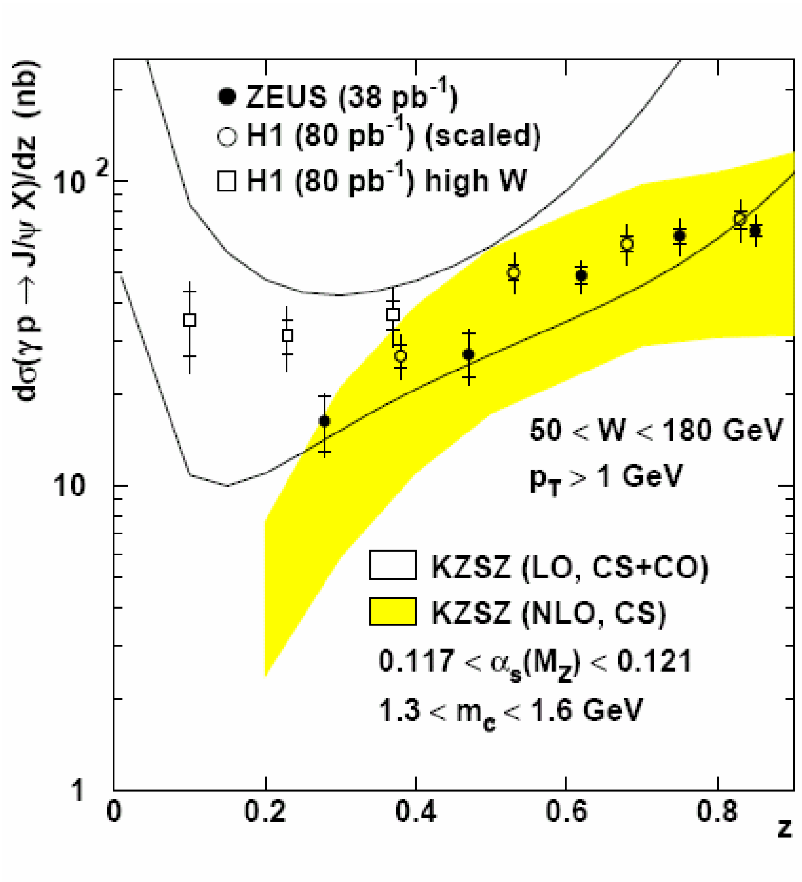




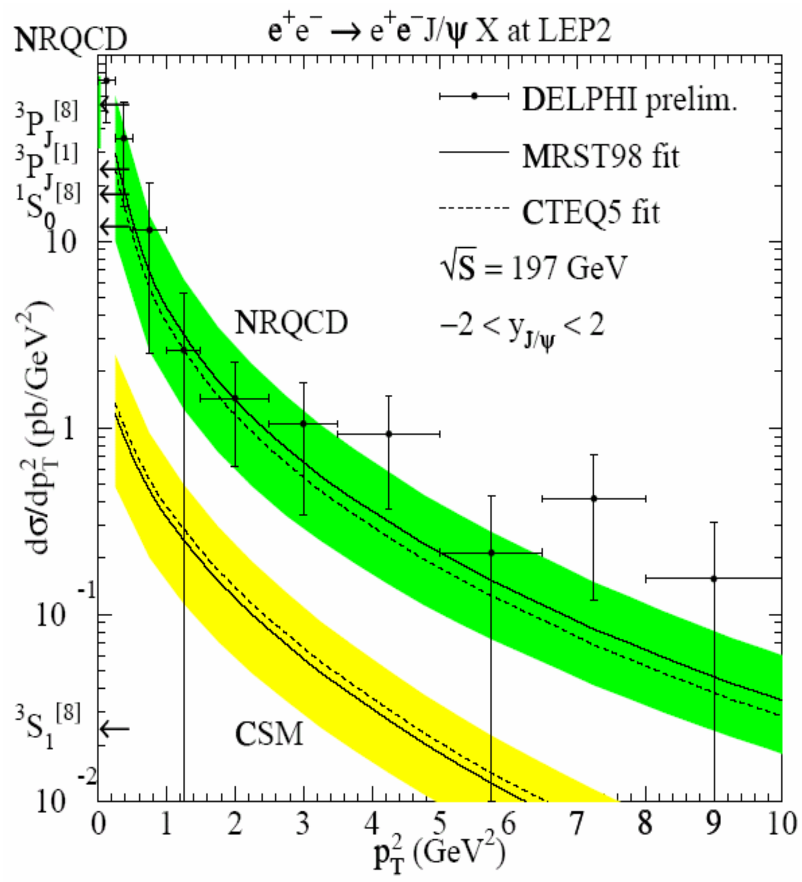
[Braaten (1995); Cho, Leibovich (1996)]

Similar situation for Ψ′ and χ_c, see M. Krämer, Prog. Part. Nucl. Phys. 47 (2001) 141

J/Ψ-Cross Section at HERA and LEP

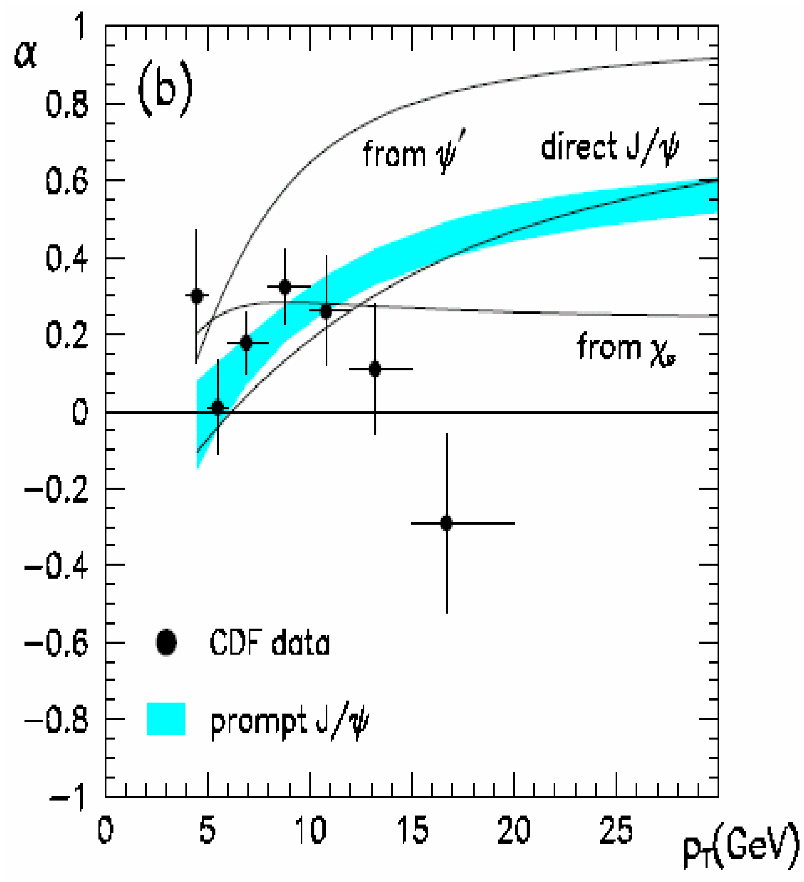


Cacciari, Krämer, PRL 76 (1996) 4128

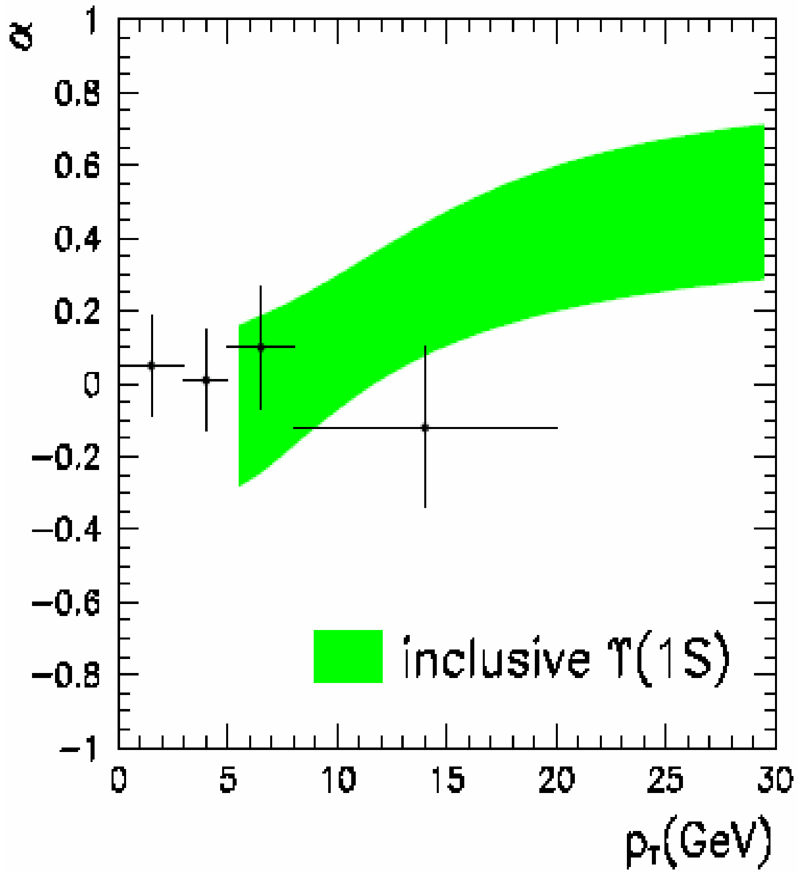


MK et al., PRL 89 (2002) 032001

J/Ψ- and Υ-Polarization at the Tevatron (1)

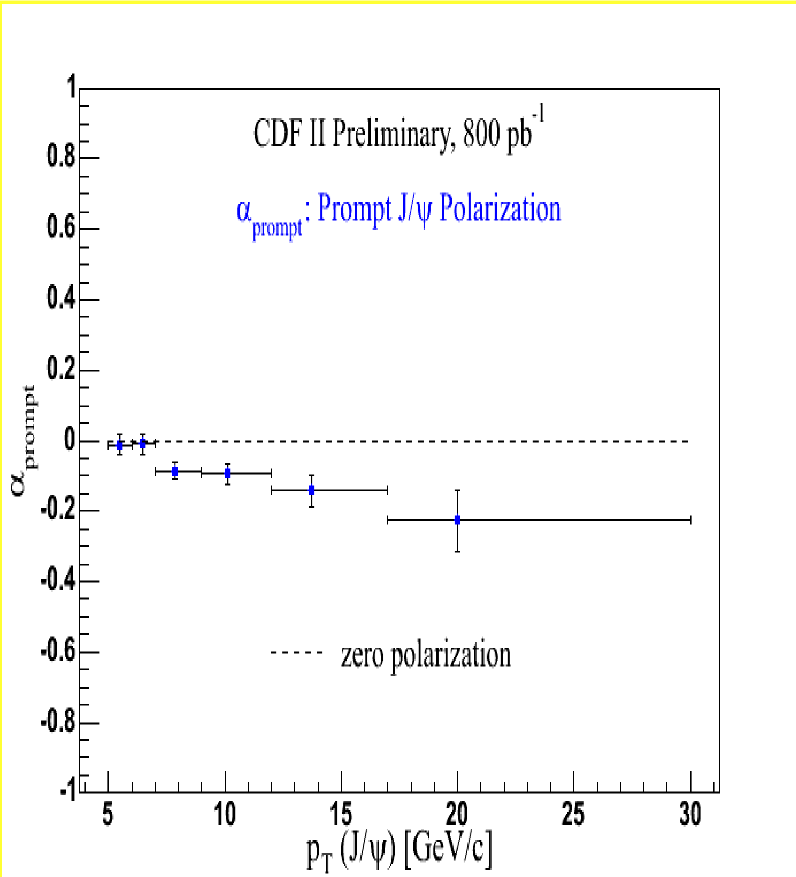


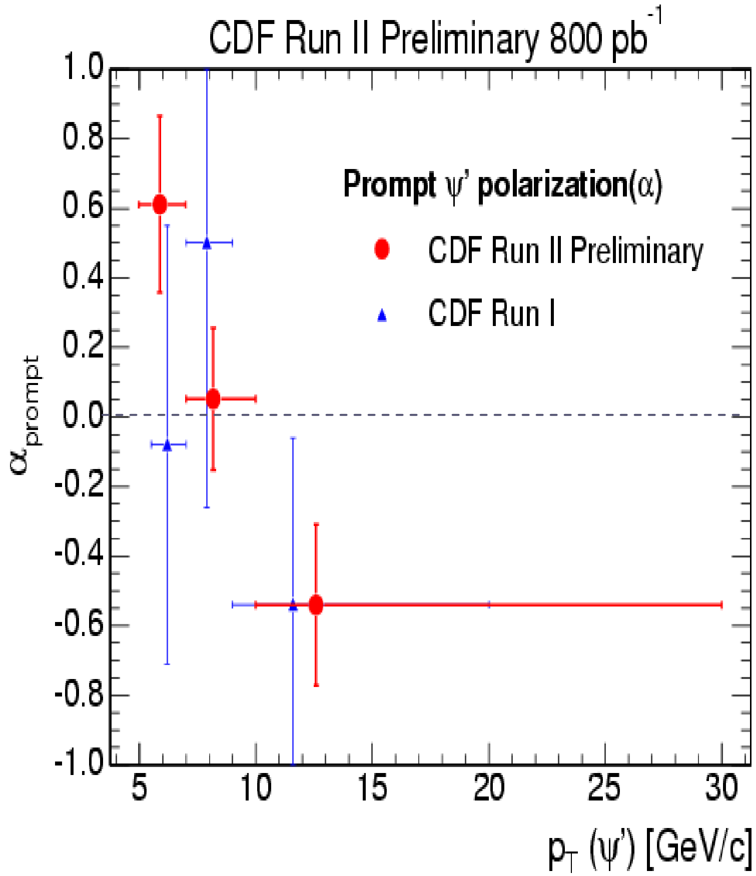
Braaten, Kniehl, Lee, PRD 62 (2000) 094005



Braaten, Lee, PRD 63 (2001) 071501

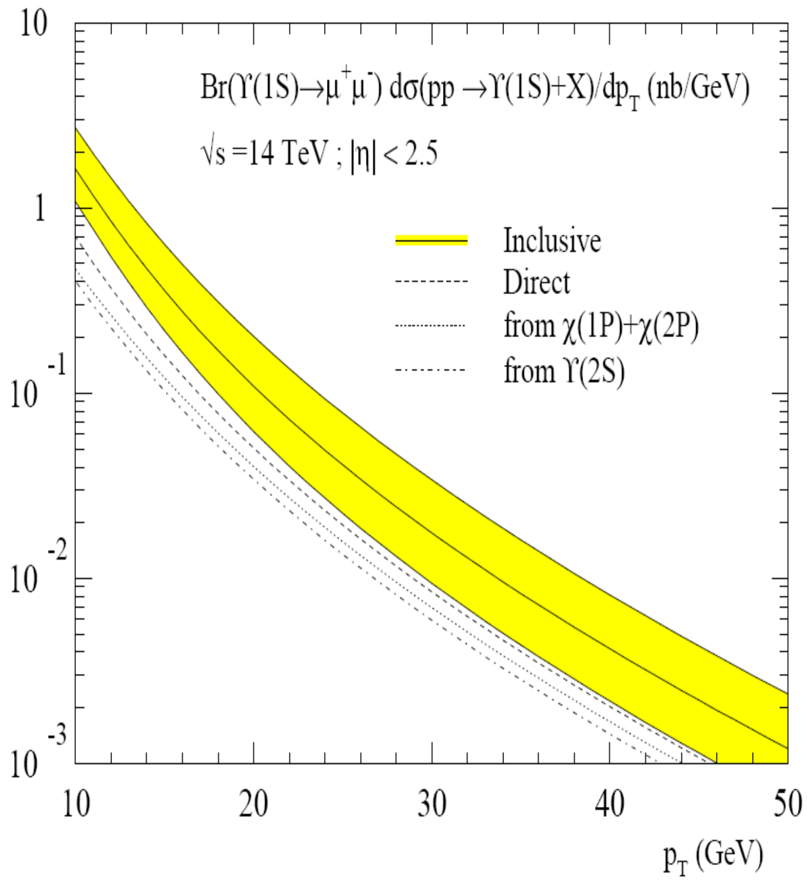
J/Ψ- and Ψ′-Polarization at the Tevatron (2)

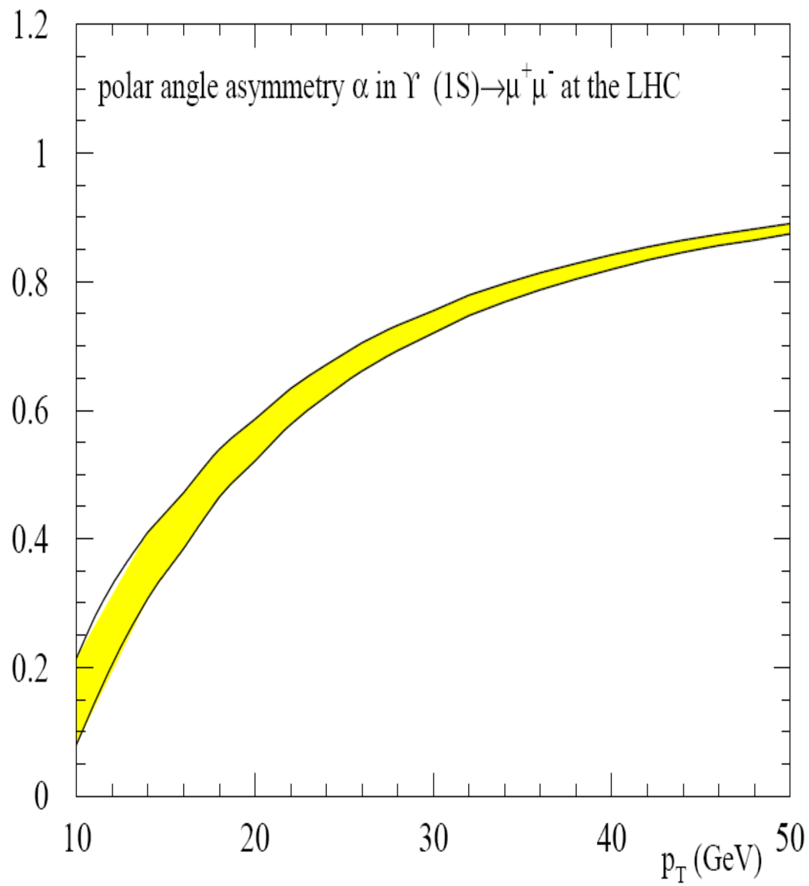




[http://www-cdf.fnal.gov/physics/new/bottom/]

Υ-Cross Section and -Polarization at the LHC





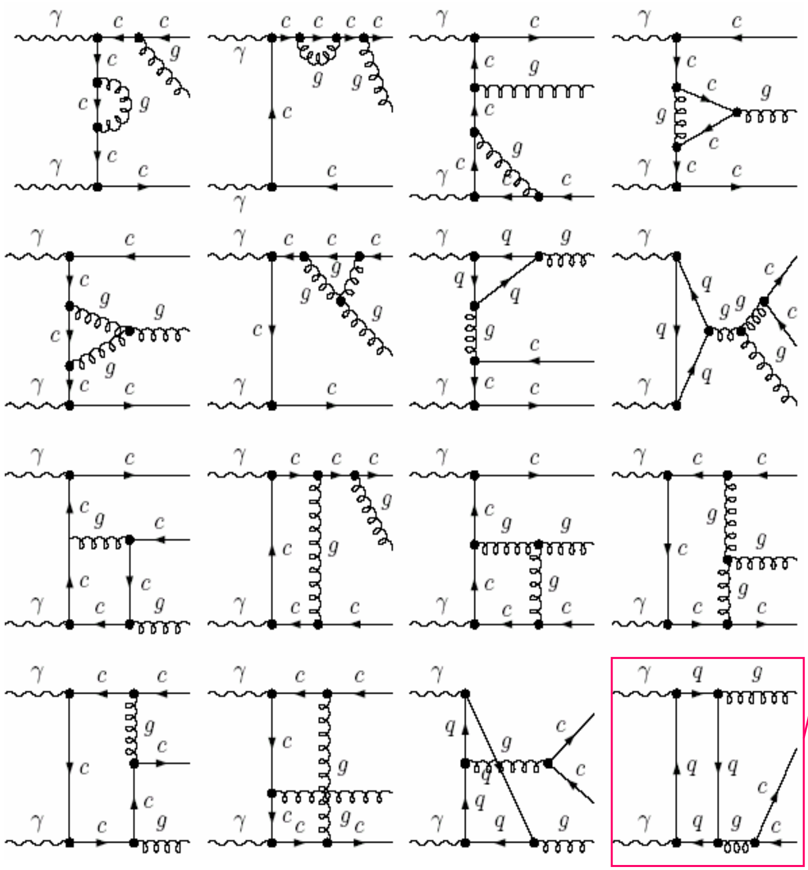
But: LO only [M. Krämer, Prog. Part. Nucl. Phys. 47 (2001) 141]

NLO Quarkonium Production

- Color-singlet model p_T-spectrum: $\gamma p \rightarrow J/\Psi$+jet
  - Krämer, Zunft, Steegborn, Zerwas, PLB 348 (1995) 657
  - Krämer, NPB 459 (1996) 3
- NRQCD σ_{tot}: pp/γp $\rightarrow$ Quarkonium
  - Petrelli et al., NPB 514 (1998) 245
  - Maltoni, Mangano, Petrelli, NPB 519 (1998) 361
- NRQCD p_T-spectrum: $\gamma\gamma \rightarrow$ Quarkonium + γ/jet
  - Klasen, Kniehl, Mihaila, Steinhauser, NPB 713 (2005) 487
  - Klasen, Kniehl, Mihaila, Steinhauser, PRD 71 (2005) 014016

Virtual Corrections

- Typical Feynman diagrams:



- Cross section:

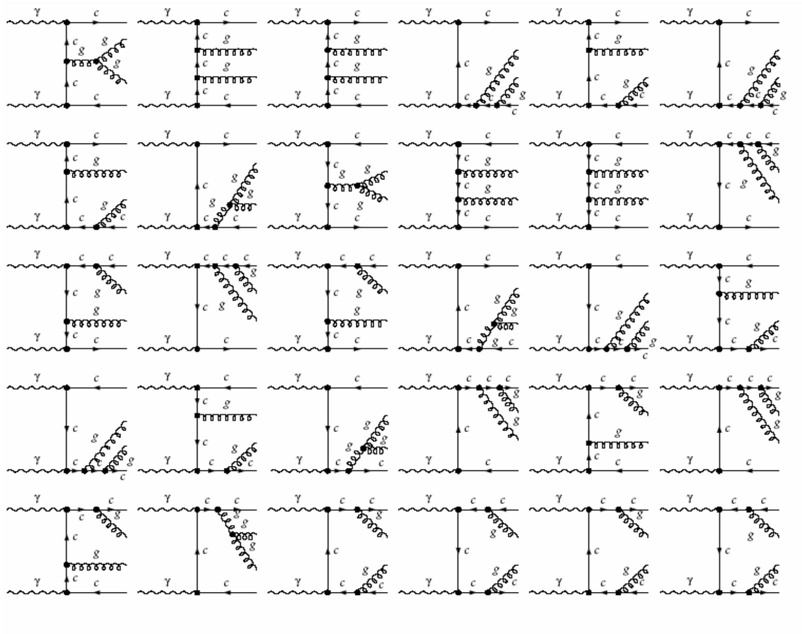
$$\frac{d\sigma_v}{dt_1du_1} = \frac{1}{16\pi s^2\Gamma(1-\epsilon)}\left(\frac{4\pi\mu^2 s}{tu}\right)^{\epsilon}\delta(s+t+u-4m^2)2\overline{\mathrm{Re}\,(\mathcal{T}_0^*\mathcal{T}_v)}$$

- Light box contribution:

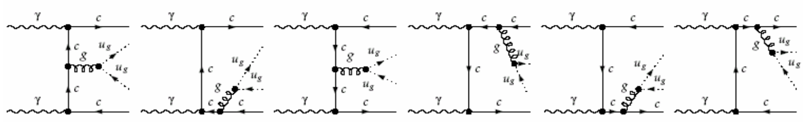
$$\begin{aligned}
2\,\mathrm{Re}\,(\mathcal{T}_0^*\mathcal{T}_q) = \frac{16e_q^2e_c^2e^4g_s^4\left\langle\mathcal{O}[^3S_1^{(8)}]\right\rangle}{3m}\Bigg\{ & \frac{stu[s^2+t^2+3tu+u^2+3s(t+u)]}{\pi^2(s+t)^2(s+u)^2(t+u)^2} \\
& - \frac{128m^4}{(s+t)^3(s+u)^3(t+u)^3}\Big[-4s^3tu(t+u)-4st^2u^2(t+u)+s^4(t^2+u^2) \\
& + t^2u^2(t^2+u^2)+s^2(t^4-4t^3u-14t^2u^2-4tu^3+u^4)\Big]B_0(4m^2,0,0) \\
& + \frac{8s[-4tu(t+u)+s(t^2+u^2)]}{(s+t)(s+u)(t+u)^3}B_0(s,0,0) \\
& + \frac{8t[s^2(t-4u)-4su^2+tu^2]}{(s+t)(s+u)^3(t+u)}B_0(t,0,0) \\
& + \frac{8u[-4st^2+t^2u+s^2(-4t+u)]}{(s+t)^3(s+u)(t+u)}B_0(u,0,0) \\
& + \frac{4s(2s^2+t^2+u^2)}{(s+t)(s+u)(t+u)}C_0(0,0,s,0,0,0) \\
& + \frac{4t(s^2+2t^2+u^2)}{(s+t)(s+u)(t+u)}C_0(0,t,0,0,0,0) \\
& + \frac{4u(s^2+t^2+2u^2)}{(s+t)(s+u)(t+u)}C_0(0,u,0,0,0,0) \\
& - \frac{4(s^2+2t^2+u^2)}{(s+t)(t+u)}C_0(4m^2,0,t,0,0,0) \\
& - \frac{4(s^2+t^2+2u^2)}{(s+u)(t+u)}C_0(4m^2,0,u,0,0,0) \\
& - \frac{4(2s^2+t^2+u^2)}{(s+t)(s+u)}C_0(4m^2,s,0,0,0,0) \\
& - \frac{4st(s^2+t^2)}{(s+t)(s+u)(t+u)}D_0(4m^2,0,0,0,t,s,0,0,0,0) \\
& - \frac{4tu(t^2+u^2)}{(s+t)(s+u)(t+u)}D_0(4m^2,0,0,0,t,u,0,0,0,0) \\
& - \frac{4su(s^2+u^2)}{(s+t)(s+u)(t+u)}D_0(4m^2,0,0,0,u,s,0,0,0,0)\Bigg\}
\end{aligned}$$

Real Corrections

- Gluon emission:

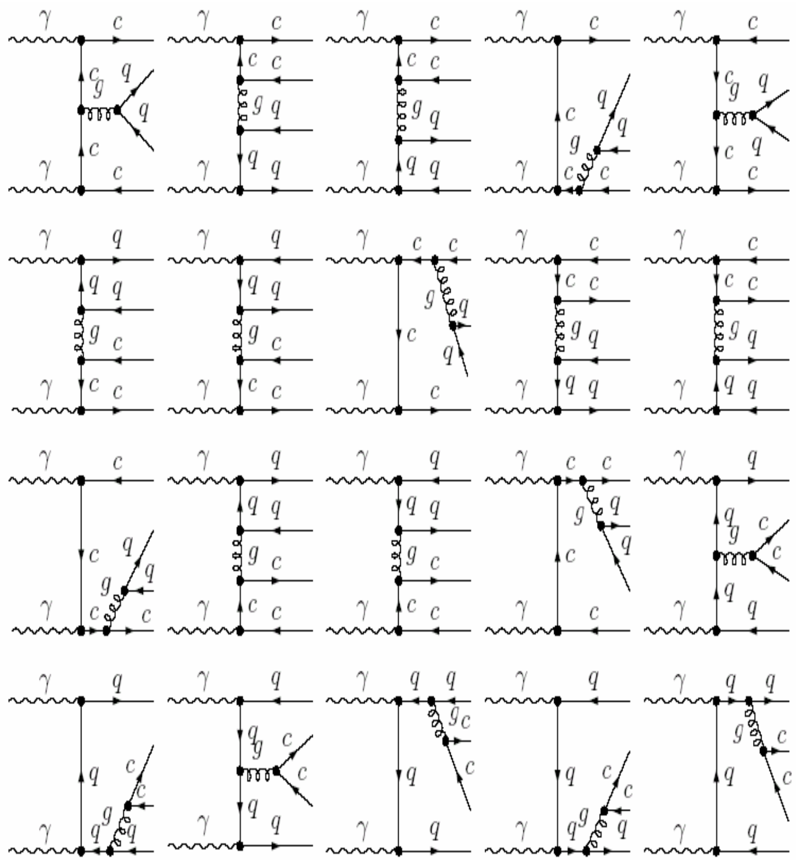


(a)



(b)

- Light quark emission:



Finite NLO Cross Section

- Cross section:

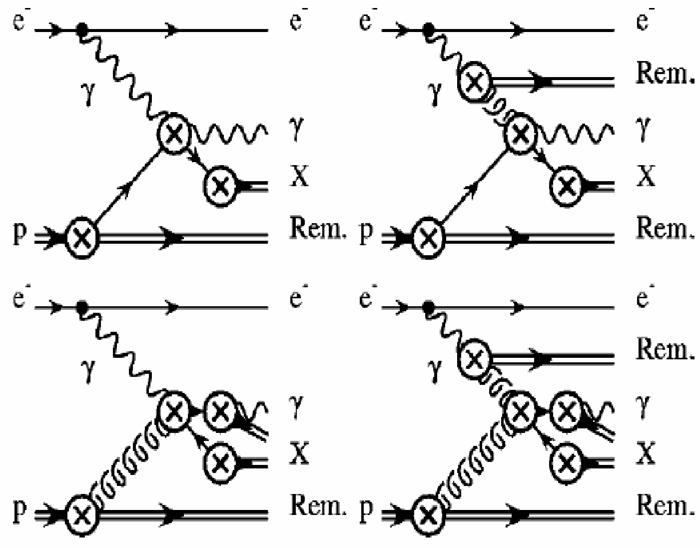
$$
\begin{aligned}
d\sigma(\mu,\lambda,M) = {}& d\sigma_0(\mu,\lambda)[1+\delta_{\rm vi}(\mu;\epsilon_{\rm UV},\epsilon_{\rm IR},v)+\delta_{\rm ct}(\mu;\epsilon_{\rm UV},\epsilon_{\rm IR})+\delta_{\rm op}(\mu,\lambda;\epsilon_{\rm IR},v)\\
&+\delta_{\rm fs}(\mu;\epsilon_{\rm IR},\delta_f)]+d\sigma_{\rm is}(\mu,\lambda,M;\delta_i)+d\sigma_{\rm so}(\mu,\lambda;\epsilon_{\rm IR},\delta_f)\\
&+d\sigma_{\rm ha}(\mu,\lambda;\delta_i,\delta_f),
\end{aligned}
$$

- Singularity cancellation:

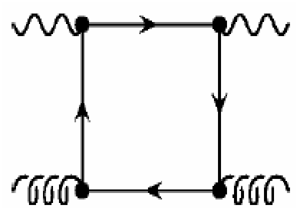
Subprocess	Source	IR-singular term
$\gamma\gamma\to c\bar{c}\left[{}^3S_1^{(8)}\right]+g$	virtual	$\frac{\alpha_s}{\pi}\left(\frac{4\pi\mu^2}{m^2}\right)^\epsilon \exp(-\epsilon\gamma_E)\left[-\frac{C_A}{\epsilon}\left(\frac{1}{2\epsilon}-\ln\frac{s_1}{2m^2}+\frac{17}{12}\right)+\frac{T_F n_f}{3\epsilon}+\left(C_F-\frac{C_A}{2}\right)\frac{\pi^2}{2v}\right]\overline{\lvert\mathcal{T}_0\rvert^2}$
	operator	$-\left(C_F-\frac{C_A}{2}\right)\frac{\pi\alpha_s}{2v}\left\langle\mathcal{O}\left[{}^3S_1^{(8)}\right]\right\rangle$
$\gamma\gamma\to c\bar{c}\left[{}^3P_J^{(1)}\right]+gg$	operator	$\frac{4C_F\alpha_s}{3\pi m^2}\left(\frac{4\pi\mu^2}{\lambda^2}\right)^\epsilon \exp(-\epsilon\gamma_E)\frac{1}{\epsilon}\left\langle\mathcal{O}\left[{}^3P_J^{(1)}\right]\right\rangle$
	soft	$-\frac{4C_F\alpha_s}{3\pi m^2}\left(\frac{4\pi\mu^2}{m^2}\right)^\epsilon \exp(-\epsilon\gamma_E)\frac{1}{\epsilon}\overline{\lvert\mathcal{T}_0\rvert^2}$
$\gamma\gamma\to c\bar{c}\left[{}^3S_1^{(8)}\right]+gg$	final-state	$\frac{C_A\alpha_s}{\pi}\left(\frac{4\pi\mu^2}{m^2}\right)^\epsilon \exp(-\epsilon\gamma_E)\frac{1}{\epsilon}\left(\ln\frac{\delta_f s}{s_1}+\frac{11}{12}\right)\overline{\lvert\mathcal{T}_0\rvert^2}$
	soft	$\frac{C_A\alpha_s}{\pi}\left(\frac{4\pi\mu^2}{m^2}\right)^\epsilon \exp(-\epsilon\gamma_E)\frac{1}{\epsilon}\left(\frac{1}{2\epsilon}-\ln\frac{\delta_f s}{2m^2}+\frac{1}{2}\right)\overline{\lvert\mathcal{T}_0\rvert^2}$
$\gamma\gamma\to c\bar{c}[{}^3P_J^{(8)}]+gg$	operator	$\frac{4B_F\alpha_s}{3\pi m^2}\left(\frac{4\pi\mu^2}{\lambda^2}\right)^\epsilon \exp(-\epsilon\gamma_E)\frac{1}{\epsilon}\left\langle\mathcal{O}\left[{}^3P_J^{(8)}\right]\right\rangle$
	soft	$-\frac{4B_F\alpha_s}{3\pi m^2}\left(\frac{4\pi\mu^2}{m^2}\right)^\epsilon \exp(-\epsilon\gamma_E)\frac{1}{\epsilon}\overline{\lvert\mathcal{T}_0\rvert^2}$
$\gamma\gamma\to c\bar{c}\left[{}^3S_1^{(8)}\right]+q\bar{q}$	final-state	$-\frac{T_F n_f\alpha_s}{3\pi}\left(\frac{4\pi\mu^2}{m^2}\right)^\epsilon \exp(-\epsilon\gamma_E)\frac{1}{\epsilon}\overline{\lvert\mathcal{T}_0\rvert^2}$
$\gamma\gamma\to c\bar{c}[n]+q\bar{q}$	initial-state	$-\left(\frac{4\pi\mu^2}{(1-z)\delta_i s}\right)^\epsilon\frac{1}{\epsilon}P_{\gamma\to q}(z)T_{\gamma q\to c\bar{c}q}[n]$
$\gamma q\to c\bar{c}[n]+q$	mass fact.	$\left(\frac{4\pi\mu^2}{M^2}\right)^\epsilon\frac{1}{\epsilon}P_{\gamma\to q}(z)T_{\gamma q\to c\bar{c}q}[n]$

Prompt-Photon Production

- QCD factorization theorem:



- Quark-box contribution:
  - NNLO
  - But important numerically



- Hadronic cross section:

$$\frac{d^2\sigma}{dp_T^2 dy} = \sum_{a,b,c} \int dx_a dx_b \frac{dz}{z^2} f_{a/A}(x_a, M_a^2) f_{b/B}(x_b, M_b^2) \times D_{H/c}(z, M_c^2) \frac{d\sigma}{dt},$$

- Partonic cross sections:
  - PHOX NLO programs
- Importance:
  - Higgs background

Fragmentation and Isolation

- Splitting functions:

$$P_{\gamma\leftarrow g}(x)=\frac{\alpha_s(Q^2)}{2\pi}\frac{e_q^2T_R}{2}\Bigg[-4+12x-\frac{164}{9}x^2+\frac{92}{9x}+\left(10+14x+\frac{16}{3}x^2+\frac{16}{3x}\right)\ln x+2(1+x)\ln^2 x\Bigg]$$

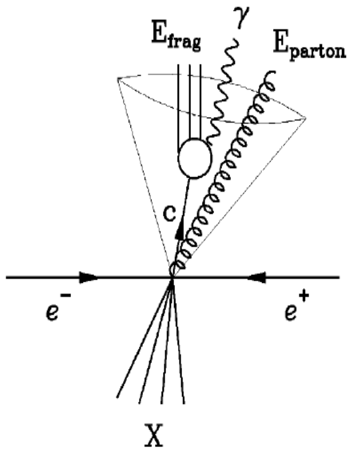
- Evolution equations:

$$\frac{dD_{\gamma/q}(Q^2)}{d\ln Q^2}=\frac{\alpha}{2\pi}P_{\gamma\leftarrow q}\otimes D_{\gamma/\gamma}(Q^2)+\frac{\alpha_s(Q^2)}{2\pi}\times[P_{q\leftarrow q}\otimes D_{\gamma/q}(Q^2)+P_{g\leftarrow q}\otimes D_{\gamma/g}(Q^2)]$$

- Fit to e^+e^- cross sections:

$$\frac{1}{\sigma_0}\frac{d\sigma(Q^2)}{dx}=\sum_q 2e_q^2\left(D_{\gamma/q}(Q^2)+\frac{\alpha}{2\pi}e_q^2C_\gamma^T+\frac{\alpha_s(Q^2)}{2\pi}[C_q^T\otimes D_{\gamma/q}(Q^2)+C_g^T\otimes D_{\gamma/g}(Q^2)]\right)$$

- Hadronic activity in cone:



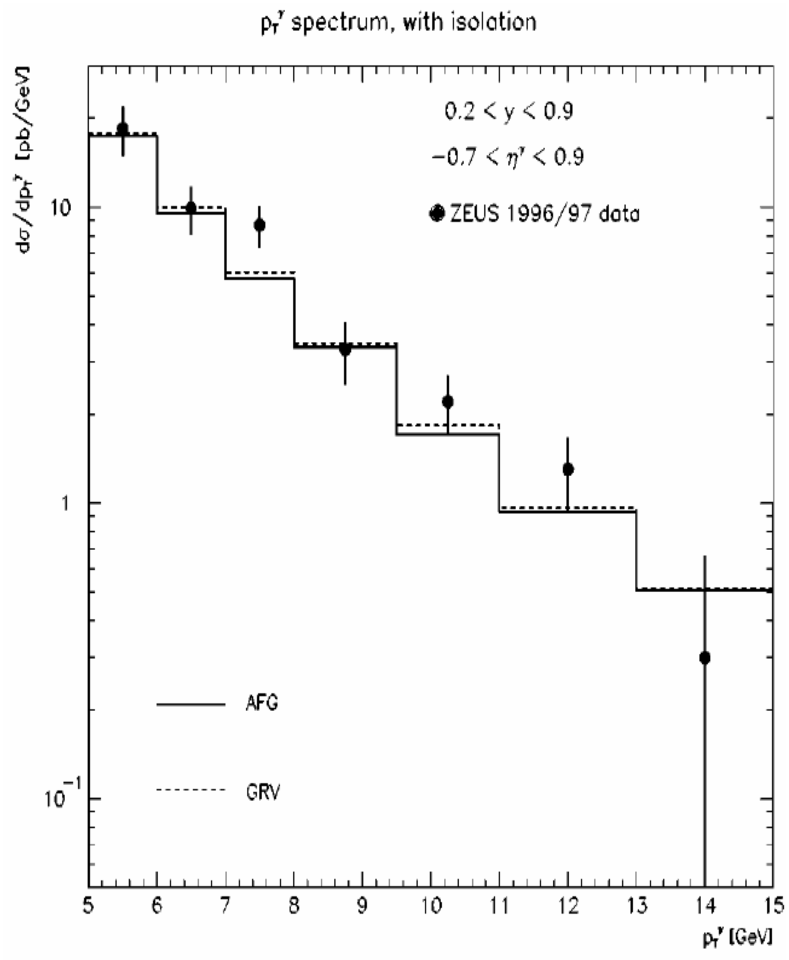
- Isolation criterion:

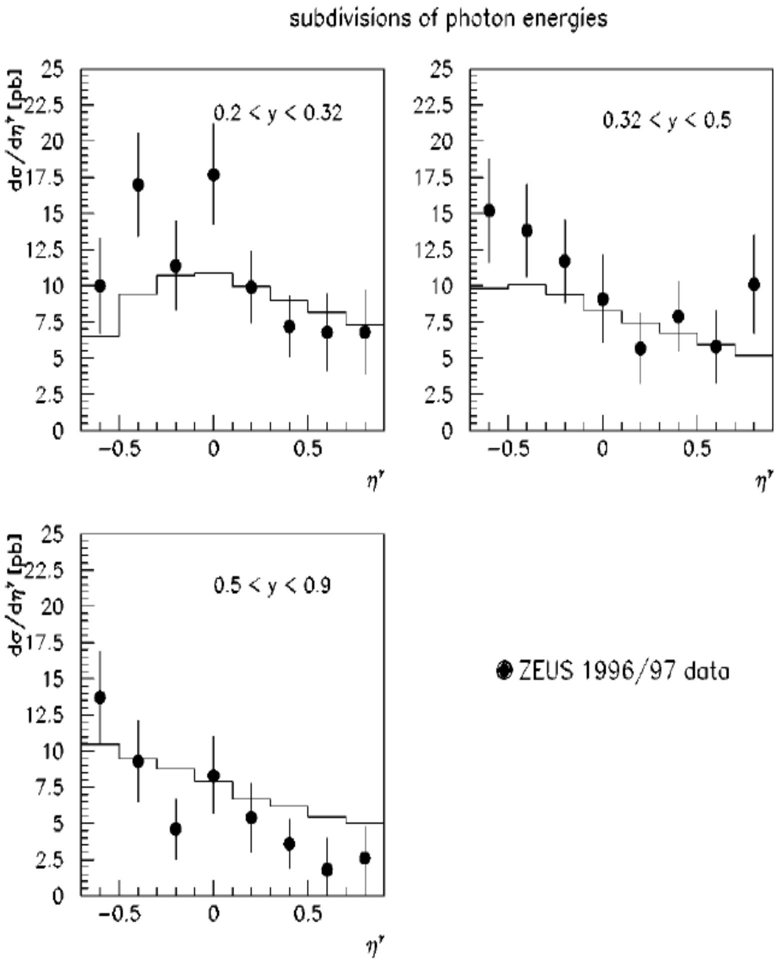
$$E_{(T)}^{\rm had}<\epsilon_{(T)}E_{(T),\gamma}$$

- IR-safe definition:

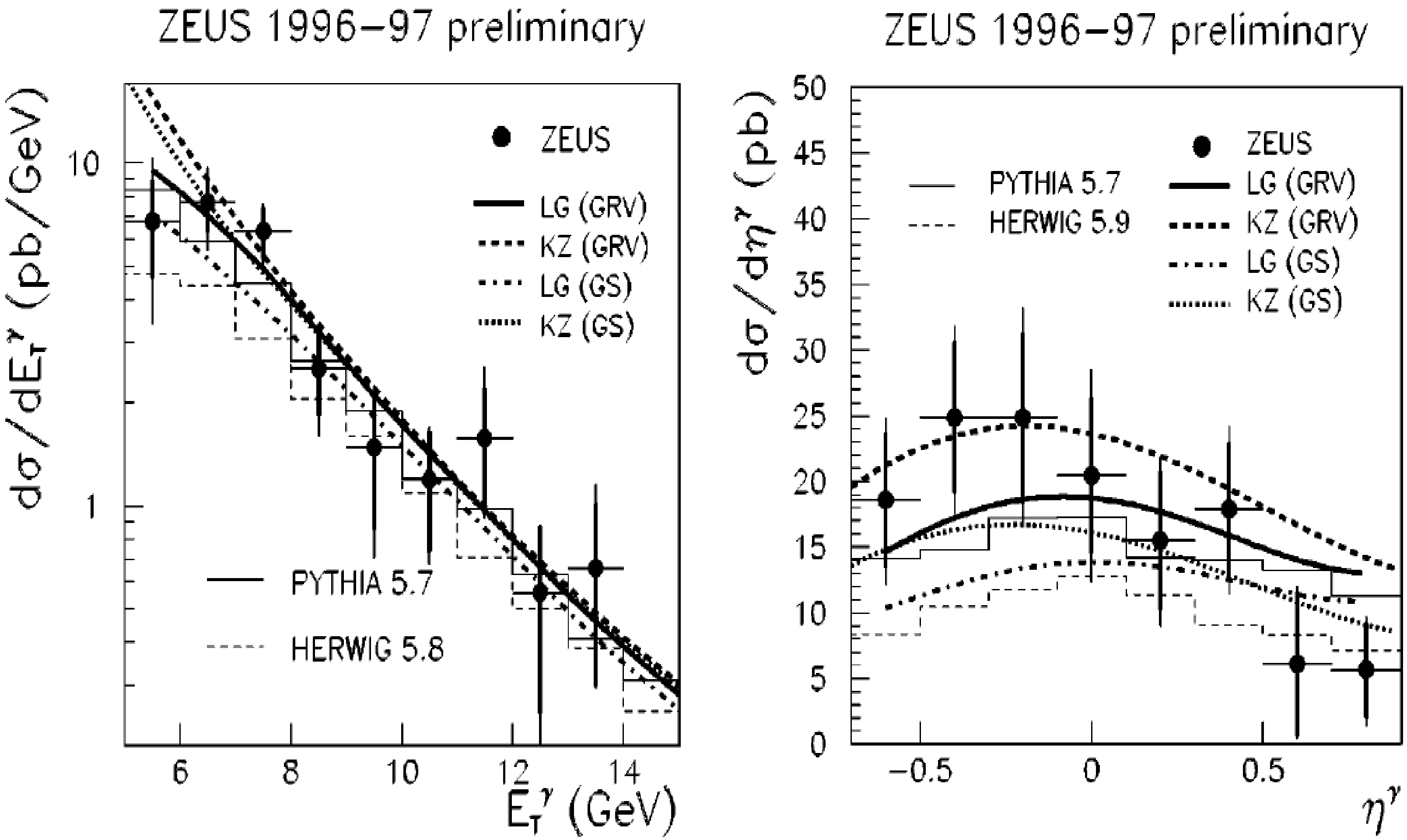
$$\sum_i E_{(T),i}^{\rm had}\theta(\delta-R_i)<\epsilon_{(T)}E_{(T),\gamma}\left(\frac{1-\cos\delta}{1-\cos\delta_0}\right)$$

Inclusive Photons





Photons with Jets



Related Topics

- Virtual photons
- Polarized photons
- Prompt photons in hadron collisions

Virtual Photons

- Transverse photon flux:

$$\frac{df_{\gamma/l}^{\rm brems}}{dP^2}(x,P^2)=\frac{\alpha}{2\pi}\left[\frac{1+(1-x)^2}{x}\frac{1}{P^2}-\frac{2m_l^2x}{P^4}\right]$$

- Longitudinal photon flux:

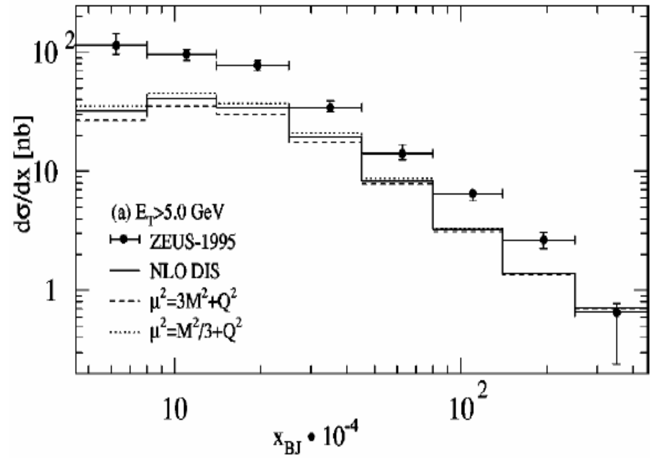
$$\frac{df_{\gamma/l}^{\rm brems}}{dP^2}(x,P^2)=\frac{\alpha}{2\pi}\left[\frac{2(1-x)}{x}\frac{1}{P^2}\right]$$

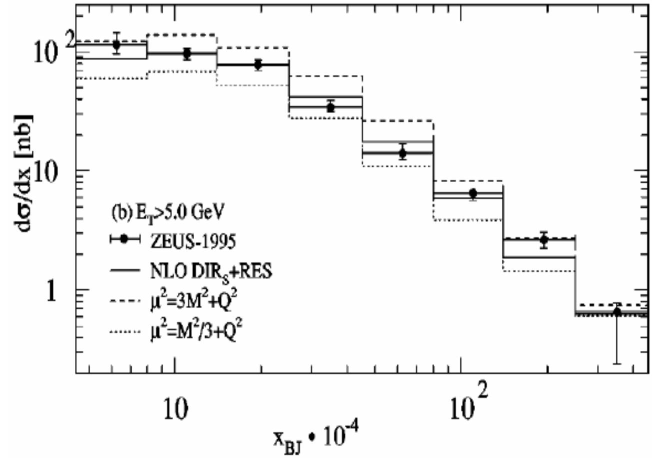
- Pert. boundary cond. $P^2=Q^2$:

$$f_{q/\gamma}^{\rm box}(x,Q^2,P^2)=3e_q^2\frac{\alpha}{2\pi}\left([x^2+(1-x)^2]\ln\frac{Q^2}{x^2P^2}+6x(1-x)-2\right).$$

- GRV interpolation: $\eta(P^2)=(1+P^2/m_\rho^2)^{-2}$

$$f_{i/\gamma}(x,\max(P^2,Q_0^2),P^2)=\eta(P^2)f_{i/\gamma}^{\rm had}(x,\max(P^2,Q_0^2))+[1-\eta(P^2)]f_{i/\gamma}^{\rm box}(x,\max(P^2,Q_0^2),P^2)$$





Polarized Photons

- Polarized photon flux:

$$\Delta f_{\gamma/l}^{\mathrm{brems}}(x)=\frac{2\lambda_l\alpha}{2\pi}\left[\frac{1-(1-x)^2}{x}\ln\frac{Q_{\max}^2(1-x)}{m_l^2x^2}+2m_l^2x^2\left(\frac{1}{Q_{\max}^2}-\frac{1-x}{m_l^2x^2}\right)\right]$$

- Polarized splitting functions:

$$\Delta P_{q\leftarrow q}(x)=C_F\left(\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\delta(1-x)\right)+\mathcal{O}(\alpha_s)$$

$$=P_{q\leftarrow q}(x),$$

$$\Delta P_{g\leftarrow q}(x)=C_F\left(\frac{1-(1-x)^2}{x}\right)+\mathcal{O}(\alpha_s),$$

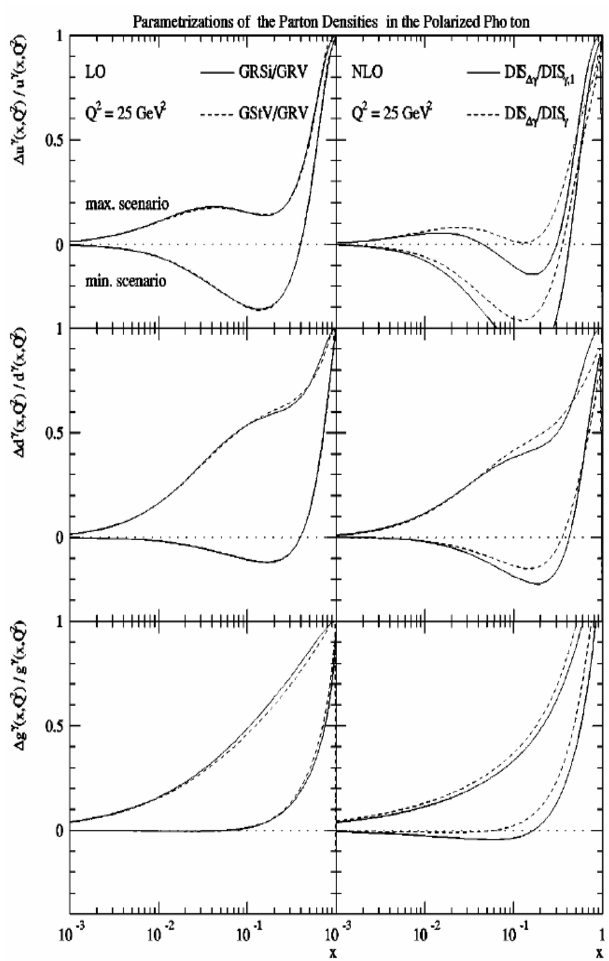
$$\Delta P_{q\leftarrow g}(x)=T_R[x^2-(1-x)^2]+\mathcal{O}(\alpha_s),$$

$$\Delta P_{g\leftarrow g}(x)=N_C\left[(1+x^4)\left(\frac{1}{x}+\frac{1}{(1-x)_+}\right)-\frac{(1-x)^3}{x}\right]$$

$$+\left(\frac{11}{6}N_C-\frac{1}{3}N_f\right)\delta(1-x)+\mathcal{O}(\alpha_s),$$

- Polarized coefficient fct.:

$$\Delta C_\gamma(x)=2N_C\Delta C_g(x)$$



Hard Photoproduction at HERA

- Strong coupling constant
- Real and virtual photon structure
- Proton, pion and pomeron structure ($\rightarrow$ Absorption!)
- Jet definition and jet shapes
- Universality of fragmentation functions
- Factorization for heavy quarks
- Color-singlet vs. color-octet quarkonium formation
- Isolation of final state photons

- Relevance for LHC-physics: Higgs, and more