



# REVIEW ON PARTON SHOWER ALGORITHMS

**ZOLTÁN NAGY**

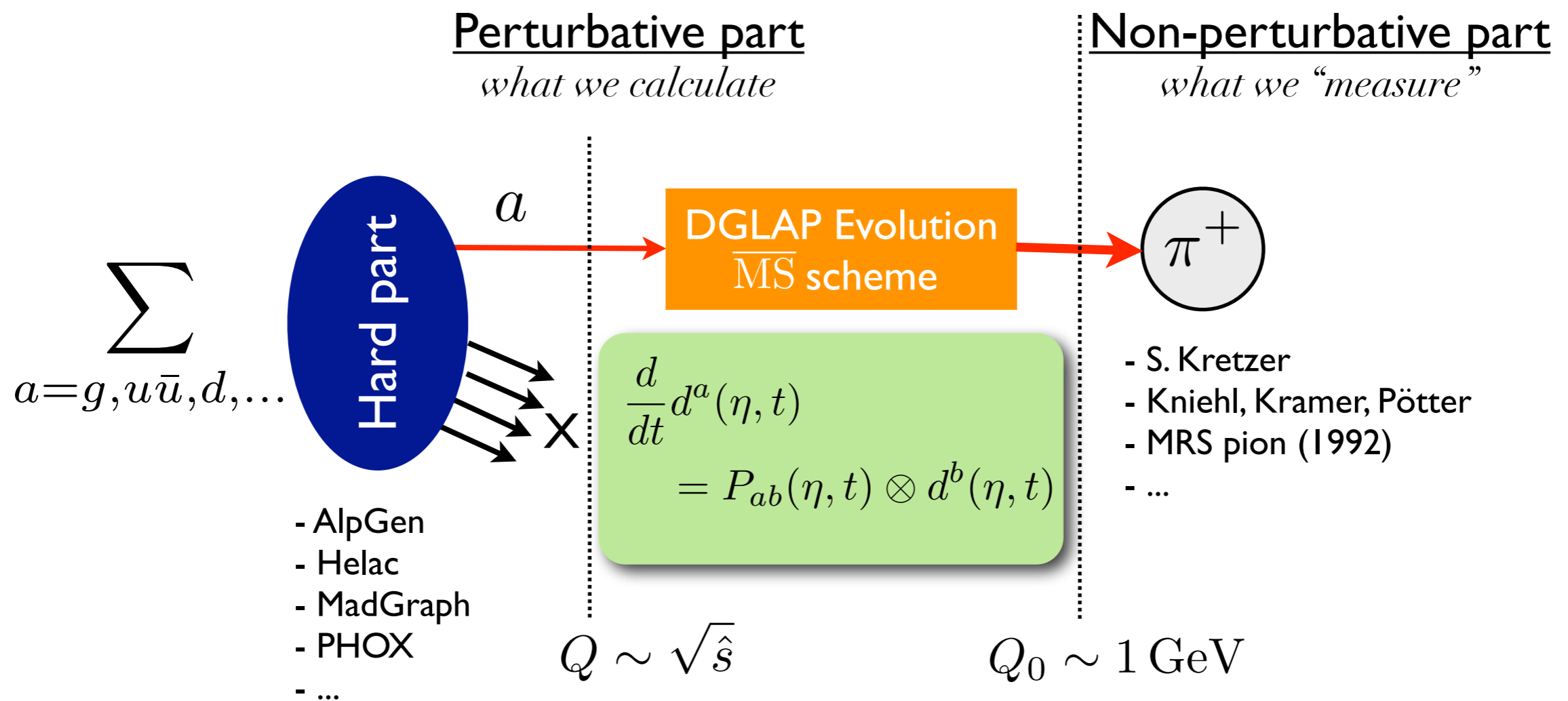
*CERN, Theoretical Physics*

Photoproduction at Collider Energies:  
from RHIC and HERA to LHC

January 15-19, 2007, Trento, Italy

# Introduction

## Structure of the one-hadron inclusive cross section



# Outline

## PART I.: Theory of the parton showers

- Parton model
- Collinear approximation
- Evolution equation
- Soft approximation, color coherence
- Initial state shower
- Color dipole model
- Matching at LO level (CKKW)
- Matching at NLO (MC@NLO)

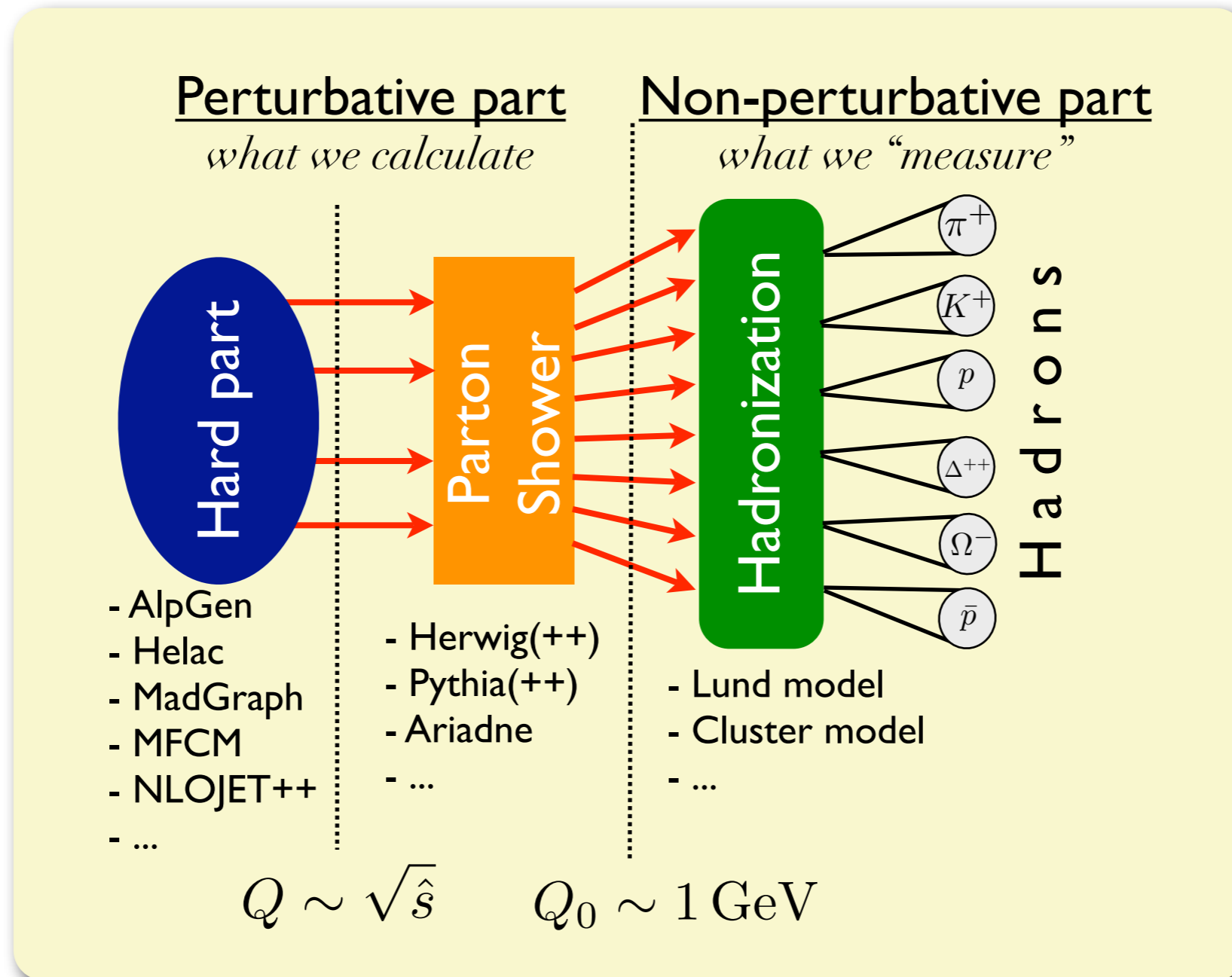
## PART II: New shower concept

*(if I have time)*

- Formalism
- Evolution, Sudakov,..
- Matching at LO and NLO level

# Introduction

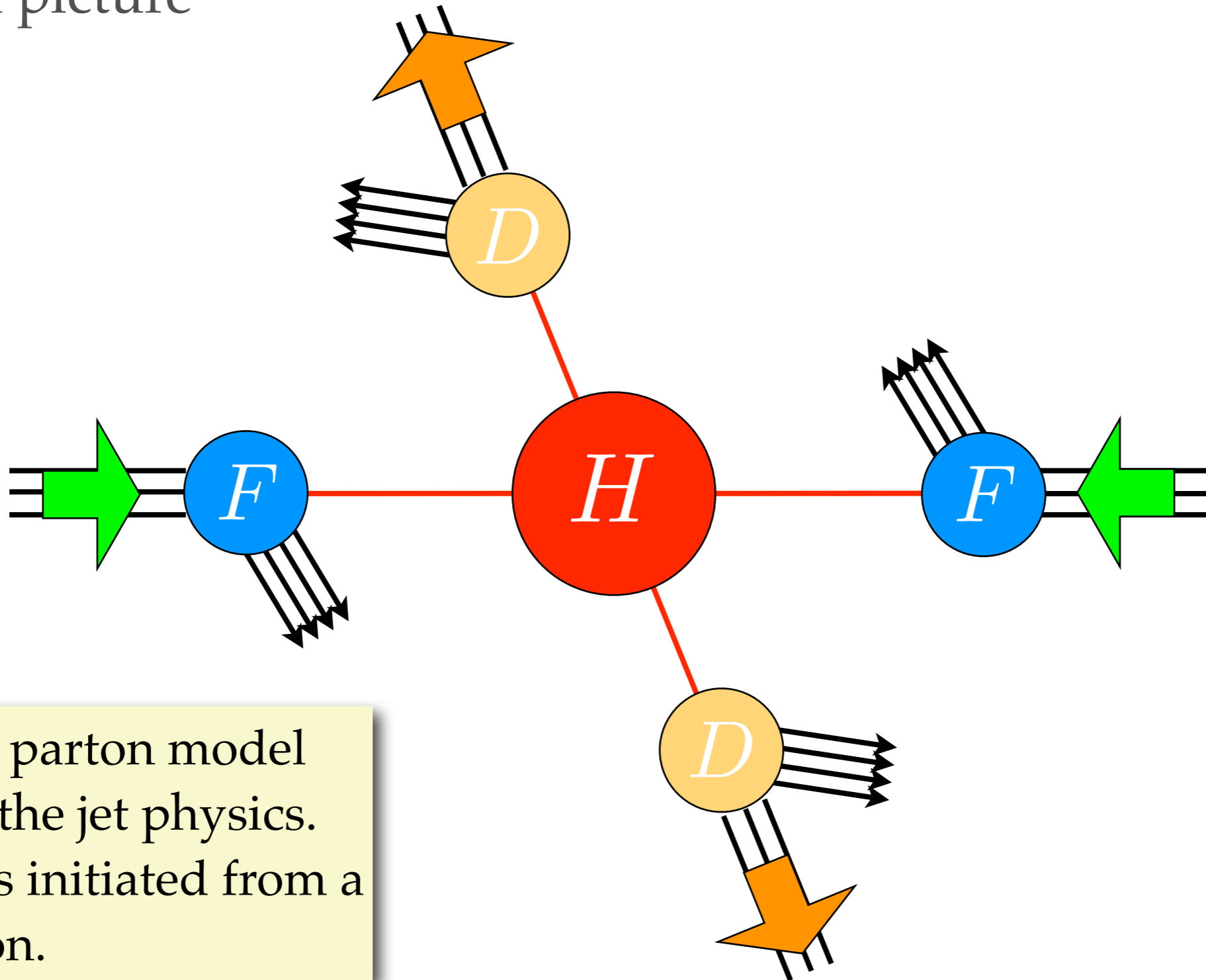
## Structure of the Monte Carlo algorithm





# Parton Model Picture

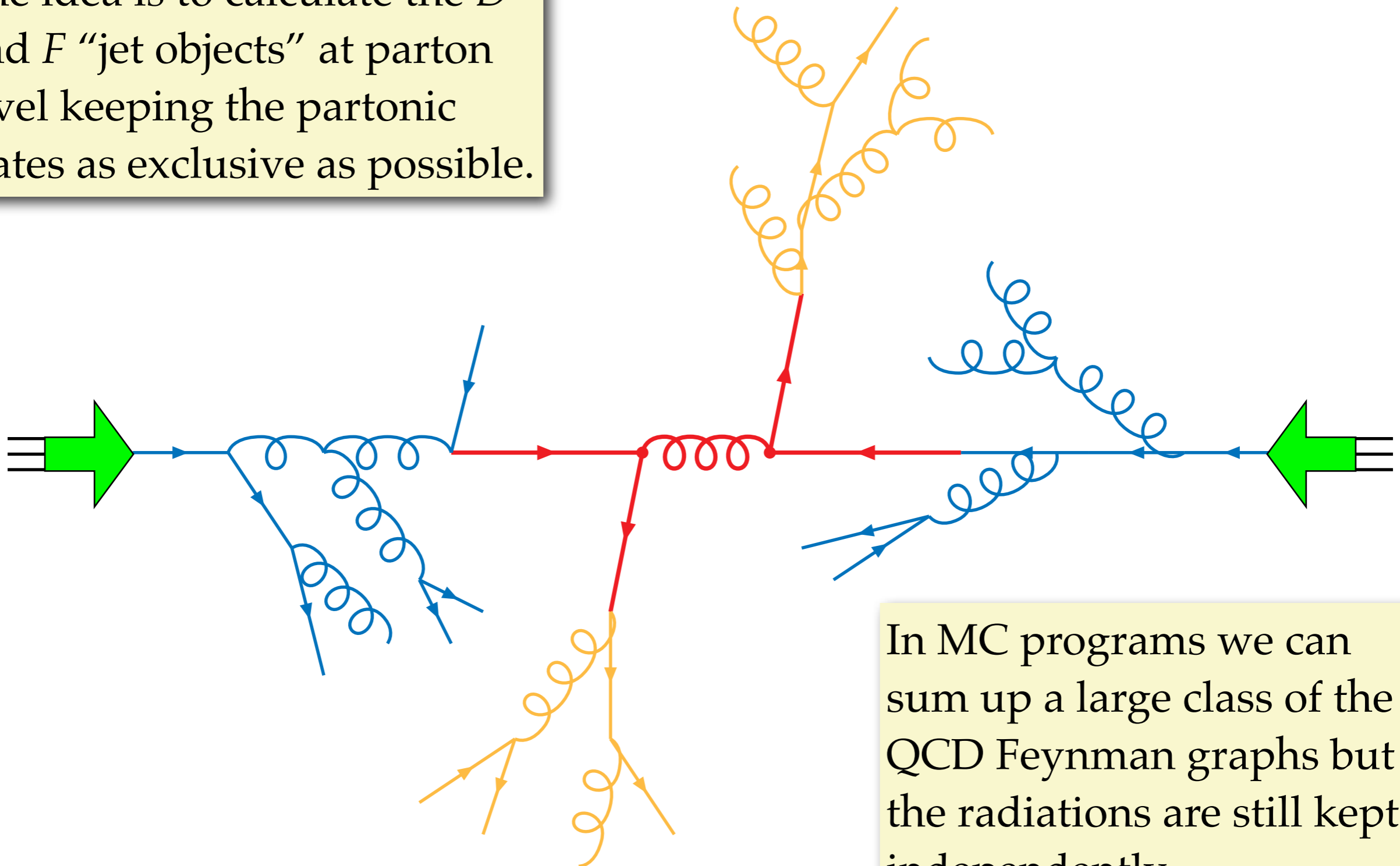
The current parton shower are based on the independent radiation picture



This is the parton model picture of the jet physics. Every jet is initiated from a hard parton.

# Parton Shower Picture

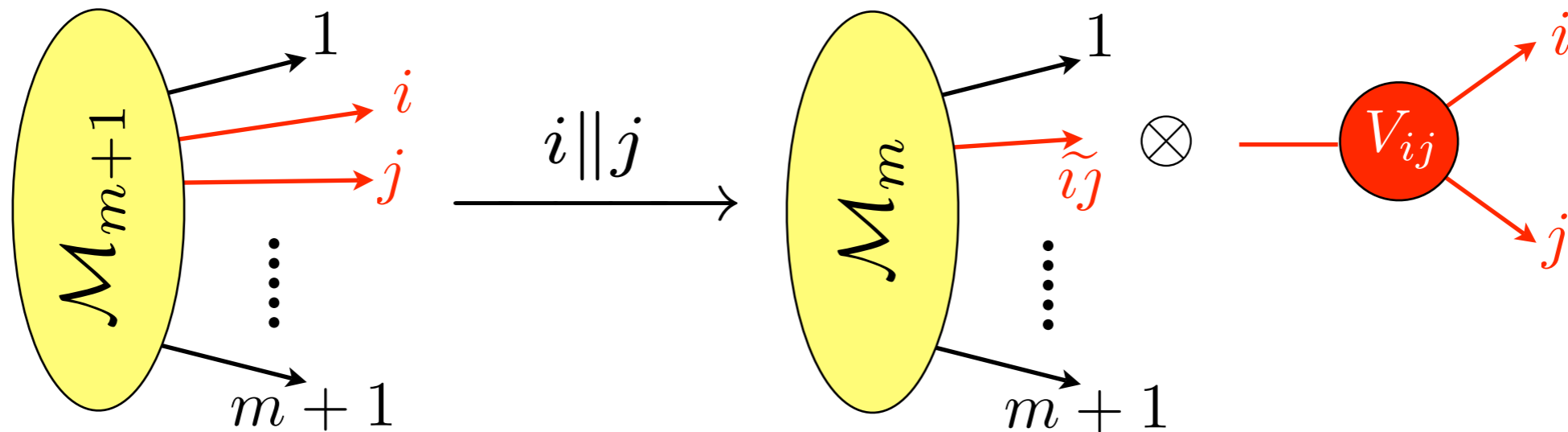
The idea is to calculate the  $D$  and  $F$  “jet objects” at parton level keeping the partonic states as exclusive as possible.



In MC programs we can sum up a large class of the QCD Feynman graphs but the radiations are still kept independently.

# Collinear Approximation

The QCD matrix elements have universal factorization property when two external partons become collinear



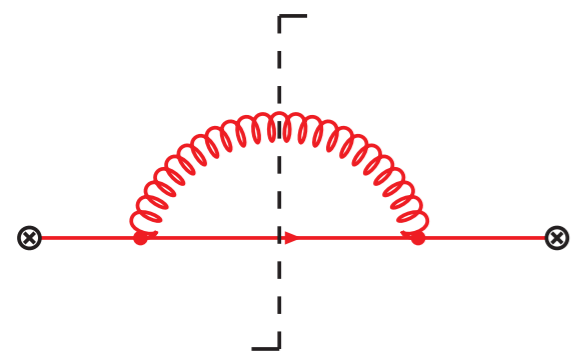
At cross section level we have

$$d\sigma_{m+1} = d\sigma_m \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} dz d\phi P(z, \phi)$$

where the  $P(z, \phi)$  is the Altarelli-Parisi splitting kernel and

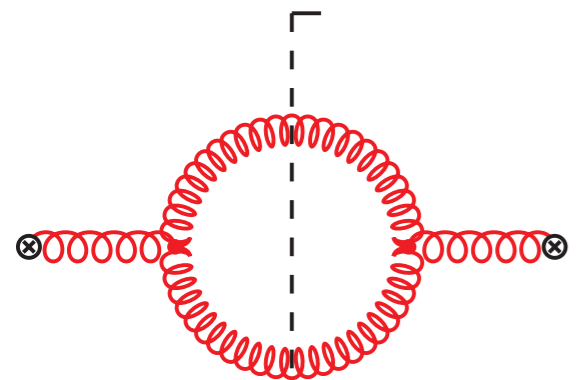
$$\frac{\alpha_s}{2\pi} \frac{1}{\theta^2} P(z, \phi) \sim \sum_{\text{colors, spins}} V_{ij}(p_i, p_j) [V_{ij}(p_i, p_j)]^*$$

# Collinear Splitting Kernels

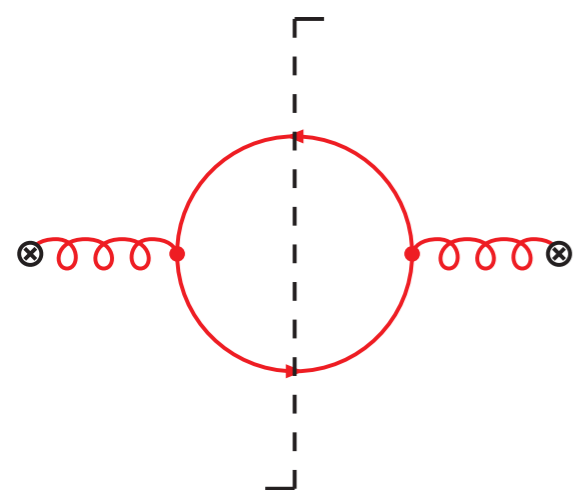


$$\sim P_{q \rightarrow qg} = C_F \frac{1+z^2}{1-z}$$

- Always diagonal in color space; no color connection between the hard partons.
- In spin space the gluon splittings are not diagonal.



$$\sim P_{g \rightarrow gg} = 2C_A \left[ -g^{\mu\nu} \left( \frac{z}{1-z} + \frac{1-z}{z} \right) - 2z(1-z) \frac{k_{\perp}^{\mu} k_{\perp}^{\nu}}{k_{\perp}^2} \right]$$

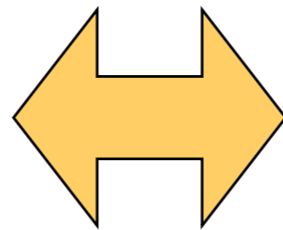


$$\sim P_{g \rightarrow q\bar{q}} = T_R \left[ -g^{\mu\nu} + 4z(1-z) \frac{k_{\perp}^{\mu} k_{\perp}^{\nu}}{k_{\perp}^2} \right]$$

# Virtual Contributions

Integrating over the phase space the cross section is singular in the collinear region. What is a parton?

Collinear parton pair



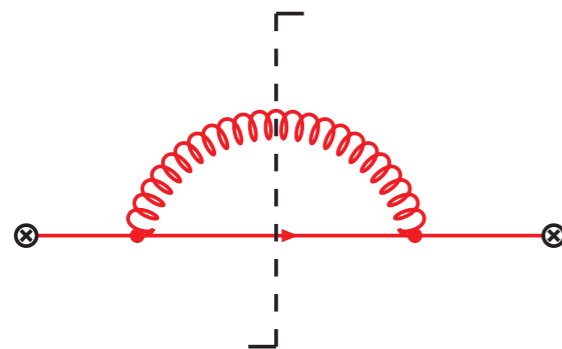
Single parton

The radiation is resolved, e.g.:

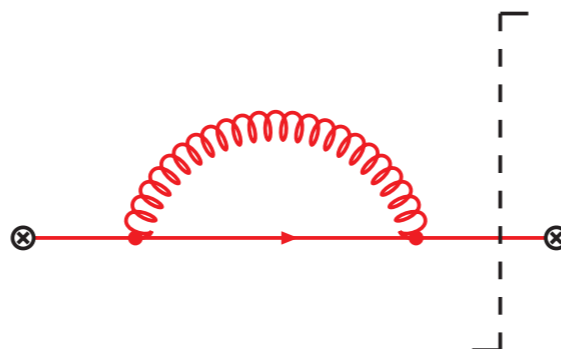
$$k_{\perp} > Q_0$$

*(any infrared sensitive variable is good)*

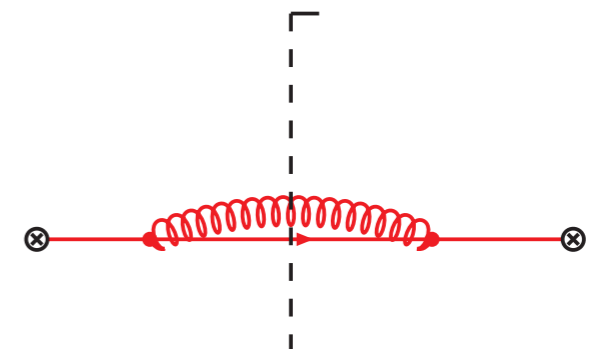
In order to cancel the IR singularities (comes from  $k_{\perp} < Q_0$ ) we have to consider the contributions of the virtual graphs



*resolvable*  
*"2 partons"*



*virtual*  
*"1 parton"*



*unresolvable*  
*"1 parton"*

# Virtual Contributions

Integrating over the phase space the cross section is singular in the collinear region

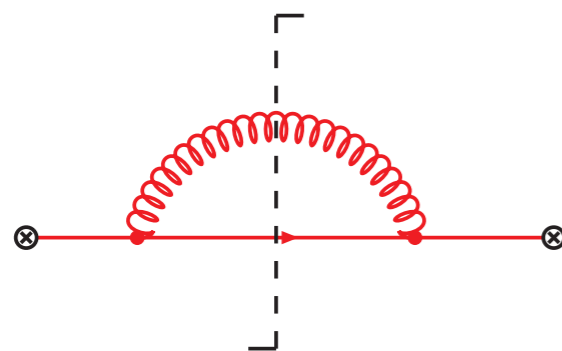
Since we don't use the exact matrix elements we have the freedom to choose the normalization.

**Unitarity:** The shower doesn't change the total cross section.

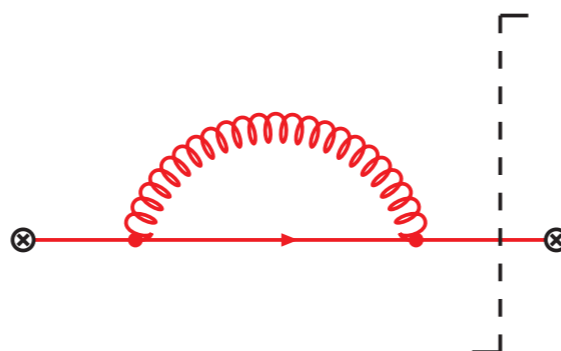
$$\text{resolvable} + [\text{virtual} + \text{unresolvable}] = 1$$

The r

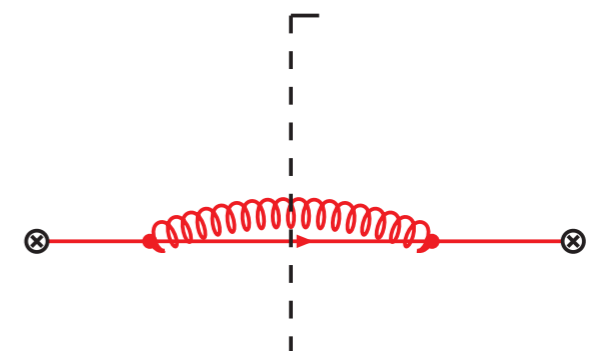
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*resolvable*  
"2 partons"



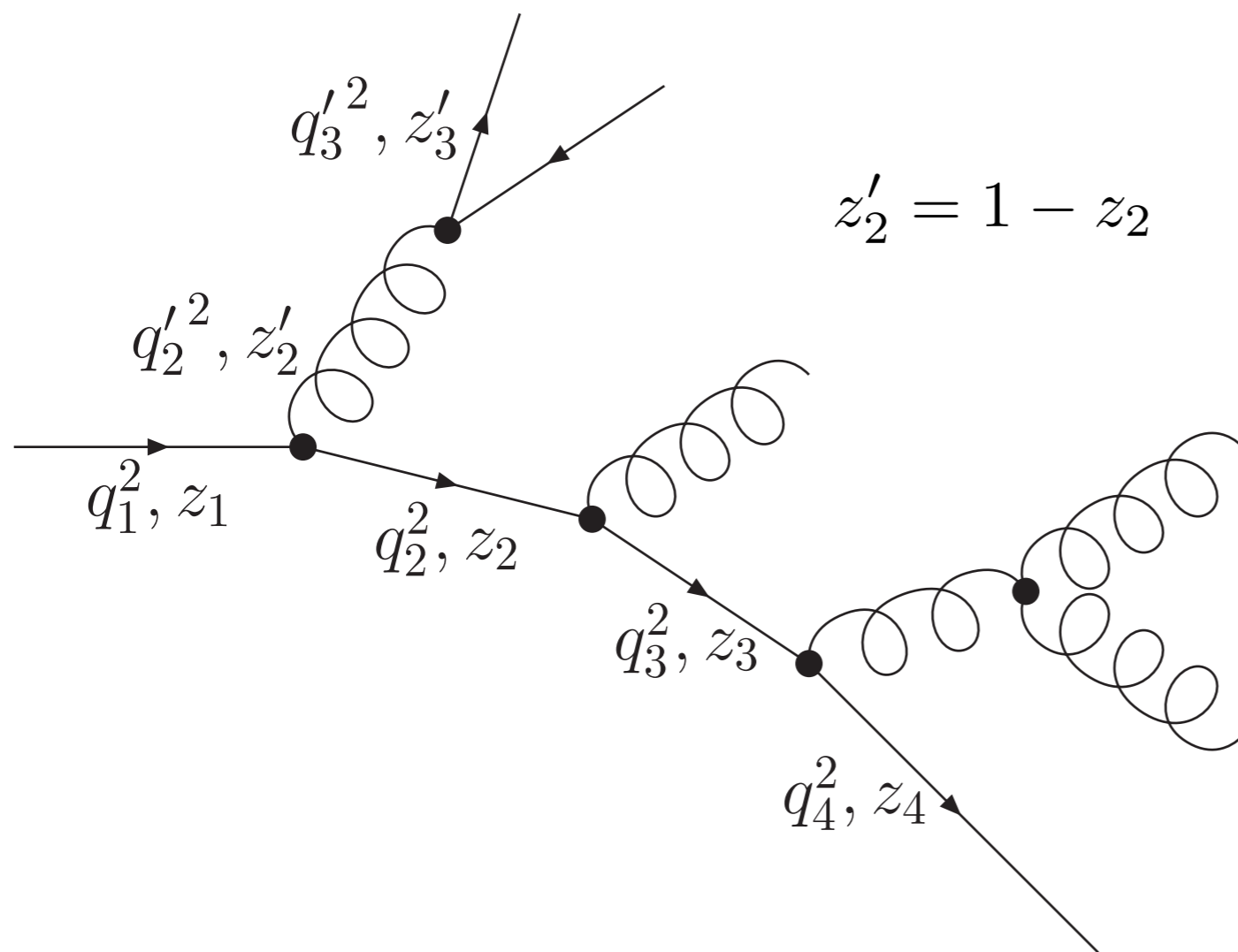
*virtual*  
"1 parton"



*unresolvable*  
"1 parton"

# Multiple Emission

To calculate the “jet object” we can apply the emission procedure based on the collinear emission successively. To avoid higher order singularities we have to introduce an ordering in a sensible variable.



$$q_1^2 > q_2^2 > q_3^2 > q_4^2 > \dots$$

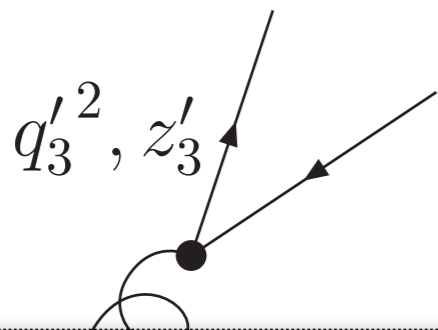
$$q_1'^2 > q_2'^2 > q_3'^2 > \dots$$

The starting condition of the evolution is fixed by the hard part and it is rather process dependent.

$$D(Q^2) = \Delta(Q^2, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \int_{\epsilon}^{1-\epsilon} dz \Delta(Q^2, q^2) \frac{\alpha_s}{2\pi} \bar{P}(z) D(zq^2) D((1-z)q^2)$$

# Multiple Emission

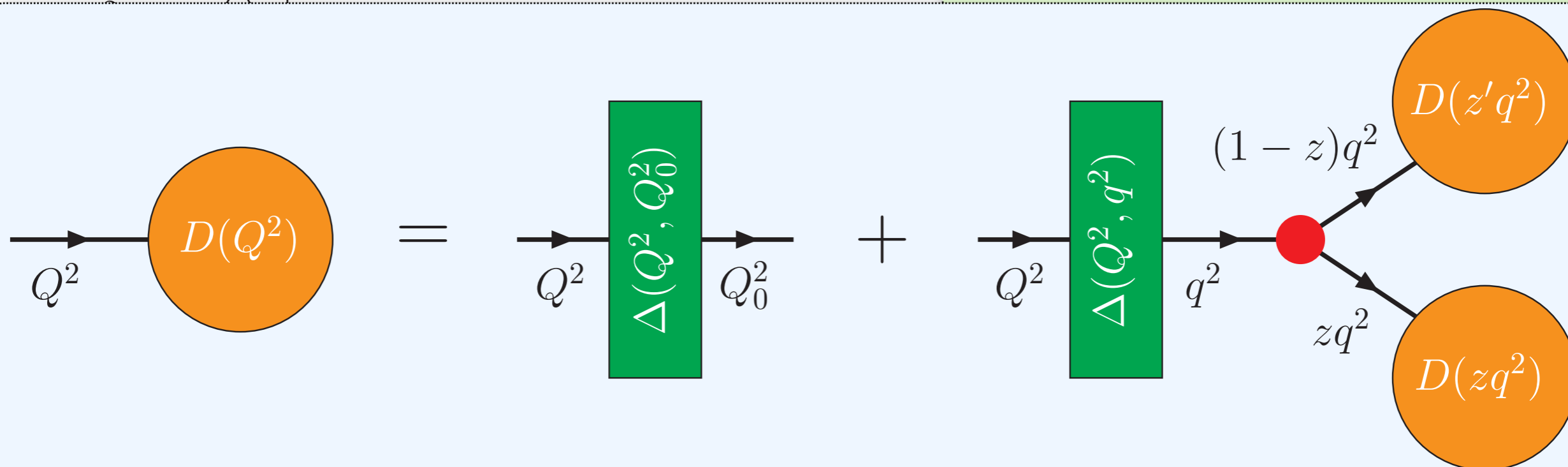
To calculate the “jet object” we can apply the emission procedure based on the collinear emission successively. To avoid higher order singularities we have to introduce an ordering in a sensible variable.



$$z_2' = 1 - z_2$$

$$q_1^2 > q_2^2 > q_3^2 > q_4^2 > \dots$$

$$q_1'^2 > q_2'^2 > q_3'^2 > \dots$$



$$D(Q^2) = \Delta(Q^2, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \int_{\epsilon}^{1-\epsilon} dz \Delta(Q^2, q^2) \frac{\alpha_s}{2\pi} \bar{P}(z) D(zq^2) D((1-z)q^2)$$



# Sudakov Form Factor

From the normalization condition we have

$$1 = \Delta(Q^2, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \int_{\epsilon}^{1-\epsilon} dz \Delta(Q^2, q^2) \frac{\alpha_s}{2\pi} \bar{P}(z)$$

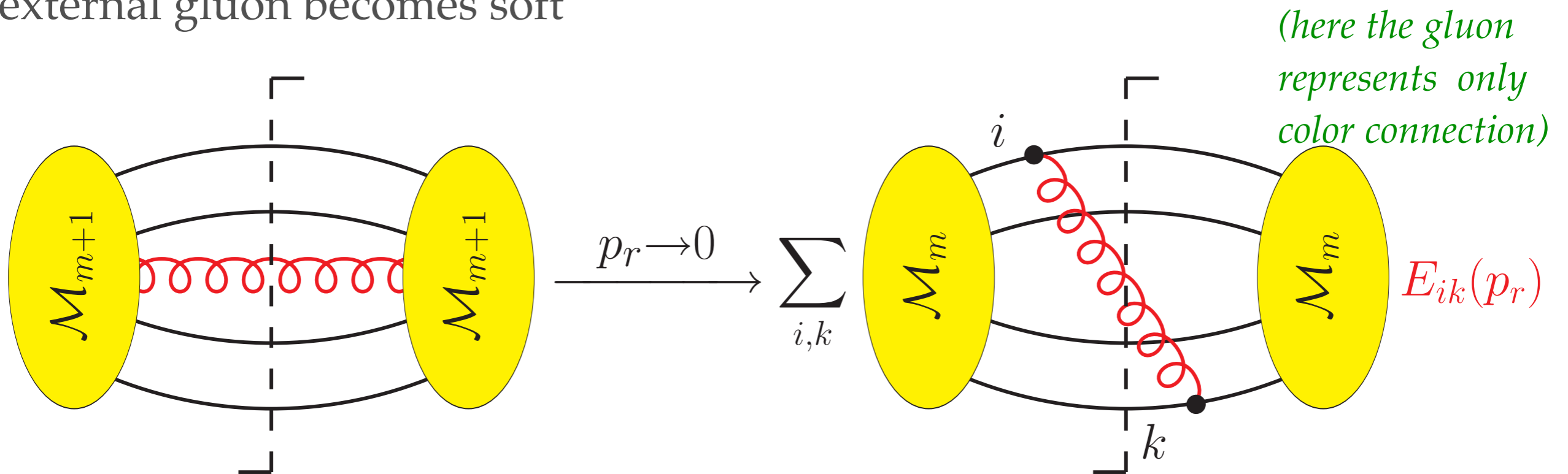
and solving it for  $\Delta(Q^2, Q_0^2)$ , we have

$$\Delta(Q^2, Q_0^2) = \exp \left( - \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \int_{\epsilon}^{1-\epsilon} dz \frac{\alpha_s}{2\pi} \bar{P}(z) \right)$$

Sudakov factor is the probability of emitting no resolvable parton; probability of “nothing happen”

# Soft Approximation

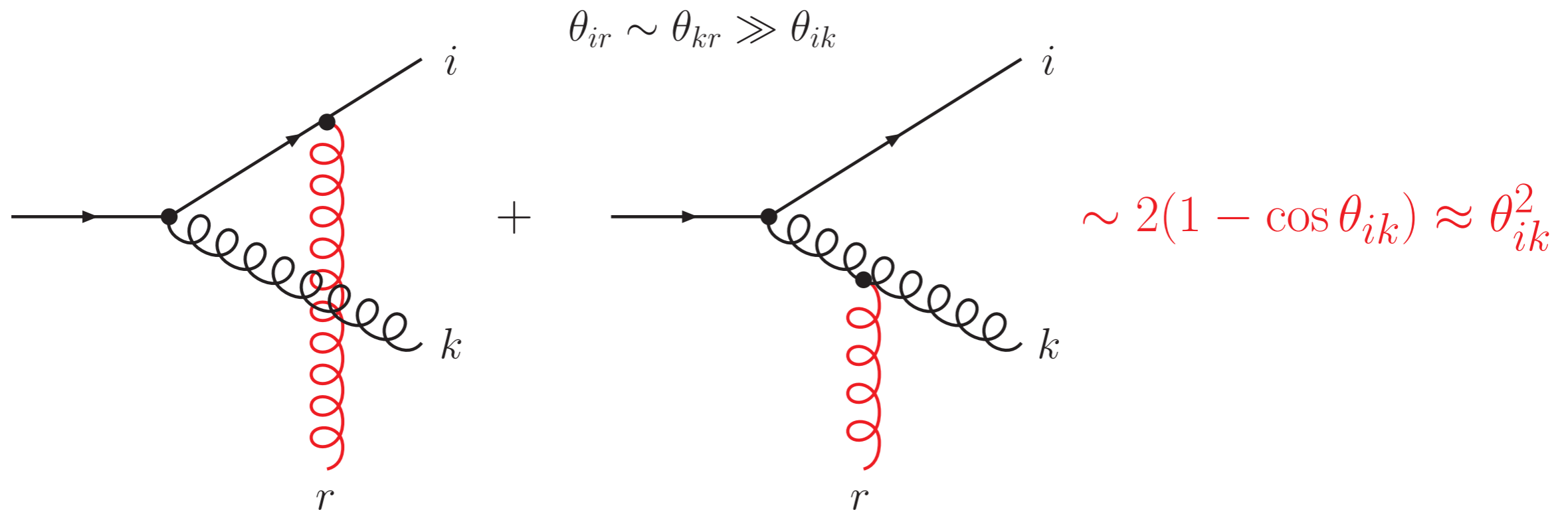
The QCD matrix elements have universal factorization property when an external gluon becomes soft



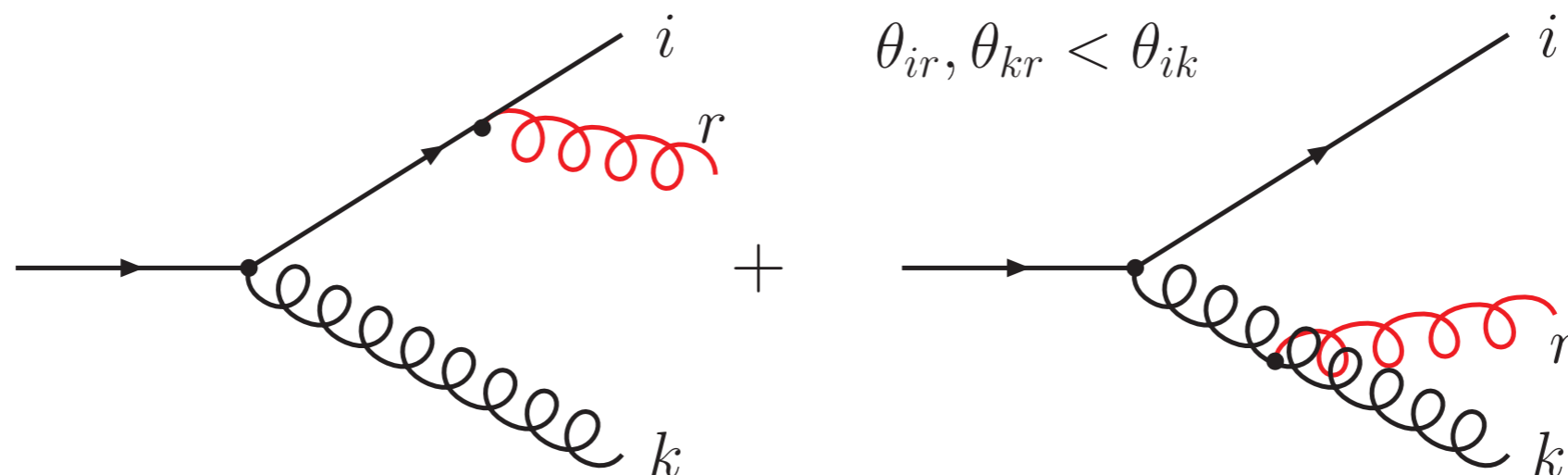
- Soft contributions produces next-to-leading logarithms.
- No spin correlation.
- Soft gluon connects everywhere and the color structure is not diagonal; quantum interferences.
- Does it spoil the independent evolution picture?

# Color Coherence

The pure soft part of the soft gluon contribution is finite in the large angle limit



Basically only the soft-collinear part contributes in the soft limit and it comes from the small angle radiations. If  $i$  and  $k$  are close to each other then the color structure is approximately diagonal.



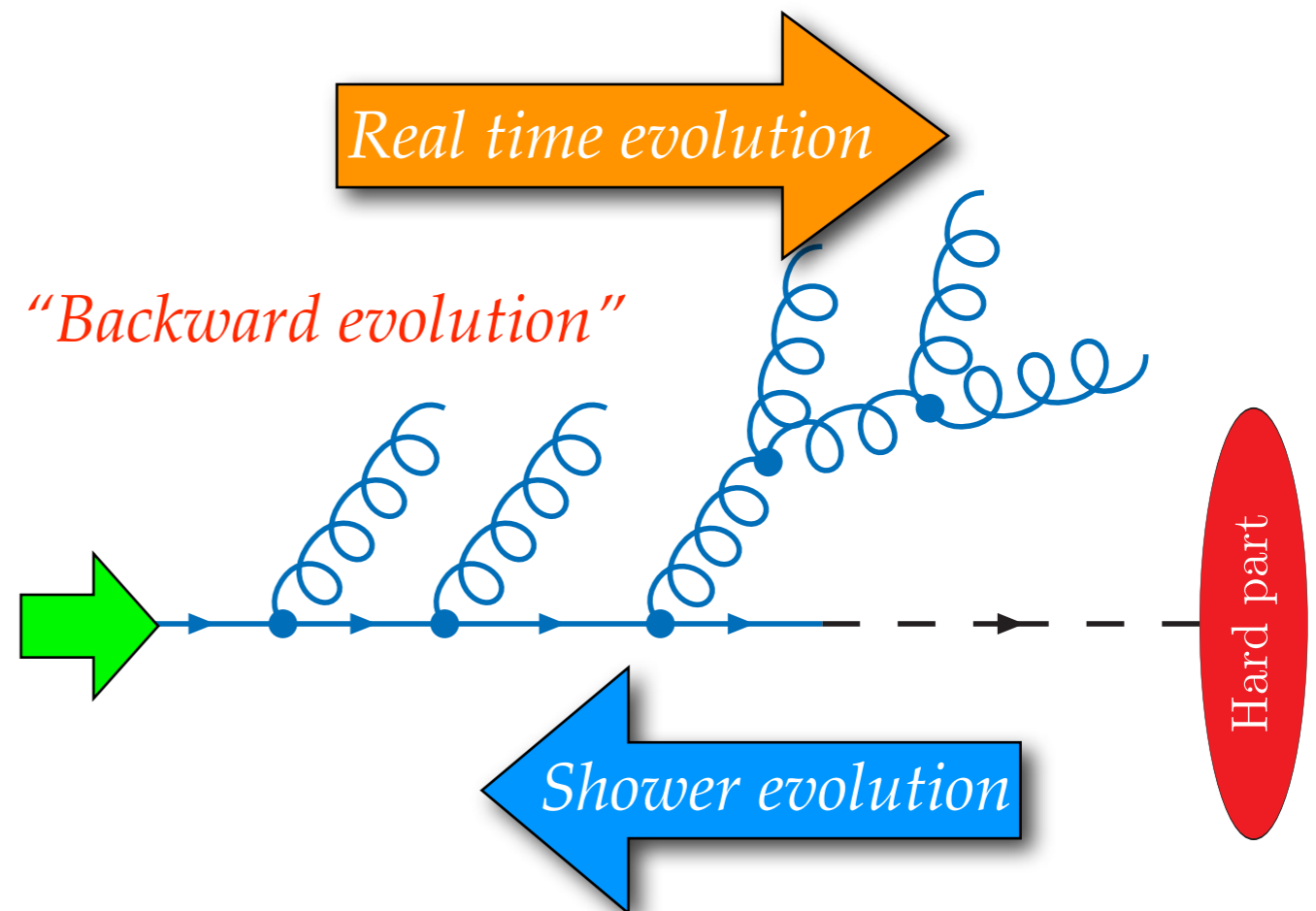
# Angular Ordering

- Soft gluon effect can be considered by using the emission angle as evolution variable.
- Remember, this is just an approximation and doesn't count properly the soft gluon effect.
- Since the emissions are ordered by the angle the first emission is not necessarily the hardest.
- The angular ordered shower is implemented in HERWIG and HERWIG++.
- The old PYTHIA is ordered by the virtuality and the color coherence is ensured by angular vetoing.
- Virtuality ordering with angular veto is not equivalent to angular ordering.

# Initial State Shower

The initial state shower is basically identical to the final state shower. In practice there is an important difference.

- The approach is based on the DGLAP equation.
- The shower is basically its exclusive version.
- The soft gluon can be considered by angular ordering (or angular veto)



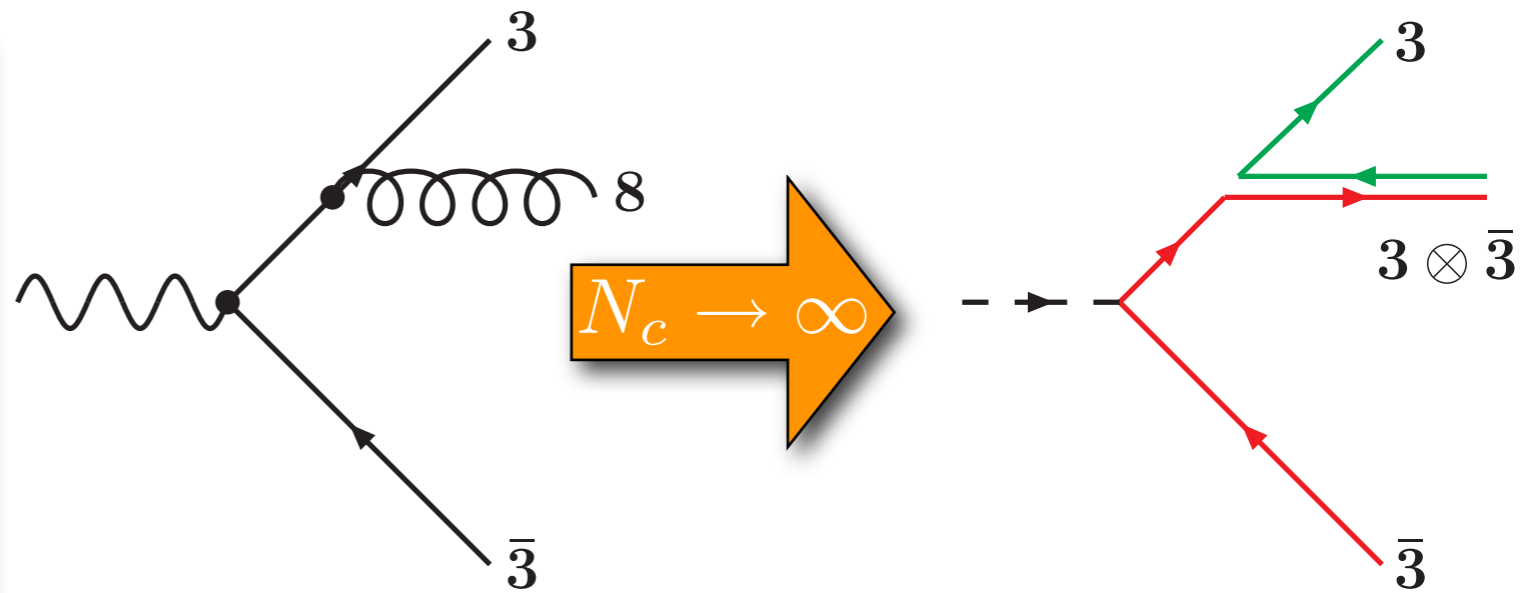
$$\Pi(Q^2, Q_0^2; \eta) = \exp\left(-\int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \int_{\eta}^{1-\epsilon} \frac{dx}{x} \frac{\alpha_s}{2\pi} \bar{P}(x) \frac{f(\eta/x, q^2)}{f(\eta, q^2)}\right)$$

$$F(Q^2, \eta) = \Pi(Q^2, Q_0^2; \eta) + \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \int_{\eta}^{1-\epsilon} \frac{dx}{x} F(xq^2, x) D((1-x)q^2) \frac{\alpha_s}{2\pi} \bar{P}(x) \frac{f(\eta/x, q^2)}{f(\eta, q^2)} \Pi(Q^2, q^2; x)$$

# Color Dipole Evolution

There is an alternative way to define shower. Rather than iterating  $1 \rightarrow 2$  splitting the gluons are emitted from the color dipoles between the color connected partons resulting  $2 \rightarrow 3$  splittings.

- ✓ Since the gluons are emitted coherently from the dipoles no need explicit angular ordering.
- ✓ Transverse momentum ordering.
- ✓ The emissions are not independent. It evolves the whole event instead of one parton.
- ✓ Implemented in ARIADNE.
- ✗ No initial state shower.



The factorization formulae is derived from the soft limit and modified to incorporate with collinear limit

$$d\sigma_{m+1} = d\sigma_m \sum_{i,j \text{ dipoles}} \frac{\alpha_s}{2\pi} D_{ij}(p_{\perp}, y) \frac{dp_{\perp}^2}{p_{\perp}^2} dy$$

# Color Dipole vs. AP Evolution

The color dipole shower has some advantage over the conventional conventional parton shower but the main disadvantage is the lack of the initial state shower.

*Is it possible to combine them?*

$$D_{ij}(p_{\perp}, y) = P_{i \rightarrow i+g}(z_i) + P_{j \rightarrow j+g}(z_j)$$

- ✓ We can build a parton shower with dipole cascade kinematics and transverse momentum ordering. (*No explicit angular ordering!*)
- ✓ The gluon is emitted from a “half color dipole” (=AP kernel) coherently.
- ✓ Initial state shower is according to conventional parton cascade.
- ✓ Implemented in new PYTHIA version 6.4.
- ✗ No color dipole between the initial and final state.



# Facts you should beware of

The shower is derived from QCD but you cannot use the shower cross sections as QCD prediction.

- ✗ Very crude approximation in the phase space. Angular ordered shower doesn't cover the whole phase space (dead cone).
  - ➔ In every step of the shower the phase space should be exact, every parton should be onshell.
- ✗ The independent emission picture is valid only in the strict collinear limit. The color correlations are not considered properly even at leading color level.
  - ➔ Color and spin correlation must be considered systematically. One should go beyond the leading color approximation.
- ✗ The parton shower algorithms use several technical parameters.
  - ➔ Since the QCD matrix elements doesn't have technical parameters, the parton shower should be free of them.
- ✗ They are not defined systematically *e.g.*: angular ordering at NLO level???
  - ➔ The core algorithm shouldn't depend on the level of the calculation.



# Facts you should beware of

The shower is derived from QCD but you cannot use the shower cross sections as QCD prediction.

x Cross sections at  $\sqrt{s} = 1960$  GeV, with structure functions, in nanobarns,  
 $p_T > 10$  GeV  $|\eta| < 2.0$ .

x

| Process                 | $\sigma_0$ : Normal | $\sigma_1$ : Large Nc component | $\frac{\sigma_1 - \sigma_0}{\sigma_0}$ |
|-------------------------|---------------------|---------------------------------|--|
| ud $\rightarrow$ W+g    | 0.1029(5)D+01       | 0.1158(5)D+01                   | 13%                                    |
| ud $\rightarrow$ W+gg   | 0.1018(8)D+00       | 0.1283(10)D+00                  | 26%                                    |
| ud $\rightarrow$ W+ggg  | 0.1119(17)D-01      | 0.1564(22)D-01                  | 40%                                    |
| ud $\rightarrow$ W+gggg | 0.1339(36)D-02      | 0.2838(71)D-02                  | 120%                                   |

x *Results are calculated by HELAC*

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x They are not defined systematically e.g.: angular ordering at NLO level???

→ The core algorithm shouldn't depend on the level of the calculation.

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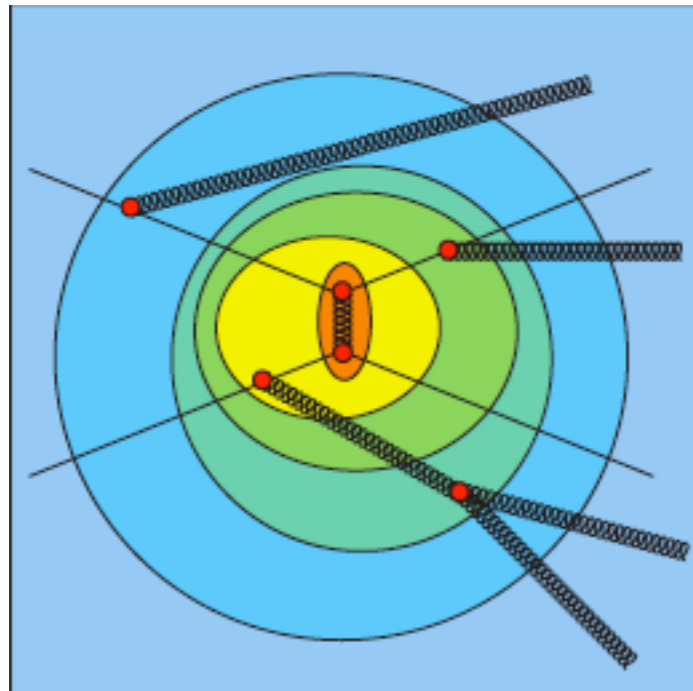
# More Questions

- ✗ The only exact matrix element in the calculation is  $2 \rightarrow 2$  like.
  - ➔ In a 3,4,..-jet calculation we should use the  $2 \rightarrow 3, 4, 5, \dots$  parton exact matrix elements at least at tree level.
- ✗ Since the strong coupling is large even the exact tree level matrix elements are not enough.
  - ➔ The shower should be matched to the NLO fix order calculation.
- ✗ ...
- ✗ Hadronization model ....
- ✗ ...

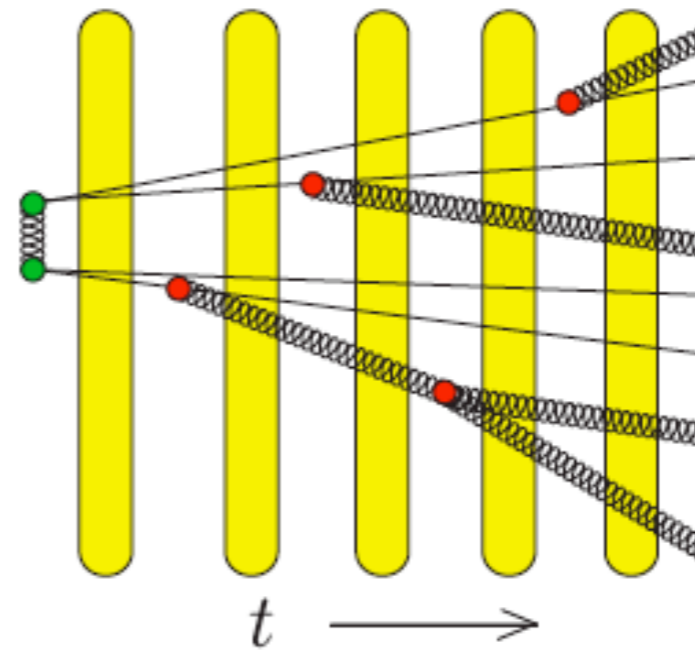
# Matching Parton Shower and Tree Level Matrix Elements

# Shower from the Inside Out

Think of shower branching as developing in a “time” that goes from most virtual to least virtual.



Real time picture

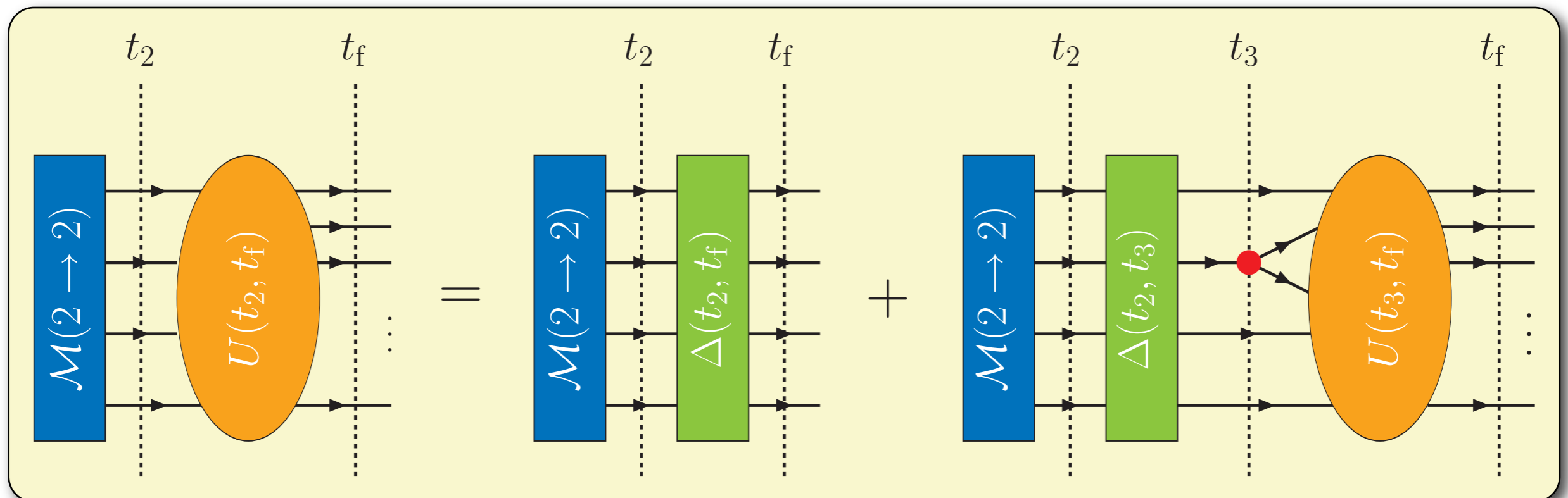


Shower time picture

Thus shower time proceeds backward in physical time for initial state radiation.

# Shower Cross Section

The shower starts from the kinematically simplest configuration, that is given by the corresponding 2->2 matrix elements.

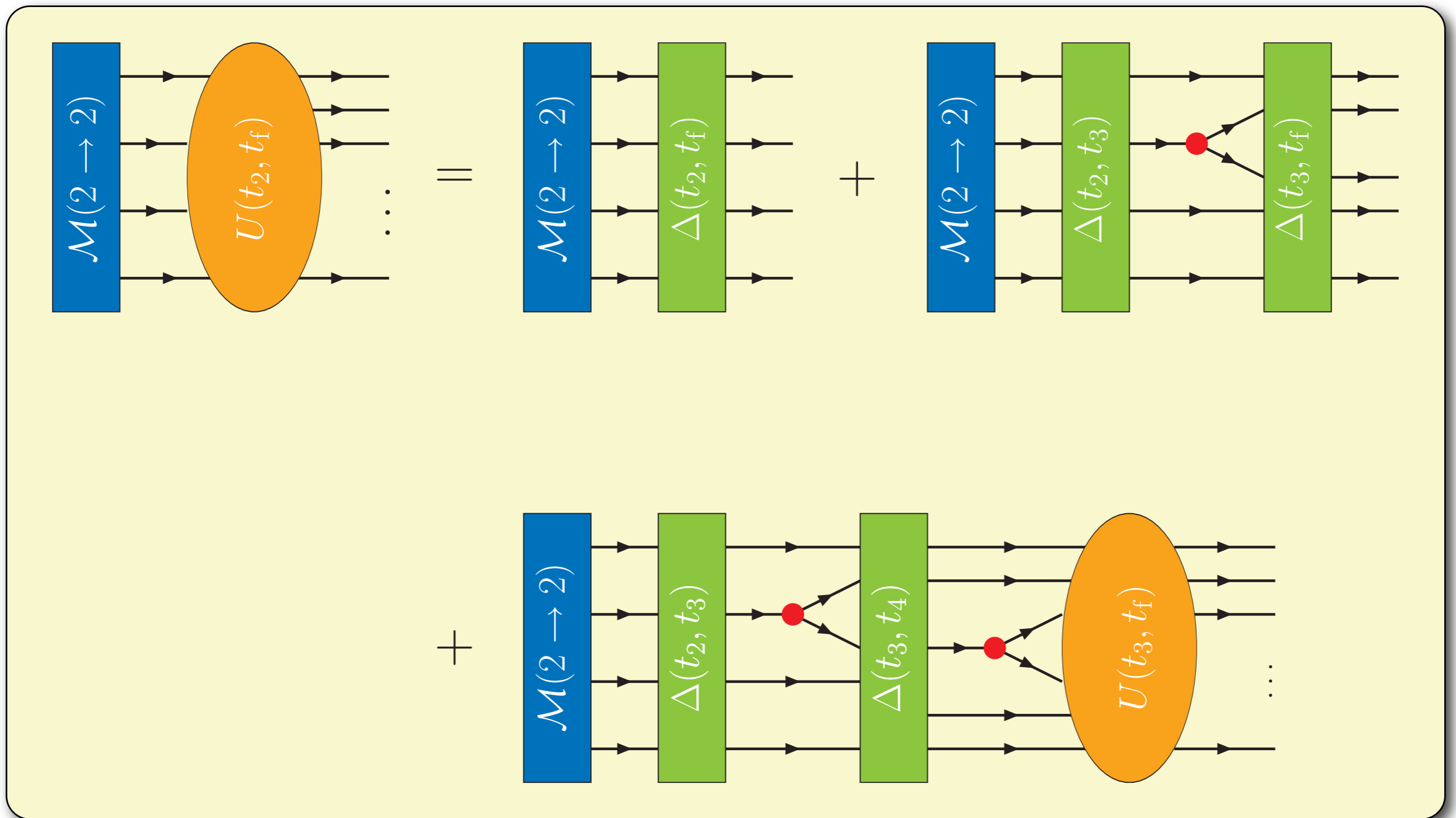


Splitting part

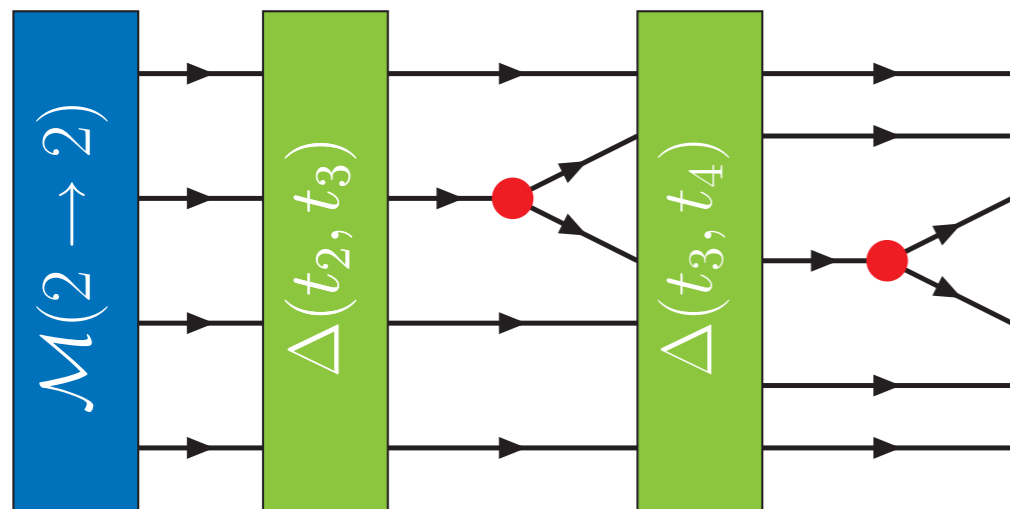
$$U(t_f, t_2) | \mathcal{M}_2 \rangle = \underbrace{\Delta(t_f, t_2) | \mathcal{M}_2 \rangle}_{\text{No-change operator}} + \int_{t_2}^{t_f} dt_3 U(t_f, t_3) \mathcal{H}(t_3) \Delta(t_3, t_2) | \mathcal{M}_2 \rangle$$

# Shower Cross Section

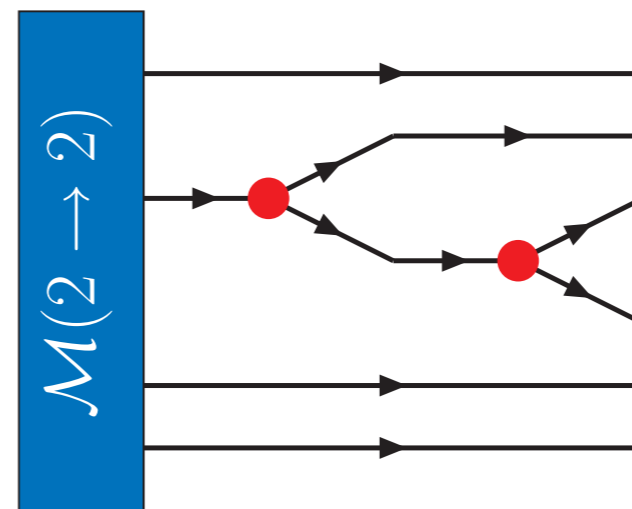
Iterating the evolution twice, then we have



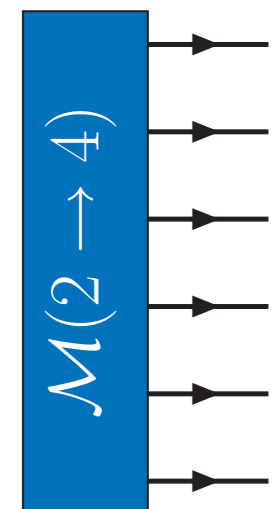
# Deficiency of the Shower



*Standard shower*



*Small  $p_T$  approximation*

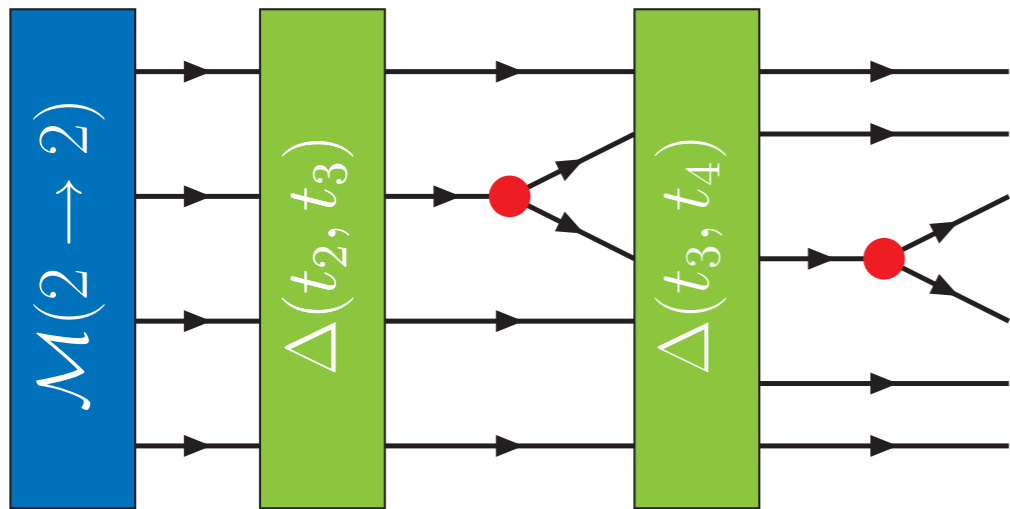


$|\mathcal{M}(2 \rightarrow 4)|^2$

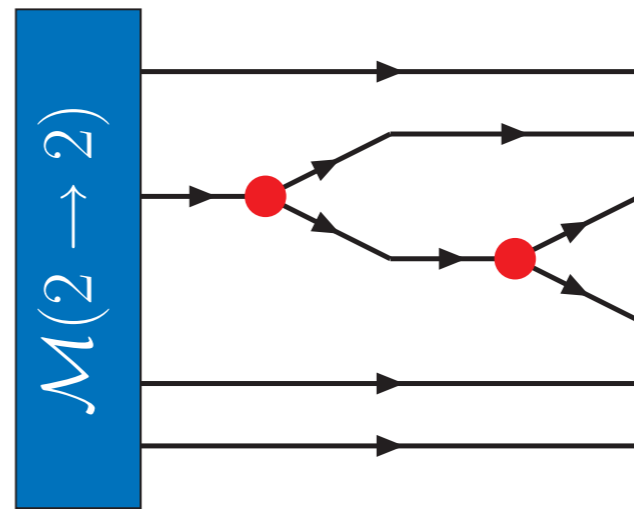
- The shower approximation relies on the small splittings.
- May be the exact matrix element would be better.
- But that lacks the Sudakov exponents.



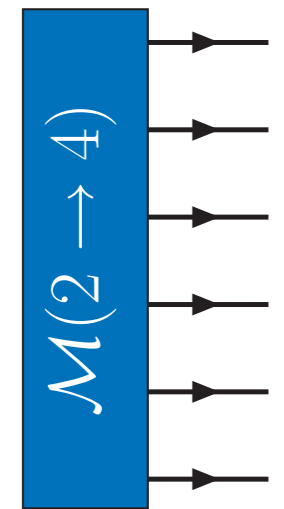
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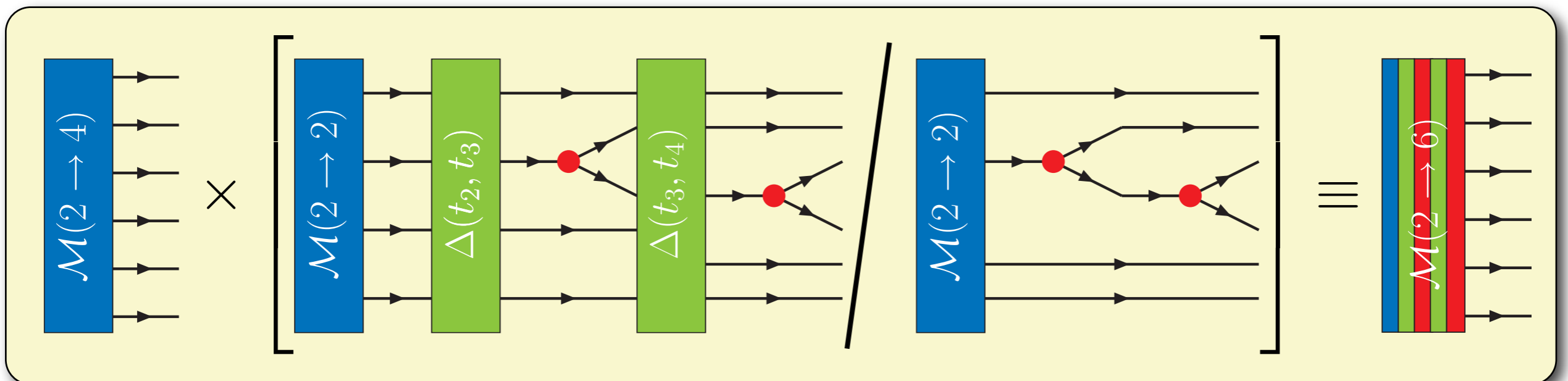
*Standard shower*



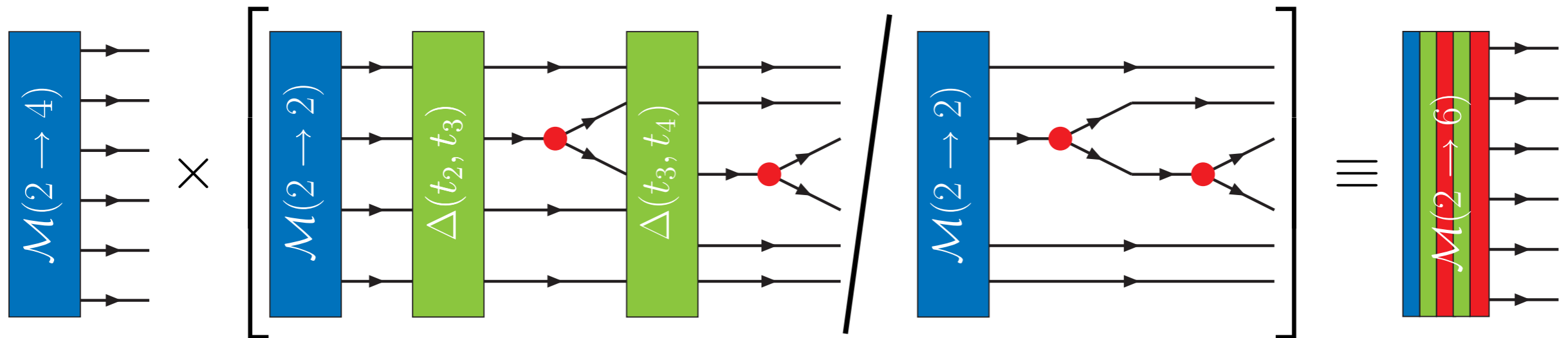
*Small  $p_T$  approximation*



$|\mathcal{M}(2 \rightarrow 4)|^2$



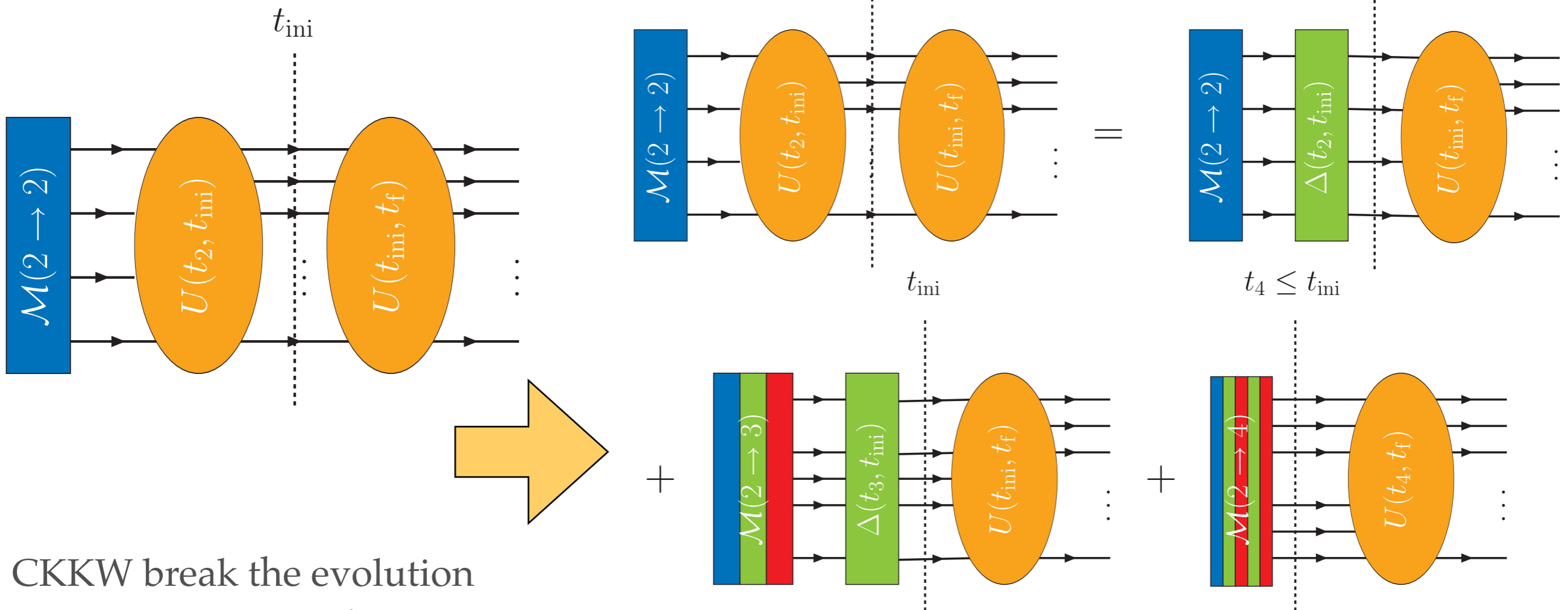
# Adding Born Matrix Elements



- This is the essential part of the CKKW matching procedure.
- In general there are many ways to get from  $2 \rightarrow 2$  configuration to  $2 \rightarrow m$  configuration.
- CKKW use the kT algorithm to find a unique history to define the Sudakov reweighting.
- The unique history requires to introduce matching scale.

# CKKW Matching

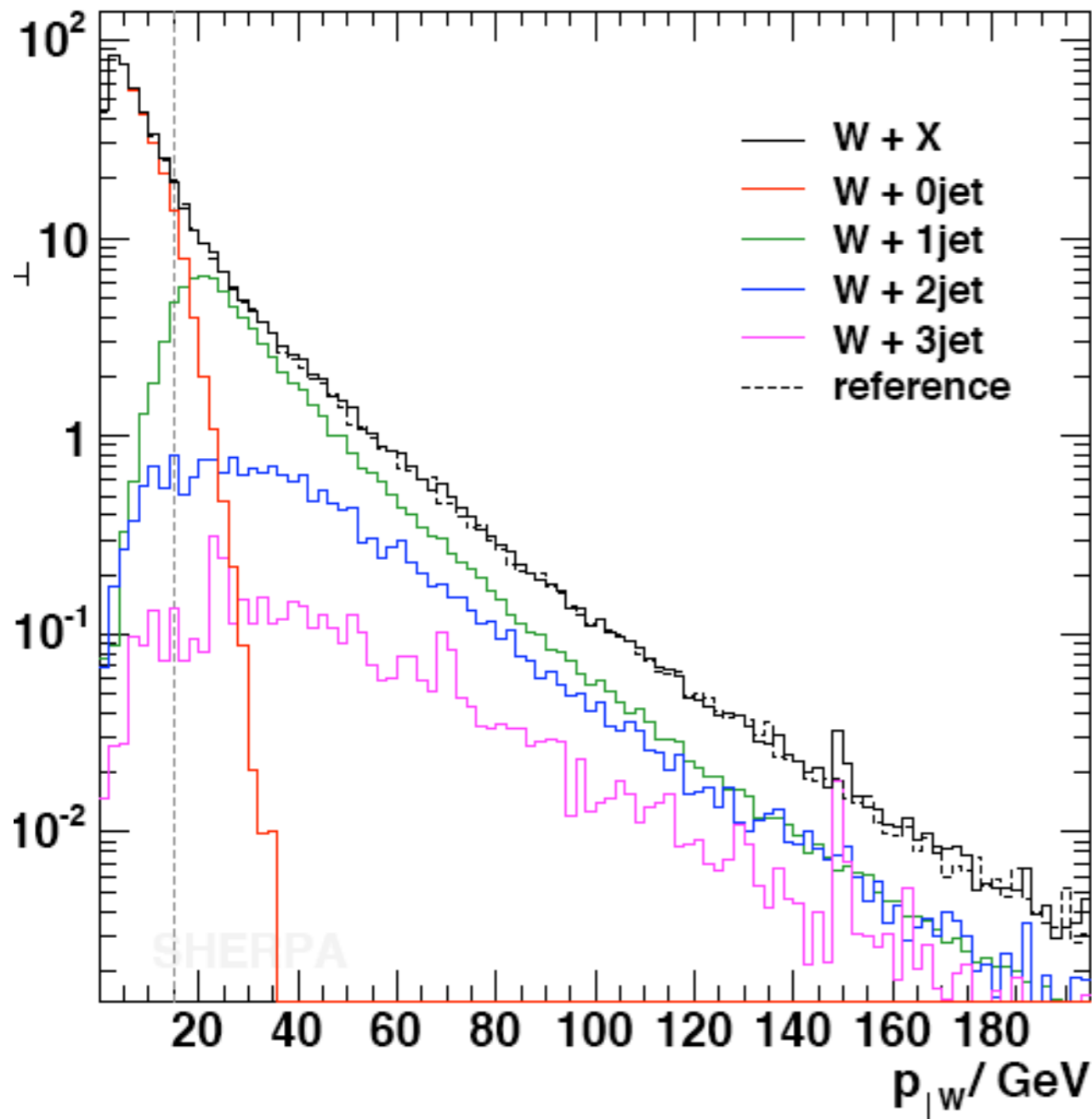
Two steps calculation:



CKKW break the evolution into  $0 < t < t_{ini}$  and  $t_{ini} < t < t_f$

- CKKW use improve weighting for  $0 < t < t_{ini}$
- For  $t_{ini} < t < t_f$  they have standard shower (in HERWIG case transverse momentum veto is needed)
- They use the kT algorithm and NLL Sudakov factors to do the reweighting.

# Adding Born Matrix Elements



- ✗ Only normalized cross sections can be calculated.
- ✗ The result could strongly depend on the matching scale.
  - ➔ It would be nice not to use matching scale.
- ✗ The matching scale dependence is cancelled at NLL level (proven only in  $e^+e^-$  annihilation)
- ✗ It is still LO order calculation thus the scale dependence is large.
  - ➔ The algorithm can be generalized at NLO level. (ZN and D. Soper, hep-ph/0503053)

# Matching Parton Shower and NLO calculations

# NLO Calculation

The NLO fix order calculations can be organized by the following way

$$\sigma_{\text{NLO}} = \int_m \left[ d\sigma^B + d\sigma^V + d\sigma^C + \int_1 d\sigma^A \right] F_J^{(m)} \\ + \int_{m+1} \left[ d\sigma_{m+1}^R F_J^{(m+1)} - d\sigma_{m+1}^A F_J^{(m)} \right]$$

The born ( $d\sigma^B$ ) and the real ( $d\sigma^R$ ) are based on the  $m$  and  $m+1$  parton matrix elements, respectively and  $d\sigma^V$  is the contribution of the virtual graphs. The universal collinear counterterm is  $d\sigma^C$ . The approximated  $m+1$  parton matrix element has universal structure

$$d\sigma^A \sim \mathcal{H}^\dagger \otimes |\mathcal{M}_m|^2$$

*It has the same singularity structure as  $d\sigma^R$*

# MC@NLO

The naive way doesn't work when we want to match the shower to NLO calculation. It leads to double counting. Frixione and Webber managed the following way:

$$\sigma_{\text{MC}} = \int_m \left[ d\sigma^B + d\sigma^V + d\sigma^C + \int_1 d\sigma^A \right] I_{\text{MC}}^{(2 \rightarrow m)} \quad \text{here } m=0,1,2 \text{ only!}$$
$$+ \int_{m+1} \left[ d\sigma_{m+1}^{\text{R}} - d\sigma_{m+1}^{\text{MC}} \right] I_{\text{MC}}^{(2 \rightarrow m+1)} + \left[ d\sigma_{m+1}^{\text{MC}} - d\sigma_{m+1}^{\text{A}} \right] I_{\text{MC}}^{(2 \rightarrow m)}$$

The  $d\sigma^{\text{MC}}$  term is extracted from the underlying shower algorithm and it is subtracted and added back in different way. The function  $I_{\text{MC}}^{(2 \rightarrow m)}$  and  $I_{\text{MC}}^{(2 \rightarrow m+1)}$  are the interface to the shower.

$$I_{\text{MC}}^{(2 \rightarrow m)} \sim U(t_f, t_2) \quad \text{and} \quad I_{\text{MC}}^{(2 \rightarrow m+1)} \sim U(t_f, t_3) \Delta(t_3, t_2)$$

*With these choices one can avoid double counting.*

# Shower and NLO Matching

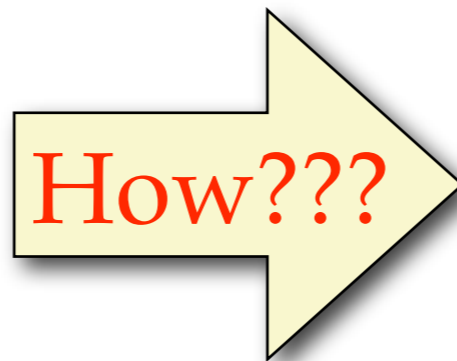
- ✗ The MC@NLO is worked out for HERWIG. If you want to use it with PYTHIA you have to redo the MC subtraction.
- ✗ MC@NLO is defined only for the simplest process like  $2 \rightarrow 2$  processes. It is more messy if we want to calculate say 3-jet cross section.
- ✗ The double counting problem is not fully solved but it is probably invisible numerically because of the Sudakov suppression.
- ✓ Several simple process is implemented in the MC@NLO framework.
- ✓ It is generate negative events.
- ✓ There are several more general approach to match shower and NLO calculation. (Krämer and Soper, ZN and Soper, Nason)
- \* The main idea is to include the first step of the shower in NLO calculation and then start the shower. This approach can cooperate any shower implementation.



# A New Shower Concept

Work with Dave Soper

*Quantum theory  
(QCD)*

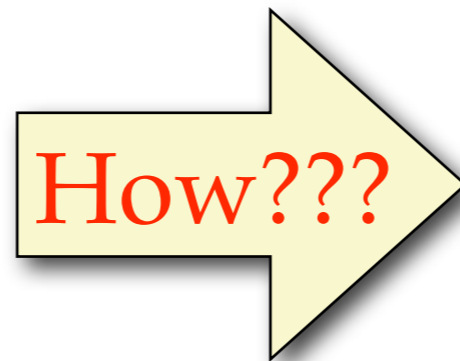


*Classical theory  
(Parton Shower)*

# A New Shower Concept

Work with Dave Soper

*Quantum theory  
(QCD)*



*Classical theory  
(Parton Shower)*

*Density Matrix*

# Density Matrix

The physical cross section is

$$\sigma[F] = \sum_m \int [d\{p, f\}_m] \overbrace{f_{a/A}(\eta_a, \mu_F^2) f_{b/B}(\eta_b, \mu_F^2)}^{\text{parton distributions}} \frac{1}{2\eta_a \eta_b p_A \cdot p_B} \\ \times \langle \mathcal{M}(\{p, f\}_m) | \underbrace{F(\{p, f\}_m)}_{\text{observable}} | \underbrace{\mathcal{M}(\{p, f\}_m)}_{\text{matrix element}} \rangle$$

It is useful to write this as trace in the color and spin space

$$\sigma[F] = \sum_m \int [d\{p, f\}_m] \text{Tr} \{ \underbrace{\rho(\{p, f\}_m)}_{\text{density operator in color } \otimes \text{ spin space}} F(\{p, f\}_m) \}$$

# Density Matrix

The physical cross section is

$$\sigma[F] = \sum_m \int [d\{p, f\}_m] \overbrace{f_{a/A}(\eta_a, \mu_F^2) f_{b/B}(\eta_b, \mu_F^2)}^{\text{parton distributions}} \frac{1}{2\eta_a \eta_b p_A \cdot p_B}$$

$$\int [d\{p, f\}_m] \equiv \frac{1}{m!} \prod_{i=1}^m \left\{ \sum_{f_i} \int \frac{d^4 p_i}{(2\pi)^4} 2\pi \delta_+(p_i^2) \right\} \sum_a \int_0^1 d\eta_a \sum_b \int_0^1 d\eta_b$$

$$\times (2\pi)^4 \delta \left( \eta_a p_A + \eta_b p_B - \sum_{i=1}^m p_i \right)$$

It  
and spin space

$$\sigma[F] = \sum_m \int [d\{p, f\}_m] \text{Tr} \left\{ \underbrace{\rho(\{p, f\}_m)}_{\text{density operator in color } \otimes \text{ spin space}} F(\{p, f\}_m) \right\}$$

density operator in color  $\otimes$  spin space

# Density Matrix

The density operator is

$$\rho(\{p, f\}_m) = |\mathcal{M}(\{p, f\}_m)\rangle \frac{f_{a/A}(\eta_a, \mu_F^2) f_{b/B}(\eta_b, \mu_F^2)}{2\eta_a \eta_b p_A \cdot p_B} \langle \mathcal{M}(\{p, f\}_m) |$$

or expanding it on a color and spin basis

$$\rho(\{p, f\}_m) = \sum_{s,c} \sum_{s',c'} |\{s, c\}_m\rangle A(\{p, f, s', c', s, c\}_m) \langle \{s', c'\}_m |$$

We use conventional treatment of spin, thus we have orthogonal basis:

$$\langle \{s'\}_m | \{s\}_m \rangle = \prod_{i=a,b,1,\dots,m} \delta_{s_i}^{s'_i}$$

# Density Matrix

In the color space we use a basis which is normalized but not orthogonal:

$$\langle \{c\}_m | \{c\}_m \rangle = 1 \quad \text{but} \quad \langle \{c'\}_m | \{c\}_m \rangle = \mathcal{O}(1/N_c^2) \quad \text{for } \{c'\}_m \neq \{c\}_m$$

It is useful to introduce a dual basis  $|\{c\}_m\rangle_D$  that is defined by

$${}_D\langle \{c'\}_m | \{c\}_m \rangle = \delta_{\{c'\}_m, \{c\}_m}$$

and the completeness relations are

$$1 = \sum_{\{c\}_m} |\{c\}_m\rangle {}_D\langle \{c\}_m| \quad \text{and} \quad 1 = \sum_{\{c\}_m} |\{c\}_m\rangle_D \langle \{c\}_m|$$

# Classical States

The set of functions  $A(\{p, f, s', c', s, c\}_m)$  forms a vector space.

**Basis:**  $|\{p, f, s', c', s, c\}_m\rangle$

**Completeness relation :**

$$1 = \sum_m \int [d\{p, f, s', c', s, c\}_m] |\{p, f, s', c', s, c\}_m\rangle \langle\{p, f, s', c', s, c\}_m|$$

where

$$\int [d\{p, f, s', c', s, c\}_m] \equiv \int [d\{p, f\}_m] \sum_{s_a, s'_a, c_a, c'_a} \sum_{s_b, s'_b, c_b, c'_b} \prod_{i=1}^m \left\{ \sum_{s_i, s'_i, c_i, c'_i} \right\}$$

**Inner product of the basis states:**

$$\langle\{p, f, s', c', s, c\}_m | \{\tilde{p}, \tilde{f}, \tilde{s}', \tilde{c}', \tilde{s}, \tilde{c}\}_{\tilde{m}}\rangle = \delta_{m, \tilde{m}} \delta(\{p, f, s', c', s, c\}_m; \{\tilde{p}, \tilde{f}, \tilde{s}', \tilde{c}', \tilde{s}, \tilde{c}\}_{\tilde{m}})$$

# Classical State

A physical state which is related to the density matrix:

$$|A\rangle = \int [d\{p, f, s', c', s, c\}_m] A(\{p, f, s', c', s, c\}_m) |\{p, f, s', c', s, c\}_m\rangle$$

or the vector corresponding to the measurement function

$$(F|\{p, f, s', c', s, c\}_m) = \langle \{s', c'\}_m | F(\{p, f\}_m) | \{s, c\}_m \rangle$$

Then the cross section that corresponding to this measurement function is

$$\sigma[F] = (F|A)$$



# Parton Shower Evolution

We use an evolution variable e.g.:

$$\log \frac{Q^2}{q^2} = t \in [0, \infty]$$

The evolution is given by a linear operator

$$|A(t)) = U(t, t_0) |A(t_0))$$

Group decomposition

$$U(t_3, t_2) U(t_2, t_1) = U(t_3, t_1)$$

Preserves the normalization

$$(1|A(t_0)) = 1 \quad \longrightarrow \quad (1|U(t, t_0)|A(t_0)) = 1$$

# Parton Shower Evolution

$$U(t_3, t_1) = \underbrace{N(t_3, t_1)}_{\text{No-change operator}} + \overbrace{\int_{t_1}^{t_3} dt_2 U(t_3, t_2) \mathcal{H}(t_2) N(t_2, t_1)}^{\text{Splitting part}}$$

Group decomposition

$$U(t_3, t_2) U(t_2, t_1) = U(t_3, t_1)$$

Preserves the normalization

$$(1|A(t_0)) = 1 \quad \longrightarrow \quad (1|U(t, t_0)|A(t_0)) = 1$$

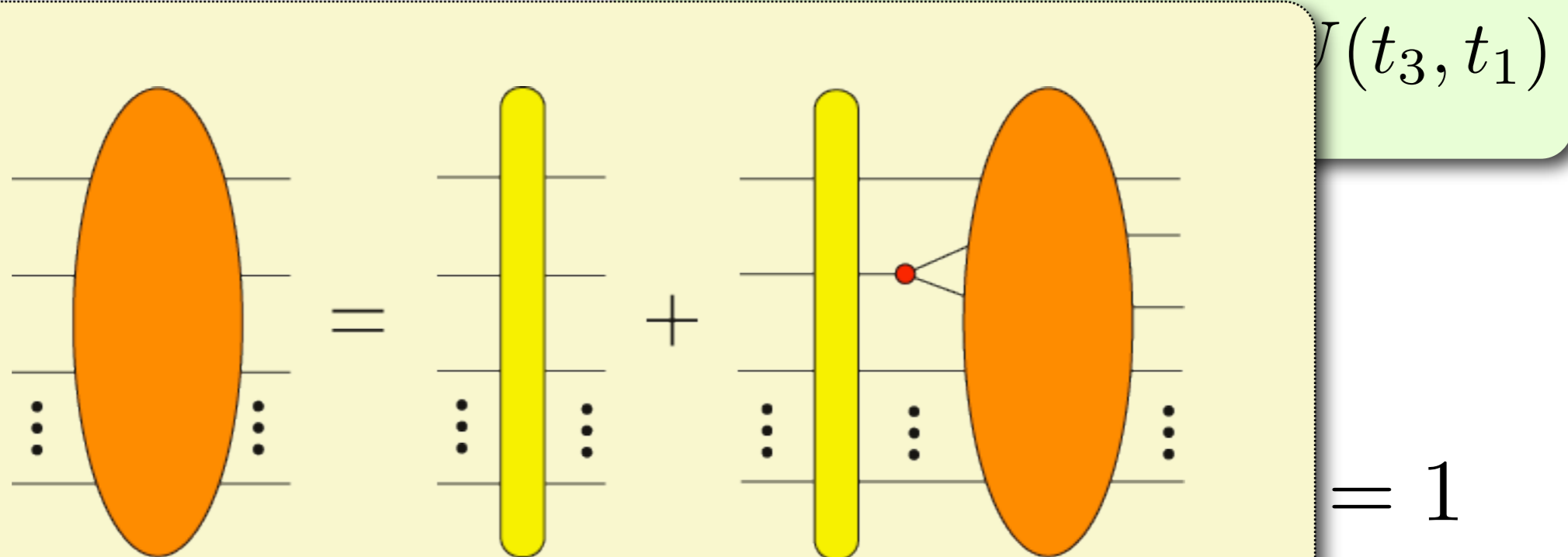
# Parton Shower Evolution

$$U(t_3, t_1) = \underbrace{N(t_3, t_1)}_{\text{No-change operator}} + \overbrace{\int_{t_1}^{t_3} dt_2 U(t_3, t_2) \mathcal{H}(t_2) N(t_2, t_1)}^{\text{Splitting part}}$$

Group d

Preserve

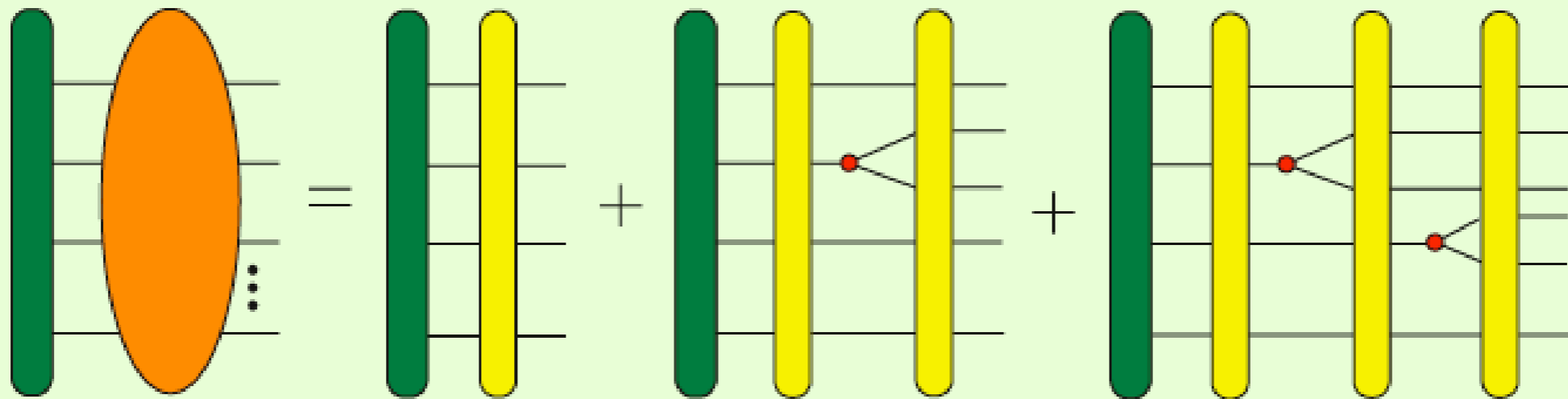
$(1|A$



# Parton Shower Evolution

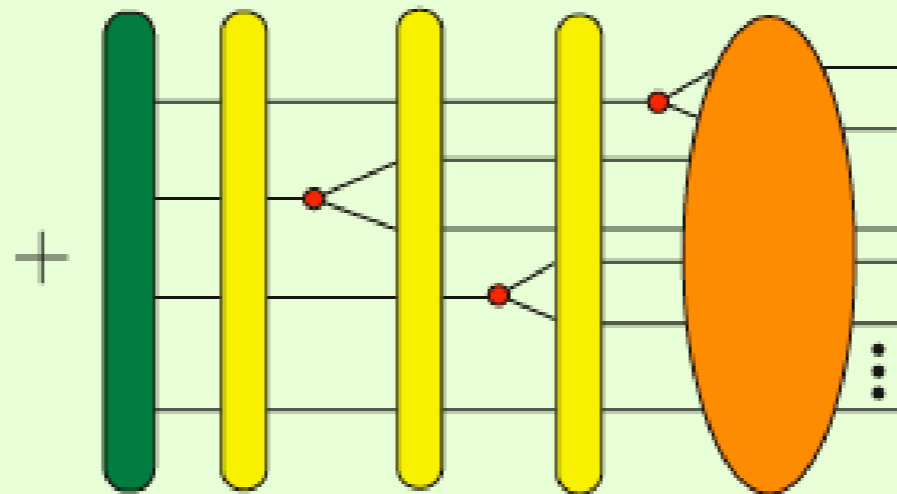
Start with  $2 \rightarrow 2$  cross section and iterate the evolution equation, say, twice:

$U(t)$



$t_1)$

Gro



$t_1)$

Pres

(1, 2, 1)

# No-change Operator

The operator  $N(t', t)$  leaves the basis states  $|\{p, f, s', c', s, c\}_m\rangle$  unchanged

$$N(t', t)|\{p, f, s', c', s, c\}_m\rangle = \underbrace{\Delta(\{p, f, c', c\}_m; t', t)}_{\text{Sudakov factor}}|\{p, f, s', c', s, c\}_m\rangle$$

Consistently with the group decomposition property and with the initial condition  $N(t, t) = 1$  for  $\Delta$  we have

$$\Delta(\{p, f, c', c\}_m; t_2, t_1) = \exp\left(-\int_{t_1}^{t_2} d\tau \underbrace{\omega(\tau; \{p, f, c', c\}_m)}_{\text{related to the parton splitting}}\right)$$

# No-change Operator

From the normalization conditions

$$(1|U(t, t')|\{p, f, \dots\}_m) = (1|\{p, f, \dots\}_m)$$

and

$$(1|\{p, f, s', c', s, c\}_m) = \langle \{s'\}_m | \{s\}_m \rangle \langle \{c'\}_m | \{c\}_m \rangle$$

the relation between the splitting operator and  $\omega(\tau; \{p, f, c', c\}_m)$  is

$$\langle \{s'\}_m | \{s\}_m \rangle \langle \{c'\}_m | \{c\}_m \rangle \omega(t, \{p, f, c', c\}_m) = (1|\mathcal{H}(t)|\{p, f, s', c', s, c\}_m)$$

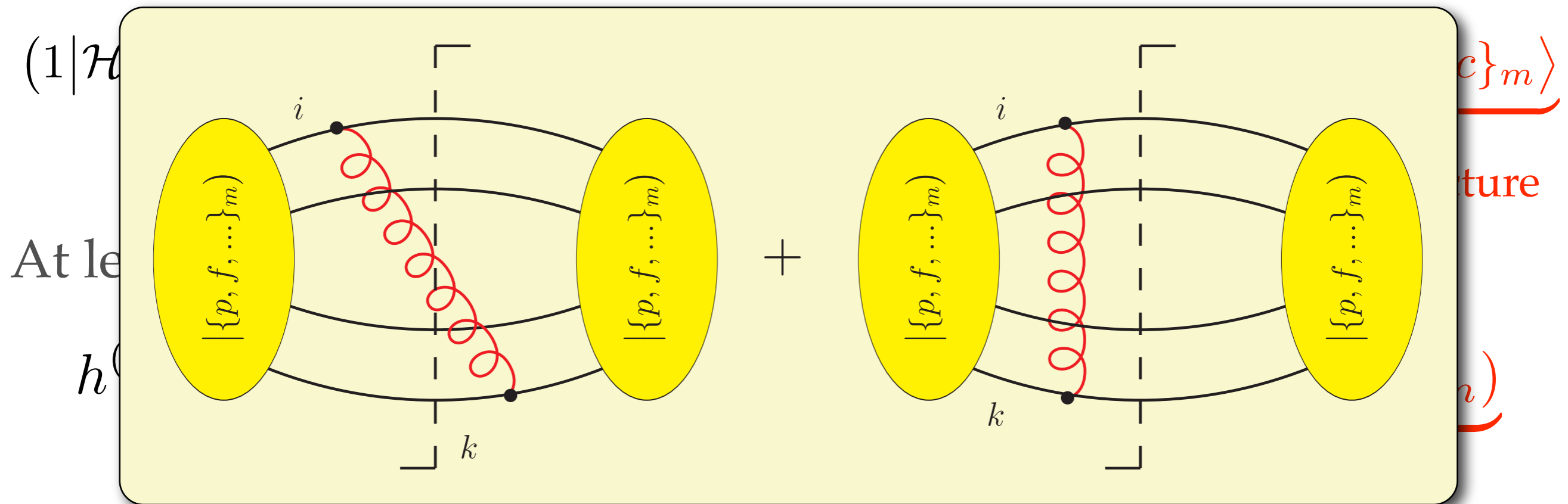
*At this point we need to discuss the structure of  $\mathcal{H}(t)$ !*

# Splitting Operator

The splitting operator consists of two terms

$$\mathcal{H}(t) = \mathcal{H}_I(t) + \mathcal{V}(t)$$

$\mathcal{H}_I(t)$  describes the parton splitting and based on the factorization property of the matrix element. It increases the number of partons and changes spins and colors.



# Splitting Operator

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$$\mathcal{H}(t) = \mathcal{H}_I(t) + \mathcal{V}(t)$$

$\mathcal{H}_I(t)$  describes the parton splitting and based on the factorization property of the matrix element. It increases the number of partons and changes spins and colors.

$$(1|\mathcal{H}_I(t)|\{p, f, s', c', s, c\}_m) = 2 \langle \{s'\}_m | \{s\}_m \rangle \underbrace{\langle \{c'\}_m | h(t, \{p, f\}_m) | \{c\}_m \rangle}_{\text{non-trivial color structure}}$$

non-trivial color structure

At leading order level:

$$h^{(0)}(t, \{p, f\}_m) = \underbrace{1P(t, \{p, f\}_m)}_{\text{collinear}} + \sum_{\substack{i, k \\ i \neq k}} \underbrace{\mathbf{T}_i \cdot \mathbf{T}_k S_{ik}(t; \{p\}_m)}_{\text{pure soft}}$$



# Splitting Operator

The operator  $\mathcal{V}(t)$  represents the contributions of the virtual graphs. It keeps the number of partons, flavors, spins unchanged but changes the color.

$$\begin{aligned} \mathcal{V}(t) &|\{p, f, s', c', s, c\}_m\rangle \\ &= \sum_{\{\tilde{c}\}_m} |\{p, f, s', c', s, \tilde{c}\}_m\rangle {}_D\langle\{\tilde{c}\}_m|v(t, \{p, f\}_m)|\{c\}_m\rangle \left[1 - \delta_{\{c\}_m}^{\{\tilde{c}\}_m}\right] \\ &+ \sum_{\{\tilde{c}'\}_m} |\{p, f, s', \tilde{c}', s, c\}_m\rangle \langle\{c'\}_m|v(t, \{p, f\}_m)|\{\tilde{c}'\}_m\rangle_D \left[1 - \delta_{\{c'\}_m}^{\{\tilde{c}'\}_m}\right] \end{aligned}$$

The singularities of the virtual graphs cancel those of the real emission graphs, thus the obvious choice is

$$v(\{p, f\}_m) = -h(\{p, f\}_m)$$

# Sudakov Exponent

The relation between the splitting operator and the Sudakov exponent is

$$\langle \{s'\}_m | \{s\}_m \rangle \langle \{c'\}_m | \{c\}_m \rangle \omega(t, \{p, f, c', c\}_m) = (1 | \mathcal{H}(t) | \{p, f, s', c', s, c\}_m)$$

From this we have

$$\omega(t, \{p, f, c', c\}_m) = {}_D \langle \{c\}_m | h(\{p, f\}_m) | \{c\}_m \rangle + \langle \{c'\}_m | h(\{p, f\}_m) | \{c'\}_m \rangle_D$$

where

$$h^{(0)}(t, \{p, f\}_m) = \underbrace{\mathbf{1}P(t, \{p, f\}_m)}_{\text{collinear}} + \sum_{\substack{i,k \\ i \neq k}} \underbrace{\mathbf{T}_i \cdot \mathbf{T}_k S_{ik}(t; \{p\}_m)}_{\text{pure soft}}$$

# Comments

- The Sudakov exponent exponentiates the color conserving part of the inclusive splitting operator  $h(t, \{p, f\}_m)$ . Fortunately the leading color part always conserves the color.
- The subleading color part is not exponentiated but it is subtracted. These contributions are treated perturbatively like the splitting terms in  $\mathcal{H}_I(t)$ .
- Thus we have two perturbative parameters:

$\alpha_s$

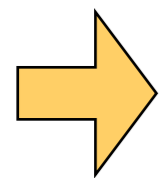
and

$\frac{1}{N_c^2}$

# Leading Color Approx.

Neglecting all the  $1/N_c^2$  contributions the color basis becomes orthogonal and the dual basis is identical to the color basis:

$$|\{c\}_m\rangle_D = |\{c\}_m\rangle + \mathcal{O}(1/N_c^2)$$



No subtraction:  $\mathcal{V}(t) = \mathcal{O}(1/N_c^2)$

The Sudakov factor exponentiates the whole inclusive splitting function

$$\omega(t, \{p, f, c', c\}_m) = 2\langle \{c\}_m | h(\{p, f\}_m) | \{c\}_m \rangle + \mathcal{O}(1/N_c^2)$$

# Splitting Operator

The splitting operator is based on the soft and collinear factorization formulas. At LO level we have

$$\mathcal{H}_I(t) = \mathcal{D}_a(t) + \mathcal{D}_b(t) + \sum_{i=1}^m \mathcal{D}_i(t) + \mathcal{O}(\alpha_s^2)$$

- The splitting kernel is organized according to the collinear splittings (even the soft radiations)
- This decomposition can be also done at higher order level.
- We need to define phase space mapping to get  $m+1$  parton configurations from  $m$  parton configurations. This can be also done systematically even for higher order.
- It is important that the phase space mapping must be exact since the classical states are defined on phase space surface in the momentum space.
- The mapping must be consistent with the higher order; *e.g.* we **cannot** use the Catani-Seymour dipole factorization and phase space mapping.

# Shower Cross Section

The evolution starts from the kinematically simplest configuration and the shower cross section is

$$\sigma[F] = (F | D(t_f) U(t_f, t_0) | \mathcal{M}_2)$$

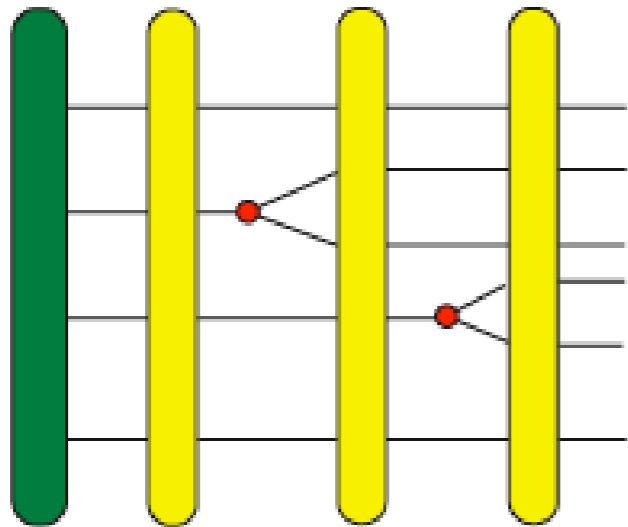
$D(t_f)$  represents the hadronization. **Tuning is allowed only here.**

If we have an leading order shower than the corresponding hard configuration should be based on the tree level matrix elements.

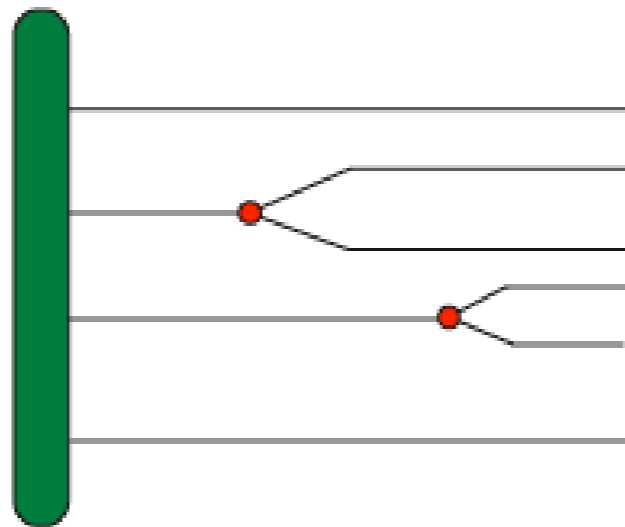
# Adding Exact Matrix Elements

- Having the LO shower defined, we can calculate any cross section, 2-jet, 3-jet,...
- Only the  $2 \rightarrow 2$  Born matrix elements are considered.
- For 3-jet we should consider at least the  $2 \rightarrow 3$  Born matrix elements.
- Since  $\alpha_s$  is large it would be useful to consider the contributions of  $2 \rightarrow 4$  Born and  $2 \rightarrow 3$  1-loop matrix elements in the 3-jet calculation.

# Matching at Born Level



Standard shower



Small  $p_T$  approximation

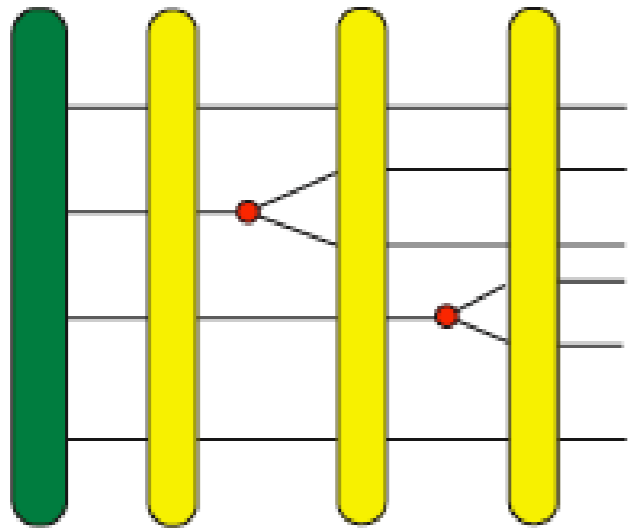


$|\mathcal{M}|^2$

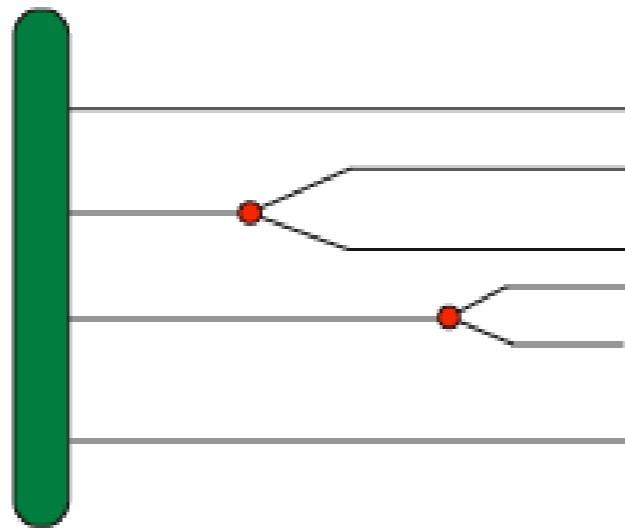
- The shower approximation relies on the small  $p_T$  splittings.
- May be the exact matrix element would be better.
- But that lacks the Sudakov exponents.



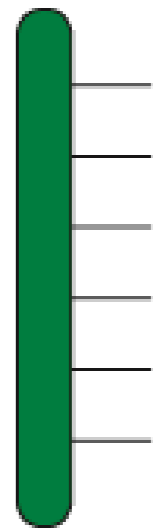
# Matching at Born Level



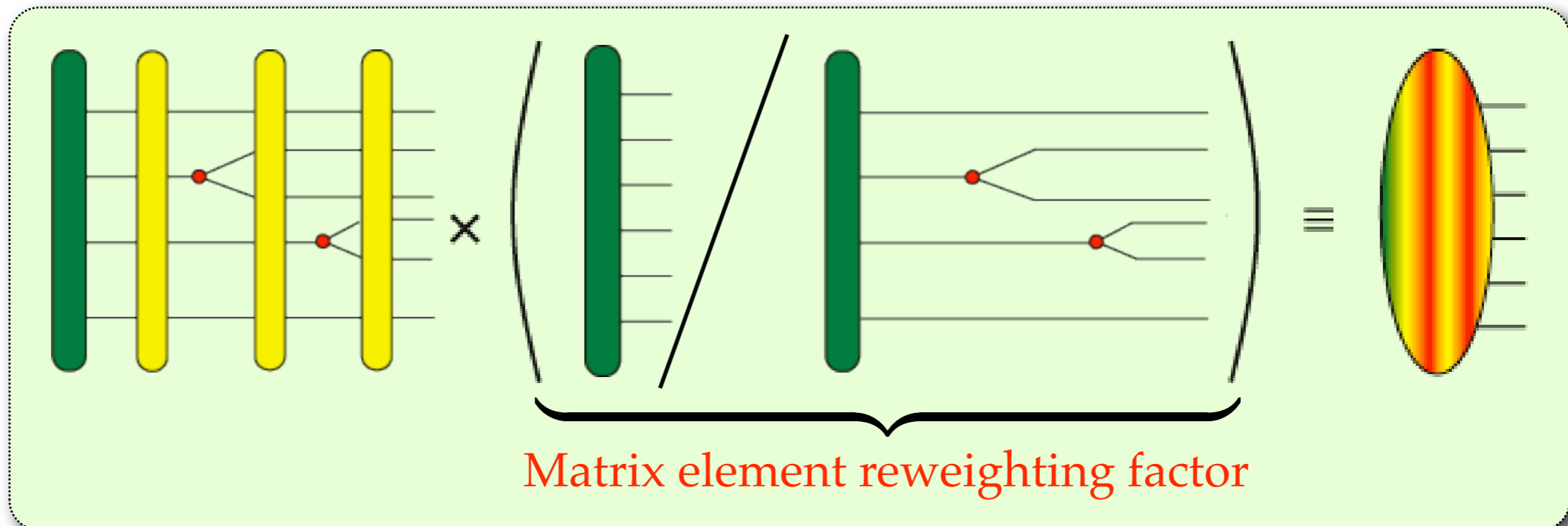
Standard shower



Small  $p_T$  approximation



$|\mathcal{M}|^2$



Matrix element reweighting factor

# Adjoint Splitting Operator

Let us define the operator  $\mathcal{H}^\dagger(t)$  according to

$$(F|\mathcal{H}(t)|A) = (A|\mathcal{H}^\dagger(t)|F)$$

$$\mathcal{H}^\dagger(t) = \mathcal{H}_I^\dagger(t) + \mathcal{V}^\dagger(t)$$

Since  $\mathcal{H}_I(t)$  always increases the number of partons  $\mathcal{H}_I^\dagger(t)$  always decreases it. **This operator is the fully exclusive version of the fix order calculation's subtraction terms.**

For multiple emission:

$$\begin{aligned} (F|\mathcal{H}(t_m)\mathcal{H}(t_{m-1})\cdots\mathcal{H}(t_1)|A) \\ = (A|\mathcal{H}^\dagger(t_1)\cdots\mathcal{H}^\dagger(t_{m-1})\mathcal{H}^\dagger(t_m)|F) \end{aligned}$$

# Approximated Matrix Element

For a given m-parton configuration the Born level approximated matrix element is

$$(\mathcal{A}_m | \{p, f, \dots\}_m) = \int_{t_2}^{t_f} dt_3 \cdots \int_{t_{m-1}}^{t_f} dt_m (\mathcal{M}_2 | \mathcal{H}_I^\dagger(t_3) \mathcal{H}_I^\dagger(t_4) \cdots \mathcal{H}_I^\dagger(t_m) | \{p, f, \dots\}_m)$$

The matrix element reweighting factor is

$$w_M = \frac{\text{Diagram with two vertical bars and branching lines}}{\text{Diagram with two vertical bars and straight lines}} = \begin{cases} \frac{(\mathcal{M}_m | \{p, f, \dots\}_m)}{(\mathcal{A}_m | \{p, f, \dots\}_m)} & \text{if } \mathcal{M}_m \text{ is known} \\ 1 & \text{otherwise} \end{cases}$$

and the reweighting operator is

$$W_M(t_f, t_2) = \sum_m \int [d\{p, f, \dots\}_m] | \{p, f, \dots\}_m \rangle (\{p, f, \dots\}_m | \times w_M(\{p, f, s', c', s, c\}_m)$$

# Approximated Matrix Element

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$$(\mathcal{A}_m | \{p, f, \dots\}_m) = \int_{t_2}^{t_f} dt_3 \cdots \int_{t_{m-1}}^{t_f} dt_m (\mathcal{M}_2 | \mathcal{H}_I^\dagger(t_3) \mathcal{H}_I^\dagger(t_4) \cdots \mathcal{H}_I^\dagger(t_m) | \{p, f, \dots\}_m)$$

The matrix element reweighting factor is

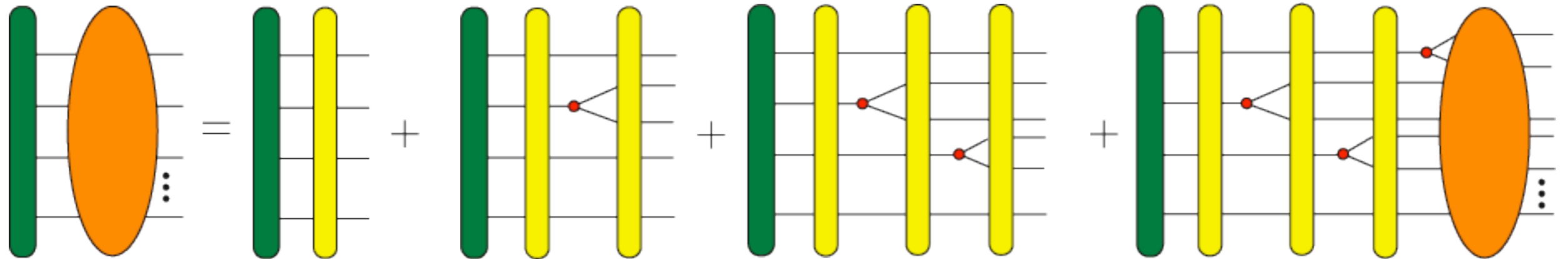
$$w_M = \frac{\text{Diagram with 5 lines and 2 vertices}}{\text{Diagram with 5 lines}} = \begin{cases} \frac{(\mathcal{M}_m | \{p, f, \dots\}_m)}{(\mathcal{A}_m | \{p, f, \dots\}_m)} & \text{if } \mathcal{M}_m \text{ is known} \\ 1 & \text{otherwise} \end{cases}$$

and the

$$V = \int_{t_2}^{t_f} dt_3 \int_{t_3}^{t_f} dt_4 N(t_f, t_4) W_M(t_f, t_2) \times \mathcal{H}_I(t_4) N(t_4, t_3) \mathcal{H}_I(t_3) N(t_3, t_2) | \mathcal{M}_2$$

# Matching at Born level

At leading color level when  $\mathcal{V}(t) = 0$  we have



Expanding one step of the shower

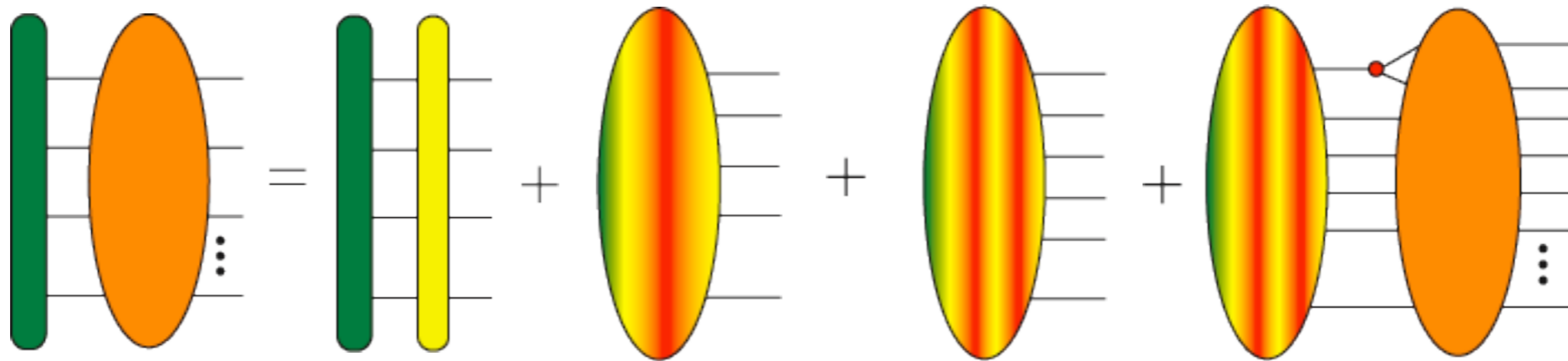
$$|A(t_f)\rangle = N(t_f, t_2)|\mathcal{M}_2\rangle + \int_{t_2}^{t_f} dt_3 U(t_f, t_3) (\mathcal{H}_I(t_3) + \mathcal{V}(t_t)) N(t_3, t_2) |\mathcal{M}_2\rangle$$

Assuming that we know the  $2 \rightarrow 3$  Born matrix elements

$$|A_M(t_f)\rangle = N(t_f, t_2)|\mathcal{M}_2\rangle + \int_{t_2}^{t_f} dt_3 U(t_f, t_3) (W_M(t_f, t_2) \mathcal{H}_I(t_3) + \mathcal{V}(t_t)) N(t_3, t_2) |\mathcal{M}_2\rangle$$

# Matching at Born level

At leading color level when  $\mathcal{V}(t) = 0$  we have



Expanding one step of the shower

$$|A(t_f)\rangle = N(t_f, t_2)|\mathcal{M}_2\rangle + \int_{t_2}^{t_f} dt_3 U(t_f, t_3) (\mathcal{H}_I(t_3) + \mathcal{V}(t_t)) N(t_3, t_2) |\mathcal{M}_2\rangle$$

Assuming that we know the  $2 \rightarrow 3$  Born matrix elements

$$|A_M(t_f)\rangle = N(t_f, t_2)|\mathcal{M}_2\rangle + \int_{t_2}^{t_f} dt_3 U(t_f, t_3) (W_M(t_f, t_2) \mathcal{H}_I(t_3) + \mathcal{V}(t_t)) N(t_3, t_2) |\mathcal{M}_2\rangle$$

# Matching at Born Level

After some algebra

$$|A_M(t_f)\rangle = \underbrace{U(t_f, t_2)|\mathcal{M}_2\rangle}_{\text{Standard shower}} + \int_{t_2}^{t_f} dt_3 U(t_f, t_3) \underbrace{[W_M(t_f, t_2), \mathcal{H}_I(t_3)]}_{W_M(t_f, t_2)\mathcal{H}_I(t_3) - \mathcal{H}_I(t_3)W_M(t_f, t_2)} N(t_3, t_2)|\mathcal{M}_2\rangle$$

The second term doesn't change the LL and NLL structure

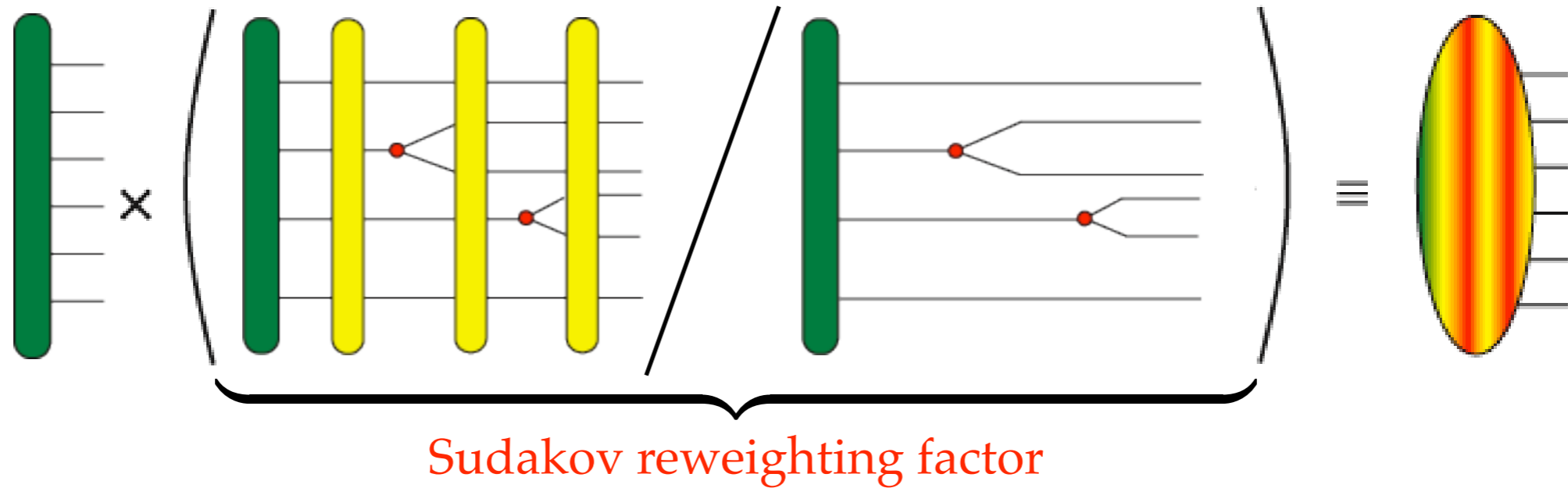
$$[W_M(t_f, t_2), \mathcal{H}_I(t_3)]|\mathcal{M}_2\rangle \sim |\mathcal{M}_3\rangle - \mathcal{H}_I(t_3)|\mathcal{M}_2\rangle$$

Assuming we know  $\mathcal{M}_3, \mathcal{M}_4, \dots, \mathcal{M}_N$  then

$$\begin{aligned} |A_M(t_f)\rangle &= U(t_f, t_2)|\mathcal{M}_2\rangle \\ &+ \sum_{m=3}^N \int_{t_2}^{t_f} dt_3 \int_{t_3}^{t_f} dt_4 \cdots \int_{t_{m-1}}^{t_f} dt_m U(t_f, t_m) [W_M(t_f, t_2), \mathcal{H}_I(t_m)] \\ &\quad \times N(t_m, t_{m-1})\mathcal{H}_I(t_{m-1})N(t_{m-1}, t_{m-2}) \cdots \mathcal{H}_I(t_3) N(t_3, t_2)|\mathcal{M}_2\rangle \end{aligned}$$

# Matching at Born Level

There is an other way to do the reweighting



$$\begin{aligned}
 W_{\Delta}(t_f, t_2; t) &= \sum_m \int [d\{p, f, s', c', s, c\}_m] |\{p, f, \dots\}_m\rangle (\{p, f, \dots\}_m| \\
 &\times \lim_{\delta \rightarrow 0} \int_{t_2}^t dt_{m-1} \int_{t_2}^{t_{m-1}} dt_{m-2} \cdots \int_{t_2}^{t_4} dt_3 \\
 &\times \frac{(\mathcal{M}_2 | N(t_3, t_2) \mathcal{H}_I^\dagger(t_3) \cdots N(t, t_{m-1}) \mathcal{H}_I^\dagger(t) | \{p, f, \dots\}_m)}{(\mathcal{A}_m(t_f, t_2) | \{p, f, s', c', s, c\}_m) + \delta}
 \end{aligned}$$



# Matching at Born Level

The improved shower formulae is

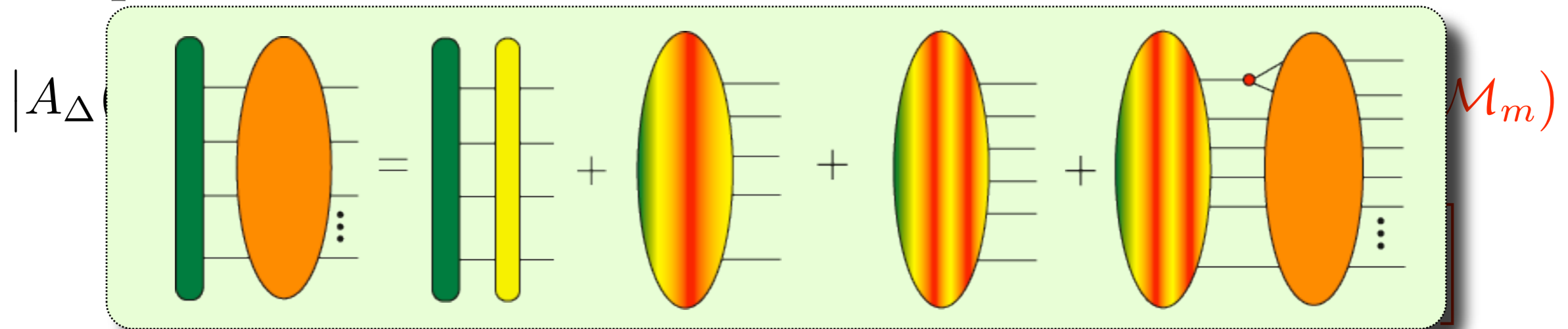
$$|A_{\Delta}(t_f)) = U(t_f, t_2)|\mathcal{M}_2) + \sum_{m=3}^N \int_{t_2}^{t_f} dt_m U(t_f, t_m) \left[ W_{\Delta}(t_f, t_2; t_m)|\mathcal{M}_m) - \mathcal{H}_I(t_m) \int_{t_2}^{t_m} dt' N(t_m, t') W_{\Delta}(t_f, t_2; t')|\mathcal{M}_{m-1}) \right]$$

and at leading color level this is even simpler

$$|A_{\Delta}(t_f)) = N(t_f, t_2)|\mathcal{M}_2) + \sum_{m=3}^{n-1} \int_{t_2}^{t_f} dt_m N(t_f, t_m) W_{\Delta}(t_f, t_2; t_m)|\mathcal{M}_m) + \int_{t_2}^{t_f} dt_n U(t_f, t_n) W_{\Delta}(t_f, t_2; t_n)|\mathcal{M}_n)$$

# Matching at Born Level

The improved shower formulae is

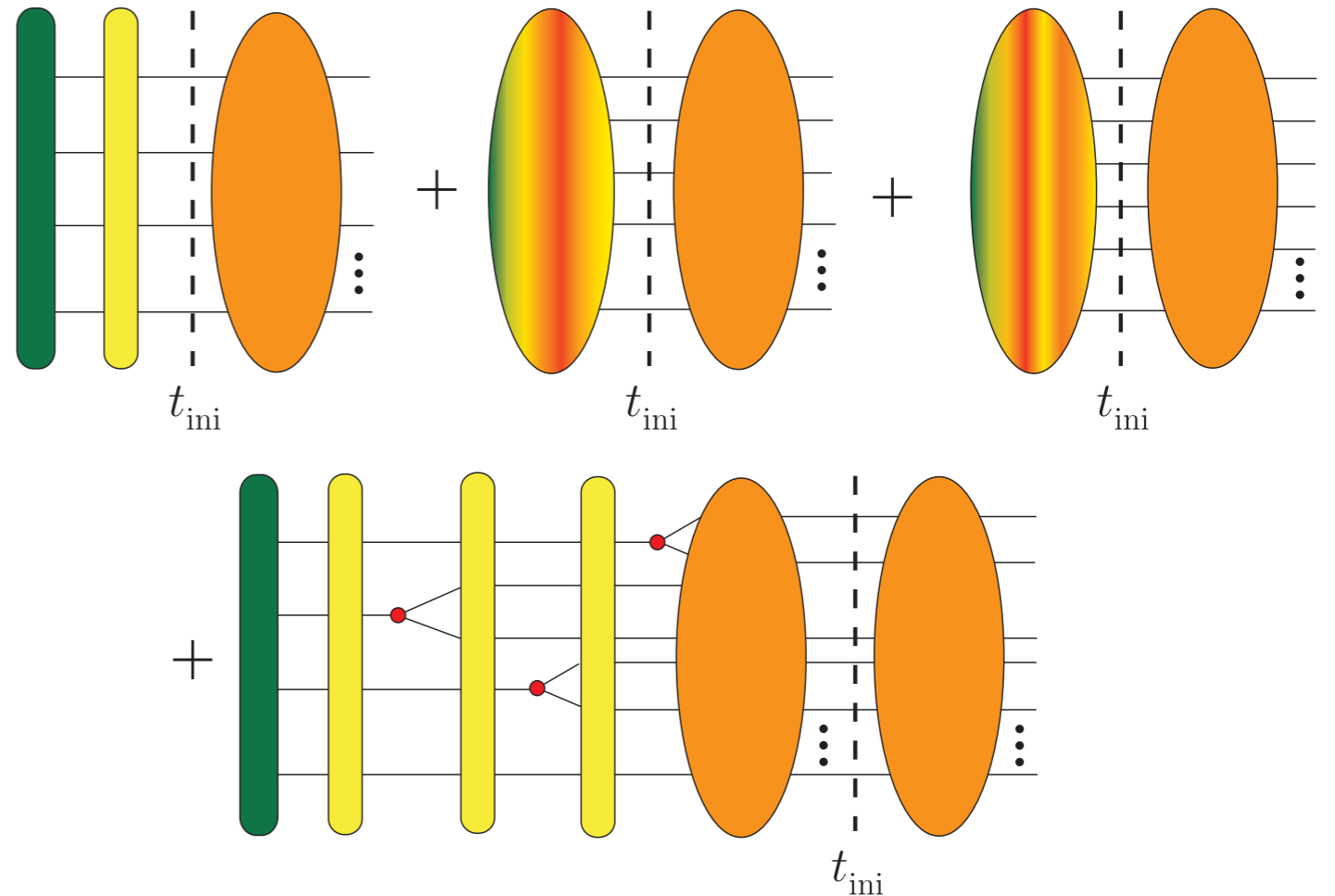
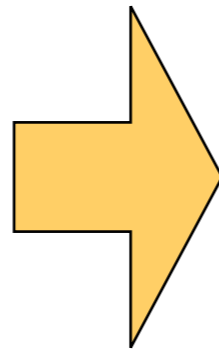
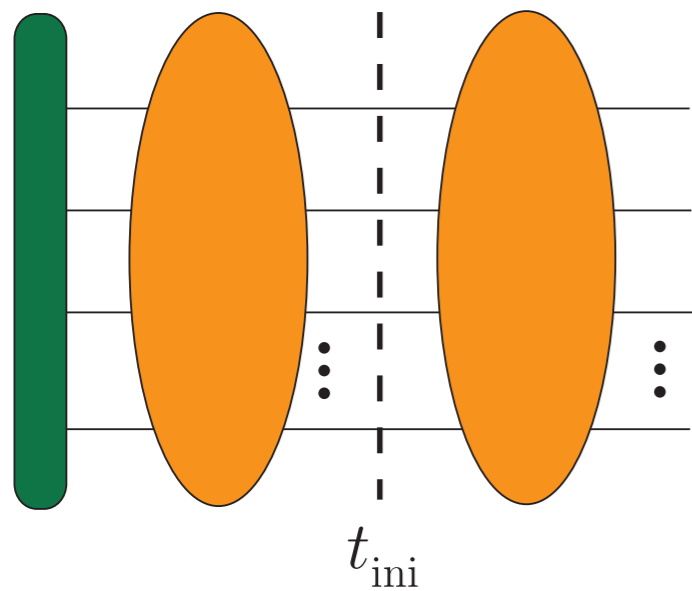


and at leading color level this is even simpler

$$|A_{\Delta}(t_f)) = N(t_f, t_2) |\mathcal{M}_2) + \sum_{m=3}^{n-1} \int_{t_2}^{t_f} dt_m N(t_f, t_m) W_{\Delta}(t_f, t_2; t_m) |\mathcal{M}_m) + \int_{t_2}^{t_f} dt_n U(t_f, t_n) W_{\Delta}(t_f, t_2; t_n) |\mathcal{M}_n)$$

# CKKW without Equations

Two steps calculation



CKKW break the evolution into  $0 < t < t_{ini}$  and  $t_{ini} < t < t_f$

CKKW@NLO  
ZN and D. Soper,  
hep-ph/0503053

- CKKW use improve weighting for  $0 < t < t_{ini}$
- For  $t_{ini} < t < t_f$  they have standard shower with transverse momentum veto
- They use the kT algorithm and NLL Sudakov factors to do the reweighting.

# NLO Calculations

The NLO fix order calculations can be organized by the following way

$$\begin{aligned} \sigma_{\text{NLO}} = & \int_N d\sigma^B + \int_{N+1} [d\sigma^R - d\sigma^A] \\ & + \int_N [d\sigma^B \otimes \mathbf{I}(\epsilon) + d\sigma^V]_{\epsilon=0} + \int_N d\sigma^B \otimes [\mathbf{K} + \mathbf{P}(\mu_F)] \end{aligned}$$

The Born ( $d\sigma^B$ ) and the real ( $d\sigma^R$ ) are based on the N and N+1 parton matrix elements, respectively and  $d\sigma^V$  is the contribution of the virtual graphs. The operators  $\mathbf{I}(\epsilon)$ ,  $\mathbf{K}$ ,  $\mathbf{P}$  are universal.

It is useful to define the subtraction term based on the shower splitting operator

$$d\sigma^A \sim \int_0^\infty dt (\mathcal{M}_N | \mathcal{H}_I^\dagger(t) | \{p, f, \dots\}_{N+1})$$

# Matching at NLO level

Let us calculate the N-jet cross section. The matrix element improved cross section is

$$\begin{aligned} (F_N | A_\Delta(t_f)) &= \int_{t_2}^{t_f} dt_N (F_N | N(t_f, t_N) W_\Delta(t_f, t_2; t_N) | \mathcal{M}_N) \\ &+ \int_{t_2}^{t_f} dt_{N+1} (F_N | U(t_f, t_{N+1}) W_\Delta(t_f, t_2; t_{N+1}) | \mathcal{M}_{N+1}) \\ &+ \underbrace{[\dots \mathcal{V}(t) \dots]} + \mathcal{O}[\mathcal{V}(t)^2] \\ &\text{[-- It is too long to display. --]} \end{aligned}$$

Expanding it in  $\alpha_s$  then we have

$$\begin{aligned} (F_N | A_\Delta) &= \int_N d\sigma^B + \int_{N+1} [d\sigma^R - d\sigma^A] \\ &+ \int_N [d\sigma^S[\mathcal{V}(t)] + d\sigma^B W_\Delta^{(1)}] + \mathcal{O}(\alpha_s^2) \end{aligned}$$

# Matching at NLO level

Fixing the shower formulae by subtracting the approximated and adding correct 1-loop contributions, thus we have

$$\begin{aligned} (F_N | A_{\Delta}^{\text{NLO}}(t_f)) &= (F_N | A_{\Delta}(t_f)) - [\dots \mathcal{V}(t) \dots] \\ &\quad + \int_{t_2}^{t_f} dt_N (F_N | U(t_f, t_N) W_{\Delta}(t_f, t_2; t_N) | \mathcal{M}_N^{(1)}) \end{aligned}$$

where

$$\begin{aligned} \int [d\{s', c', s, c\}_N] (M_N^{(1)} | \{p, f, \dots\}_N) &\sim -\frac{\alpha_s}{2\pi} W_{\Delta}^{(1)} |\mathcal{M}_N|^2 \\ &\quad + |\mathcal{M}_N|^2 \otimes (\mathbf{K} + \mathbf{P}(\mu_F)) + \left[ |\mathcal{M}_N|_{1-loop}^2 + |\mathcal{M}_N|^2 \otimes \mathbf{I}(\epsilon) \right]_{\epsilon=0}, \end{aligned}$$

# Conclusions

## The things I talked about:

- We defined a very general formalism to define and implement parton shower algorithm.
- We can consider spin correlation and color correlation beyond the leading color approximation.
- The treatment of the subleading color part is still perturbatively (not exponentiated).
- The algorithm is capable to deal with the higher order corrections in  $\alpha_s$  once the splitting kernels are known.
- We have general method to match the LO shower to Born and NLO matrix elements.

# Conclusion

The things I didn't talk about:

- The algorithm is Lorentz covariant / invariant.
- Based on exact phase space.
- No technical cuts, parameters. Only the infrared cutoff parameter.
- Color coherence (angular ordering) is naturally provided without angular ordering, vetoing, or other tricks. No azimuthal averaging.
- The evolution parameter doesn't have to be special. Any infrared sensitive parameter is good. Use the simplest, say virtuality.
- Since we defined  $\mathcal{H}_I^\dagger(t)$  operator, we defined a new NLO subtraction scheme for fix order NLO calculation.



# Conclusions

## About this new NLO scheme

- The algorithm is Lorentz covariant/invariant.
- Based on exact phase space factorization.
- Splitting operator is defined fully exclusive way. One can do MC helicity and color sum in the NLO calculation.
- The number of the subtraction terms is  $(N + 1)(N + 4)/2$  . Compared to the Catani-Seymour dipole method  $(N + 1)^2(N + 4)/2$ .
- Since we have less counter-terms we expect better numerical behavior.