# Onium photoproduction
# and
# Models of diffractive exclusive production

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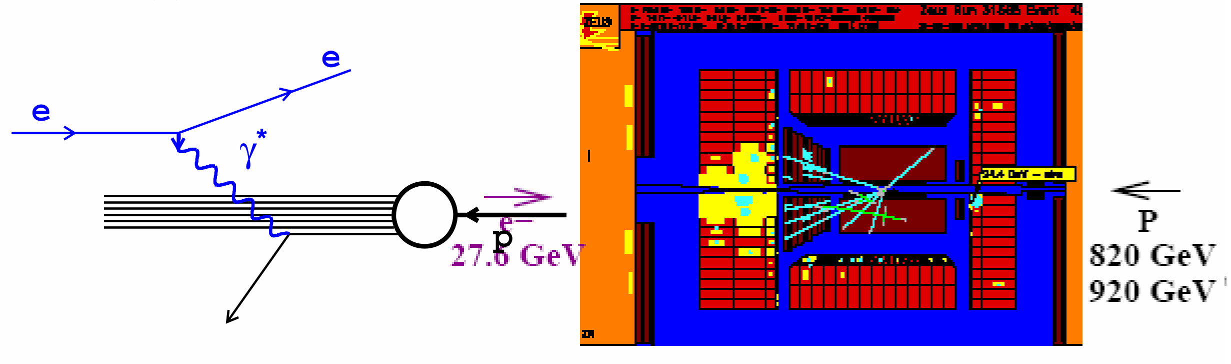
DESY

ECT
Trento, 17th of January 2007

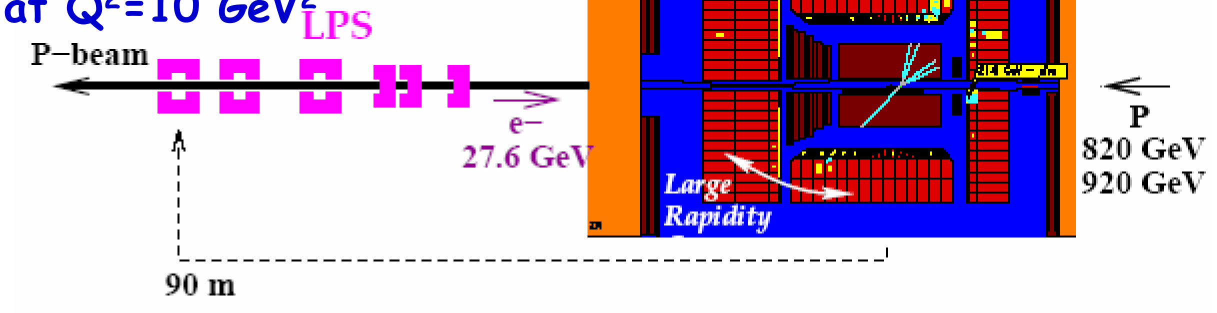
# Hard Diffraction – the HERA surprise

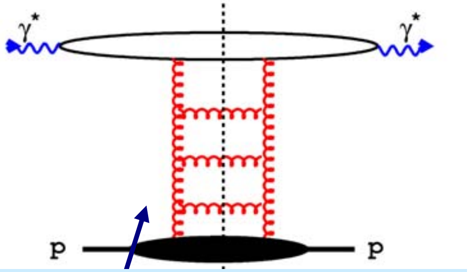
**Non-Diffractive Event**



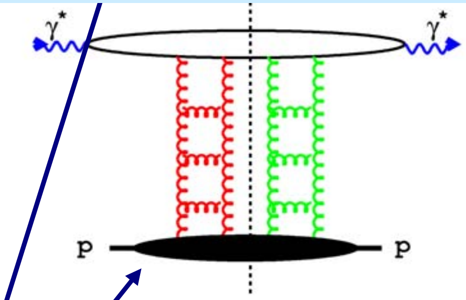
**Diffractive Event**

**expected before HERA**
**<0.01%, seen over 10%**
**at $Q^2$=10 GeV$^2$**





$$\tau_{qq} \approx \frac{1}{\Delta E} \approx \frac{1}{m_p x} \approx 10-1000 \text{ fm}$$



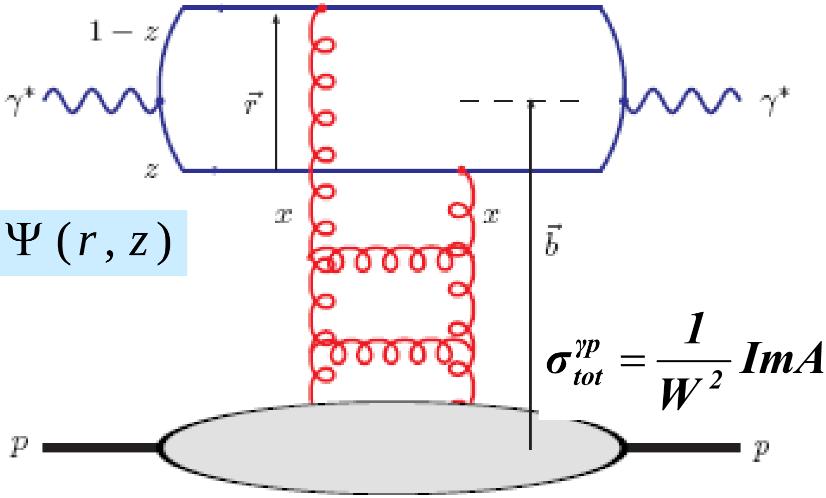
**Diffraction at HERA is so large because it is a shadow of DIS (i.e. inelastic processes) ➔ dipole picture**

$$\sigma_{tot}^{\gamma^* p} = \frac{1}{W^2} Im A_{el}(W^2, t=0)$$

# Dipole Models

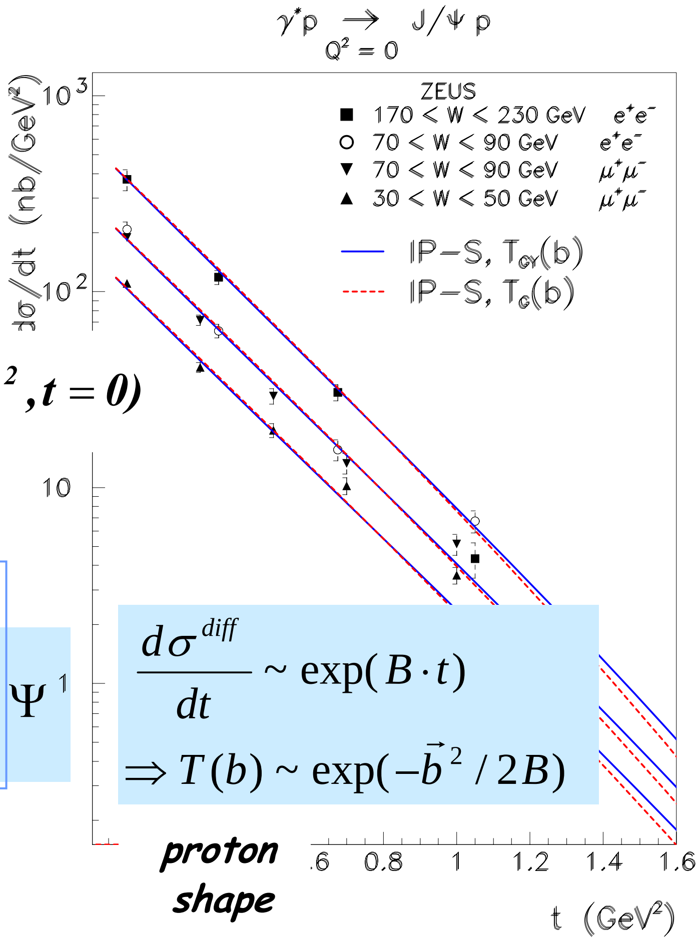
equivalent to LO perturbative QCD for small dipoles

NNPZ, GLM, FKS, GBW, MMS
DGKP, BGBK, KT, IIM, FSS……
KMW - Kowalski, Motyka, Watt



$\Psi(r,z)$

$$\sigma_{tot}^{\gamma p} = \frac{1}{W^2} Im A_{el}(W^2, t=0)$$



$$\frac{d\sigma^{diff}}{dt} \sim \exp(B \cdot t)$$

$$\Rightarrow T(b) \sim \exp(-\vec{b}^2/2B)$$

**proton shape**

**Glauber Mueller**

$$\sigma_{tot}^{\gamma^* p} = \int d^2\vec{r} \int_0^1 dz \int d^2 b\, \Psi^* \cdot 2\left\{1 - \exp\left(-\frac{\Omega}{2}\right)\right\} \Psi$$

**Optical Theorem**

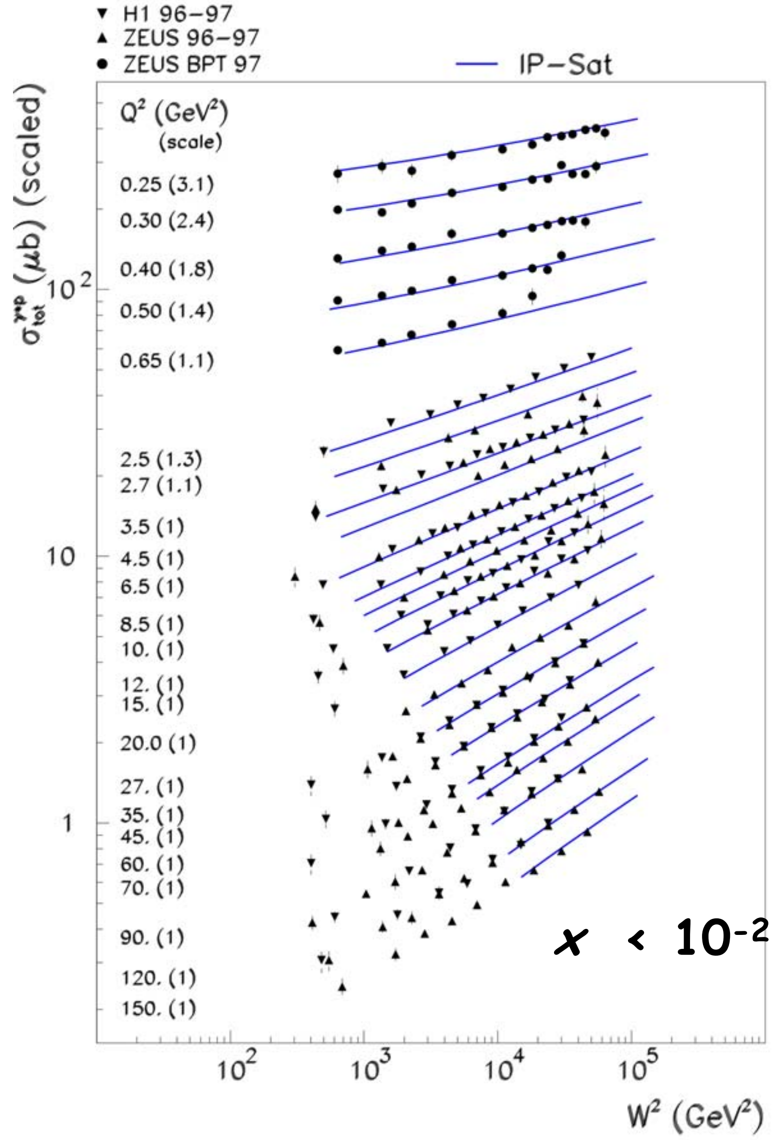
$$\Omega = \frac{\pi^2}{N_C} r^2 \alpha_s(\mu^2) x g(x,\mu^2) T(b)$$

$$\frac{d\sigma_{VM}^{\gamma^* p}}{dt} = \frac{1}{16\pi} \left| \int d^2\vec{r} \int d^2 b\, e^{-i\vec{b}\cdot\vec{\Delta}} \int_0^1 dz \Psi_{VM}^*\ 2\left\{1 - \exp\left(-\frac{\Omega}{2}\right)\right\} \Psi \right|^2$$

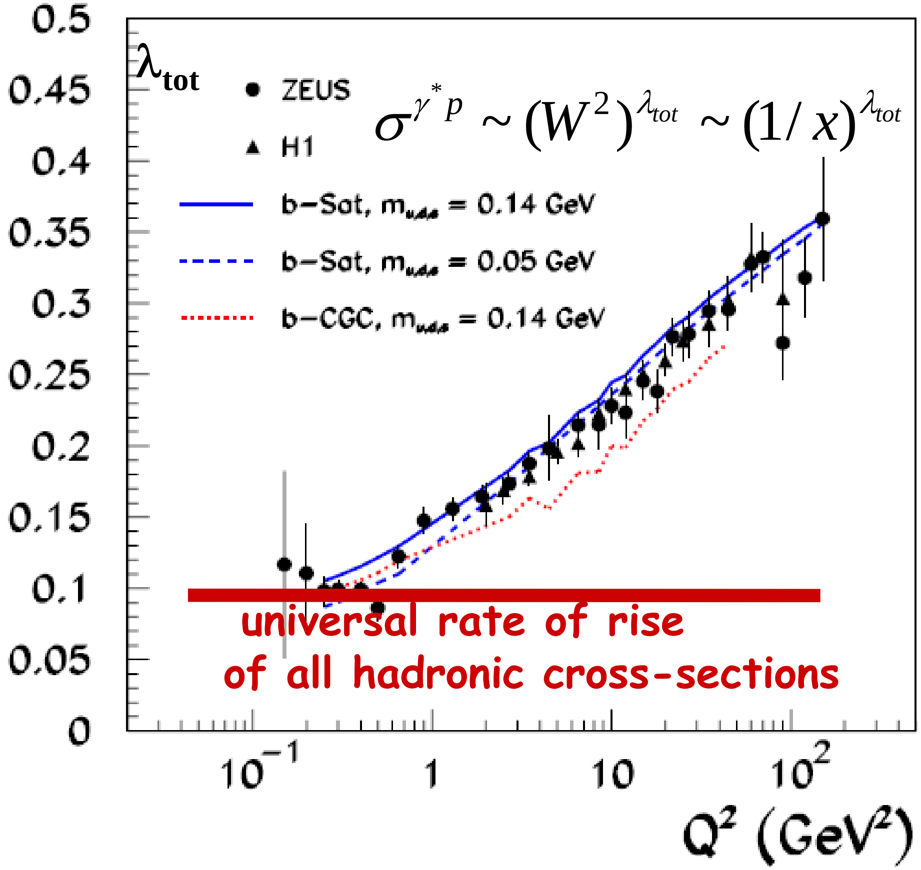
# Total $\gamma^* p$ cross-section

KMW

KT



$x < 10^{-2}$



$$\sigma^{\gamma^* p} \sim (W^2)^{\lambda_{tot}} \sim (1/x)^{\lambda_{tot}}$$

universal rate of rise
of all hadronic cross-sections

$$\mu^2 = \frac{C}{r^2} + \mu_0^2$$

$$xg(x,\mu_0^2) = A_g \left(\frac{1}{x}\right)^{\lambda_g} (1-x)^{5.6}$$

# Fits to $F_2$ with the b-Sat model

$$\mu^2 = \frac{C}{r^2} + \mu_0^2 \qquad\qquad xg(x,\mu_0^2) = A_g \left(\frac{1}{x}\right)^{\lambda_g} (1-x)^{5.6}$$

| Model | $T(b)$ | $Q^2/\mathrm{GeV}^2$ | $m_{u,d,s}/\mathrm{GeV}$ | $m_c/\mathrm{GeV}$ | $\mu_0^2/\mathrm{GeV}^2$ | $A_g$ | $\lambda_g$ | $\chi^2/\mathrm{d.o.f.}$ |
|---|---|---|---|---|---|---|---|---|
| b-Sat | Gaussian | [0.25,650] | 0.14 | 1.4 | 1.17 | 2.55 | 0.020 | $193.0/160 = 1.21$ |
| b-Sat | Gaussian | [0.25,650] | 0.14 | 1.35 | 1.20 | 2.51 | 0.024 | $190.2/160 = 1.19$ |
| b-Sat | Gaussian | [0.25,650] | 0.14 | 1.5 | 1.11 | 2.64 | 0.011 | $198.1/160 = 1.24$ |
| b-Sat | Gaussian | [0.25,650] | 0.05 | 1.4 | 0.77 | 3.61 | −0.118 | $144.7/160 = 0.90$ |
| b-Sat | Step | [0.25,650] | 0.14 | 1.4 | 1.50 | 2.20 | 0.071 | $199.6/160 = 1.25$ |

$$\frac{\partial xg(x,\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_S(\mu^2)}{2\pi} \int_x^1 \mathrm{d}z\, P_{gg}(z) \frac{x}{z} g\left(\frac{x}{z}, \mu^2\right)$$

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\boldsymbol{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_S(\mu^2) xg(x,\mu^2) T(b)\right)\right] \qquad T_G(b) = \frac{1}{2\pi B_G} \mathrm{e}^{-\frac{b^2}{2B_G}}$$

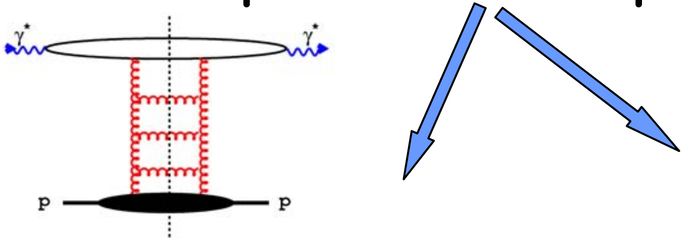
# Fits to $F_2$ with the b-CGC model

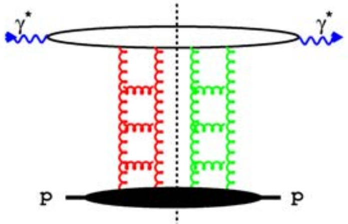
$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\boldsymbol{b}} \equiv 2\mathcal{N}(x,r,b) = 2 \times \begin{cases} \mathcal{N}_0 \left(\frac{rQ_s}{2}\right)^{2\left(\gamma_s + \frac{1}{\kappa\lambda Y}\ln\frac{2}{rQ_s}\right)} & : \quad rQ_s \leq 2 \\ 1 - \mathrm{e}^{-A\ln^2(BrQ_s)} & : \quad rQ_s > 2 \end{cases}$$

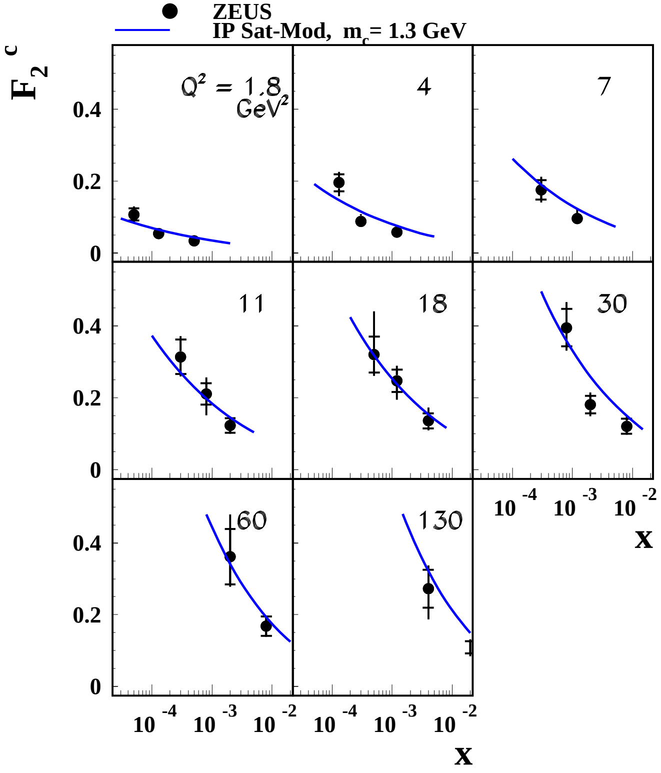
$$Q_s \equiv Q_s(x,b) = \left(\frac{x_0}{x}\right)^{\frac{\lambda}{2}} \left[\exp\left(-\frac{b^2}{2B_{\mathrm{CGC}}}\right)\right]^{\frac{1}{2\gamma_s}}$$

| Model | $Q^2/\mathrm{GeV}^2$ | $m_{u,d,s}/\mathrm{GeV}$ | $m_c/\mathrm{GeV}$ | $\mathcal{N}_0$ | $x_0/10^{-4}$ | $\lambda$ | $\chi^2/\mathrm{d.o.f.}$ |
|---|---|---|---|---|---|---|---|
| b-CGC | [0.25,45] | 0.14 | 1.4 | 0.417 | 5.95 | 0.159 | $211.2/130 = 1.62$ |

## Dipole Model - gluon density convoluted with dipole wave functions simultaneous prediction/description of many reactions

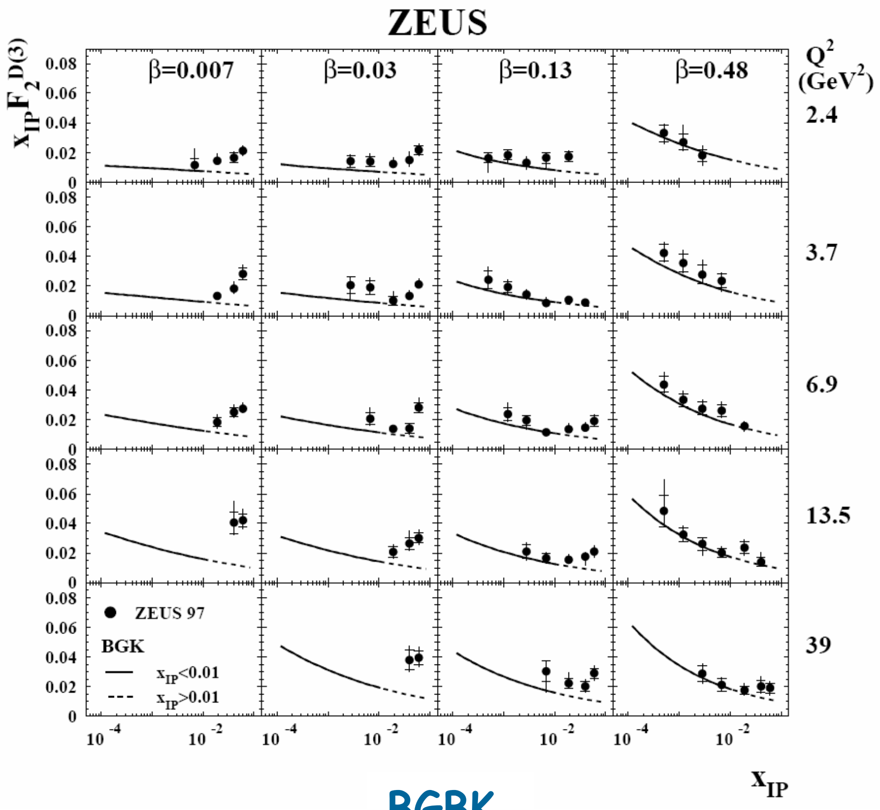






**KT**

*$F_2{}^c$*

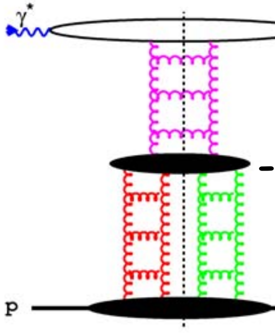


**BGBK**

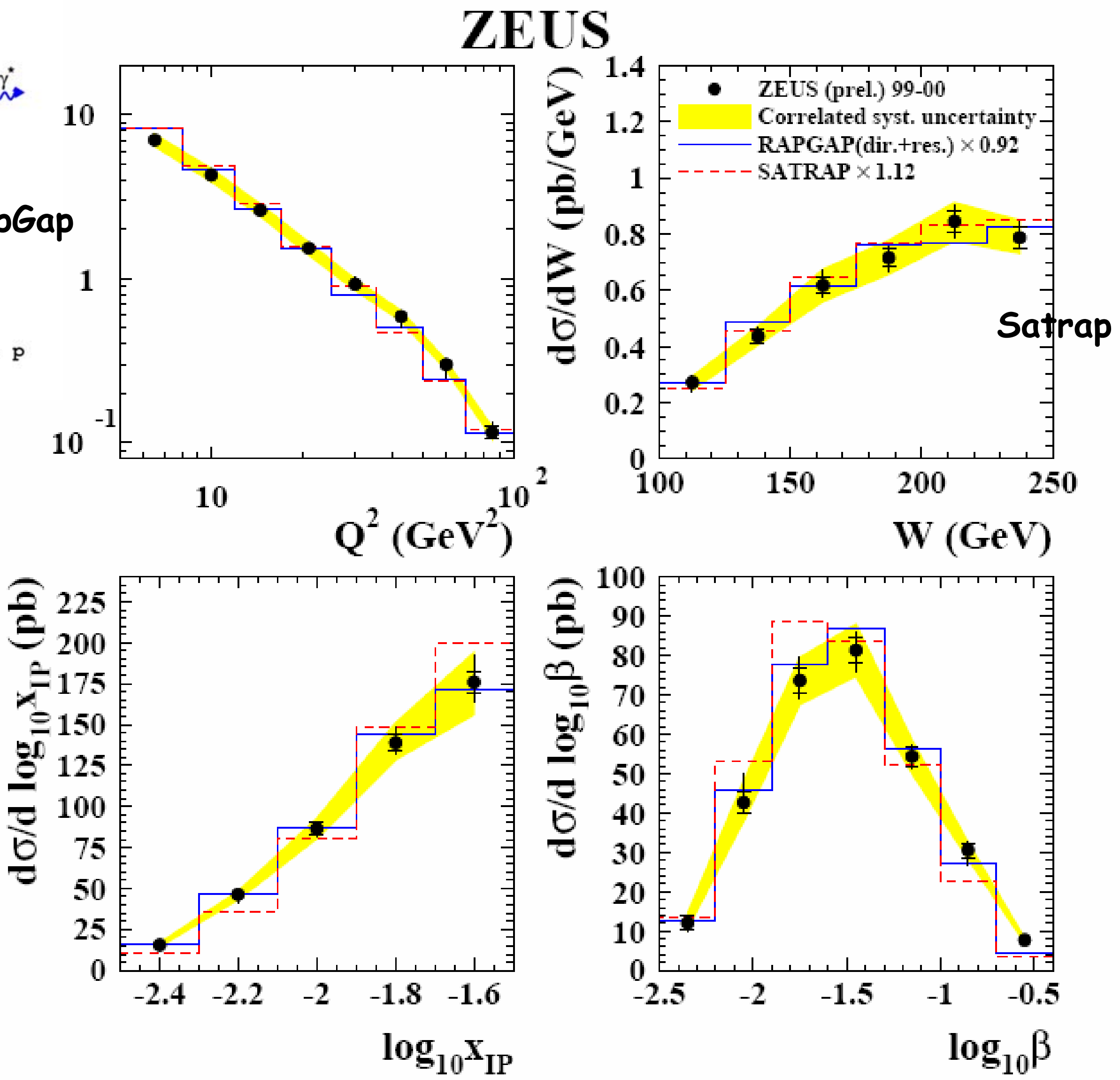
**Inclusive Diffraction**

# Diffractive Di-jets

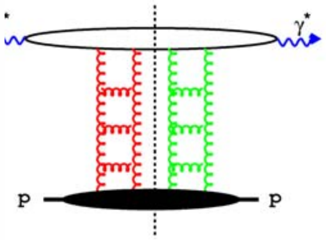
## $Q^2 > 5\ \mathrm{GeV}^2$

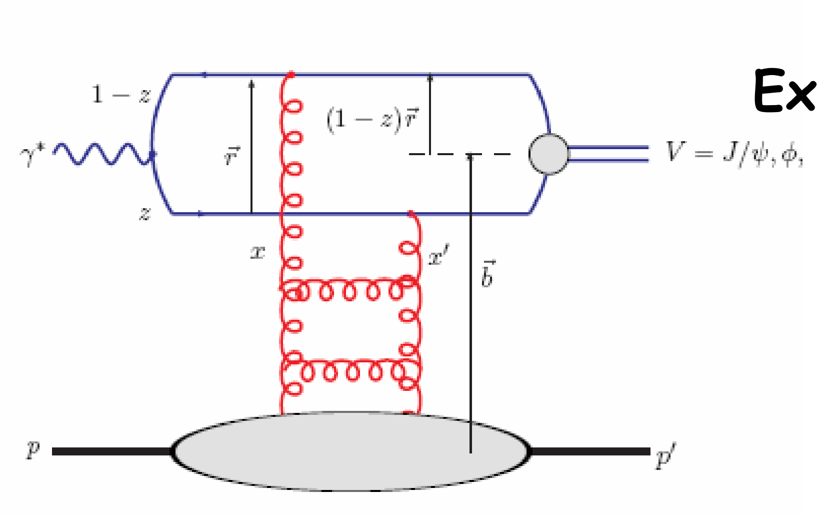


-RapGap



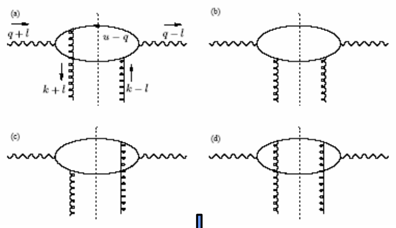
Satrap





# Exclusive Vector Meson Production

**H. Kowalski, L. Motyka, G. Watt**



**Effective modification of Fourier Trans by Bartels, Golec-Biernat, Peters**

$$\frac{d\sigma^{\gamma^* p\to Vp}_{T,L}}{dt} = \frac{1}{16\pi}\left|\mathcal{A}^{\gamma^* p\to Vp}_{T,L}\right|^2 = \frac{1}{16\pi}\left|\int d^2\boldsymbol{r}\int_0^1\frac{dz}{4\pi}\int d^2\boldsymbol{b}\,(\Psi_V^*\Psi)_{T,L}\,e^{-i[\boldsymbol{b}-(1-z)\boldsymbol{r}]\cdot\boldsymbol{\Delta}}\,\frac{d\sigma_{q\bar q}}{d^2\boldsymbol{b}}\right|^2(1+\beta^2)$$

$$\beta = \tan(\pi\lambda/2), \quad \text{with} \quad \lambda \equiv \frac{\partial\ln\left(\mathcal{A}^{\gamma^* p\to Vp}_{T,L}\right)}{\partial\ln(1/x)}.$$

**Real part correction**

$$\boxed{\frac{d\sigma}{d^2b} = 2(1-\exp(-\Omega/2))}$$

$$\Omega = \frac{\pi^2}{N_C} r^2\alpha_s(\mu^2)xg(x,\mu^2)R_g T(b)$$

**Skewedness correction Martin, Ryskin Teubner**

$$R_g(\lambda) = \frac{2^{2\lambda+3}}{\sqrt{\pi}}\frac{\Gamma(\lambda+5/2)}{\Gamma(\lambda+4)}, \quad \text{with} \quad \lambda \equiv \frac{\partial\ln[xg(x,\mu^2)]}{\partial\ln(1/x)}.$$

# Wave Functions

## WF Overlaps

$$(\Psi_V^*\Psi)_T = \hat{e}_f e \frac{N_C}{\pi z(1-z)} \left\{ m_f^2 K_0(\epsilon r)\phi_T(r,z) - \left[z^2 + (1-z)^2\right] \epsilon K_1(\epsilon r)\partial_r \phi_T(r,z) \right\}$$

$$(\Psi_V^*\Psi)_L = \hat{e}_f e \frac{N_C}{\pi M_V} 2Q K_0(\epsilon r) \left\{ \left[z(1-z)M_V^2 + m_f^2\right] \phi_L(r,z) - \nabla_r^2 \phi_L(r,z) \right\},$$

## Boosted Gaussian – NNPZ, FKS, FS

Gaussian distribution of quark 3-momentum in the meson rest frame

then boosted to LC

$$p^2 = \frac{k^2 + m_f^2}{4z(1-z)} - m_f^2$$

$$\phi_{T,L}(r,z) = \mathcal{N}_{T,L} 4z(1-z)\sqrt{2\pi\mathcal{R}^2} \exp\left( -\frac{m_f^2\mathcal{R}^2}{8z(1-z)} - \frac{2z(1-z)r^2}{\mathcal{R}^2} + \frac{m_f^2\mathcal{R}^2}{2} \right)$$

## Gauss LC - KT

Gaussian distribution of quark 2-momentum in LC, factorization of r, z components

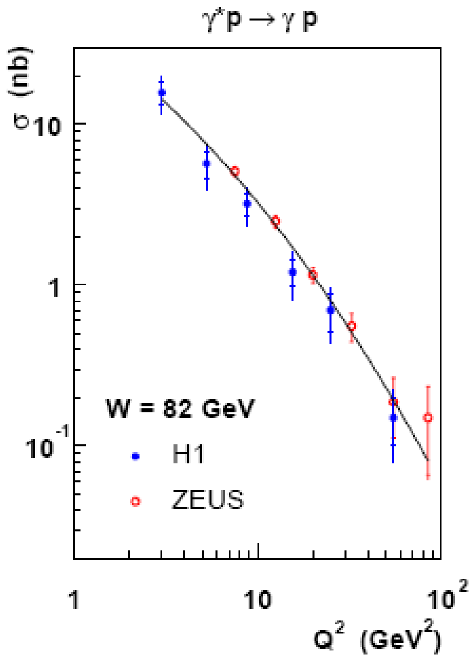
- strong endpoint suppression in $\phi_T$

$$\varphi_L(r,z) = \frac{N}{2\pi R_L^2} z(1-z)\exp\left(-\frac{r^2}{2R_L^2}\right) \qquad \varphi_T(r,z) = \frac{N}{2\pi R_T^2} z^2(1-z)^2 \exp\left(-\frac{r^2}{2R_T^2}\right)$$

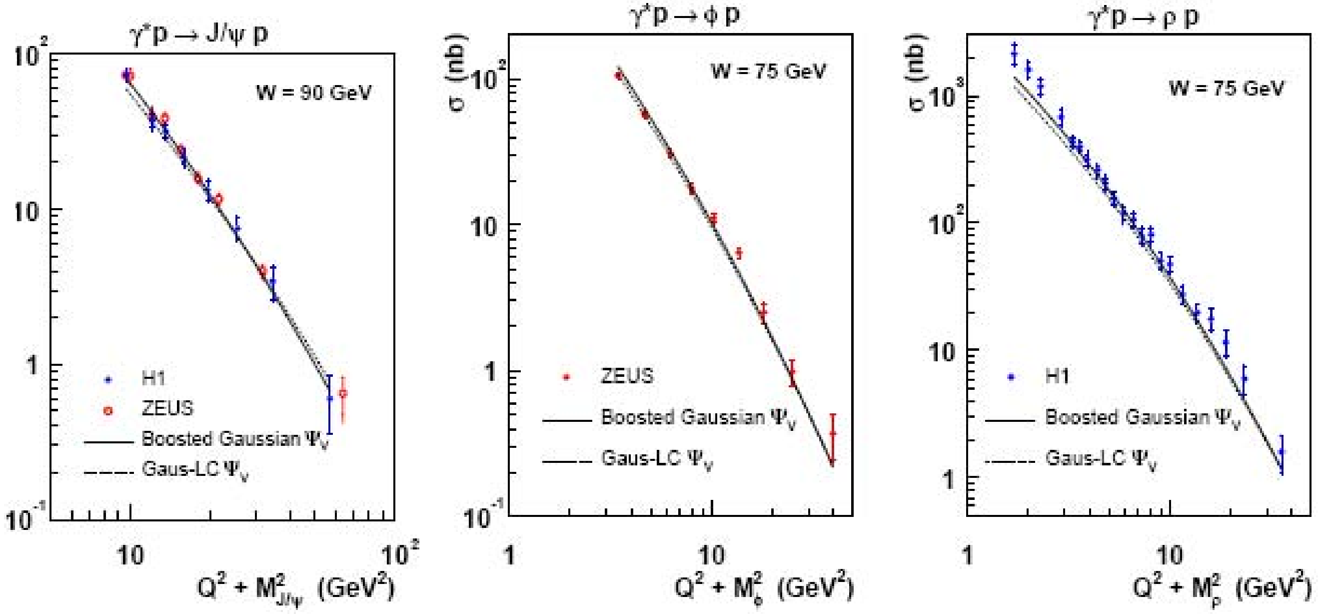
**Parameters of WF fixed by normalization conditions and the values of mesons decay constant, $f_V$**

from gluon density convoluted with dipole wave functions we obtain simultaneous prediction/description of many reactions

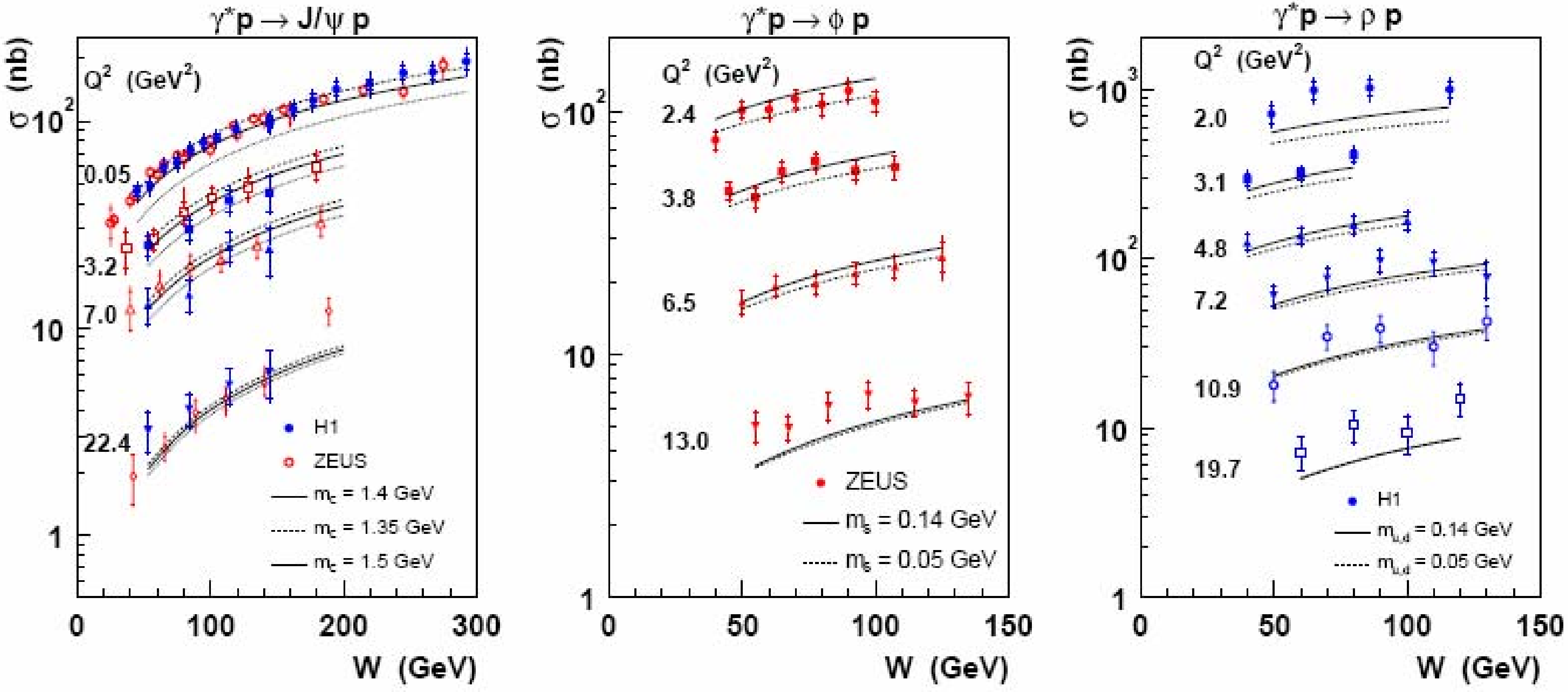
## DVCS

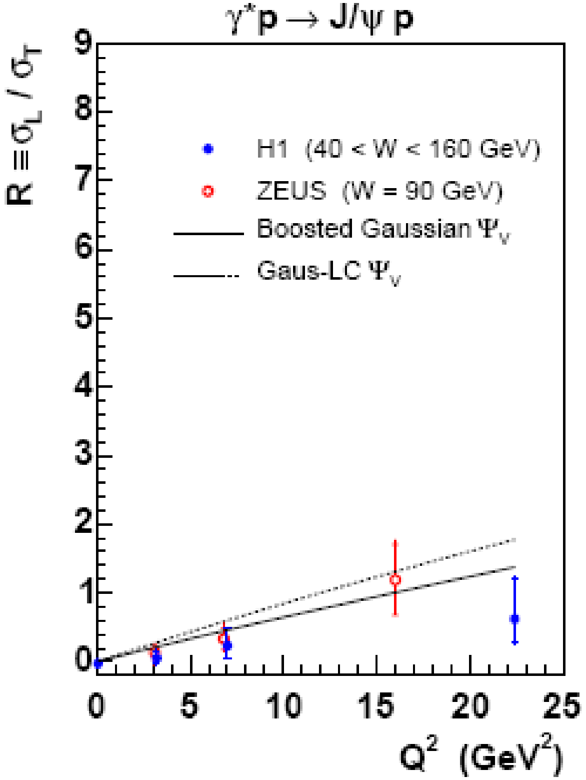


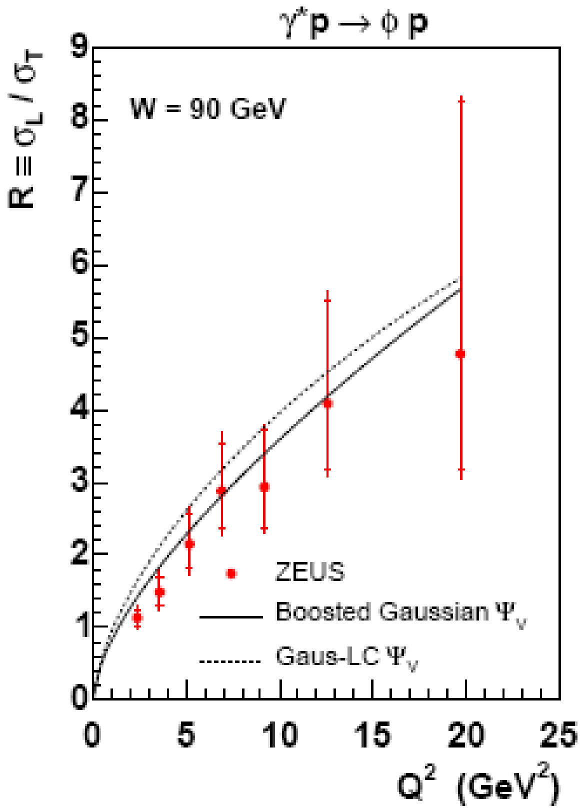
## Vector Mesons

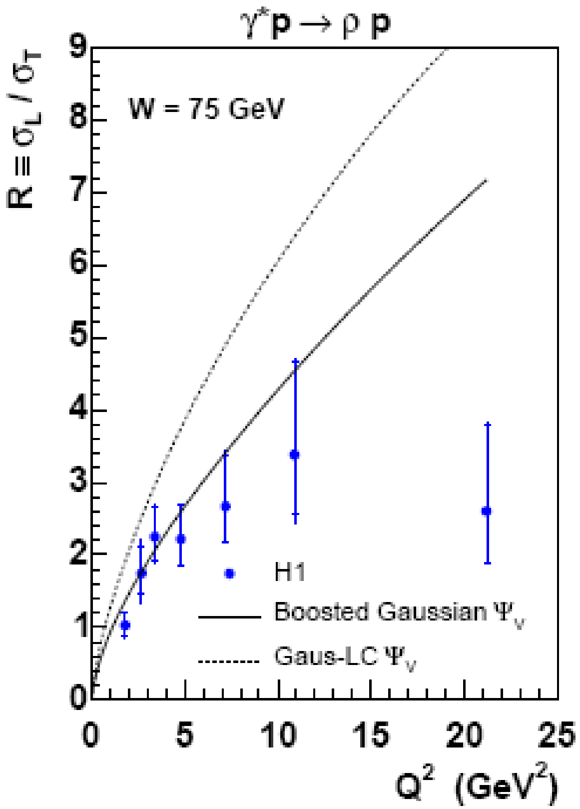


Note: educated guesses for VM wf work surprisingly well

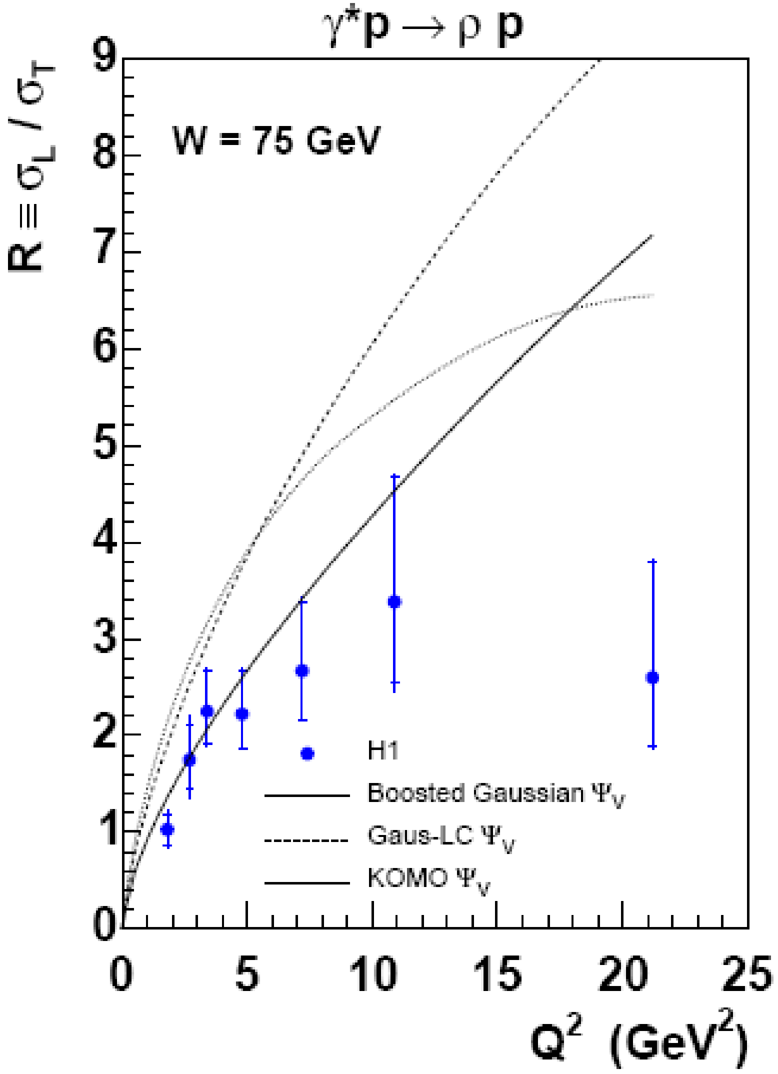

γ*p → J/ψ p
σ (nb)
Q² (GeV²)
0.05
3.2
7.0
22.4
H1
ZEUS
m_c = 1.4 GeV
m_c = 1.35 GeV
m_c = 1.5 GeV
1
10
10²
0
100
200
300
W (GeV)
γ*p → φ p
σ (nb)
Q² (GeV²)
2.4
3.8
6.5
13.0
ZEUS
m_s = 0.14 GeV
m_s = 0.05 GeV
1
10
10²
0
50
100
150
W (GeV)
γ*p → ρ p
σ (nb)
Q² (GeV²)
2.0
3.1
4.8
7.2
10.9
19.7
H1
m_u,d = 0.14 GeV
m_u,d = 0.05 GeV
1
10
10²
10³
0
50
100
150
W (GeV)


$\gamma^* p \to J/\psi\ p$
$R \equiv \sigma_L / \sigma_T$
0
1
2
3
4
5
6
7
8
9
H1 (40 < W < 160 GeV)
ZEUS (W = 90 GeV)
Boosted Gaussian $\Psi_V$
Gaus-LC $\Psi_V$
0
5
10
15
20
25
$Q^2$ (GeV$^2$)


$\gamma^* p \to \phi\ p$
$R \equiv \sigma_L / \sigma_T$
0
1
2
3
4
5
6
7
8
9
W = 90 GeV
ZEUS
Boosted Gaussian $\Psi_V$
Gaus-LC $\Psi_V$
0
5
10
15
20
25
$Q^2$ (GeV$^2$)

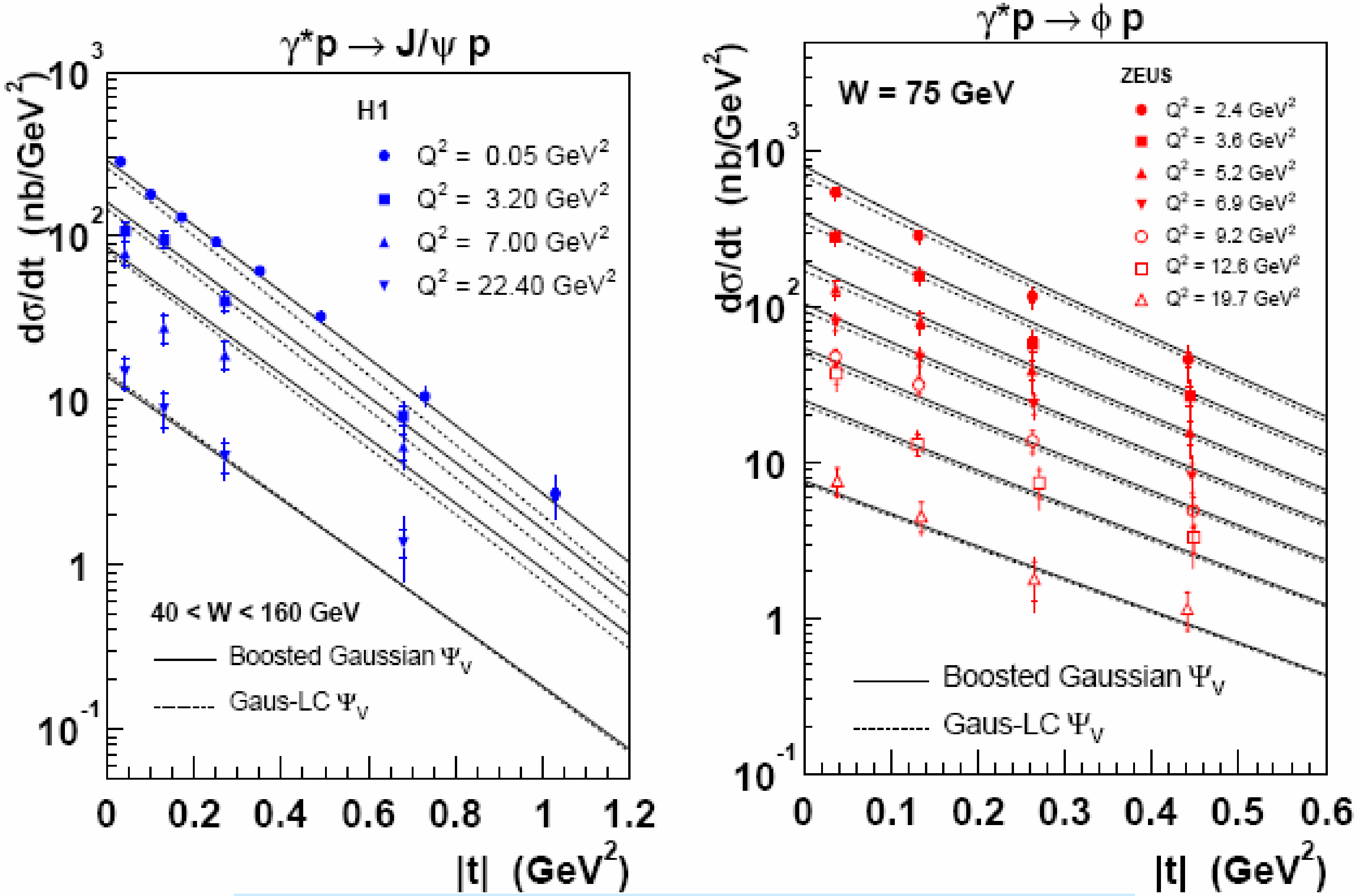

$\gamma^* p \to \rho\ p$
$R \equiv \sigma_L / \sigma_T$
0
1
2
3
4
5
6
7
8
9
W = 75 GeV
H1
Boosted Gaussian $\Psi_V$
Gaus-LC $\Psi_V$
0
5
10
15
20
25
$Q^2$ (GeV$^2$)

# Sensitivity to end points suppression of the ρ wave function



$$\varphi_{T,L}^{KOMO}(r,z) = \frac{N}{2\pi R_L^2}\sqrt{z(1-z)}\,z(1-z)\exp(-\frac{r^2}{2R_L^2})$$

# Exponential fall of t-distributions



$$\frac{d\sigma^{diff}}{dt} \sim \exp(B_D \cdot t) \qquad \Rightarrow T(b) \sim \exp(-\vec{b}^{\,2} / 2B_G)$$

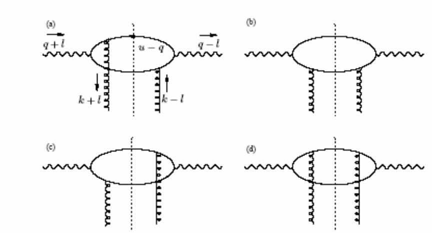
➔ **gaussian shape of the proton in the impact parameter b**

# Description of the size of interaction region $B_D$

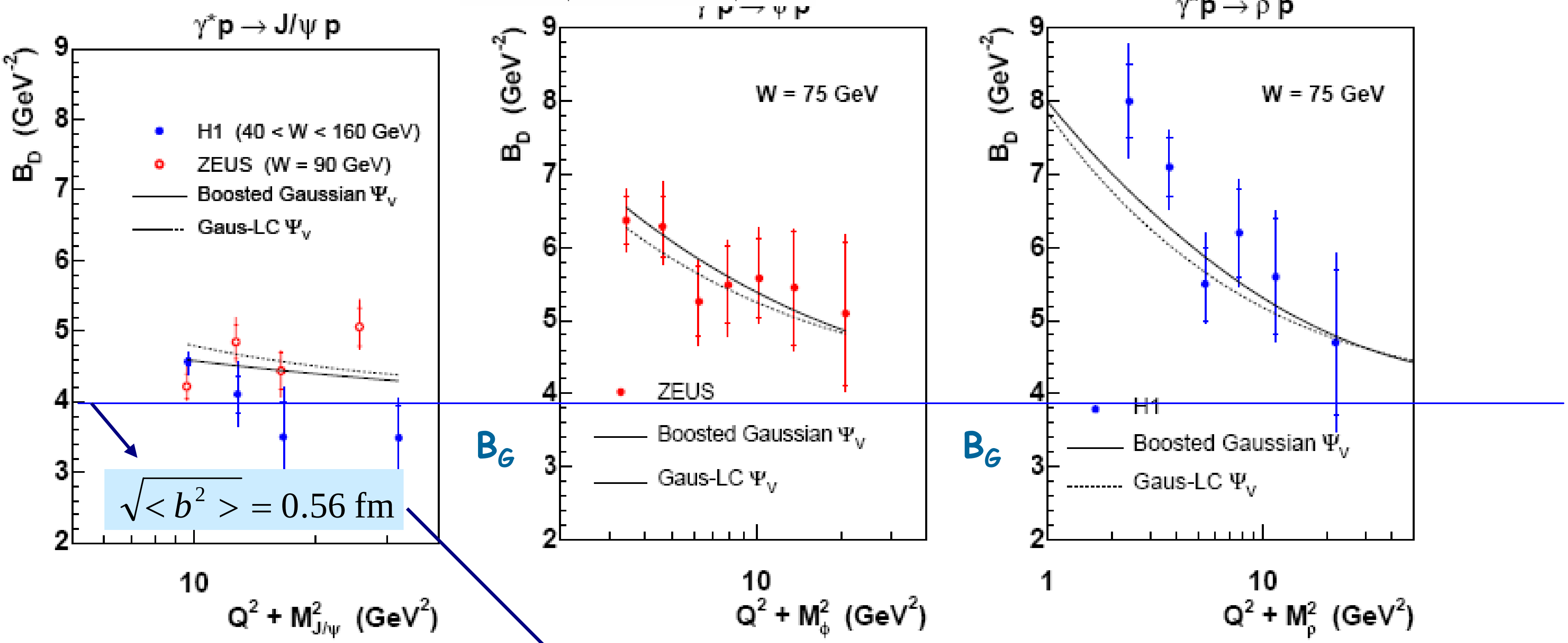
$$\frac{d\sigma^{diff}}{dt} \sim \exp(B_D \cdot t) \qquad \Rightarrow T(b) \sim \exp(-\vec{b}^2 / 2B_G)$$

**Modification by Bartels, Golec-Biernat, Peters**

$$e^{i\vec{b}\cdot\vec{\Delta}} \rightarrow e^{i(\vec{b}+(1-z)\vec{r})\cdot\vec{\Delta}}$$



**KMW**



$B_G$

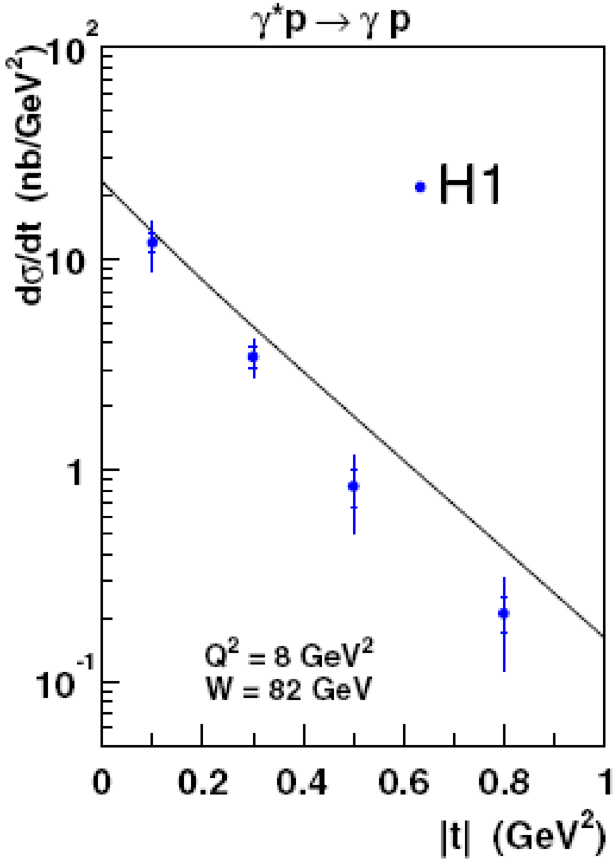
$B_G$

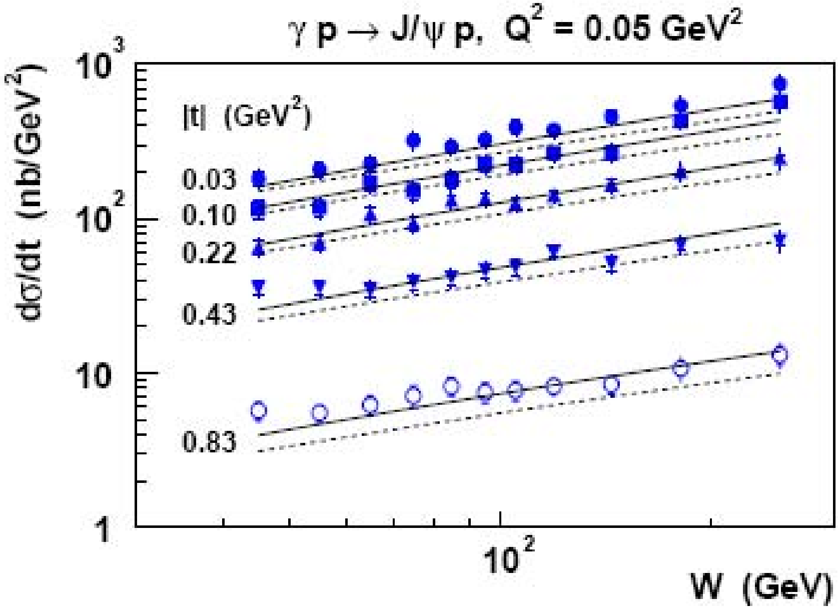
$$\sqrt{<b^2>} = 0.56 \text{ fm}$$

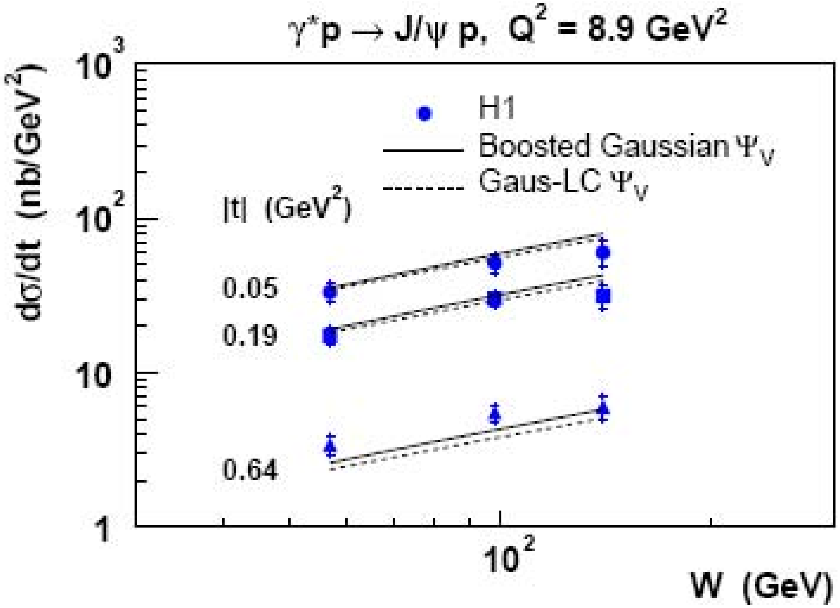
$$R_p = 0.870 \pm 0.008 \text{ fm}$$

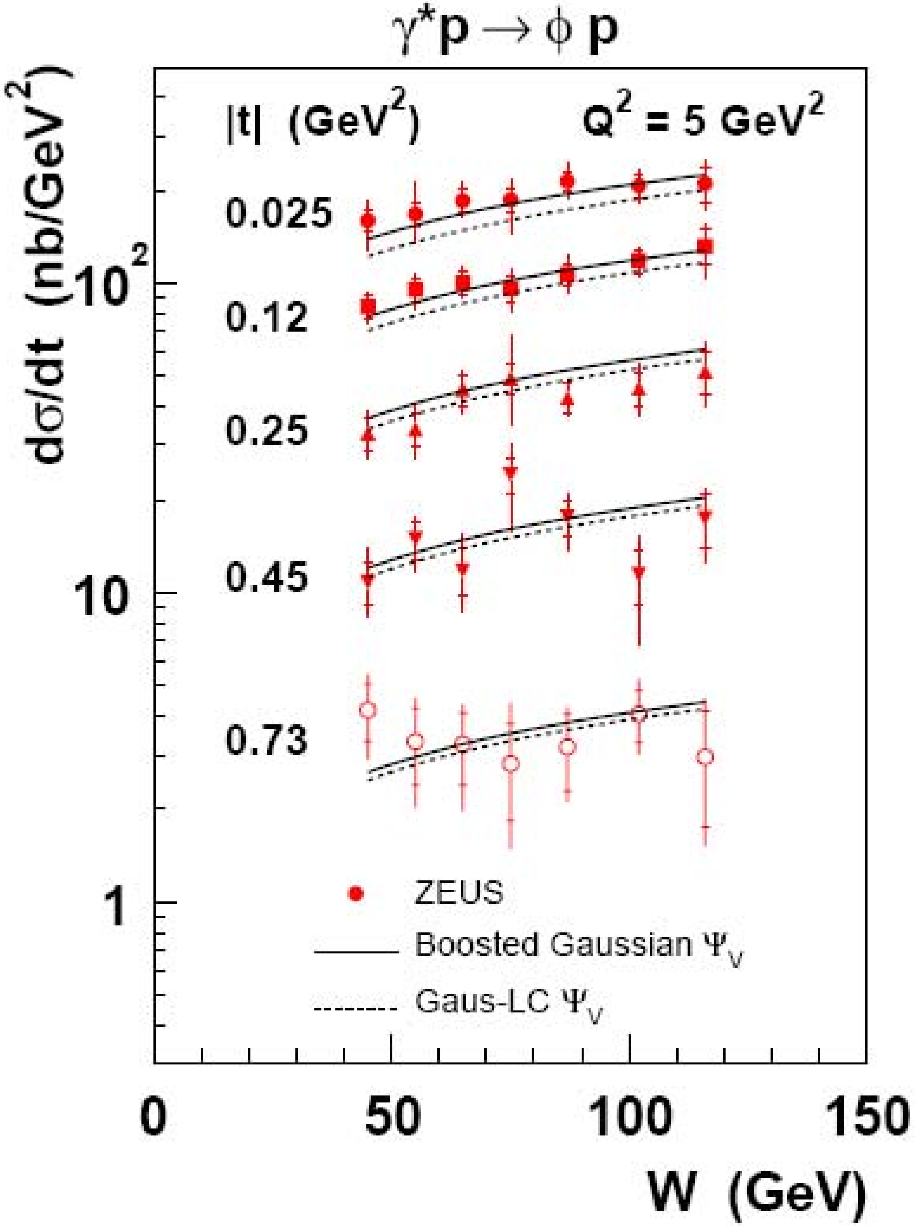
**the gluonic proton radius smaller than the quark radius**

# t-distributions of DVCS



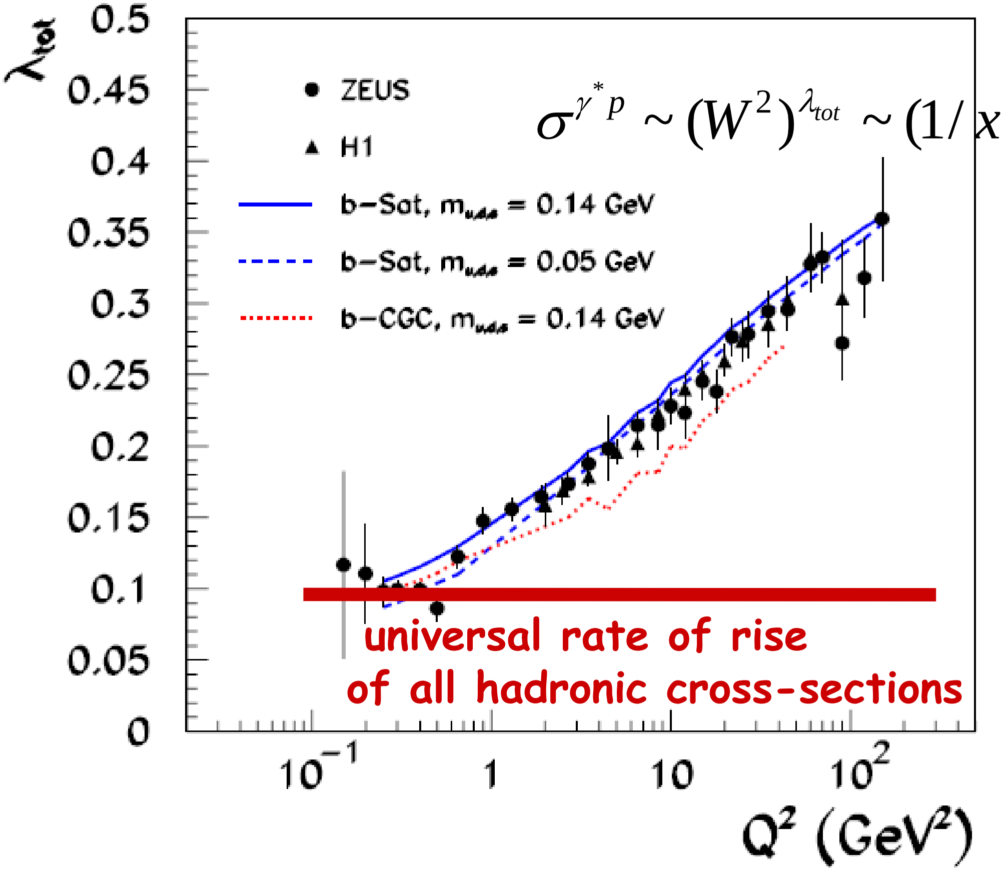

$\gamma p \rightarrow J/\psi p$, $Q^2 = 0.05$ GeV$^2$
dσ/dt (nb/GeV$^2$)
$10^3$
$10^2$
10
1
|t| (GeV$^2$)
0.03
0.10
0.22
0.43
0.83
$10^2$
W (GeV)


$\gamma^* p \rightarrow J/\psi p$, $Q^2 = 8.9$ GeV$^2$
dσ/dt (nb/GeV$^2$)
$10^3$
$10^2$
10
1
H1
Boosted Gaussian $\Psi_V$
Gaus-LC $\Psi_V$
|t| (GeV$^2$)
0.05
0.19
0.64
$10^2$
W (GeV)


$\gamma^* p \rightarrow \phi p$
dσ/dt (nb/GeV$^2$)
|t| (GeV$^2$)
$Q^2 = 5$ GeV$^2$
0.025
0.12
0.25
0.45
0.73
$10^2$
10
1
ZEUS
Boosted Gaussian $\Psi_V$
Gaus-LC $\Psi_V$
0
50
100
150
W (GeV)

# What have we learnt from HERA about small-x

Rate of rise of the $\gamma^* p$ cross-section

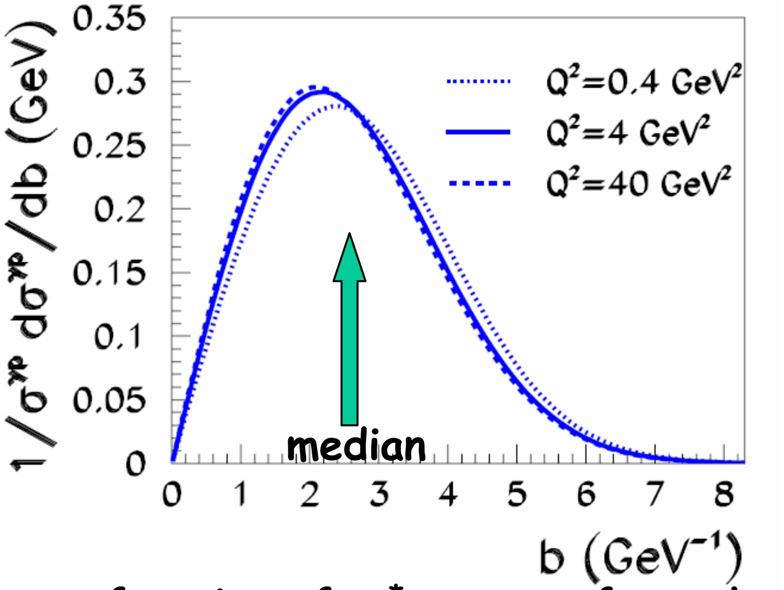


$$\sigma^{\gamma^* p} \sim (W^2)^{\lambda_{tot}} \sim (1/x)^{\lambda_{tot}}$$

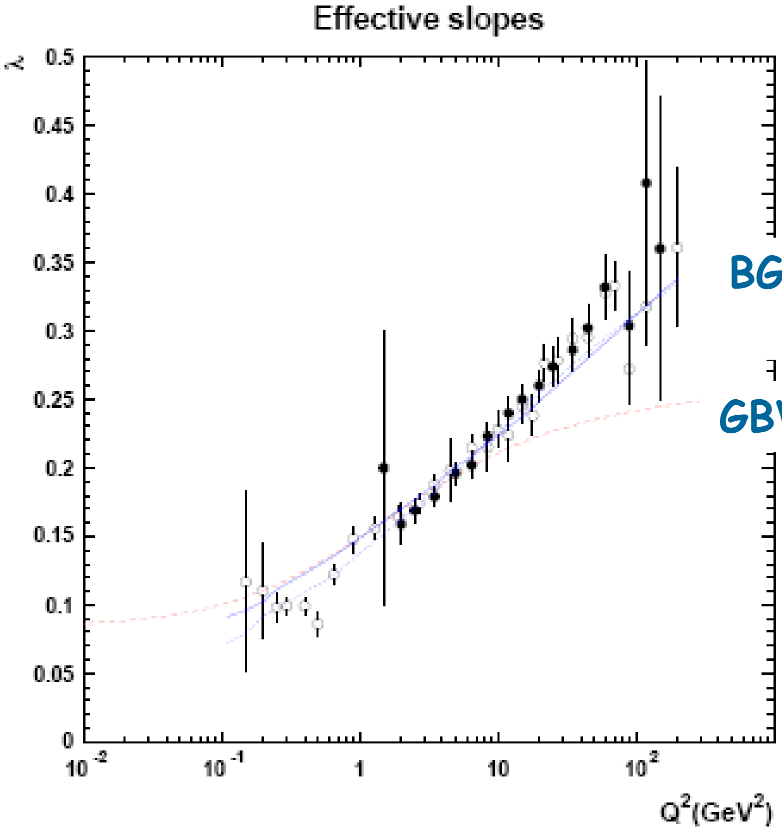
universal rate of rise
of all hadronic cross-sections

In the impact parameter dipole models of DIS with the gaussian proton shape the fit of the rate of rise of $\sigma^{\gamma^* p}$ requires QCD evolution which is DGLAP-like

DGLAP-like ←→ strong interdependence between $\lambda$ and $Q^2$



Large fraction of $\sigma^{\gamma^* p}$ comes from the region of large *b* where matter density is low $T(b) \sim \exp(-b^2_{MEDIAN} / 2 \cdot B_G) \approx 40\%$



BGBK – DGLAP evolution
& saturation

GBW - saturation only

# Saturation scale

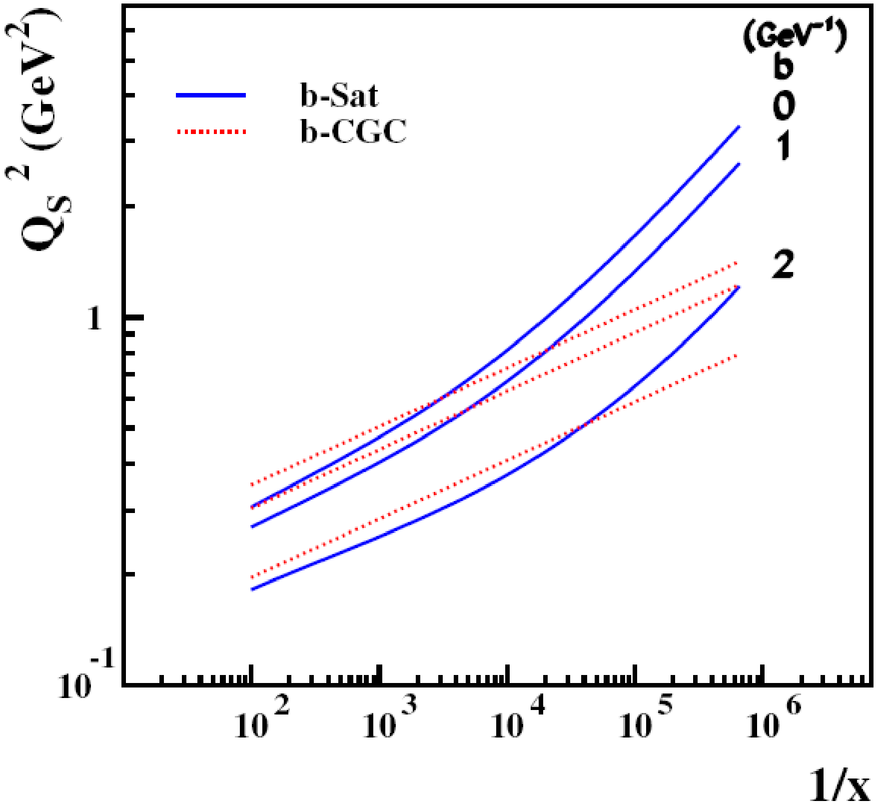
(a measure of gluon density at which gluon re-scattering starts to be substantial)

$$\frac{d\sigma_{qq}(x,r)}{d^2b} = 2\cdot\left\{1-\exp\left(-\frac{\pi^2}{2\cdot 3}r^2\alpha_s xg(x,C/r^2+Q_0^2)T(b)\right)\right\}$$

$$(Q_S^2)_g = \frac{N_C}{C_F}(Q_S^2)_q = \frac{9}{4}(Q_S^2)_q$$

$$Q_S^2 = \frac{2}{r_S^2}$$

## HERA



## RHIC

$$Q_S^2 = \frac{4\pi^2\alpha_s N_C}{N_C^2-1}\frac{1}{\pi R^2}\frac{dN}{dy}$$

$$\frac{dN}{dy}\approx 1000 \qquad R\approx 7\ \mathrm{fm}$$

$$Q_S^2 \approx 1\ \mathrm{GeV}^2$$

$Q_S^{RHIC} \sim Q_S^{HERA}$

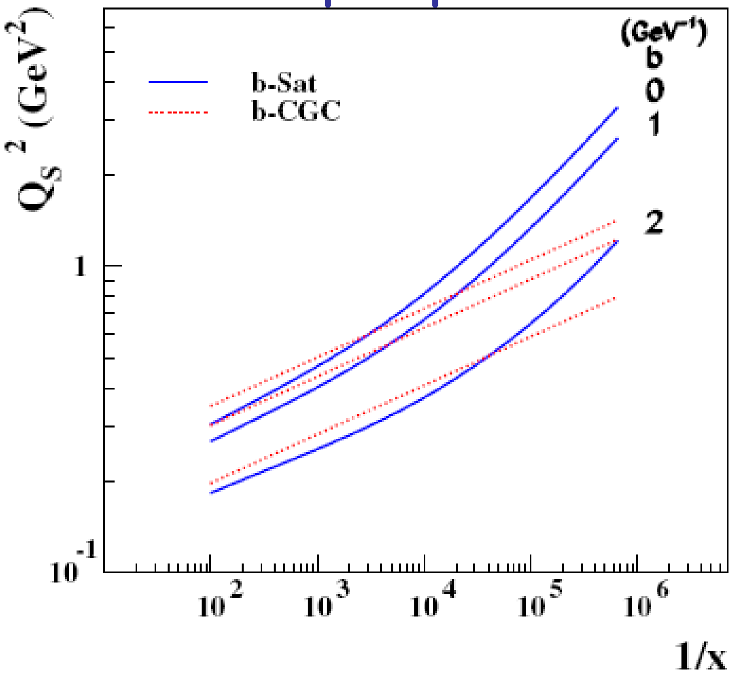
# More about saturation

$$Q_S^2 \equiv 2/r_S^2$$

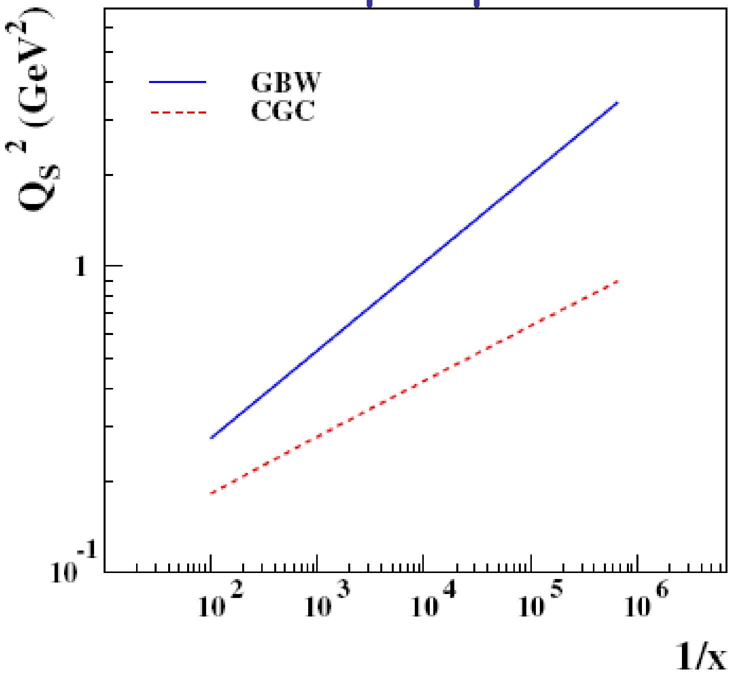
$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\boldsymbol{b}} \equiv 2\mathcal{N}(x, r, b)$$

$$\mathcal{N}(x, r_S, b) = 1 - \mathrm{e}^{-(1/2)}$$

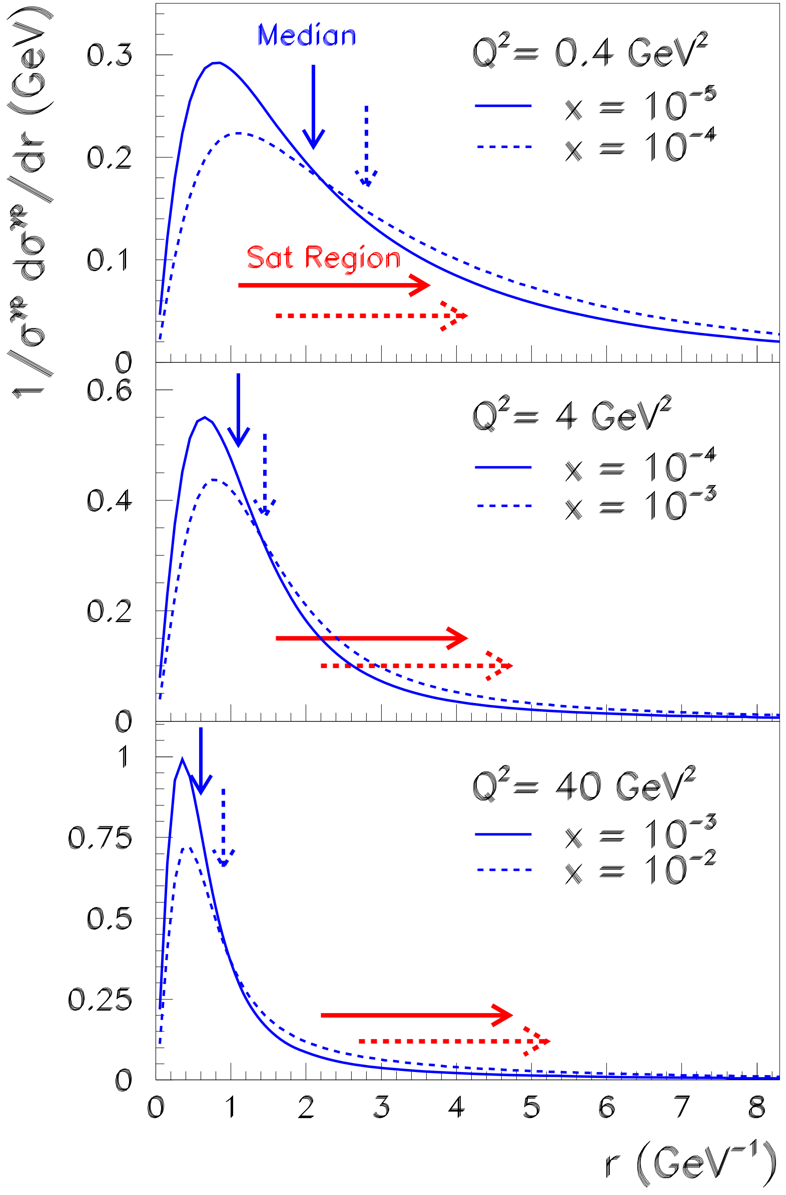
with impact parameter



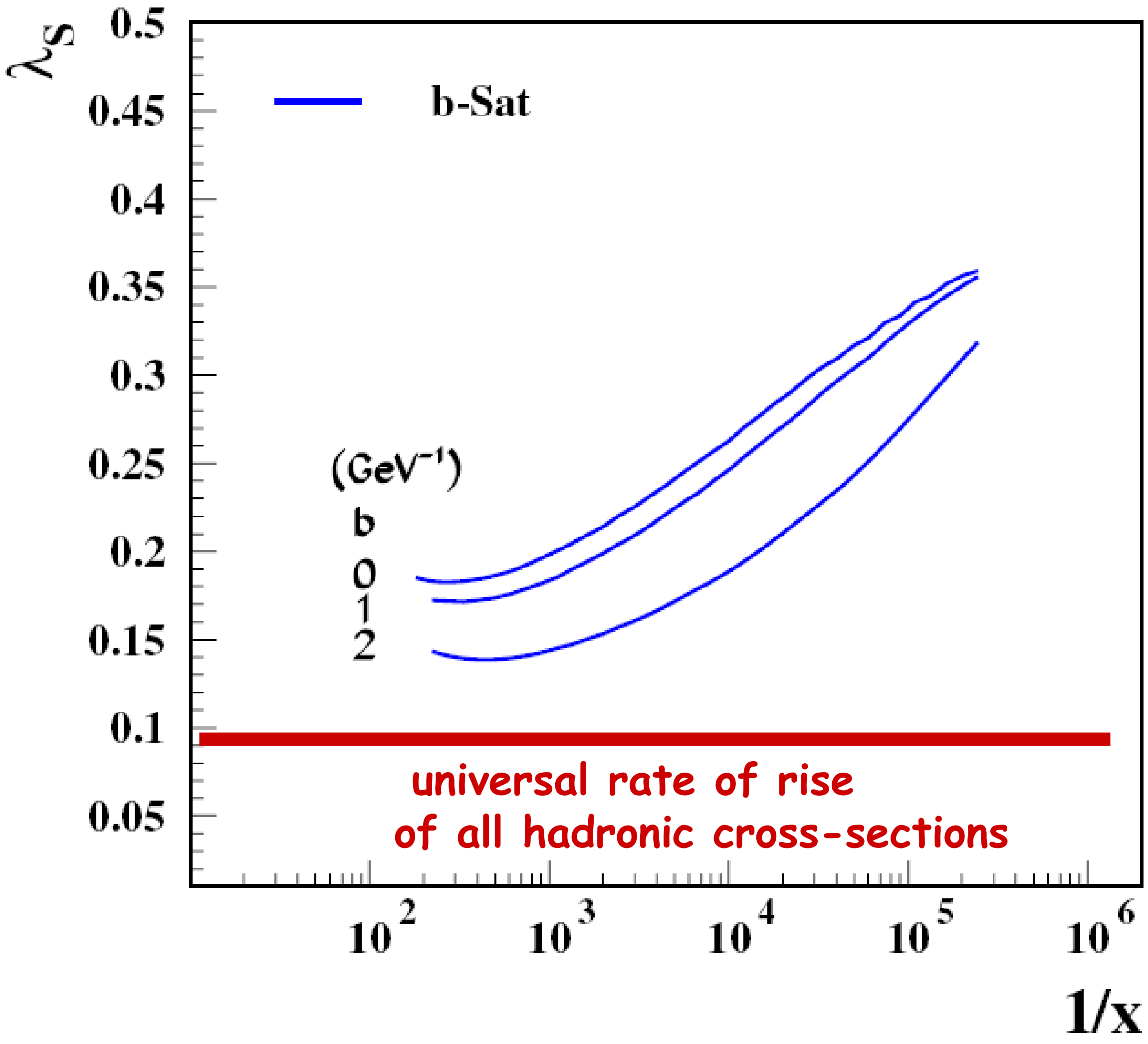
without impact parameter



# Is saturated state observed at HERA perturbative?



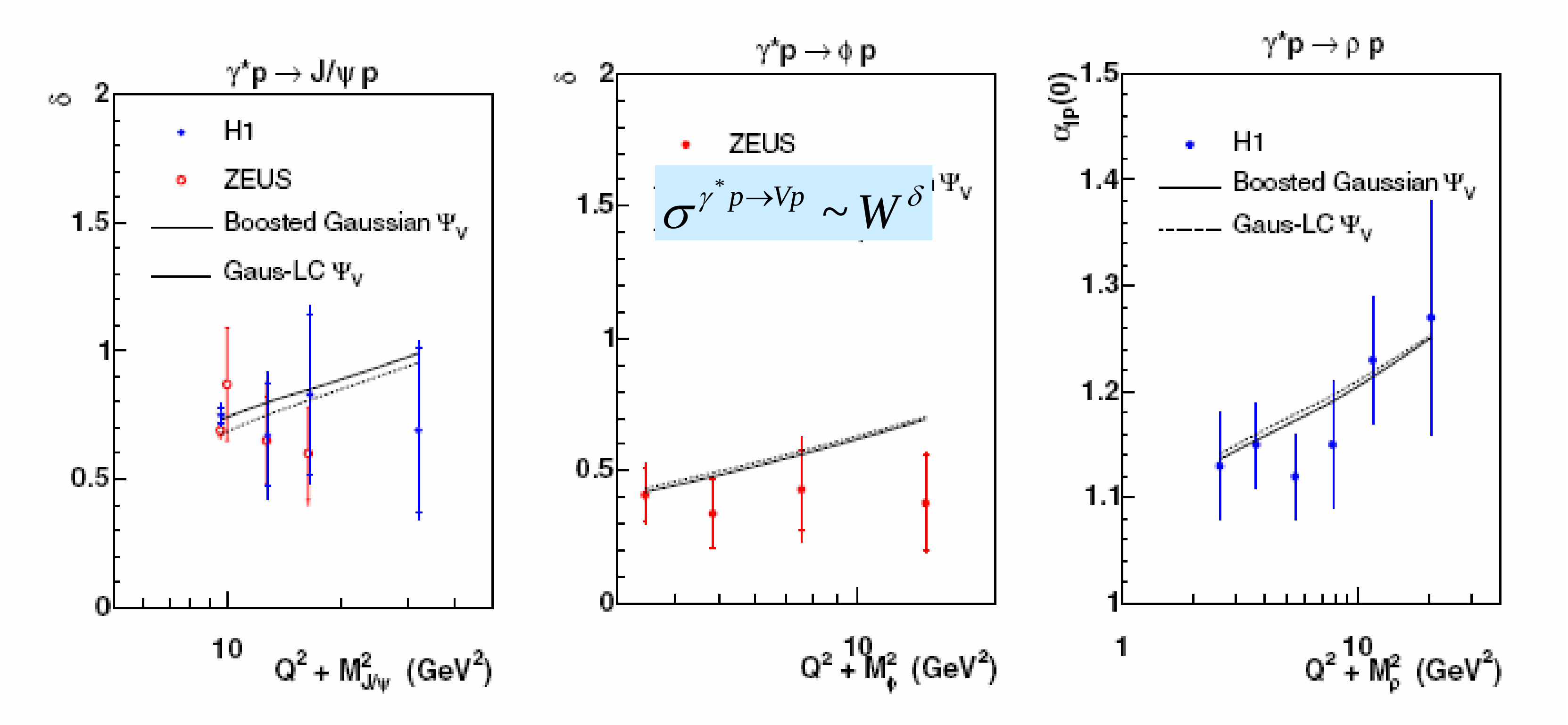
# Is saturated state observed at HERA perturbative?



KMW

# properties of gluon density

## Rates of rise of the VM cross-sections



$$\sigma^{\gamma^* p \to Vp} \sim W^{\delta}$$

At EIC it should be possible to reduce the errors by a large factor, O(100), ➔ study of t-dependent rate of rise

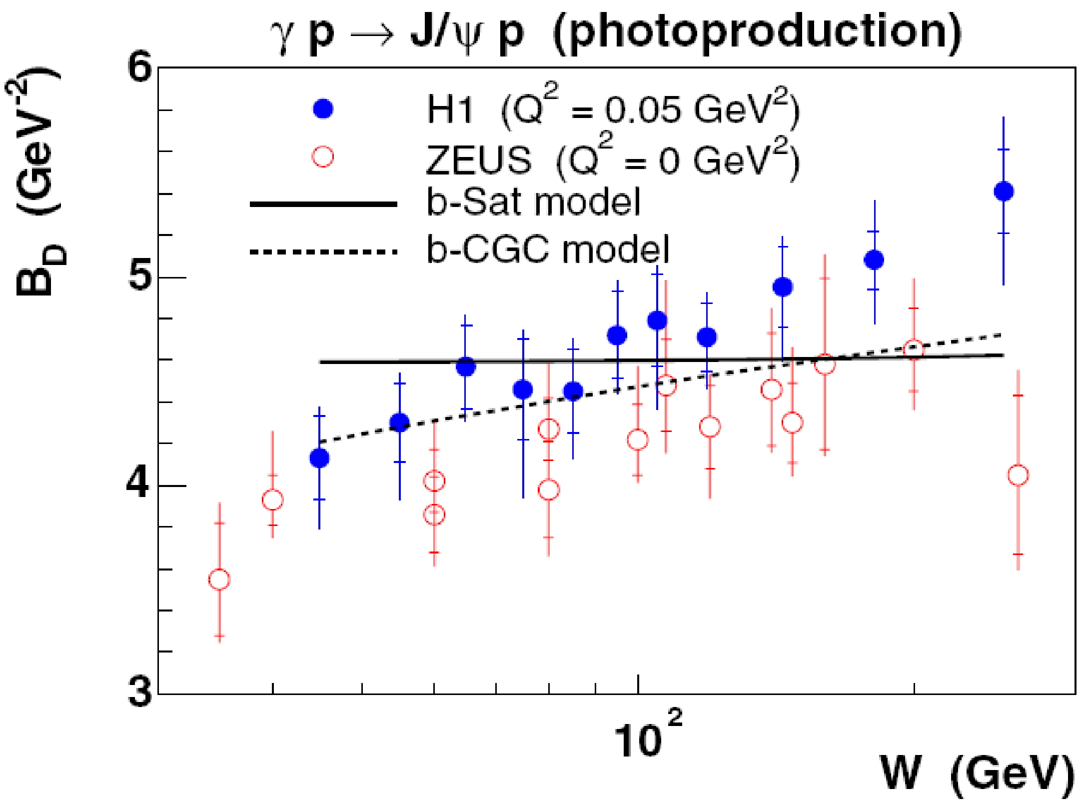
➔

study of b-dependent Pomeron evolution

- direct insight into saturation inside the proton or nuclei

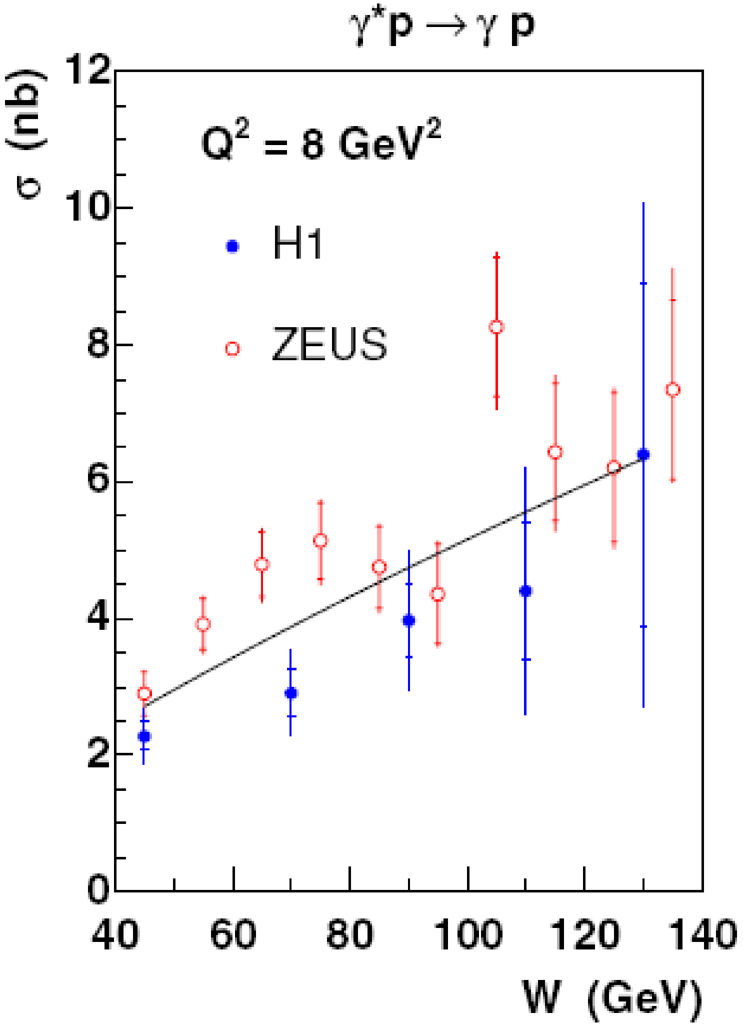
# properties gluon density

## Random walk in the impact parameter



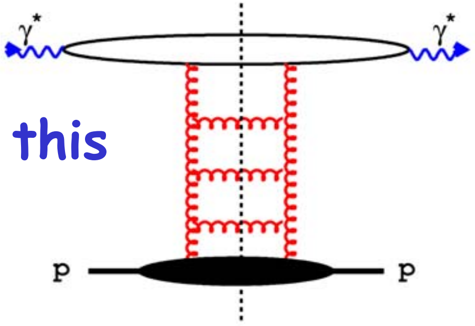
# properties of the gluon density

## Rise of the DVCS cross-sections

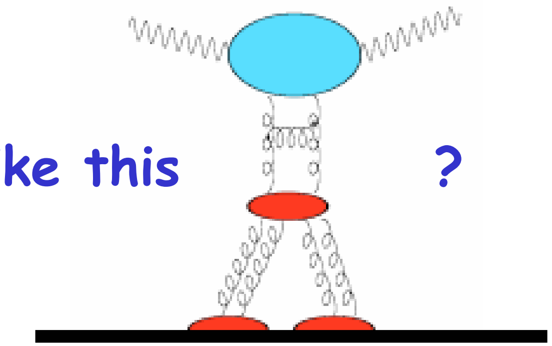


# More about the Pomeron

Is Pomeron like this

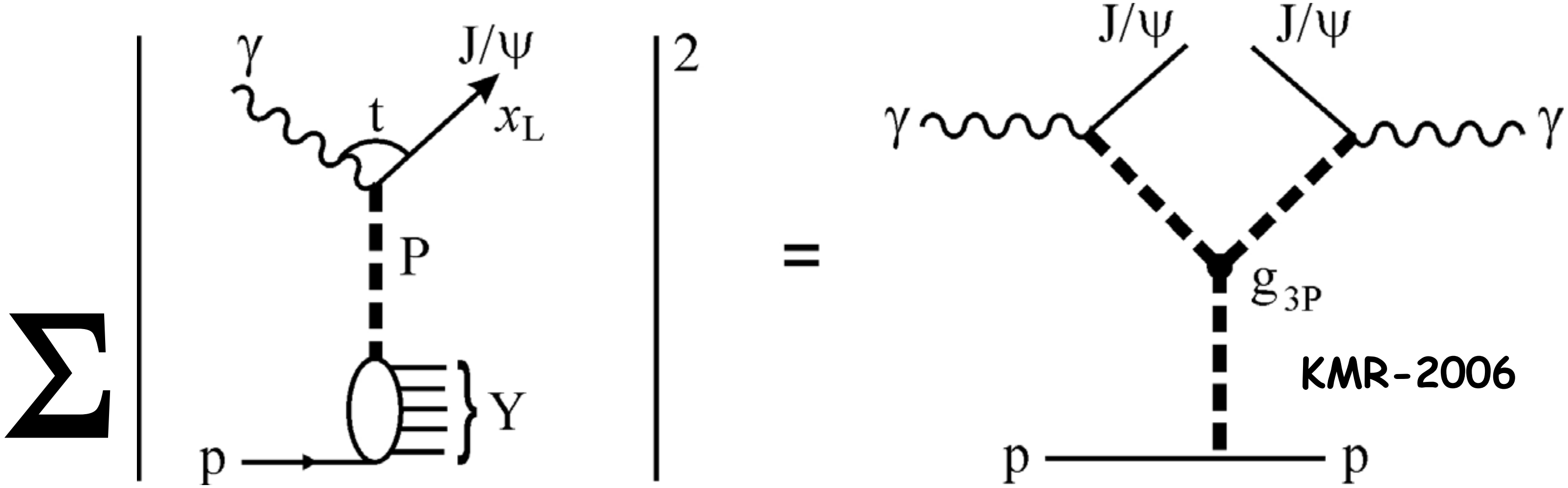


or like this



?

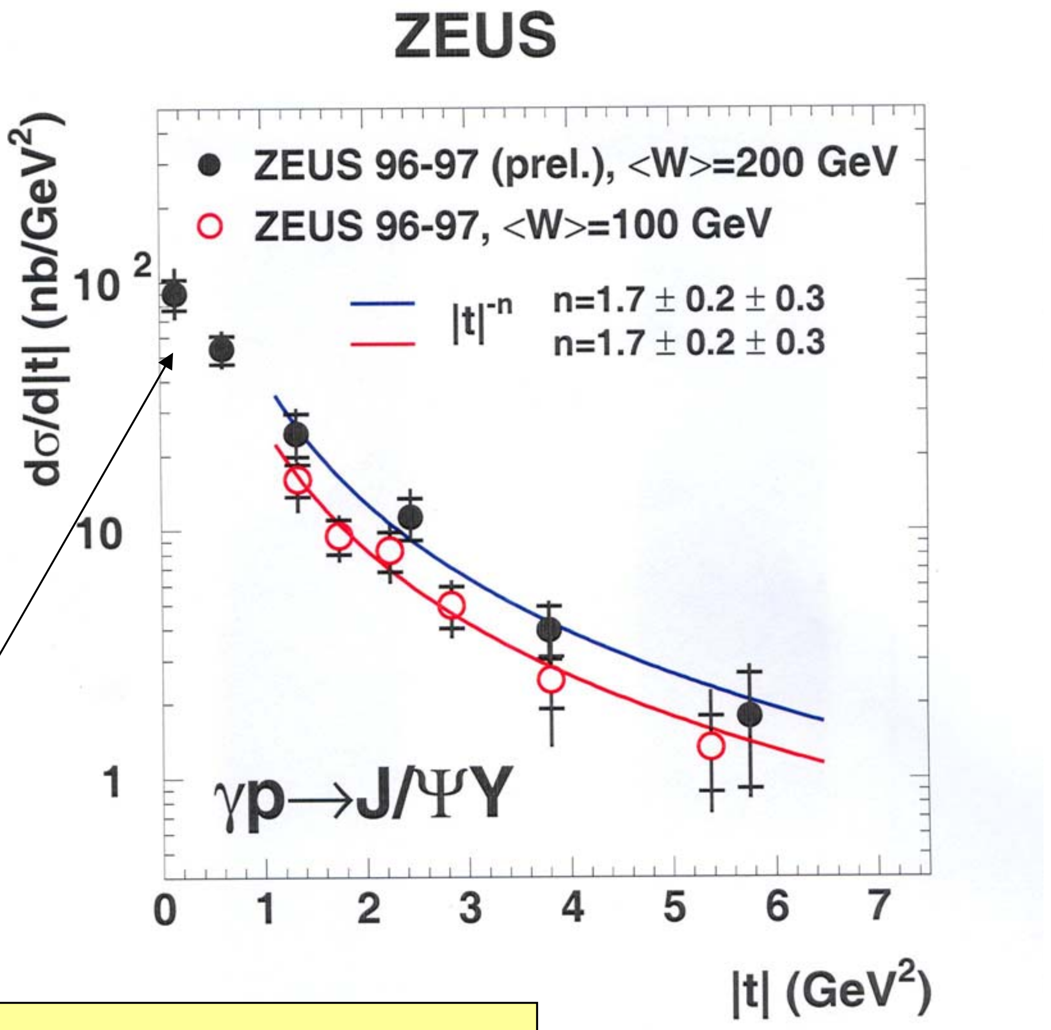
## Estimate of bare triple-Pomeron coupling $g_{3P}$



**KMR-2006**

cc system very compact ➔ need $\gamma p \rightarrow J/\psi\ Y$ data as a func. of $M_Y$

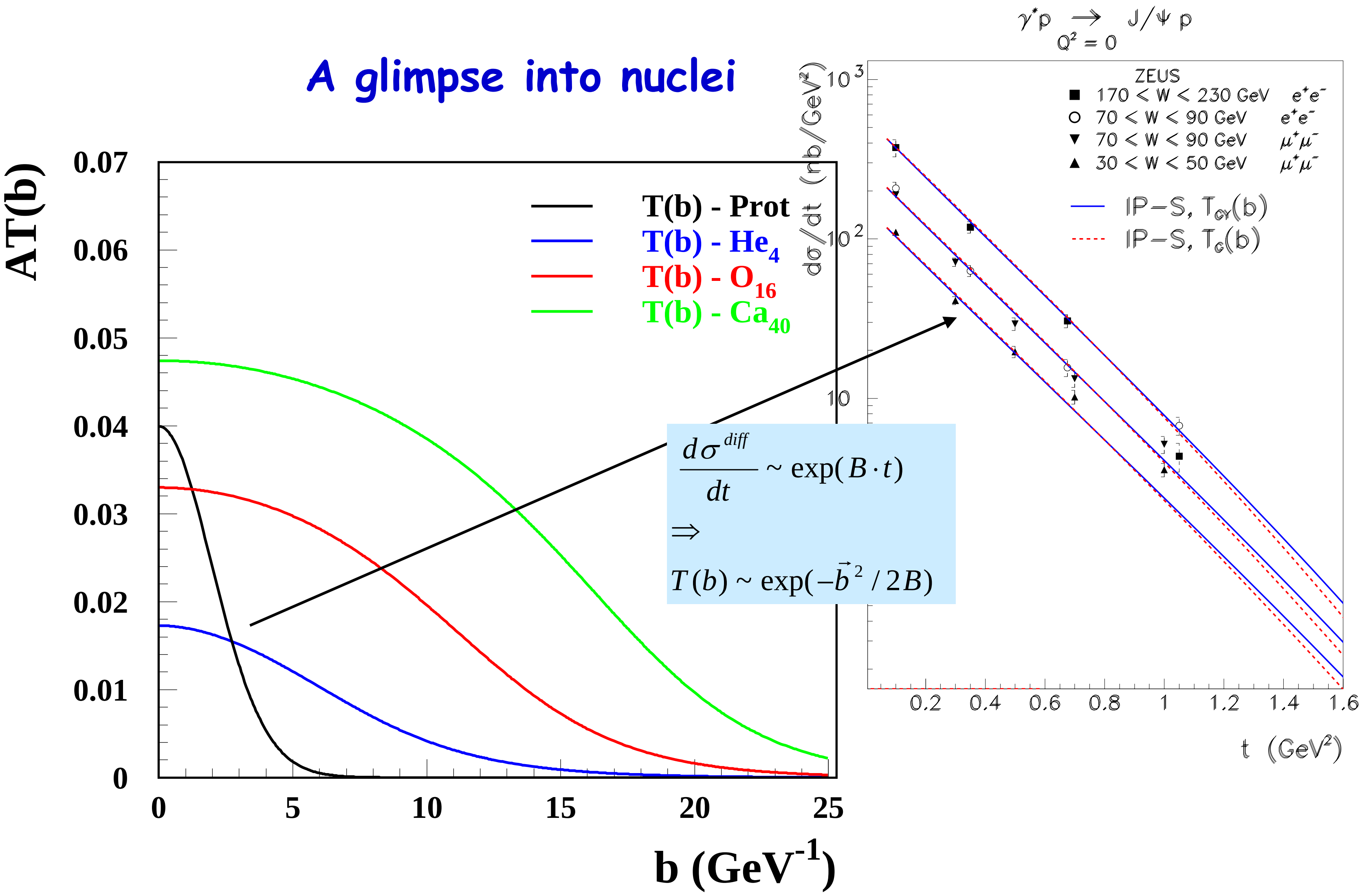
➔ detector should cover a large rapidity range



first evidence that $g_{3P}(t)$ does not vanish as t→0.

first exptal. evidence of “strong coupling” of Pomeron

# A glimpse into nuclei



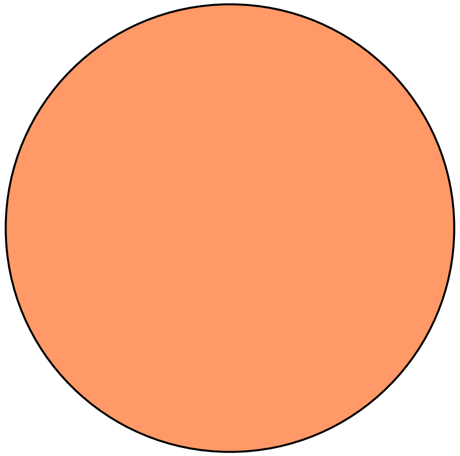
$$\frac{d\sigma^{diff}}{dt} \sim \exp(B \cdot t)$$

$$\Rightarrow$$

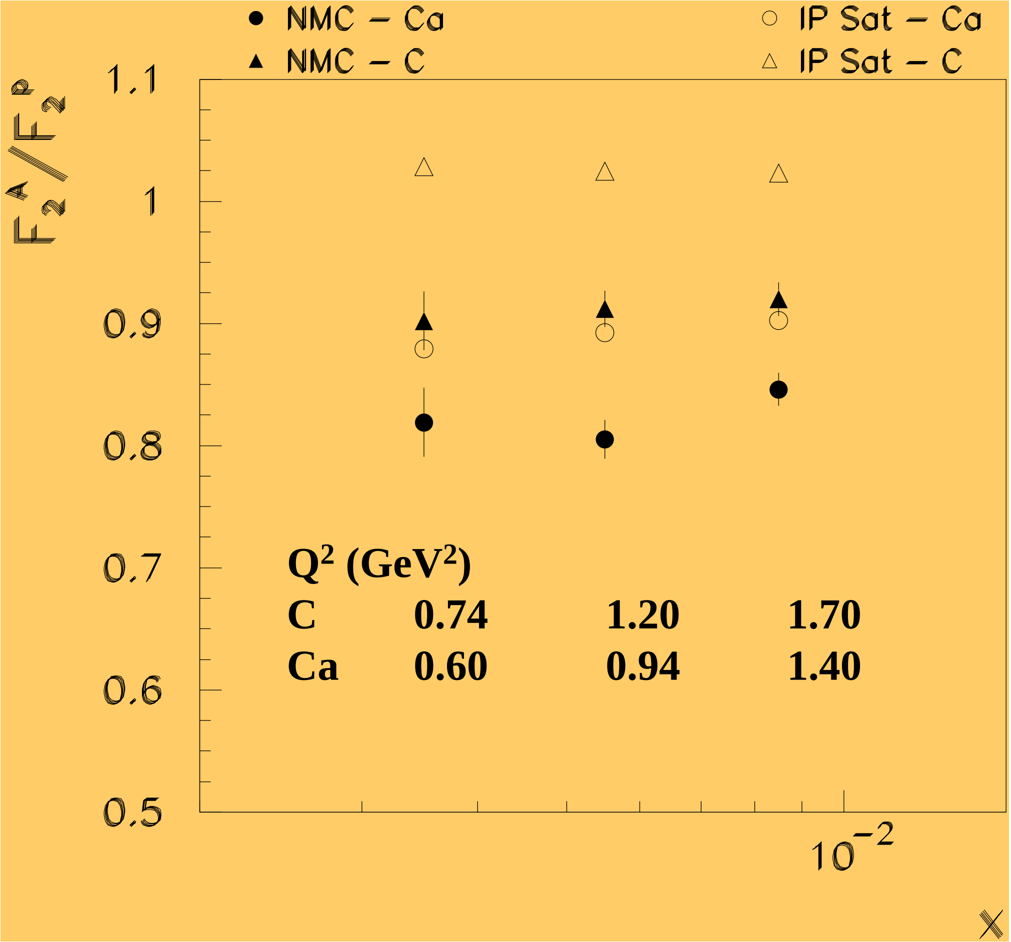
$$T(b) \sim \exp(-\vec{b}^{\,2}/2B)$$

Naïve assumption for T(b):
Wood-Saxon like, homogeneous, distribution of nuclear matter

$$\frac{d\sigma^{A}_{qq}(x,r)}{d^{2}b} = \frac{2}{A}\cdot\left\{1-\exp\left(-\frac{\pi^{2}}{2\cdot 3}r^{2}\alpha_{s}\,xg\,(x,\mu^{2})\,AT_{WS}\,(b)\right)\right\}$$



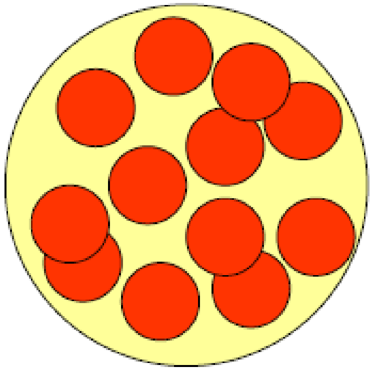
**Smooth Gluon Cloud**



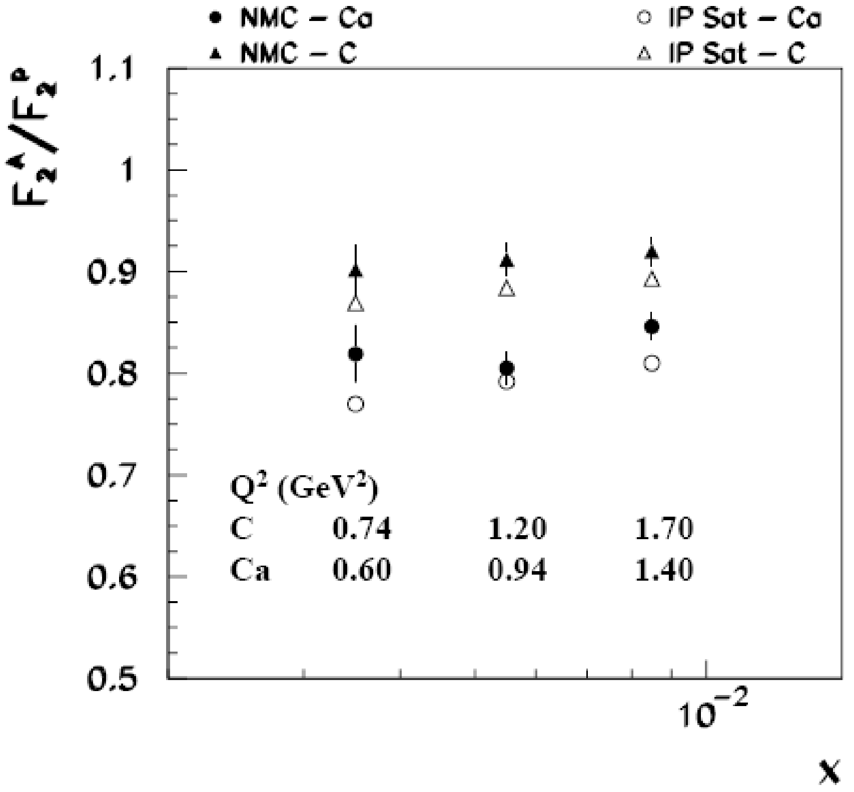
| Q² (GeV²) | | | |
|---|---|---|---|
| C | 0.74 | 1.20 | 1.70 |
| Ca | 0.60 | 0.94 | 1.40 |

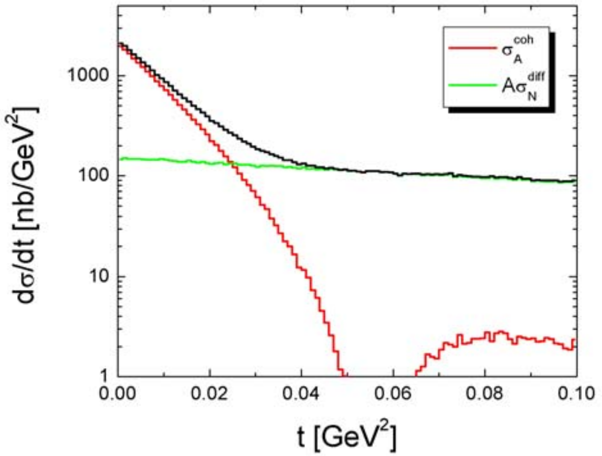
# DIS on Nuclei

$$\frac{d\sigma_{qq}^{A}(x,r)}{d^{2}b} = \frac{2}{A}\cdot\left\{1-\left(1-T_{WS}(b)\sigma_{qq}(x,r)/2\right)^{A\cdot}\right\}$$



Lumpy Gluon Cloud

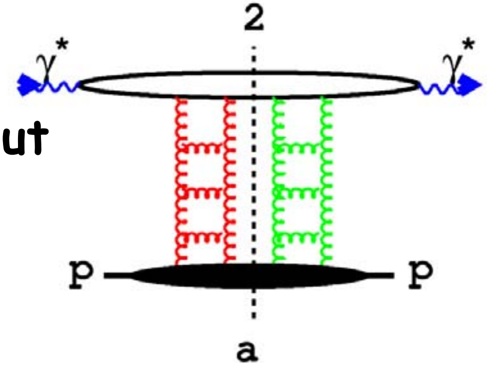


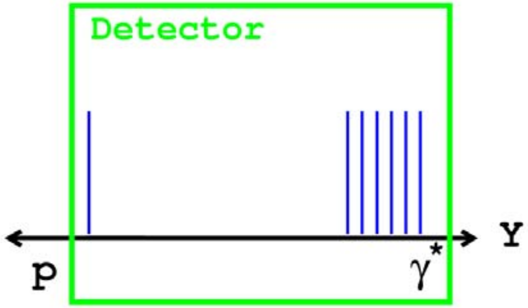


## 2-Pomeron exchange in QCD

## Final States (naïve picture)

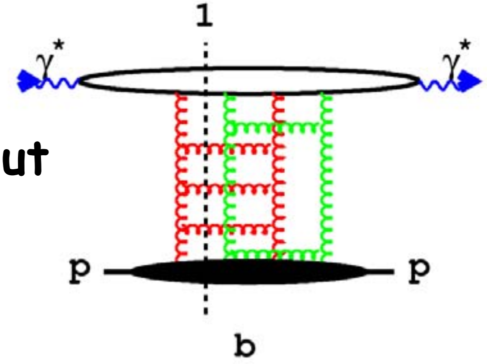
0-cut

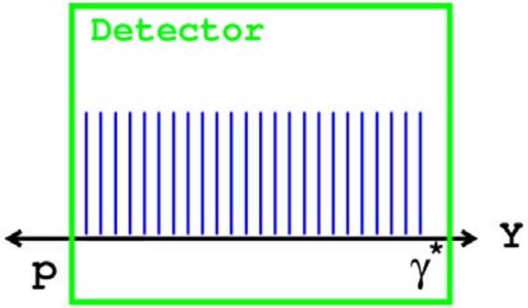




Diffraction

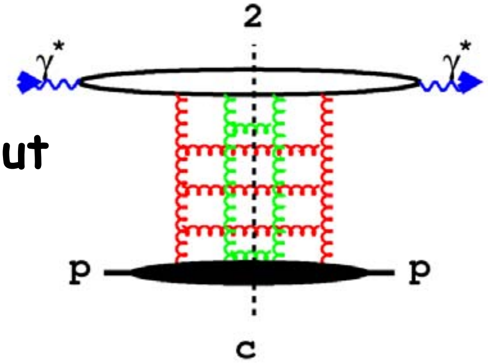
1-cut

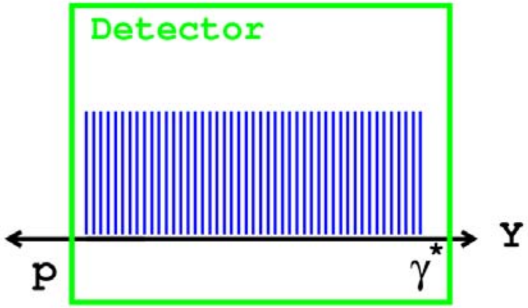




<n>

2-cut





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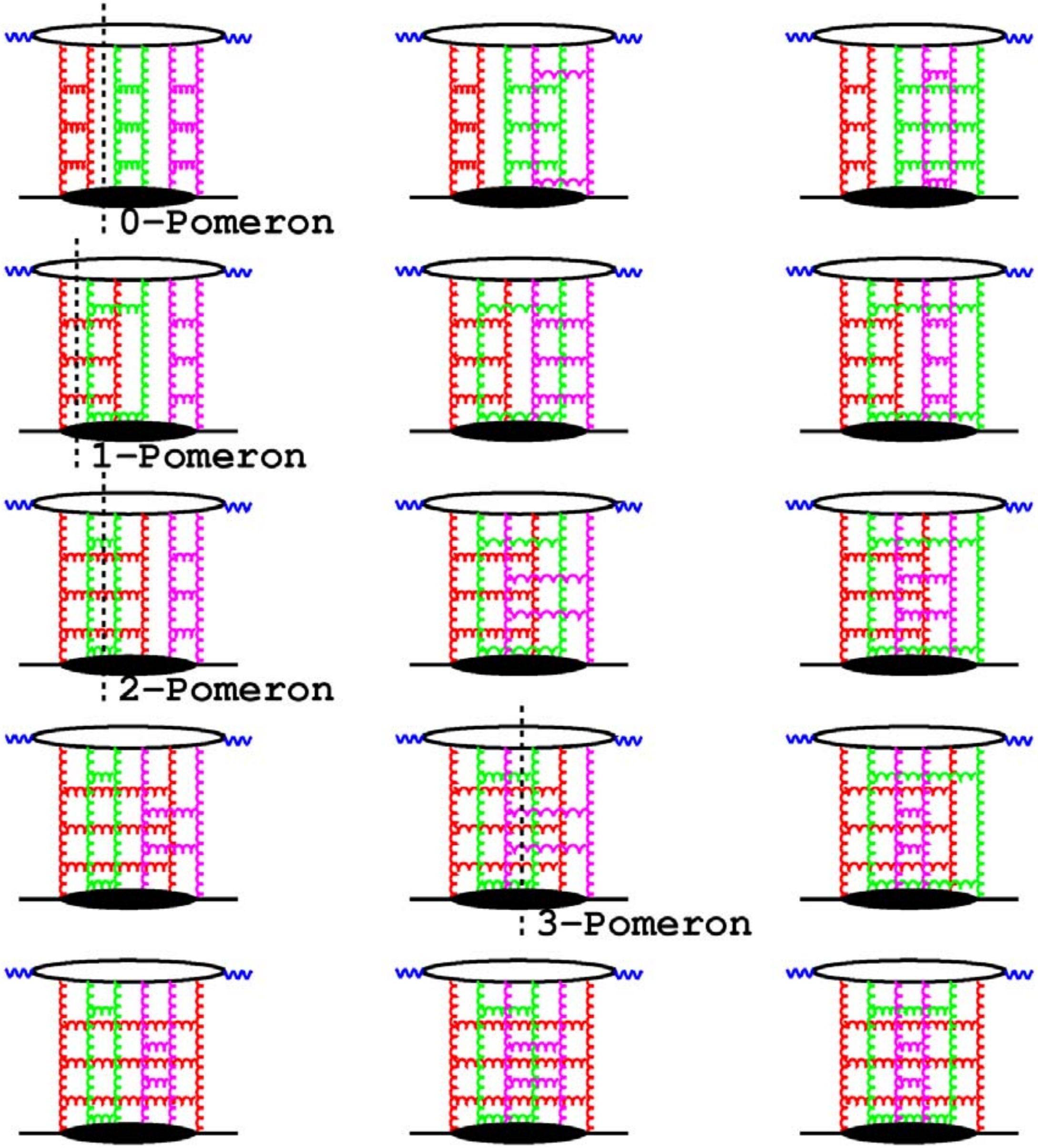
# QCD diagrams



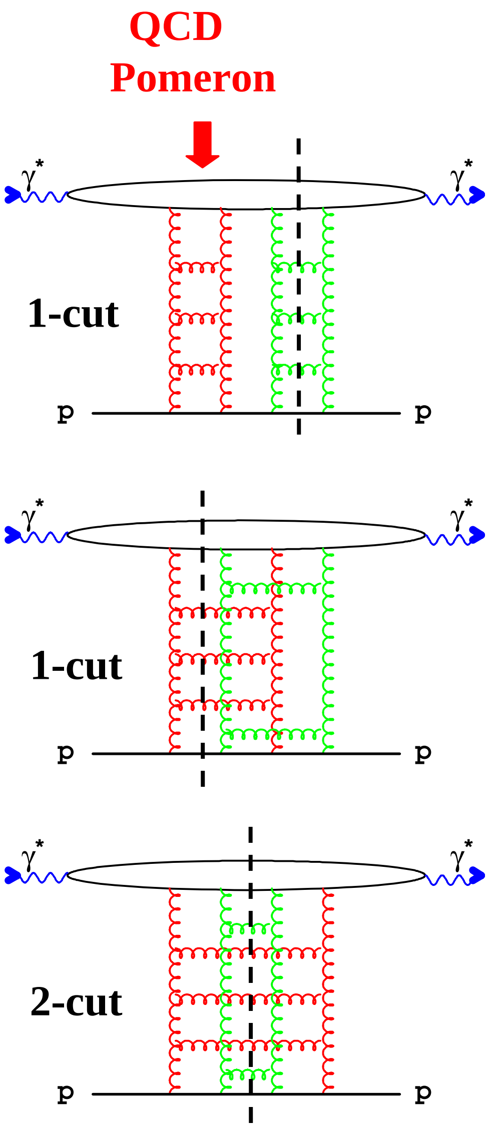
# AGK Rules

**The cross-section for k-cut pomerons:**

Abramovski, Gribov, Kancheli
Sov. ,J., Nucl. Phys. 18, p308 (1974)

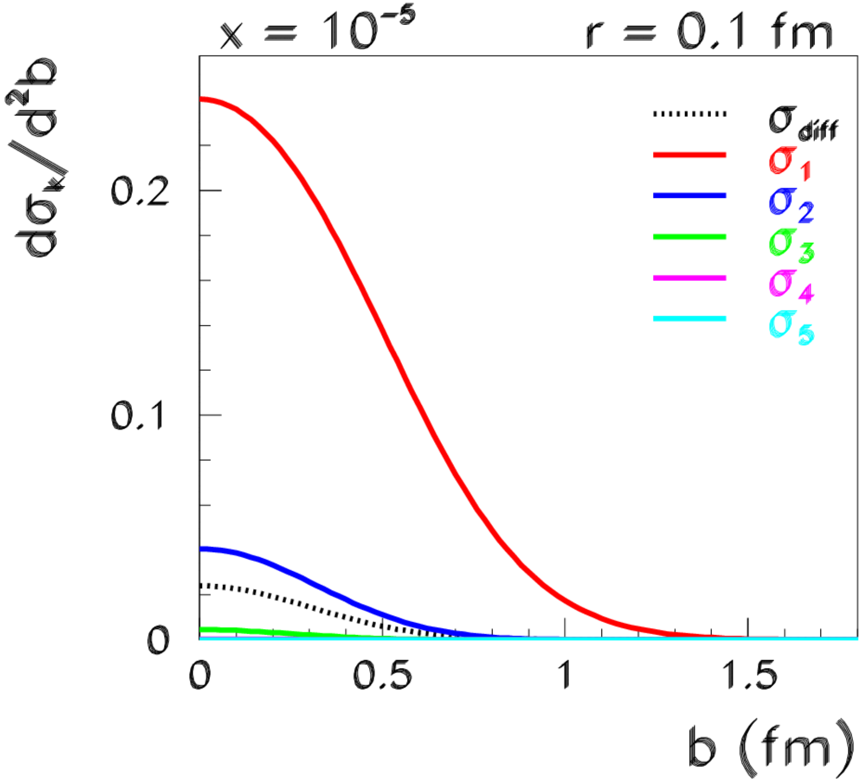
$$\sigma_k = \sum_{m=k}^{\infty} (-1)^{m-k}\, 2^m \frac{m!}{k!(m-k)!} F^{(m)}$$

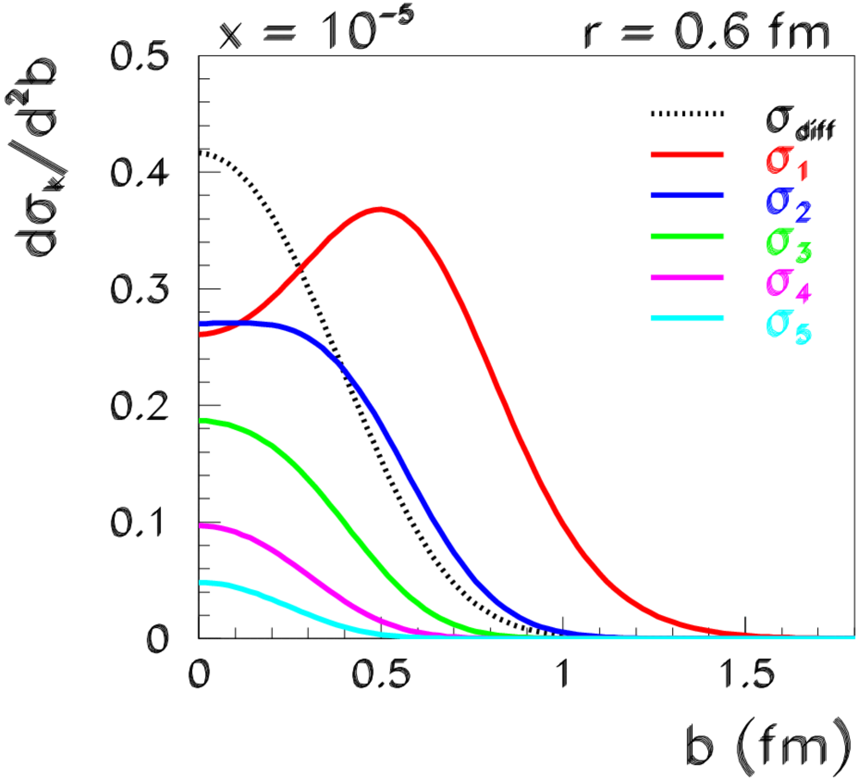
$F^{(m)}$ – amplitude for the exchange of *m* Pomerons



$$\frac{d\sigma_{qq}}{d^2b} = 2\cdot\left\{1-\exp\left(-\frac{\Omega}{2}\right)\right\}$$

$$\frac{d\sigma_k}{d^2b} = \frac{\Omega^k}{k!}\exp(-\Omega)$$



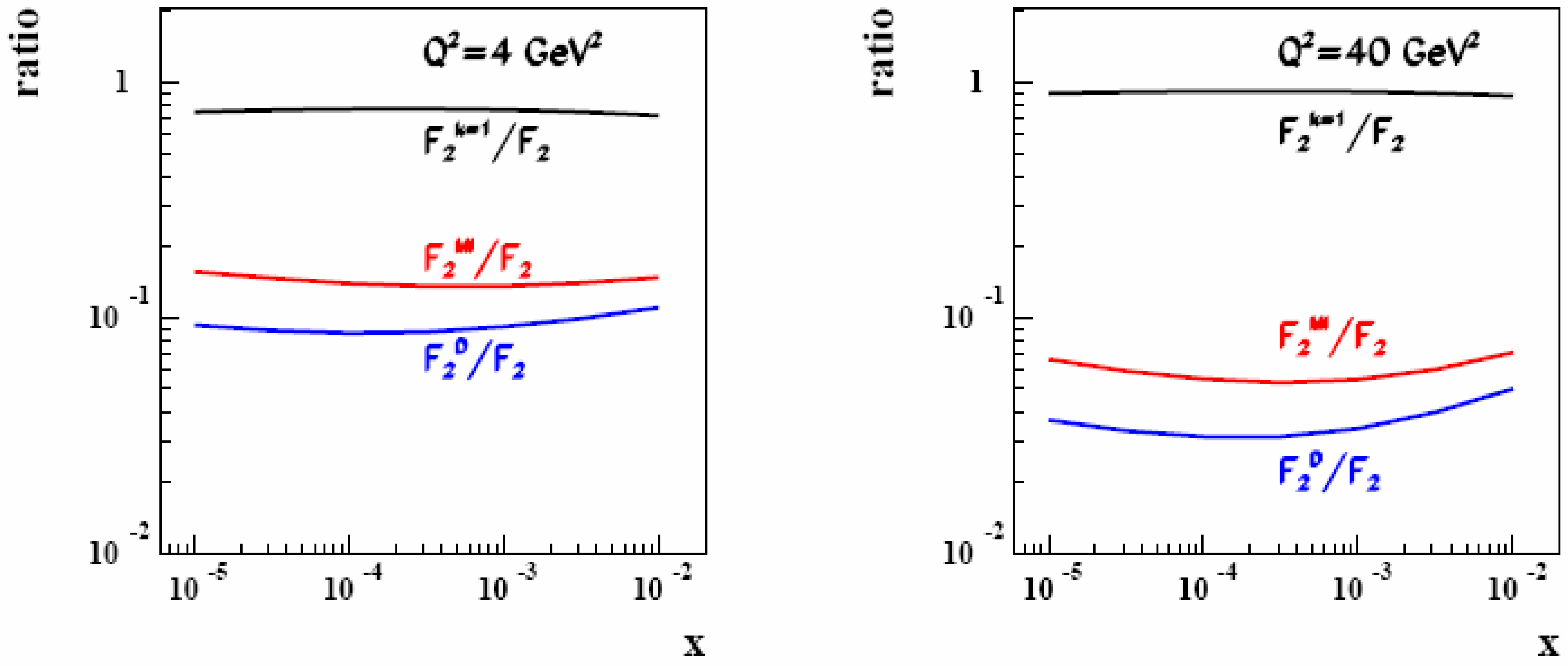


$$\Omega = \frac{\pi^2}{N_C} r^2 \alpha_s(\mu^2)\, x g(x,\mu^2)\, T(b)$$

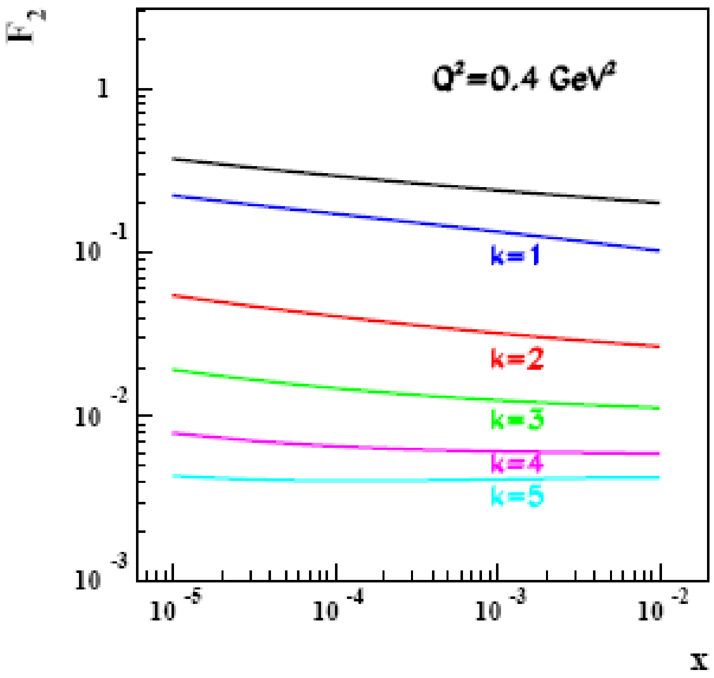
## AGK rules in the Dipole Model ⇒

$$\frac{d\sigma_k}{d^2b} = \frac{\Omega^k}{k!}\exp(-\Omega)$$

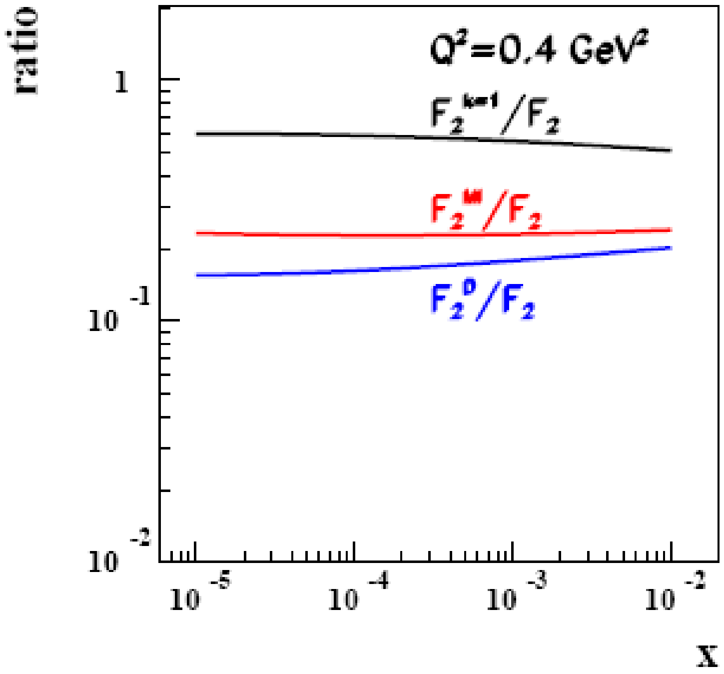
$$\Omega = \frac{\pi^2}{N_C} r^2 \alpha_s(\mu^2) x g(x,\mu^2) T(b)$$



Note: AGK rules underestimate the amount of diffraction in DIS

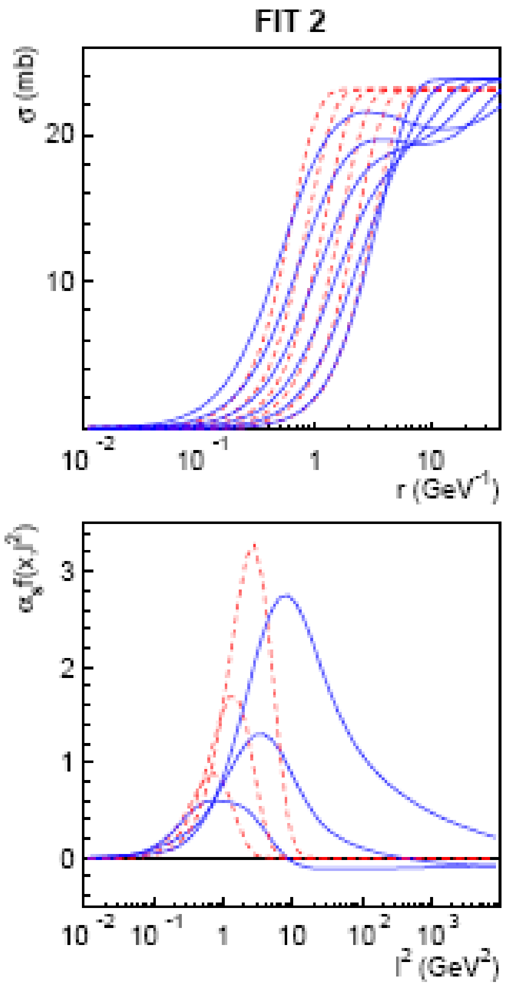


$F_2$
$Q^2$=0.4 GeV$^2$
k=1
k=2
k=3
k=4
k=5
1
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-5}$
$10^{-4}$
$10^{-3}$
$10^{-2}$
x



ratio
$Q^2$=0.4 GeV$^2$
$F_2^{k=1}/F_2$
$F_2^{M}/F_2$
$F_2^{D}/F_2$
1
$10^{-1}$
$10^{-2}$
$10^{-5}$
$10^{-4}$
$10^{-3}$
$10^{-2}$
x

# BGBK



Dipole cross section
for various x ~ $10^{-2}$, $10^{-3}$, $10^{-4}$

corresponding un-integrated gluon density

$$\frac{\alpha_s f(x,l^2)}{l^4} = \frac{3}{4\pi}\int \frac{d^2\mathbf{r}}{(2\pi)^2}\exp\{i\mathbf{l}\cdot\mathbf{r}\}\,\{\hat{\sigma}_\infty(x) - \hat{\sigma}(x,r)\} = $$
$$= \frac{3}{8\pi^2}\int_0^\infty dr\, r\, J_0(lr)\,\{\hat{\sigma}_\infty(x) - \hat{\sigma}(x,r)\}\ .$$

➔ Pt cutoffs for MC x and b dependent?

# A High Luminosity, High Energy Electron-Ion Collider: A New Experimental Quest to Study the Glue which Binds Us All

*A. Bruell, A. Deshpande, R. Ent, R. Milner,*
*R. Venugopalan, W. Vogelsang, and B. Surrow*

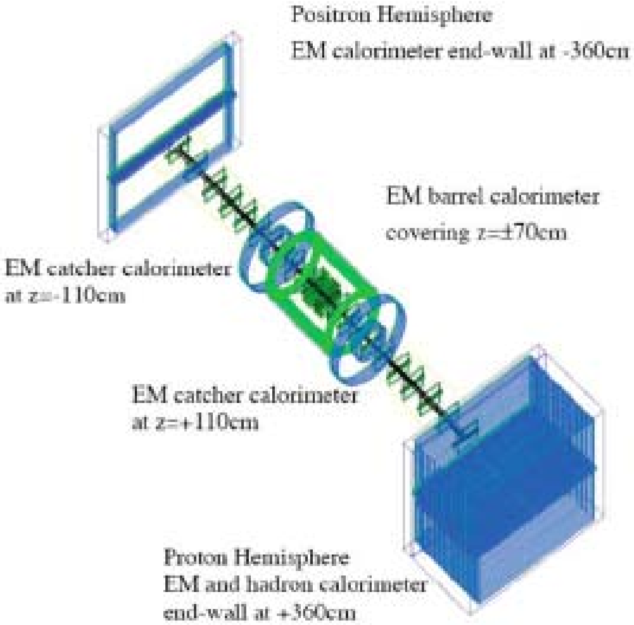
**for the EIC Collaboration**

**DRAFT**

**January 7th 2007**

# What are the lessons/open questions for EIC from exclusive diffraction at HERA

- **eRHIC**
  - Variable beam energy
  - P-U ion beams
  - Light ion polarization
  - Huge luminosity



Solve the gluon density puzzle –
why DGLAP like properties (dilute, short distance evolution)
are neighboring saturation and diffusion effects?

Diffractive vector mesons scattering - an excellent probe of nuclear matter,
why is the gluonic radius smaller than the quark radius??
e.g. follow $\phi$ cross section to low energies and look for
a transition in t-behavior

\>>>>> Measure *t* distribution on polarized nuclei <<<<<<
\>>>>> Obtain holographic picture of nuclei !!!! <<<<<<