Physics
of
Ultra-Peripheral Nuclear Collisions

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Particle production from EM Fields

- ***Lepton-pair production:***

  When heavy ions collide central or near-central in relativistic velocities, the Lorentz contracted high EM fields produce lepton pairs.

- ***Beam Lifetime (electron capture):***

  Some created electron-positron pair can not go on their way as a pair, the electron can be captured by the ion. As a result, charge-mass ratio of the ion and the beam lifetime differs.

- ***Non-perturbative and perturbative approach:***

  In single pair-production, to solve 2nd order Feynman diagrams,perturbative methods have been used. In multi-pair production, this approach have been modified by using higher order Feynman diagrams.

- ***Impact parameter dependence:***

  We can calculate the impact parameter dependence cross section.

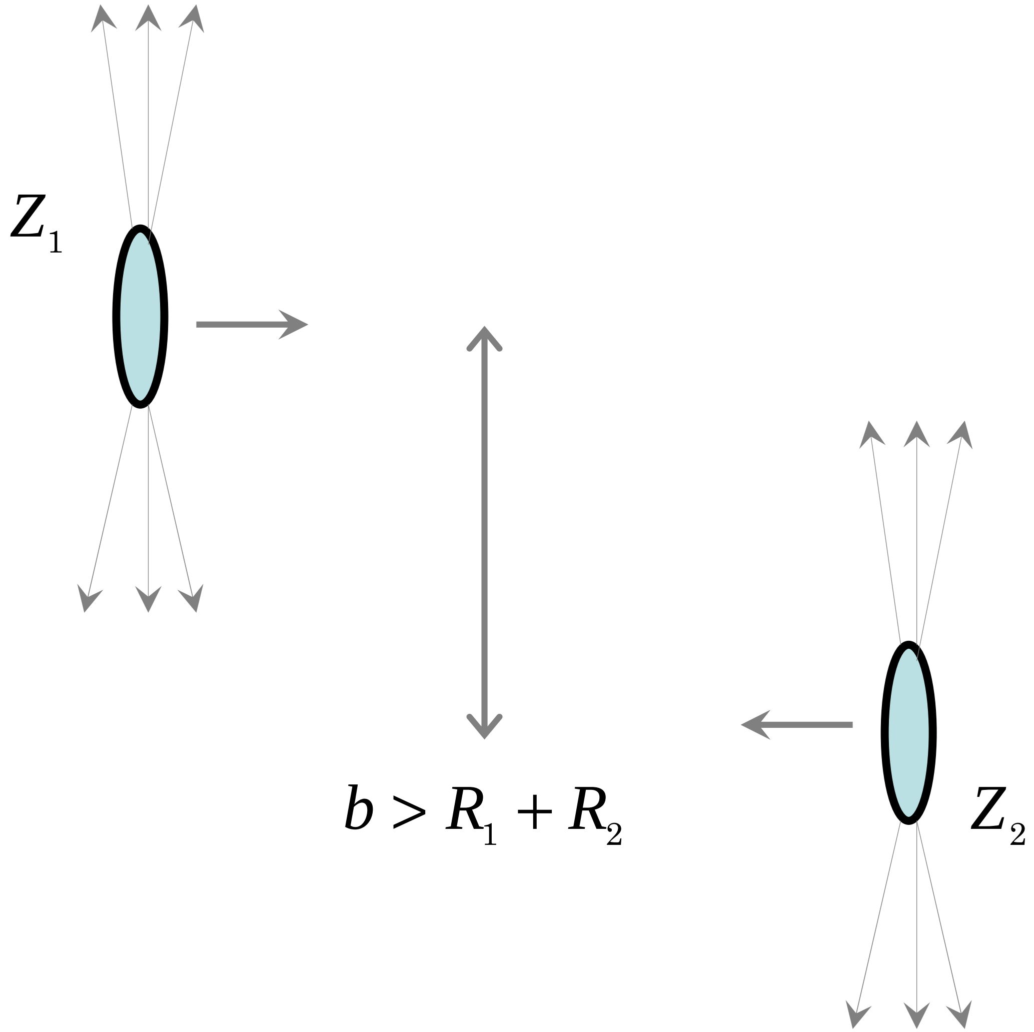
- ***Multi-pair production:***

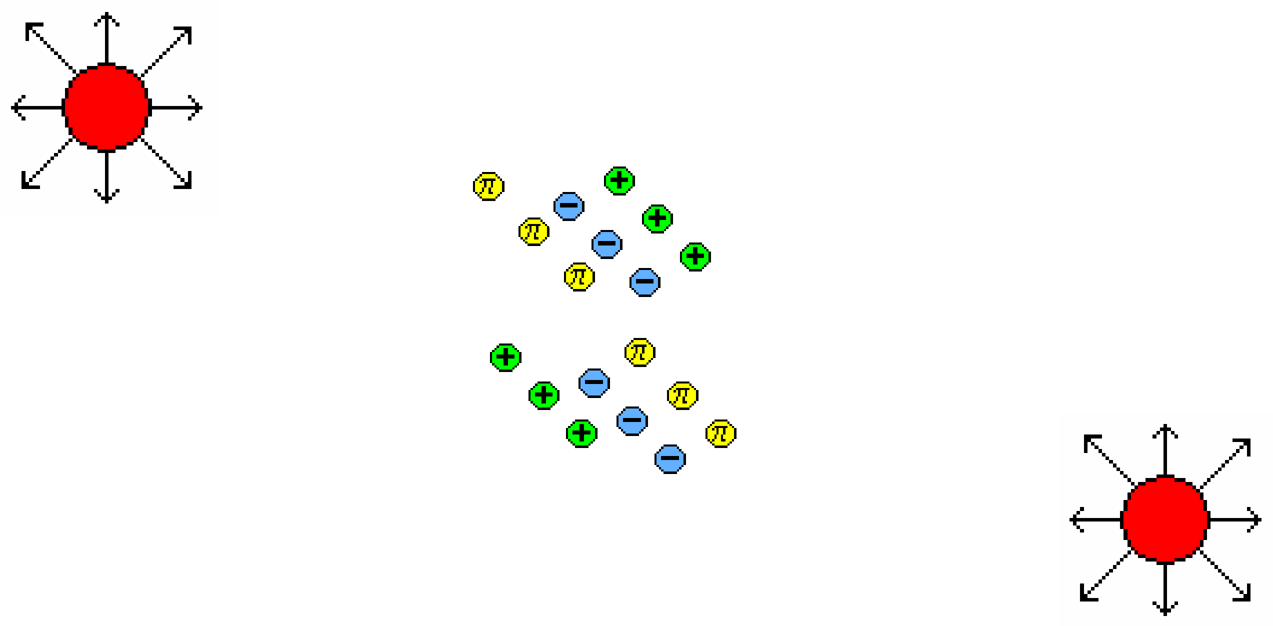
  For high energies and small impact parameters, the perturbative results for single-pair production violates the unitarity. Therefore, lowest order calculation is not sufficient to describe the pair production. Multi-pair production must be studied.

- ***Test of strong QED:***

  Heavy ion collisions in relativistic velocities gives us chance to test the strong QED.

Collisions of Heavy Ions





Lepton-Pair Production

Semi Classical Action

$$S = \int d^4x < \Phi(t) | : L_0(x) + L_{int}(x) : | \Phi(t) >$$

Free Lagrangian

$$L_0(x) = \overline{\Psi}(x)(\gamma_\mu i\partial^\mu - m)\Psi(x)$$

Interaction Lagrangian :

$$L_{int}(x) = \overline{\Psi}(x)\gamma_\mu \Psi(x) A^\mu(x)$$

Four Vector Potentials of the Colliding Ions

A_μ Electromagnetic vector potential

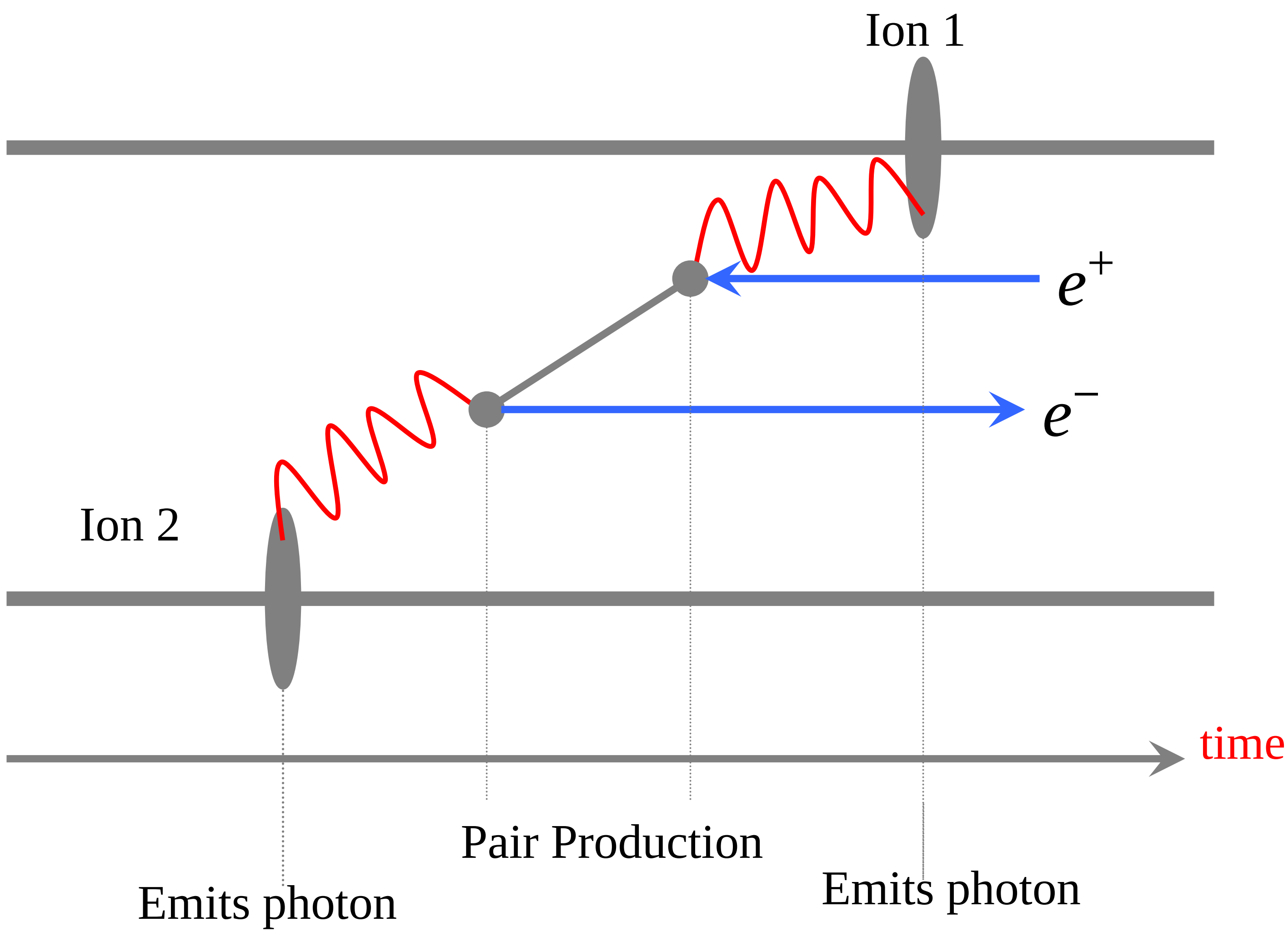
$$A^\mu = A^\mu(1) + A^\mu(2)$$

$$A^0(1) = -8\pi^2 Z\gamma^2 \frac{\delta(q_0 - \beta q_z)}{q_z^2 + \gamma^2(q_x^2 + q_y^2)} \exp[i\vec{q}_\perp \cdot \frac{\vec{b}}{2}]$$

$$A^z(1) = \beta A^0(1)$$

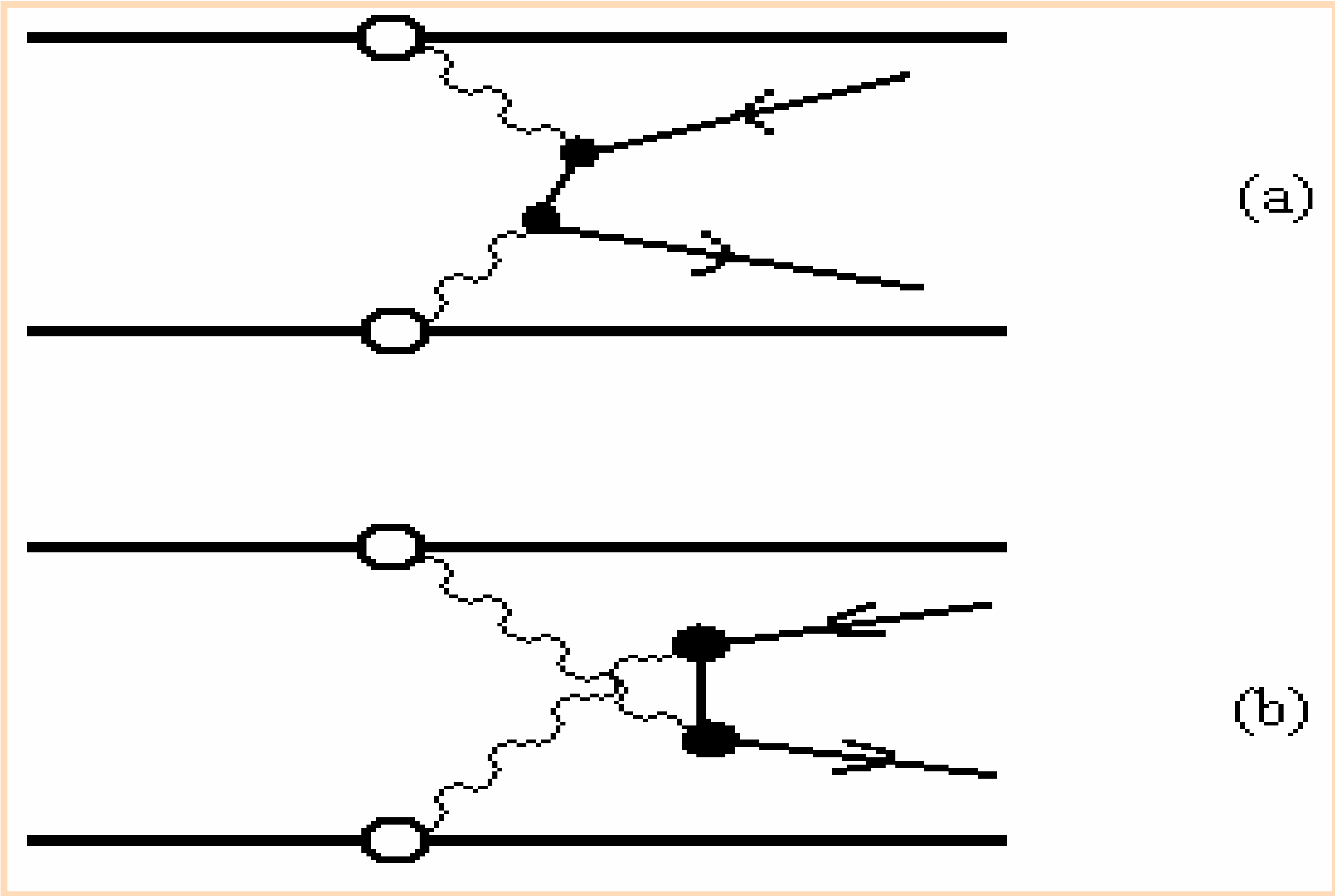
$$A^x(1) = A^x(2) = 0$$

$$A^y(1) = A^y(2) = 0$$



Ion 1
e^+
e^-
Ion 2
time
Pair Production
Emits photon
Emits photon

Direct and Exchange Diagrams



Total Cross Section

$$\sigma = \frac{1}{4\pi}\sum_{\sigma_q \sigma_k}\int \frac{d^3k d^3q d^2 p_\perp}{(2\pi)^8}\left| A^{(+)}\left(k, q : \mathbf{p}_\perp\right) + A^{(-)}\left(k, q : \mathbf{k}_\perp + \mathbf{q}_\perp - \mathbf{p}_\perp\right)\right|^2$$

$$A^{(+)}(\vec{k}, \vec{q} : \vec{p}_\perp) = F(\vec{k}_\perp - \vec{p}_\perp : \omega_1) F(\vec{p}_\perp - \vec{q}_\perp : \omega_2) T_{kq}(\vec{p}_\perp : \beta)$$

$$A^{(-)}(\vec{k}, \vec{q} : \vec{p}_\perp) = F(\vec{k}_\perp - \vec{p}_\perp : \omega_2) F(\vec{p}_\perp - \vec{q}_\perp : \omega_1) T_{kq}(\vec{p}_\perp : -\beta)$$

Scalar part of EM Fields in momentum space of moving heavy ions

$$F(\vec{q}:\omega) = \frac{4\pi Z\gamma^2\beta^2}{\omega^2 + \beta^2\gamma^2 q^2} G_E(q^2) f_Z(q^2)$$

$$T_{kq}(\vec{p}_\perp:\beta) = \sum_s \sum_{\sigma_p} \left[E_p^{(s)} - \left(\frac{E_k^{(+)} + E_q^{(-)}}{2} \right) + \beta \left(\frac{k_z - q_z}{2} \right) \right]^{-1}$$
$$\times \left\langle u_{\sigma_k}^{(+)} \middle| (1 - \beta\alpha_z) \middle| u_{\sigma_p}^{(-)} \right\rangle \left\langle u_{\sigma_p}^{(s)} \middle| (1 + \beta\alpha_z) \middle| u_{\sigma_q}^{(-)} \right\rangle$$

Impact Parameter Dependence Cross Sections

$$\frac{d\sigma}{db} = \int_0^\infty dq\, q\, b\, J_0(qb)\, F(q)$$

$$F(q) = \frac{\pi}{8\beta^2} \sum_{\sigma_k} \sum_{\sigma_q} \int_0^{2\pi} d\varphi_q \int \frac{dk_z dq_z d^2k_\perp d^2K d^2Q}{(2\pi)^{10}}$$

$$\times F[\frac{1}{2}(\vec{Q}-\vec{q});\omega_1]F[-\vec{K};\omega_2]F[\frac{1}{2}(\vec{Q}+\vec{q});\omega_1]F[-\vec{q}-\vec{K};\omega_2]$$

$$\times \left\{T_{kq}[\vec{k}_\perp - \frac{1}{2}(\vec{Q}-\vec{q});\beta] + T_{kq}[\vec{k}_\perp - \vec{K};-\beta]\right\}$$

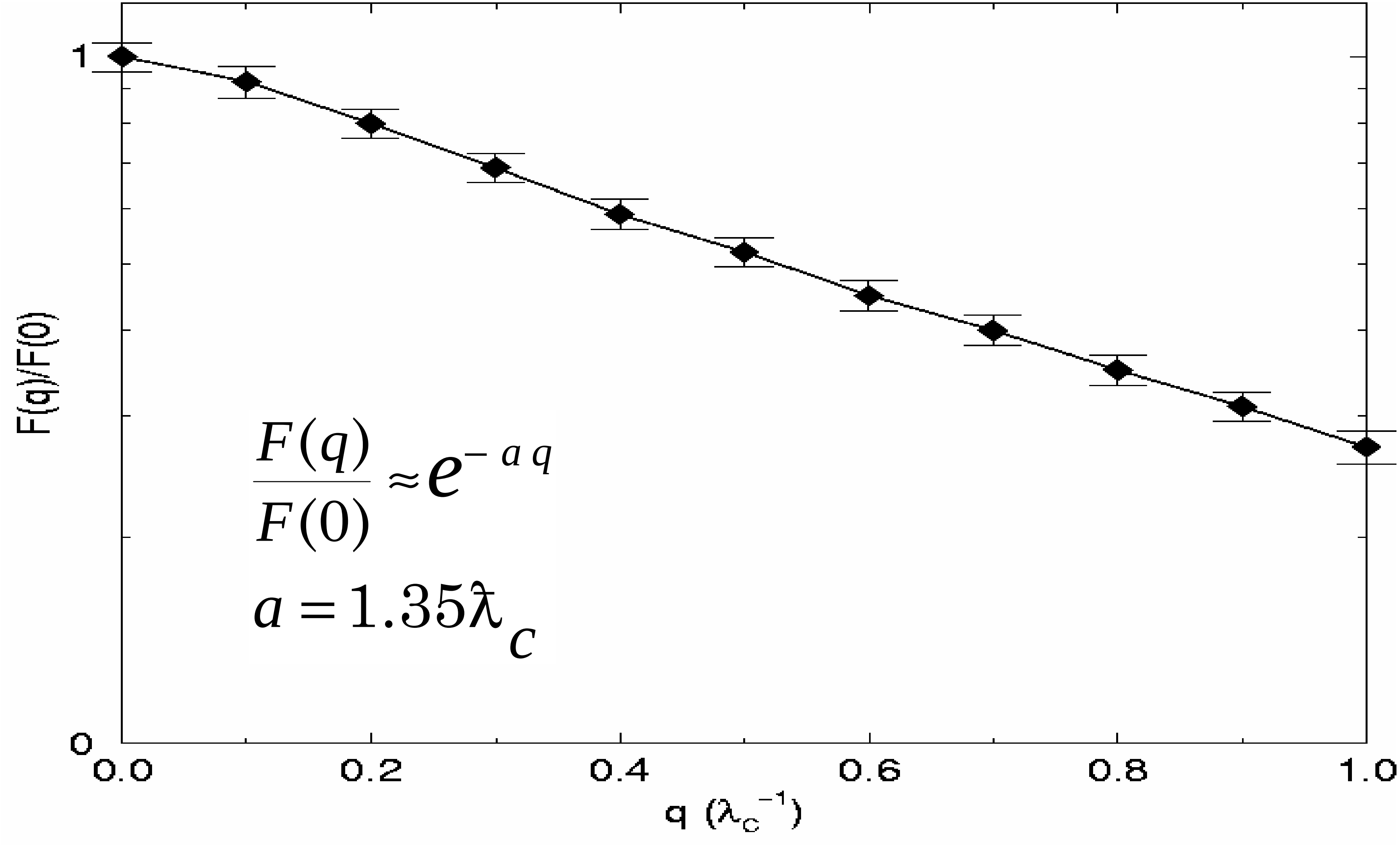
$$\times \left\{T_{kq}[\vec{k}_\perp - \frac{1}{2}(\vec{Q}+\vec{q});\beta] + T_{kq}[\vec{k}_\perp + \vec{q}_\perp - \vec{K};-\beta]\right\}$$

Monte-Carlo Method

$$\frac{d\sigma}{db} = \sigma_T \int_0^\infty dq\, q\, b\, J_0(q\,b)\, e^{-a\,q}$$

$$= \sigma_T \int_0^\infty dq\, q\, b\, \frac{2}{\pi} \int_0^1 \frac{\cos(qbt)}{\sqrt{1-t^2}} e^{-a\,q}\, dt$$

$$= \sigma_T \frac{a\,b}{(a^2+b^2)^{\frac{3}{2}}}$$



The function of *F(q)* verses *q* for the charges of heavy-ions between $Z_{1,2} = 20 - 90$ and for the energies between γ =*10-3400* . Each point has converged to within five percent. From the slope of this function the value of *a* can be determined as *a=1.35* $ƛ_C$.

Monte Carlo Method :

$$P(b) = \frac{1}{2\pi b}\frac{d\sigma}{db} = \frac{1}{2\pi} C_\infty \lambda_C^2 Z_1^2 Z_2^2 \alpha^4 \ln^3(\gamma) \frac{a}{(a^2+b^2)^{\frac{3}{2}}}$$

Equivalent Photon Method:

$$P(b) \approx \frac{14}{9\pi^2} Z_1^2 Z_2^2 \alpha^4 \left[\frac{\lambda_C}{b}\right]^2 \ln^2\left(\frac{\gamma_{lab}\delta\lambda_C}{2b}\right) + \Delta(Z)$$

M. C. Güçlü, Nucl. Phys. A, Vol. 668, 207-217 (2000)

Born Approximation with Coulomb Corrections

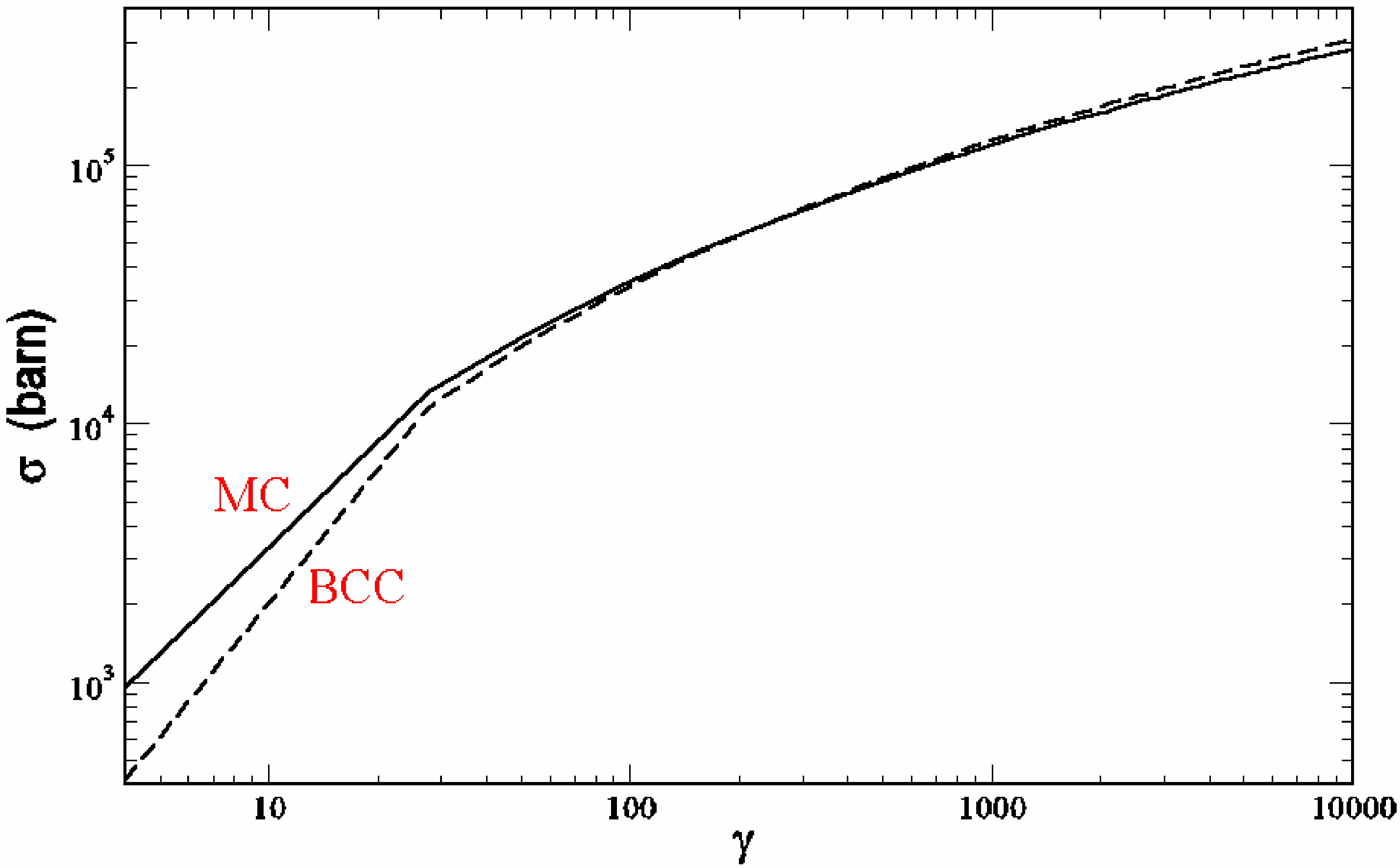
$$\sigma_{total} = \sigma^b + \sigma_A^C + \sigma_B^C + \sigma_{AB}^C$$

$$\sigma^b = \frac{28\left(Z_A\alpha\right)^2\left(Z_B\alpha\right)^2}{27\pi m^2}\ln^3\left(\gamma^2\right)$$

$$\sigma_A^C = -\frac{28\left(Z_A\alpha\right)^2\left(Z_B\alpha\right)^2}{9\pi m^2} f\left(Z_A\alpha\right)\ln^2\left(\gamma^2\right)$$

$$\sigma_B^C = -\frac{28\left(Z_A\alpha\right)^2\left(Z_B\alpha\right)^2}{9\pi m^2} f\left(Z_B\alpha\right)\ln^2\left(\gamma^2\right)$$

$$\sigma_{AB}^C = \frac{56\left(Z_A\alpha\right)^2\left(Z_B\alpha\right)^2}{9\pi m^2} f\left(Z_A\alpha\right) f\left(Z_B\alpha\right)\ln\left(\gamma^2\right)$$



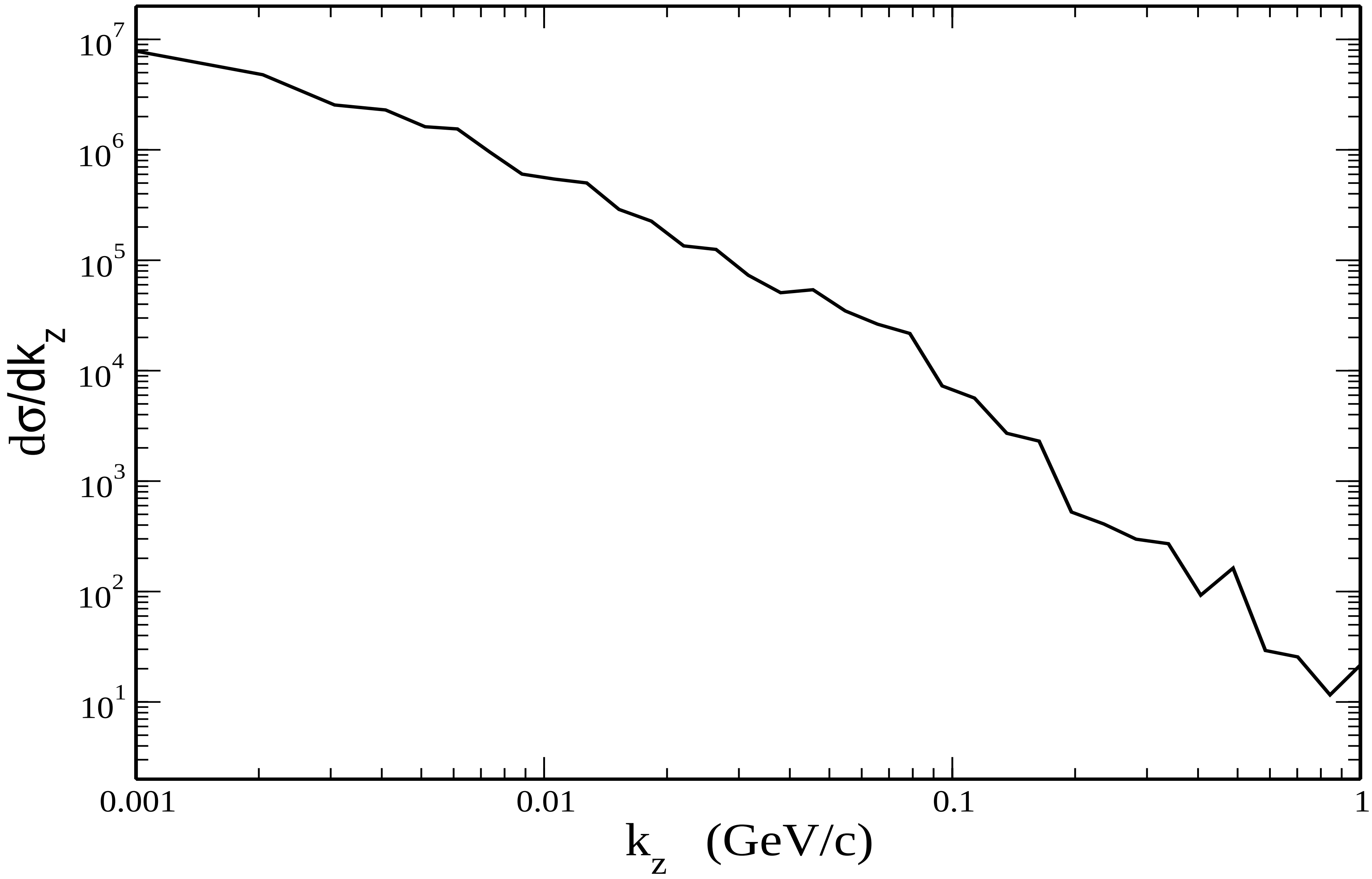
Total cross section of electron-positron pair production as a function of energy. The solid line is the Monte Carlo calculation, the dashed-dot line is the Born approximation with Coulomb corrections.

Small Momentum Approximation

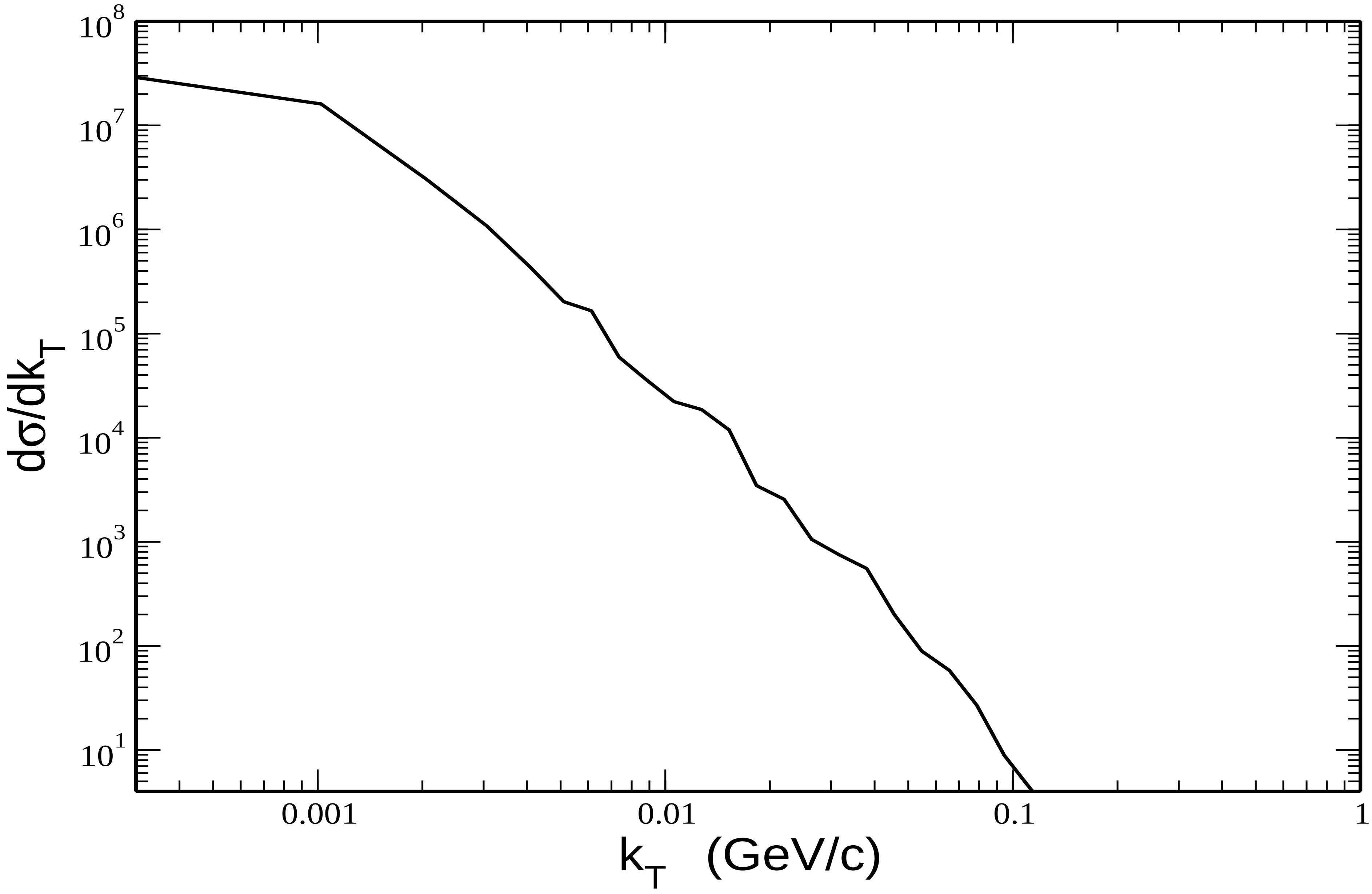
$$k_z << \gamma m \qquad\qquad k_\perp \approx m$$

$$\frac{\int_0^{\gamma m} dk'_z \frac{d\sigma}{dk'_z}}{\int_0^{\infty} dk'_z \frac{d\sigma}{dk'_z}} \approx 0.2 \qquad \frac{\int_0^{m} dk'_\perp \frac{d\sigma}{dk'_\perp}}{\int_0^{\infty} dk'_\perp \frac{d\sigma}{dk'_\perp}} \approx 0.4$$

M.C. Güçlü at al. Phys. Rev. A 72 022724 (2005).



Monte Carlo calculation of the differential cross section of pair production as a function of the longitudinal momentum of produced pairs.



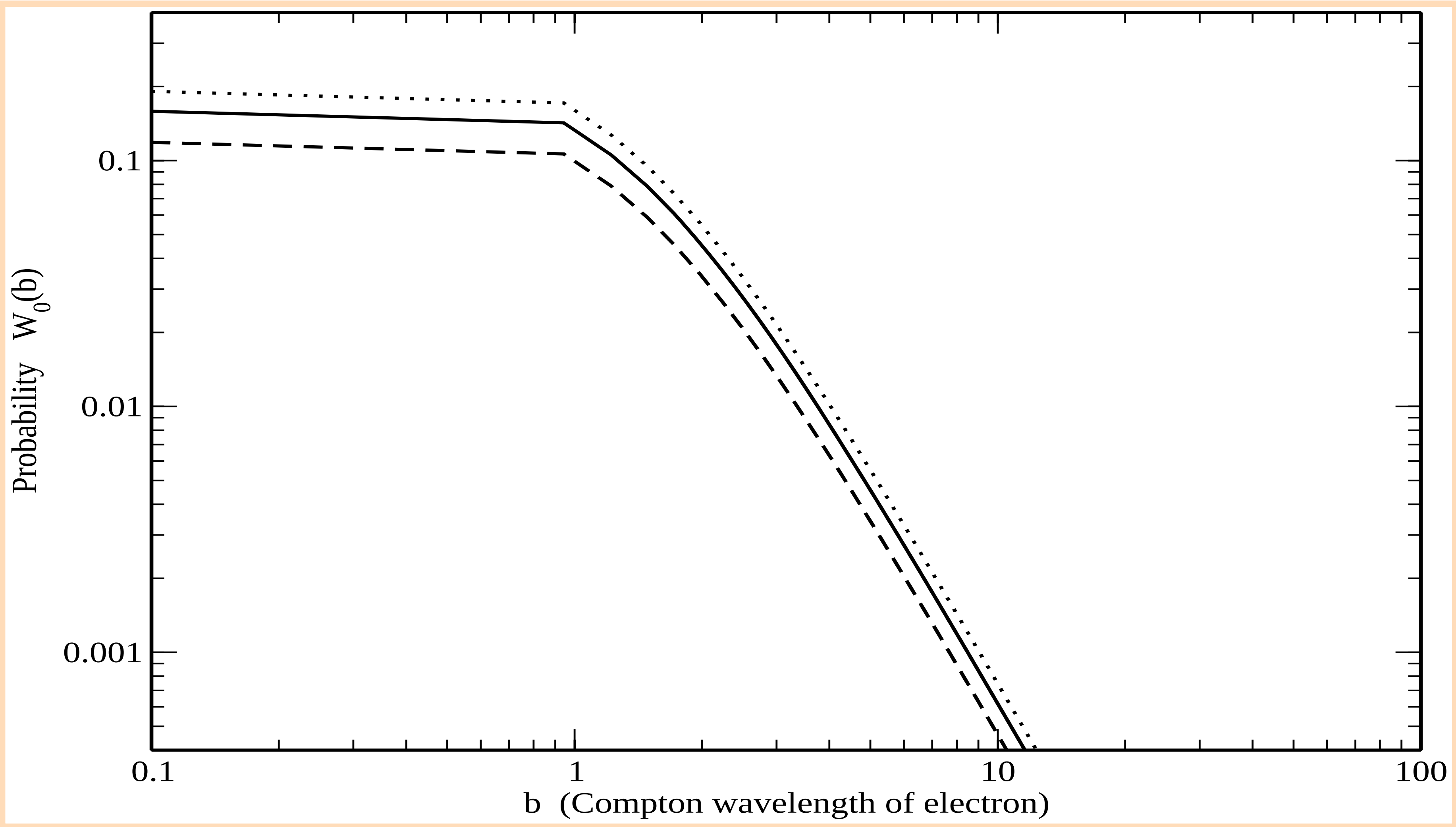
Monte Carlo calculation of the differential cross section of pair production as a function of the transverse momentum of produced pairs.

Ansatz:

$$W_0^{Born}(\rho) = \frac{1}{2\pi\rho}\frac{d\sigma}{d\rho}$$

$$= \frac{1}{2\pi}\left(\sigma^b + \sigma_A^C + \sigma_B^C + \sigma_{AB}^C\right)\frac{\rho_0}{\left(\rho_0^{\ 2} + \rho^2\right)^{3/2}}$$

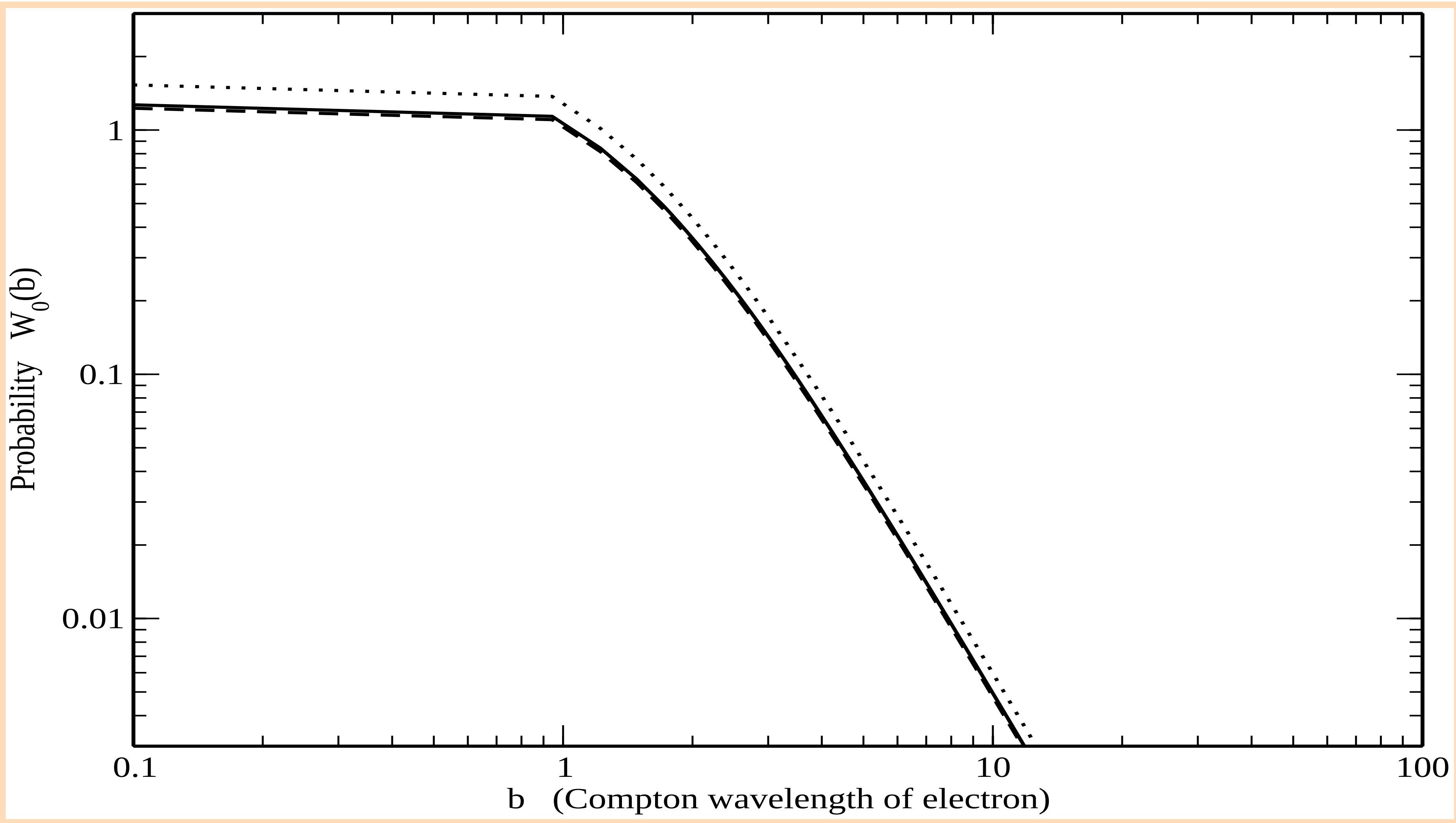
$\gamma = 10$



γ

Au+Au collisions. The solid line is the Monte Carlo calculation, the dotted line is the Born approximation, and the dashed line is the Born approximation with Coulomb corrections.

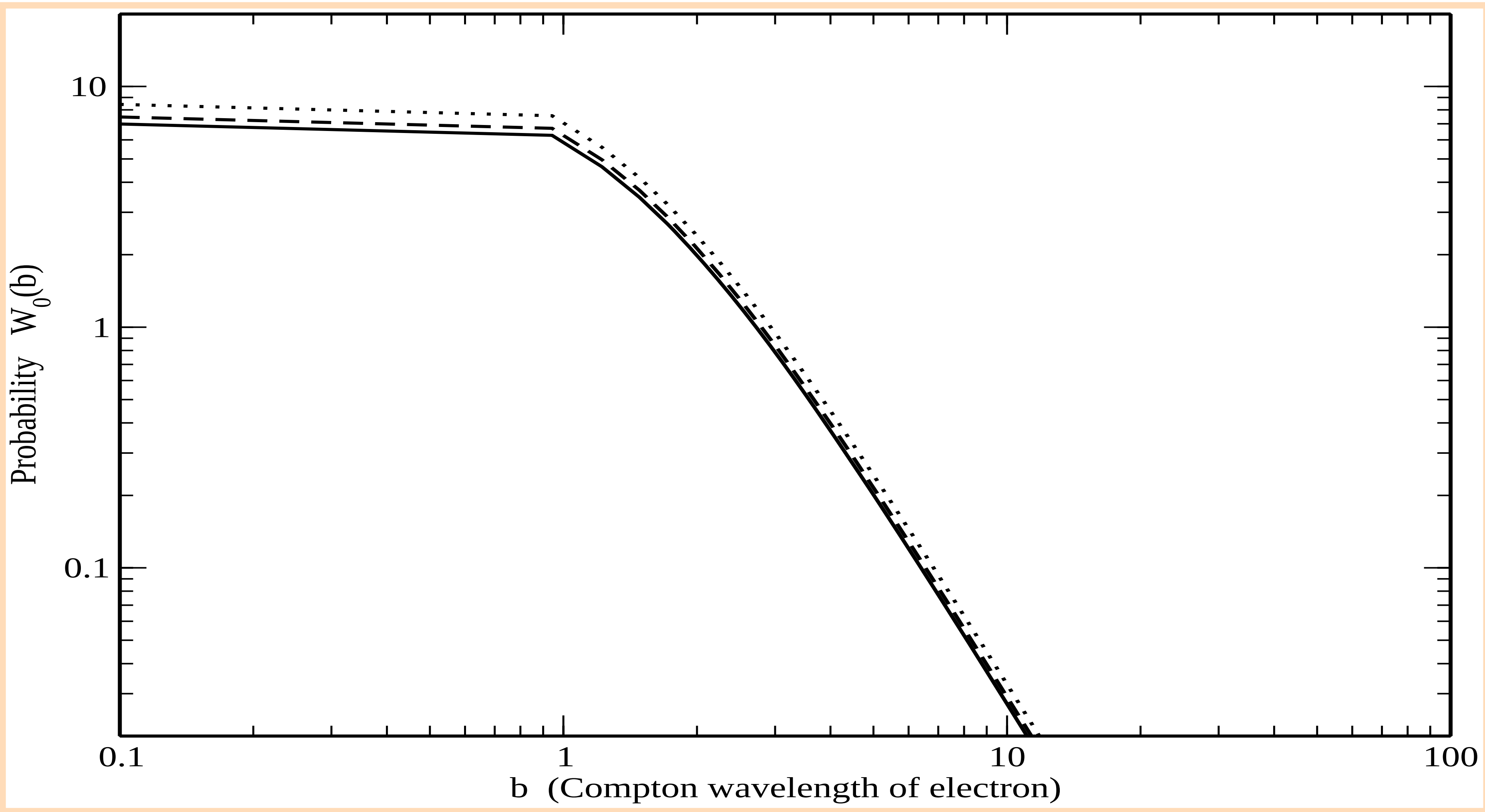
$\gamma = 100$



γ

and Au+Au collisions. The solid line is the Monte Carlo calculation, the dotted line is the Born approximation, and the dashed line is the Born approximation with Coulomb corrections.

$$\gamma = 3400$$



γ

and Au+Au collisions. The solid line is the Monte Carlo calculation, the dotted line is the Born approximation, and the dashed line is the Born approximation with Coulomb corrections.

CONCLUSIONS

1. We have obtained impact parameter dependence of lepton-pair production probability by using the semi-classical Monte Carlo Method.
2. In single-pair production, for high energies and small impact parameters, second order Feynman diagrams violates the unitarity. Therefore, higher order diagrams must be included.
3. When we include the higher order diagrams, we obtain multi-pair production probability as a Poisson distribution.
4. Born approximation with Coulomb corrections gives anomalous results for high energies.
5. Small-momentum approximation is not adequate to express pair-production probability.
6. We are successful to write a general expression for pair production probability of Born approximation.
7. Can we use this method to calculate the production of other particles such as mesons, heavy leptons, may be Higgs particles ?