

$b \rightarrow sy$ and $b \rightarrow dy$ (B factories)



Wenfeng Wang*
(on behalf of Babar* and Belle)



UNIVERSITY OF NOTRE DAME



Outline

❖ Introduction

❖ Direct CP asymmetry (A_{CP}) in $B \rightarrow X_{s+d} \gamma$ **New**

❖ Inclusive $b \rightarrow s \gamma$ branching fraction & spectral moments

❖ Measurement of $b \rightarrow d \gamma$ and determination of $|V_{\text{td}}/V_{\text{ts}}|$ **New**

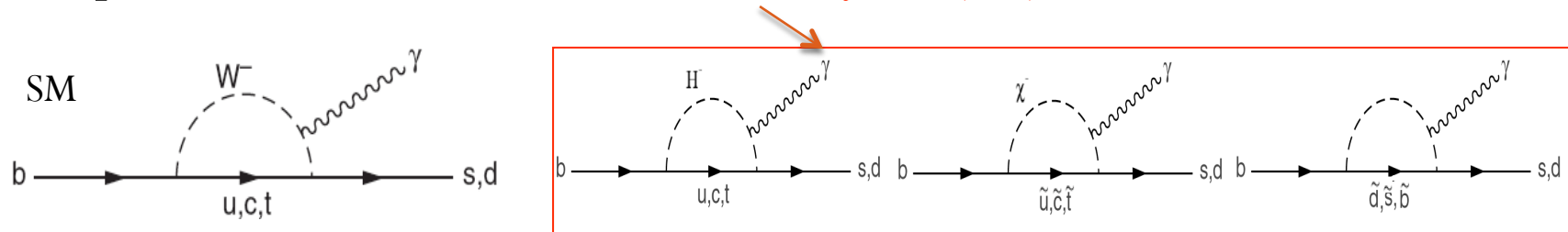
❖ Summary



Motivation

- ❖ $b \rightarrow s(d)\gamma$, flavor-changing neutral currents, forbidden at tree-level in Standard Model (SM)—lowest order requires one loop.

Ideal process for indirect search for **New Physics (NP)**.



- **Branching fraction:** SM prediction Next-to-Next-Leading-Log

$$(B \rightarrow X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}} = (3.15 \pm 0.23) \times 10^{-4} \quad (\text{Misiak et al. Phys.Rev.Lett.98:022002,2007})$$

- **Direct CP asymmetry**

$$A_{CP}(B \rightarrow X_s \gamma) \approx +0.0044$$

$$A_{CP}(B \rightarrow X_d \gamma) \approx -0.102$$

$$A_{CP}(B \rightarrow X_{s+d} \gamma) \sim 10^{-6}$$

$$A_{CP}(B \rightarrow X_q \gamma) = \frac{\Gamma(\bar{B} \rightarrow X_q \gamma) - \Gamma(B \rightarrow X_{\bar{q}} \gamma)}{\Gamma(\bar{B} \rightarrow X_q \gamma) + \Gamma(B \rightarrow X_{\bar{q}} \gamma)}$$

(Hurth et al, Nucl.Phys.B704:56-74,2005)

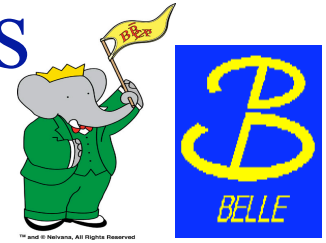
- Spectral moments => **Heavy quark parameters:** m_b , μ_π^2 .



Experimental technique

- ❖ Exclusive: Fully reconstructed resonance states, e.g. $K^*\gamma$, $(\rho, \omega)\gamma$, low background, large theoretical uncertainty from form factors.
- ❖ Inclusive: Better theoretical control from quark-hadron duality.
 - Sum of exclusive: reconstruct as many as possible exclusive modes, background understood, extrapolation of missing modes.
 - Fully Inclusive: identify one high energy photon, no missing modes, estimation of very large background is challenging.

Inclusive $b \rightarrow s(d)\gamma$ & B factories

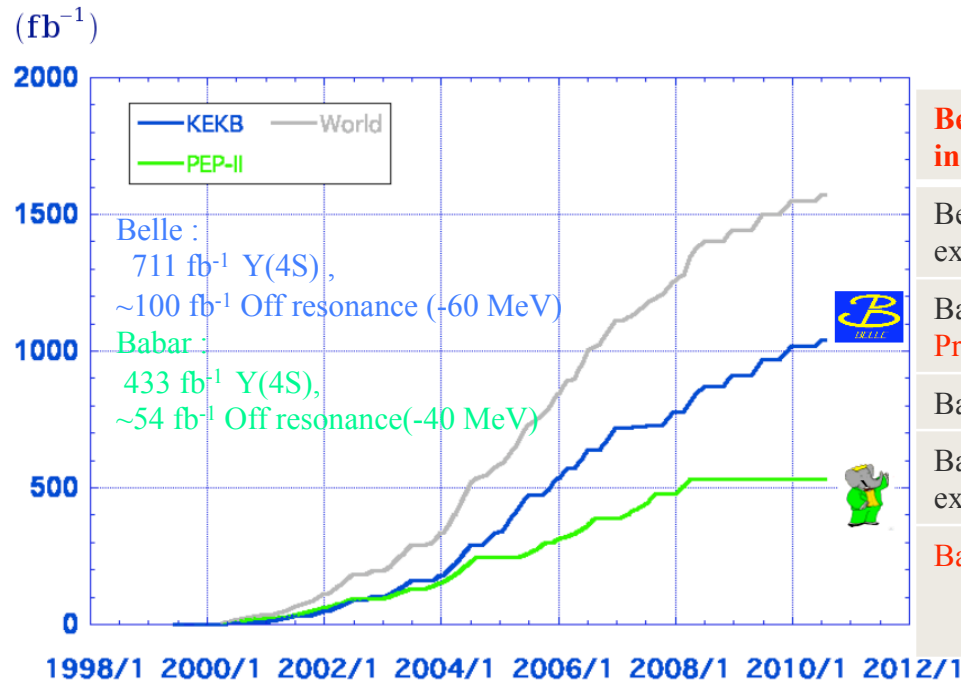


❖ $b \rightarrow s(d)\gamma$ pioneered by CLEO

PRL 74: 2885 (1995), PRL 87:251807 (2001)

❖ Huge datasets from B factories (Babar & Belle)

Luminosity at B factories



Belle inclusive*	PRL 103:241801(2009) PRL 93:061803(2004)	605 fb⁻¹ 140 fb⁻¹
Belle Σ exclusive	PL B511:151(2001)	5.8 fb ⁻¹
Babar l-tagged Preliminary*	PRL 97:171803(2006) *****	81fb ⁻¹ 347 fb⁻¹
Babar B-tagged	PRD77:151103 (2008)	210 fb ⁻¹
Babar Σ exclusive	PR D72:052004(2005)	82 fb ⁻¹
Babar Xs(d) *	PRL 102: 161803(2009) arXiv:1005.4087 (accepted by PRD-RC)	347 fb ⁻¹ 433fb⁻¹

*after CKM2008

Babar: $B \rightarrow X_s \gamma$ (New, preliminary)



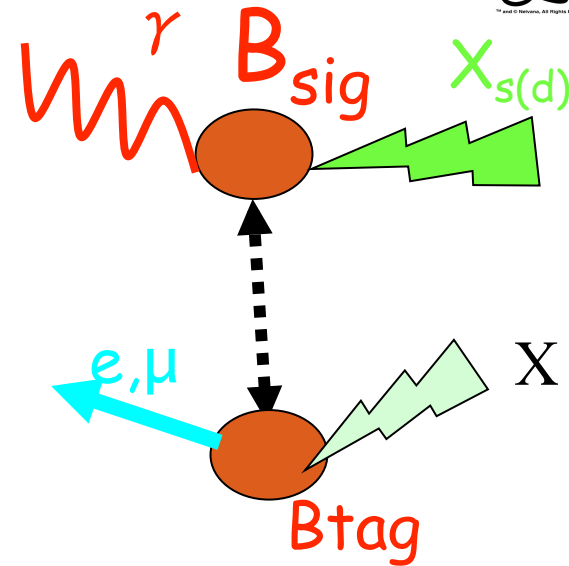
347 fb⁻¹ Y(4S) Data and 36 fb⁻¹ Off resonance Data

❖ Signal signature:

One isolated High Energy photon (γ_{HE}),
do not reconstruct the hadronic system.

Veto γ_{HE} from π^0/η .

❖ Lepton tag and event topology criteria used to suppress continuum.



❖ Measurements

🍏 Branching fraction

Does not distinguish X_s and X_d , $\sim 4\%$ X_d subtracted at the end.

🍏 CP asymmetry $A_{\text{CP}}(B \rightarrow X_{s+d} \gamma)$: Lepton charge identifies B flavor.

🍏 Spectral moments.

Present
today

Analysis method



Key of this analysis is to subtract backgrounds.

❖ Continuum

Event selection optimized, by lepton tagging and by exploiting event topology (Neural Network). Remained subtracted with off resonance data (scaled).

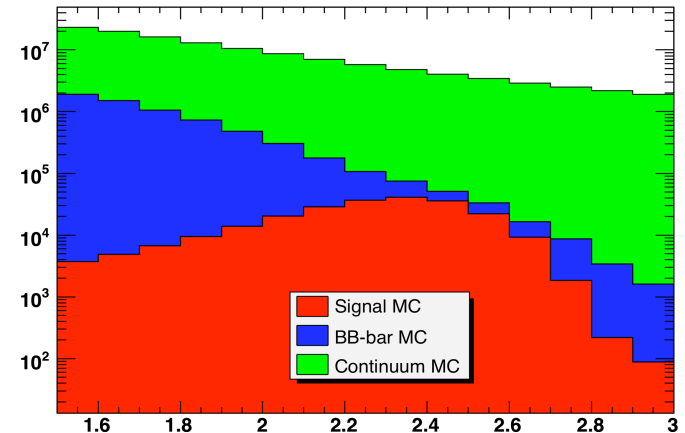
⇒ dominant statistical uncertainty.
Continuum control region (2.9-3.5) GeV.

❖ $B\bar{B}$ background (non-signal)

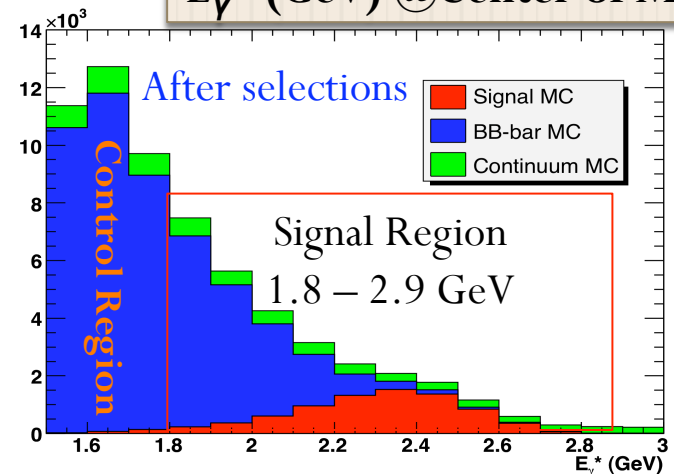
Rely on MC, correct each component with data control sample.

⇒ dominant systematic uncertainty.
 $B\bar{B}$ control region (1.53-1.8 GeV).

Monte Carlo (MC) after select HE photons + reject QED events



E_γ^* (GeV) @ Center of Mass



B \bar{B} background components



MC truth		Control region		Signal region	
Truth-match	Parent	MC fraction	Corr. factor	MC fraction	Corr. factor
Photon	π^0	0.5390	1.05	0.6127	1.09
	η	0.2062	0.79	0.1919	0.75
	ω	0.0386	0.80	0.0270	0.80
	η'	0.0112	0.52	0.0082	1.13
	B	0.0362	1.00	0.0194	1.00
	J/ψ	0.0061	1.00	0.0071	1.00
	e^\pm	0.0967	1.07	0.0619	1.07
	Other	0.0035	1.00	0.0032	1.00
	Total	0.9375	---	0.9315	---
e^\pm	Any	0.0411	1.65	0.0333	1.68
\bar{n}	Any	0.0170	0.35	0.0243	0.15
Other	Any	0.0029	1.00	0.0028	1.00
None		0.0015	1.00	0.0079	1.00

❖ Largest $\pi^0 + \eta$: 75-80%:

❖ $\omega, \eta' \sim 5\%$

❖ Electron bremsstrahlung,

❖ Fake photon $\sim 6-7\%$

➤ Electron mismatched or track lost.

➤ Cluster produced by \bar{n} .

❖ Nearly all components corrected/checked with various data control sample.

B \bar{B} background components-- π^0/η

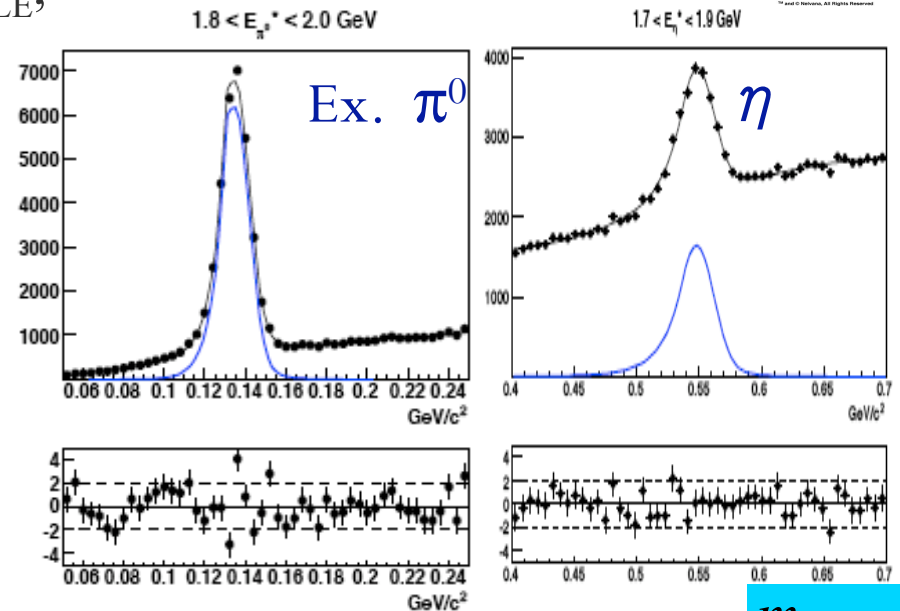


❖ Control sample $B \rightarrow X\pi^0(\eta)$, $\pi^0(\eta) \rightarrow \gamma_{HE}\gamma_{LE}$,
same selection applied except the veto.

❖ Yield fitted in $E^*_{\pi^0(\eta)}$ bins,
translated into E^*_{γ} bins.

❖ Corrections for data/MC differences
including:

- Produced yield after selection;
- Efficiency of high energy photon;
- Efficiency of lepton tag.



Veto: ($115 < m_{\gamma\gamma} < 155$ GeV, $508 < m_{\gamma\gamma} < 588$ GeV)

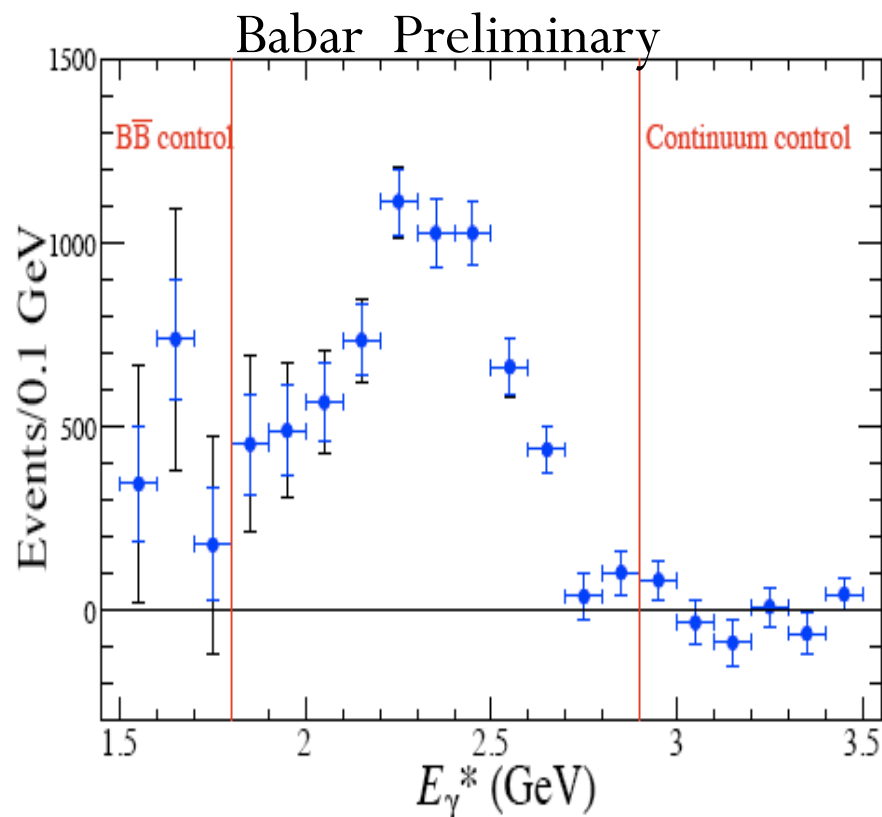
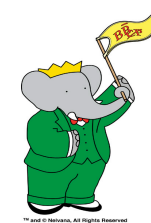
❖ However, $\pi^0(\eta)$ samples (**Low Energy photon found**)

⇒ the $\pi^0(\eta)$ events pass the π^0/η vetoes (mostly true **LE photon NOT found**).

Any data/MC difference on LE photon efficiency

⇒ **opposite direction in control sample and normal sample,**
Additional correction applied.

Unblinded spectrum after bg. subtraction



❖ Continuum Control Region:

OnPeak On – Off Data:
 1825 => -100 ± 138 events

❖ $B\bar{B}$ Control Region:

OnPeak On – Off Data – BB MC
 3.6×10^4 => $1252 \pm 272 \pm 841$

(1.4σ IF no signal)

a tail of signal $\sim 100-400$ (models)

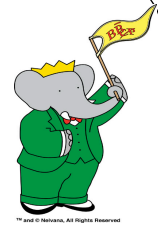
=> $0.9-1.3\sigma$

❖ Control region checks show good understanding of backgrounds.

❖ A_{CP} is insensitive to photon energy cut, statistical optimization

=> $(2.1-2.8)$ GeV for the A_{CP} .

$A_{CP}(B \rightarrow X_{s+d}\gamma)$ measurement



❖ Lepton charge gives the flavor of B.

❖ Sometimes mistags B by mistag rate (ω):

$$A_{CP} = \frac{A_{CP}^{meas}}{1 - 2\omega}$$

❖ Statistical errors, dominated by off peak data subtraction.

❖ Most of the systematic uncertainties in the branching fraction measurement are common for +/- leptons, cancelled for A_{CP} .

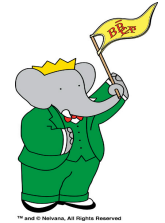
❖ Potential bias (ΔA_{CP}) from

- BB Background subtraction, estimated from control region.
- Detector charge asymmetry: Lepton ID.

	$\omega \pm \Delta\omega$
$B^0 - \bar{B}^0$ oscillation	$(0.1824 \pm 0.0024)/2$
$B^0 \bar{B}^0 : B^+ B^-$	0.000 ± 0.0030
Non-direct-semileptonic	0.0318 ± 0.0035
Fake ID	0.0073 ± 0.0037
sum	0.131 ± 0.0064

	$\Delta A_{CP} (10^{-2})$
$B\bar{B}$ background	-0.4 ± 0.6
Detection asymmetry	0.0 ± 1.1
Sum	-0.4 ± 1.3

$A_{CP}(B \rightarrow X_{s+d}\gamma)$ preliminary



$N(I^+) = 2623 \pm 158$

$N(I^-) = 2397 \pm 151$

Babar preliminary

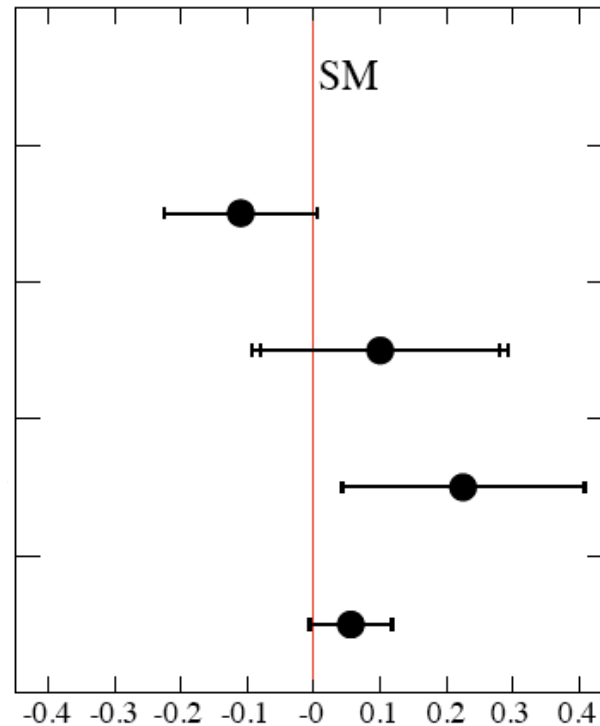
$A_{CP} = 0.056 \pm 0.060 \pm 0.018$

Babar l-tag

Babar B-tag

CLEO l-tag

Babar preliminary



$A_{CP} \pm \text{stat} \pm \text{syst}$

$-0.110 \pm 0.115 \pm 0.017$

$0.10 \pm 0.18 \pm 0.05$

$0.225 \pm 0.181 \pm 0.027$

$0.056 \pm 0.060 \pm 0.018$

- ✓ No significant asymmetry is observed, most precise to date.
- ✓ Consistent with SM and previous measurements.
- ✓ All measurements dominated by statistical uncertainty.

Belle: Fully Inclusive $b \rightarrow s\gamma$

Phys.Rev.Lett.103:241801,2009.

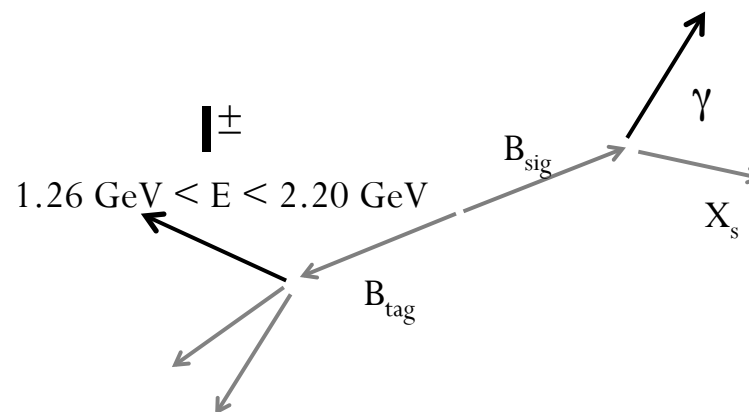


❖ 657M $B\bar{B}$ and 68 fb⁻¹ offpeak (60MeV below).

❖ Only high energy γ reconstructed, veto π^0/η .

❖ Mix of two sub samples:

➤ **Tagged**: similar to Babar method, well defined electron/muon.



➤ **Untagged**: Fischer discriminant based on energy flow and event-shape for continuum suppress.

❖ Two methods: Similar sensitivity, largely statistically independent, profit from combination.

❖ Remaining background

➤ Continuum taken from off-peak data;

➤ $B\bar{B}$ estimated from MC.

B \bar{B} Background



Phys.Rev.Lett.103:241801,2009.

❖ BB Background: 6 background categories taken from MC.

1.7-2.8 GeV, photon sources (B \bar{B})

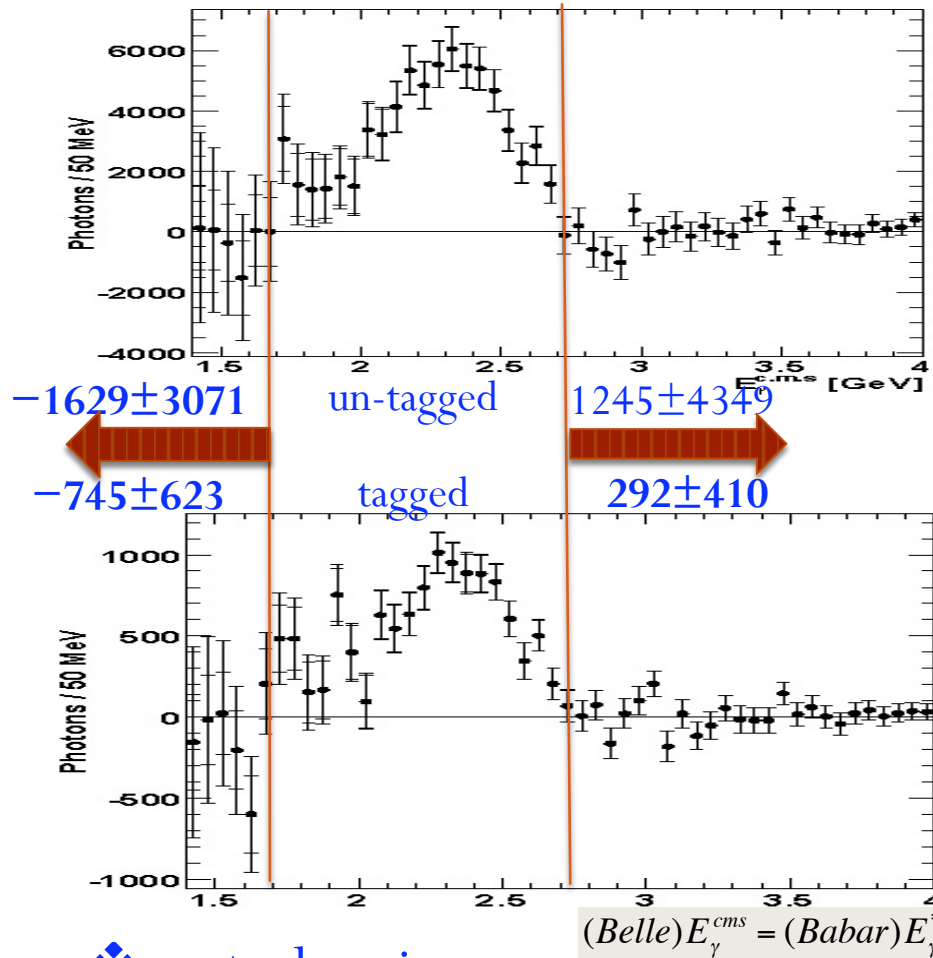
	Untagged (%)	Tagged(%)
π^0	47.4	48.0
η	16.3	16.0
ω η' J/ Ψ	8.1	8.9
K_L, \bar{n}	1.7	1.6
Electron	6.1	3.3
Beam bg	1.3	2.6
Signal	19.1	19.7

➤ inclusive $B \rightarrow X\pi^0(\eta)$ samples reconstructed in data and MC \Rightarrow correct MC yield;
veto efficiency measured by partially reconstructed $D^0 \rightarrow K^-\pi^+\pi^0$;

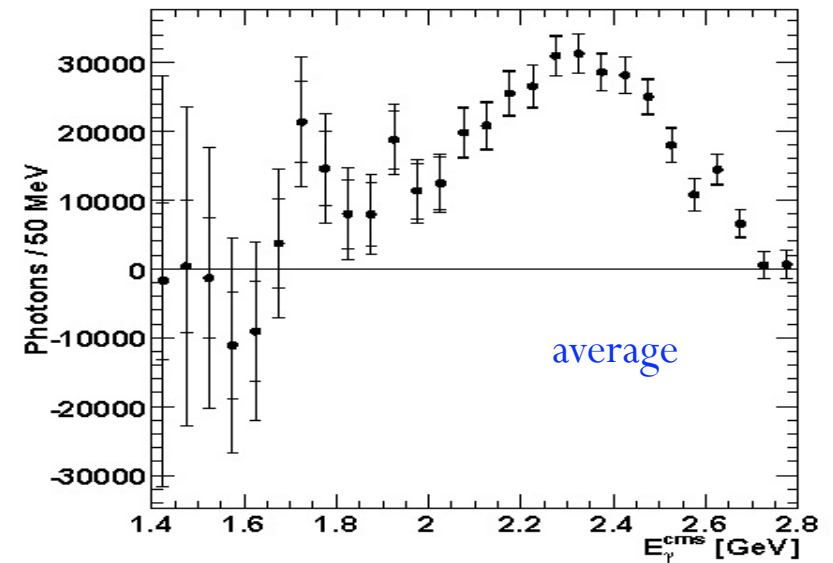
➤ Others B components,
shape and yield corrected by data control samples,
 $\pm 20\%$ assigned to systematic uncertainty \Rightarrow largest systematic.

❖ $\sim 60\%$ of data with EM cluster timing information used to remove overlapping events (hadronic + Bhabha).

Measured spectra



❖ Corrected and combined spectra
 correct for select efficiency
 ($\epsilon_{\text{untag}} \sim 15\%$, $\epsilon_{\text{tag}} \sim 2.5\%$);
 statistic correlation into account.



❖ control region

- High-energy region (2.8-4.0 GeV): agree with zero $< 0.5 \sigma$
- Low-energy region (1.4-1.7 GeV): 0.5σ (untagged) 1.2σ if no signal

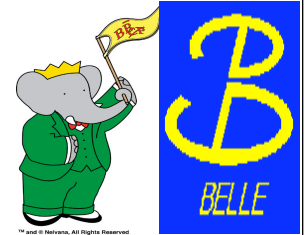
Branching fraction



E_{γ^B} (GeV)	>1.7	>1.8	>1.9	>2.0
BF ($b \rightarrow s \gamma$)	3.45	3.36	3.21	3.02
Statistical error	4.4%	3.9%	3.4%	3.3%
Systematic error	11.6%	7.4%	5.0%	3.6%
Sum	12.4%	8.4%	6.0%	4.9%
HFAG extrapolation (%)	98.5 ± 0.4	96.7 ± 0.6	93.6 ± 1.0	89.4 ± 1.6

- ❖ Smallest total measured error is 5% for $>2.0\text{GeV}$.
- ❖ First time down to 1.7 GeV, cover more than 98% of Xs, smallest model error, systematic error increased by a factor of 3, from large B background subtraction.

$b \rightarrow s\gamma$ Branching Fractions



	E^{\min} (GeV)	$B(E_{\gamma}^B > E^{\min}) (\times 10^{-4})$
CLEO (2001)	2.0	$3.06 \pm 0.41 \pm 0.26$
Belle Σ exclusive (2001)	2.24	$3.36 \pm 0.53 \pm 0.42^{+0.50}_{-0.54}$
Babar Σ exclusive (2005)	1.9	$3.27 \pm 0.18^{+0.56}_{-0.41} \quad ^{+0.04}_{-0.09}$
Babar l-tag (2006)	1.9	$3.67 \pm 0.29 \pm 0.34 \pm 0.29$
Babar B-tag (2008)	1.9	$3.66 \pm 0.85 \pm 0.60$
Bell inclusive (2009)	1.7	$3.45 \pm 0.15 \pm 0.40$
HFAG average (2010)	1.6	$3.55 \pm 0.24 \pm 0.09$
SM NNLL	1.6	3.15 ± 0.23

} **1.2 σ**

- ❖ Good agreement with SM prediction.
- ❖ Provide stringent constraints on NP: e.g. $M_{H^\pm} > 295 \text{ GeV} @ 95\% \text{ CL}$.
- ❖ Babar lepton tag (New) results will come soon.
- ❖ Important to measure the $B \rightarrow X_s \gamma$ with reduce error in future!

$b \rightarrow s\gamma$ spectral moments

❖ Ideal laboratory for the studying dynamics of b-quark inside the B meson, because of the universal motion of b-quark inside the B meson => apply to B semileptonic decays.

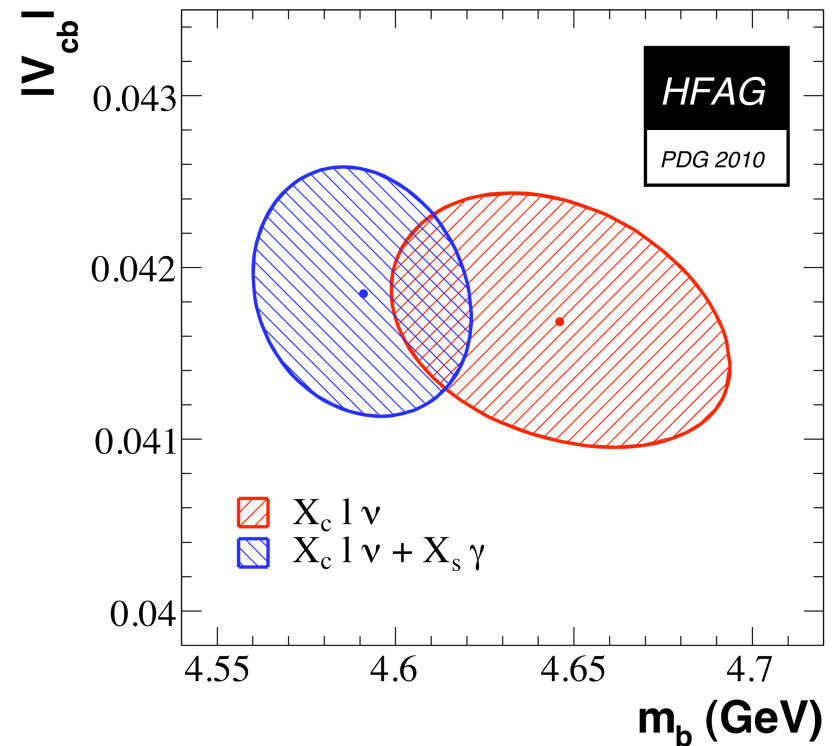
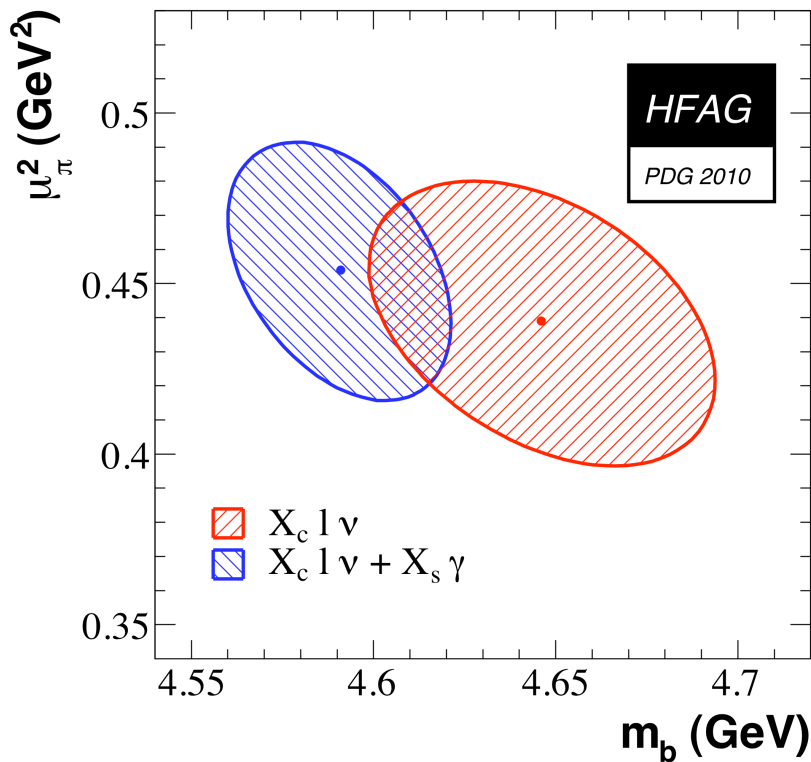
First Moments $\langle E_\gamma \rangle$ in $\text{GeV} \times 10^3$

E_γ threshold	> 1.7	> 1.8	> 1.9	> 2.0	> 2.1
Belle inclusive	2282 ± 53	2294 ± 30	2311 ± 18	2334 ± 11	2350 ± 14
BaBar l-tagged			2288 ± 33	2316 ± 22	2355 ± 19
BaBar B-tagged			2289 ± 64	2315 ± 41	2371 ± 27
BaBar Σ exclusive			2321 ± 47	2314 ± 31	2357 ± 21
B factory average	2282 ± 53	2294 ± 30	2304 ± 16	2324 ± 10	2357 ± 10

Second Moments $\langle E_\gamma^2 - \langle E_\gamma \rangle^2 \rangle$ in $\text{GeV}^2 \times 10^4$

E_γ threshold	> 1.7	> 1.8	> 1.9	> 2.0	> 2.1
Belle inclusive	428 ± 207	370 ± 86	302 ± 36	230 ± 21	170 ± 22
BaBar l-tagged			328 ± 60	266 ± 34	191 ± 25
BaBar B-tagged			334 ± 139	265 ± 62	142 ± 39
BaBar Σ exclusive			253 ± 107	273 ± 40	183 ± 25
B factory average	428 ± 207	370 ± 86	305 ± 32	253 ± 17	174 ± 13

$b \rightarrow s \gamma$ spectral moments & Global Fit



Input	$m_b(\text{GeV})$	$\mu_\pi^2(\text{GeV}^2)$	$ V_{cb} (10^{-3})$
$X_c 1 \nu$	4.646 ± 0.047	0.439 ± 0.042	$41.68 \pm 0.44_{\text{fit}} \pm 0.09_{\tau_B} \pm 0.58_{\text{th}}$
$X_c 1 \nu + X_s 1 \nu$	4.591 ± 0.031	0.454 ± 0.038	$41.85 \pm 0.42_{\text{fit}} \pm 0.09_{\tau_B} \pm 0.59_{\text{th}}$

$B \rightarrow X_d \gamma$ Analysis & $|V_{td}/V_{ts}|$

- ❖ $b \rightarrow d\gamma$ suppressed by a factor of ~ 20 w.r.t. $b \rightarrow s\gamma$ in the SM. New Physics may appear differently for $b \rightarrow s\gamma$ and $b \rightarrow d\gamma$, change the branching fraction ratio.

- ❖ The ratio used to extract $|V_{td}/V_{ts}|$.

$$\frac{\Gamma(b \rightarrow d\gamma)}{\Gamma(b \rightarrow s\gamma)} = \xi^2 \left| \frac{V_{td}}{V_{ts}} \right|^2 (1 + \Delta R)$$

Ali et al PRB 429,87 (1998)

- ξ is the remaining SU(3) breaking effects and ΔR is the annihilation amplitudes correction, often calculated in term of CKM parameters (ρ and η), both related to $|V_{td}/V_{ts}|$;
 - Reformulate the equation to express the ratio as function of $|V_{td}/V_{ts}|$ and a orthogonal coordinate, i.e. well-known CKM angle β .
- ❖ Independent check of value from B_s vs. B_d mixing
 - ❖ Study from exclusive decay well established $\sim 7\%$ theory uncertainty; inclusive decay $\sim 1\%$ theory uncertainty.

Babar: $B \rightarrow X_d \gamma$ Analysis



arXiv:1005.4087

(accepted by PRD-RC)

- ❖ 471M $B\bar{B}$ (full Babar dataset), supersedes the previous measurements with 383 M BB : PRL102:161803 (2009)

- ❖ Measure the partial branching fraction of sum of seven modes: X_d and X_s

$B \rightarrow X_d \gamma$	$B \rightarrow X_s \gamma$
$B^0 \rightarrow \pi^+ \pi^- \gamma$	$B^0 \rightarrow K^+ \pi^- \gamma$
$B^+ \rightarrow \pi^+ \pi^0 \gamma$	$B^+ \rightarrow K^+ \pi^0 \gamma$
$B^+ \rightarrow \pi^+ \pi^- \pi^+ \gamma$	$B^+ \rightarrow K^+ \pi^+ \pi^- \gamma$
$B^0 \rightarrow \pi^+ \pi^- \pi^0 \gamma$	$B^0 \rightarrow K^+ \pi^- \pi^0 \gamma$
$B^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^- \gamma$	$B^0 \rightarrow K^+ \pi^- \pi^+ \pi^- \gamma$
$B^+ \rightarrow \pi^+ \pi^- \pi^+ \pi^0 \gamma$	$B^+ \rightarrow K^+ \pi^- \pi^+ \pi^0 \gamma$
$B^+ \rightarrow \pi^+ \eta \gamma$	$B^+ \rightarrow K^+ \eta \gamma$

X_s and X_d related by kaon-pion substitution

Charge conjugation implied throughout

- ❖ Extrapolate to inclusive BF, to include un-reconstructed decay modes, requires knowledge of fragmentation of hadronic systems

$B \rightarrow X_d \gamma$ Fit to data



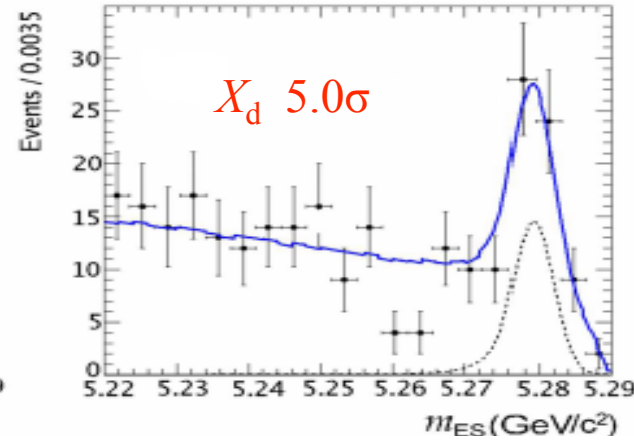
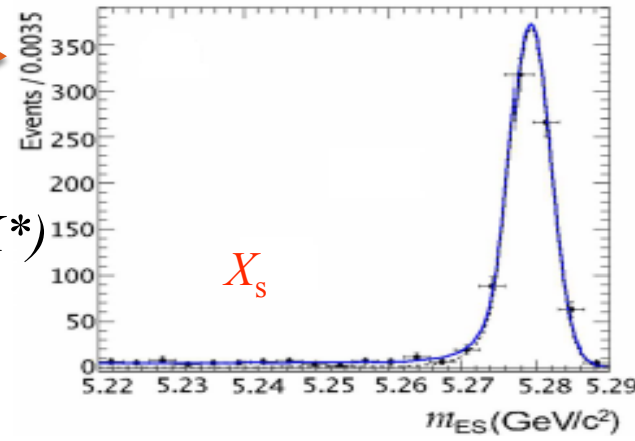
- ❖ Yield determined from simultaneous fit to m_{ES} (below) and ΔE
- ❖ Two mass bins to cover 0.5-2.0 GeV

$$m_{ES} = \sqrt{E_{Beam}^2 - p_B^2}$$

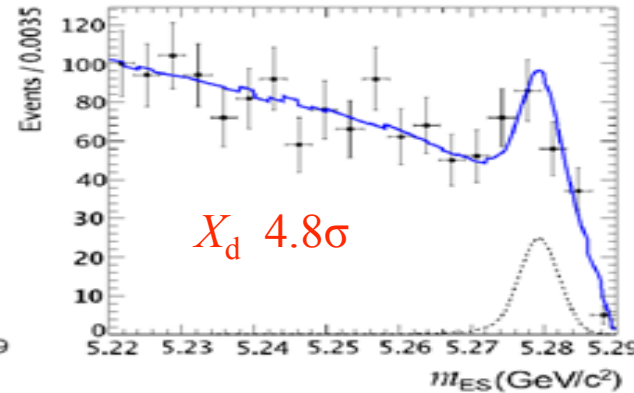
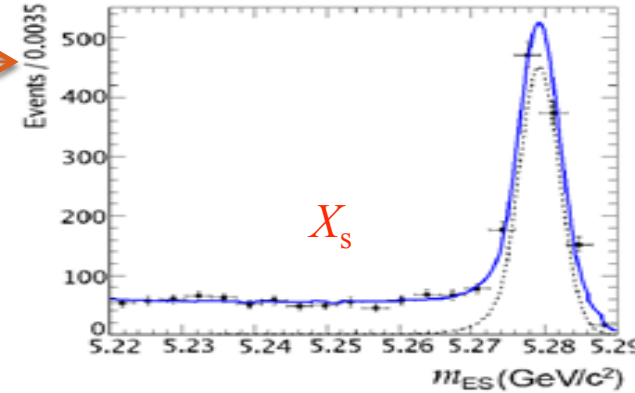
$$\Delta E = E_B - E_{Beam}$$

-0.15 ΔE 0.1 GeV

Top:
Low mass bin
0.5 -1.0 GeV/c²
Dominated by ρ/ω (K^*)
resonances



Bottom:
High mass bin
1.0 -2.0 GeV/c²
Non-resonant



- Large signal yield X_s , allow to study the fragmentation of the non-resonant decay, adjust MC.
- First significant observation of $b \rightarrow d \gamma$ in non-resonant modes.

Systematic errors



- ❖ Systematic error on the measured partial branching fraction calculated from data/MC comparisons.
- ❖ To extrapolate to inclusive BF, correct **the missing states**
 - Low mass region: resonance dominated, simply correct unreconstructed $\rho/\omega(K^*)$
 - High mass region: based on MC fragmentation model, corrected for measured $b \rightarrow s\gamma$.
 - Vary fragmentation with physics-motivated bounds to obtain uncertainty
=> **Largest systematic errors.**

Systematic Error Source	$M(X_s)$		$M(X_d)$	
	0.5-1.0	1.0-2.0	0.5-1.0	1.0-2.0
Error on partial \mathcal{B}	4.0%	9.0%	9.3%	14.2%
Missing ≥ 5 body		9.6%		18.2%
Other missing states		7.5%		15.3%
Spectrum Model		1.8%		1.6%
Error on inclusive \mathcal{B}	4.0%	15.2%	9.3%	27.7%

$B \rightarrow X_d \gamma$ Results



	Branching Fraction
$b \rightarrow s\gamma$ low mass	$3.83 \pm 0.16 \pm 0.15 \times 10^{-5}$
$b \rightarrow s\gamma$ high mass	$19.2 \pm 0.8 \pm 1.7 \pm 2.3 \times 10^{-5}$
$b \rightarrow s\gamma$ ($m_{had} < 2.0 \text{ GeV}/c^2$)	$23.0 \pm 0.8 \pm 1.9 \pm 2.3 \times 10^{-5}$
$b \rightarrow d\gamma$ low mass	$1.25 \pm 0.32 \pm 0.12 \times 10^{-6}$
$b \rightarrow d\gamma$ high mass	$7.90 \pm 1.98 \pm 1.12 \pm 1.88 \times 10^{-6}$
$b \rightarrow d\gamma$ ($m_{had} < 2.0 \text{ GeV}/c^2$)	$9.15 \pm 2.01 \pm 1.24 \pm 1.88 \times 10^{-6}$
$\frac{B \rightarrow (\rho, \omega)\gamma}{B \rightarrow K^*\gamma}$	$0.033 \pm 0.009 \pm 0.003$
$\frac{b \rightarrow d\gamma}{b \rightarrow s\gamma}$ ($m_{had} < 2.0 \text{ GeV}/c^2$)	$0.040 \pm 0.009 \pm 0.005 \pm 0.010$

World Average (HFAG):
 $\text{BF}(B \rightarrow K^*\gamma) = (4.21 \pm 0.18) \times 10^{-5}$

$\text{BF}(B \rightarrow (\rho, \omega)\gamma) = (1.30 \pm 0.19) \times 10^{-5}$

Combined: the first measurement $< 2.0 \text{ GeV}$

Unchanged for the full spectra

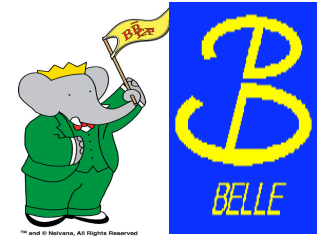
$$|V_{td}/V_{ts}| = 0.199 \pm 0.022_{stat} \pm 0.012_{syst} \pm 0.027_{extrap} \pm 0.002_{th}$$

$$\text{B-Mixing average : } |V_{td}/V_{ts}| = 0.2059 \pm 0.001_{stat} \pm 0.008_{th}$$

- ❖ Consistent with previously measured results.
- ❖ Smaller theoretical uncertainty!



Summary



- ❖ **Babar: New** CP asymmetry ($B \rightarrow X_{s+d}\gamma$) measurement: consistence with SM, the most precise to date.
- ❖ **Belle** provides the best experimental measurements on the branching fraction & spectral moments, first experiment down to 1.7 GeV, the smallest experimental error > 2.0 GeV.
- ❖ **Babar:** Sum of exclusive $b \rightarrow d\gamma$ measurement. First significant measurement of non-resonant $b \rightarrow d\gamma$. $|V_{td}/V_{ts}|$, comparable with previous measurements, smallest theoretical uncertainty
- ❖ **No significant evidence for NP** from B factories, BaBar and Belle, consistent with SM predictions within experimental error.
=> CKM is the dominant (only?) source of CP violation in B decay.
- ❖ **LHC-b and Super B factories! Hope for the future!**

Backup

- ▶ Extract $X=|V_{td}/V_{ts}|$ from **ratio** of inclusive BFs

- ▶ Use NLO **calculation** of Ali, *et al.* [Phys. Lett. B429 87]

$$R = \lambda^2 [1 + \lambda^2 (1 - 2\bar{\rho})] \left[(1 - \bar{\rho})^2 + \bar{\eta}^2 + \frac{D_u}{D_t} (\bar{\rho}^2 + \bar{\eta}^2) + \frac{D_r}{D_t} (\bar{\rho}(1 - \bar{\rho}) - \bar{\eta}^2) \right]$$

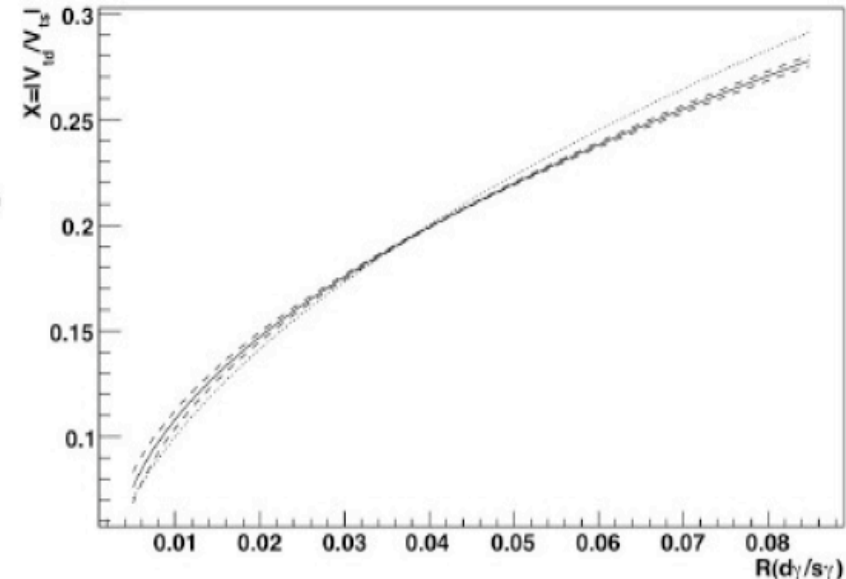
- ▶ Rewrite in terms of **X** and UT angle β

$$R = \kappa_1 X^2 + \kappa_2 X + \kappa_3,$$

$$\kappa_1 = 1 + \frac{D_u}{D_t} (1 - 2\lambda^2 \cos^2 \beta) - \frac{D_r}{D_t} (\lambda^2 \cos^2 \beta + 1),$$

$$\kappa_2 = \lambda \cos \beta \left[\frac{D_u}{D_t} (3\lambda^2 - 2) + \frac{D_r}{D_t} \left(1 + \frac{\lambda^2}{2} \right) \right],$$

$$\kappa_3 = \lambda^2 \frac{D_u}{D_t} (1 - \lambda^2).$$



- ▶ Uncertainties from PDG and numerical calculations of **D** factors