

Creation of an intense source of  $\pi\pi$ ,  $\pi K$  and other exotic atoms at SPS proton beam and using them for accurate measurements of  $\pi\pi$ ,  $\pi K$  scattering length and other low energy parameters to check the precise low energy  $QCD$  predictions



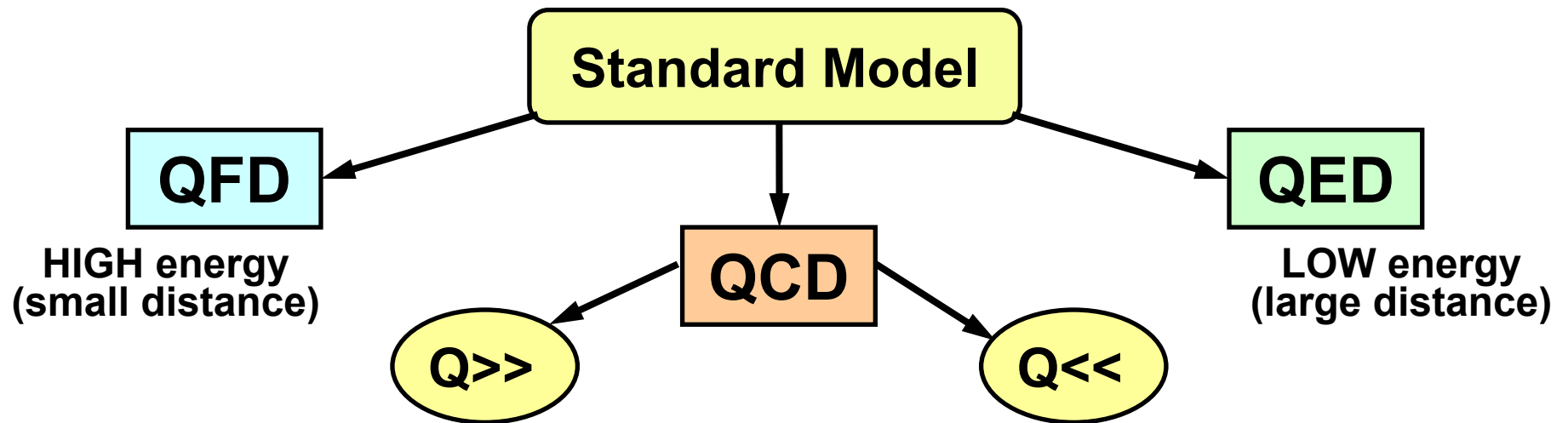
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*25 September 2004*



# Theoretical motivation



## perturbative QCD:

$$L_{QCD}(q,g)$$

interaction  $\rightarrow$  „weak“ (asympt. freedom): expansion in coupling

Check only  $L_{sym}$

## chiral sym. & break:

$$L_{eff}(GB: \pi, K, \eta)$$

interaction  $\rightarrow$  „strong“ (confinement)  
- **but**: expansion in energy

Check  $L_{sym}$  as well as

$L_{break-sym} \rightarrow$  **q-condensate**



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# $\pi\pi$ scattering

ChPT predicts s-wave scattering lengths:

$$a_0 = 0.220 \pm 0.005 (2.3\%)$$

$$a_2 = -0.0444 \pm 0.0010 (2.3\%)$$

$$a_0 - a_2 = 0.265 \pm 0.004 (1.5\%)$$

Chiral expansion of the  $\pi$  mass:

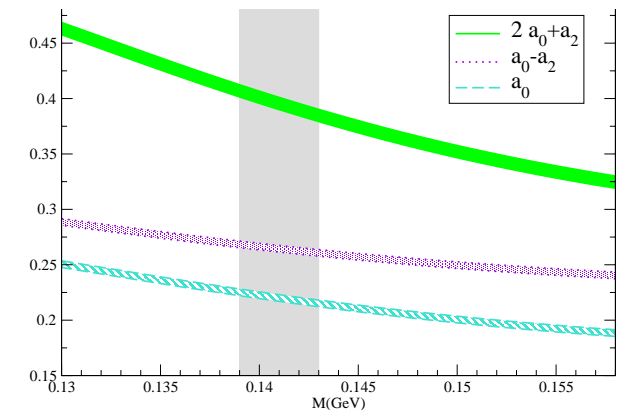
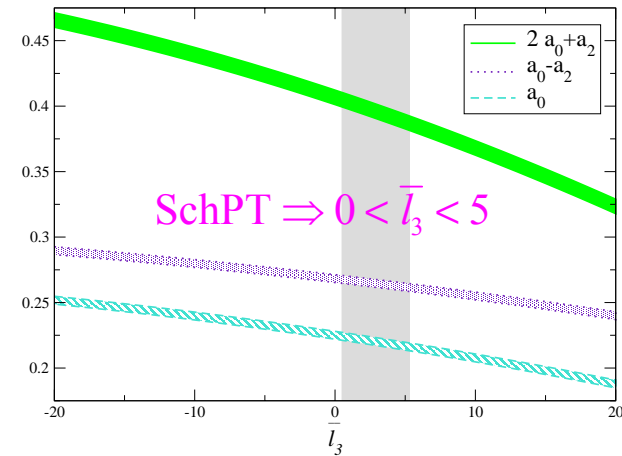
$$M_\pi^2 = (m_u + m_d) B - \left[ (m_u + m_d) B \right]^2 \frac{\bar{l}_3}{32\pi^2 F^2} + O((m_u + m_d)^3)$$

where  $BF_\pi^2 = |\langle 0 | \bar{u}u | 0 \rangle|$   
is the *quark condensate*, reflecting  
a property of the *QCD vacuum*.

Measurement of  $\bar{l}_3^{(1)} \Rightarrow$  estimate of  $(m_u + m_d) |\langle 0 | \bar{u}u | 0 \rangle|$ :

e.g.:  $a_0 - a_2 = 0.260 \pm 3\% \Rightarrow 1 < \bar{l}_3 < 11$  or  $1.00 < M / M_\pi < 1.06$

E865:  $a_0 = 0.216 \pm 6\%$  (BNL)  $\Rightarrow -4 < \bar{l}_3 < 12$  or  $0.98 < M / M_\pi < 1.06$



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# Experimental Status

$$K^+ \rightarrow \pi^+ \pi^- e^+ \nu_e (K_{e4}) \Rightarrow \begin{cases} \text{measurement of the phase difference} \\ \delta(s) \equiv \delta_0^0(s) - \delta_1^1(s) \text{ for } 4M_\pi^2 < s < M_K^2 \end{cases}$$

1)  $a_0 = 0.26 \pm 0.05$ , using Roy eq.

Rosselet et al. CERN, 1977

2a)  $a_0 = 0.203 \pm 0.033$   
 $a_2 = -0.055 \pm 0.023$

Pislak et al.  
E865/BNL,  
2001/03

2b)  $a_0 = 0.216 \pm 0.013$  (stat)  $\pm$   
 $0.004$  (sys)  $\pm 0.002$  (th),  
using Roy eq. &  $a_2 = f_{\text{ChPT}}(a_0)$

DIRAC after analysis of all collected data

$$\delta(a_0 - a_2) = \pm 5\%(\text{stat}) \pm 3\%(\text{syst}) \pm 2\%(\text{theor})$$

preliminary



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# $\pi K$ scattering

## I. ChPT predicts s-wave scattering lengths:

$$a_0^{1/2} = 0.19 \pm 0.2 \quad a_0^{3/2} = -0.05 \pm 0.02$$

$L^{(2)}, L^{(4)}$  and 1-loop

V. Bernard, N. Kaiser,  
U. Meissner. – 1991

$$a_0^{1/2} - a_0^{3/2} = 0.23 \pm 0.01$$

A. Rossel. – 1999

J. Bijnens, P. Talaver. – April 2004

$L^{(2)}, L^{(4)}, L^{(6)}$  and 2-loop

## II. Roy-Steiner equations:

$$a_0^{1/2} - a_0^{3/2} = 0.269 \pm 0.015$$

## III. $A_{\pi K}$ lifetime:

$$A_{\pi^+ K^-} \rightarrow \pi^0 \bar{K}^0 \quad (A_{K^+ \pi^-} \rightarrow \pi^0 K^0)$$

$$\Gamma(\pi^0 \bar{K}^0) \sim |a_0^{1/2} - a_0^{3/2}|^2 \quad \text{precision} \sim 1\%$$

J. Schweizer. – 2004

$$\tau = (3.7 \pm 0.4) \cdot 10^{-15} \text{ s}$$



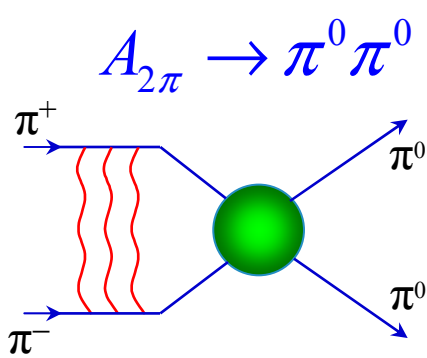
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# 1. $A_{2\pi}$ time of life



$$\frac{1}{\tau} = R_{\pi} (a_0 - a_2)^2$$

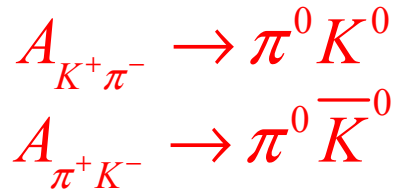
$$\frac{\Delta R_{\pi}}{R_{\pi}} \approx 1\%^{(*)}$$

DIRAC ADDENDUM

(\*) A. Gall et al. PR(2001)

$$\frac{\Delta\tau}{\tau} = 4\% (stat), \quad \frac{\Delta |a_0 - a_2|}{a_0 - a_2} = 2\% (stat) \pm 1\% (syst) \pm 1\% (theor)$$

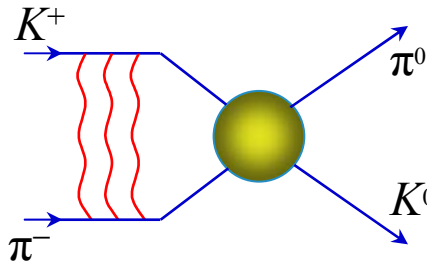
# 2. $A_{K^+\pi^-}$ and $A_{K^+\pi^-}$ time of life



$$\frac{1}{\tau} = R_K |a_{1/2} - a_{3/2}|^2$$

$$\frac{\Delta R_K}{R_K} \approx 2\%^{(**)}$$

(\*\*) J. Schweizer (2004)



DIRAC ADDENDUM

$$\frac{\Delta\tau}{\tau} = 20\% (stat), \quad \frac{\Delta |a_{1/2} - a_{3/2}|}{a_{1/2} - a_{3/2}} = 10\% (stat) \pm 1\% (syst) \pm 1.5\% (theor)$$



# Energy Splitting between $np$ - $ns$ states in ( $\pi^+$ - $\pi^-$ ) atom

$$\Delta E_n \equiv E_{ns} - E_{np}$$

$$\Delta E_n \approx \Delta E_n^{vac} + \Delta E_n^s \quad \Delta E_n^s \sim 2a_0 + a_2$$

For  $n=2$

$$\Delta E_2^{vac} = -0.107 \text{ eV} \text{ from QED calculations}$$

$$\Delta E_2^s \approx -0.45 \text{ eV} \text{ numerical estimated value from ChPT}$$

$$a_0 = 0.220 \pm 0.005$$

$$a_2 = -0.0444 \pm 0.0010$$

(2001) *G. Colangelo, J. Gasser and H. Leutwyler*

$$\Rightarrow \boxed{\Delta E_2 \approx -0.56 \text{ eV}}$$

(1979) A. Karimkhodzhaev and R. Faustov

(1983) G. Austen and J. de Swart

(1986) G. Efimov *et al.*

(1999) A. Gashi *et al.*

(2000) D. Eiras and J. Soto

A. Rusetsky, *priv. comm.*



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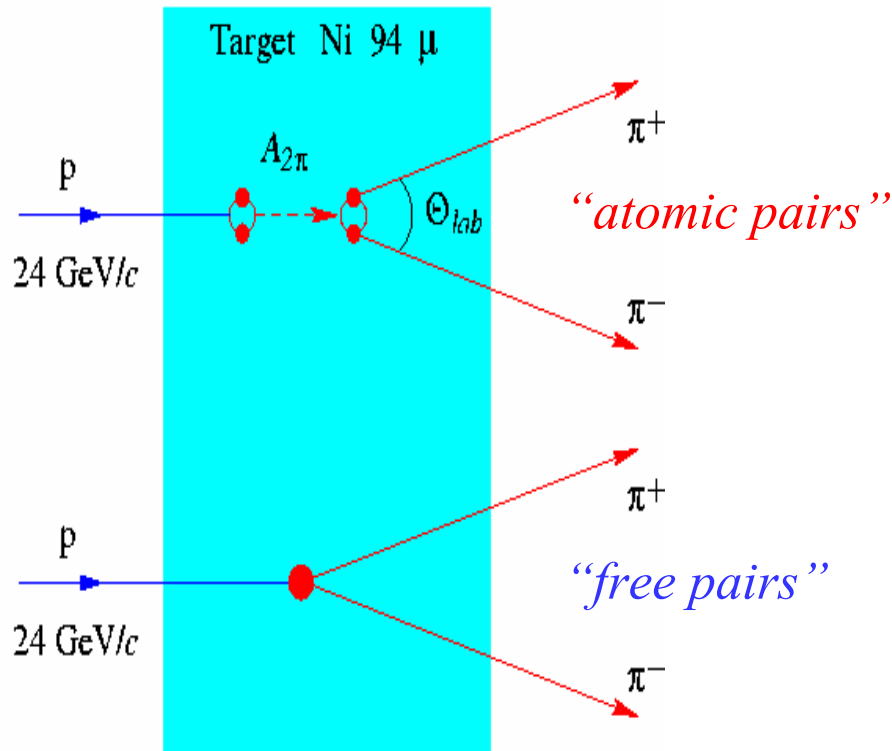
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# Method of $A_{2\pi}$ Observation

$\tau(A_{2\pi})$  too small to be measured directly



*e.m. interaction of  $A_{2\pi}$  in the target*

$$A_{2\pi} \rightarrow \pi^+ \pi^-$$

$$Q < 3 \text{ MeV}/c \quad E_+ \approx E_-$$

$$\Theta_{lab} < 2.5 \text{ mrad}$$

*Coulomb from short-lived sources*  
*non-Coulomb from long-lived sources*



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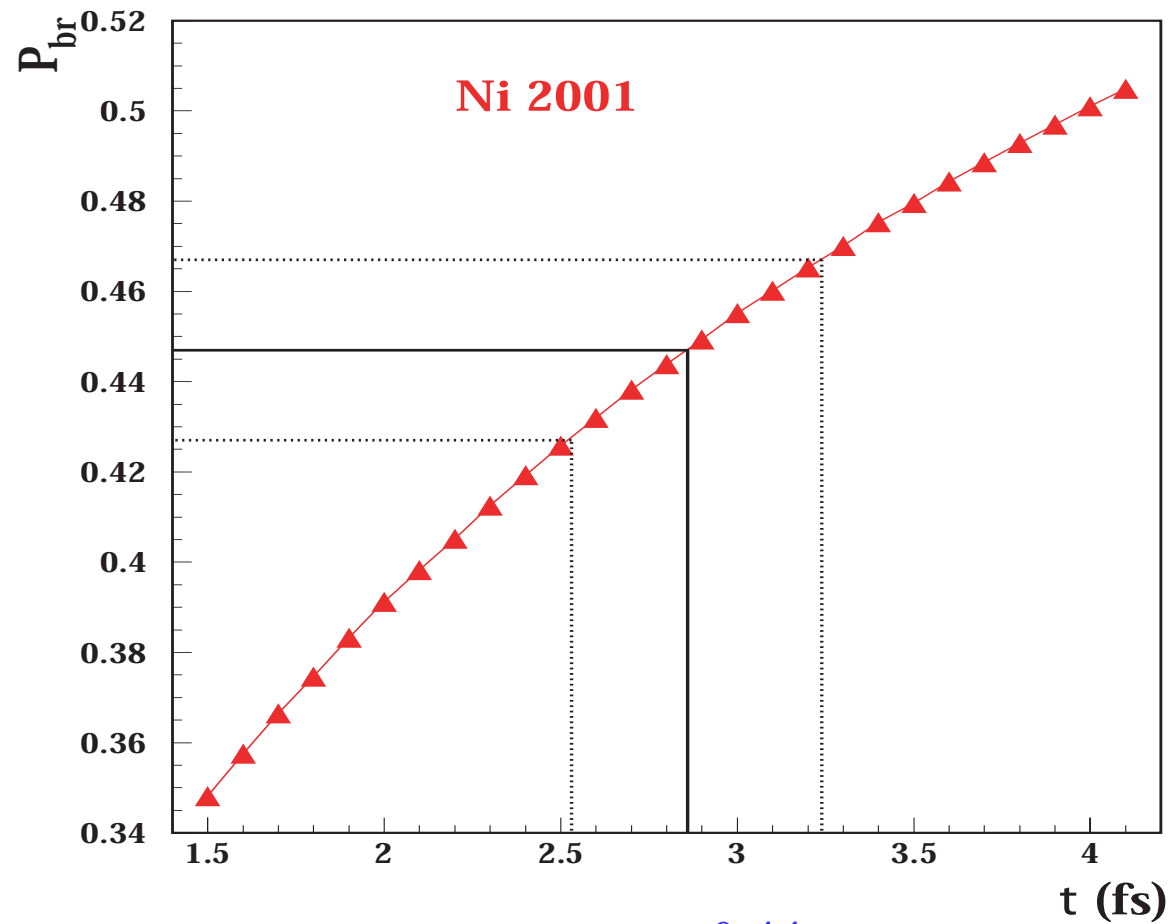
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$$P_{Br} = 0.447 \pm 0.023_{stat}$$



$$\tau = 2.85^{+0.44}_{-0.38}_{stat} \text{ [fs]}$$



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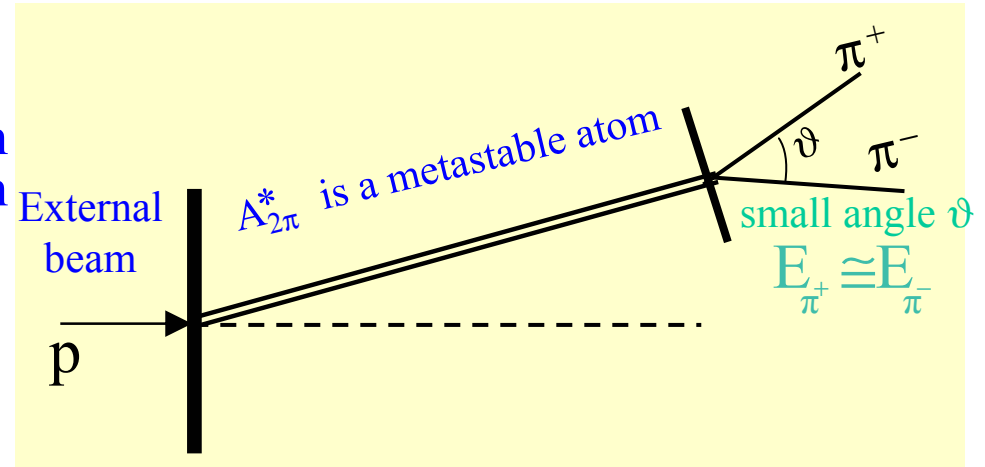
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# Metastable atoms

For  $p_A = 5.6 \text{ GeV}/c$  and  $\gamma = 20$

$$\left\{ \begin{array}{ll} \tau_{1s} = 2.9 \times 10^{-15} \text{ s}, & \lambda_{1s} = 1.7 \times 10^{-3} \text{ cm} \\ \tau_{2s} = 2.3 \times 10^{-14} \text{ s}, & \lambda_{2s} = 1.4 \times 10^{-2} \text{ cm} \\ \tau_{2p} = 1.17 \times 10^{-11} \text{ s}, & \lambda_{2p} = 7 \text{ cm} \\ & \lambda_{3p} \approx 23 \text{ cm} \\ & \lambda_{4p} \approx 54 \text{ cm} \end{array} \right.$$



Probabilities of the  $A_{2\pi}$  breakup (Br) and yields of the long-lived states for different targets provided the maximum yield of summed population of the long-lived states:  $\Sigma(l \geq 1)$

Target Z	Thickness Mm	Br	$\Sigma$ ( $l \geq 1$ )	$2p_0$	$3p_0$	$4p_0$	$\Sigma$ ( $l=1, m=0$ )
04	100	4.45%	5.86%	1.05%	0.46%	0.15%	1.90%
06	50	5.00%	6.92%	1.46%	0.51%	0.16%	2.52%
13	20	5.28%	7.84%	1.75%	0.57%	0.18%	2.63%
28	5	9.42%	9.69%	2.40%	0.58%	0.18%	3.29%
78	2	18.8%	10.5%	2.70%	0.54%	0.16%	3.53%



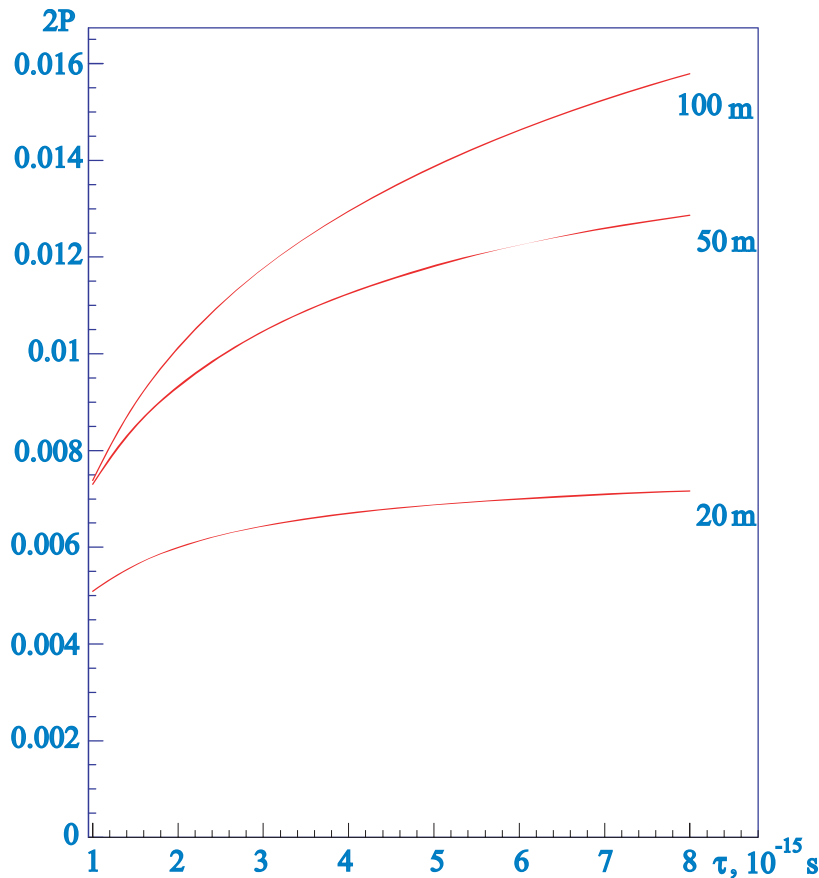
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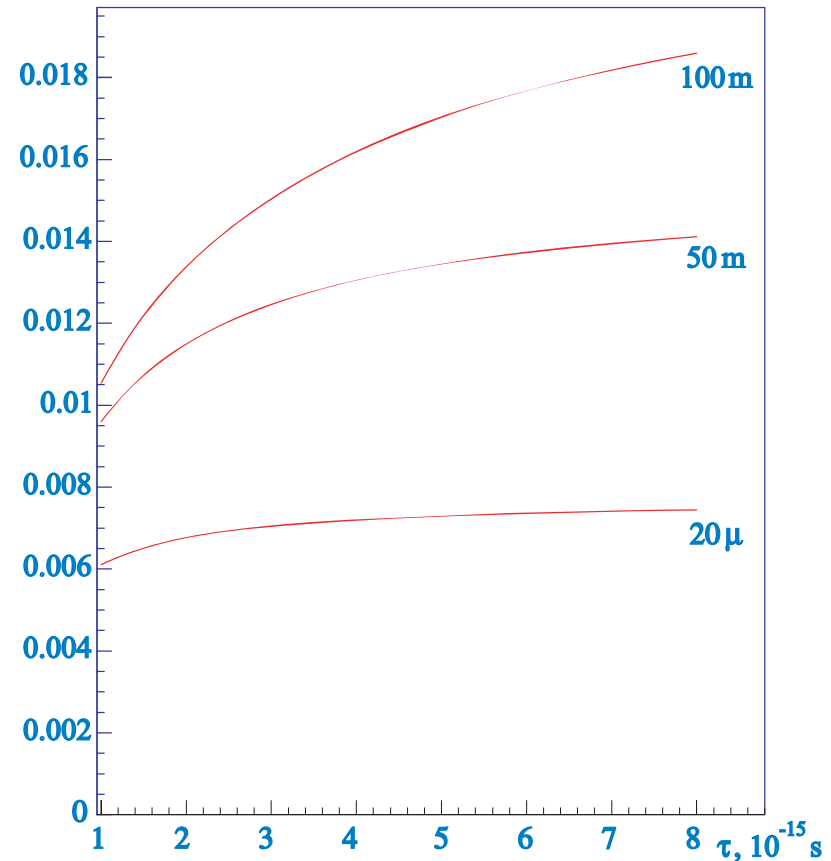
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$$p(A_{2\pi}) = 4.5 \text{ GeV} / c$$



$$p(A_{2\pi}) = 10 \text{ GeV} / c$$



Yields of the long-lived states  $2p$  ( $m = 0$ ) as a function of the  $A_{2\pi}$  lifetime for Beryllium targets ( $Z = 04$ ). Target thicknesses are given in microns on the right side of the picture.



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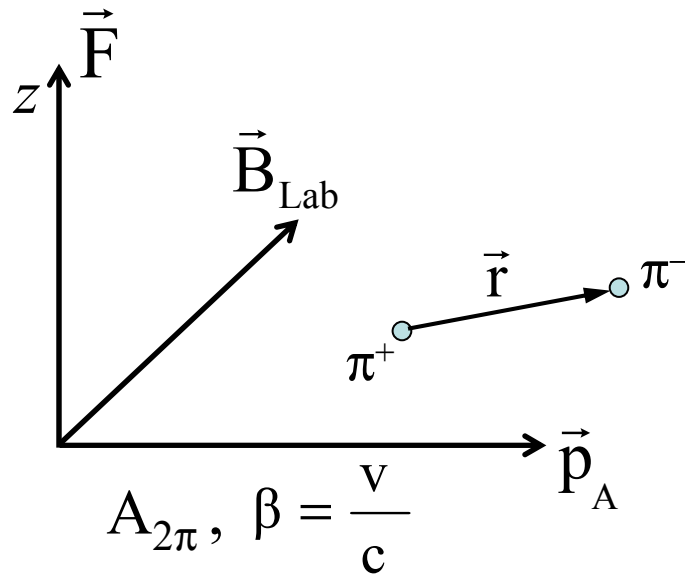
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# External magnetic and electric fields

*Atoms in a beam are influenced by external magnetic field and the relativistic Lorentz factor*



$\vec{r} \equiv$  relative distance between  $\pi^+$  and  $\pi^-$  mesons in  $A_{2\pi}$  atom

$\vec{B}_{\text{Lab}} \equiv$  laboratory magnetic field

$\vec{F} \equiv$  electric field in the CM system of an  $A_{2\pi}$  atom

$$F = \beta \gamma B_{\text{Lab}} \approx \gamma B_{\text{Lab}}$$



# The dependence of $A_{2\pi}$ life time in 2p-states $\tau_{\text{eff}}$ from a strength of the electric field F

$$\tau_{\text{eff}} = \frac{\tau_{2p}}{1 + \frac{|\xi|^2}{4} \frac{\tau_{2p}}{\tau_{2s}}} = \frac{\tau_{2p}}{1 + 120 |\xi|^2}$$

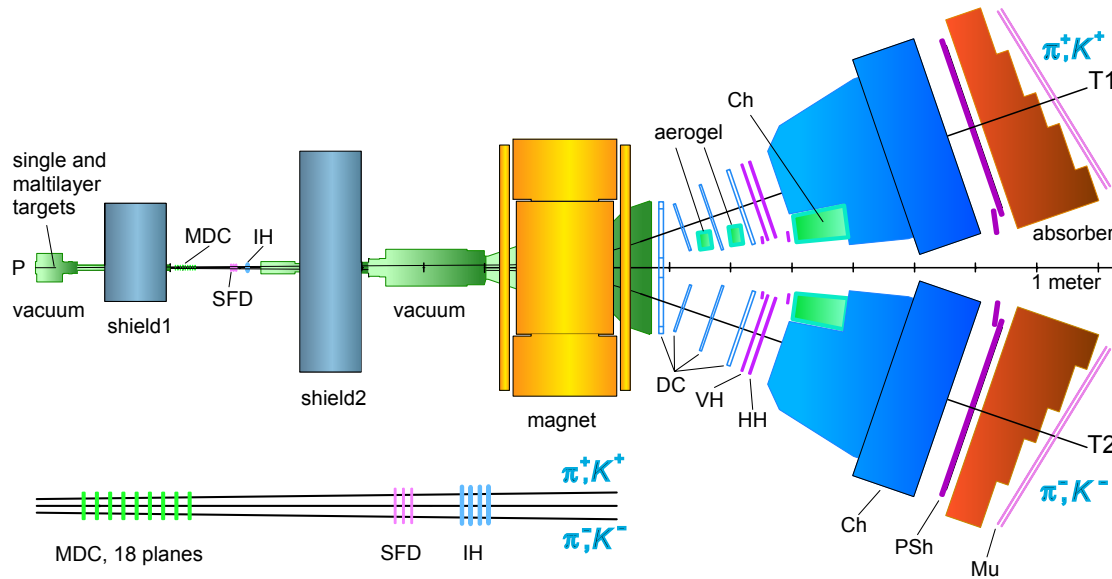
where:  $|\xi|^2 \approx \frac{F^2}{(E_{2p} - E_{2s})^2}$

$B_{\text{Lab}} = 4 \text{ Tesla}$	{	$\gamma = 20$ ,	$ \xi  = 0.1$	$\Rightarrow$	$\tau_{\text{eff}} = \frac{\tau_{2p}}{2.2}$
		$\gamma = 40$ ,	$ \xi  = 0.2$	$\Rightarrow$	$\tau_{\text{eff}} = \frac{\tau_{2p}}{6}$



# Upgraded DIRAC setup

(Addendum to DIRAC/PS212 (CERN) (13/04/04):  
CERN-SPSC-2004-009 (SPSC-P-284 Add.4))



Schematic top view of the updated DIRAC spectrometer.

Downstream of the magnet, in each spectrometer arm:  
**drift chambers (DC)**,  
**vertical and horizontal** scintillation hodoscopes (VH, HH),  
 gas **Cherenkov** counters (Ch),  
**preshower** detector (PSh) and, behind the iron absorber,  
**muon** detector (Mu).

**In the left arm:** Aerogel **Cherenkov** counters.

Upstream of the spectrometer magnet:

**microdrift chambers (MDC)** ,  
**scintillating fiber detectors (SFD)** ,  
**ionization hodoscopes (IH)**.



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## $A_{2\pi}$ and $A_{\pi K}$ production

$$\frac{d\sigma_{nlm}^A}{d\vec{P}} = (2\pi)^3 \frac{E}{M} \left| \psi_{nlm}^{(C)}(0) \right|^2 \frac{d\sigma_s^0}{dp_1 dp_2} \square \frac{d\sigma}{dp_1} \cdot \frac{d\sigma}{dp_2}$$

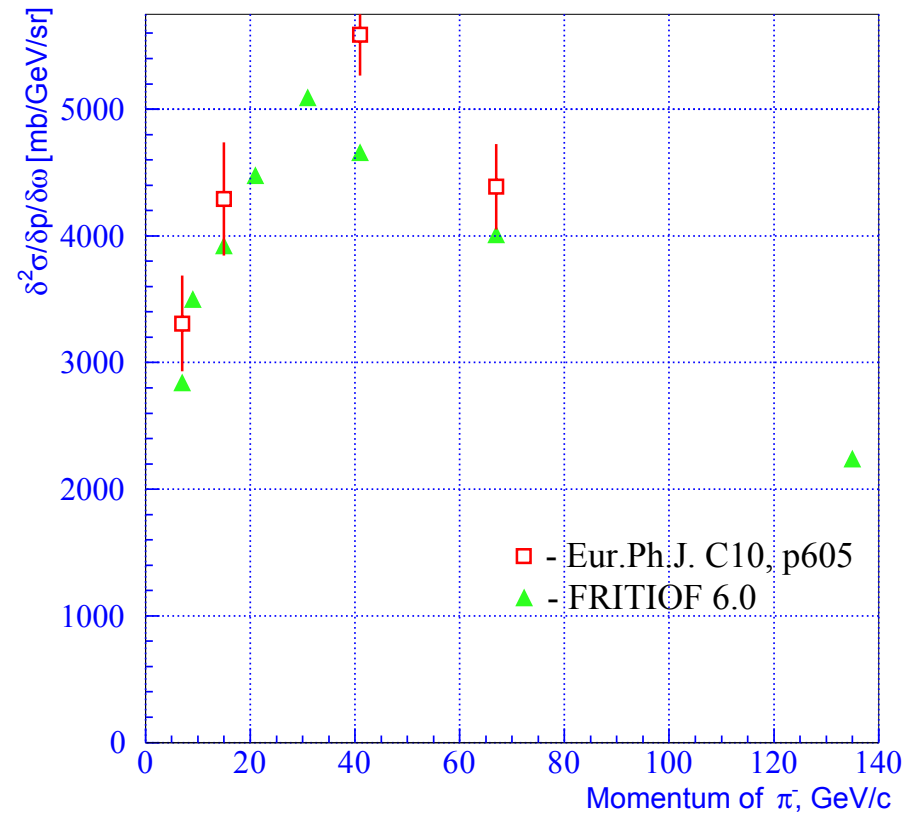
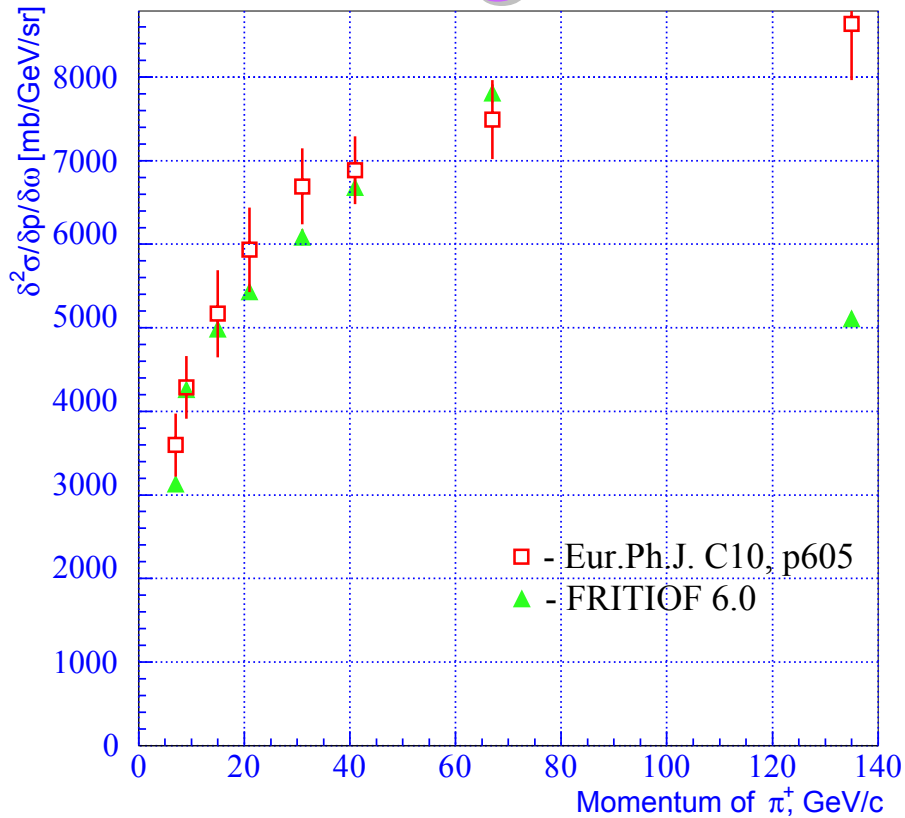
*for atoms  $\vec{v}_1 = \vec{v}_2$  where  $\vec{v}_1, \vec{v}_2$  – velocities of particles in the L.S. for all types of atoms*

*for  $A_{2\pi}$  production  $\vec{p}_1 = \vec{p}_2$*

*for  $A_{\pi K}$  production  $\vec{p}_\pi = \frac{m_\pi}{m_K} \vec{p}_K$*



# Inclusive cross-sections for $\pi^+$ , $\pi^-$ - mesons generation



$$E_p = 450 \text{ GeV} \quad \theta_L = 0^\circ$$



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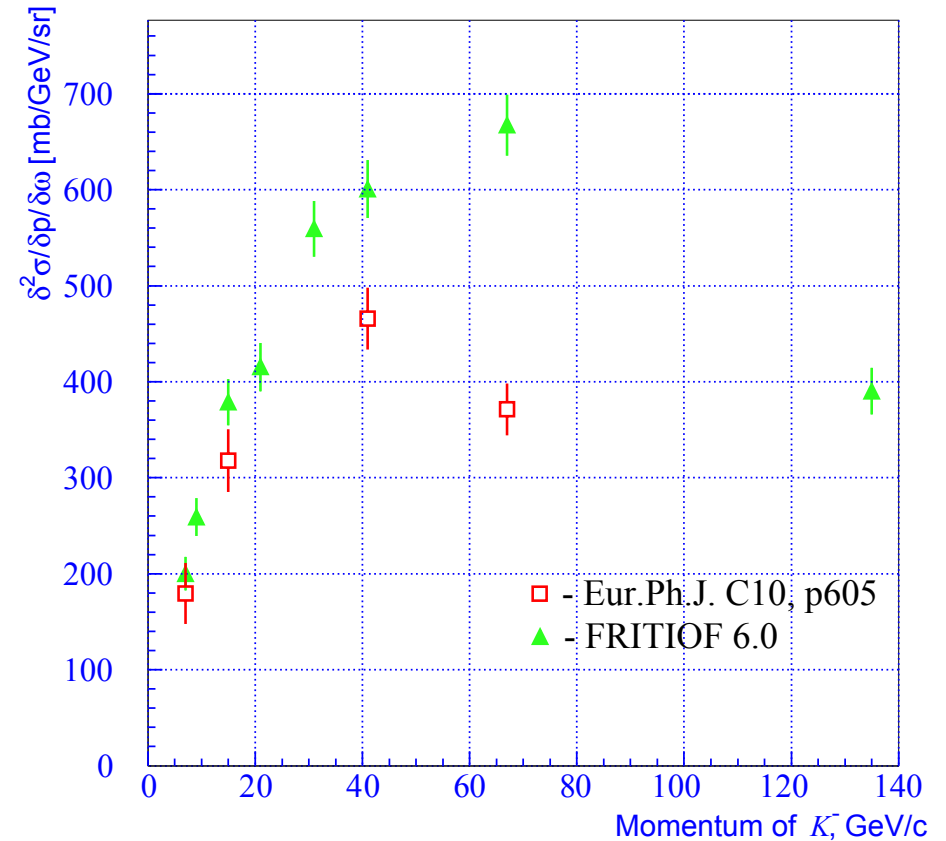
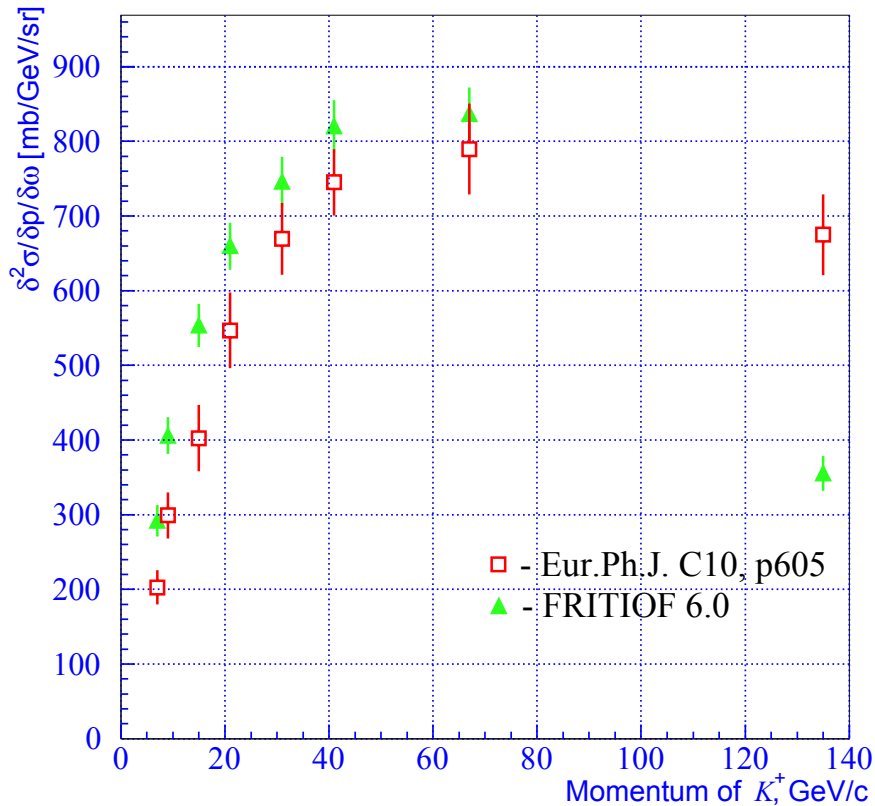
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# Inclusive cross-sections for $K^+$ , $K^-$ - mesons generation



$$E_p = 450 \text{ GeV} \quad \theta_L = 0^\circ$$



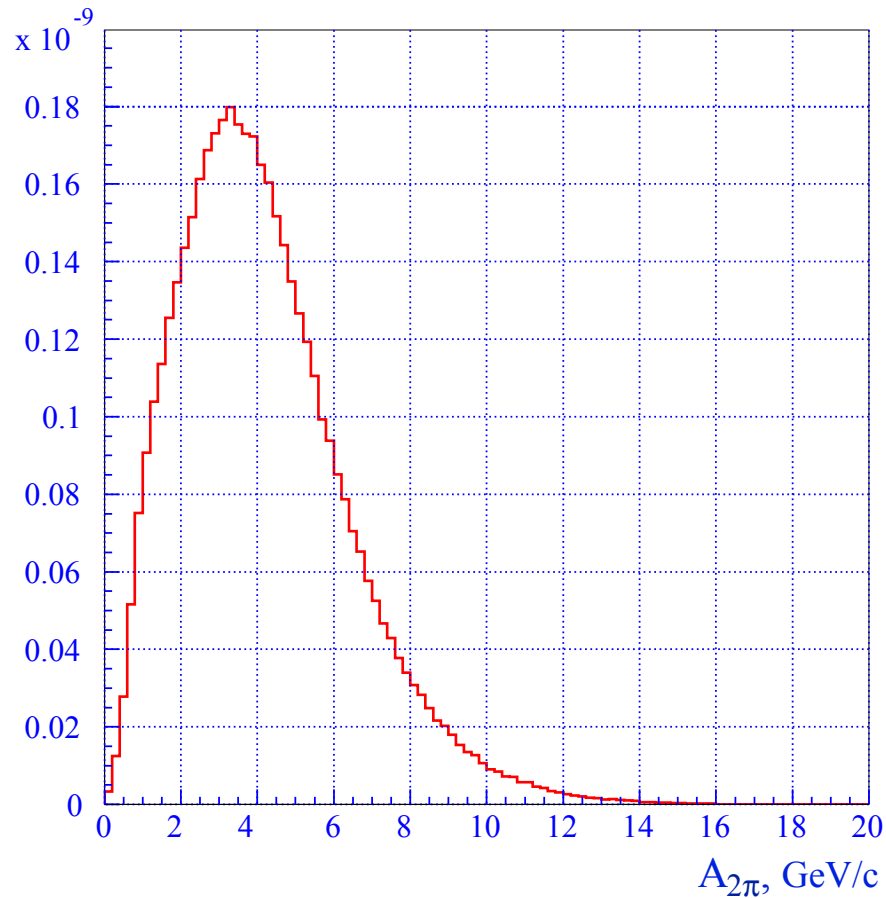
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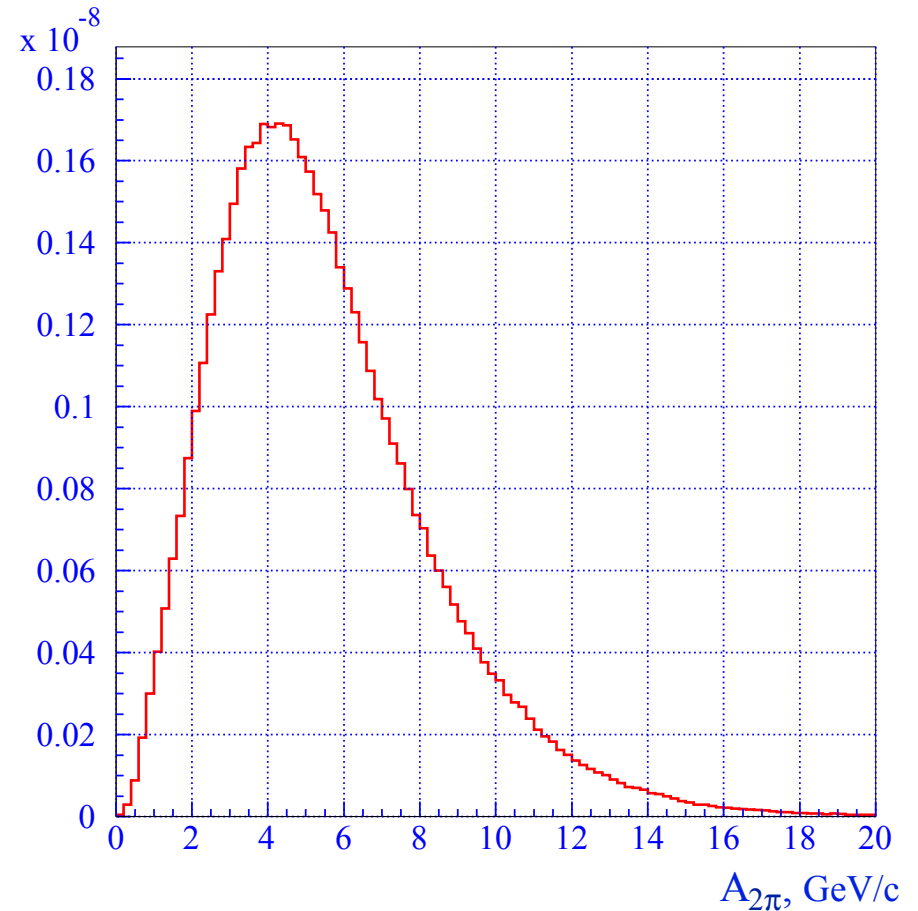
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# $A_{2\pi}$ momentum distributions



$$\theta_L = 5.7^\circ \pm 1.3^\circ \quad E_p = 24 \text{ GeV}$$



$$\theta_L = 5.7^\circ \pm 1.3^\circ \quad E_p = 450 \text{ GeV}$$



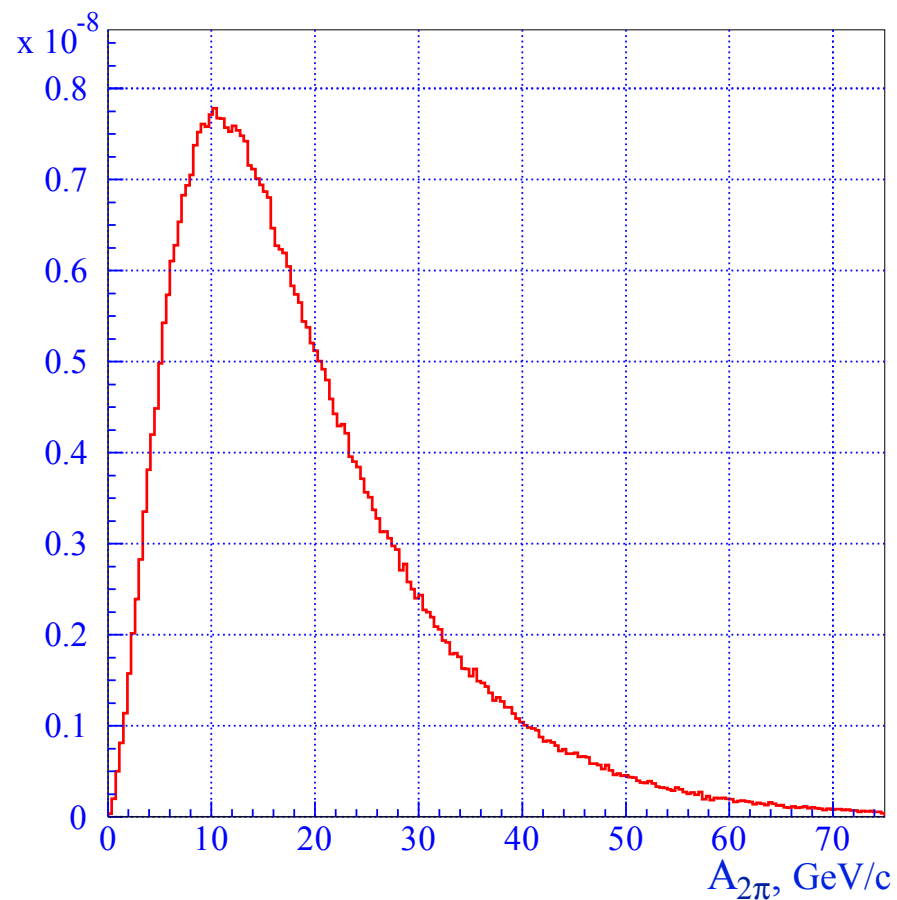
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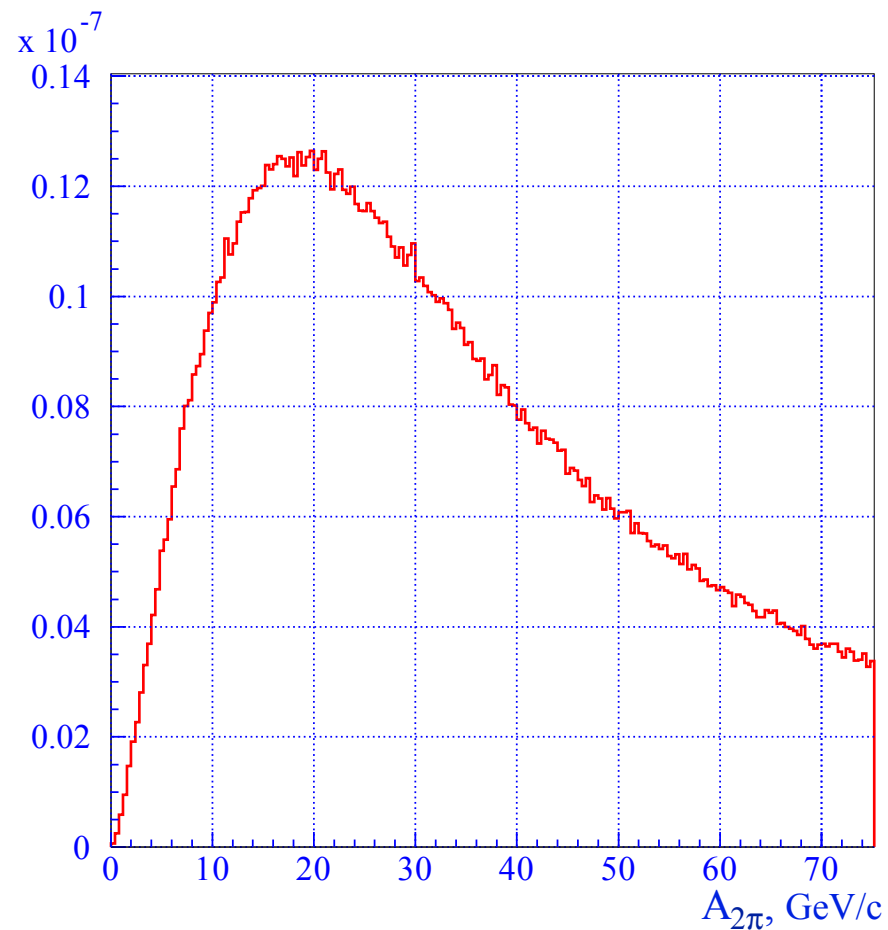
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# $A_{2\pi}$ momentum distributions



$$\theta_L = 2^\circ \pm 1.3^\circ \quad E_p = 450 \text{ GeV}$$



$$\theta_L = 0^\circ \pm 1.3^\circ \quad E_p = 450 \text{ GeV}$$



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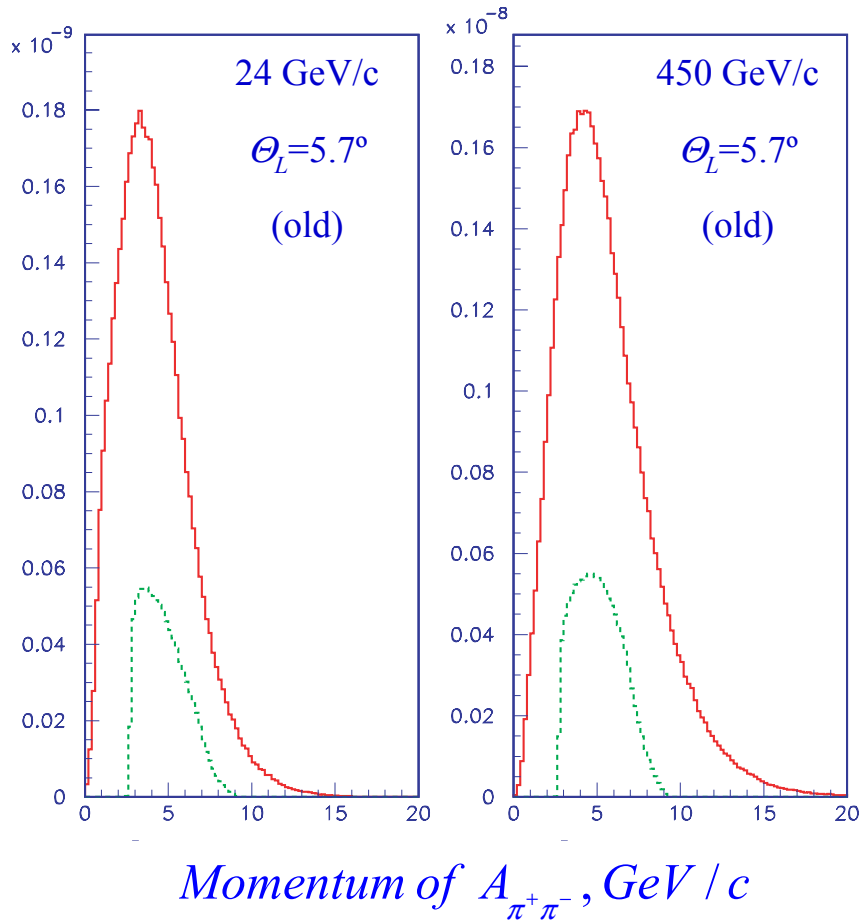


**Yield of  $A_{2\pi}$  and  $\pi^\pm$  into the aperture of  $1.7 \cdot 10^{-3}$  sr per one proton interaction**

$\Theta_L$	$5.7^\circ \pm 1.3^\circ$	$5.7^\circ \pm 1.3^\circ$	$2^\circ \pm 1.3^\circ$	$0^\circ \pm 1.3^\circ$
$E_p$	<b>24 GeV</b>	<b>450 GeV</b>	<b>450 GeV</b>	<b>450 GeV</b>
$W_A$	$0.46 \cdot 10^{-8}$	$0.53 \cdot 10^{-7}$	$0.47 \cdot 10^{-6}$	$0.18 \cdot 10^{-5}$
$W_A^N$	1.	12.	103.	389.
$W_\pi$	0.032	0.1	1.1	6.6
$W_\pi^N$	1.	3.1	34.	206.
$W_A/W_\pi$	$1.4 \cdot 10^{-7}$	$5.3 \cdot 10^{-7}$	$4.3 \cdot 10^{-7}$	$2.7 \cdot 10^{-7}$
$W_A^N/W_\pi^N$	1.	3.7	3.	1.9



# $A_{2\pi}$ momentum distributions



— - red curve  $A_{2\pi}$  spectra in channel the aperture  
 ●●● - green curve  $A_{2\pi}$  spectra registered by the set-up

Yield of $A_{2\pi}$ per one proton interaction, detectable by DIRAC setup at $\Theta_L = 5.7^\circ$			
$E_p$	24 GeV (old)	450 GeV (old)	450 GeV (new)
$W_A$	$0.96 \cdot 10^{-9}$	$0.11 \cdot 10^{-7}$	$0.13 \cdot 10^{-7}$
$W_A^N$	1.	12.	13.
$W_A/W_\pi$	$3 \cdot 10^{-8}$	$1.1 \cdot 10^{-7}$	$1.3 \cdot 10^{-7}$
$W_A^N/W_\pi^N$	1.	3.7	4.3
		<b>A multiplier due to different spill duration ~4</b>	
<b>Total gain</b>	<b>1.</b>	<b>15.</b>	<b>17.</b>



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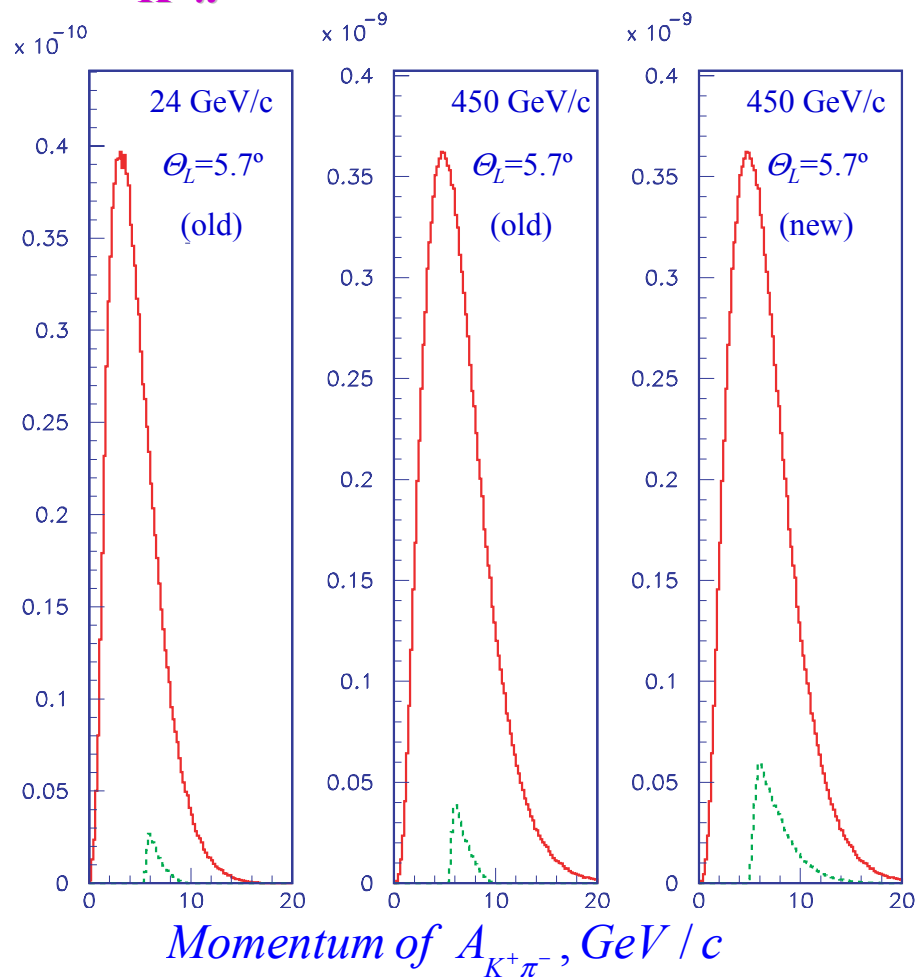


**Yield of  $A_{K^+\pi^-}$  and  $A_{K^-\pi^+}$  into the aperture of  $1.7 \cdot 10^{-3}$  sr  
per one proton interaction**

	$A_{K^+\pi^-}$				$A_{K^-\pi^+}$			
$\theta_L$	$5.7^\circ \pm 1.3^\circ$	$5.7^\circ \pm 1.3^\circ$	$2^\circ \pm 1.3^\circ$	$0^\circ \pm 1.3^\circ$	$5.7^\circ \pm 1.3^\circ$	$5.7^\circ \pm 1.3^\circ$	$2^\circ \pm 1.3^\circ$	$0^\circ \pm 1.3^\circ$
$E_p$	24GeV	450GeV	450GeV	450GeV	24GeV	450GeV	450GeV	450GeV
$W_A$	$0.11 \cdot 10^{-8}$	$0.13 \cdot 10^{-7}$	$0.16 \cdot 10^{-6}$	$0.65 \cdot 10^{-6}$	$0.60 \cdot 10^{-9}$	$0.10 \cdot 10^{-7}$	$0.11 \cdot 10^{-6}$	$0.54 \cdot 10^{-6}$
$W_A^N$	1.	12.	146.	610.	1.	17.	192.	897.
$W_A/W_\pi$	$3.4 \cdot 10^{-8}$	$1.3 \cdot 10^{-7}$	$1.5 \cdot 10^{-7}$	$9.8 \cdot 10^{-8}$	$1.9 \cdot 10^{-8}$	$1 \cdot 10^{-7}$	$1 \cdot 10^{-7}$	$8.2 \cdot 10^{-8}$
$W_A^N/W_\pi^N$	1.	3.8	4.2	2.9	1.	5.3	5.3	4.4



# $A_{K^+\pi^-}$ - momentum distributions



— - red curve  $A_{2\pi}$  spectra in the channel aperture  
 ●●● - green curve  $A_{2\pi}$  spectra registered by the set-up

**Yield of  $A_{K^+\pi^-}$  per one proton interaction, detectable by DIRAC setup at  $\theta_L=5.7^\circ$**

$E_p$	24 GeV (old)	450 GeV (old)	450 GeV (new)
$W_A$	$0.20 \cdot 10^{-10}$	$0.34 \cdot 10^{-9}$	$0.10 \cdot 10^{-8}$
$W_A^N$	1.	17.	49.
$W_A/W_\pi$	$6.2 \cdot 10^{-10}$	$3.4 \cdot 10^{-9}$	$1 \cdot 10^{-8}$
$W_A^N/W_\pi^N$	1.	5.4	16
		<b>A multiplier due to different spill duration ~4</b>	
<b>Total gain</b>	<b>1.</b>	<b>22.</b>	<b>64.</b>



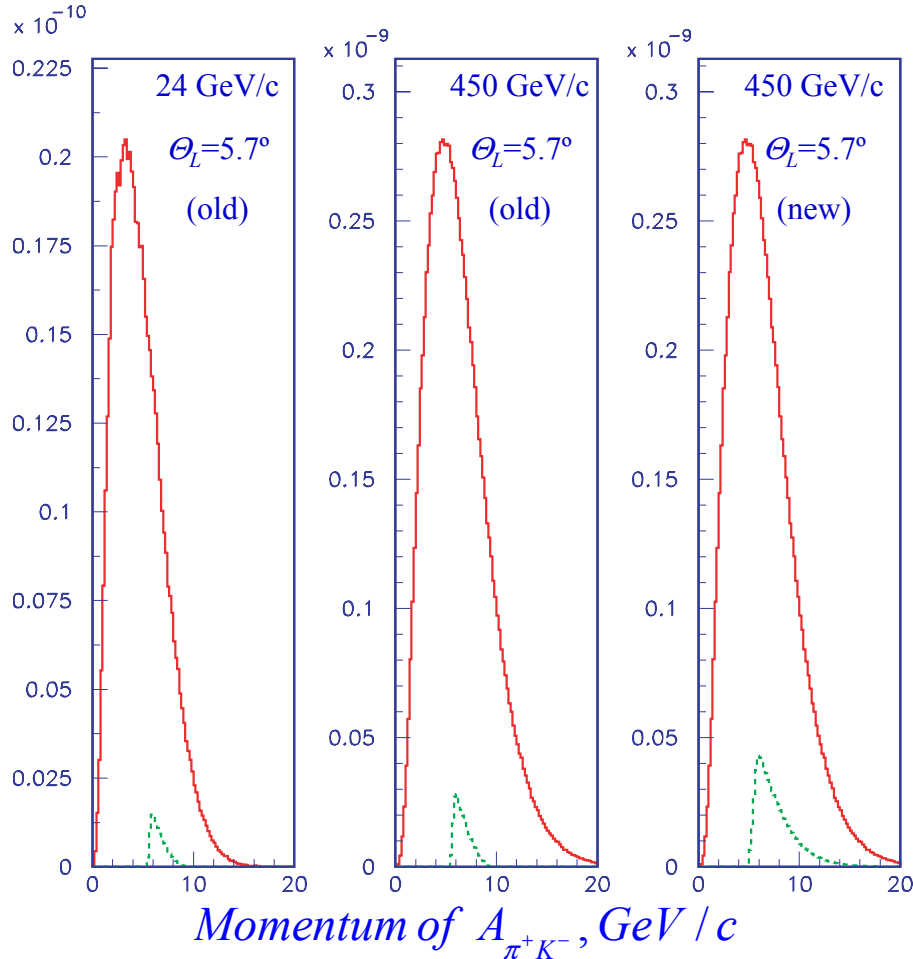
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# $A_{\pi^+K^-}$ momentum distributions



— - red curve  $A_{2\pi}$  spectra in the channel aperture  
 ●●● - green curve  $A_{2\pi}$  spectra registered by the set-up

Yield of $A_{\pi^+K^-}$ per one proton interaction, detectable by DIRAC setup at $\Theta_L=5.7^\circ$			
$E_p$	24 GeV (old)	450 GeV (old)	450 GeV (new)
$W_A$	$0.11 \cdot 10^{-10}$	$0.23 \cdot 10^{-9}$	$0.71 \cdot 10^{-9}$
$W_A^N$	1.	21.	63.
$W_A/W_\pi$	$3.4 \cdot 10^{-10}$	$2.3 \cdot 10^{-9}$	$7.1 \cdot 10^{-9}$
$W_A^N/W_\pi^N$	1.	6.7	20.6
		<b>A multiplier due to different spill duration ~4</b>	
<b>Total gain</b>	<b>1.</b>	<b>27.</b>	<b>82.</b>





# Conclusions

Present low energy *QCD* predictions for  $\pi\pi$  and  $\pi K$  scattering lengths

$$\begin{aligned} \pi\pi \quad \delta a_0 &= 2.3\% & \delta a_2 &= 2.3\% & \delta(a_0 - a_2) &= 1.5\% \\ \pi K \quad \delta(a_{1/2} - a_{3/2}) &\approx 10\% \end{aligned}$$

Expected results of DIRAC ADDENDUM at PS CERN

$$\begin{aligned} \tau(A_{2\pi}) &\rightarrow \delta(a_0 - a_2) = \pm 2\%(\text{stat}) \pm 1\%(\text{syst}) \pm 1\%(\text{theor}) \\ \tau(A_{\pi K}) &\rightarrow \delta(a_{1/2} - a_{3/2}) = \pm 10\%(\text{stat}) \pm \dots \pm 1.5\%(\text{theor}) \end{aligned}$$

*Observation of metastable  $A_{2\pi}$*

DIRAC at SPS CERN

$$\begin{aligned} \tau(A_{2\pi}) &\rightarrow \delta(a_0 - a_2) = \pm 0.5\%(\text{stat}) \\ \tau(A_{\pi K}) &\rightarrow \delta(a_{1/2} - a_{3/2}) = \pm 2.5\%(\text{stat}) \\ (E_{np} - E_{ns})_{\pi\pi} &\rightarrow \delta(2a_0 + a_2) \approx \pm 2.5\%(\text{stat}) \\ (E_{np} - E_{ns})_{\pi K} &\rightarrow \delta(2a_{1/2} + a_{3/2}) \end{aligned}$$

*Possibility of the observation  $(\pi^\pm \mu^\mp)$  – atoms  
and  $(K^+ K^-)$  – atoms will be studied.*

