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Critical Behaviour in QCD

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Fundamental Problems of Physics

constituents	forces
quarks	strong
leptons	e-m
gluons, photons	weak
vector bosons (Z, $W^{\pm}$)	gravitation
Higgs	unification, TOE

elementary interactions

$\Downarrow$

complex systems, critical behaviour

states of matter	transitions
solid, liquid, gas, plasma	thermal phase transitions
insulator, conductor	percolation transitions
superconductor, ferromagnet	scaling, renormalization
fluid, superfluid	critical exponents
glass, gelatine, network	universality classes

Complex Systems $\Rightarrow$ New Direction in Physics

Statistical QCD:

$\exists$ **transition hadronic matter $\rightarrow$ quark-gluon plasma**

High Energy Heavy Ion Programme

study in the laboratory

- **deconfinement transition**
- **properties of QGP**

Capabilities

- **deconfinement transition: SPS, RHIC**
- **properties of QGP: SPS, RHIC, LHC**

Contents

1. What is critical behaviour?

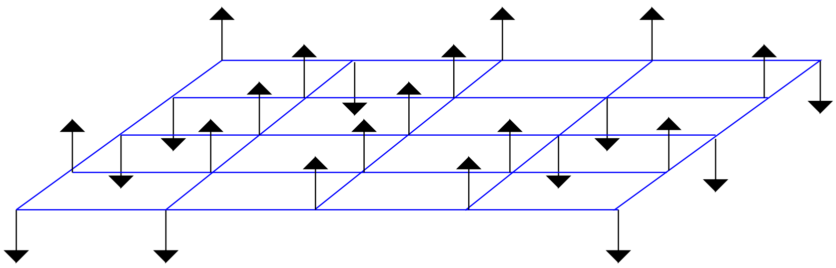
divergent or discontinuous behavior of observables

(natura facit saltum)

example:

magnetization transition in a spin system

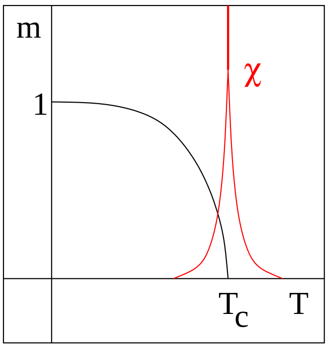
Ising model: $s_i = \pm 1 \; \forall i = 1, \ldots, N$



average spin $m(T)$ **in thermodynamic limit** $(N \to \infty)$:

$m(T)$ **is not analytic ('not smooth')**

$$m(T) \sim \begin{cases} (T - T_c)^{\beta} > 0 & \forall \; T < T_c \\ 0 & \forall \; T > T_c \end{cases}$$



discontinuous change of $m(T)$ **at** $T = T_c$:

$\Rightarrow$ **critical exponent** β

higher derivatives: susceptibility

$\chi(T) \sim |T - T_c|^{-\gamma}$

$\Rightarrow$ **critical exponent** γ

and other observables diverge as well, give more critical exponents

critical behaviour of a system fully specified by the set of critical exponents $\alpha, \beta, \gamma, \ldots$; can be reduced to two independent exponents (universality class)

But why is there singular behaviour?

$\Rightarrow$ **spontaneous symmetry breaking**

Ising Hamiltonian is invariant under $\uparrow \leftrightarrow \downarrow$ flips

at $T = T_c$, state of system spontaneously breaks flip symmetry, chooses either $\uparrow$ or $\downarrow$.

breaking symmetry is "either-or": you cannot do it "a little" $\Rightarrow$ singular observables

$\Rightarrow$ **Thermodynamic Critical Behaviour** $\Leftarrow$

- **onset of spontaneous symmetry breaking**
- **singular behaviour of thermodynamic observables***

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* divergence : continuous transition
  discontinuity : first order transition

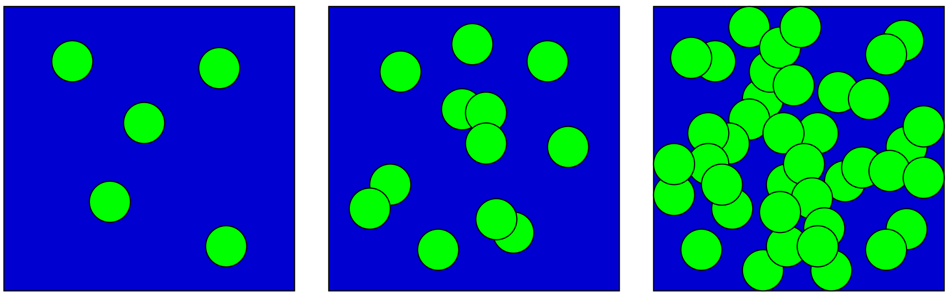
thermal transitions, critical behaviour:
dynamics $\rightarrow$ non-analytic partition function

given constituents with intrinsic scale,
$\exists$ more general form of critical behaviour:

$\Rightarrow$ formation of infinite cluster, network

example: 2-d disk percolation (lilies on a pond)

distribute small disks of area $a = \pi r^2$ randomly on large area $F = L^2$, $L \gg r$, with overlap allowed



isolated disks **clusters** **percolation (network)**

for N disks, disk density $n = N/F$

average cluster size $S(n)$ increases with increasing density n

$\exists$ critical density: for

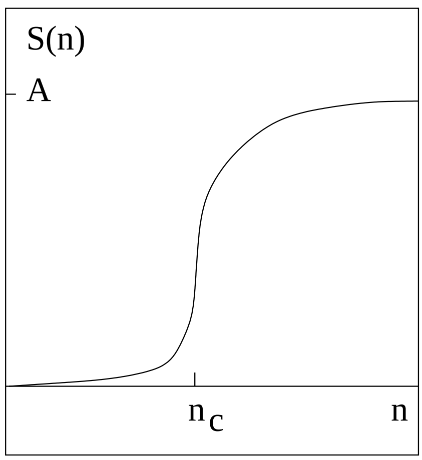
$$n \rightarrow n_c = 1.13/a$$

$S(n)$ spans area F: $S \sim F$

for $N \rightarrow \infty, F \rightarrow \infty$:

$S(n_c)$ and $(dS(n)/dn)_{n=n_c}$

diverge: $\Rightarrow$ percolation



probability $P(n)$ that given disk in infinite cluster

$$P(n) \begin{cases} = 0 & \forall\ n < n_c \\ \sim (n - n_c)^{\beta} & \text{for } n \to n_c \text{ from above} \end{cases}$$

$\Rightarrow$ order parameter for percolation

average cluster size diverges

$$\tilde{S}(n) \simeq |n - n_c|^{-\gamma}$$

so do other observables: again singular behaviour, as function of density n instead of temperature T

$\Rightarrow$ critical exponents, universality classes

Again, why is there singular behaviour?

$\Rightarrow$ spontaneous global connection

connected or disconnected, not "gradual"

$\Rightarrow$ Geometric Critical Behaviour $\Leftarrow$

- **onset of infinite cluster/network formation**
- **singular behaviour of geometric observables**

- **Thermodynamic critical behaviour:**

spontaneous symmetry breaking as function of T

- **Geometric critical behaviour:**

spontaneous global connection as function of n

geometric critical behaviour can occur even if the partition function is analytic

$\Rightarrow$ geometric without thermodynamic criticality

(spin systems in external magnetic field)

2. Critical Behaviour in QCD

What happens to strongly interacting matter at high temperatures and/or densities?

- colour deconfinement

  hadronic matter:

  colourless constituents of hadronic dimension

  $\Downarrow$

  quark-gluon plasma:

  pointlike coloured constituents

- chiral symmetry restoration

  hadronic matter:

  quarks acquire effective mass $M_q \neq 0$

  $\Downarrow$

  quark-gluon plasma:

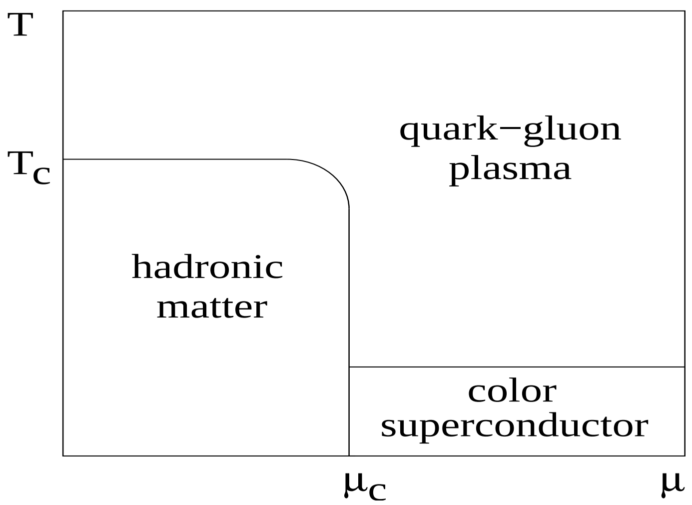
  $M_q \to m_q = 0$, chiral symmetry restored

- colour superconductivity

  deconfined quarks $\to$ coloured bosonic 'diquarks'

  diquark condensation $\to$ colour superconductor

- phase diagram of QCD:



baryochemical potential $\mu \sim$ baryon density.

given QCD as dynamics input, calculate resulting thermodynamics, based on QCD partition function

Ab initio calculation:

$\Rightarrow$ finite temperature/finite density lattice QCD

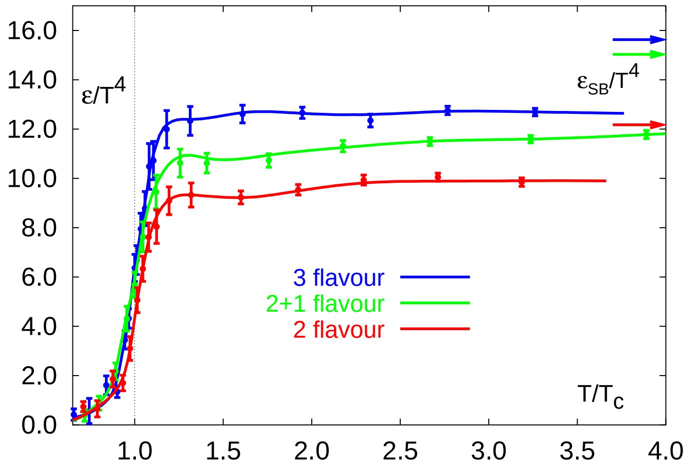
at zero net baryon density ($\mu = 0$, $N_b = N_{\bar{b}}$), finite T lattice QCD with dynamical quarks gives

- deconfinement and chiral symmetry restoration coincide, determine critical temperature T_c

$$\boxed{N_f = 2, 2+1: \; T_c \simeq \; 175 \text{ MeV}}$$

in chiral limit ($m_q \to 0$).

- **energy density increases sharply by the latent heat of deconfinement**

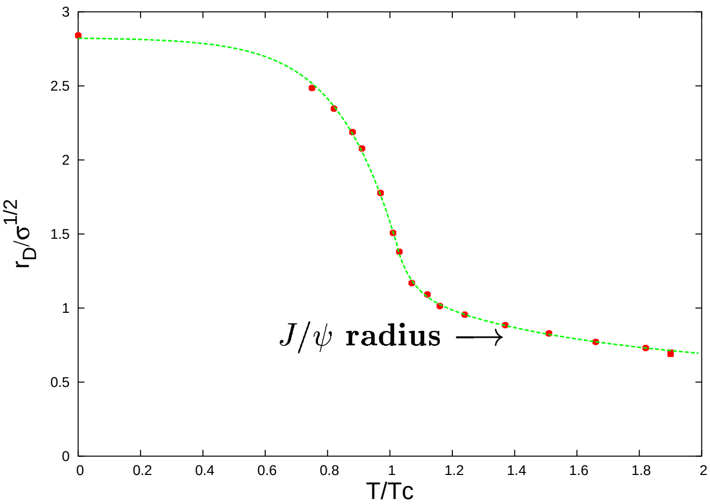


with

$$\boxed{N_f = 2, 2+1 : \ \epsilon(T_c) \simeq 0.5 - 1.0 \text{ MeV}}$$

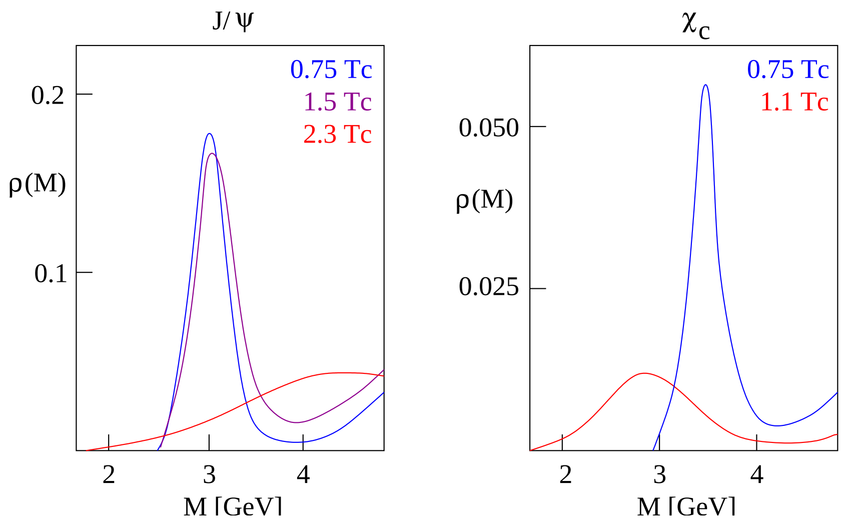
for deconfinement energy density.

- **interaction range (from string breaking) drops sharply as** $T \to T_c$



$\Rightarrow$ **colour screening**

- **consequence: charmonium suppression**



χ_c suppressed essentially at T_c

J/ψ survives until 1.5–2.0 T_c

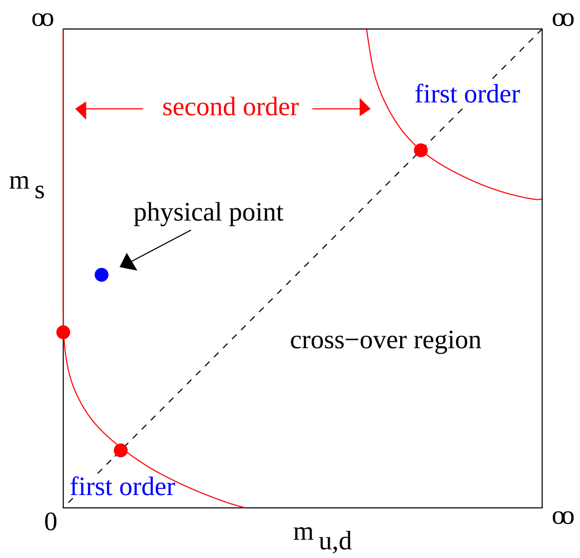
NB: equilibrium QCD thermodynamics

nature of transition depends on N_f and m_q

$\Downarrow$

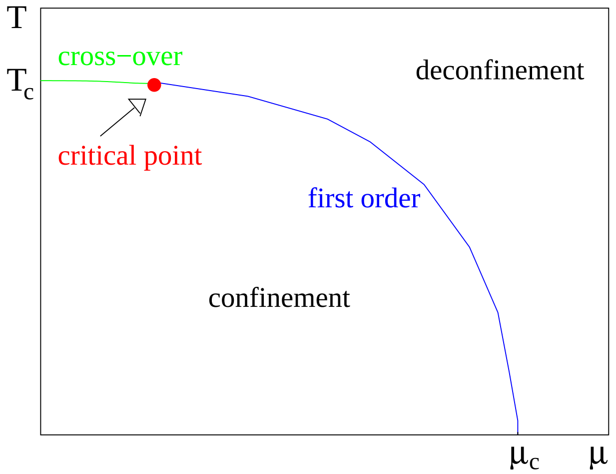
continuous, first order, cross-over (percolation)

structure for $\mu = 0$



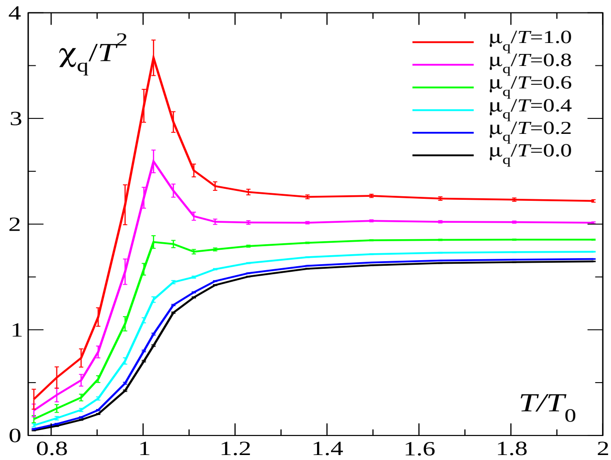
at **non-zero net baryon density** **(**$\mu \neq 0, N_b > N_{\bar{b}}$**),**
computer algorithms break down, power series...

conjecture for $\mu \neq 0,\ N_f = 2+1$



critical point **in** $T-\mu$ **plane depends on position of**
physical point **in** $m_s - m_{u,d}$ **plane**

preliminary results (m_q**, power series, ...)**



net baryon density fluctuations increase with μ**,**
$\rightarrow$ **approach to critical point** $\mu_c \simeq 0.3 - 0.7$ GeV

3. QCD Transitions in Nuclear Collisions

Expectation:

high energy nucleus-nucleus collisions → strongly interacting matter

multiple collisions → thermalization, QGP

at high energy:

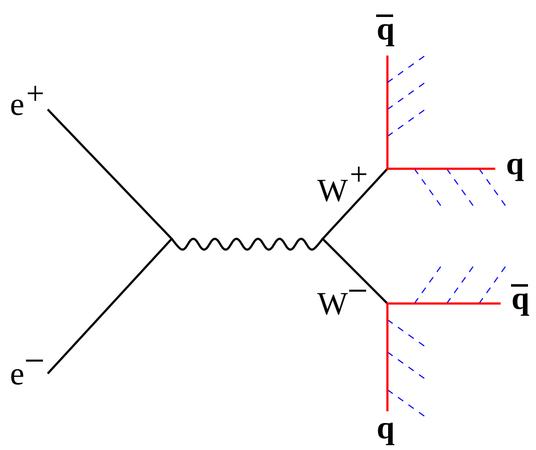
nucleon interactions ∼ parton interactions

⇒ conditions for thermalization on partonic level?

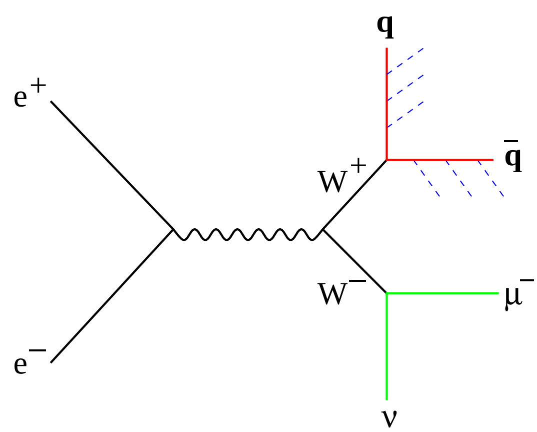
prerequisite :

∃ communication ('cross talk', 'colour connection') between partons from different nucleon interactions

counterexample : hadron production at LEP



(a)



(b)

consider hadron multiplicity from jet decay of W's

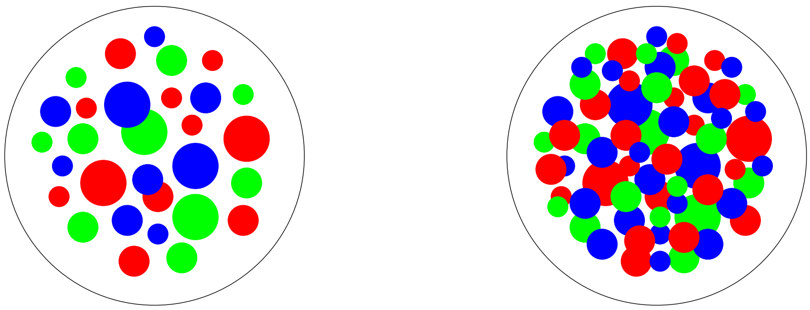
– cross talk:
$\Rightarrow N_h(a) < 2N_h(b)$

– no cross talk:
$\Rightarrow N_h(a) = 2N_h(b)$ $\Leftarrow$ **3 LEP expts.**

same space-time region, but no cross talk

$\Rightarrow$ pre-equilibrium initial state conditions crucial for final state of high energy nuclear collisions

partons in transverse plane of nuclear collision:



increasing density $\rightarrow$ superposition $\rightarrow$ clustering

percolation: parton cluster spans whole system

$\Rightarrow$ partonic network, global colour connection

$\Rightarrow$ parton picture breaks down: saturation, classical field $\sim$ colour glass condensate

When does that occur?

percolation in nuclear collisions

nuclear overlap area F
N partons of transverse size $a \ll F$
parton density $n = N/F$

$\Rightarrow$ **threshold for geometric critical behavior**

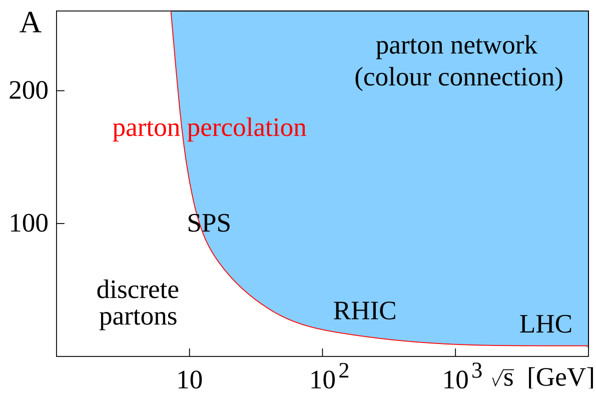
$$n = n_c = 1.13/a$$

defines critical density n_c

N**/nucleon from PDF's in DIS**
N**/nuclear interaction from nuclear source density**
$a \sim 1/k_T^2$ **determined by intrinsic** k_T **of partons**

$\Rightarrow$ n_c **depends on** $\underline{A}$, **<u>centrality</u>**, **<u>collision energy</u>**

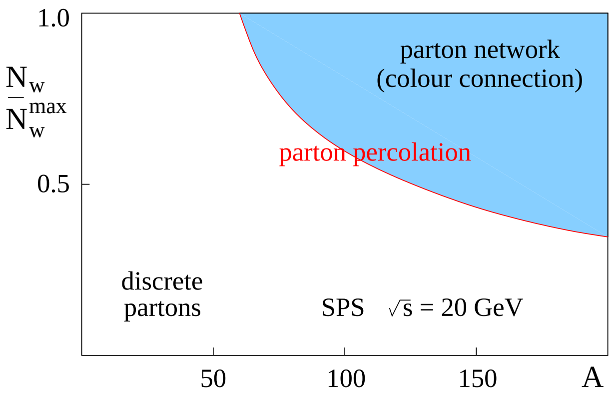
schematic:
central $A-A$ **collisions**
vs. A **and** $\sqrt{s}$



$\Rightarrow$ **<u>onset</u> of percolation**
best accessible at SPS

schematic:
$Pb-Pb$ **collisions**
vs.centrality

SPS, $\sqrt{s} = 20$ GeV



parton network:
initial state satisfies prerequisite for thermalization

<u>necessary</u>, but not necessarily sufficient

assume: parton network thermalizes $\rightarrow$ **QGP**

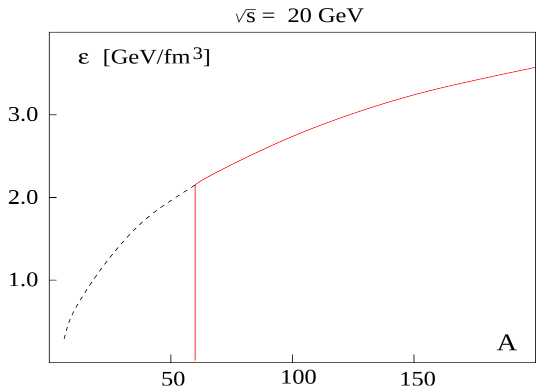
energy density [Bjorken estimate]

$$\epsilon_0 \simeq \frac{p_0}{\pi R_A^2 \tau_0} \left(\frac{dN_h^{AA}}{dy} \right)_{y=0} \simeq \frac{p_0}{\pi \tau_0} A^{0.43} \ln(\sqrt{s}/2)$$

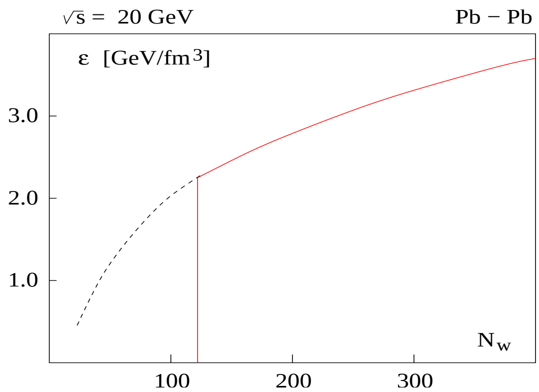
$\Rightarrow \tau_0$: time needed to reach thermalization

if partons do not form network, they cannot thermalize, $\tau_0 = \infty$

schematic:
central collisions
energy density
vs. A
for $\sqrt{s} = 20$ GeV



schematic:
$Pb-Pb$ collisions
energy density
vs. centrality
for $\sqrt{s} = 20$ GeV



$\Rightarrow$ hot QGP, well above deconfinement

$(\epsilon(T_c) \simeq 0.5 - 1.0$ **GeV/fm^3**)

in $Pb-Pb$ at $\sqrt{s} = 20$ **GeV**,
formation threshold at mid-centrality ($b \simeq 6$ fm)

experimental consequences:

$\exists$ sharp variation of observables?

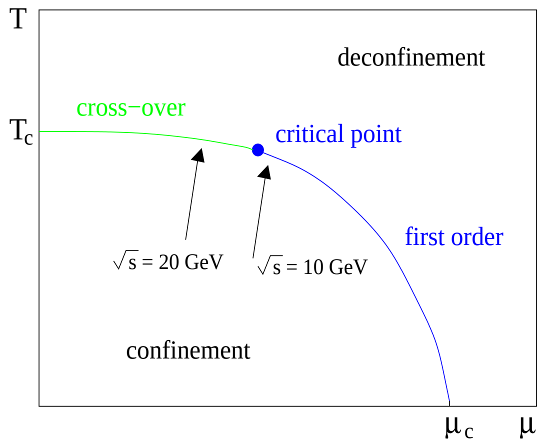
$\Rightarrow J/\psi$ **suppression vs. centrality,** A, $\sqrt{s}$

critical behaviour from confined (hadronic) side:

$\Rightarrow$ diverging fluctuations

possible scenario:

variation with $\sqrt{s}$
$\rightarrow$ variation with μ
$\rightarrow$ critical point



observables:

$\Rightarrow$ net baryon density vs. rapidity, A, $\sqrt{s}$
$\Rightarrow$ strangeness vs. $\sqrt{s}$?

4. Summary

- Critical behaviour, thermodynamic or geometric, implies abrupt change of physical observables.
- Statistical QCD $\rightarrow$ thermodynamic critical behaviour for equilibrium QCD matter.
- Parton physics $\rightarrow$ geometric critical behaviour for pre-equilibrium partons in nuclear collisions.
- Onset in both cases accessible best (perhaps only) at SPS.