Effect of Q'' on beam stability

N. Mounet, A. Burov, G. Rumolo and E. Métral

Effect of Q'' on beam stability

- Theoretical considerations
- Comparison between HEADTAIL and theory
- Conclusion
- Some bonus

Theoretical considerations

- Definition: single-particle detuning due to longitudinal motion:

  $$Q_x = Q_{x0} + Q'_x \delta + Q''_x \delta^2/2$$

  (relative momentum deviation $\delta = (p-p_0)/p_0$)

  Q'' is in particular created by octupoles (see e.g. S. Fartoukh).

- Up to now, this source of tune spread not included in instability theories (maybe it is in MOSES by Y. Chin – to be checked).

- With the approach of J. Scott Berg and F. Ruggiero for Landau damping, it is possible to get a simple (approximate) theory for beam stability with Q''. Ideas:

  - Compute the coherent tune shift due to the dipolar (drving) impedance, w/o detuning, from e.g Sacherer's formula (or Laclare's theory),
  - Get the Landau damping stability diagram due to the incoherent tune spread, thanks to dispersion integral,
  - If complex coherent tune shift "inside" the diagram → stable.

Theoretical considerations

- Stability diagram: e.g. horizontal plane, if Q'' is the only source of detuning, solve equation over complex tune shift Q:

$$1=\frac{-\Delta Q_c}{N_m}\int dJ_z \frac{J_z^{|m|}\Psi_0(J_z)}{Q-a_{xz}J_z-i0}$$

with:

  - J_z → longitudinal action,
  - $a_{xz}= Q''_x\,\sigma_\delta^2/(2\,\varepsilon_z)$ → detuning coefficient (σ_δ =momentum spread, ε_z= long. emittance),
  - m → headtail mode,
  - ΔQ_c → coherent tune shift from the impedance (no nonlinearities), w.r.t $Q_{x0}+ mQ_s$
  - N_m → normalization factor,
  - $\Psi_0(J_z)$ → unperturbed longitudinal distribution.

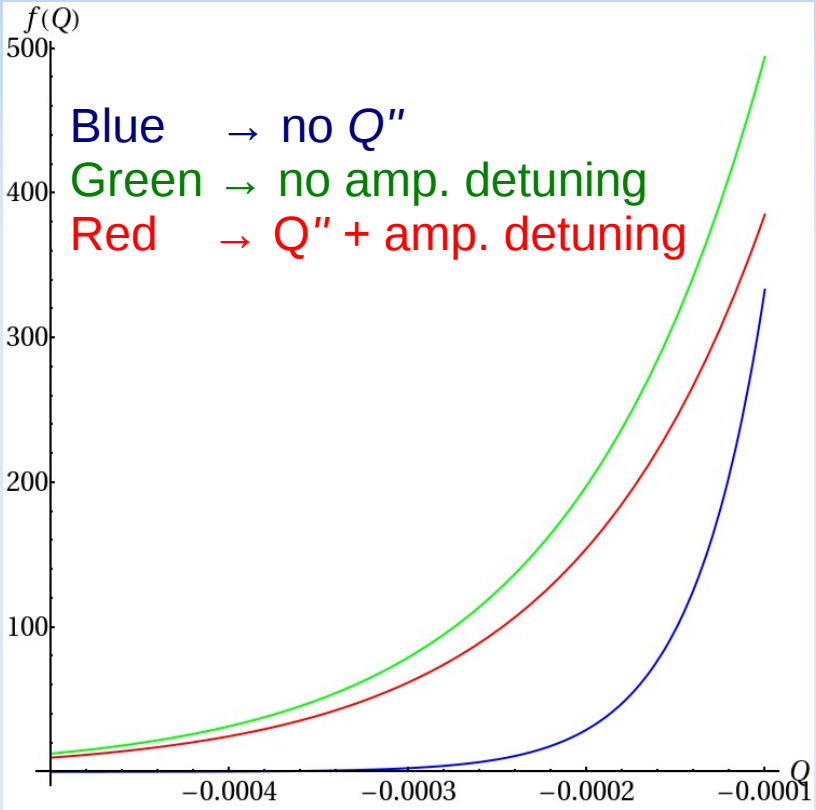
(note: Q' does not provide any source of damping as $Q'\delta$ averages to zero over a synchrotron period)

- This can be solved analytically for a Gaussian distribution.
- Also, with additional amplitude detuning (from octupoles), get similar equation (with triple integral) → can also be solved analytically for Gaussian distributions.

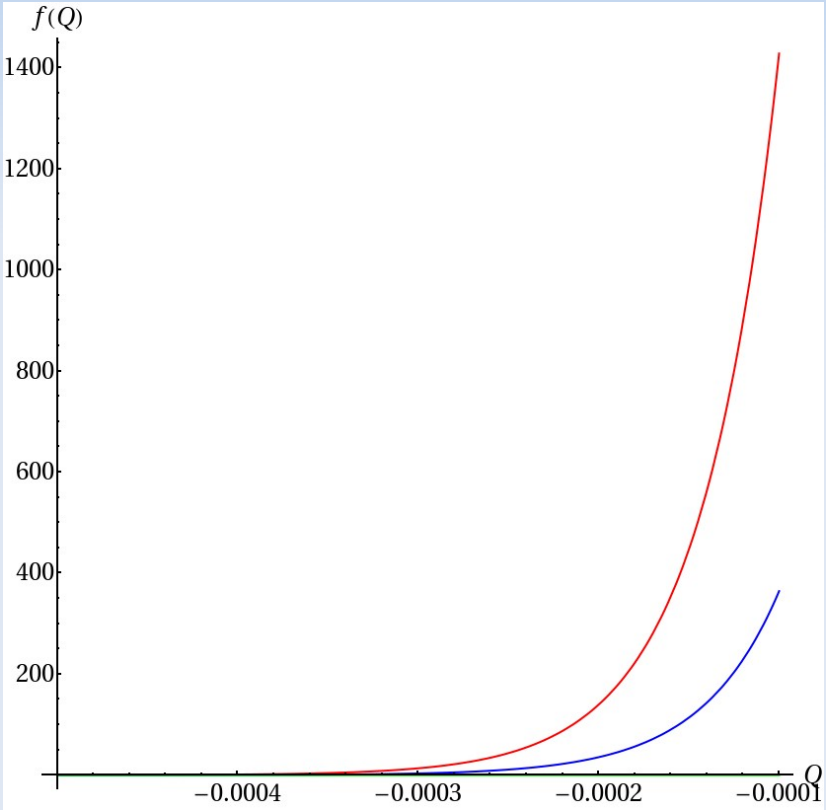
Theoretical considerations

- To get a first idea of what is going on: plot tune shift distribution function

  Example: LHC with +/- 100 A in octupoles (>0 in defocusing), 4 TeV/c, emittances 2 mm.mrad & 2.5 eVs, headtail mode *m=1*



Horizontal plane (*Q″<0*)

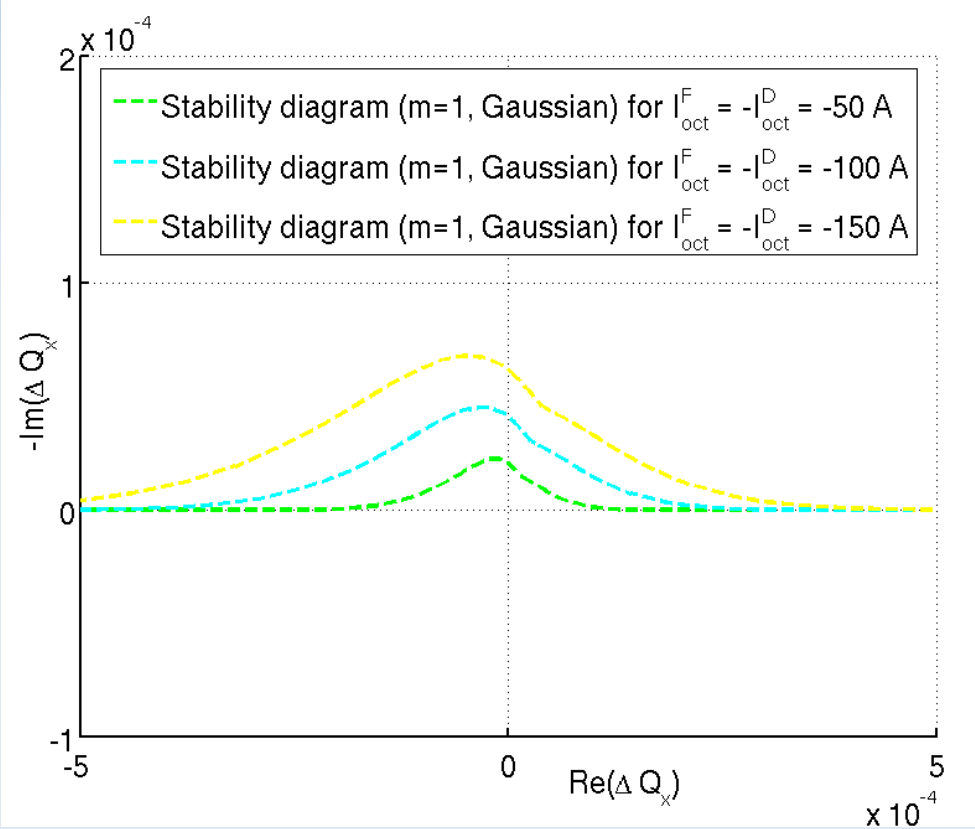


Vertical plane (*Q″>0*)

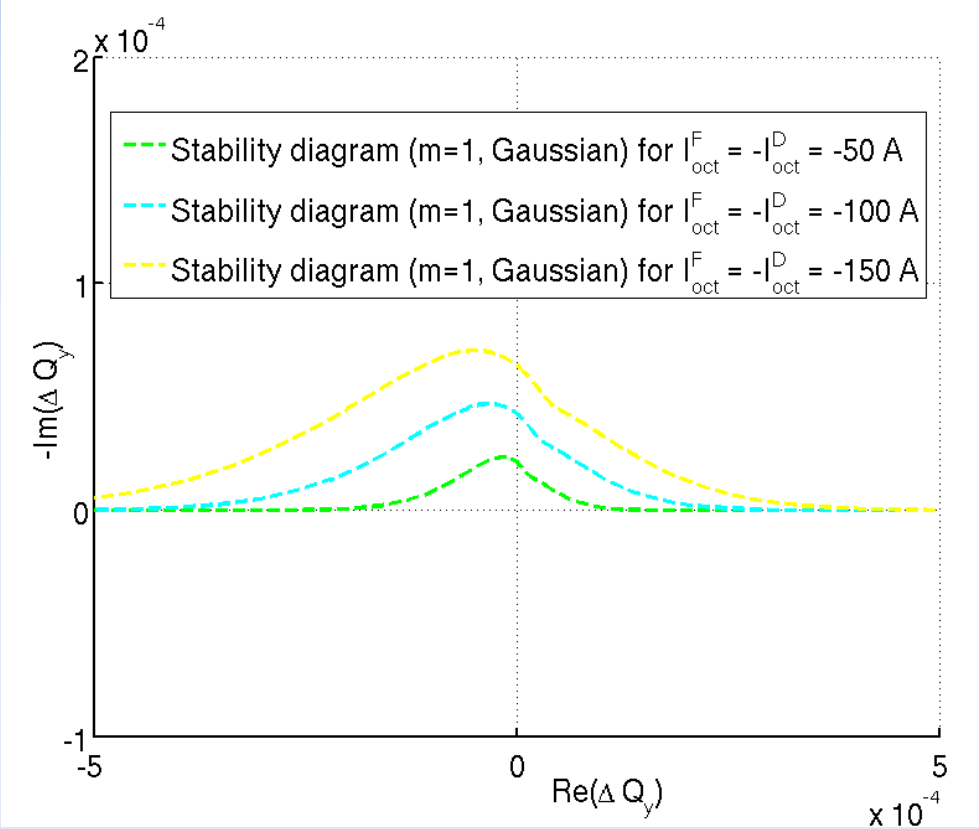
⇒ Q'' should always help, more in horizontal than in vertical (sign difference).

Theoretical considerations

- Stability diagrams: without Q'', LHC, 4 TeV/c, emittances 2 mm.mrad & 2.5 eVs



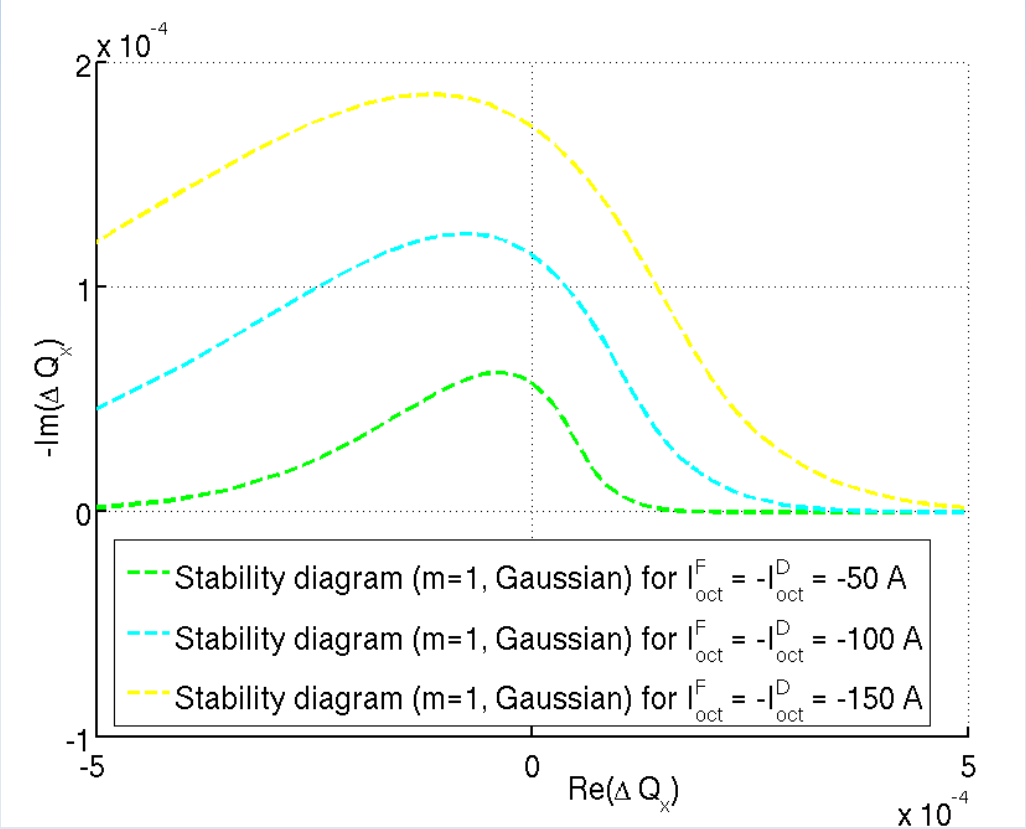
Horizontal plane

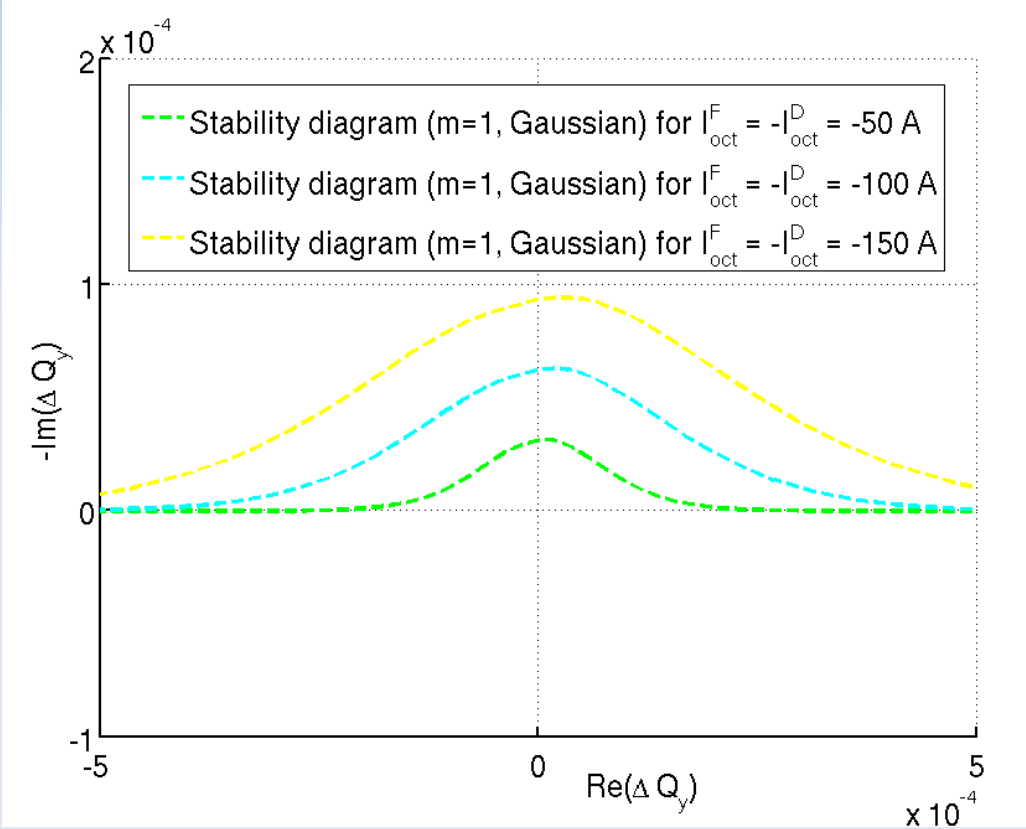


Vertical plane

Theoretical considerations

- Stability diagrams: with Q'', LHC, 4 TeV/c, emittances 2 mm.mrad & 2.5 eVs





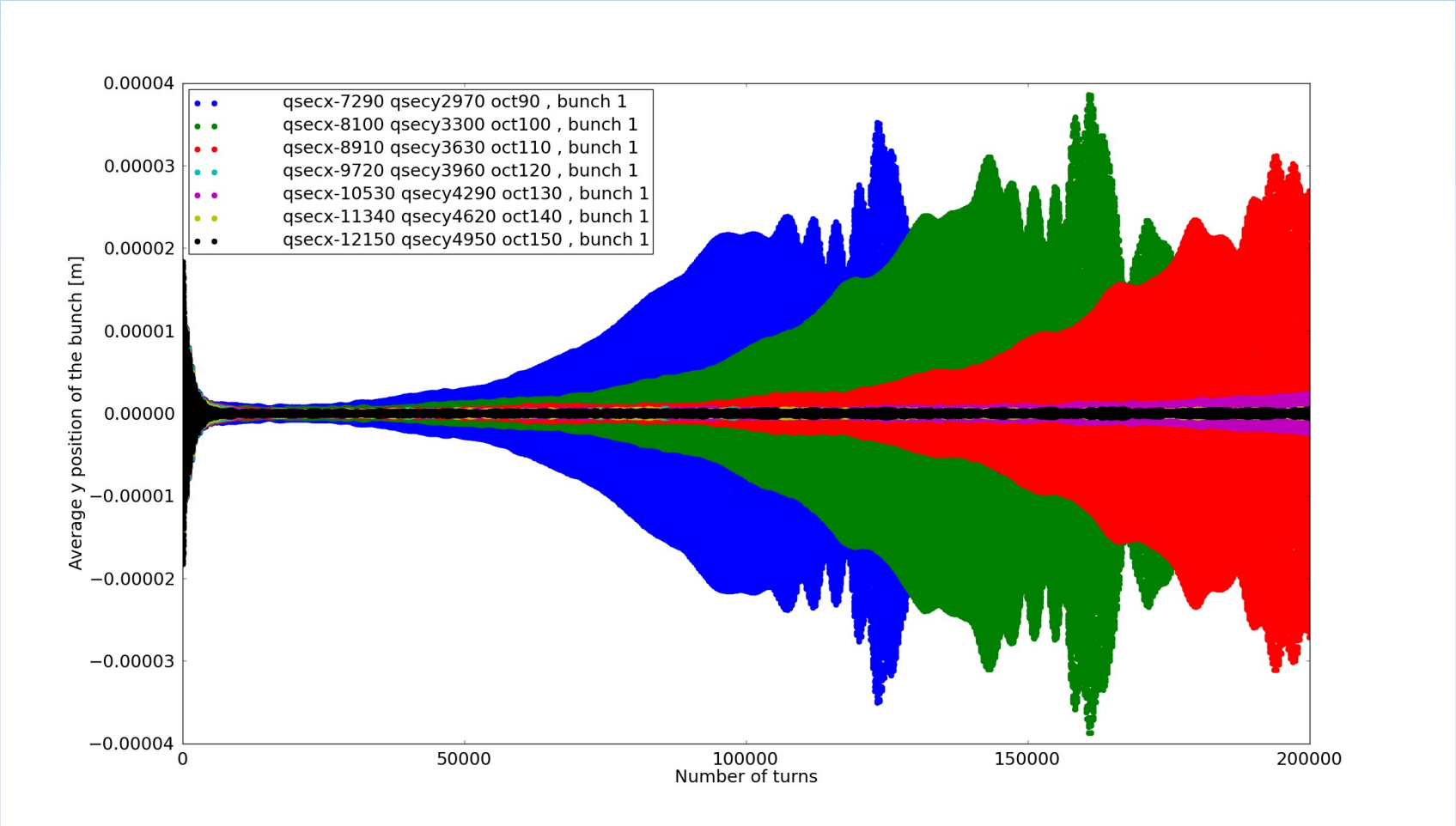
Horizontal plane (*Q''<0*)

Vertical plane (*Q''>0*)

⇒ Confirm that Q'' much more favorable in horizontal than in vertical.

HEADTAIL simulations with *Q''*

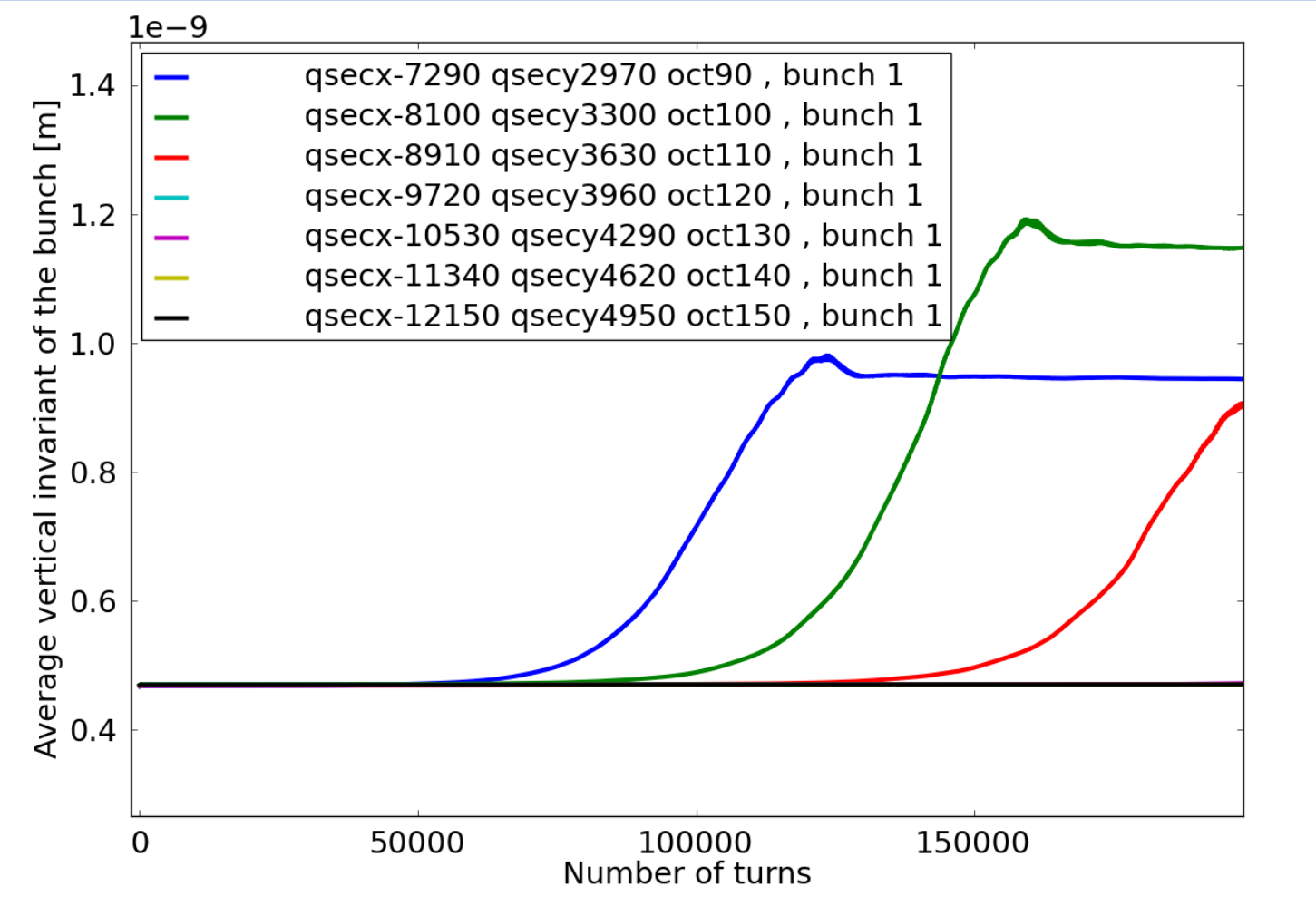
- Single-bunch, 4 TeV/c, linear bucket, 2.5 eV.s & 2mm.mrad, 1.7e11 p+: bunch centroid position vs. turns for *Q'* = *6* and various octupole currents:



⇒ 120 A < stabilizing I_{oct} < 130 A ?

HEADTAIL simulations with *Q''*

- Same run as previous slide: bunch average action (taking away the centroid motion) vs. turns:



$\Rightarrow$ 110 A < stabilizing I_{oct} < 120 A ?

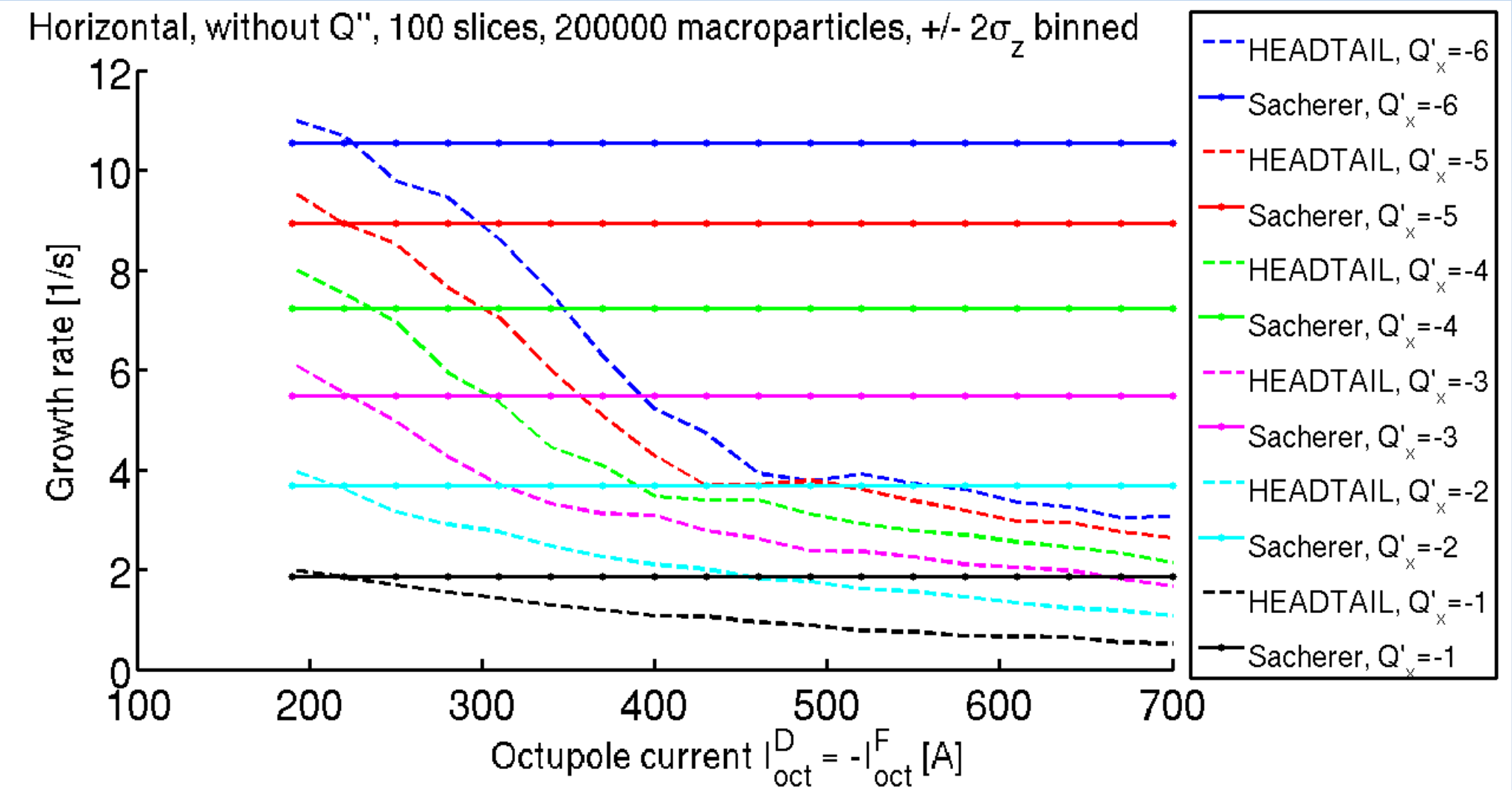
Landau damping threshold determination

- Several possible methods to determine if a run is stable or not:
  - ”visually” , from the bunch centroid data → quite subjective, and very tedious when many data,
  - with a criterion on the average invariant (or beam size), e.g. ”if it has increased by more than 10 % over the run, it's unstable” → quick but still arbitrary,
  - Alexey's idea: plot growth rate vs. octupole current, and extrapolate to get current at which it would cross zero → extrapolation always hazardous + when instability is weak, poor accuracy on growth rate.

We chose second method, but not clear if it's the best possible one.

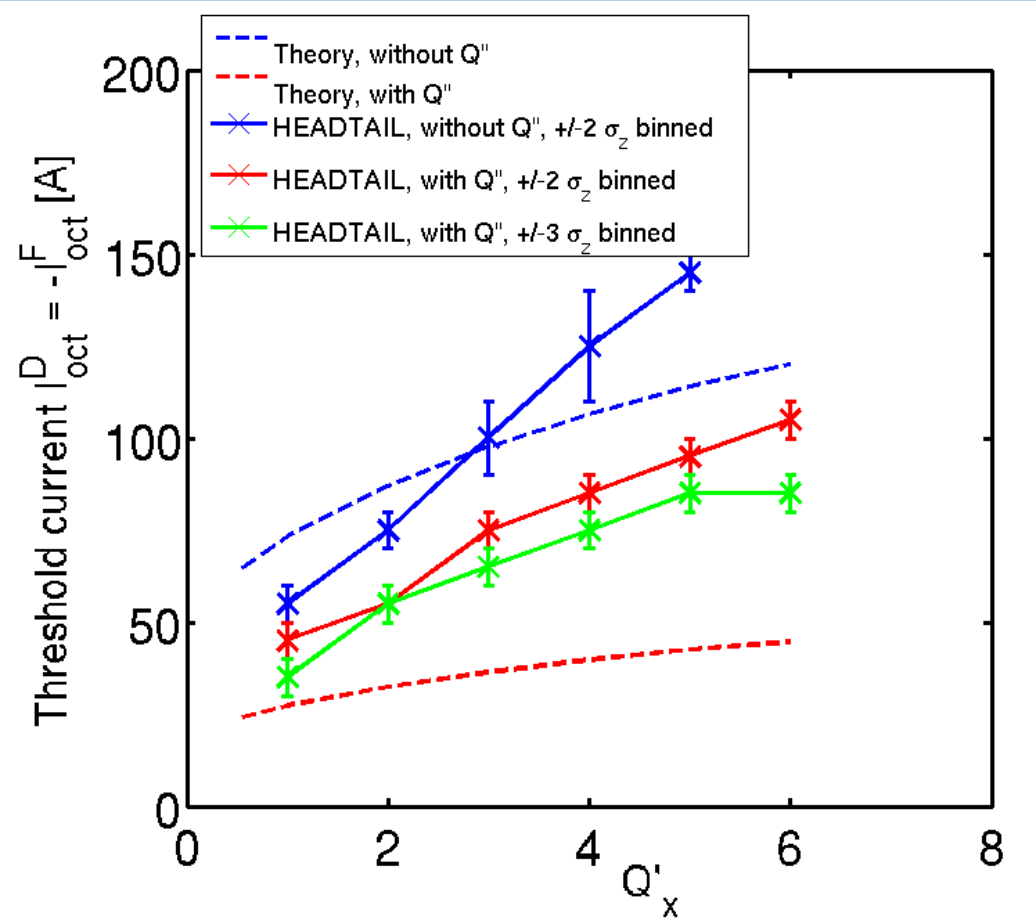
Comparison HEADTAIL / theory

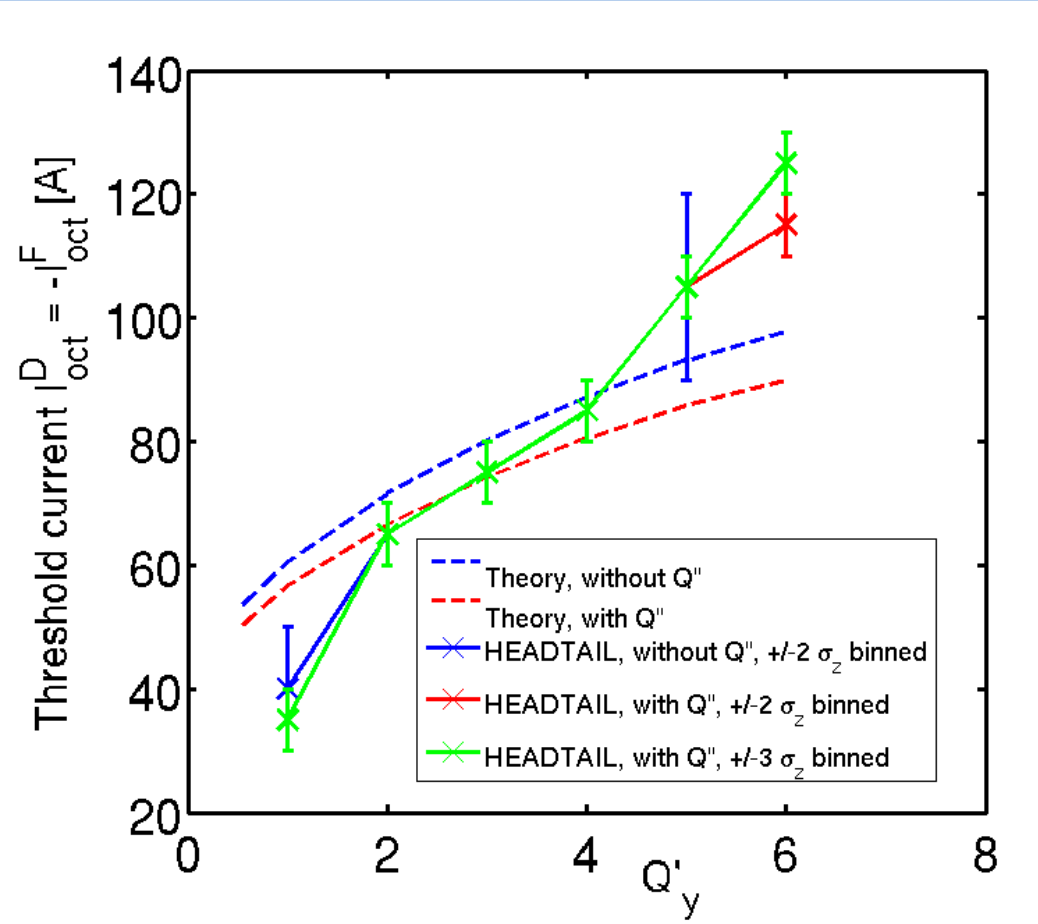
- First important difference: growth rate (so coherent tune shift) depends on octupole current even much below stability theshold (here *Q'<0, m=0*):



Comparison HEADTAIL / theory

- Landau damping threshold for positive chromaticity (headtail mode *m=1*):



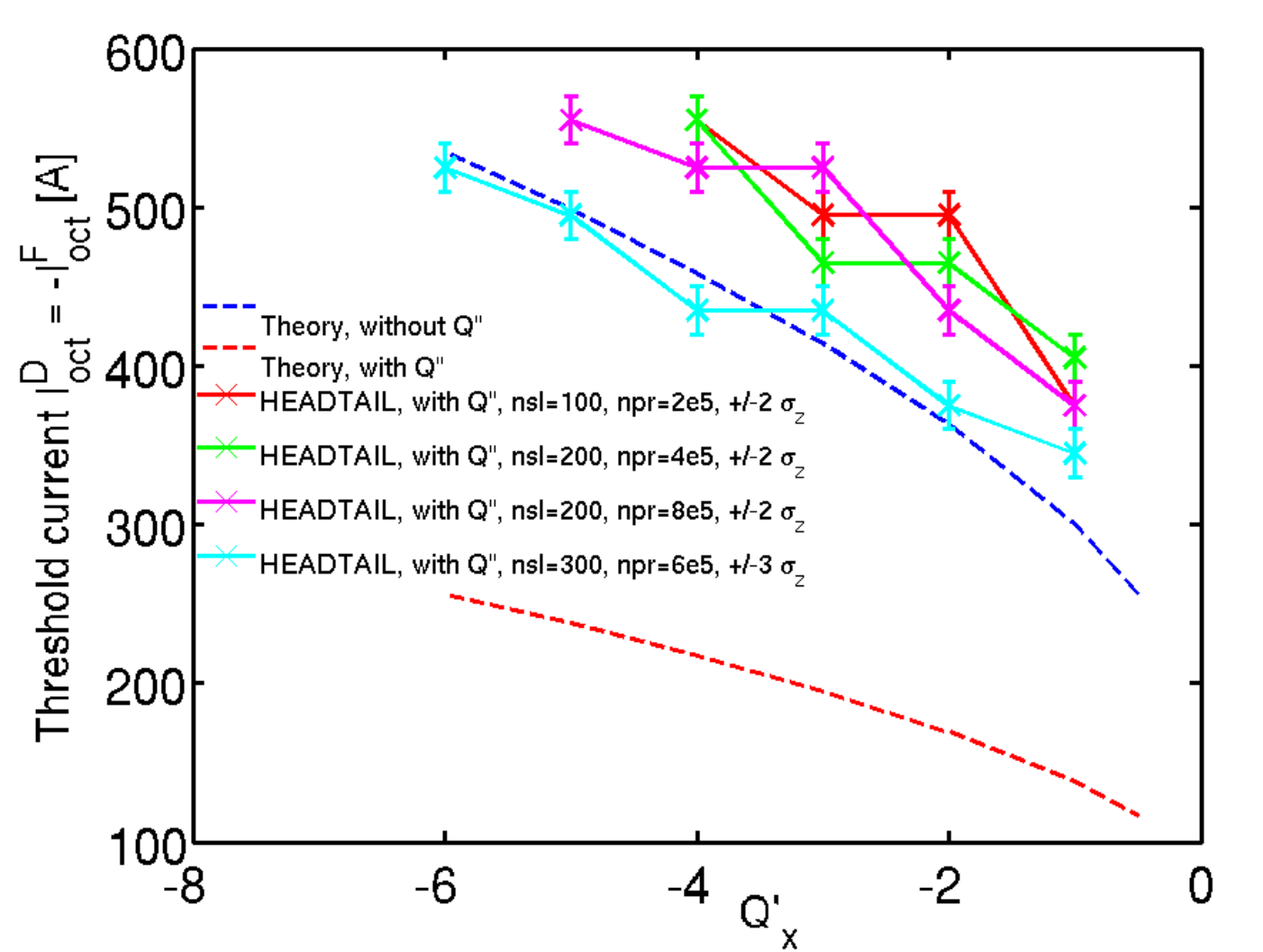


Horizontal plane (*Q''<0*) Vertical plane (*Q''>0*)

→ Impact of *Q''* clearly less important in the simulations than in the theory.

Comparison HEADTAIL / theory

- Landau damping threshold for $Q'_x<0$, in horizontal (headtail mode $m=0$):



→ Higher discrepancy for $Q'_x<0$ with m=0, than for $Q'_x>0$ with $m=1$.

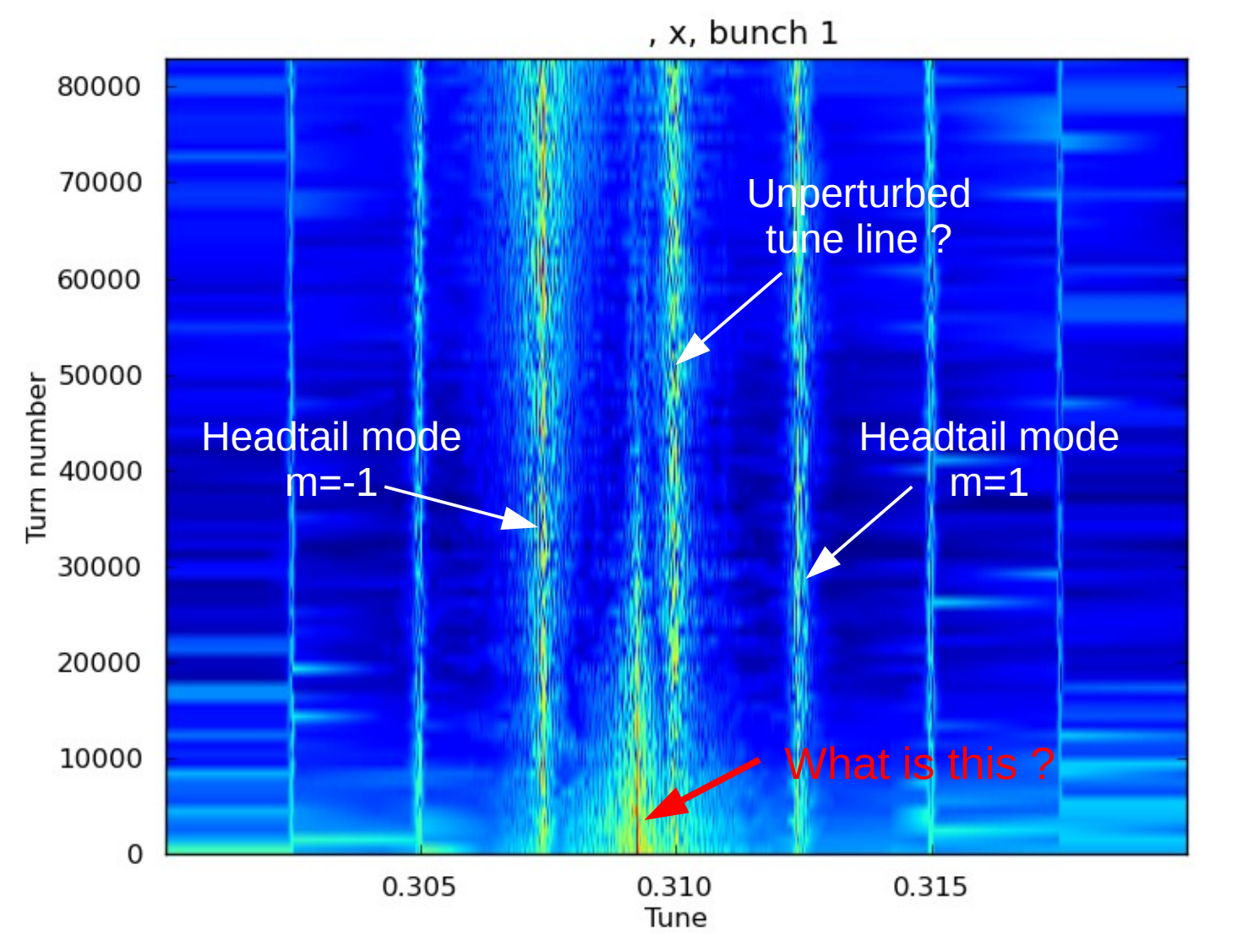
→ Number of slices and macroparticles has a rather small impact, but number of σ_z binned slightly more significant.

Conclusion on *Q″*

- Landau damping threshold from amplitude detuning in reasonable agreement between theory and simulations.
- On the contrary, effect of *Q″<0* much more important (stabilization) in the theory w.r.t to simulations.
- Impact of number of macroparticles and number of slices rather unimportant, number of σ_z binned has some effect on the stabilization threshold (but not enough to explain the discrepancy).
- Theory is at this stage quite approximate – a more reliable one would be to solve Sacherer's integral equation w/o any assumptions, with detuning from *Q″* taken into account.
- Note: HEADTAIL should be adapted to slow growth rates → apply kick only every several turns, to speed up.

Some bonus

- Strange line in the tune spectrum at the beginning of simulations:



→ amplitude decreases more quickly with higher chromaticity,
→maybe it's actually simply the main tune line with coherent tune shift from impedance... Then why the unperturbed tune is visible ??