Recursive equations for arbitrary scattering processes at tree order *and beyond*

Costas G. Papadopoulos

HP^2, September 2006, ETH, Zurich

HEP - NCSR Democritos

- **Reliable cross section computation and event generation for multiparticle processes, with $\sim$ 10-12 particles in the final state.**

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hep-ph/0012004 and Tokyo 2001,(CPP2001) Computational particle physics, p. 20-25

T. Gleisberg, *et al.* Eur. Phys. J. C 34 (2004) 173

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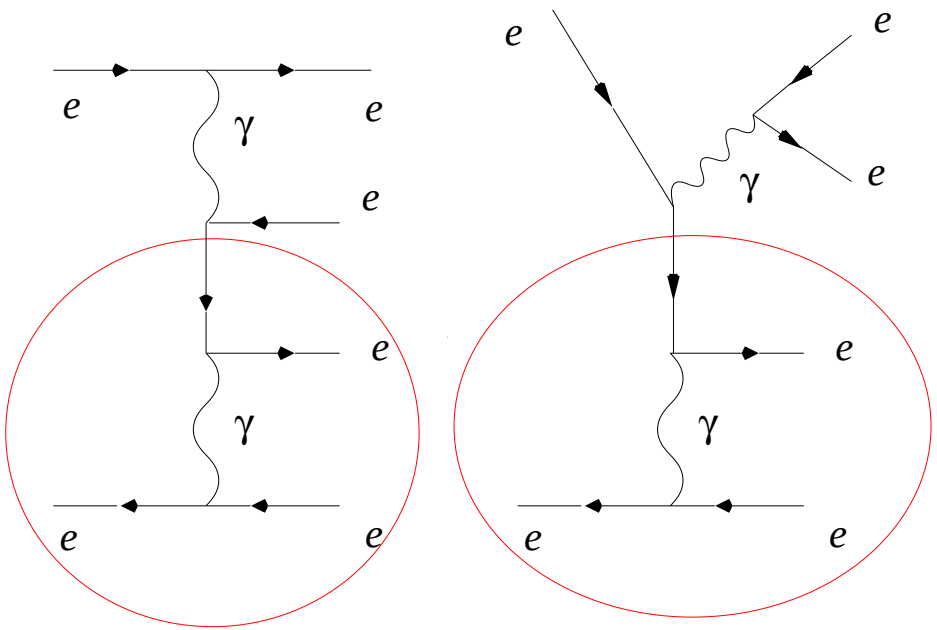
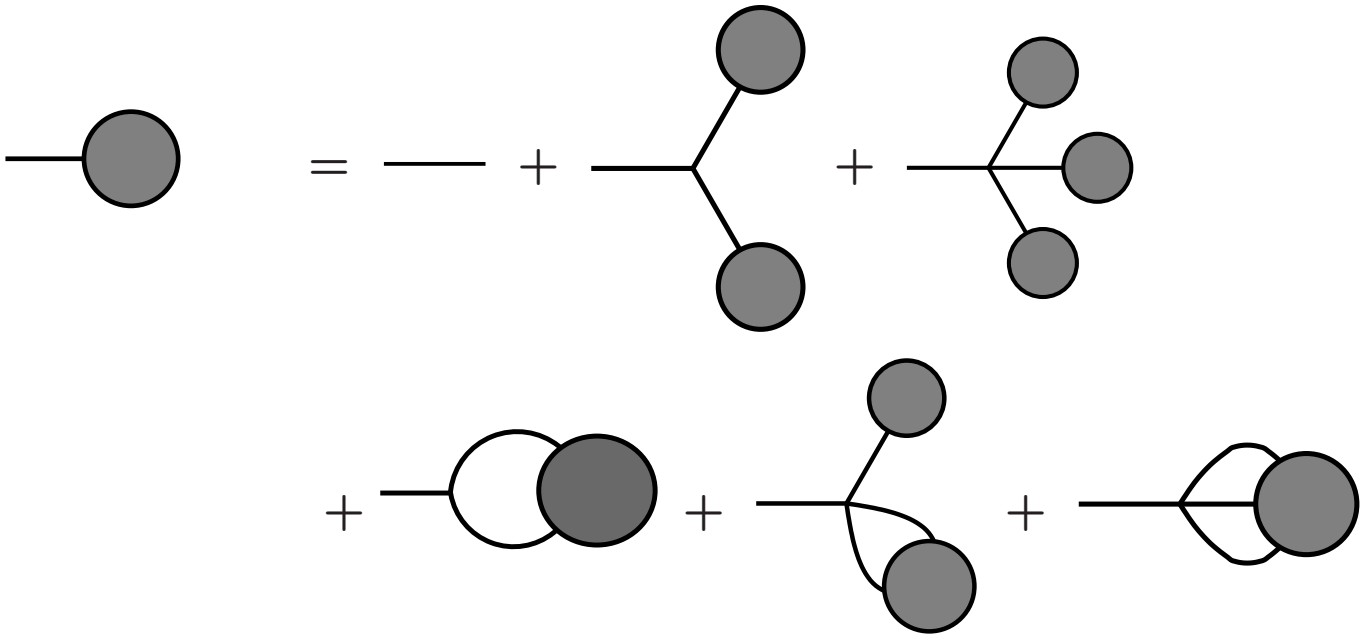
- ***Example:*** $e^- e^+ \longrightarrow e^- e^+ e^- e^+$ **in QED:**



The Dyson-Schwinger recursion

- **Imagine a theory with 3- and 4- point vertices and just one field.**

  Then it is straightforward to write an equation that gives the amplitude for $1 \longrightarrow n$



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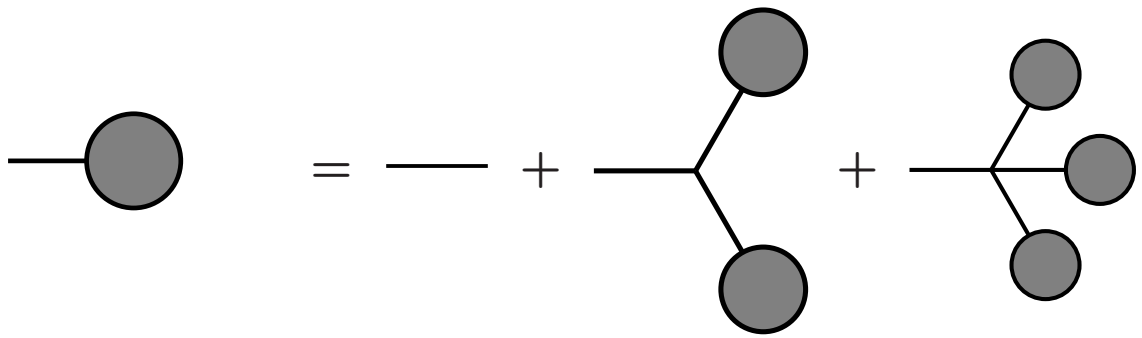

$$a(n) = \delta_{n,1} + \sum \frac{n!}{n_1! n_2!} a(n_1) a(n_2) \delta_{n_1+n_2,n}$$

$$+ \frac{n!}{n_1! n_2! n_3!} \sum a(n_1) a(n_2) a(n_3) \delta_{n_1+n_2+n_3,n}$$

HELAC

- Construction of the skeleton solution of the Dyson-Schwinger equations. At this stage only integer arithmetic is performed. This is part of the initialization phase.
- Dressing-up the skeleton with momenta, provided by `PHEGAS` and wave functions, propagators, n-point functions in general.
- Unitary and Feynman gauges implemented. Due to multi-precision arithmetic,tests of gauge invariance can be extended to arbitrary precision.
- All fermions masses can be non-zero.
- All Electroweak and QCD vertices are implemented, including Higgs and would-be Goldstone bosons.

Colour Configuration - EWK$\oplus$QCD

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- **Ordinary approach $SU(N)$-type**

$$\mathcal{A}^{a_1 \ldots a_n} = \sum Tr(T^{a_{\sigma_1}} \ldots T^{a_{\sigma_n}}) \quad A(\sigma_1 \ldots \sigma_n)$$

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Quarks and gluons treated differently

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- **New approach $U(N)$-type**

Each color-configuration amplitude is proportional to

$$D_i = \delta_{1,\sigma_i(1)}\delta_{2,\sigma_i(2)}\cdots\delta_{n,\sigma_i(n)}$$

where σ_i represents the i-th permutation of the set $1, 2, \ldots, n$.

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- ⋆ **quarks $1\ldots n$**
- ⋆ **antiquarks $\sigma_i(1\ldots n)$ and**
- ⋆ **gluons $= q\bar{q}$**

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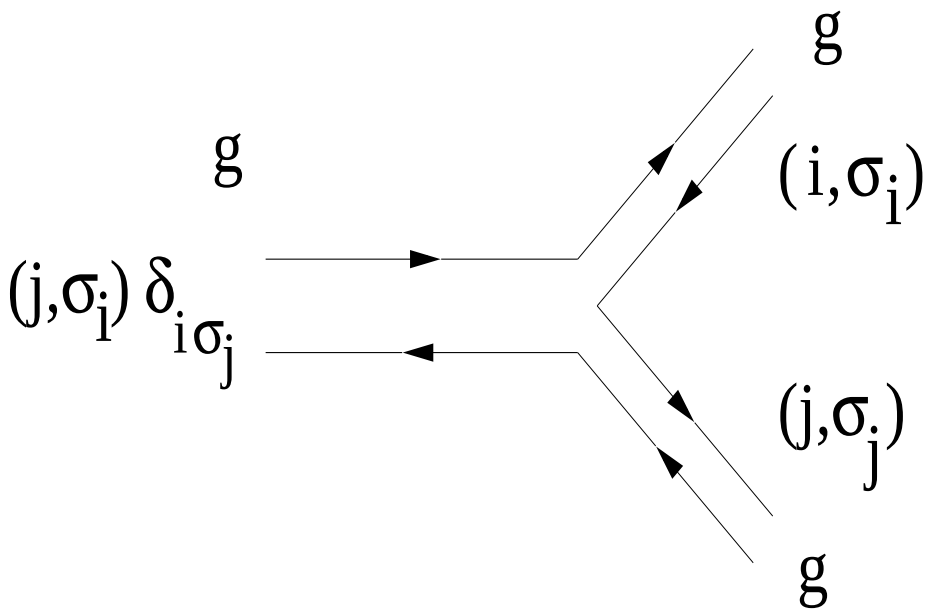
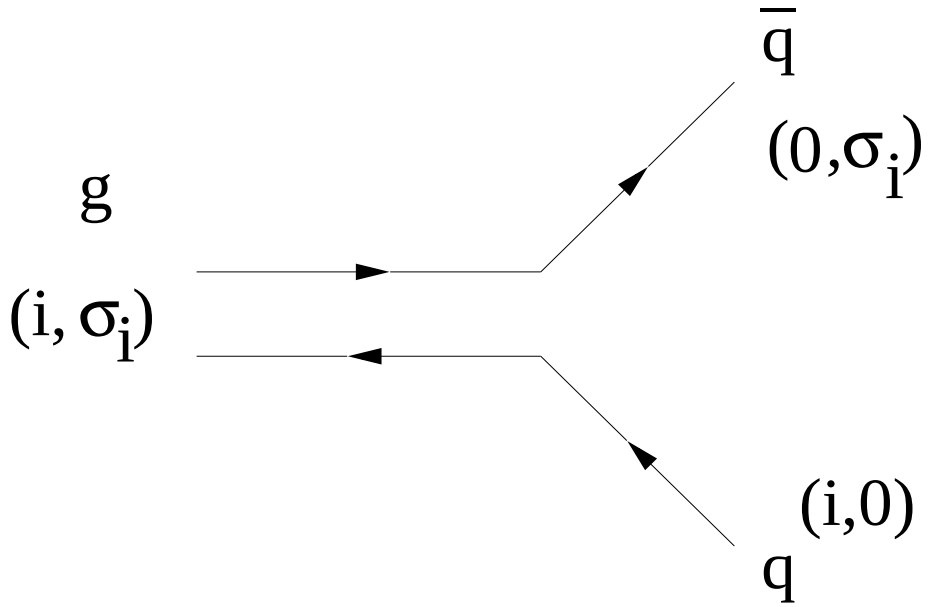
♠ **exact color treatment $\Rightarrow$ low color charge**

Problem: number of colour connection configurations: $\sim n!$ where n is the number of gluons or $q\bar{q}$ pairs. $\Rightarrow$ Monte-Carlo over continuous colour-space.



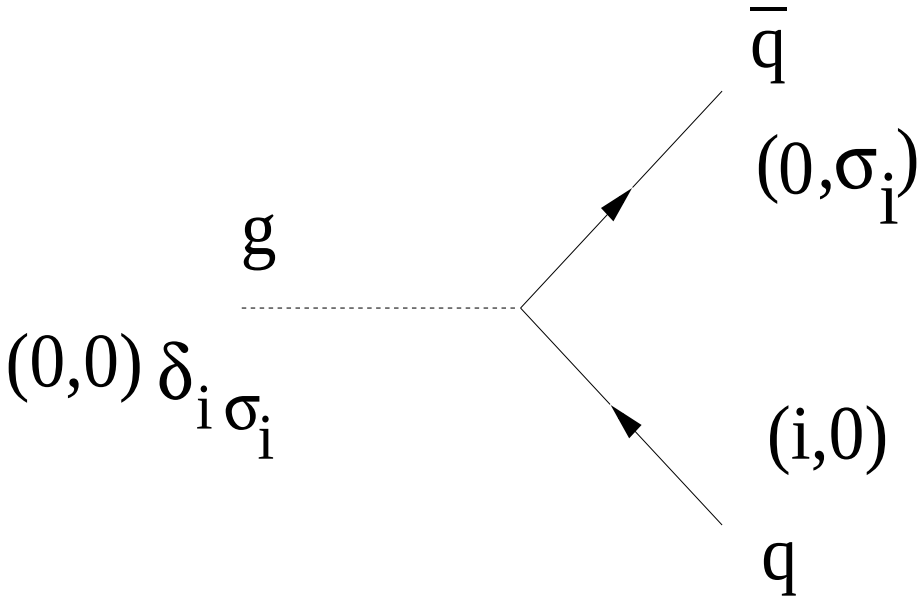
$$\sum f^{abc} t^a_{AB} t^b_{CD} t^c_{EF} = -\frac{i}{4}(\delta_{AD}\delta_{CF}\delta_{EB} - \delta_{AF}\delta_{CB}\delta_{ED})$$

$$\delta_{1\sigma_2}\delta_{2\sigma_3}\delta_{3\sigma_1}$$



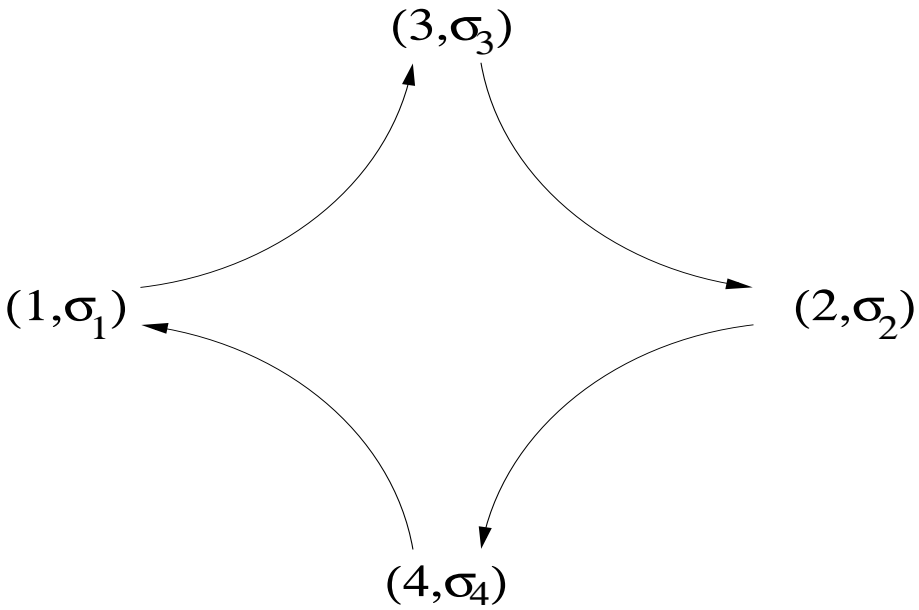
$$\sum t^a_{AB} t^b_{CD} = \frac{1}{2}(\delta_{AD}\delta_{CB} - \frac{1}{N_c}\delta_{AB}\delta_{AC})$$

$$\frac{1}{\sqrt{2}}$$



$$\sum t^a_{AB} t^b_{CD} = \frac{1}{2}(\delta_{AD}\delta_{CB} - \frac{1}{N_c}\delta_{AB}\delta_{AC})$$

$$\frac{1}{\sqrt{2N_c}}$$



$$\delta_{1\sigma_3}\delta_{3\sigma_2}\delta_{2\sigma_4}\delta_{4\sigma_1}$$

$$2g_{12}g_{34} - g_{13}g_{24} - g_{14}g_{23}$$

Summation/Integration over color

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$$\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{a_i\}_1^n) \sim \sum_{P(2,\ldots,n)} Tr(t^{a_1} \ldots t^{a_n}) \mathcal{A}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n)$$

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$$\sum_{\{a_i\}_1^n \{\varepsilon_i\}_1^n} |\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{a_i\}_1^n)|^2 = g^{2n-4} \sum_{\varepsilon} \sum_{ij} \mathcal{A}_i \mathcal{C}_{ij} \mathcal{A}_j^*$$

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$$\sum_{P(2,\ldots,n)} \sim n!$$

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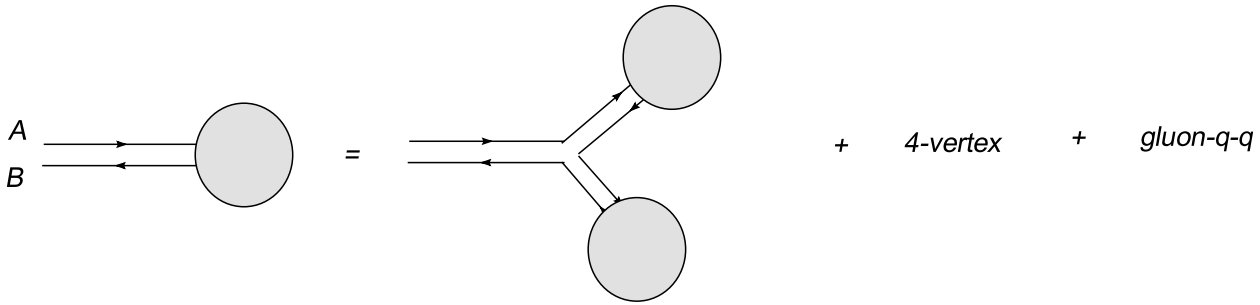
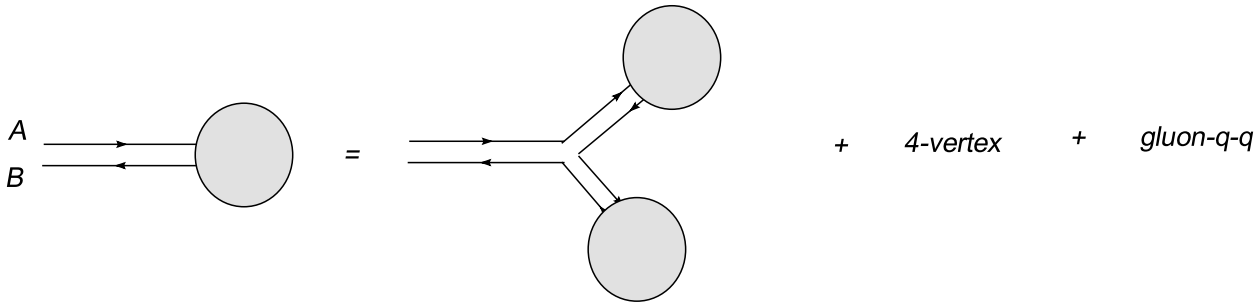
$$\sum_{\{I_i,J_i\}_1^n} \sim 3^n \times 3^n$$


A
B
=
+
4-vertex
+
gluon-q-q



$$[A^\mu(P);(A,B)] = \sum_{i=1}^{n} [\ \delta(P-p_i)\ \ A^\mu(p_i);(A,B)_i] +$$

$$\sum [\ (ig)\ \Pi^\mu_\rho\ V^{\rho\nu\lambda}(P,p_1,p_2)A_\nu(p_1)A_\lambda(p_2)\sigma(p_1,p_2);(A,B) = (C,D)_1 \otimes (E,F)_2]$$

$$-\sum [\ (g^2)\ \Pi^\mu_\sigma\ G^{\sigma\nu\lambda\rho}(P,p_1,p_2,p_3)A_\nu(p_1)A_\lambda(p_2)A_\rho(p_3)\sigma(p_1,p_2+p_3);$$

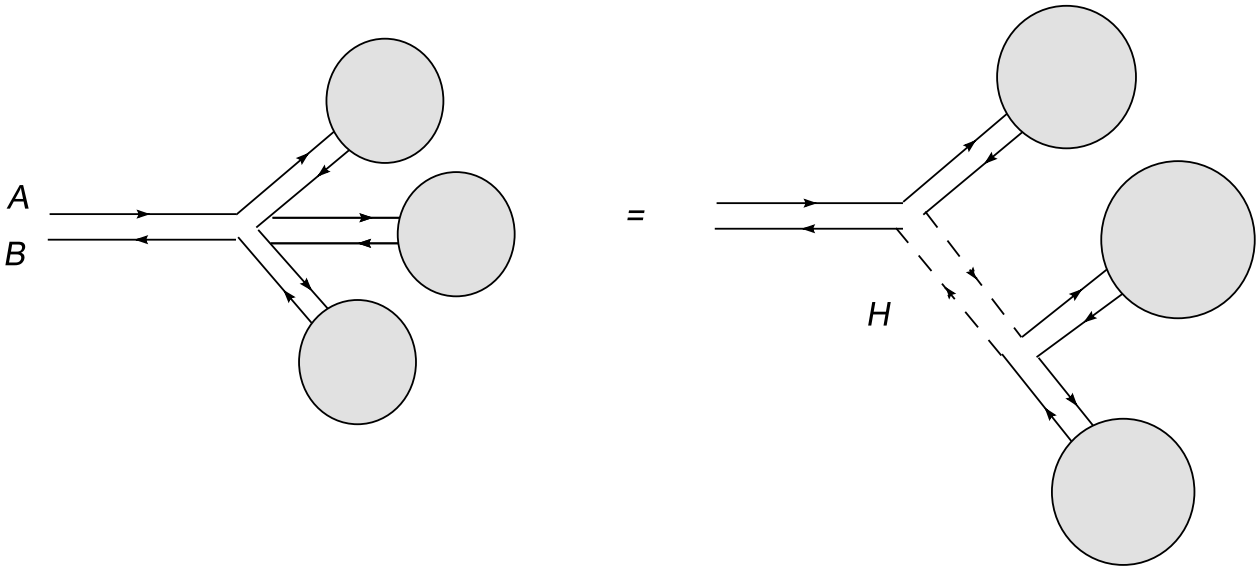
$$(A,B) = (C,D)_1 \otimes (E,F)_2 \otimes (G,H)_3]$$

$$+ \sum_{P=p_1+p_2} [\ (ig)\ \Pi^\mu_\nu\ \bar{\psi}(p_1)\gamma^\nu\psi(p_2)\sigma(p_1,p_2);(A,B) = (0,D)_1 \otimes (C,0)_2]$$

where $A,B,C,D,E,F,G,H = 1,2,3.$

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a}, \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$\mathcal{L} = -\frac{1}{2} H^a_{\mu\nu} H^{\mu\nu a} + \frac{1}{4} H^a_{\mu\nu} F^{\mu\nu a}.$$



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$$+$$

$$[\ (ig)\ \Pi^\mu_\sigma\ \ (g^{\sigma\lambda}g^{\nu\rho} - g^{\nu\lambda}g^{\sigma\rho})\ \ A_\nu(p_1)H_{\lambda\rho}(p_2)\sigma(p_1,p_2);(A,B) = (C,D)_1 \otimes (E,F)_2]$$

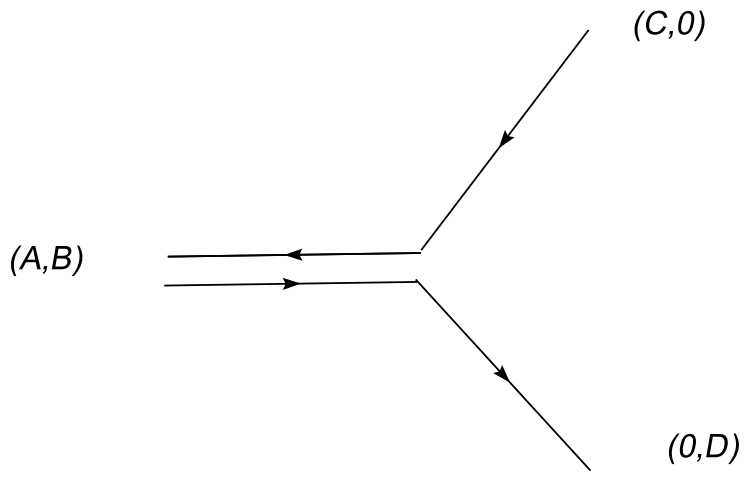
$$+$$

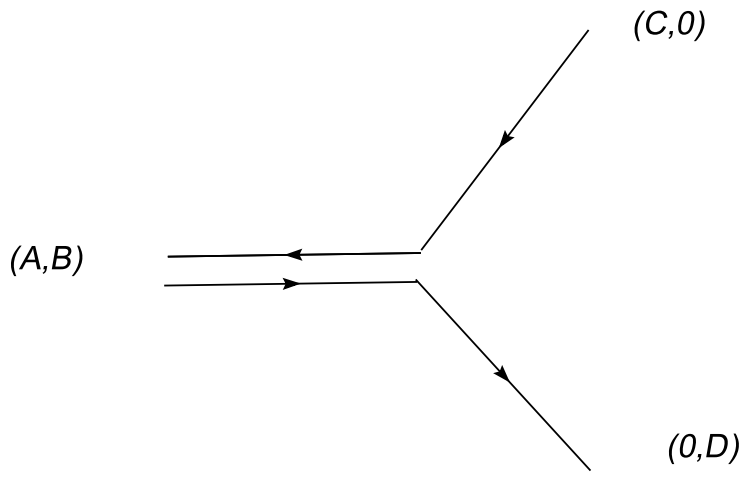
$$[\ (ig)\ \Pi^\mu_\nu\ \ \bar{\psi}(p_1)\gamma^\nu\psi(p_2)\sigma(p_1,p_2);(A,B) = (0,D)_1 \otimes (C,0)_2]$$

and

$$[H^{\mu\nu}(P);(A,B)] = \sum_{P=p_1+p_2} [\ (ig)\ (g^{\mu\lambda}g^{\nu\rho} - g^{\nu\lambda}g^{\mu\rho})\ \ A_\lambda(p_1)A_\rho(p_2)\sigma(p_1,p_2);$$

$$(A,B) = (C,D)_1 \otimes (E,F)_2].$$


(C,0)
(A,B)
(0,D)



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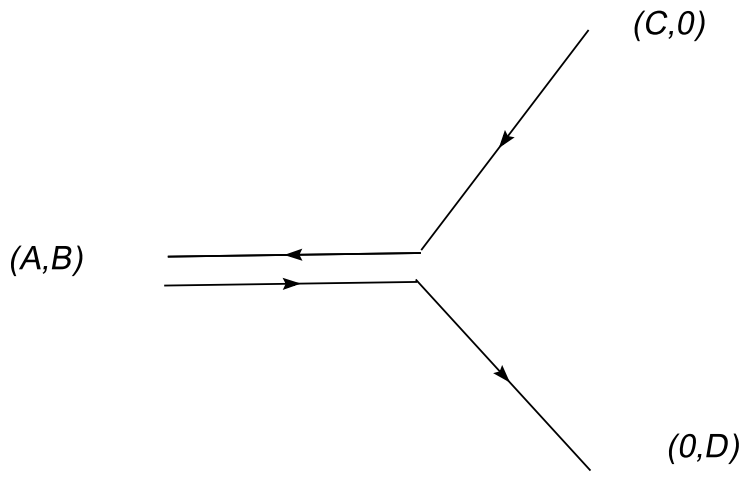

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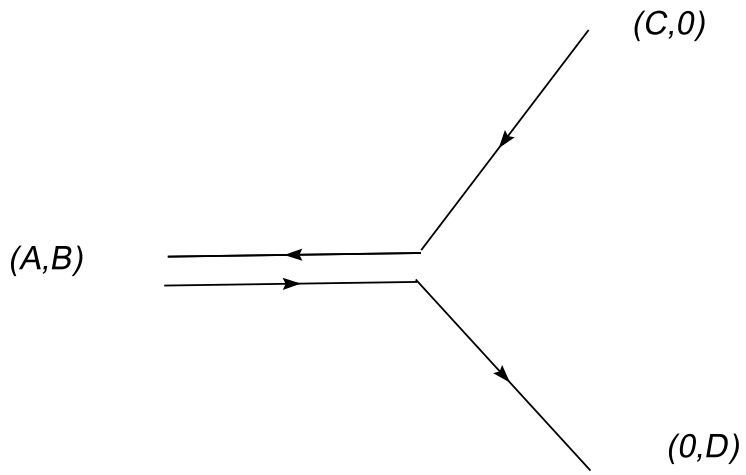

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$$(1,0) \otimes (0,1) = (1,1)_{2/3} \oplus (2,2)_{-1/3} \oplus (3,3)_{-1/3}$$

$$N_{\mathrm{CC}} = \sum_{A=0}^{n_q} \sum_{B=0}^{n_q-1} \sum_{C=0}^{n_q-A-B} \left(\frac{n_q!}{A!B!C!} \right)^2 \delta(n_q = A + B + C)$$

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Process	N_{CC}^{ALL}	N_{CC}	N_{CC}^{F} (%)
$gg \to 2g$	6561	639	59.1
$gg \to 3g$	59049	4653	68.4
$gg \to 4g$	531441	35169	77.4
$gg \to 5g$	4782969	272835	85.0
$gg \to 6g$	43046721	2157759	90.4
$gg \to 7g$	387420489	17319837	94.0
$gg \to 8g$	3486784401	140668065	96.4

Process	N_{CC}^{ALL}	N_{CC}	N_{CC}^{F} (%)
$gg \to u\bar{u}$	729	93	93.5
$gg \to gu\bar{u}$	6561	639	91.6
$gg \to 2gu\bar{u}$	59049	4653	92.6
$gg \to 3gu\bar{u}$	531441	35169	94.6
$gg \to 4gu\bar{u}$	4782969	272835	96.4
$gg \to 5gu\bar{u}$	43046721	2157759	97.8
$gg \to 6gu\bar{u}$	387420489	17319837	98.6
$gg \to c\bar{c}c\bar{c}$	6561	639	99.1
$gg \to gc\bar{c}c\bar{c}$	59049	4653	98.8
$gg \to 2gc\bar{c}c\bar{c}$	531441	35169	99.0
$gg \to 3gc\bar{c}c\bar{c}$	4782969	272835	99.3
$gg \to 4gc\bar{c}c\bar{c}$	43046721	2157759	99.6

Process	$\sigma_{\mathrm{MC}} \pm \varepsilon$ (nb)	ε (%)
$gg \to 7g$	$(0.53185 \pm 0.01149)\times 10^{-2}$	2.1
$gg \to 8g$	$(0.33330 \pm 0.00804)\times 10^{-3}$	2.4
$gg \to 9g$	$(0.17325 \pm 0.00838)\times 10^{-4}$	4.8
$gg \to 5gu\bar{u}$	$(0.38044 \pm 0.01096)\times 10^{-3}$	2.8
$gg \to 3gc\bar{c}c\bar{c}$	$(0.95109 \pm 0.02456)\times 10^{-5}$	2.6
$gg \to 4gc\bar{c}c\bar{c}$	$(0.81400 \pm 0.02583)\times 10^{-6}$	3.2

Process	$\sigma_{\rm MC} \pm \varepsilon$ (nb)	ε (%)
$gg \to Zu\bar{u}gg$	$(0.18948 \pm 0.00344)\times 10^{-3}$	1.8
$gg \to W^+\bar{u}dgg$	$(0.62704 \pm 0.01458)\times 10^{-3}$	2.3
$gg \to ZZu\bar{u}gg$	$(0.16217 \pm 0.00420)\times 10^{-6}$	2.6
$gg \to W^+W^-u\bar{u}gg$	$(0.27526 \pm 0.00752)\times 10^{-5}$	2.7
$d\bar{d} \to Zu\bar{u}gg$	$(0.38811 \pm 0.00569)\times 10^{-5}$	1.5
$d\bar{d} \to W^+\bar{c}sgg$	$(0.18765 \pm 0.00453)\times 10^{-5}$	2.4
$d\bar{d} \to ZZgggg$	$(0.99763 \pm 0.02976)\times 10^{-7}$	2.9
$d\bar{d} \to W^+W^-gggg$	$(0.52355 \pm 0.01509)\times 10^{-6}$	2.9

PHEGAS

- Phase space

$$d\Phi_n = (2\pi)^{4-3n} \prod_{i=1}^{n} \frac{d^3 p_i}{2E_i} \delta\left(\sum E_i - w\right) \delta^3\left(\sum \vec{p}_i\right)$$

- RAMBO, VEGAS-based nice but completely inefficient!

$$d\sigma_n = \text{FLUX} \times |\mathcal{M}_{2\to n}|^2 d\Phi_n$$

need appropriate mappings of peaking structures, plus optimization!

- Efficiency $\Rightarrow$ to a large number of generators, each one for a specific class of processes.

Multichannel approach

$$\mathcal{I} = \int f(\vec{x}) d\mu(\vec{x}) = \int \frac{f(\vec{x})}{p(\vec{x})} p(\vec{x}) d\mu(\vec{x})$$

$$p(\vec{x}) = \sum_{i=1}^{M_{ch}} \alpha_i \, p_i(\vec{x}) \qquad \sum_{i=1}^{M_{ch}} \alpha_i = 1$$

$$\mathcal{I} \rightarrow \left\langle \frac{f(\vec{x})}{p(\vec{x})} \right\rangle \qquad \mathcal{E}^2 N \rightarrow \left\langle \left(\frac{f(\vec{x})}{p(\vec{x})} \right)^2 - \mathcal{I}^2 \right\rangle$$

$\star$ **Optimize $\alpha_i \Rightarrow$ Minimize $\mathcal{E}$** $\star$

R.Kleiss and R.Pittau, Comput. Phys. Commun. 83, 141 (1994).

New **Dyson-Schwinger equations: subamplitude is a combination of several peaking structures!**

problem unsolved?

QCD antennas

P.D.Draggiotis, A.van Hameren and R.Kleiss, hep-ph/0004047.

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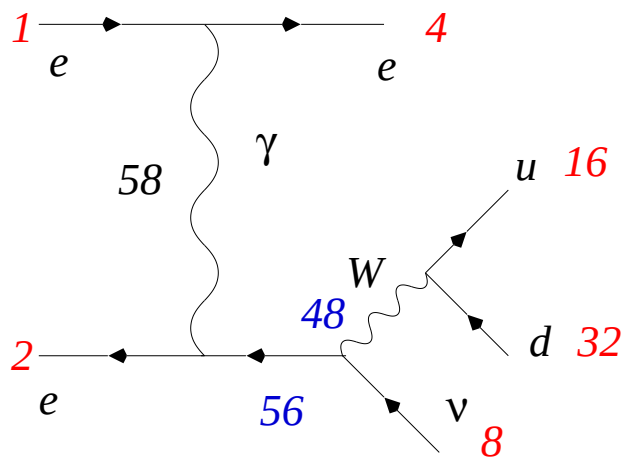
Old **Feynman graphs: exhibit single peaking structure!**

problem solved

Back to Feynman graphs:



The corresponding intrinsic representation looks like

62	**-2**	**4**	**-2**	**58**	**31**
58	**31**	**2**	**-2**	**56**	**2**
56	**2**	**48**	**33**	**8**	**1**
48	**33**	**16**	**-3**	**32**	**4**

Higher-order corrections

- **Fermion-loop corrections have been implemented and studied for up to 3-point vertices.**
- **We have started the computation of 4-point contributions.**
  - `FORM` **has been used to reduce the expressions to Passarino-Veltman coefficient functions**
  - `FF` **has been updated to include level-4 tensor coefficient functions for 4-point integrals.**
  - **Implementation and checking in** `HELAC` **is in progress.**
- **This implementation will allow to study 4 fermion+γ and 6 fermion production including the running of electroweak couplings.**

- **Fermion-loop corrections to six-fermion production process**

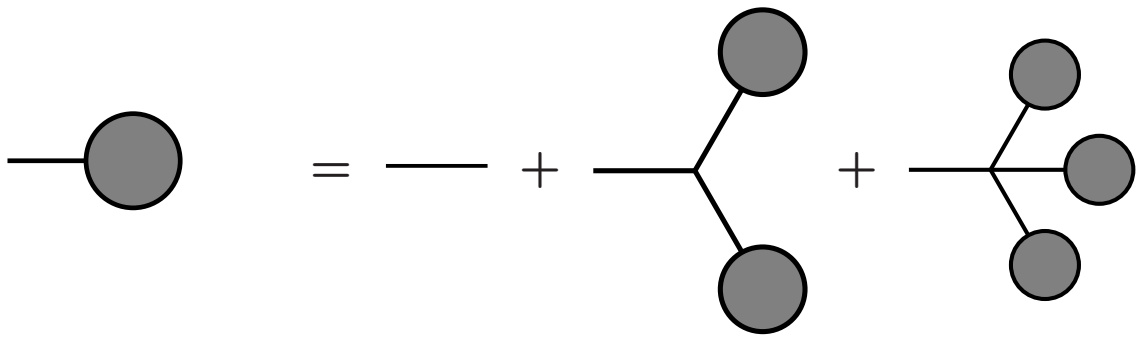
$$e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d} \tau^- \tau^+$$

- **Number of Feynman Graphs: 208**
- **Number of DS vertices: 140**
- **Cuts:** $E_l, E_q > 5\text{GeV}$ **and** $m_{ll}, m_{qq} > 10\text{GeV}$
- **Results:** $E = 500\text{GeV}$
- $\sigma_0/ab = 54,96(26)$ $\sigma_1/ab = 57,31(28)$ $K/100 = 4.28(2)$
- **MC data: generated: 1M(961792) used: 404842 time:6 1/2 h**

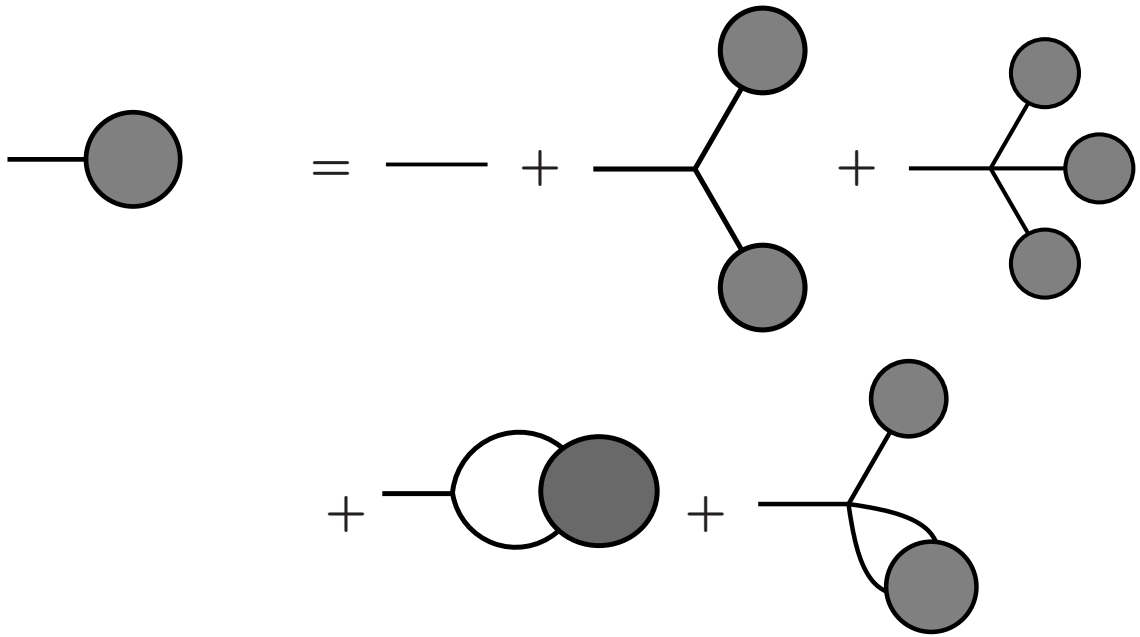
Higher-order corrections - NEW

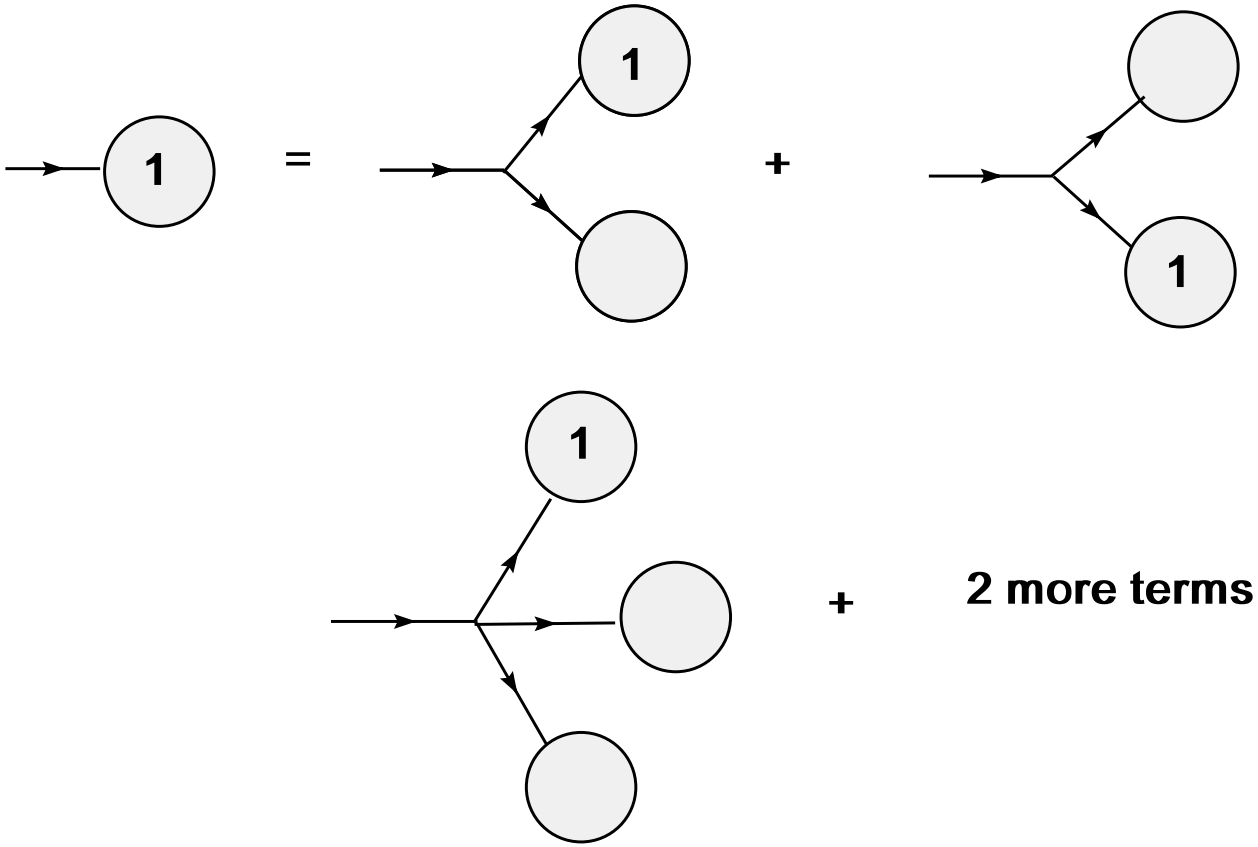
Lets go back to the DS equation

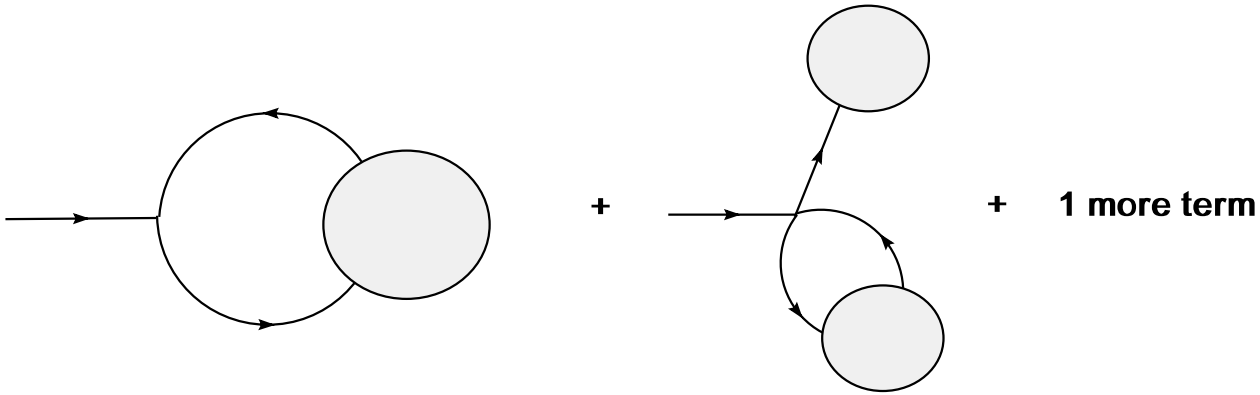
- **Tree order:**

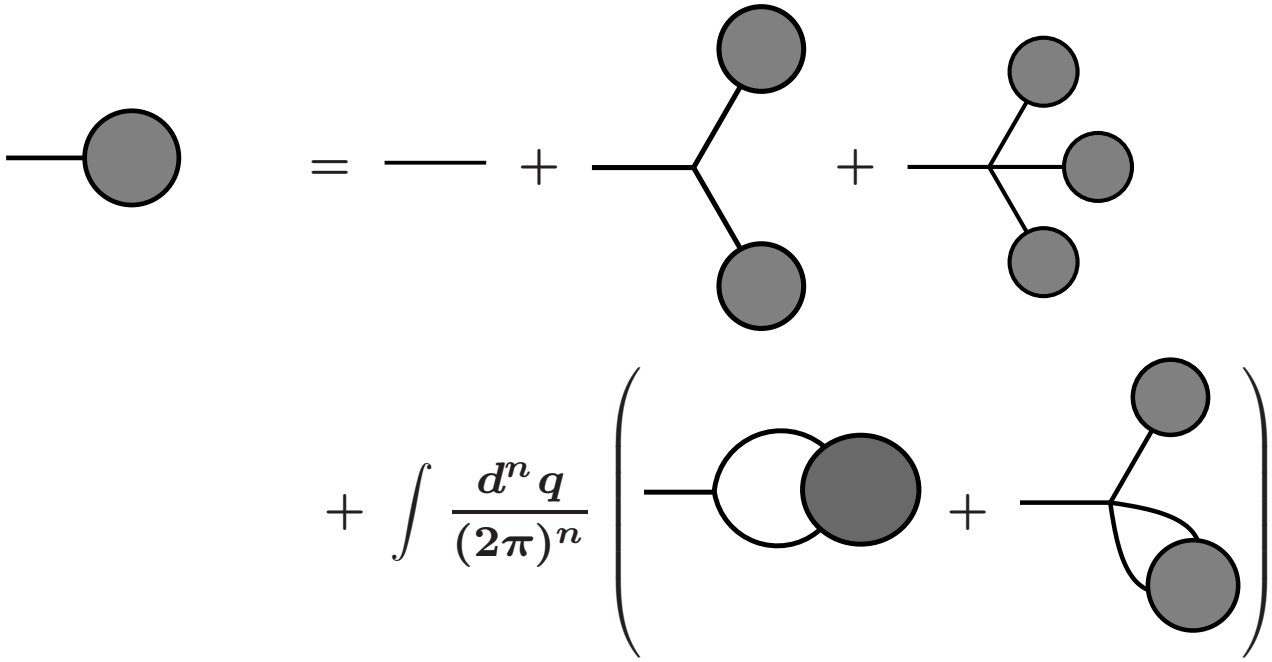


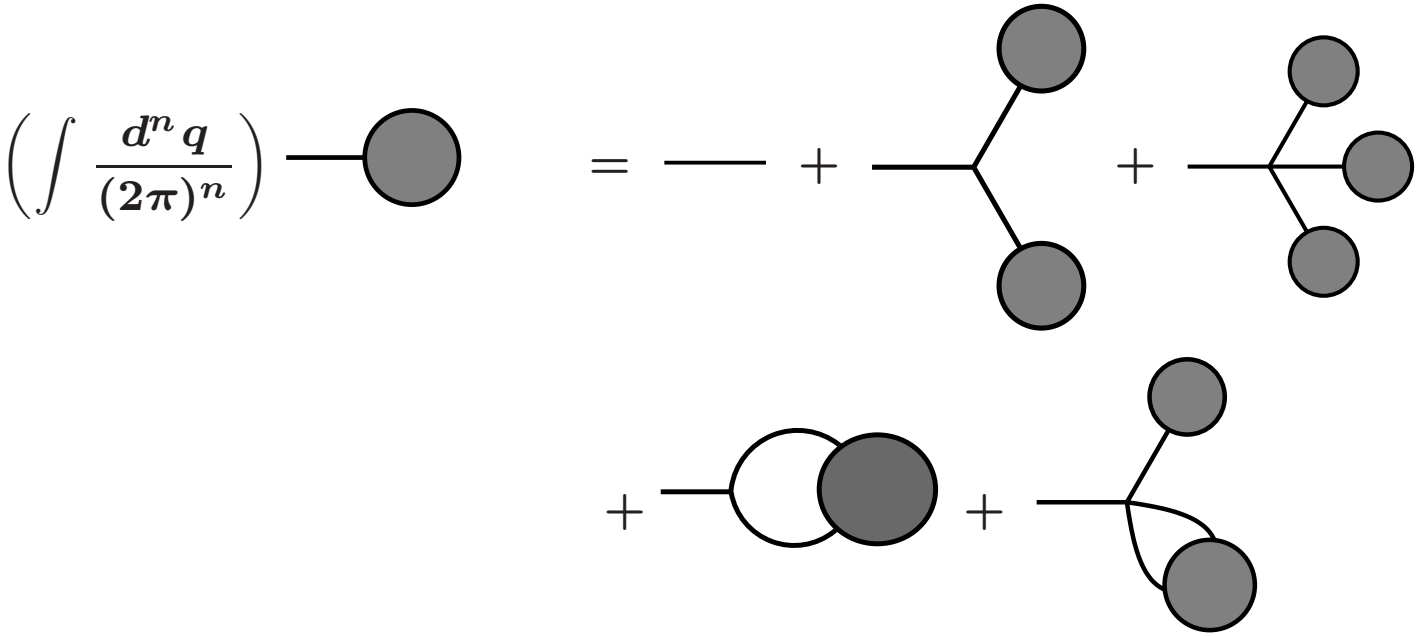
- **One loop:**



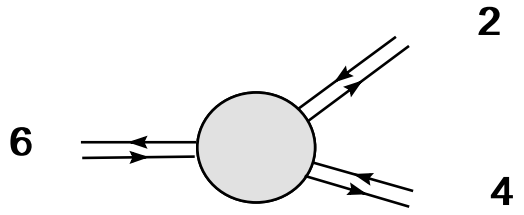

1
=
1
+
1
1
+
2 more terms

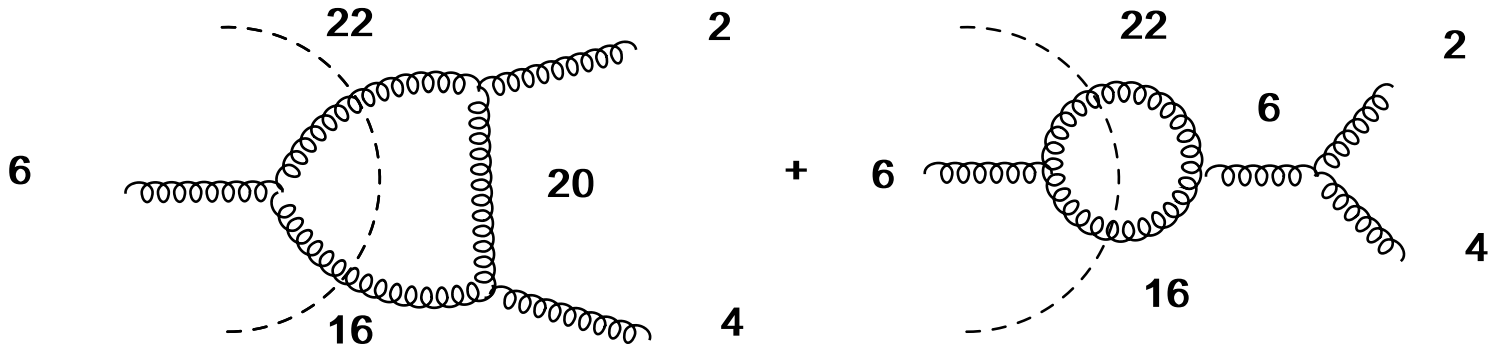

+
+
1 more term


$= \quad + \quad + \quad + \int \frac{d^n q}{(2\pi)^n} \left(\quad + \quad \right)$


$\left(\int \frac{d^n q}{(2\pi)^n}\right)$
=
+
+
+
+

g(1)+g(2) → g(4)+g(8) q=16



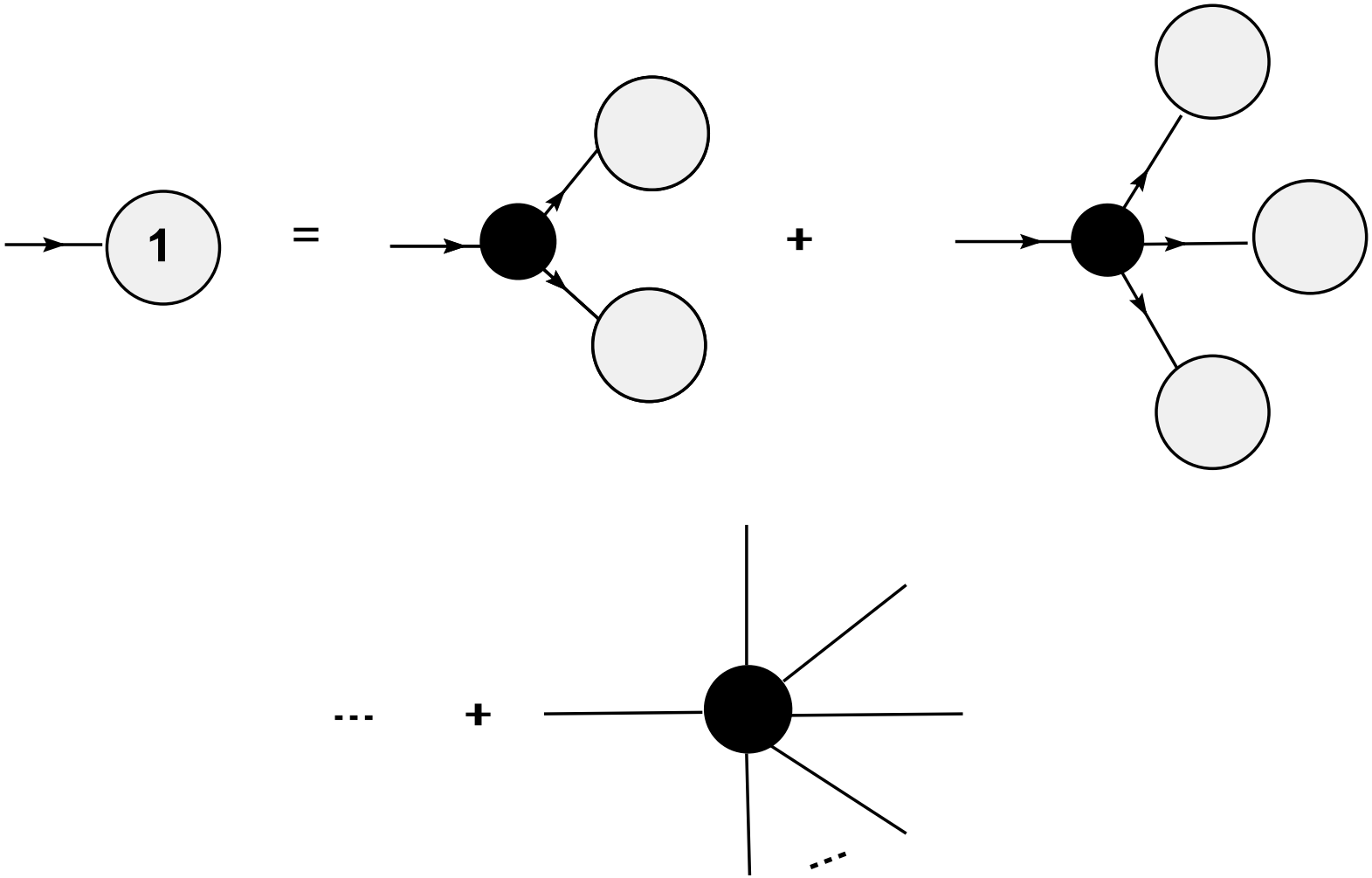


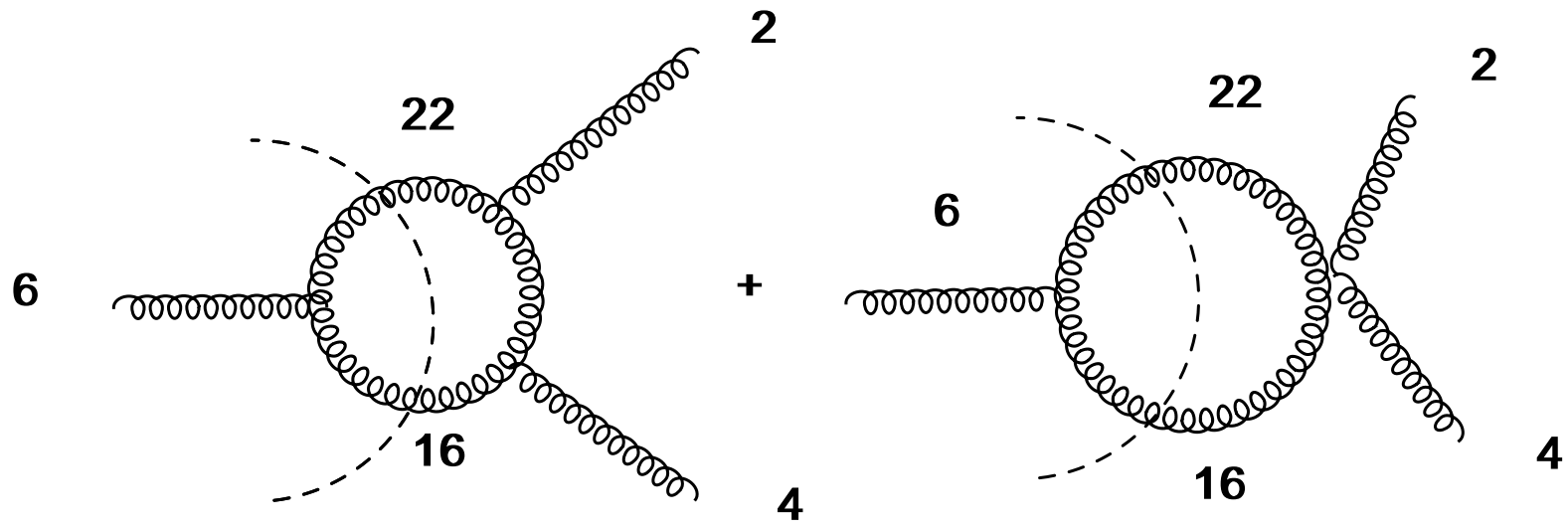
$N(q) = N_1(q) + N_2(q) \times D(20)$

Generalized vertices approach

- **Tree structure with effective quantum action vertices**
- **Re-normalized vertices**

One-particle irreducible (1PI) graphs !


1
=
+
...
+
...


2
22
6
+
16
4
2
22
6
16
4

where $q_k^{(\sigma)}$ for $\sigma = 1, 2$ are the two solutions of

$$(q_k^{(\sigma)} - Q)^2 + b = 0, \qquad (q_k^{(\sigma)} - p_k)^2 + a_k = 0. \tag{28b}$$

We note from Eq. (27a) that F_1 is the desired result for the numerator $N(p, q)q$ needed for the induction proof. Therefore, we must only show that F_2 vanishes. From Eqs. (28b), using the solution (21), we find that each component of both of the vectors $q_k^{(\sigma)}$ as well as the $(q_k^{(\sigma)})^2$, are of the first order in b when $b \to \infty$. An elementary estimate then shows that, if the integral in Eq. (27) is convergent, the term F_2 vanishes at least as rapidly as $b^{-1} \log b$. This completes the proof.

5. DISCUSSION

The result just obtained can be summarized in the following way.

The contribution of the Feynman diagram (1) or (26) in two-dimensional space–time can be written as a sum of terms where each term is obtained by applying rules:

(i) Pick out all possible pairs of denominators $(q - p_j)^2 + a_j$ and $(q - p_k)^2 + a_k$ in (1) and find the two vectors $q_{jk}^{(\sigma)}$ which make both these denominators equal to zero. An explicit formula for these vectors is given by Eq. (21).

(ii) Substitute $q_{jk}^{(\sigma)}$ in all of the remaining factors in (1) or (26). This quantity is denoted by $A_{jk}^{(\sigma)}$.

(iii) Take the average of $A_{jk}^{(\sigma)}$ for the two possible values of σ.

(iv) Multiply by the function L_{jk} defined in Eqs. (7) and (8) and corresponding to a simple "bubble" with $p_j - p_k$ as external vector.

(v) Sum over all $j < k$.

In passing, we mention that a special case of Eq. (25) with $n = 3$ and corresponding to a "triangle" diagram has been noticed before.[4]

The possibility of generalizing our result to higher dimensions has already been mentioned.[3] Another generalization to be investigated is concerned with diagrams in two-dimensional space–time but of more complicated structure involving two or more closed loops. Even if we have no definite results to report for such cases we want to remark that the result (25) together with the representation (8) for the functions L_{ij} may be useful also in such cases. By performing one of the integrations with the aid of (25) and representing all the logarithms in the form (8) one brings the function in a shape where the second integration can again be performed with the aid of (25). Proceeding in this way it is possible to write the function of such a Feynman diagram in terms of a ν-fold integral, where ν is the number of closed loops in the diagram and not the number of internal lines. In certain cases, this might be a convenient intermediate step in the evaluation of the integral.

[4] G. Källén, A. Wightman, Dan. Mat. Fys. Skr. 1, No. 6 (1958). Cf. also Ref. 3.

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Outlook

- **PHEGAS / HELAC: a framework for high-energy phenomenology**
- **Standard Model fully included**

⋆ **High color charge processes: multijet production**

P.Draggiotis, R.H.Kleiss and C.G.Papadopoulos, Phys. Lett. B439 (1998) 157;
Eur. Phys. J. C 24 (2002) 447 hep-ph/0202201
C. G. Papadopoulos and M. Worek, arXiv:hep-ph/0512150

⋆ **Higher order corrections**

- Direct approach. Ongoing work to better understand Dyson-Schwinger equations and loop calculations: stepping equations, recursive actions, etc.
- Running couplings and masses: 4-point FL contributions and BBC non-local approch to go beyond 4-fermion final states.

- **SUSY and new particles**

wait and see