Heavy quarkonium through a Quark-Gluon plasma

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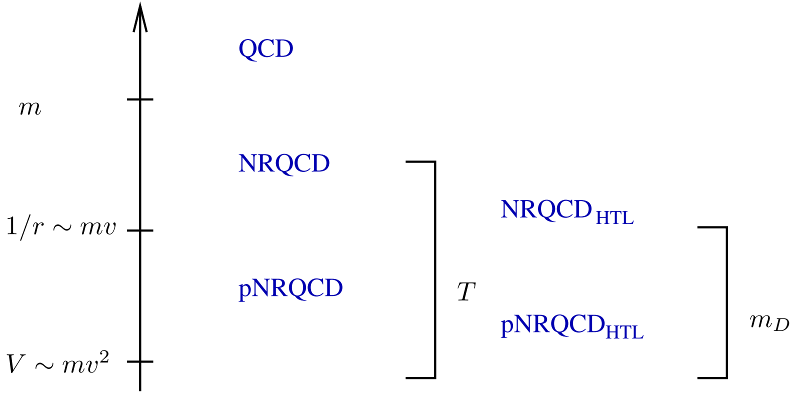
Work done in collaboration with Floriana Gianuzzi, Massimo Mannarrelli and Joan Soto. In preparation.

Outline

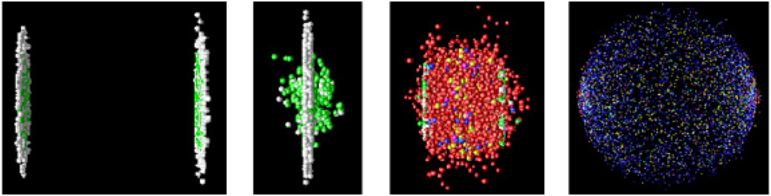
Introduction

EFT for bound states at finite temperature



Ideal conditions

- The EFTs for HQ at finite temperature and the imaginary part of the potential were obtained assuming thermal equilibrium and that the bound state is at rest.
- This is not what happens in heavy-ion collisions.



Relax this conditions

- Anisotropic plasma
- Quarkonium is moving
- ...

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- Anisotropic plasma
  Burnier, Laine and Vepsäläinen. Dumitru, Guo and Strickland. Philipsen and Tassler.
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Medium effects on a moving quarkonium

For a heavy quark in NRQCD

$$P^\mu = m_Q u^\mu + k^\mu$$

- $m_Q u^\mu$ information about the center of mass momentum. $u^2 = 1$ and $u = (1, 0, 0, 0)$ in the bound state rest frame.
- k^μ information about other properties, as for example the binding.

Medium effects on a moving quarkonium

For a heavy quark in NRQCD

$$P^\mu = m_Q u^\mu + k^\mu$$

- Medium may modify $m_Q u^\mu$. Heavy quark energy loss. Only happens when there is a finite momentum.
- Medium may modify k^μ. Existence or not of heavy quarkonium states. Happens in the comoving case but it may also be modified when there is a finite momentum.

General framework

We choose the frame where the bound state is at rest and the thermal bath is moving.

$$f(\beta^\mu k_\mu) = \frac{1}{e^{|\beta^\mu k_\mu|} \pm 1},$$

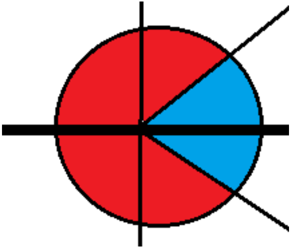
$$\beta^\mu = \frac{\gamma}{T}(1, \mathbf{v}) = \frac{u^\mu}{T},$$

We use a generalization of the real-time formalism called Non-equilibrium field theory (Zhou, Su, Han and Liu). At tree level substitute the equilibrium distribution functions by the non-equilibrium ones in the propagator.

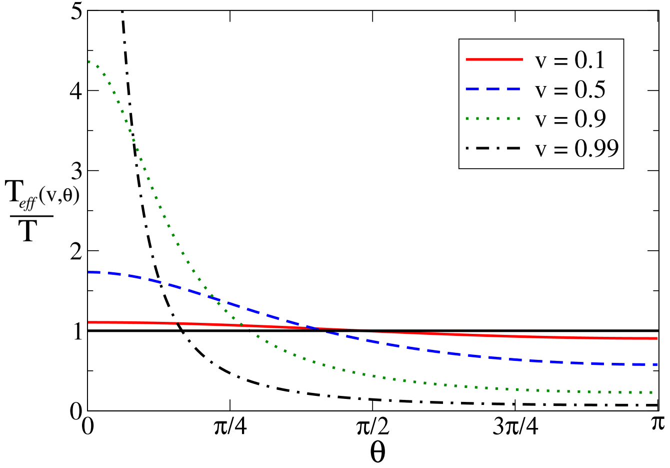
Massless particles

We can define an *effective temperature* depending on the incidence angle.

$$T_{\mathrm{eff}}(\theta, v) = \frac{T\sqrt{1-v^2}}{1 - v\cos\theta}\,.$$



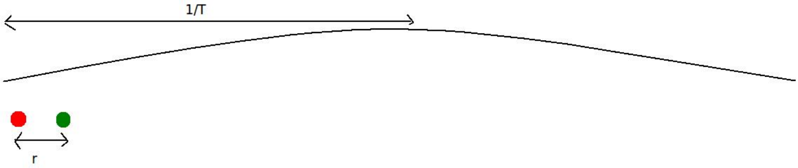
Effective temperature



$$1/r \gg T \gg E \gg m_D$$

The $1/r \gg T \gg E \gg m_D$ regime

- $1/r \gg T$. The medium sees heavy quarkonium as a color dipole.



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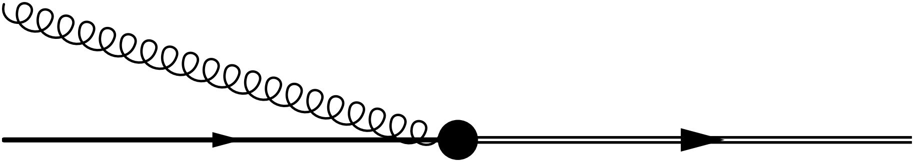
- $1/r \gg T$. The medium sees heavy quarkonium as a color dipole.
- Decay width is dominated by the process $HQ + g \to octet$. Cross-section does not depend on incidence angle. Decay width is expected to decrease with increasing velocity because for most angles $T_{eff} < T$.



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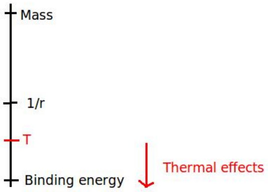
- Decay width is dominated by the process $HQ + g \to octet$. Cross-section does not depend on incidence angle. Decay width is expected to decrease with increasing velocity because for most angles $T_{eff} < T$.
- A QED study was previously done. M. A. E, Mannarelli and Soto.

EFT framework



- The starting point can be pNRQCD.
- Matching from pNRQCD to $pNRQCD_{HTL}$. Effects of the scale T are encoded in a modification of the potential.
- Computation of the scale E effects in $pNRQCD_{HTL}$. Modifications to the decay width and binding energy. Not necessary potential like.

Matching to $pNRQCD_{HTL}$. Modification of the potential.

All effects encoded in a modification of the potential.

$$\delta V_s = \frac{2\pi C_F \alpha_s T^2}{3m_Q} + \frac{\pi N_c C_F \alpha_s^2 T^2 r}{12}\left(\frac{4}{3} + f(v) - \frac{1}{3} + \frac{(\mathbf{r}\cdot\mathbf{v})^2}{r^2 v^2}(1 - 3f(v))\right)$$

where

$$f(v) = \frac{1}{v^3}\left(v(2 - v^2) - 2(1 - v^2)\tanh^{-1}(v)\right)$$

Computation in $pNRQCD_{HTL}$. Binding energy

$$\delta E_{nlm} = \frac{2\pi C_F T^2}{3}\left[\frac{\alpha_s}{m_Q} + \frac{N_c\alpha_s^2}{2}\langle r\rangle_{nlm} + \frac{N_c\alpha_s^2}{2}\langle r\rangle_{nlm}(1-3f(v))\langle 2l00|l0\rangle\langle 2l0m|lm\rangle\right]$$

where $\langle ll'mm'|lm\rangle$ are the Clebsch-Gordan coefficients.
In the s-wave case

$$\delta E_n^{s-wave} = \frac{2\pi C_F \alpha_s T^2}{3m_Q} + \frac{\pi N_c C_F \alpha_s^2 T^2 a_0 n^2}{6}$$

No momentum effects in the s-wave.

Computation in $pNRQCD_{HTL}$. Decay width

$$\Gamma_{nlm} = \frac{\alpha_s C_F T\sqrt{1-v^2}}{3v}\left[4\left(-\frac{2E_n^c}{m_Q} + \frac{\alpha_s N_c}{m_Q a_0^2 n^2} + \frac{\alpha_s^2 N_c^2}{8}\right)\log\left(\frac{1+v}{1-v}\right) + \right.$$
$$\left. + \left(-\frac{4E_n^c}{m_Q} - \frac{\alpha_s N_c}{m_Q a_0 n^2} + \frac{\alpha_s^2 N_c^2}{4}\right) h(v)\langle 2l00|l0\rangle\langle 2l0m|lm\rangle\right]$$

where

$$h(v) = \left[\left(1 - \frac{3}{v^2}\right)\log\left(\frac{1+v}{1-v}\right) + \frac{6}{v}\right]$$

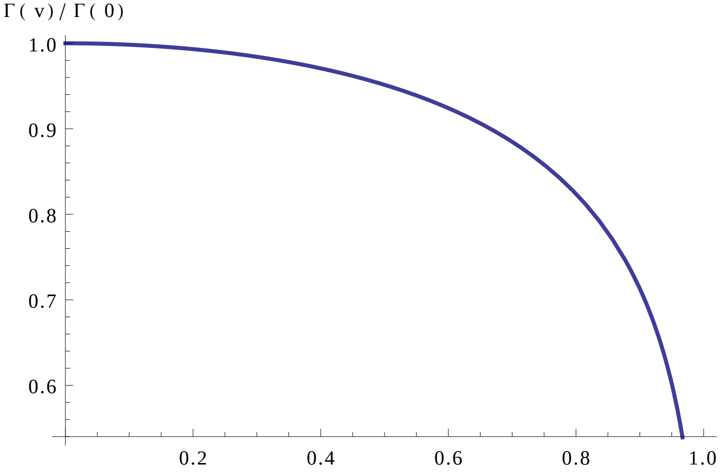
Computation in $pNRQCD_{HTL}$. Decay width

In the s-wave case

$$\Gamma_n^{s-wave} = \frac{4\alpha_s C_F T\sqrt{1-v^2}}{3v}\left(-\frac{2E_n^c}{m_Q} + \frac{\alpha_s N_c}{m_Q a_0 n^2} + \frac{\alpha_s^2 N_c^2}{8}\right)\log\left(\frac{1+v}{1-v}\right)$$

- Decreasing function with velocity.
- Goes to 0 as $v \to 1$.

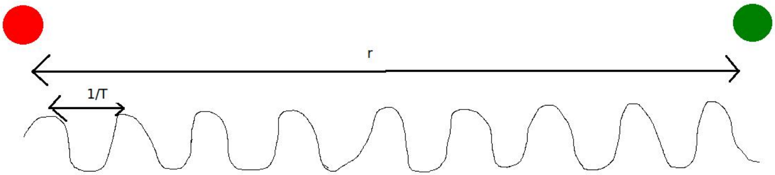
Computation in $pNRQCD_{HTL}$. Decay width



$$T \gg 1/r \gg m_D$$

The $T \gg 1/r \gg m_D$ regime

- Effects at the energy scale T are going to see heavy quarks as elements that are very far away from each other.



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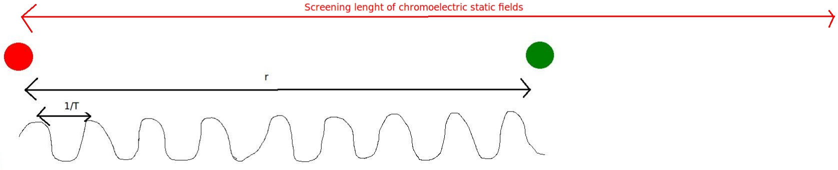
- Effects at the energy scale T are going to see heavy quarks as elements that are very far away from each other.
- Averaging this thermal fluctuations is going to introduce a screening of long distance fields (HTL). For those fields HQ behaves as a dipole.



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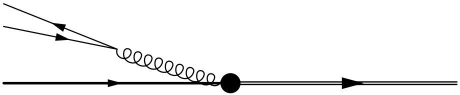
- Averaging this thermal fluctuations is going to introduce a screening of long distance fields (HTL). For those fields HQ behaves as a dipole.
- Decay width is dominated by the process $HQ + parton \rightarrow octet + parton$. The tri-momentum of the interchanged gluon is perpendicular to the one of the incident parton. No isotropic potential.



Modification of the potential

$$V = V_c + \delta V_r + \delta V_{m_D}$$

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Modification of the potential

$$V = V_c + \delta V_r + \delta V_{m_D}$$

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- δV_{m_D} is a correction coming from the scale m_D. Ultraviolet divergence $r^2 \log(m_D\mu)$.
- Deviations from Coulomb potential can be computed with quantum mechanical perturbation theory.

Decay width

$$\Gamma_{1s} = -2\langle n00|\Im V_s(r)|n00\rangle = -\int d^3r|\psi_n(r)|^2 \int \frac{d^3k}{(2\pi)^3}(e^{i\mathbf{k}\cdot\mathbf{r}}-1)\Delta_S(\mathbf{k},\mathbf{v})$$

- Thermal effects are encoded in the symmetric propagator. Complicated form.
- An analytical approximation can be obtained for moderate velocities and one only keeps the logarithmically enhanced term.

Decay width. Approximation

$$\Gamma_{1s} \sim \frac{2\alpha_s C_F T m_D^2 a_0^2}{\sqrt{1-v^2}} \log\left(\frac{2}{m_D a_0}\right)$$

or, equivalently,

$$\frac{\Gamma_{1s}(v)}{\Gamma_{1s}(v=0)} \sim \frac{1}{\sqrt{1-v^2}}$$

For moderate velocities. Dissociation increases with velocity.

Modification of the dissociation temperature

Temperature in which the imaginary part is not a perturbation any more.
For $v = 0$. (M. A. E and Soto, Laine)

$$T^{diss} \sim g^{4/3} m_Q$$

For finite v, using previous approximation

$$T^{diss}(v) = T^{diss}(0)(1 - v^2)^{1/6}$$

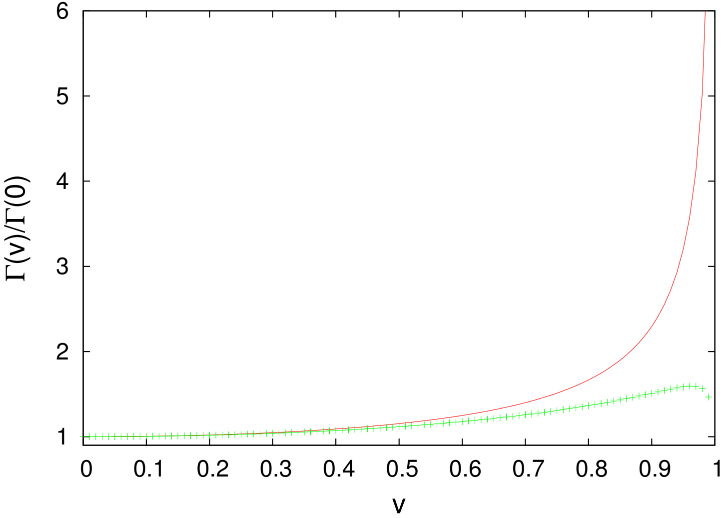
Numerical computation

$$\Gamma_{1s} = -2\langle n00|\Im V_s(r)|n00\rangle = -\int d^3r|\psi_n(r)|^2\int \frac{d^3k}{(2\pi)^3}(e^{i\mathbf{k}\cdot\mathbf{r}}-1)\Delta_S(\mathbf{k},\mathbf{v})$$

can also be computed numerically.
Valid for all velocities as long as the imaginary part can be considered a perturbation.

Numerical computation

$T = 250$ MeV



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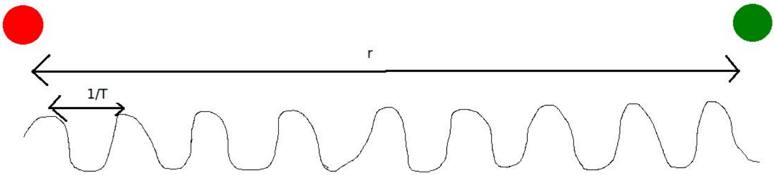
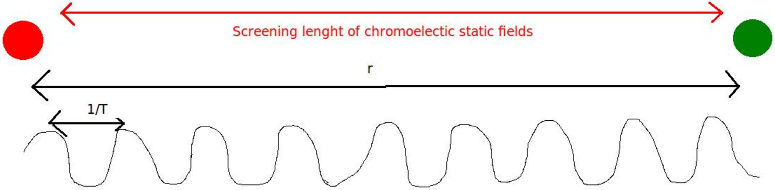


The $T \gg 1/r \sim m_D$ regime

- Effects at the energy scale T are going to see heavy quarks as elements that are very far away from each other.
- Now the screening length is of the same order as the size of the bound state.



The real part of the potential, normalization

At $v = 0$

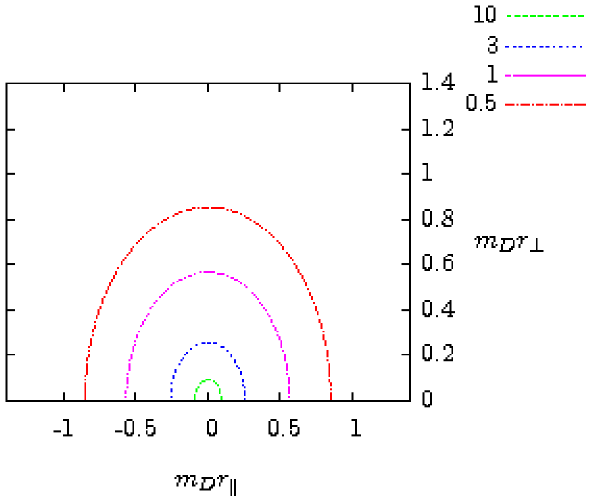
$$Re\,V(r) = -\frac{4\alpha_s e^{-m_D r}}{3r} = -\frac{4\alpha_s C_F m_D g(m_D r)}{3}$$

at any velocity we can define

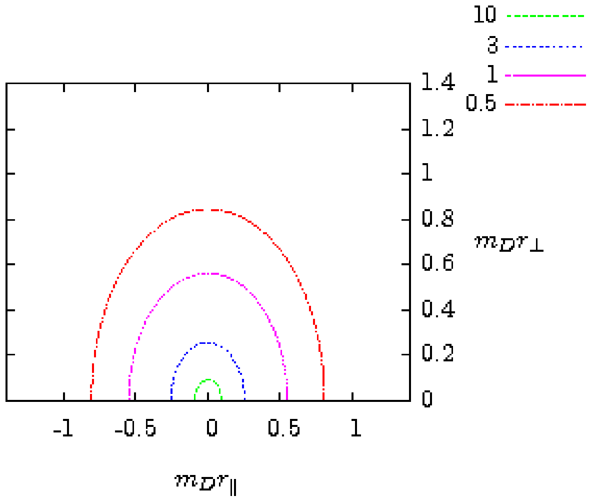
$$g(m_D r) = -\frac{3ReV(r)}{4\alpha_s m_D}$$

$g(x)$ does not depend on T, it is useful to compare the same T with different v. This is what we are going to plot.
Computed by Matsui and Chu.

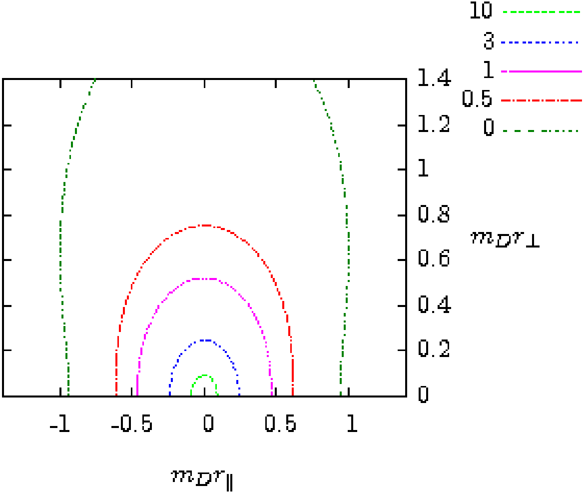
The real part of the potential at $v = 0$



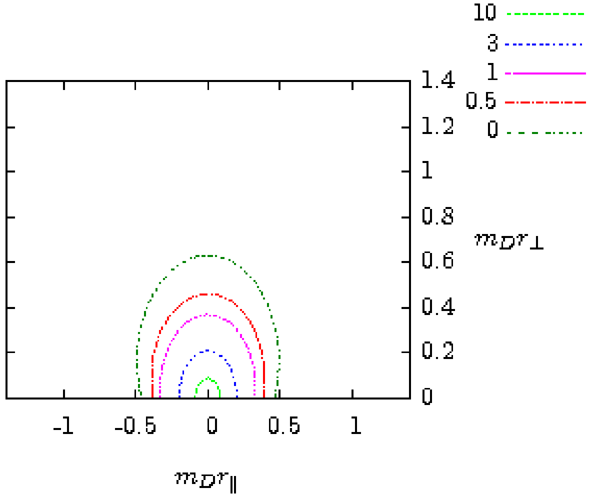
The real part of the potential at $v = 0.5$



The real part of the potential at $v = 0.9$



The real part of the potential at $v = 0.99$



The imaginary part of the potential at $v = 0$

$$Im\,V(r) = V_S(r) = -\frac{4\alpha_s T \phi(m_D r)}{3},$$

with

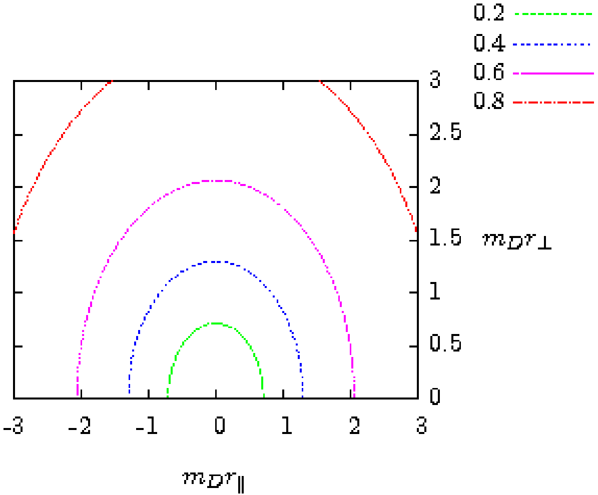
$$\phi(x) = 2\int_0^\infty \frac{dzz}{(z^2+1)^2}\left(1 - \frac{\sin(zx)}{zx}\right).$$

(Laine, Philipsen, Romatschke and Tassler). At any velocity we can define

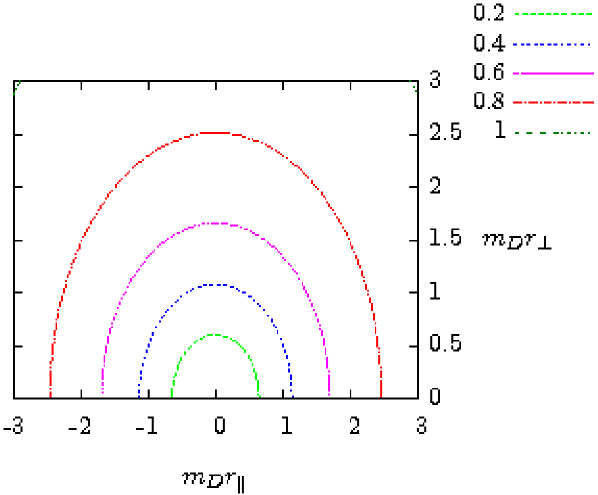
$$\phi(m_D r) = -\frac{3 Im\,V(r)}{4\alpha_s T}.$$

This is what we are going to plot. (Computed for muonic hydrogen in M.A.E, Mannarelli and Soto).

The imaginary part of the potential at $v = 0$



The imaginary part of the potential at $v = 0.5$



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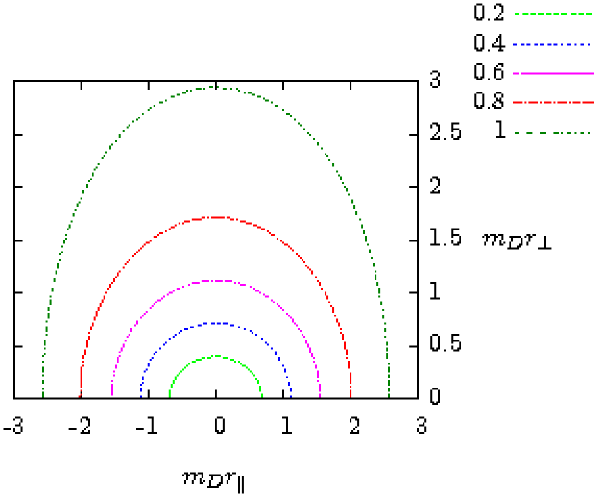

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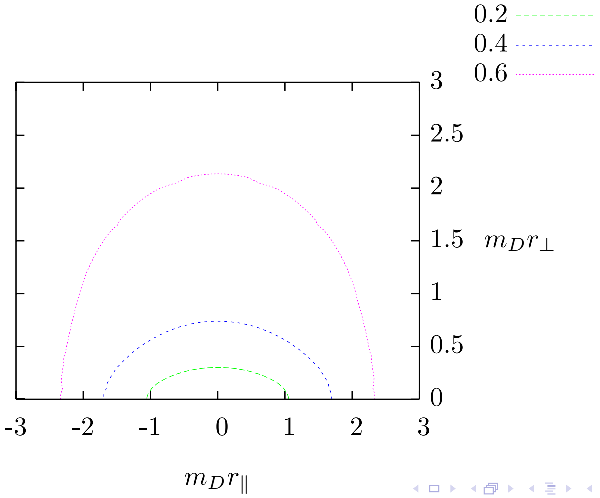


Spectral function

- Allow to do more quantitative statements about dissociation.
- Directly related with dilepton production.
- Allow to compare result with lattice computations.

Computation at $v = 0$ was already available (Laine (2007)). For finite v not spherical symmetry, only cylindrical symmetry.

Running coupling constant

The potential can be written as

$$V(r) = \frac{\alpha_s(\mu_1)}{r} f(m_D(\mu_2) r)$$

In the original $v = 0$ computation $\mu_1 \sim 2\pi T$ and $\mu_2 \sim 2\pi T$.
We call this choice running 1.

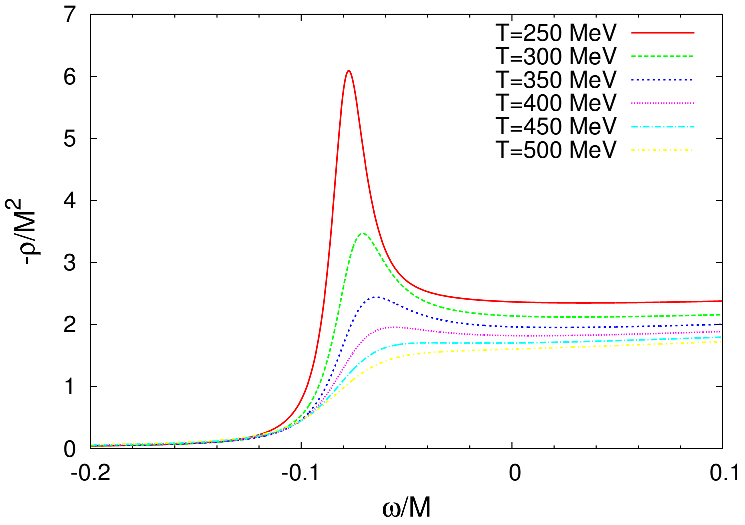
Running coupling constant

The potential can be written as

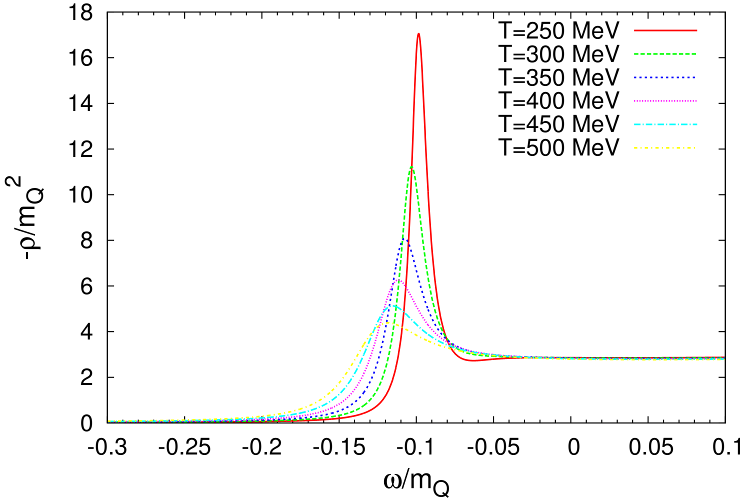
$$V(r) = \frac{\alpha_s(\mu_1)}{r} f(m_D(\mu_2) r)$$

In pNRQCD at $T = 0$ one uses $\mu_1 = 1/r$ or $\mu_1 = 1/a_0$.
Use $\mu_1 = 1/a_0$ and $\mu_2 = 2\pi T$.
We call this choice running 2. Used at $v = 0$ to determine dissociation temperature for $\Upsilon(1S)$. M.A.E and Soto.

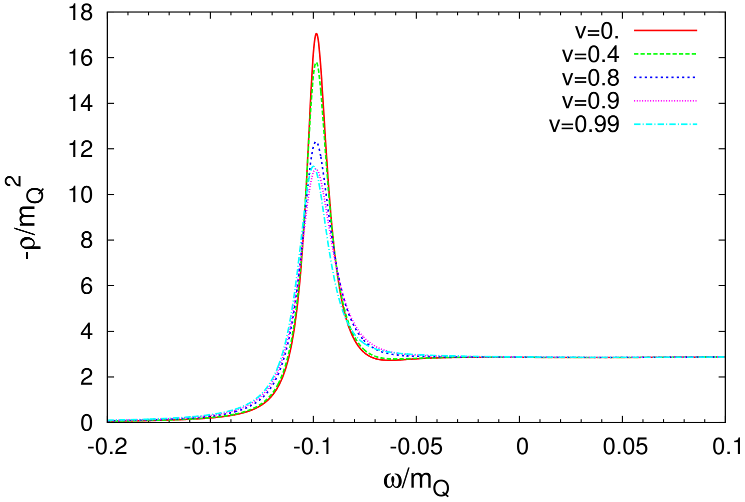
Spectral function at $v = 0$. Running 1



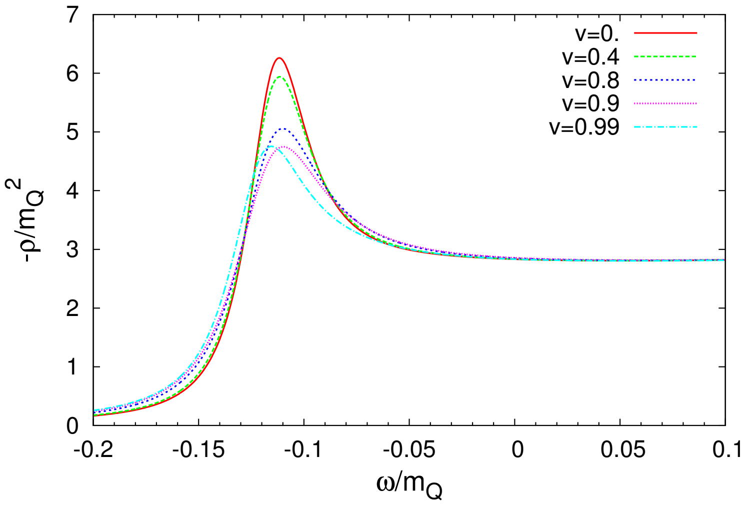
Spectral function at $v = 0$. Running 2



Spectral function at $T = 250$ MeV



Spectral function at $T = 400$ MeV



Comparisons and conclusions.

Comparison with Lattice QCD

- In the regime $1/r \gg T \gg E \gg m_D$ there are computations to which we can compare (Aarts, Allton, Kim, Lombardo, Oktay, Ryan, Sinclair, Skullerud (2012)), but the velocity is small to see important effects.
- In the regime $T \gg 1/r \sim m_D$ (Nonaka, Asakawa, Kitazawa and Kohno (2011)). However, a more precise statistical analysis is needed to determine the width of the peaks.(Also work presented yesterday by Skullerud, but we were not aware).

Comparison with lattice computations. $1/r \gg T \gg E \gg m_D$

Aarts, Allton, Kim, Lombardo, Oktay, Ryan, Sinclair, Skullerud (2012).
Plasma rest frame.

- They observe no additional effects due to the finite momentum.
- Results are still compatible with our predictions because the maximum velocity achieved in the lattice still fulfils $v \ll 1$.
- If our prediction is right an important effect should be observed for $v > 0.5$.

Comparison with lattice computations. $T \gg 1/r \gg m_D$

(Nonaka, Asakawa, Kitazawa and Kohno (2011))
To do the comparison

- Assume peaks to be of Breit-Wigner form, extract width and transform them to the plasma rest frame.
- Assume that $1/r \gg m_D$ so that the imaginary part of the potential is a perturbation.

Comparison with lattice computations. $T \gg 1/r \gg m_D$

(Nonaka, Asakawa, Kitazawa and Kohno (2011))

p	v	Γ_{plot} (MeV)	Γ_{pred} (MeV)
0	0	106	X
6	0.6	135	132
7	0.65	134	139
8	0.67	128	142

Comparison with AdS/CFT computations

- There exist several computations (Ejaz, Faulkner, Liu, Rajagopal and Wiedemann (2007), Chernicoff, Fernandez, Mateos and Trancanelli (2012)).
- They focus only in the real part.
- Screening increases with velocity.
- Isotropy.

Conclusions

- In the regime where gluo-dissociation dominates the decay width decreases with velocity.
- In the regime relevant for dissociation and for moderate velocities dissociation increases with velocity, this is also what is found in AdS/CFT.
- In the regime relevant for dissociation and for very large velocities the width decreases with velocity.
- We confirm that for very large velocities modifications of the real part of the potential are very important.