# Optimizing the basis of $B \rightarrow K^{*} I^{+} I^{-}$observables and understanding its tensions 

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Based on: S. Descotes-Genon, T. Hurth, JM, J. Virto, JHEP 1305 (2013) 137<br>S. Descotes-Genon, JM, J. Virto, in preparation

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## PLAN of the TALK

- Why is so important the measurement of $B \rightarrow K^{*}(\rightarrow K \pi) I^{+} I^{-}$?
- The path towards an optimized basis of observables to describe this 4-body decay.
- First analysis of new data on $P_{1,2}$ and understanding of its tensions (3 $\sigma$ ).
- Conclusions
$\Rightarrow$ In the short term the best paradigm to unveil New Physics will be an accurate analysis of Wilson coefficients.
- UT for CPV $\leftrightarrow$ Wilson Coefficient correlations for Rare Decays
- Wilson Coefficients are tested $C_{i}=C_{i}^{S M}+\delta \mathbf{C}_{\mathbf{i}}\left\{\begin{array}{l}\text { different levels of accuracy } \\ \text { allow different ranges of NP }\end{array}\right.$

Wilson coefficients

$$
\begin{aligned}
& \mathbf{C}_{7}^{\text {eff }}\left(\mu_{\mathbf{b}}\right) \\
& \mathbf{C}_{9}\left(\mu_{\mathbf{b}}\right) \\
& \mathbf{C}_{10}\left(\mu_{\mathbf{b}}\right) \\
& \mathbf{C}_{7}^{\prime}\left(\mu_{\mathbf{b}}\right) \\
& \mathbf{C}_{9}^{\prime}\left(\mu_{\mathbf{b}}\right) \\
& \mathbf{C}_{10}^{\prime}\left(\mu_{\mathbf{b}}\right)
\end{aligned}
$$

## Observables

$\mathcal{B}\left(\bar{B} \rightarrow X_{s} \gamma\right), A_{l}\left(B \rightarrow K^{*} \gamma\right), S_{K^{*} \gamma}, A_{F B}, F_{L}$
$\mathcal{B}\left(B \rightarrow X_{s} \ell \ell\right), A_{F B}, F_{L}$
$\mathcal{B}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right), \mathcal{B}\left(B \rightarrow X_{s} \ell \ell\right), A_{F B}, F_{L} \quad-4.308$
$\mathcal{B}\left(\bar{B} \rightarrow X_{s} \gamma\right), A_{l}\left(B \rightarrow K^{*} \gamma\right), S_{K^{*} \gamma}, A_{F B}, F_{L} \quad-0.006$
$\mathcal{B}\left(B \rightarrow X_{s} \ell \ell\right), A_{F B}, F_{L}$
$\mathcal{B}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right), A_{F B}, F_{L}$
4.075

SM values

$$
-0.292
$$

$$
\begin{equation*}
-0.006 \tag{0}
\end{equation*}
$$

0

High Precision Observables are necessary to disentangle NP and to overconstrain the deviations $\delta \mathbf{C}_{\mathbf{i}}$ of Wilson Coefficients from SM in order to reduce allowed regions.
$\Rightarrow$ In the short term the best paradigm to unveil New Physics will be an accurate analysis of Wilson coefficients.

- UT for CPV $\leftrightarrow$ Wilson Coefficient correlations for Rare Decays
- Wilson Coefficients are tested $C_{i}=C_{i}^{S M}+\delta \mathbf{C}_{\mathbf{i}}\left\{\begin{array}{l}\text { different levels of accuracy } \\ \text { allow different ranges of NP }\end{array}\right.$


## Wilson coefficients

## Observables

## SM values

$$
\begin{array}{lcr}
\mathbf{C}_{7}^{\text {eff }}\left(\mu_{\mathbf{b}}\right) & \mathcal{B}\left(\bar{B} \rightarrow X_{s} \gamma\right), A_{l}\left(B \rightarrow K^{*} \gamma\right), S_{K^{*} \gamma}, A_{F B}, F_{L}, P_{2}, P_{4,5}^{\prime} & -0.292 \\
\mathbf{C}_{9}\left(\mu_{\mathbf{b}}\right) & \mathcal{B}\left(B \rightarrow X_{s} \ell \ell\right), A_{F B}, F_{L}, P_{2}, P_{4,5}^{\prime} & 4.075 \\
\mathbf{C}_{10}\left(\mu_{\mathbf{b}}\right) & \mathcal{B}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right), \mathcal{B}\left(B \rightarrow X_{\mathbf{s}} \ell \ell\right), A_{F B}, F_{L}, P_{4}^{\prime} & -4.308 \\
\mathbf{C}_{7}^{\prime}\left(\mu_{\mathbf{b}}\right) & \mathcal{B}\left(\bar{B} \rightarrow X_{\mathbf{s}} \gamma\right), A_{l}\left(B \rightarrow K^{*} \gamma\right), S_{K^{*} \gamma}, A_{F B}, F_{L}, P_{1} & -0.006 \\
\mathbf{C}_{9}^{\prime}\left(\mu_{\mathbf{b}}\right) & \mathcal{B}\left(B \rightarrow X_{s} \ell \ell\right), A_{F B}, F_{L}, P_{1} & 0 \\
\mathbf{C}_{10}^{\prime}\left(\mu_{\mathbf{b}}\right) & \mathcal{B}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right), A_{F B}, F_{L}, P_{1}, P_{4}^{\prime} & 0
\end{array}
$$

High Precision Observables are necessary to disentangle NP and to overconstrain the deviations $\delta \mathbf{C}_{\mathbf{i}}$ of Wilson Coefficients from SM in order to reduce allowed regions.
$\Rightarrow B \rightarrow K^{*}(\rightarrow K \pi) \mu^{+} \mu^{-}$fulfills the requirements by means of clean observables $P_{1,2,3}, P_{4,5,6,8}^{\prime}$ improving the precision in not very accurately constrained coefficients like $C_{9}$ or $C_{7,9,10}^{\prime}$ (soon). New Physics in phases of Wilson Coefficients: $P_{3}, P_{6,8}^{\prime}$.

All those observables come from the decay $\overline{\mathbf{B}}_{\mathbf{d}} \rightarrow \overline{\mathbf{K}}^{* 0}\left(\rightarrow \mathbf{K}^{-} \pi^{+}\right) \mathbf{I}^{+} \mathbf{I}^{-}$with the $K^{* 0}$ on the mass shell. It is described by $s=q^{2}$ and three angles $\theta_{\mathrm{I}}, \theta_{\mathrm{K}}$ and $\phi$

$$
\frac{d^{4} \Gamma\left(\bar{B}_{d}\right)}{d q^{2} d \cos \theta_{l} d \cos \theta_{K} d \phi}=\frac{9}{32 \pi} J\left(q^{2}, \theta_{l}, \theta_{K}, \phi\right)
$$

The differential distribution splits in $J_{i}$ coefficients:

$$
\begin{gathered}
J\left(q^{2}, \theta_{l}, \theta_{K}, \phi\right)= \\
J_{1 s} \sin ^{2} \theta_{K}+J_{1 c} \cos ^{2} \theta_{K}+\left(J_{2 s} \sin ^{2} \theta_{K}+J_{2 c} \cos ^{2} \theta_{K}\right) \cos 2 \theta_{l}+J_{3} \sin ^{2} \theta_{K} \sin ^{2} \theta_{l} \cos 2 \phi \\
+J_{4} \sin 2 \theta_{K} \sin 2 \theta_{l} \cos \phi+J_{5} \sin 2 \theta_{K} \sin \theta_{l} \cos \phi+\left(J_{6 s} \sin ^{2} \theta_{K}+J_{6 c} \cos ^{2} \theta_{K}\right) \cos \theta_{l} \\
+J_{7} \sin 2 \theta_{K} \sin \theta_{l} \sin \phi+J_{8} \sin 2 \theta_{K} \sin 2 \theta_{l} \sin \phi+J_{9} \sin ^{2} \theta_{K} \sin ^{2} \theta_{l} \sin 2 \phi .
\end{gathered}
$$

There is a corresponding $C P$ - conjugate distribution for $\mathbf{B}_{\mathbf{d}} \rightarrow \mathbf{K}^{* \mathbf{0}}\left(\rightarrow \mathbf{K}^{-} \pi^{+}\right) \mathbf{I}^{+} \mathbf{I}^{-}$function of $\bar{J}$.
The information on

- the transversity amplitudes of the $K^{*}\left(A_{\perp, \|, 0}\right)$ is inside the coefficients $J_{i}$.
- short distance physics $C_{i}$ is encoded in $\left(A_{\perp, \|, 0}=C_{i} \times\right.$ form factors $)$

$$
\begin{aligned}
& J_{1 s}=\frac{\left(2+\beta_{\ell}^{2}\right)}{4}\left[\left|A_{\perp}^{L}\right|^{2}+\left|A_{\|}^{L}\right|^{2}+(L \rightarrow R)\right]+\frac{4 m_{\ell}^{2}}{q^{2}} \operatorname{Re}\left(A_{\perp}^{L} A_{\perp}^{R^{*}}+A_{\|}^{L} A_{\|}^{R^{*}}\right), \\
& J_{1 c}=\left|A_{0}^{L}\right|^{2}+\left|A_{0}^{R}\right|^{2}+\frac{4 m_{\ell}^{2}}{q^{2}}\left[\left|A_{t}\right|^{2}+2 \operatorname{Re}\left(A_{0}^{L} A_{0}^{R^{*}}\right)\right]+\beta_{\ell}^{2}\left|A_{S}\right|^{2}, \\
& J_{2 s}=\frac{\beta_{\ell}^{2}}{4}\left[\left|A_{\perp}^{L}\right|^{2}+\left|A_{\|}^{L}\right|^{2}+(L \rightarrow R)\right], \quad J_{2 c}=-\beta_{\ell}^{2}\left[\left|A_{0}^{L}\right|^{2}+(L \rightarrow R)\right], \\
& J_{3}=\frac{1}{2} \beta_{\ell}^{2}\left[\left|A_{\perp}^{L}\right|^{2}-\left|A_{\|}^{L}\right|^{2}+(L \rightarrow R)\right], \quad J_{4}=\frac{1}{\sqrt{2}} \beta_{\ell}^{2}\left[\operatorname{Re}\left(A_{0}^{L} A_{\|}^{L *}\right)+(L \rightarrow R)\right], \\
& J_{5}=\sqrt{2} \beta_{\ell}\left[\operatorname{Re}\left(A_{0}^{L} A_{\perp}^{L}{ }^{*}\right)-(L \rightarrow R)-\frac{m_{\ell}}{\sqrt{q^{2}}} \operatorname{Re}\left(A_{\|}^{L} A_{S}^{*}+A_{\|}^{R} A_{S}^{*}\right)\right], \\
& J_{6 s}=2 \beta_{\ell}\left[\operatorname{Re}\left(A_{\|}^{L} A_{\perp}^{L *}\right)-(L \rightarrow R)\right], \quad J_{6 c}=4 \beta_{\ell} \frac{m_{\ell}}{\sqrt{q^{2}}} \operatorname{Re}\left[A_{0}^{L} A_{S}^{*}+(L \rightarrow R)\right], \\
& J_{7}=\sqrt{2} \beta_{\ell}\left[\operatorname{lm}\left(A_{0}^{L} A_{\|}^{L *}\right)-(L \rightarrow R)+\frac{m_{\ell}}{\sqrt{q^{2}}} \operatorname{lm}\left(A_{\perp}^{L} A_{S}^{*}+A_{\perp}^{R} A_{S}^{*}\right)\right], \\
& J_{8}=\frac{1}{\sqrt{2}} \beta_{\ell}^{2}\left[\operatorname{lm}\left(A_{0}^{L} A_{\perp}^{L *}\right)+(L \rightarrow R)\right], \quad J_{9}=\beta_{\ell}^{2}\left[\operatorname{lm}\left(A_{\|}^{L *} A_{\perp}^{L}\right)+(L \rightarrow R)\right]
\end{aligned}
$$

In red lepton mass terms $\left(\beta_{\ell}^{2}=1-4 m_{\ell}^{2} / q^{2}\right)$.

An important step forward to find a complete description of the distribution was the identification of the symmetries of the distribution:

Transformation of amplitudes leaving distribution invariant.
Symmetries determine the minimal \# observables for each scenario:

$$
n_{o b s}=2 n_{A}-n_{S}
$$

| Case | Coefficients | Amplitudes | Symmetries | Observables |
| :---: | :---: | :---: | :---: | :---: |
| $m_{\ell}=0, A_{S}=0$ | 11 | 6 | 4 | $\mathbf{8} \Leftarrow$ |
| $m_{\ell}=0$ | 11 | 7 | 5 | $\mathbf{9}$ |
| $m_{\ell}>0, A_{S}=0$ | 11 | 7 | 4 | $\mathbf{1 0}$ |
| $m_{\ell}>0$ | 12 | 8 | 4 | $\mathbf{1 2}$ |

All symmetries (massive and scalars) were found explicitly later on. [JM, Mescia, Ramon, Virto'12]
Symmetries $\Rightarrow$ \# of observables $\Rightarrow$ determine a basis: each angular observable constructed can be expressed in terms of this basis.
Main criteria to define this basis: minimize the form factor sensitivity

For a long time huge efforts were devoted (still now) to measure the position of the zero of the forwardbackward asymmetry $A_{F B}$ of $B \rightarrow K^{*} \mu^{+} \mu^{-}$.


Reason:

- At LO the soft form factor dependence $\left(\xi_{\perp}\left(q^{2}\right), \xi_{\|}\left(q^{2}\right)\right)$ cancels exactly at the position of the zero $q_{0}^{2}$ (dependence appears at NLO).
- A relation among $\mathbf{C}_{9}^{\text {eff }}$ and $\mathbf{C}_{7}^{\text {eff }}$ arises at the zero (at LO):

$$
\mathbf{C}_{9}^{\text {eff }}\left(q_{0}^{2}\right)+2 \frac{m_{b} M_{B}}{q_{0}^{2}} \mathbf{C}_{7}^{\text {eff }}=0
$$

A similar idea was incorporated in the construction of the transverse asymmetry
[Kruger, J.M'05]

$$
P_{1}=A_{T}^{(2)}\left(q^{2}\right)=\frac{\left|A_{\perp}\right|^{2}-\left|A_{\|}\right|^{2}}{\left|A_{\perp}\right|^{2}+\left|A_{\| \mid}\right|^{2}}
$$

[Becirevic et al.'12]

$$
P_{2}=\frac{A_{T}^{r e}}{2}=\frac{\operatorname{Re}\left(A_{\perp}^{L *} A_{\|}^{L}-A_{\perp}^{R} A_{\|}^{R *}\right)}{\left|A_{\perp}\right|^{2}+\left|A_{\| \mid}\right|^{2}}
$$

where $A_{\perp, \|}$ correspond to two transversity amplitudes of the $K^{*}$.


- Both asymmetries exhibits an exact cancellation of soft form factors not only at a point (like $A_{F B}$ ) but in the full low- $q^{2}$ range $\left(0.1-6 \mathrm{GeV}^{2}\right)$.
- First examples of clean observables that could be measured.
- $A_{T}^{(2)}$ is constructed to detect presence of RH currents ( $A_{\perp} \sim-A_{\| \mid}$in the SM), $A_{T}^{r e}$ complements (partly supersedes) $A_{F B}$ since it contains similar information, but in a theoretically better controlled way.
- Later on a set of transverse asymmetries called $\mathbf{A}_{\mathrm{T}}^{(3,4,5)}$ were proposed

$$
\mathbf{A}_{\mathrm{T}}^{(3)}=\frac{\left|A_{0}^{L} A_{\|}^{L *}+A_{0}^{R *} A_{\|}^{R}\right|}{\sqrt{\left|A_{0}\right|^{2}\left|A_{\perp}\right|^{2}}} \quad \mathbf{A}_{\mathrm{T}}^{(4)}=\frac{\left|A_{0}^{L} A_{\perp}^{L *}-A_{0}^{R *} A_{\perp}^{R}\right|}{\left|A_{0}^{L} A_{\|}^{L *}+A_{0}^{R *} A_{\|}^{R}\right|} \quad \mathbf{A}_{\mathbf{T}}^{(5)}=\frac{\left|A_{\perp}^{L} A_{\|}^{R *}+A_{\perp}^{R *} A_{\|}^{L}\right|}{\left|A_{\perp}\right|^{2}+\left|A_{\|}\right|^{2}}
$$

[Bobeth, Hiller, Dyk,'10]

- Also at the low-recoil a set of clean observables called $\mathbf{H}_{\mathrm{T}}^{(\mathbf{1 , 2 , 3})}$ were proposed that correspond to $P_{4,5,6}$ at large-recoil.

$$
\mathbf{H}_{\mathrm{T}}^{(1)}=\frac{\operatorname{Re}\left(A_{0}^{L} A_{\|}^{L *}+A_{0}^{R *} A_{\|}^{R}\right)}{\sqrt{\left|A_{0}\right|^{2}\left|A_{\|}\right|^{2}}}, \mathbf{H}_{\mathrm{T}}^{(2)}=\frac{\operatorname{Re}\left(A_{0}^{L} A_{\perp}^{L *}-A_{0}^{R *} A_{\perp}^{R}\right)}{\sqrt{\left|A_{0}\right|^{2}\left|A_{\perp}\right|^{2}}}, \mathbf{H}_{\mathrm{T}}^{(3)}=\frac{\operatorname{Re}\left(A_{\|}^{L} A_{\perp}^{L *}-A_{\|}^{R *} A_{\perp}^{R}\right)}{\sqrt{\left|A_{\|}\right|^{2}\left|A_{\perp}\right|^{2}}}
$$

[Altmannshofer, Ball, Bharucha, Buras, Straub, Wick'09] - In parallel a set of CP-conserving and CP-violating observables $\mathbf{S}_{\mathbf{i}}$ and $\mathbf{A}_{\mathbf{i}}$ were constructed directly from the coefficients of the distribution, easy to measure but not following the criteria of clean observables:

$$
\mathbf{S}_{\mathbf{i}}=\frac{\int_{b i n} d q^{2}\left[J_{i}+\bar{J}_{i}\right]}{d \Gamma / d q^{2}+d \bar{\Gamma} / d q^{2}}, \quad \mathbf{A}_{\mathbf{i}}=\frac{\int_{b i n} d q^{2}\left[J_{i}-\bar{J}_{i}\right]}{d \Gamma / d q^{2}+d \bar{\Gamma} / d q^{2}}
$$

Finally we arrived to an Optimal Basis of observables, a compromise between:

- Excellent experimental accessibility and simplicity of the fit.
- Reduced FF dependence (in the large-recoil region: $0.1 \leq q^{2} \leq 8 \mathrm{GeV}^{2}$ ).

Our proposal for CP-conserving basis:

$$
\left\{\frac{\mathrm{d} \Gamma}{\mathrm{dq}}, \mathbf{A}_{\mathrm{FB}}, \mathbf{P}_{\mathbf{1}}, \mathbf{P}_{2}, \mathbf{P}_{\mathbf{3}}, \mathbf{P}_{4}^{\prime}, \mathbf{P}_{5}^{\prime}, \mathbf{P}_{6}^{\prime}\right\} \text { or } \mathbf{P}_{3} \leftrightarrow \mathbf{P}_{8}^{\prime} \text { and } \mathbf{A}_{\mathrm{FB}} \leftrightarrow \mathbf{F}_{\mathrm{L}}
$$

where $P_{1}=A_{T}^{2}$ [Kruger, J.M'05], $P_{2}=\frac{1}{2} A_{T}^{\text {re }}, P_{3}=-\frac{1}{2} A_{T}^{\mathrm{im}}$ [Becirevic, Schneider'12] and $P_{4,5,6}^{\prime}$ [Descotes, JM, Ramon, Virto'13]) given by

$$
P_{i}^{\prime}=\frac{1}{k_{i} N_{i}}\left[J_{i}+\bar{J}_{i}\right] \quad N_{i}=\sqrt{-\left(J_{2 s}+\bar{J}_{2 s}\right)\left(J_{2 c}+\bar{J}_{2 c}\right)} \quad k_{4}=1, k_{5}=2, k_{6}=-2
$$

and the corresponding CP-violating basis $\left(J_{i}+\bar{J}_{i} \rightarrow J_{i}-\bar{J}_{i}\right.$ in numerators):

$$
\left\{\mathbf{A}_{\mathrm{CP}}, \mathbf{A}_{\mathrm{FB}}^{\mathrm{CP}}, \mathbf{P}_{1}^{\mathrm{CP}}, \mathbf{P}_{2}^{\mathrm{CP}}, \mathbf{P}_{3}^{\mathrm{CP}}, \mathbf{P}_{4}^{\prime \mathrm{CP}}, \mathbf{P}_{5}^{\prime \mathrm{CP}}, \mathbf{P}_{6}^{\prime \mathrm{CP}}\right\} \text { or } \mathbf{P}_{3}^{\mathrm{CP}} \leftrightarrow \mathbf{P}_{8}^{\prime \mathrm{CP}} \text { and } \mathbf{A}_{\mathrm{FB}}^{\mathrm{CP}} \leftrightarrow \mathbf{F}_{\mathrm{L}}^{\mathrm{CP}}
$$

Large-recoil: NLO QCDfactorization $+\mathcal{O}\left(\Lambda / m_{b}\right)$. Soft form factors $\xi_{\perp, \|}\left(q^{2}\right)$ from

$$
\xi_{\perp}\left(q^{2}\right)=m_{B} /\left(m_{B}+m_{K^{*}}\right) \mathbf{V}\left(\mathbf{q}^{2}\right) \quad \xi_{\|}\left(q^{2}\right)=\left(m_{B}+m_{K^{*}}\right) /(2 E) \mathbf{A}_{1}\left(\mathbf{q}^{2}\right)-\left(m_{B}-m_{K^{*}}\right) /\left(m_{B}\right) \mathbf{A}_{2}\left(\mathbf{q}^{2}\right)
$$

- FF at $q^{2}=0$ and slope parameters are computed by [Khodjamirian et al.'10] (KMPW) using LCSR.

Tensor form factors $\mathcal{T}_{\perp, \|}$ are computed in QCDF following [Beneke, Feldmann, Seidel'01,'05] including factorizable and non-factorizable contributions.

Low-recoil: LCSR are valid up to $q^{2} \leq 14 \mathrm{GeV}^{2}$. We extend FF determination [Bobeth \& Hiller \& Dyk'10] till $19 \mathrm{Gev}^{2}$ and cross check the consistency with lattice QCD.
In HQET one expects the ratios to be near one

$$
\mathbf{R}_{1}=\frac{\mathbf{T}_{1}\left(\mathbf{q}^{2}\right)}{\mathbf{V}\left(\mathbf{q}^{2}\right)}, \quad \mathbf{R}_{2}=\frac{\mathbf{T}_{2}\left(\mathbf{q}^{2}\right)}{\mathbf{A}_{1}\left(\mathbf{q}^{2}\right)}, \quad \mathbf{R}_{3}=\frac{q^{2}}{m_{B}^{2}} \frac{T_{3}\left(q^{2}\right)}{A_{2}\left(q^{2}\right)}
$$

Our approach at low-recoil: we determine $T_{1,2}$ by exploiting the ratios $R_{1,2}$ allowing for up to a $20 \%$ breaking, i.e., $R_{1,2}=1+\delta_{1,2}$. All other form factors extrapolated from KMPW. We find perfect agreement between our determination of $T_{1,2}$ using $R_{1,2}$ and lattice data.

Contact between theory and experiment:
Indeed the observables are measured in bins.
Present bins: $[0.1,2],[2,4.3],[4.3,8.68],[1,6],[14.18,16],[16,19] \mathrm{GeV}^{2}$.
This requires a redefinition of observables in bins: $\left\langle J_{i}\right\rangle_{\text {bin }}=\int_{\text {bin }}\left[J_{i}+\overline{J_{i}}\right] d q^{2}$

$$
\begin{gathered}
\left\langle A_{T}^{(2)}\right\rangle_{\text {bin }} \equiv\left\langle P_{1}\right\rangle_{\text {bin }}=\frac{\left\langle J_{3}\right\rangle_{\text {bin }}}{2\left\langle J_{2 s}\right\rangle_{\text {bin }}} \quad\left\langle P_{2}\right\rangle_{\text {bin }}=\frac{\left\langle J_{6 s}\right\rangle_{\text {bin }}}{8\left\langle J_{2 s}\right\rangle_{\text {bin }}} \quad\left\langle P_{3}\right\rangle_{\text {bin }}=-\frac{\left\langle J_{9}\right\rangle_{\text {bin }}}{4\left\langle J_{2 s}\right\rangle_{\text {bin }}} \\
\left\langle P_{4}^{\prime}\right\rangle_{\text {bin }}=\frac{\left\langle J_{4}\right\rangle_{\text {bin }}}{\sqrt{-\left\langle J_{2 s}\right\rangle_{\text {bin }}\left\langle J_{2 c}\right\rangle_{\text {bin }}}} \quad\left\langle P_{5}^{\prime}\right\rangle_{\text {bin }}=\frac{\left\langle J_{5}\right\rangle_{\text {bin }}}{2 \sqrt{-\left\langle J_{2 s}\right\rangle_{\text {bin }}\left\langle J_{2 c}\right\rangle_{\text {bin }}}} \quad\left\langle P_{6}^{\prime}\right\rangle_{\text {bin }}=\frac{-\left\langle J_{7}\right\rangle_{\text {bin }}}{2 \sqrt{-\left\langle J_{2 s}\right\rangle_{\text {bin }}\left\langle J_{2 c}\right\rangle_{\text {bin }}}} .
\end{gathered}
$$

Similar definitions for $\left\langle P_{i}^{C P}\right\rangle_{\text {bin }}$ with $J_{i}-\bar{J}_{i}$.
$P_{1,2,3}$ were first indirectly measured via $S_{3}, A_{i m}, A_{F B}, F_{L}$
(and already provide constraints).
First results on $P_{1,2}$ available since Beauty 2013.
BUT it is urgent to get experimental measurements of $P_{i}^{\prime}$




R. Aaij et al. LHCb, 1304.6325 [hep-ex]

At Beauty $P_{1,2}$ were presented. Conclusion: Results consistent with SM predictions. BUT ...
Regarding measurement of $P_{1}$ at LHCb:


- Three first bins same 'shape' as CDF.
- Why error bars so large?
- Too early to draw any definite conclusion on existence or not of right-handed currents.

We suggest a new folding to measure uniquely $P_{1}$.
$d \Gamma\left(\hat{\phi}, \hat{\theta}_{\ell}, \hat{\theta}_{K}\right)+d \Gamma\left(\hat{\phi}, \hat{\theta}_{\ell}, \pi-\hat{\theta}_{K}\right)+d \Gamma\left(-\hat{\phi}, \pi-\hat{\theta}_{\ell}, \hat{\theta}_{K}\right)+d \Gamma\left(-\hat{\phi}, \pi-\hat{\theta}_{\ell}, \pi-\hat{\theta}_{K}\right)=f\left(P_{1}, F_{L}\right)+g\left(A_{S}^{5}, A_{S}^{8}\right)$

At Beauty $P_{1,2}$ were presented. Conclusion: Results consistent with SM predictions. BUT ...
Regarding measurement of $P_{1}$ at LHCb:

- Three first bins same 'shape' as CDF.

- Why error bars so large?
- Too early to draw any definite conclusion on existence or not of right-handed currents.
- $P_{1}$ can discriminate clearly at large recoil on the presence of $\delta C_{7}^{\prime}, \delta C_{9}^{\prime}$ and $\delta C_{10}^{\prime}$ if error bars reduced:
- $\delta C_{7}^{\prime}>0$ (a bit large) BLUE
- $\delta C_{9}^{\prime}>0$ also can generate it. RED
- $\delta C_{10}^{\prime}<0$ together with ( $\delta C_{7}^{\prime}>0$ GREEN or $\delta C_{9}^{\prime}>0$ ORANGE) can reproduce the shape easily.

We suggest a new folding to measure uniquely $P_{1}$.
$d \Gamma\left(\hat{\phi}, \hat{\theta}_{\ell}, \hat{\theta}_{K}\right)+d \Gamma\left(\hat{\phi}, \hat{\theta}_{\ell}, \pi-\hat{\theta}_{K}\right)+d \Gamma\left(-\hat{\phi}, \pi-\hat{\theta}_{\ell}, \hat{\theta}_{K}\right)+d \Gamma\left(-\hat{\phi}, \pi-\hat{\theta}_{\ell}, \pi-\hat{\theta}_{K}\right)=f\left(P_{1}, F_{L}\right)+g\left(A_{S}^{5}, A_{S}^{8}\right)$

Concerning the forward-back asymmetry $\left(A_{F B}\right)$ and $P_{2}$ :



- $P_{2}$ is the evolved version of $A_{F B}$, but, they play a complementary role.
- It magnifies a tiny tension in the second bin of $A_{F B}$.
- Both zeroes prefer a higher value $q_{0}^{2 e x p}=4.9 \pm 0.9 \mathrm{GeV}^{2}$ compared to $q_{0}^{2 S M}=3.95 \pm 0.38 \mathrm{GeV}^{2}$.

At LO how to move the position of the zero to the right?

$$
q_{0}^{2 L O}=-2 m_{b} M_{B} \frac{C_{7}^{\text {eff }} C_{10}-C_{7}^{\prime} C_{10}^{\prime}}{C_{9}^{e f f}\left(q_{0}^{2}\right) C_{10}-C_{9}^{\prime} C_{10}^{\prime}}
$$

where

$$
C_{i}=C_{i}^{S M}+\delta C_{i}
$$

Four main possibilities on how to test them:

| Mechanism | Constraint: <br> $A_{F B}$ in 3 <br> bins | Constraint: <br> $P_{2}$ in 3 <br> bins | Constraint: <br> $P_{1}$ in 3 <br> bins |
| :---: | :---: | :---: | :---: |
| I. $\delta \mathbf{C}_{7}<0$ | OK | OK | $\sim$ |
| II. $\delta \mathbf{C}_{9}<0$ | OK | OK | $\sim$ |
| III. $\left(\delta \mathbf{C}_{7}^{\prime}>\mathbf{0}, \delta \mathbf{C}_{10}^{\prime}<\mathbf{0}\right)$ | OK | $\sim$ | OK |
| IV. $\left(\delta C_{7}^{\prime}<0, \delta C_{10}^{\prime}>0\right)$ | NO | $\sim$ | NO |
| V. $\left(\delta \mathbf{C}_{9}^{\prime}>0, \delta \mathbf{C}_{10}^{\prime}<0\right)$ | OK | $\sim$ | OK |
| VI. $\left(\delta C_{9}^{\prime}<0, \delta C_{10}^{\prime}>0\right)$ | NO | $\sim$ | NO |

## Mechanism I, II, III and V preferred.

- $\delta \mathbf{C}_{7}<\mathbf{0}$ preferred by radiative constraints.
- $\delta \mathbf{C}_{9}<\mathbf{0}$, mechanism mainly tested with $P_{5}^{\prime}$
- Mec. III-VI sign of $\delta C_{10}^{\prime}$ tested by $P_{4}^{\prime}$ and $P_{1}$.
- Mec. III-IV sign of $\delta C_{7}^{\prime}$ tested by $P_{1}$
- Mec. $\mathrm{V}, \delta C_{9}^{\prime}$ can be tested by $P_{1}$.


At LO how to move the position of the zero to the right?

$$
q_{0}^{2 L O}=-2 m_{b} M_{B} \frac{C_{7}^{\text {eff }} C_{10}-C_{7}^{\prime} C_{10}^{\prime}}{C_{9}^{\text {eff }}\left(q_{0}^{2}\right) C_{10}-C_{9}^{\prime} C_{10}^{\prime}} \quad \text { where } \quad C_{i}=C_{i}^{S M}+\delta C_{i}
$$

Six main possibilities and how to test them:

| Mechanism | Constraint: <br> $A_{F B}$ in 3 <br> bins | Constraint: <br> $P_{2}$ in 3 <br> bins | Constraint: <br> $P_{1}$ in 3 <br> bins |
| :---: | :---: | :---: | :---: |
| I. $\delta \mathbf{C}_{7}<0$ | OK | OK | $\sim$ |
| II. $\delta \mathbf{C}_{9}<0$ | OK | OK | $\sim$ |
| III. $\left(\delta \mathbf{C}_{7}^{\prime}>\mathbf{0}, \delta \mathbf{C}_{10}^{\prime}<\mathbf{0}\right)$ | OK | $\sim$ | OK |
| IV. $\left(\delta C_{7}^{\prime}<0, \delta C_{10}^{\prime}>0\right)$ | NO | $\sim$ | NO |
| V. $\left(\delta \mathbf{C}_{9}^{\prime}>\mathbf{0}, \delta \mathbf{C}_{10}^{\prime}<0\right)$ | OK | $\sim$ | OK |
| VI. $\left(\delta C_{9}^{\prime}<0, \delta C_{10}^{\prime}>0\right)$ | NO | $\sim$ | NO |

## Mechanism I, II, III and V preferred.

- $\delta \mathbf{C}_{7}<\mathbf{0}$ preferred by radiative constraints.
- $\delta \mathbf{C}_{9}<\mathbf{0}$, mechanism mainly tested with $\boldsymbol{P}_{5}^{\prime}$
- Mec. III-VI sign of $\delta C_{10}^{\prime}$ tested by $P_{4}^{\prime}$ and $P_{1}$.
- Mec. III-IV sign of $\delta C_{7}^{\prime}$ tested by $P_{1}$
- Mec. $\mathrm{V}, \delta C_{9}^{\prime}$ can be tested by $P_{1}$.


After analyzing different scenarios we have perform a frequentist analysis with asymmetric errors and NP error bars to an scenario with $\delta C_{7}$ and $\delta C_{9}$ including: I. $\mathcal{B}\left(B \rightarrow X_{s} \gamma\right), A_{l}\left(B \rightarrow K^{*} \gamma\right), S_{K^{*} \gamma}$, $\mathcal{B}\left(B \rightarrow X_{s} \mu^{+} \mu^{-}\right)$and $\mathcal{B}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right)$together with $P_{1}, P_{2}, A_{F B}$ of $B \rightarrow K^{*} \mu^{+} \mu^{-}$.

Result in $\delta \mathbf{C}_{\mathbf{7}}-\delta \mathbf{C}_{\mathbf{9}}$ :


We find $3 \sigma$ deviation from SM prediction for $C_{9}$ (check the rest of basis $P_{i}^{\prime}$ !)

- 3 large-recoil bins (colored)
$\mathrm{C}_{7} \in(-0.332,-0.287)$ and $\mathrm{C}_{9} \in(2.58,3.38)$
- ONLY 1-6 bin at large recoil (orange)
- 3 large-recoil and 2 low-recoil bins (dashed)

Robustness tests:

- We have check using naive factorization that the effect on $C_{9}$ is confirmed.
- Also the bin 1-6 confirms the deviation.
- We have analyzed two types of charm effects:
- $m_{c}$ value: Increasing $m_{c}$ up to 1.4 GeV reduces significance to $2.3 \sigma$.
- non-perturbative $c-\bar{c}$ contribution (KMPW) increases slightly the significance above $3 \sigma$.
$\mathbf{P}_{2}$ for New Physics $\delta C_{9}=-1.5$ (red box) SM binned prediction in gray $\Rightarrow$ LHCb data crosses in blue


For completeness we show also the result of full fit to all:

- $\delta C_{10}, \delta C_{7,9,10}^{\prime}$ are already consistent with SM at $1 \sigma$
- $\delta C_{7}$ at $2 \sigma$
- $\delta C_{9}$ at $3 \sigma$

| Coefficient | $1 \sigma$ | $2 \sigma$ | $3 \sigma$ |
| :--- | :---: | :---: | :---: |
| $\delta C_{7}$ | $[-0.04,-0.01]$ | $[-0.06,0.01]$ | $[-0.08,0.03]$ |
| $\delta C_{9}$ | $[-1.2,-0.5]$ | $[-1.5,-0.1]$ | $[-1.8,0.4]$ |
| $\delta C_{10}$ | $[0,+1.8]$ | $[-0.8,2.4]$ | $[-1.8,3.4]$ |
| $\delta C_{7}^{\prime}$ | $[-0.05,0.03]$ | $[-0.1,0.08]$ | $[-0.14,0.13]$ |
| $\delta C_{9}^{\prime}$ | $[-0.2,1]$ | $[-0.8,1.4]$ | $[-1.4,1.8]$ |
| $\delta C_{10}^{\prime}$ | $[-0.8,0.2]$ | $[-1.4,0.6]$ | $[-2.0,1.0]$ |

Table : $68.3 \%(1 \sigma), 95.5 \%(2 \sigma)$ and $99.7 \%(3 \sigma)$ confidence intervals for the NP contributions to Wilson coefficients resulting from the global analysis.

- We have combined recent LHCb measurements on the first two theoretically clean observables $P_{1,2}$ of the optimal basis together with $A_{F B}$, other radiative modes and $B_{s} \rightarrow \mu^{+} \mu^{-}$. We work in the framework of NLO QCDF at large-recoil and HQET at low-recoil.
- We have found a strong indication for a negative possible New Physics contribution to the coefficient $\mathcal{C}_{9}$ at $3 \sigma$ using large-recoil data and $2.6 \sigma$ using both large and low-recoil data. This result corresponds to a range for $C_{9}$ inside a $68 \%$ CL of $2.6 \leq C_{9} \leq 3.4$ to be compared with the SM value for $C_{9}^{S M}=4.075$ at same $\mu_{b}$ scale. Different robustness tests have been included.
- A too large error bars on $P_{1}$ does not allow yet to draw any definite conclusion on the existence or not of right-handed currents. Still in our global fit we do not see clear indications of the need to introduce them.

Prospects: A measurement of the rest of the basis $P_{i}^{\prime}$ is essential to disprove or confirm this result

## BACK-UP SLIDES

## Computation of Primary Observables

Large-recoil: NLO QCDfactorization $+\mathcal{O}\left(\Lambda / m_{b}\right)$. Soft form factors $\xi_{\perp, \|}\left(q^{2}\right)$ from

$$
\xi_{\perp}\left(q^{2}\right)=m_{B} /\left(m_{B}+m_{K^{*}}\right) \mathbf{V}\left(\mathbf{q}^{2}\right) \quad \xi_{\|}\left(q^{2}\right)=\left(m_{B}+m_{K^{*}}\right) /(2 E) \mathbf{A}_{1}\left(\mathbf{q}^{2}\right)-\left(m_{B}-m_{K^{*}}\right) /\left(m_{B}\right) \mathbf{A}_{2}\left(\mathbf{q}^{2}\right)
$$

- FF at $q^{2}=0$ and slope parameters are computed by [Khodjamirian et al.'10] (KMPW) using LCSR.

Tensor form factors $\mathcal{T}_{\perp, \|}$ are computed in QCDF following [Beneke, Feldmann, Seidel'01,'05] including factorizable and non-factorizable contributions.
The wide spread of different errors in literature associated to FF:

$$
\begin{aligned}
& V(0)=0.31 \pm 0.04 \text { and } A(0)=0.33 \pm 0.03 \text { [W. Altmannshofer et al. '09] } \\
& V(0)=0.36 \pm 0.17 \text { and } A(0)=0.29 \pm 0.10 \text { [A. Khodjamirian et al. '10]. }
\end{aligned}
$$

Even central values have shifted significantly $V(0)=0.41 \pm 0.05$



Figure: Predictions in SM and for one benchmark point of NP for $P_{1}$ (left) and $S_{3}$ (right). The yellow boxes are the SM predictions integrated in five $1 \mathrm{GeV}^{2}$ bins. The blue curve corresponds to the central values for the NP scenario. The green/grey band is the total uncertainty considering two different FF determinations (BZ/KMPW).

Low-recoil: LCSR are valid up to $q \leq 14 \mathrm{GeV}^{2}$. We extend FF determination [Bobeth \& Hiller \& Dyk'10] till $19 \mathrm{Gev}^{2}$ and cross check the consistency with lattice QCD.
In HQET one expects the ratios to be near one

$$
\mathbf{R}_{1}=\frac{\mathbf{T}_{1}\left(\mathbf{q}^{2}\right)}{\mathbf{V}\left(\mathbf{q}^{2}\right)}, \quad \mathbf{R}_{2}=\frac{\mathbf{T}_{2}\left(\mathbf{q}^{2}\right)}{\mathbf{A}_{1}\left(\mathbf{q}^{2}\right)}, \quad \mathbf{R}_{3}=\frac{q^{2}}{m_{B}^{2}} \frac{T_{3}\left(q^{2}\right)}{A_{2}\left(q^{2}\right)}
$$

- BZ was problematic with $R_{3}$.

Our approach: we determine $T_{1,2}$ by exploiting the ratios $R_{1,2}$ allowing for up to a $20 \%$ breaking, i.e., $R_{1,2}=1+\delta_{1,2}$. All other form factors extrapolated from KMPW.



- We find excellent agreement between our determination of $T_{1,2}$ using $R_{1,2}$ and lattice data.

Contact between theory and experiment:
Indeed the observables are measured in bins.
Present bins: $[0.1,2],[2,4.3]$, $[4.3,8.68],[1,6],[14.18,16],[16,19] \mathrm{GeV}^{2}$.
Comments on the bins:

- Ultralow bin region $[\mathbf{0 . 1}, \mathbf{1}$ ] including light-resonances analyzed in [S. Jager, JM Camalich]'12. Binning tends to wash out the resonances.
- The region $q^{2} \sim 6-8.68 \mathrm{GeV}^{2}$ can be affected by charm-loop effects. [Khodjamirian, Mannel, Pivovarov, Wang'10]
- The middle bin $[\mathbf{1 0 . 0 9}, \mathbf{1 2 . 8 9}] \mathrm{GeV}^{2}$ between $J / \Psi$ and $\Psi(2 s)$. Charm-loop effects lead to a destructive interference (raw estimate).
We treat it as a simple interpolation.
- Suggestion to experimentalists on binning: [1,2], [2,4.3], [4.3,6]
- Another possible source of uncertainty is the S-wave contribution coming from $B \rightarrow K_{0}^{*} I^{+} I^{-}$decay. [Becirevic, Tayduganov '13], [Blake et al.'13]
- We will assume that both $P$ and $S$ waves are described by $q^{2}$-dependent FF times a Breit-Wigner function.
- The distinct angular dependence of the S-wave terms in folded distributions allow to disentangle the signal of the P-wave from the S-wave: $P_{i}^{(\prime)}$ can be disentangled from $S$-wave pollution [JM'12].
Problem: Changing the normalization used for the distribution from

$$
\frac{d \Gamma_{K}^{*}}{d q^{2}} \equiv \Gamma_{K^{*}}^{\prime} \rightarrow \Gamma_{\text {full }}^{\prime}
$$

introduces a $\left(1-F_{S}\right)$ in front of the $P$-wave.

$$
\Gamma_{\text {full }}^{\prime}=\Gamma_{K^{*}}^{\prime}+\Gamma_{S}^{\prime}
$$

and the longitudinal polarization fraction associated to $\Gamma_{S}^{\prime}$ is

$$
F_{S}=\frac{\Gamma_{S}^{\prime}}{\Gamma_{\text {full }}^{\prime}} \quad \text { and } \quad 1-F_{S}=\frac{\Gamma_{K^{*}}^{\prime}}{\Gamma_{\text {full }}^{\prime}}
$$

The modified distribution including the S-wave and new normalization $\Gamma_{\text {full }}^{\prime}$ :

$$
\begin{aligned}
& \frac{1}{\Gamma_{\text {full }}^{\prime}} \frac{d^{4} \Gamma}{d q^{2} d \cos \theta_{K} d \cos \theta_{l} d \phi}=\frac{9}{32 \pi}\left[\frac{3}{4} \mathrm{~F}_{\mathrm{T}} \sin ^{2} \theta_{K}+\mathrm{F}_{\mathrm{L}} \cos ^{2} \theta_{K}\right. \\
& \quad+\left(\frac{1}{4} \mathrm{~F}_{\mathrm{T}} \sin ^{2} \theta_{K}-F_{L} \cos ^{2} \theta_{K}\right) \cos 2 \theta_{l}+\frac{1}{2} \mathrm{P}_{1} \mathrm{~F}_{\mathrm{T}} \sin ^{2} \theta_{K} \sin ^{2} \theta_{l} \cos 2 \phi \\
& +\sqrt{\mathrm{F}_{\mathrm{T}} \mathrm{~F}_{\mathrm{L}}}\left(\frac{1}{2} \mathrm{P}_{4}^{\prime} \sin 2 \theta_{K} \sin 2 \theta_{l} \cos \phi+\mathrm{P}_{5}^{\prime} \sin 2 \theta_{K} \sin \theta_{l} \cos \phi\right) \\
& -\sqrt{\mathrm{F}_{\mathrm{T}} \mathrm{~F}_{\mathrm{L}}}\left(\mathrm{P}_{6}^{\prime} \sin 2 \theta_{K} \sin \theta_{l} \sin \phi-\frac{1}{2} \mathrm{Q}^{\prime} \sin 2 \theta_{K} \sin 2 \theta_{l} \sin \phi\right) \\
& \left.+2 \mathrm{P}_{2} \mathrm{~F}_{\mathrm{T}} \sin ^{2} \theta_{K} \cos \theta_{l}-\mathrm{P}_{3} \mathrm{~F}_{\mathrm{T}} \sin ^{2} \theta_{K} \sin ^{2} \theta_{l} \sin 2 \phi\right]\left(1-\mathrm{F}_{\mathrm{S}}\right)+\frac{1}{\Gamma_{\text {full }}^{\prime}} W_{\mathrm{S}}
\end{aligned}
$$

in the massless case and where the polluting terms are

$$
\begin{aligned}
\frac{\mathbf{W}_{\mathrm{S}}}{\Gamma_{\text {full }}^{\prime}}= & \frac{3}{16 \pi}\left[\mathbf{F}_{\mathrm{S}} \sin ^{2} \theta_{\ell}+\mathbf{A}_{\mathbf{S}} \sin ^{2} \theta_{\ell} \cos \theta_{K}+\mathbf{A}_{\mathrm{S}}^{4} \sin \theta_{K} \sin 2 \theta_{\ell} \cos \phi\right. \\
& \left.+\mathbf{A}_{\mathrm{S}}^{5} \sin \theta_{K} \sin \theta_{\ell} \cos \phi+\mathbf{A}_{\mathrm{S}}^{7} \sin \theta_{K} \sin \theta_{\ell} \sin \phi+\mathbf{A}_{\mathrm{S}}^{8} \sin \theta_{K} \sin 2 \theta_{\ell} \sin \phi\right]
\end{aligned}
$$

We can get bounds on the size of the S-wave polluting terms.
Let's take for instance $A_{S}$

$$
A_{\mathrm{S}}=2 \sqrt{3} \frac{1}{\Gamma_{\text {full }}^{\prime}} \int \operatorname{Re}\left[\left(A_{0}^{\prime} A_{0}^{L *}+A_{0}^{\prime R} A_{0}^{R *}\right) B W_{K_{0}^{*}}\left(m_{K \pi}^{2}\right) B W_{K^{*}}^{\dagger}\left(m_{K \pi}^{2}\right)\right] d m_{K \pi}^{2}
$$

where

$$
\mathrm{F}_{\mathrm{S}}=\frac{8}{3} \frac{\tilde{\mathcal{J}}_{12}^{c}}{\Gamma_{\text {full }}^{\prime}}=\frac{\left|A_{0}^{\prime} L^{2}+\left|A_{0}^{\prime} R\right|^{2}\right.}{\Gamma_{\text {full }}^{\prime}} \mathbf{Y} \quad \mathbf{Y}=\int d m_{K \pi}^{2}\left|B W_{K_{0}^{*}}\left(m_{K \pi}^{2}\right)\right|^{2}
$$

$\mathbf{Y}$ factor included to take into account the width of scalar resonance $K_{0}^{*}$
A bound is obtained once we define the $S-P$ interference integral

$$
\mathbf{Z}=\int\left|B W_{K_{0}^{*}}\left(m_{K \pi}^{2}\right) B W_{K^{*}}^{\dagger}\left(m_{K \pi}^{2}\right)\right| d m_{K \pi}^{2}
$$

and use the bound from the Cauchy-Schwartz inequality

$$
\begin{gathered}
\left|\int(\operatorname{Re}, \operatorname{Im})\left[\left(A_{0}^{\prime L} A_{j}^{L *} \pm A_{0}^{\prime R} A_{j}^{R *}\right) B W_{K_{0}^{*}}\left(m_{K \pi}^{2}\right) B W_{K^{*}}^{\dagger}\left(m_{K \pi}^{2}\right)\right] d m_{K \pi}^{2}\right| \\
\leq \mathbf{Z} \times \sqrt{\left[\left|A_{0}^{\prime} L\right|^{2}+\left|A_{0}^{\prime} R\right|^{2}\right]\left[\left|A_{j}^{L}\right|^{2}+\left|A_{j}^{R}\right|^{2}\right]}
\end{gathered}
$$

From the definitions of $F_{S}$ and $F_{L}$ and $P_{1}$ one gets the following bound:

$$
\left|A_{S}\right| \leq 2 \sqrt{3} \sqrt{F_{S}\left(1-F_{S}\right) F_{L}} \frac{Z}{\sqrt{X_{Y}}}
$$

the factor $\left(1-F_{S}\right)$ in the bound arises due to the fact that $\mathrm{F}_{\mathrm{L}}$ is defined with respect to $\Gamma_{K^{*}}^{\prime}$ rather than $\Gamma_{\text {full }}^{\prime}$.

$$
\begin{aligned}
&\left|A_{S}^{4}\right| \leq \sqrt{\frac{3}{2}} \sqrt{F_{S}\left(1-F_{S}\right)\left(1-F_{L}\right)\left(\frac{1-P_{1}}{2}\right)} \frac{Z}{\sqrt{X Y}} \\
&\left|A_{S}^{5}\right| \leq 2 \sqrt{\frac{3}{2}} \sqrt{F_{S}\left(1-F_{S}\right)\left(1-F_{L}\right)\left(\frac{1+P_{1}}{2}\right)} \frac{Z}{\sqrt{X Y}} \\
&\left|A_{S}^{7}\right| \leq 2 \sqrt{\frac{3}{2}} \sqrt{F_{S}\left(1-F_{S}\right)\left(1-F_{L}\right)\left(\frac{1-P_{1}}{2}\right)} \frac{Z}{\sqrt{X Y}} \\
&\left|A_{S}^{8}\right| \leq \sqrt{\frac{3}{2}} \sqrt{F_{S}\left(1-F_{S}\right)\left(1-F_{L}\right)\left(\frac{1+P_{1}}{2}\right)} \frac{Z}{\sqrt{X Y}}
\end{aligned}
$$

| Coefficient | Large <br> recoil <br> $\infty$ <br> Range | Low recoil <br> $\infty$ Range | Large Recoil <br> Finite Range | Low Recoil <br> Finite Range |
| :---: | :---: | :---: | :---: | :---: |
| $\left\|A_{S}\right\|$ | 0.33 | 0.25 | 0.67 | 0.49 |
| $\left\|A_{S}^{4}\right\|$ | 0.05 | 0.10 | 0.11 | 0.19 |
| $\left\|A_{S}^{5}\right\|$ | 0.11 | 0.11 | 0.22 | 0.23 |
| $\left\|A_{S}^{7}\right\|$ | 0.11 | 0.19 | 0.22 | 0.38 |
| $\left\|A_{S}^{8}\right\|$ | 0.05 | 0.06 | 0.11 | 0.11 |

Table: Illustrative values of the size of the bounds for the choices of $F_{S}, F_{L}, P_{1}$ and $\mathbf{F}=\mathbf{Z} / \sqrt{\mathbf{X Y}}$

- Large-recoil: $F_{S} \sim 7 \%$ (like $B^{0} \rightarrow J / \psi K^{+} \pi^{-}$), $F_{L} \sim 0.7$ and $P_{1} \sim 0$
- Low-recoil: $F_{S} \sim 7 \%, F_{L} \sim 0.38$ and $P_{1} \sim-0.48$.

We take the maximal value for $Z / \sqrt{X Y}$ factor in two cases:
"infinite range" $\rightarrow$ integrals in the whole $m_{K \pi}$ range
"finite range" $\rightarrow$ integrals around $m_{K^{*}} \pm 0.1 \mathrm{GeV}$.
This may help in estimating the systematics associated to S-wave.

There is a correspondence between $\mathbf{P}_{\mathrm{i}}^{(\prime)}$ and $\mathbf{J}_{\mathrm{k}}\left(\beta_{\ell}^{2}\right.$ absorbed here in $\left.F_{L, T}\right)$

$$
\begin{aligned}
& \left(\mathbf{J}_{2 \mathrm{~s}}+\overline{\mathbf{J}}_{2 \mathrm{~s}}\right)=\frac{1}{4} \mathbf{F}_{\mathbf{T}} \frac{\mathbf{d} \boldsymbol{\Gamma}+\mathbf{d} \overline{\boldsymbol{\Gamma}}}{\mathbf{d q ^ { 2 }}} \quad\left(\mathbf{J}_{2 \mathrm{c}}+\overline{\mathbf{J}}_{2 \mathrm{c}}\right)=-\mathbf{F}_{\mathbf{L}} \frac{\mathbf{d} \boldsymbol{\Gamma}+\mathbf{d} \overline{\boldsymbol{\Gamma}}}{\mathbf{d q ^ { 2 }}} \\
& \mathrm{J}_{3}+\bar{J}_{3}=\frac{1}{2} \mathrm{P}_{1} \mathrm{~F}_{\mathrm{T}} \frac{\mathrm{~d} \boldsymbol{\Gamma}+\mathrm{d} \overline{\boldsymbol{\Gamma}}}{\mathrm{dq}}{ }^{2} \quad \mathrm{~J}_{3}-\bar{J}_{3}=\frac{1}{2} \mathrm{P}_{1}^{\mathrm{CP}} \mathrm{~F}_{\mathrm{T}} \frac{\mathrm{~d} \boldsymbol{\Gamma}+\mathrm{d} \bar{\Gamma}}{\mathrm{dq}}{ }^{2}
\end{aligned}
$$

$$
\begin{aligned}
& \mathbf{J}_{9}+\bar{J}_{9}=-\mathrm{P}_{3} \mathrm{~F}_{\mathbf{T}} \frac{\mathbf{d \Gamma}+\mathrm{d} \overline{\boldsymbol{\Gamma}}}{\mathrm{dq}^{2}} \quad \mathrm{~J}_{9}-\bar{J}_{9}=-\mathrm{P}_{3}^{\mathrm{CP}} \mathrm{~F}_{\mathrm{T}} \frac{\mathrm{~d} \mathrm{\Gamma}+\mathrm{d} \overline{\mathbf{\Gamma}}}{\mathrm{dq}^{2}}
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{J}_{5}+\bar{J}_{5}=\mathrm{P}_{5}^{\prime} \sqrt{\mathrm{F}_{\mathrm{T}} \mathrm{~F}_{\mathrm{L}}} \frac{\mathrm{~d} \mathrm{\Gamma}+\mathrm{d} \bar{\Gamma}}{d q^{2}} \quad \mathrm{~J}_{5}-\bar{J}_{5}=\mathrm{P}_{5}^{\prime C P} \sqrt{\mathrm{~F}_{\mathrm{T}} \mathrm{~F}_{\mathrm{L}}} \frac{\mathrm{~d} \Gamma+\mathrm{d} \bar{\Gamma}}{\mathrm{dq}^{2}} \\
& \mathrm{~J}_{7}+\bar{J}_{7}=-\mathrm{P}_{6}^{\prime} \sqrt{\mathrm{F}_{\mathrm{T}} \mathrm{~F}_{\mathrm{L}}} \frac{\mathrm{~d} \boldsymbol{\Gamma}+\mathrm{d} \bar{\Gamma}}{\mathrm{dq}^{2}} \quad \mathrm{~J}_{7}-\bar{J}_{7}=-\mathrm{P}_{6}^{\prime \mathrm{CP}} \sqrt{\mathrm{~F}_{\mathrm{T}} \mathrm{~F}_{\mathrm{L}}} \frac{\mathrm{~d} \boldsymbol{\Gamma}+\mathrm{d} \bar{\Gamma}}{\mathrm{dq}}
\end{aligned}
$$

where each $\mathbf{P}_{i}^{(\prime)}$ and $\mathbf{P}_{i}^{(\prime) C P}$ encodes the information that can be extracted cleanly at large-recoil inside each $\mathbf{J}_{k}$ and define the simplest possible fit besides $S_{i}, \boldsymbol{A}_{i}$. The brown and blue pieces are strongly FF-dependent pieces.

