Superstring Amplitudes:

Formal Progress and Implications for LHC

Stephan Stieberger, MPP München

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Strings 2008, CERN, August, 18–23, 2008

Outline

I. Progress in higher point open superstring amplitudes (tree)
(in particular multi–gluon scattering)

- SUSY Ward identities relating various amplitudes to all orders in α'
- Compact representation for amplitudes with many external massless string states

II. Application: Universal string predictions for LHC

- Class of parton amplitudes: universal for any string background and relevant for QCD jets

Review: perturbative QCD amplitudes (tree)

- Compact representation for amplitudes with many external particle legs, by use of spinor basis $(\lambda, \tilde{\lambda})$

  e.g.: MHV N–gluon amplitude

$$A(g_1^-,g_2^-,g_3^+,g_4^+,\ldots,g_N^+) = (\sqrt{2}g_{YM})^{N-2}\ \mathrm{Tr}(T^1 \ldots T^N)\ \frac{\langle 12\rangle^4}{\prod_{k=1}^{N}\langle k\, k{+}1\rangle}$$

Parke, Taylor 1986
Berends, Giele 1989

$\langle ij\rangle \sim \sqrt{k_i k_j}$

- SUSY Ward identities relating various amplitudes

  Grisaru, Pendleton, van Nieuwenhuizen (1977); Parke, Taylor, (1985); . . . ; Bianchi, Elvang, Freedman (2008)

  e.g.: $A(g_1^-,g_2^-,g_3^+,g_4^+,\ldots,g_N^+) = \frac{\langle 12\rangle^2}{\langle 34\rangle^2}\ A(\phi_1^-,\phi_2^-,\phi_3^+,\phi_4^+,g_5^+,\ldots,g_N^+)$

- More efficient techniques to compute QCD amplitudes; CSW, BCFW rules

Do similar simple properties also hold for superstring (tree)–amplitudes ?

(Space–time) SUSY transformation on world–sheet disk

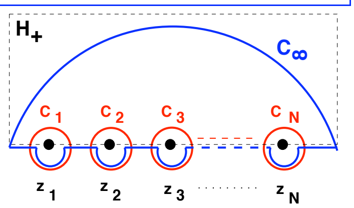
Setup: World–sheets with boundaries and $\mathcal{N}$ conserved SUSY charges $\mathcal{Q}_\alpha^I$
$I = 1, \ldots, \mathcal{N}$, with $\mathcal{Q}_\alpha^I = \oint \frac{dz}{2\pi i}\ V_\alpha^I(z)$

$V_\alpha^I(z) = \{$ $\mathcal{N}$ supercurrents, extended into double cover

Variation of open string vertex operator $\mathcal{O}(z)$ under (inf.) SUSY transformation

$$[\ Q^I(\eta_I)\ ,\ \mathcal{O}(z)\ ] := \oint_{C_z} \frac{dw}{2\pi i}\ \eta_I^\alpha\ V_\alpha(w)\ \mathcal{O}(z)$$

$$\oint_{C_\infty} \frac{dw}{2\pi i}\ \eta_I^\alpha\ \langle V_\alpha^I(w)\ V_1(z_1)\ V_2(z_2) \ldots V_N(z_N)\rangle \sim \oint_{C_\infty} \frac{dw}{2\pi i}\ w^{-2} = 0$$



Generate SUSY Ward identity in superstring theory

$$\sum_{l=1}^{N} \langle V_1(z_1) \ldots V_{l-1}(z_{l-1})\ [Q^I(\eta_I^\alpha), V_l(z_l)]\ V_{l+1}(z_{l+1}) \ldots V_N(z_N)\rangle = 0$$

N–gluon MHV amplitude in superstring theory

From supersymmetric Ward identities in string theory:

$$\boxed{\begin{aligned} &A(g_1^-, g_2^-, g_3^+, g_4^+, \ldots, g_N^+) \;= \\ &\qquad = \frac{\langle 12\rangle^2}{\langle 34\rangle^2}\, A(\phi_1^-, \phi_2^-, \phi_3^+, \phi_4^+, g_5^+, \ldots, g_N^+) \end{aligned}}$$

- valid to all orders in α'
- universal to all string compactifications
- any numbers of supersymmetries

This allows for very short expressions for the N–gluon MHV amplitude

E.g.: $A(g_1^-, g_2^-, g_3^+, g_4^+, g_5^+) \;=\; \mathrm{Tr}(T^1 \ldots T^5)\, (\sqrt{2}\, g_{YM})^3\, \alpha'$ St. St., Taylor arXiv:0708.0574

$$\times\; \frac{\langle 1\,2\rangle^2}{\langle 3\,4\rangle^2 \langle 4\,5\rangle} \left(\langle 4\,1\rangle [1\,5]\; \boldsymbol{K}_1 + \langle 4\,2\rangle [2\,5]\; \boldsymbol{K}_2 \right)$$

6–gluon NMHV amplitude in superstring theory

From SUSY Ward identities:

Full 6–gluon NMHV amplitude can be constructed from three partial subamplitudes (Mangano, Parke 1991)

$$\boxed{\begin{aligned} A(g_1^-, g_2^-, g_3^+, g_4^+, g_5^-, g_6^+) &= \frac{\alpha'^4}{s_{12}^2 s_{34}^2}\left(y^2\; A_g^Y - 2\, y\, \alpha_Y\; A_\lambda^Y + \alpha_Y^2\; A_s^Y \right) \\ A(g_1^+, g_2^+, g_3^-, g_4^-, g_5^-, g_6^+) &= \frac{\alpha'^4}{s_{12}^2 s_{34}^2}\left(x^2\; A_g^X - 2\, x\, \alpha_X\; A_\lambda^X + \alpha_X^2 A_s^X \right) \\ A(g_1^-, g_2^+, g_3^-, g_4^+, g_5^-, g_6^+) &= \frac{\alpha'^4}{s_{13}^2 s_{24}^2}\left(z^2\; A_g^Z - 2\, z\, \alpha_Z\; A_\lambda^Z + \alpha_Z^2\; A_s^Z \right) \end{aligned}}$$

$$\begin{aligned} A_g^Y &= A(\phi_1^-, \phi_2^-, \phi_3^+, \phi_4^+, g_5^-, g_6^+) \\ A_\lambda^Y &= A(\phi_1^-, \phi_2^-, \phi_3^+, \phi_4^+, \lambda_5^-, \lambda_6^+) \\ A_s^Y &= A(\phi_1^-, \phi_2^-, \phi_3^+, \phi_4^+, \phi_5^-, \phi_6^+) \end{aligned}$$

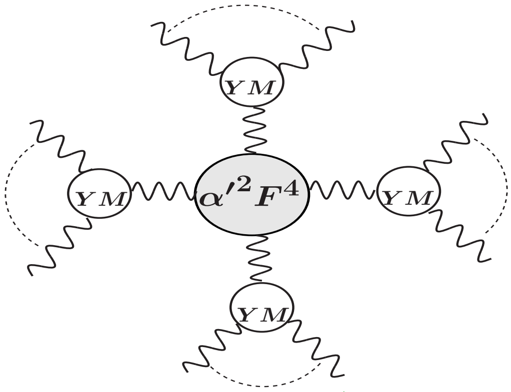
St. St., Taylor, arXiv:0711.4354

N–gluon MHV amplitude in superstring theory

$$\boxed{A(g_1^-,g_2^-,g_3^+,g_4^+,\ldots,g_N^+;\alpha') = \left(1-\alpha'^2\frac{\zeta(2)}{2}F^{(N)}\right) A(g_1^-,g_2^-,g_3^+,g_4^+,\ldots,g_N^+) + \mathcal{O}(\alpha'^3)}$$

$F^{(N)}$ polynomial in kinematic invariants:

$$\begin{aligned} F^{(4)} &= s_1 s_2 \\ F^{(5)} &= s_1s_2+s_2s_3+s_3s_4+s_4s_5+s_5s_1+4i\ \epsilon_{1234} \\ F^{(6)} &= s_1s_2+s_2s_3+s_3s_4+s_4s_5+s_5s_6+s_6s_1 \\ &+ t_1t_2+t_2t_3+t_3t_1-s_1s_4-s_2s_5-s_3s_6 \\ &+ 4i\ [\epsilon_{1234}+\epsilon_{1235}+\epsilon_{1245}+\epsilon_{1345}+\epsilon_{2345}] \\ &\vdots \end{aligned}$$



St. St., Taylor, hep-th/0609175

Non–trivial duality: $\zeta(2)$ term agrees with one–loop field–theory result !

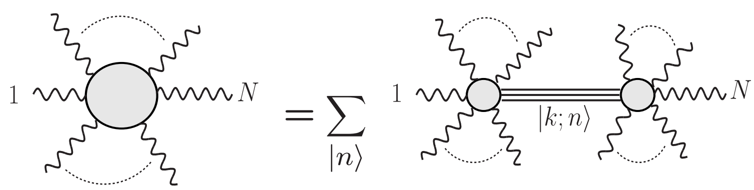
see also Dixon, Schabinger, to appear

Amplitudes from first principles

Analytic properties: Scattering amplitudes are computed *directly*

use:
- cyclic invariance
- soft–boson limit
- factorizing into collinear limits
- ...

String tree–level recursion relations:



$\Longrightarrow$ Construct amplitudes from first principles

work in progress

Relevance to low–energy physics

Question:

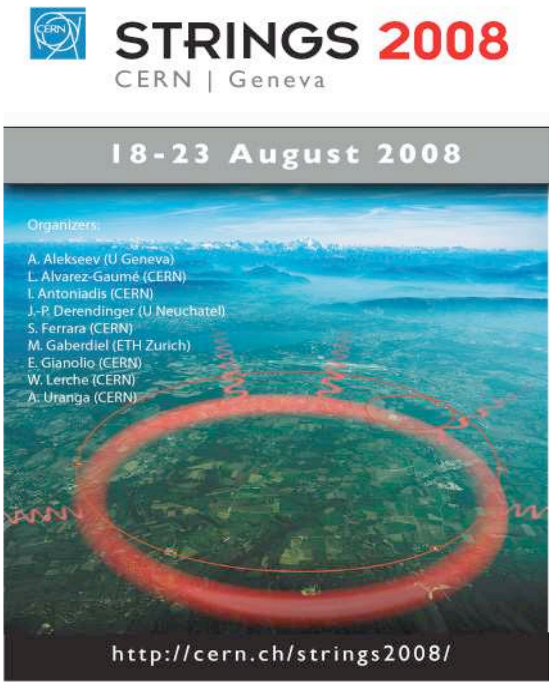
Can we make **model–independent**

low–energy **string predictions**

from parton amplitudes

in superstring theory ?

String signatures at LHC ?



Model–independent parton amplitudes

N–point parton superstring amplitudes in $D = 4$:

$$VM \begin{cases} g = \text{gluon} \\ \chi = \text{gaugino} \end{cases}$$

$$CM \begin{cases} \psi = \text{fermion} \\ \phi = \text{scalar} \end{cases}$$

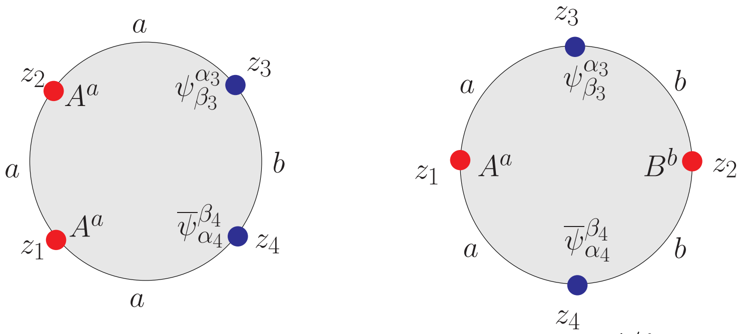
in D=4

$$\left. \begin{array}{c} A(g^{a_1} \ldots g^{a_N}) \\ A(\chi^{a_1} \overline{\chi}^{a_2} g^{a_1} \ldots g^{a_{N-2}}) \\ A(\psi^{a_1} \overline{\psi}^{a_2} g^{a_1} \ldots g^{a_{N-2}}) \\ A(\phi^{a_1} \overline{\phi}^{a_2} g^{a_1} \ldots g^{a_{N-2}}) \end{array} \right\}$$

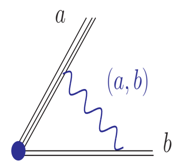
- completely model independent
- for any string compactification
- any number of supersymmetries
- even with broken supersymmetry

No intermediate exchange of KKs, windings nor emmission of graviton !

Scattering of two gluons and two chiral fermions



Vertex operator of chiral fermion (a,b) $\quad [\ g_\psi = (2\alpha')^{1/2}\alpha'^{1/4}\ e^{\phi_{10}/2}\ ]$



$$V^{(-1/2)}_{\psi^{\alpha}_{\beta}}(z,u,k) = g_\psi\ [T^{\alpha}_{\beta}]^{\beta_1}_{\alpha_1}\ \ e^{-\frac{1}{2}\phi(z)}\ \ u^\lambda S_\lambda(z)\ \ \Xi^{a\cap b}(z)\ e^{ik_\rho X^\rho(z)}$$

with boundary changing operator $\Xi^{a\cap b}(z)$ ($h=\frac{3}{8}$), with:

$$\langle \Xi^{a\cap b}(z_1)\ \overline{\Xi}^{a\cap b}(z_2)\rangle = \frac{1}{(z_1-z_2)^{3/4}}$$

String S–matrix of two gluons and two chiral fermions

Lüst, St. St., Taylor, arXiv:0807.3333

Result:

$$\mathcal{M}(A_1^-,A_2^+,q_3^-,\overline{q}_4^+) \ =\ 2\ g^2_{Dp_a}\ \delta^{\beta_4}_{\beta_3}\frac{\langle 13\rangle^2}{\langle 23\rangle\langle 24\rangle}\ \left[(T^{a_1}T^{a_2})^{\alpha_3}_{\alpha_4}\ \frac{t}{s}\ \boldsymbol{V_t} + (T^{a_2}T^{a_1})^{\alpha_3}_{\alpha_4}\ \frac{u}{s}\ \boldsymbol{V_u}\right]$$

$$\mathcal{M}(A_1^-,B_2^+,q_3^-,\overline{q}_4^+) \ =\ 2\ g_{Dp_a}\ g_{Dp_b}\ \frac{\langle 13\rangle^2}{\langle 23\rangle\langle 24\rangle}\ (T^a)^{\alpha_3}_{\alpha_4}(T^b)^{\beta_4}_{\beta_3}\ \boldsymbol{V_s}$$

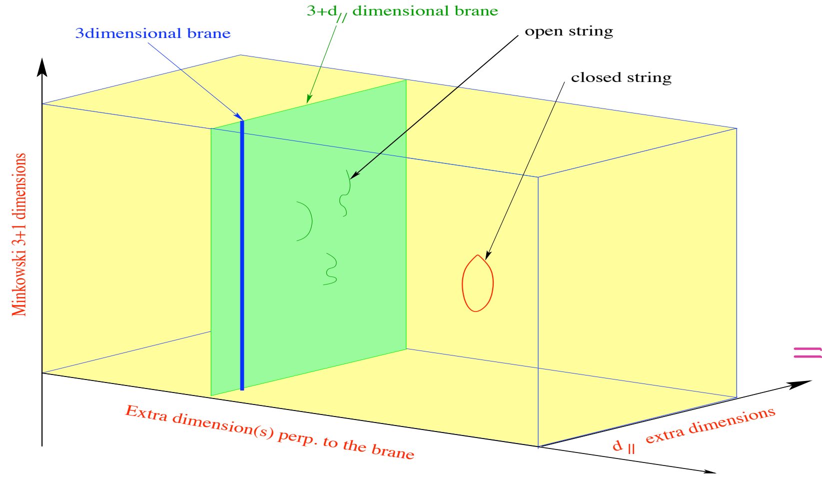
Similarly for gauginos χ or scalars ϕ

Functions:
$$\begin{cases} V_t = -\frac{su}{t}\ B(s,u) = 1-\zeta(2)\ s\ u + \mathcal{O}(\alpha'^3)\ , \\ V_s = -\frac{tu}{s}\ B(t,u) = 1-\zeta(2)\ t\ u + \mathcal{O}(\alpha'^3)\ , \\ V_u = -\frac{st}{u}\ B(s,t) = 1-\zeta(2)\ s\ t + \mathcal{O}(\alpha'^3)\ . \end{cases}$$

Physics of large extra dimensions

Open and closed strings (*e.g.* type II superstring with Dp–branes):

$$\boxed{g_{Dp}^2\ M_{\text{Planck}} = 2^{5/2}\pi\ M_{\text{string}}^{7-p}\ \left(\prod_{j=1}^{d_\perp} R_j^\perp\right)^{\frac{1}{2}}\ \left(\prod_{i=1}^{d_\parallel} R_i^\parallel\right)^{-1/2}}$$



$$\Longrightarrow \boxed{\boldsymbol{R_j^\perp} \uparrow \Longleftrightarrow \ \boldsymbol{M_{\text{string}}} \downarrow}$$

Antoniadis, Arkani–Hamed
Dimopoulos, Dvali

Physics of large extra dimensions and low string scale

States:

- string Regge **(SR)** excitations: $M_{SR} \sim 1$ TeV
- **KK** modes w.r.t. $\boldsymbol{R_i^\parallel}$: $M_{KK^\parallel} \sim M_{\text{string}}$
- **winding** modes w.r.t. $\boldsymbol{R_j^\perp}$: $M_{W^\perp} \sim M_{\text{string}}$
- **KK** modes w.r.t. $\boldsymbol{R_j^\perp}$: $M_{KK^\perp} \sim 10^{-3}$eV

Couplings:

$\boldsymbol{g_{\text{string}}}$	disk	SM tree–processes ($4pt$) including **exchange of SR**
$\boldsymbol{g_{\text{string}}^{3/2}}$	sphere	gravitational couplings (bulk)
$\boldsymbol{g_{\text{string}}^{2}}$	annulus	–exchange of graviton and KK –new contact interactions due to SUSY breaking:$F^3, \ldots$

$$\boxed{g_{\text{string}} \sim g_{YM}^2 < 1}$$

Physics of large extra dimensions and low string scale

What about strong gravity effects ?

Black hole production at energies $\sim \frac{M_{\text{string}}}{g^2_{\text{string}}}$

Horowitz, Polchinski 1996
Meade, Randall 2007

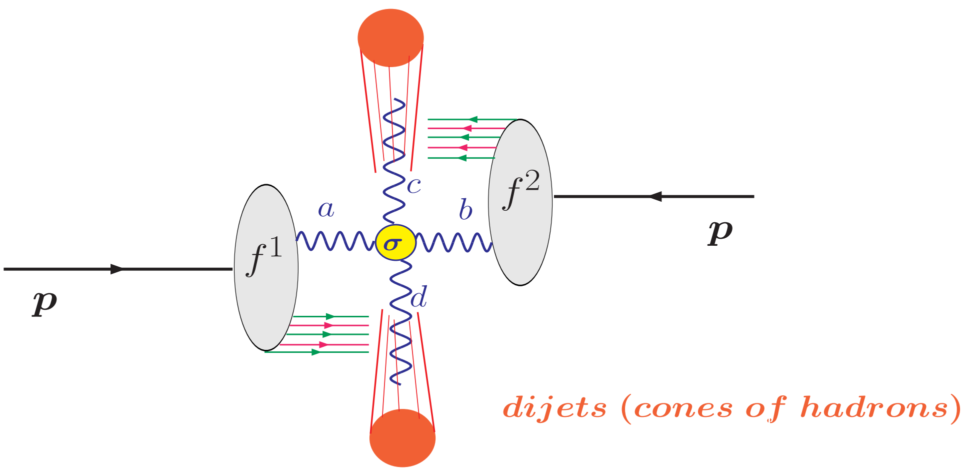
$$\boxed{n \sim g_{\text{string}}^{-2} > 1}$$

$\Longrightarrow$ For $g_{\text{string}} < 1$ strong gravity effects occur above M_{string}

$\Longrightarrow$ We may first see SR's from 1-st, $\ldots, n$-th level

Dijet signals for low M_{string} at LHC

Two jets:



$$\boxed{\sigma(pp \to 2 \text{ jets}) = \sum_{a,b,c,d} \int dx_1 \; dx_2 \; f_a^1(x_1; Q^2) \; f_b^2(x_2; Q^2) \; \sigma_{ab \to cd}(\underbrace{x_1 x_2 s}_{\hat{s}}, \underbrace{Q^2}_{\hat{t}}; \alpha')}$$

Look for **resonances of string Regge excitations** propagating in s–channel

Cross sections

Compute cross sections:

$$\boxed{\begin{array}{lcl} |\mathcal{M}(gg \to gg)|^2 & , & |\mathcal{M}(gg \to q\bar{q})|^2 \\ |\mathcal{M}(q\bar{q} \to gg)|^2 & , & |\mathcal{M}(qg \to qg)|^2 \end{array}} \Bigg\}$$

completely model–independent:
for any CY orientifold !

Result:

tabulated in Lüst, St. St., Taylor, arXiv:0807.3333

$$\boxed{\begin{aligned} |\mathcal{M}(gg \to gg)|^2 \; = \; & g_{Dp_a}^4 \left(\frac{1}{s^2} + \frac{1}{t^2} + \frac{1}{u^2} \right) \\ \times & \left[\frac{9}{4} \left(s^2 V_s^2 + t^2 V_t^2 + u^2 V_u^2 \right) - \frac{1}{3} \left(sV_s + tV_t + uV_u \right)^2 \right] \end{aligned}}$$

$$\boxed{|\mathcal{M}(gg \to q\bar{q})|^2 = g_{Dp_a}^4 \, \frac{t^2 + u^2}{s^2} \left[\frac{1}{6} \, \frac{1}{tu} \, (tV_t + uV_u)^2 - \frac{3}{8} \, V_t \; V_u \right]}$$

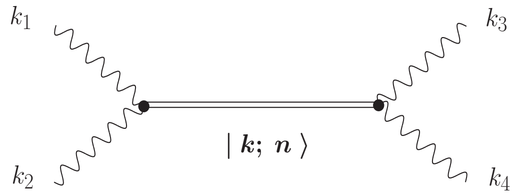
YM–limits agree with book "*Collider Physics*" by Barger, Phillips

Exchange of string Regge (SR) excitations

Universal sum over infinite s–channel poles:

$$\mathcal{M}(k_1, k_2, k_3, k_4; \alpha') \sim \sum_{n=0}^{\infty} \frac{\gamma(n)}{s - M_n^2} = -\frac{\Gamma(-\alpha' s) \; \Gamma(1 - \alpha' u)}{\Gamma(-\alpha' s - \alpha' u)}$$

s–channel exchange
of **SR excitations**
of SM particles:



$s = -2k_1k_2$,
$t = -2k_1k_3$,
$u = -2k_1k_4$,
$s + t + u = 0$

with
$$\left\{ \begin{array}{rl} \text{intermediate masses:} & M_n^2 = M_{\text{string}}^2 \; n \\ \text{residua:} & \gamma(n) = t \, \dfrac{(u \; \alpha', n)}{n!} \\ \text{maximal spin:} & n + 1 \end{array} \right.$$

Cross sections

In addition we need:

$$\boxed{\begin{array}{lcl} |\mathcal{M}(q\bar{q} \to q\bar{q})|^2 & , & |\mathcal{M}(qq \to qq)|^2 \\ |\mathcal{M}(q\bar{q} \to q'\bar{q}')|^2 & , & \begin{array}{l} |\mathcal{M}(qq' \to qq')|^2 \\ |\mathcal{M}(q\bar{q}' \to q\bar{q}')|^2 \end{array} \end{array}} \Bigg\}$$

depend on geometry:
KK and windings

tabulated in Lüst, St. St., Taylor, arXiv:0807.3333

however they are suppressed:

- QCD $SU(3)$ color group factors favor gluons over quarks in the initial state

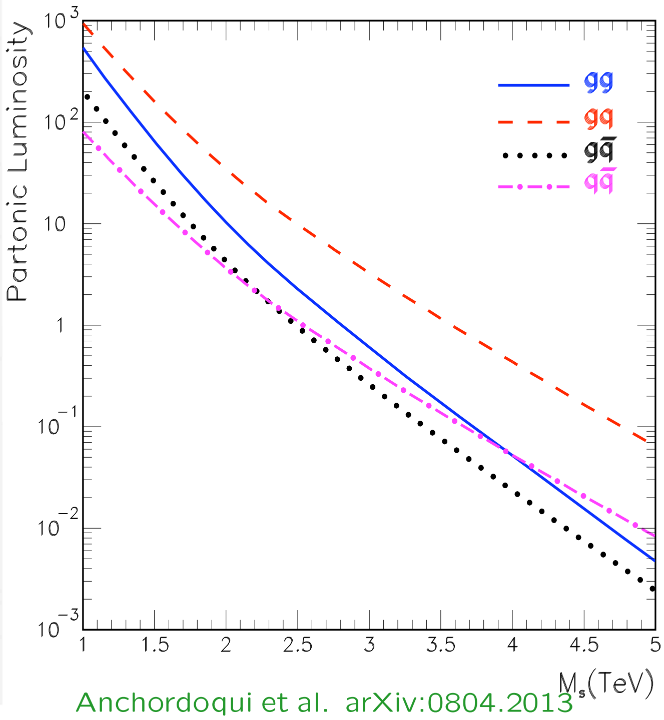
- Parton luminosities in pp–collisions, at the parton center of mass energies above 1TeV, are significantly lower for $q\bar{q}$ subprocesses than for gg or gq

At any rate: they may be used to probe the internal geometry
("precision tests")

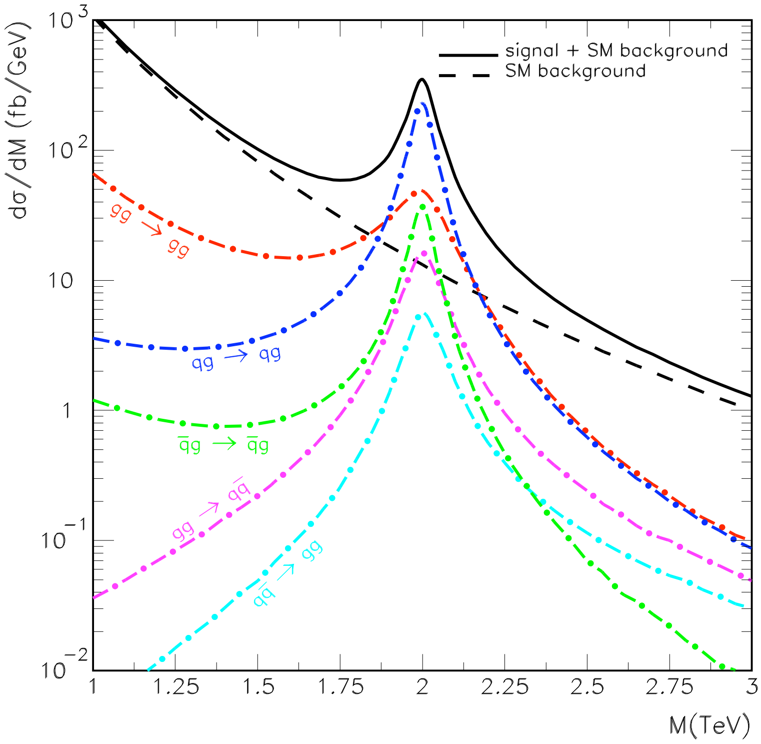
Table 9.1. *Squared matrix elements for $2 \to 2$ parton-parton subprocesses in QCD: q and q' denote distinct flavors of quark, $g_s^2 = 4\pi\alpha_s$ is the coupling squared.*

Subprocess	$\lvert\mathcal{M}\rvert^2/g_s^4$	$\lvert\mathcal{M}(90^\circ)\rvert^2/g_s^4$
$qq' \to qq'$, $q\bar{q}' \to q\bar{q}'$	$\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$	2.2
$qq \to qq$	$\frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right)-\frac{8}{27}\frac{\hat{s}^2}{\hat{u}\hat{t}}$	3.3
$q\bar{q} \to q'\bar{q}'$	$\frac{4}{9}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$	0.2
$q\bar{q} \to q\bar{q}$	$\frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right)-\frac{8}{27}\frac{\hat{u}^2}{\hat{s}\hat{t}}$	2.6
$q\bar{q} \to gg$	$\frac{32}{27}\frac{\hat{u}^2+\hat{t}^2}{\hat{u}\hat{t}}-\frac{8}{3}\frac{\hat{u}^2+\hat{t}^2}{\hat{s}^2}$	1.0
$gg \to q\bar{q}$	$\frac{1}{6}\frac{\hat{u}^2+\hat{t}^2}{\hat{u}\hat{t}}-\frac{3}{8}\frac{\hat{u}^2+\hat{t}^2}{\hat{s}^2}$	0.1
$gq \to gq$	$\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}-\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{u}\hat{s}}$	6.1
$gg \to gg$	$\frac{9}{4}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}+\frac{\hat{u}^2+\hat{t}^2}{\hat{s}^2}+3\right)$	30.4

from "Collider Physics" by Barger, Phillips



Anchordoqui et al. arXiv:0804.2013



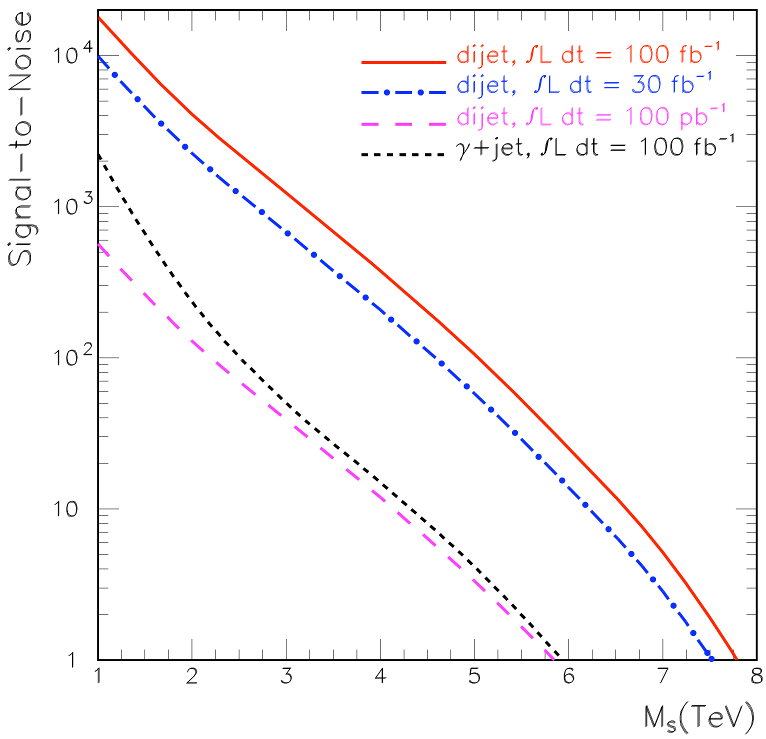
Any superstring theory with low M_{string} and $g_{\text{string}} < 1$

Universal deviation from SM in jet distribution

$M_{\text{string}} = 2$ TeV

$\Gamma_{SR} = 15 - 150$ GeV

Anchordoqui, Goldberg, Lüst, Nawata, Taylor, St. St., arXiv:0808.0497



Discovery reach