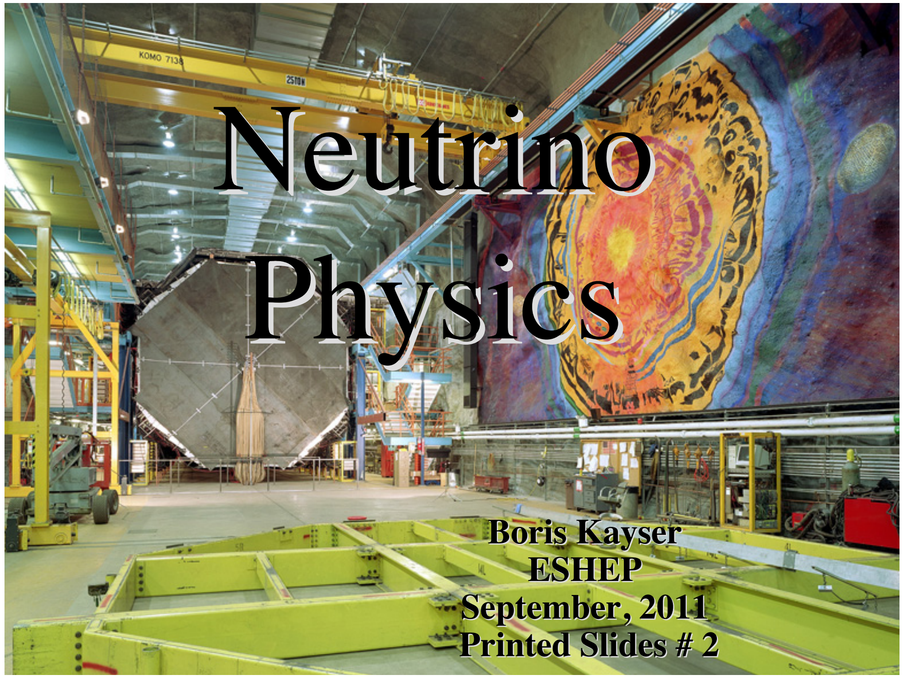
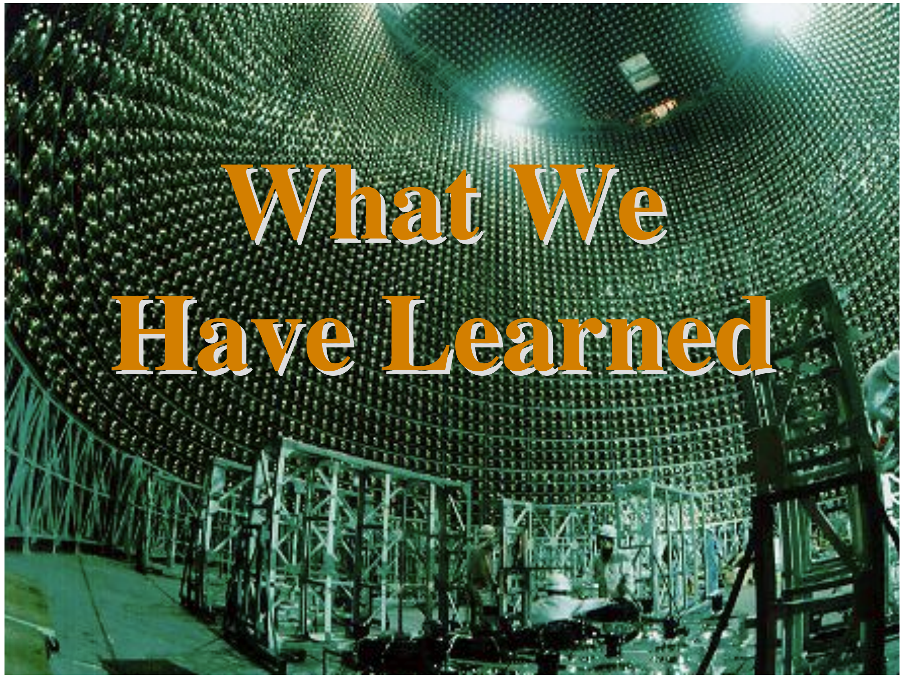
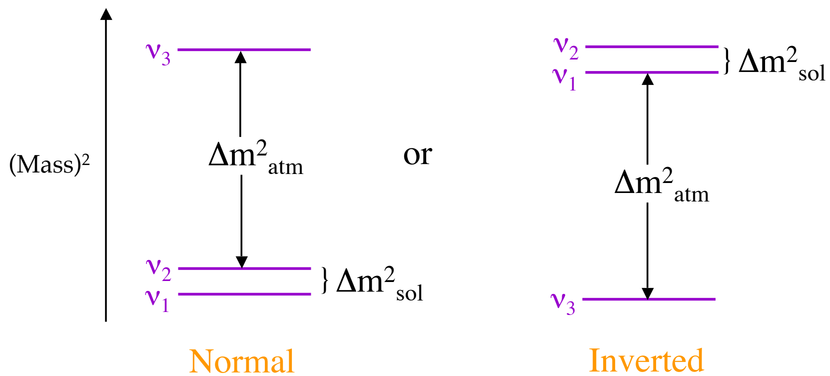


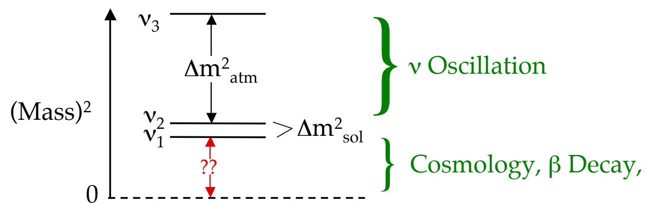


The (Mass)^2 Spectrum



$\Delta m^2_{sol} \cong 7.5 \times 10^{-5}\ eV^2$, $\Delta m^2_{atm} \cong 2.3 \times 10^{-3}\ eV^2$

The Absolute Scale of Neutrino Mass



How far above zero is the whole pattern?

Oscillation Data $\Rightarrow \sqrt{\Delta m^2_{atm}} <$ Mass[Heaviest ν_i]

The Upper Bound From Cosmology

Neutrino mass affects large scale structure.

Cosmological Data + Cosmological Assumptions ⇒

$\Sigma\, m_i < (0.17 - 1.0)$ eV .

Mass(ν_i)

(Seljak, Slosar, McDonald Hannestad; Pastor)

If there are only 3 neutrinos,

0.04 eV $\lesssim$ Mass[Heaviest ν_i] $< (0.07 - 0.4)$ eV

$\sqrt{\Delta m^2_{atm}}$ Cosmology

The Upper Bound From Tritium

Cosmology is wonderful, but there are known loopholes in its argument concerning neutrino mass.

The absolute neutrino mass can in principle also be measured by the kinematics of β decay.

Tritium decay: $^3H \rightarrow {}^3He + e^- + \overline{\nu_i}$; $i = 1, 2,$ or 3

$$BR\left({}^3H \rightarrow {}^3He + e^- + \overline{\nu_i}\right) \propto |U_{ei}|^2$$

In $^3H \rightarrow {}^3He + e^- + \overline{\nu_i}$, the bigger m_i is, the smaller the maximum electron energy is.

There are 3 separate thresholds in the β energy spectrum.

The β energy spectrum is modified according to —

$$(E_0 - E)^2 \Theta[E_0 - E] \Rightarrow \sum_i |U_{ei}|^2 (E_0 - E)\sqrt{(E_0 - E)^2 - m_i^2}\, \Theta[(E_0 - m_i) - E]$$

{Maximum β energy when there is no neutrino mass — β energy

Present experimental energy resolution is insufficient to separate the thresholds.

Measurements of the spectrum bound the average neutrino mass —

$$\langle m_\beta \rangle = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$

Presently: $\langle m_\beta \rangle < 2$ eV

Mainz & Troitzk

Leptonic Mixing

This has the consequence that —

Mass eigenstate — Flavor eigenstate

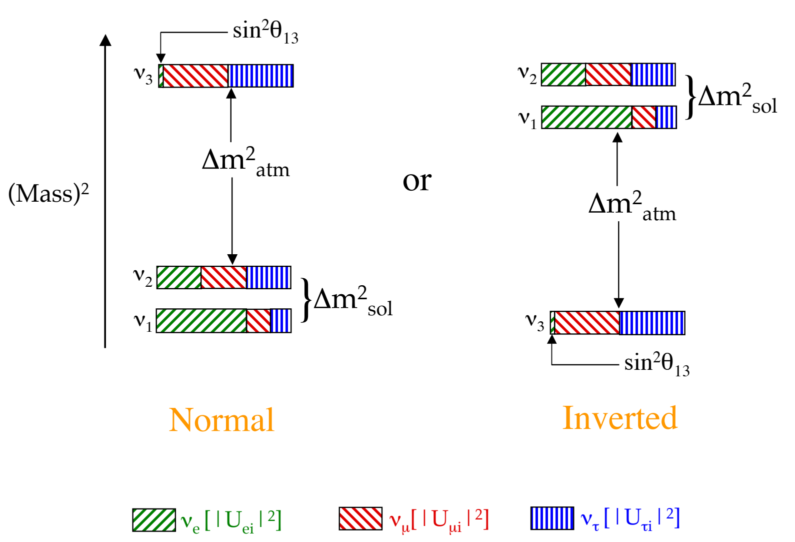
$|\nu_i\rangle = \sum_\alpha U_{\alpha i}\, |\nu_\alpha\rangle$.

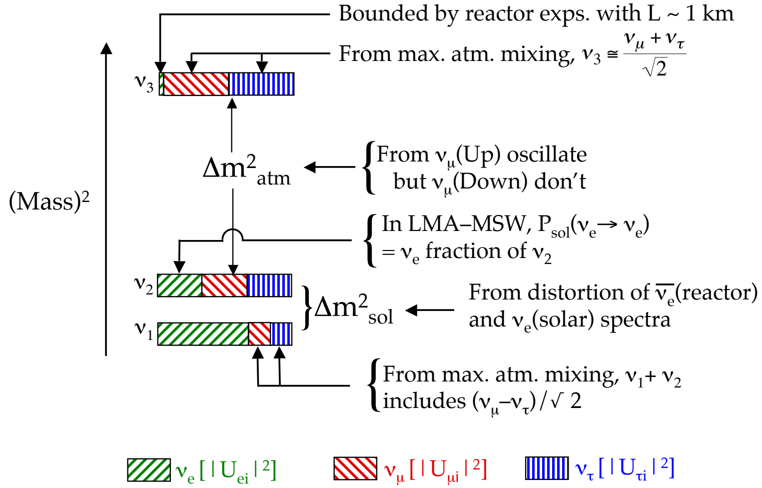
e, μ, or τ — Leptonic Mixing Matrix

Flavor-α fraction of $\nu_i = |U_{\alpha i}|^2$.

When a ν_i interacts and produces a charged lepton, the probability that this charged lepton will be of flavor α is $|U_{\alpha i}|^2$.

The spectrum, showing its approximate flavor content, is





The Mixing Matrix

Atmospheric — Cross-Mixing — Solar

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$c_{ij} \equiv \cos\theta_{ij}$
$s_{ij} \equiv \sin\theta_{ij}$

Hints??

$$\times \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Majorana ~~CP~~ phases

$\theta_{12} \approx \theta_{sol} \approx 34°$, $\theta_{23} \approx \theta_{atm} \approx 39\text{-}51°$, $\theta_{13} \lesssim 12°$

δ would lead to $P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) \neq P(\nu_\alpha \to \nu_\beta)$. ~~CP~~

But note the crucial role of $s_{13} \equiv \sin\theta_{13}$.

Recent Evidence For Non-Zero θ_{13}

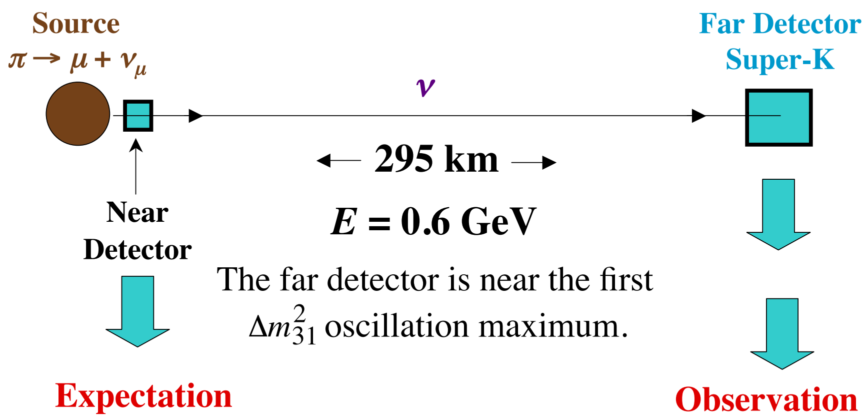
In an experiment where *L/E* is too small for the small splitting $\Delta m^2_{21} \equiv m^2_2 - m^2_1$ to be seen,

$$P(\nu_\mu \to \nu_e) \cong 4|U_{\mu 3}U_{e3}|^2 \sin^2\left(\Delta m^2_{31}\frac{L}{4E}\right)$$

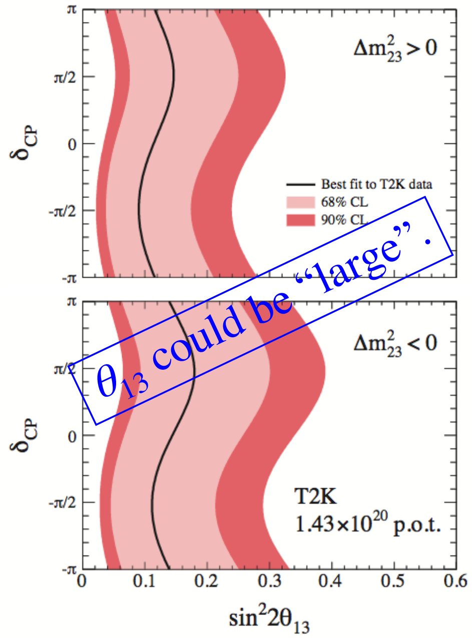
$$= \sin^2 2\theta_{13} \sin^2\theta_{23} \sin^2\left(\Delta m^2_{31}\frac{L}{4E}\right)$$

T2K has looked for $\nu_\mu \to \nu_e$ in a long-baseline experiment:

The T2K experiment (Designed to seek $\nu_\mu \to \nu_e$)



T2K sees 6 ν_e candidate events in the far detector, whereas 1.5 are expected if $\theta_{13} = 0$.



These take the Δm^2_{21} contributions and matter effects into account.

MINOS, not designed to look for $\nu_\mu \to \nu_e$, sees 62 candidate events where 50 are expected if $\theta_{13} = 0$.

While not highly significant by itself, this result is consistent with that from T2K.

There Is Nothing Special About θ_{13}

All mixing angles must be nonzero for $\not{CP}$ in oscillation.

For example —

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) - P(\nu_\mu \to \nu_e) = 2\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin\delta$$
$$\times \sin\left(\Delta m^2{}_{31}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{32}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{21}\frac{L}{4E}\right)$$

In the factored form of U, one can put δ next to θ_{12} instead of θ_{13}.

The Majorana ~~CP~~ Phases

The phase α_i is associated with neutrino mass eigenstate ν_i:

$U_{\alpha i} = U^0_{\alpha i} \exp(i\alpha_i/2)$ for all flavors α.

$\mathrm{Amp}(\nu_\alpha \to \nu_\beta) = \sum_i U_{\alpha i}^* \exp(-\, i m_i^2 L/2E)\, U_{\beta i}$

is insensitive to the Majorana phases α_i.

Only the phase δ can cause CP violation in neutrino oscillation.

- What is the absolute scale of neutrino mass?
- Are neutrinos their own antiparticles?
- Are there ***more*** than 3 mass eigenstates?
- Are there “sterile” neutrinos?
- What are the neutrino magnetic and electric dipole moments?

What is θ_{13}?
How close to maximal is θ_{23}?

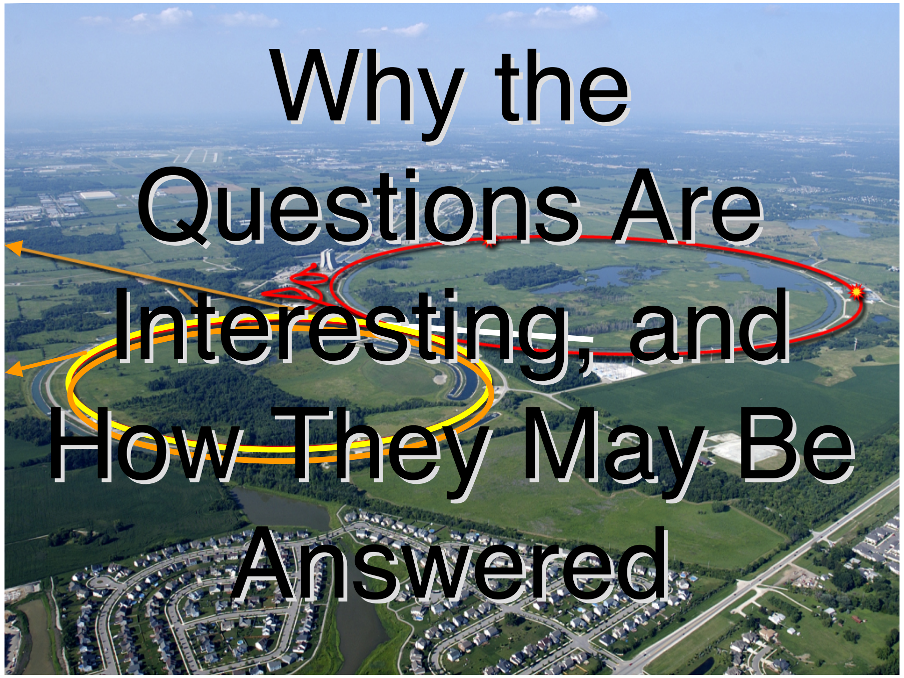
- Is the spectrum like  or  ?
- Do neutrino interactions violate CP?
Is $P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \neq P(\nu_\alpha \to \nu_\beta)$?

- What can neutrinos and the universe tell us about one another?

- Is CP violation involving neutrinos the key to understanding the matter – antimatter asymmetry of the universe?

- What physics is behind neutrino mass?

- What *surprises* are in store?



Does $\bar{\nu} = \nu$?

What Is the Question?

For each ***mass eigenstate*** ν_i , and ***given helicty*** h, does —

- $\overline{\nu_i}(h) = \nu_i(h)$ (Majorana neutrinos)

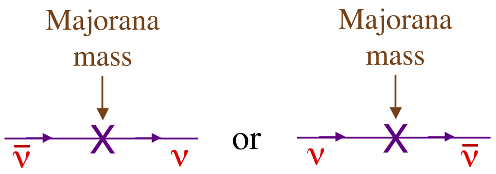
or

- $\overline{\nu_i}(h) \neq \nu_i(h)$ (Dirac neutrinos) ?

Equivalently, do neutrinos have *Majorana masses*? If they do, then the mass eigenstates are *Majorana neutrinos*.

Majorana Masses

Their effect:



Majorana masses mix ν and $\bar{\nu}$, so they do not conserve the Lepton Number L that distinguishes leptons from antileptons:

$$L(\nu) = L(\ell^-) = -L(\bar{\nu}) = -L(\ell^+) = 1$$

A Majorana mass for any fermion f causes f ⟷ $\bar{f}$.

Quark and *charged-lepton* Majorana masses are forbidden by electric charge conservation.

***Neutrino* Majorana masses would make the neutrinos *very* distinctive.**

Majorana neutrino masses have a different origin than the quark and charged-lepton masses.

In the SM, the top quark mass comes from —

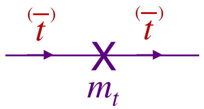
Higgs field → ; Top quark mass m_t

$$G_t H^0 \bar{t}_R t_L \Rightarrow G_t \langle H^0 \rangle_0 \bar{t}_R t_L$$

Coupling constant ; Vacuum expectation value

Such an operator does ***not*** mix quark and antiquark.

Its effect:



A Majorana mass term ***does*** mix neutrino and antineutrino.

A Majorana mass term must have a different origin than the quark and charged-lepton masses.

Why Majorana Masses ⟹ Majorana Neutrinos

As a result of $K^0 \longleftrightarrow \overline{K^0}$ mixing, the neutral K mass eigenstates are —

$$K_{S,L} \cong (K^0 \pm \overline{K^0})/\sqrt{2} \ . \qquad \overline{K_{S,L}} = K_{S,L} \ .$$

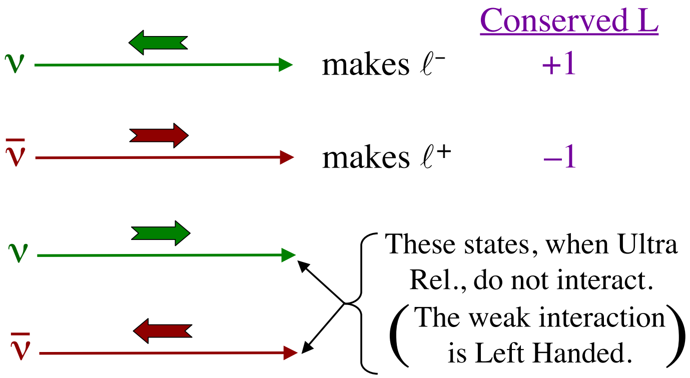
Majorana masses induce $\nu \leftrightarrow \bar{\nu}$ mixing.

As a result of $\nu \leftrightarrow \bar{\nu}$ mixing, the neutrino mass eigenstate is —

$$\nu_i = \nu + \bar{\nu} \ . \qquad \bar{\nu}_i = \nu_i \ .$$

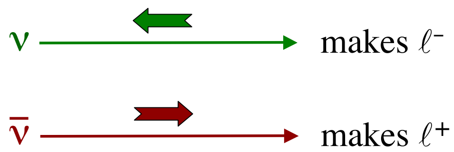
SM Interactions Of A Dirac Neutrino

We have 4 mass-degenerate states:



SM Interactions Of A Majorana Neutrino

We have only 2 mass-degenerate states:



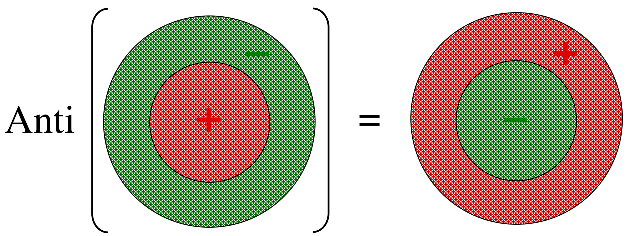
The weak interactions violate *parity*.
(They can tell *Left* from *Right*.)

An incoming left-handed neutral lepton makes ℓ^-.

An incoming right-handed neutral lepton makes ℓ^+.

Can a Majorana Neutrino Have an Electric Charge *Distribution*?

No!

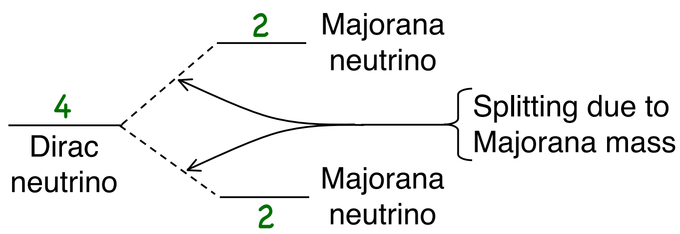


But for a Majorana neutrino —

Anti (ν) = ν

Majorana Masses Split Dirac Neutrinos

A Majorana mass term splits a Dirac neutrino into two Majorana neutrinos.



Why Most Theorists Expect Majorana Masses

The Standard Model (SM) is defined by the fields it contains, its symmetries (notably weak isospin invariance), and its renormalizability.

Leaving neutrino masses aside, anything allowed by the SM symmetries occurs in nature.

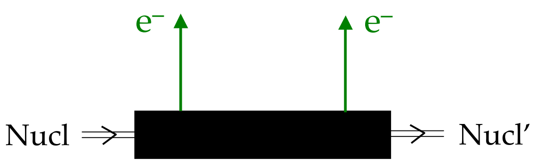
Majorana mass terms
are allowed by the SM symmetries.

Then quite likely *Majorana masses* occur in nature too.

To Determine Whether Majorana Masses Occur in Nature

The Promising Approach — Seek

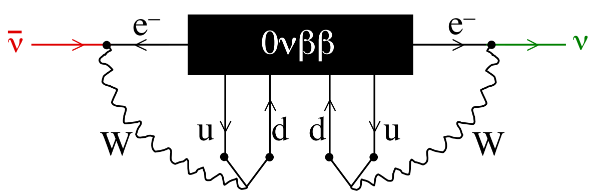
Neutrinoless Double Beta Decay [0νββ]



We are looking for a ***small*** Majorana neutrino mass. Thus, we will need ***a lot*** of parent nuclei (say, one ton of them).

Whatever diagrams cause 0νββ, its observation would imply the existence of a Majorana mass term:

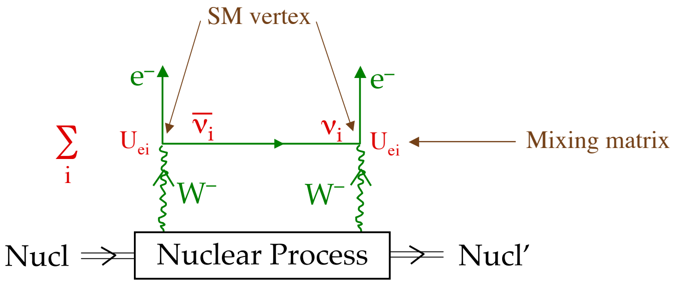
(Schechter and Valle)



$\bar{\nu} \rightarrow \nu$: A (tiny) Majorana mass term

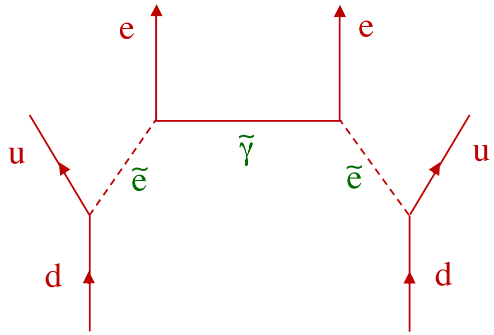
$\therefore$ 0νββ ➡ $\bar{\nu}_i = \nu_i$

We anticipate that $0\nu\beta\beta$ is dominated by a diagram with Standard Model vertices:

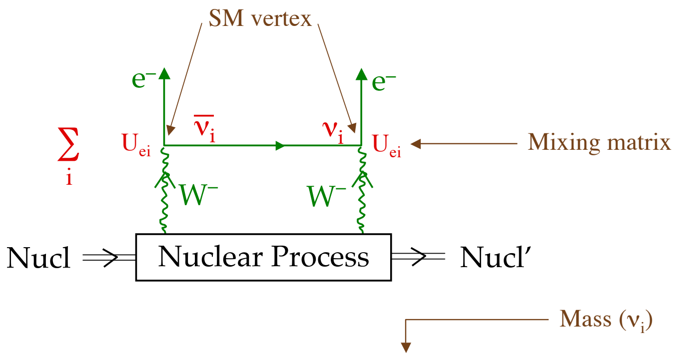


But there could be other contributions to $0\nu\beta\beta$, which at the quark level is the process $dd \rightarrow uuee$.

An example from Supersymmetry:



Assume the dominant mechanism is —



The $\overline{\nu}_i$ is emitted [RH + O{m_i/E}LH].

Thus, Amp [ν_i contribution] $\propto m_i$

$$\mathrm{Amp}[0\nu\beta\beta] \propto \left|\sum m_i U_{ei}^2\right| \equiv m_{\beta\beta}$$

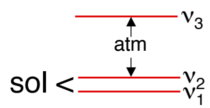
How Large is $m_{\beta\beta}$?

How sensitive need an experiment be?

Note: $\Gamma = m_{\beta\beta}^2\, |\text{Nuclear M.E.}|^2\, \text{Phase Space}$

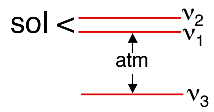
Suppose there are only 3 neutrino mass eigenstates. (More might help.)

Then the spectrum looks like —



Normal hierarchy

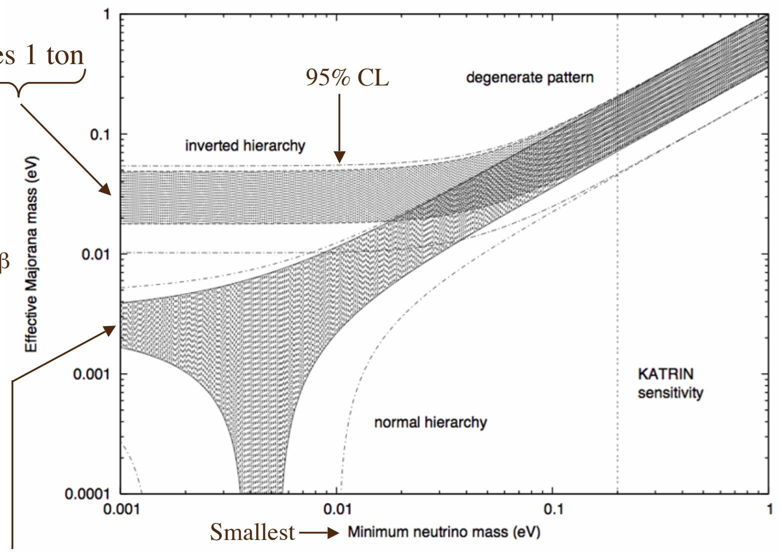
or



Inverted hierarchy

Takes 1 ton

$m_{\beta\beta}$



Takes
100 tons

$m_{\beta\beta}$ For Each Hierarchy

There is no clear theoretical preference
for either hierarchy.

If the hierarchy is **inverted**—

then $0\nu\beta\beta$ searches with sensitivity
to $m_{\beta\beta} = 0.01$ eV have
a very good chance to see a signal.

Sensitivity in this range is the target
for the next generation of experiments.