The currents

$$J_m \; = \; \bar{\Psi}\gamma_m\Psi \quad , \quad J_m^5 \; = \; \bar{\Psi}\gamma_m\gamma_5\Psi$$

satisfy

$$\partial^m J_m = 0 \quad , \quad \partial^m J_m^5 = 2iM\bar{\Psi}\gamma_5\Psi - \frac{g^2}{16\pi^2}\epsilon^{mnpq}\; F_{mn}\; F_{pq}$$

The last term is the quantum anomaly. Even if they are both classically conserved for $M = 0$, there is no regularization preserving both the vector and the axial conservation.

This explain why the $\eta'$ meson is not a pseudo-Goldstone for $U(2)_L \times U(2)_L = SU(2)_L \times SU(2)_R \times U(1)_B \times U(1)_A \to SU(2)_V \times U(1)_B$. Indeed,

$$J_m^{U(1)_A} = \bar{u}\gamma_m\gamma_5 u + \bar{d}\gamma_m\gamma_5 d$$

$$\partial^m J_m^{U(1)_A} \;=\; 2i(m_u\bar{u}u + m_d\bar{d}d) - \frac{3g^2}{16\pi^2}\epsilon^{mnpq}\; F_{mn}^A\; F_{pq}^A$$

Another manifestation of the axial anomaly is $\pi^0 \to \gamma\gamma$. Define the $SU(2)$ currents

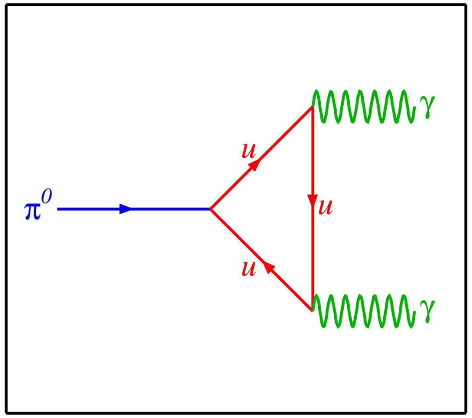
$$J_m^a = \bar{q}\gamma_m\tau^a q \quad , \quad J_m^{5a} = \bar{q}\gamma_m\gamma_5\tau^a q$$

Pions are Goldstone's $\Leftrightarrow \langle | J_m^{5a}(x) | \pi^b(p) \rangle = -ip_m f_\pi \delta^{ab} e^{-ipx}$. Axial isospin currents have no QCD anomalies, but $J_m^{5a}$ has an electromagnetic anomaly.

$$\partial^m J_m^{53} = -\frac{e^2}{32\pi^2}\epsilon^{mnpq}\, F_{mn}\, F_{pq}$$

$\pi^0 \to \gamma\gamma$ is related to the axial $U(1)_A$ anomaly.

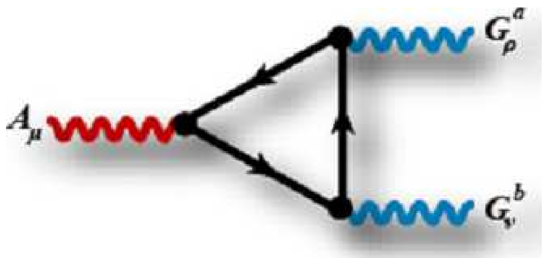
$\pi^0$ Decay




$\Rightarrow \Gamma(\pi^0 \to \gamma\gamma) = \frac{\alpha^2}{64\pi^3}\frac{m_\pi^3}{f_\pi^2}$, agreement with experiment.

- For gauge symmetries, if present, they generate inconsistencies, since it would violate gauge invariance of the theory :

$$\delta\mathcal{L} \sim \alpha_A \, \partial^m \, J_m^A$$



The corresponding currents are of chiral type

$$J_m^A \;=\; \bar{\Psi}\;\gamma_m\;\gamma_5\;T^a\;\Psi \;=\; \bar{\Psi}_R\gamma_m T^a \Psi_R \;-\; \bar{\Psi}_L\gamma_m T^a \Psi_L$$

and its divergence is proportional to

$$\partial^m\; J_m^A \;\sim\; \frac{g_A g_B}{16\pi^2}\; A^{ABC}\epsilon^{mnpq}\; F_{mn}^B\; F_{pq}^C\;,$$

where the anomaly coeff. that has to vanish is

$$A^{ABC} \;=\; tr\;(\{T^A, T^B\}T^C)_L - tr\;(\{T^A, T^B\}T^C)_R \;=\; 0\;,$$

where the trace is taken over all the fermions. For the SM, the only possible anomalies are (**Homework:**) $SU(2)_L^2 U(1)_Y$, $U(1)_Y^3$ and $SU(3)_c^2 U(1)_Y$. The results in the SM are

$$tr\ (\{\frac{\tau^a}{2},\frac{\tau^b}{2}\}Y)_L \ = \ \frac{1}{2}\delta^{ab}(trY)_L = 3\times(N_c\times\frac{1}{3}-1)=0\ ,$$
$$tr\ (\{Y,Y\}Y)_{L-R} = \cdots = 6(-2N_c+6)=0$$
$$tr\ (\{\frac{\lambda^A}{2},\frac{\lambda^B}{2}\}Y)_{L-R} \ = \ \frac{1}{3}\delta^{AB}(trY)_{L-R} = \cdots = 0$$

• Anomaly cancelation happens precisely for $N_c = 3$ !

• Provides a deep connection between quarks and leptons in the SM, hint towards Grand Unified Theories ?

Strong constraint on new chiral particles.

Homework : fourth lepton generation $l_4$, $E_R$ alone is inconsistent.

Similar diagrams generate new terms in the SM lagrangian from the redefs. of quarks we did to get the CKM matrix :

$$\mathcal{L}_\theta \sim \theta \, \frac{g^2}{16\pi^2} \, \epsilon^{mnpq} \, Tr(F_{mn} \, F_{pq})$$

The gluonic term violates CP and unless $\theta < 10^{-9}$, it generates a neutron dipole moment in conflict with exp. data $\rightarrow$ the strong CP problem.

One of possible solutions is the axion $a$. If :

- there is a new $U(1)_{PQ}$,spont. broken global symmetry, pseudo-Goldston boson $a$, symmetry breaking scale $f$.

- which has triangle anomalies $U(1)_{PQ} SU(3)_c^2$

then the anomaly generates new couplings

$$\frac{g^2}{16\pi^2} \frac{a(x)}{f} \epsilon^{mnpq} Tr(F_{mn} F_{pq}) \rightarrow \theta_{\text{eff}} = \theta + \frac{a}{f}$$

Non-perturbative QCD effects then generate an axion potential

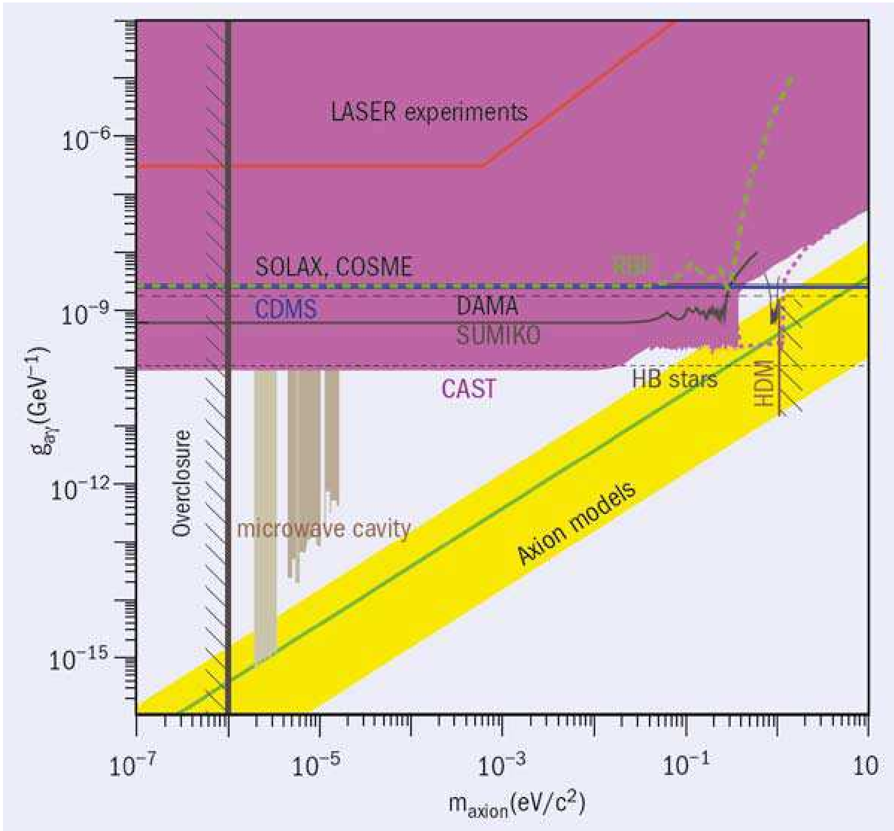
$$V \sim \Lambda^4_{QCD} \left[1 - \cos\left(\frac{a(x)}{f} + \theta\right)\right] .$$

The minimum is then for

$$\theta_{\text{eff}} = 0 , \quad \text{and the axion mass } m_a \sim \frac{\Lambda^2_{QCD}}{f} .$$

Axions were intensively searched since the 80's. They are also present in most SUSY and string extensions of the SM.

## Axion searches and constraints :



Comment : the anomaly is actually a total derivative :

$$\epsilon^{mnpq}\ Tr(F_{mn}\ F_{pq})\ =\ \partial^m\ K_m\ ,$$

where

$$K_\mu = 2\epsilon_{\mu\nu\alpha\beta}\left(A^{\nu a}\partial^\alpha A^{\beta a} + \frac{1}{3}f^{abc}A^{\nu a}A^{\alpha b}A^{\beta c}\right),$$

Despite this, classical configurations generate effects like theta angle, B and L number nonconservation.

## 6. The Higgs / Symmetry breaking sector of the Standard Model.

### 6.1.1 Perturbativity bounds

The RGE for the Higgs self-coupling in the SM is

$$16\pi^2 \frac{d\lambda}{d\ln\mu} = 24\lambda^2 - (3g'^2 + 3g^2 - 12h_t^2)\ \lambda$$
$$+\frac{3}{8}(g'^4 + 2g^2g'^2 + 3g^4) - 6h_t^4\ +\ \cdots\ ,$$

where $\cdots$ denote smaller Yukawas. In the large Higgs mass limit $\lambda >> g^2, h_t^2$, it reduces to

$$\frac{d\lambda}{\lambda^2} = \frac{3}{2\pi^2}\ d\ln\mu\ \rightarrow\ \frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2}\ln\frac{\Lambda}{\mu}\ .$$

This can be interpreted in two alternative ways :

i) If the Higgs mass is known, SM has a Landau pole (non-pert. regime) $\lambda(\Lambda) >> 1$ for

$$\Lambda \;=\; v\; e^{\frac{2\pi^2}{3\lambda}} \;=\; v\; e^{\frac{4\pi^2 v^2}{3M_h^2}}$$

ii) Conversely, asking for perturbativity up to scale $\Lambda$ (say $M_{GUT}$), we obtain an upper bound on the Higgs mass (**homework**)

$$M_h^2 \;\leq\; \frac{4\pi^2 v^2}{3\ln\frac{\Lambda}{v}}\;.$$

### 6.1.2 Stability bounds

SM has another instability in the small Higgs mass limit,

since $\lambda$ can become negative at high-energy.

If $\lambda << h_t^2$, the leading RGE's are

$$16\pi^2 \frac{d\lambda}{d\ln\mu} \;=\; -6h_t^4 \;,\; 16\pi^2 \frac{dh_t}{d\ln\mu} \;=\; \frac{9h_t^3}{2}$$

which integrate to (**homework**)

$$\lambda(\mu) \;=\; \lambda(\lambda) + \frac{\frac{3h_t^4(\Lambda)}{8\pi^2}\ln\frac{\Lambda}{\mu}}{1+\frac{9h_t^2(\Lambda)}{16\pi^2}\ln\frac{\Lambda}{\mu}} \;,$$

$$h_t^2(\mu) \;=\; \frac{h_t^2}{1+\frac{9h_t^2(\Lambda)}{16\pi^2}\ln\frac{\Lambda}{\mu}} \;.$$

This can be interpreted in two ways :

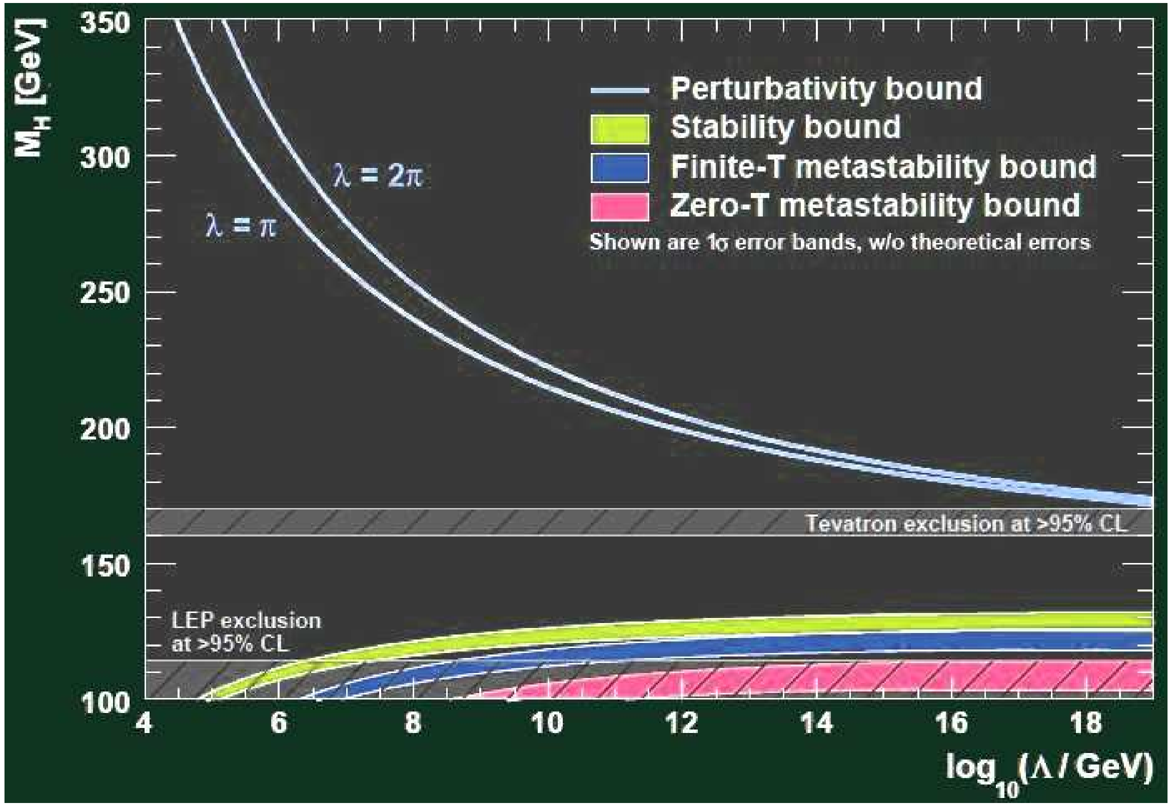
i) For a fixed, known value of the Higgs mass : take $\mu = v$. Then, new physics should show up before the scale $\Lambda$ where $\lambda(\Lambda) = 0$

$$\Lambda \;\leq\; v\; e^{\frac{8\pi^2\lambda}{3h_t^4}} \;=\; v\; e^{\frac{4\pi^2 M_h^2}{3h_t^4 v^2}}$$

ii) For a fixed $\Lambda$, we get a lower bound on the Higgs mass (**homework**)

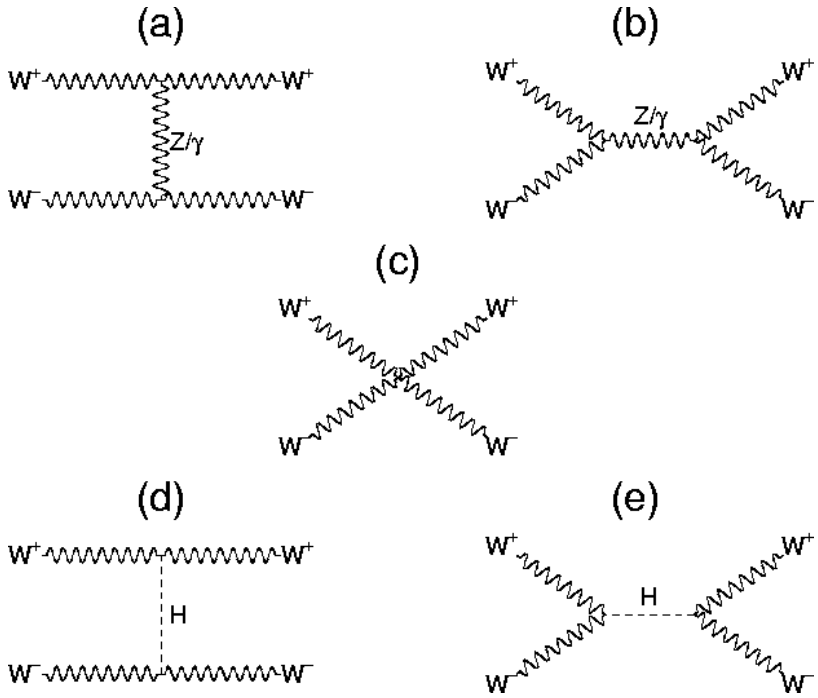
$$M_h^2 \;\geq\; \frac{3h_t^4 v^2}{4\pi^2}\ln\frac{\Lambda}{v} \;=\; \frac{3m_t^4}{\pi^2 v^2}\ln\frac{\Lambda}{v}$$

These theoretical Higgs mass limits are summarized in the following plot



## - 6.2. $W\ W$ scattering and unitarity.

Let us consider the longitudinal $W_L W_L \to W_L W_L$ scattering



For a massive gauge particle of momentum $k$ and mass $M_W$, $A_m = \epsilon_m \; e^{ikx}$, the three polarizations satisfy $\epsilon_m \epsilon^m = -1$, $k_m \epsilon^m = 0$. For $k^m = (E, 0, 0, k)$, they are

$$\begin{aligned} \text{transverse} \;\; &: \;\; \epsilon_1^m = (0,1,0,0) \quad , \quad \epsilon_2^m = (0,0,1,0) \; , \\ \text{longitudinal} \;\; &: \;\; \epsilon_L^m = (\frac{k}{M_W}, 0, 0, \frac{E}{M_W}) \sim \frac{k^m}{M} + \mathcal{O}(\frac{E}{M_W}) \; . \end{aligned}$$

Since the longitudinal polarization is proportional to the energy, we expect a tree-level amplitude behaving as

$$\mathcal{A} \;=\; \mathcal{A}^{(4)} (\frac{E}{M_W})^4 \;+\; \mathcal{A}^{(2)} (\frac{E}{M_W})^2 + \cdots$$

Actually, the diagrams a),b) and c) give $\mathcal{A} = g^2 (\frac{E}{M_W})^2$.
On the other hand, unitarity constrains the amplitude to stay small enough at any energy.

Start with the unitarity of the S-matrix $S^\dagger S = 1$. Then

$$S = 1 + i\mathcal{A} \quad \rightarrow \quad i(\mathcal{A} - \mathcal{A}^\dagger) + \mathcal{A}^\dagger \mathcal{A} = 0$$

Let us sandwich this eq. between a two-particle state $|i>$ :

$$i(\mathcal{A} - \mathcal{A}^\dagger)_{ii} + \sum_f |\mathcal{A}_{fi}|^2 = 0 \qquad (73)$$

which is the optical theorem : the imaginary part of the forward amplitude of the process $i \rightarrow i$ is proportional to the total cross section of $i \rightarrow$ anything.

Let us decompose the scattering amplitude into partial waves

$$\mathcal{A} = \sum_{l=0}^{\infty} (2l+1)\ P_l(\cos\theta) a_l\ ,$$

where $a_l$ are partial wave amplitudes of elastic scattering of two particles. Projecting (73) into the partial wave $l$ gives $Im\ a_l = |a_l|^2$ . This is only possible if

$$|Re\ a_l| \leq 1/2\ ,\ 0 \leq Im\ a_l \leq 1 \quad \rightarrow \quad |a_l|^2 \leq 5/4\ ,$$

which is the unitarity bound we were searching for.

- For the SM without the Higgs boson

$a_0 = \dfrac{g^2 E^2}{M_W^2} \quad \rightarrow \quad$ unitarity breaks down for $\sqrt{s} \sim 1.2\, TeV$

With the Higgs boson, amplitudes d),e) cancel the raising energy term, such that

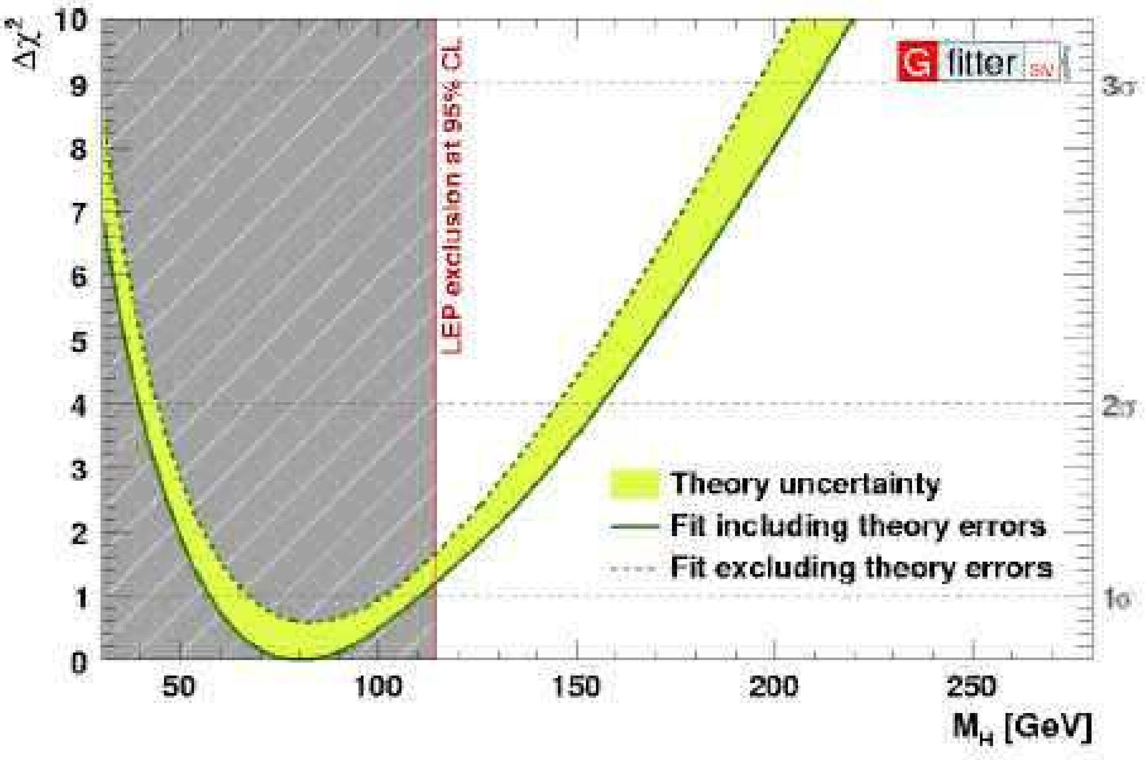
$$a_0 = \frac{g^2 M_H^2}{4 M_W^2} \quad \rightarrow \quad \text{unitarity breaks down unless } M_H \leq 1.2 \text{ TeV}$$

By considering other channels, one get the stronger bound $M_H \leq 800$ GeV.

Intepretation :

- If LHC finds no Higgs with a mass $M_H \leq 800 GeV$, unitarity of S-matrix will be violated ! New light degrees of freedom should exist in order to restore unitarity $\rightarrow$ the no-loose "theorem" for LHC.

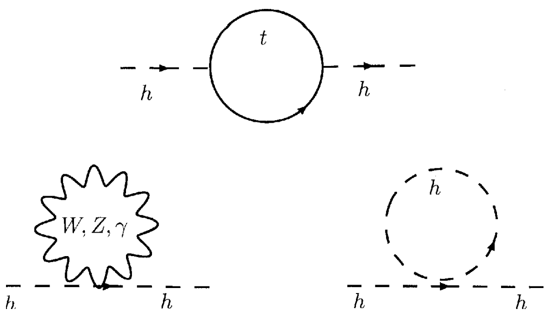
Most theories have a biased towards a light Higgs, since it provides a better fit for the SM precision tests.



## Higgs and the hierarchy problem

Quantum corrections to the Higgs mass in the SM are quadratically divergent

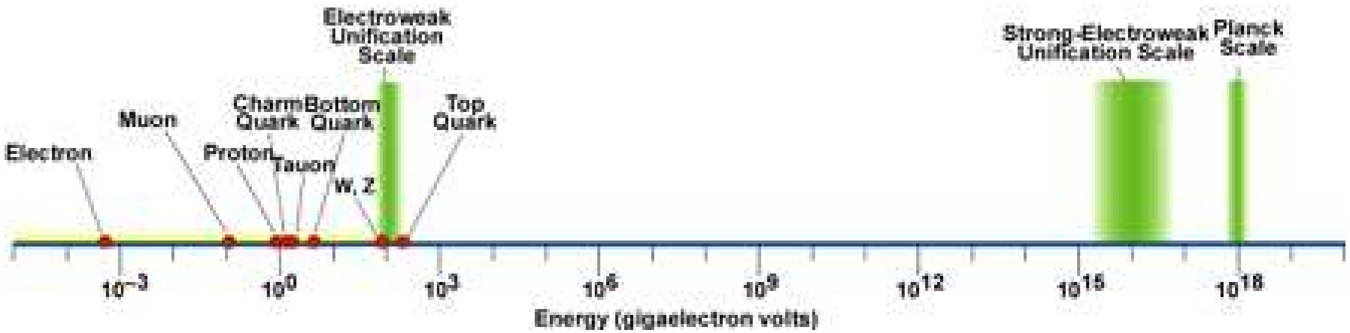
$$\delta m_h^2 \simeq \frac{3\Lambda^2}{8\pi^2 v^2}(4m_t^2 - 4M_W^2 - 2M_Z^2 - m_h^2)$$

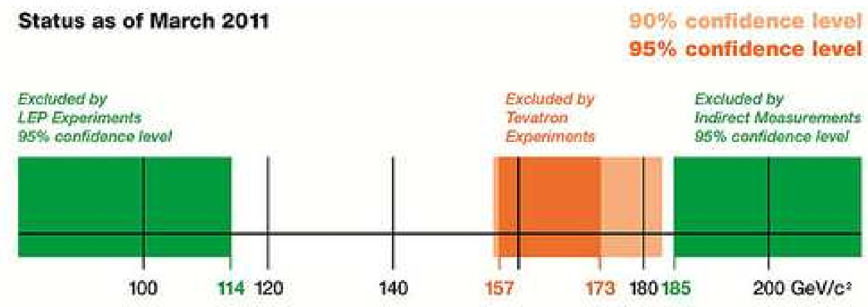


In a theory including gravity or GUT's, $\Lambda$ is physical mass scale $\Lambda = M_P, M_{GUT}$. It is then difficult to understand why

$$m_h^2 = (m_h^0)^2 + \frac{3\Lambda^2}{8\pi^2 v^2}(4m_t^2 - 4M_W^2 - 2M_Z^2 - m_h^2) \sim v^2 << \Lambda^2$$

$\rightarrow$ the hierarchy problem.






Latest news ("Lepton-Photon", august 2011): Both ATLAS+CMS exclude the SM Higgs at 95 % CL for

$145 \leq M_H \leq 446\ GeV$ except $288-296\ GeV$

M. Peskin (LP2011) "There is therefore strong evidence that either :

- Higgs is light, compatible with electroweak precision tests and theoretical prejudice, or
- the Higgs boson is very heavy and strongly self-coupled".

## Can Standard Model be the final theory ?

# NO

- No neutrino masses at the renormalizable level (lect. Boris).

- misterious hierarchies in the quarks/lepton masses and mixings (lect. Yuval).

- No Dark Matter candidate (lect. Bogdan).

- problem with the radiative stability of the electroweak scale ("the hierarchy problem").

- no accurate gauge coupling unification.

Last three problems $\Rightarrow$ SUPERSYMMETRY ?

- the strong CP problem.

- gravity not incorporated into a renormalizable framework $\Rightarrow$ STRING THEORY ?

- cosmological constant problem

$\Lambda \sim 10^{-4}\ eV^4 \sim\ 10^{-120}\ M_P^4.$

**YES**

- no signal of new physics yet... But if no SM higgs the next year, something else must replace it...