# Top Quark Theoretical Cross Sections and $p_T$ and Rapidity Distributions
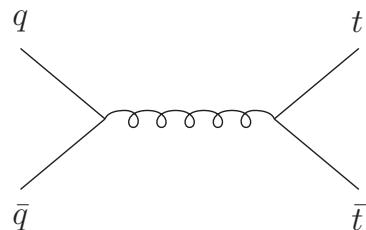
## Nikolaos Kidonakis

**(Kennesaw State University)**

- $t\bar{t}$ and single top production channels

- Higher-order two-loop corrections

- $t\bar{t}$ cross section at Tevatron and LHC

- Top $p_T$ and $Y$ distributions at Tevatron and LHC

- Top forward-backward asymmetry at Tevatron

- $t$-channel production at Tevatron and LHC

- $s$-channel production at Tevatron and LHC

- Associated production of a top with a $W^-$ or $H^-$

# Partonic processes at LO
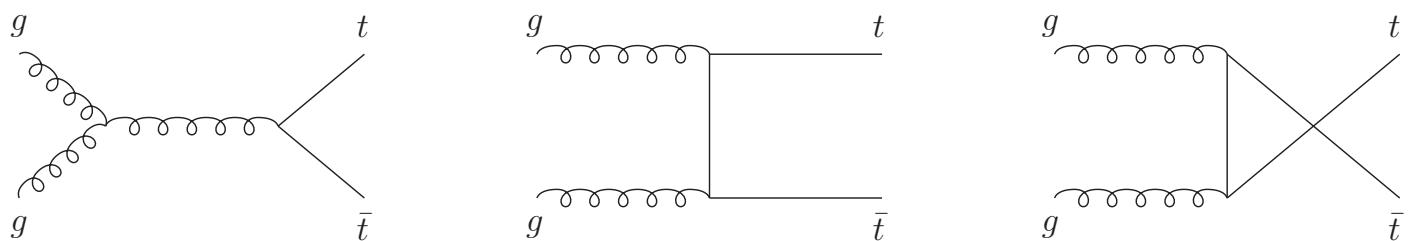
## Top-antitop pair production
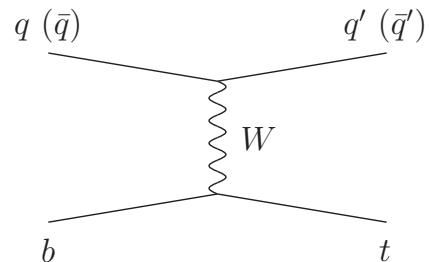
- $q\bar{q} \to t\bar{t}$

  **dominant at Tevatron**



- $gg \to t\bar{t}$

  **dominant at LHC**
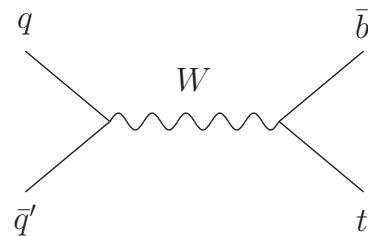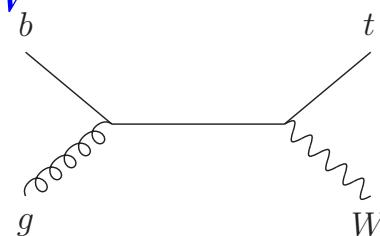
# Single top quark production

- **$t$ channel:** $qb \to q't$ **and** $\bar{q}b \to \bar{q}'t$
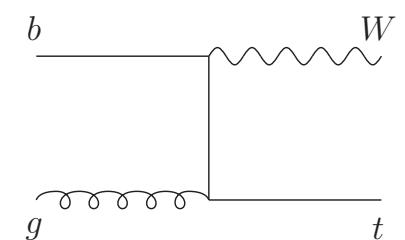
**dominant at Tevatron and LHC**



- **$s$ channel:** $q\bar{q}' \to \bar{b}t$

**small at Tevatron and LHC**



- **associated $tW$ production:** $bg \to tW^-$

**very small at Tevatron, significant at LHC**

**Related process:** $bg \to tH^-$

# Higher-order corrections

**QCD corrections significant for top pair and single top quark production**

**Soft-gluon corrections from emission of soft (low-energy) gluons**

**Soft corrections:** $\left[\frac{\ln^{k}(s_4/m^2)}{s_4}\right]_{+}$ **with** $k \leq 2n-1$, $s_4$ **distance from threshold**

**Soft-gluon corrections are dominant near threshold**

**Resum these soft corrections - factorization and RGE**

**At NLL accuracy requires one-loop calculations in eikonal approximation**

**Complete results at NNLL–two-loop soft anomalous dimension**
NK, Phys. Rev. Lett. 102, 232003 (2009), arXiv:0903.2561 [hep-ph];

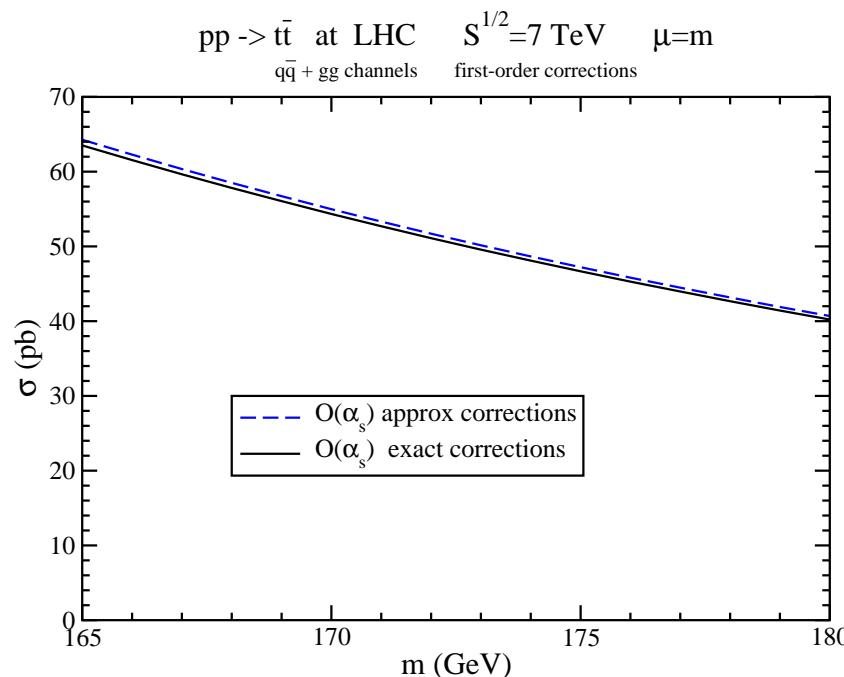Phys. Rev. D 82, 114030 (2010), arXiv:1009.4935 [hep-ph]

**Approximate NNLO cross section from expansion of resummed cross section**
Calculation is at differential cross section level
1PI kinematics refer to partonic threshold (not just absolute threshold)
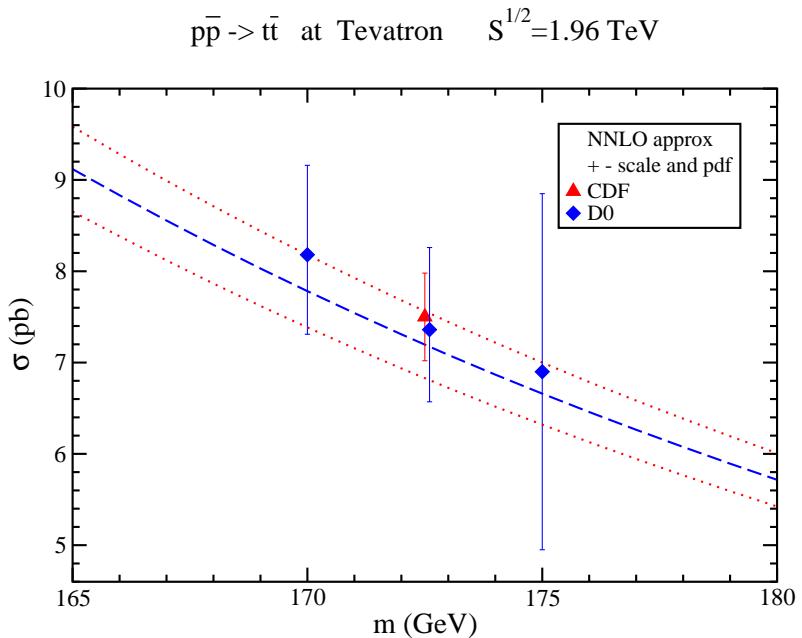
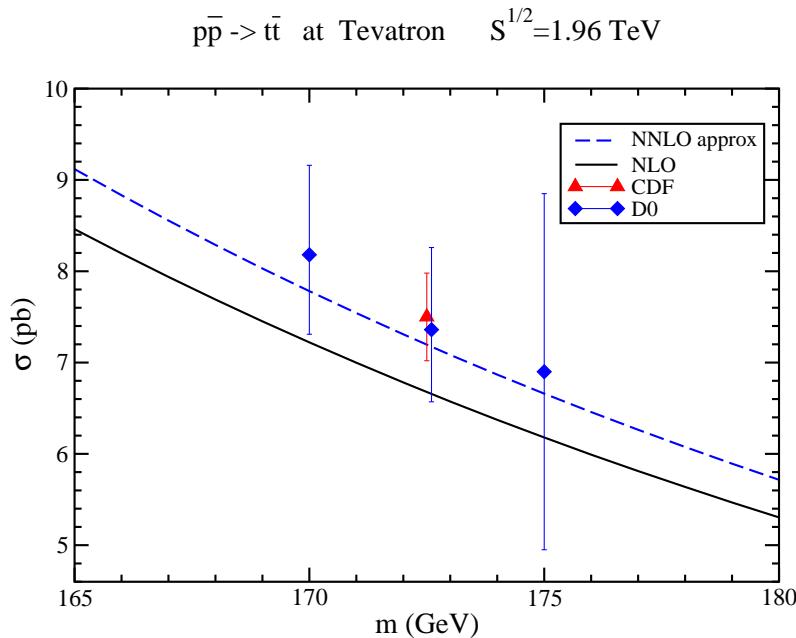# Threshold approximation

**Approximation works very well not only for Tevatron but also for LHC energies because partonic threshold is still important**



only 1% difference between first-order approximate and exact corrections
$\rightarrow$ less than 1% difference between NLO approximate and exact cross sections

For best prediction add NNLO approximate corrections to exact NLO cross section
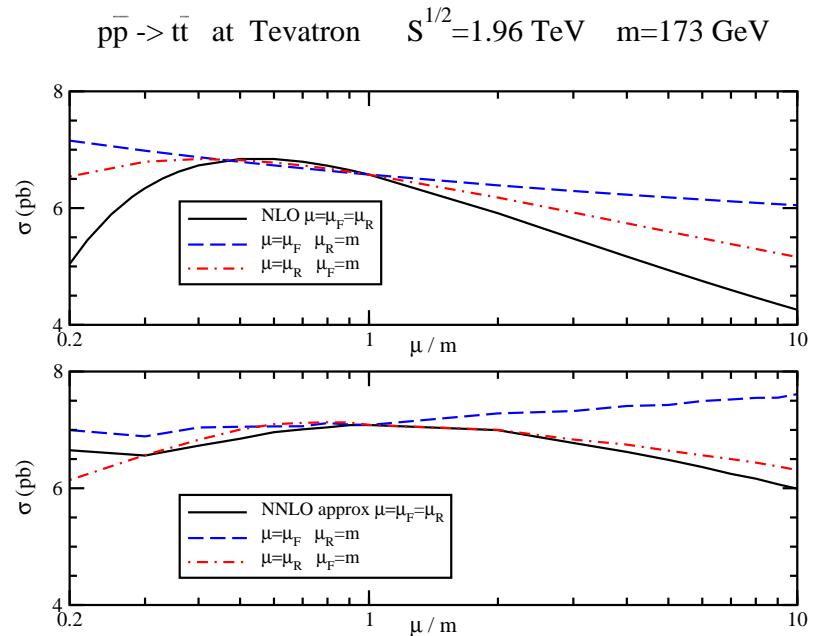
# $t\bar{t}$ cross section at the Tevatron

$p\bar{p} \rightarrow t\bar{t}$ at Tevatron $\quad$ S$^{1/2}$=1.96 TeV

$p\bar{p} \rightarrow t\bar{t}$ at Tevatron $\quad$ S$^{1/2}$=1.96 TeV
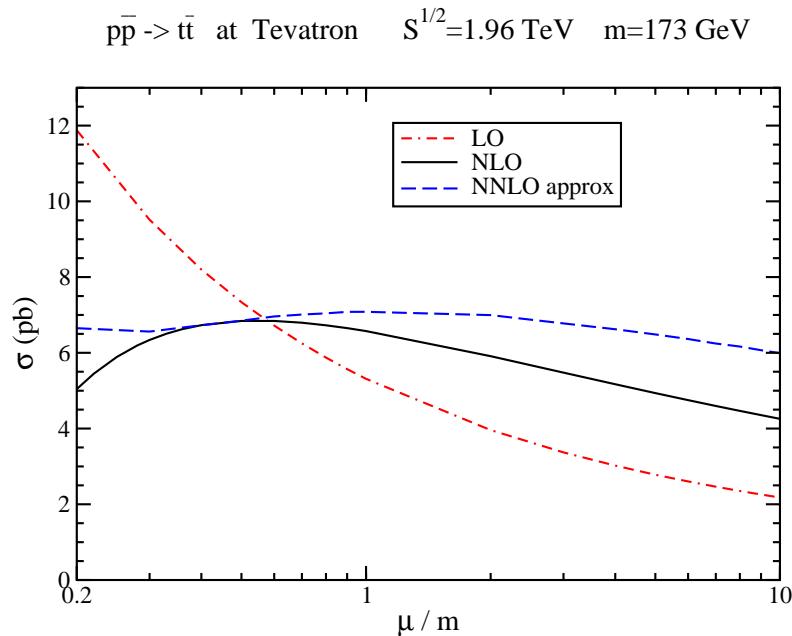


$$\sigma_{t\bar{t}}^{\text{NNLOapprox}}(m_t = 173\,\text{GeV},\, 1.96\,\text{TeV}) \quad = \quad 7.08^{+0.00+0.36}_{-0.24-0.27}\,\text{pb}$$

$$\text{scale} \quad \text{pdf}$$

## NNLO approx: 7.8% enhancement over NLO
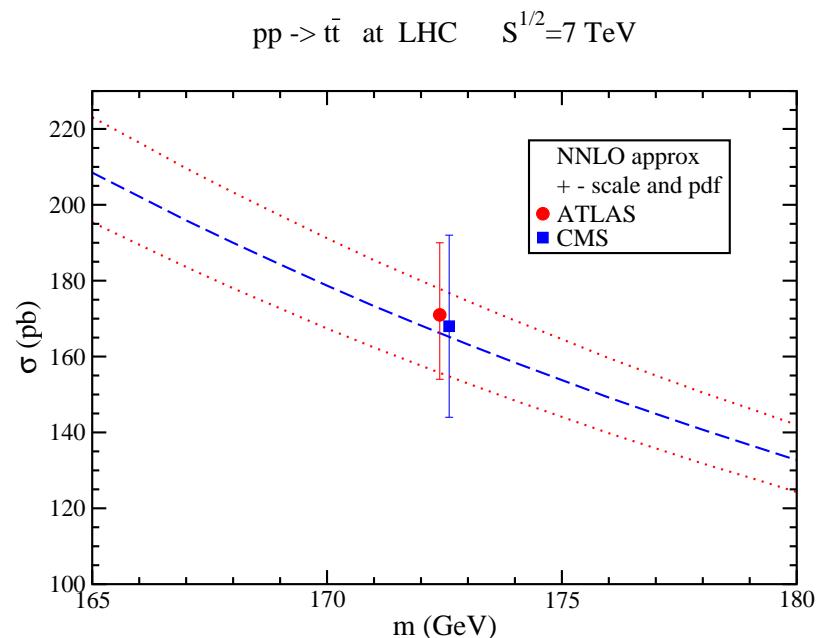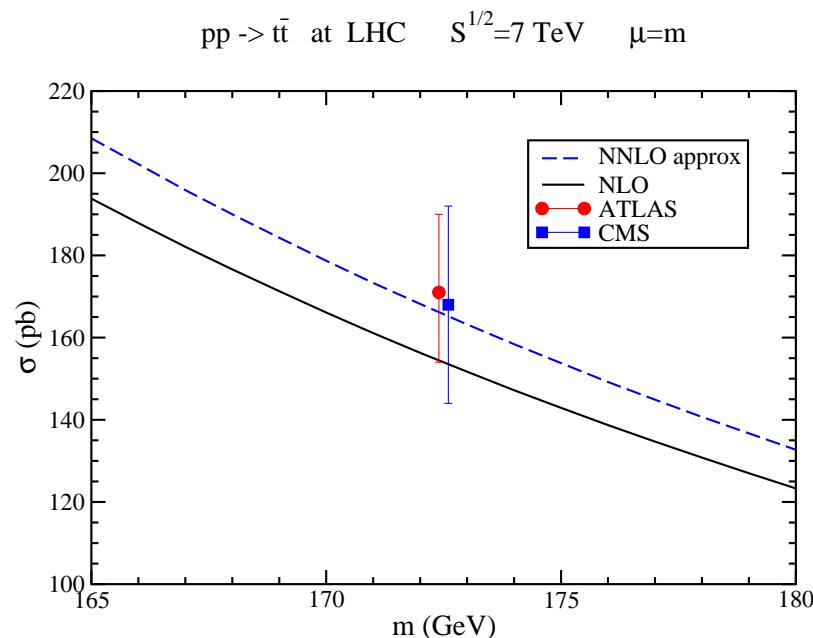
**used MSTW 2008 NNLO pdf**

# $t\bar{t}$ cross section at the Tevatron



pp̄ -> tt̄  at  Tevatron     $S^{1/2}$=1.96 TeV    m=173 GeV

LO
NLO
NNLO approx

σ (pb)

μ / m

pp̄ -> tt̄  at  Tevatron     $S^{1/2}$=1.96 TeV    m=173 GeV

NLO $\mu=\mu_F=\mu_R$
$\mu=\mu_F$  $\mu_R=m$
$\mu=\mu_R$  $\mu_F=m$

σ (pb)

NNLO approx $\mu=\mu_F=\mu_R$
$\mu=\mu_F$  $\mu_R=m$
$\mu=\mu_R$  $\mu_F=m$

σ (pb)

μ / m

**scale dependence greatly reduced over a large range**
**separate factorization and renormalization scale dependence calculated**
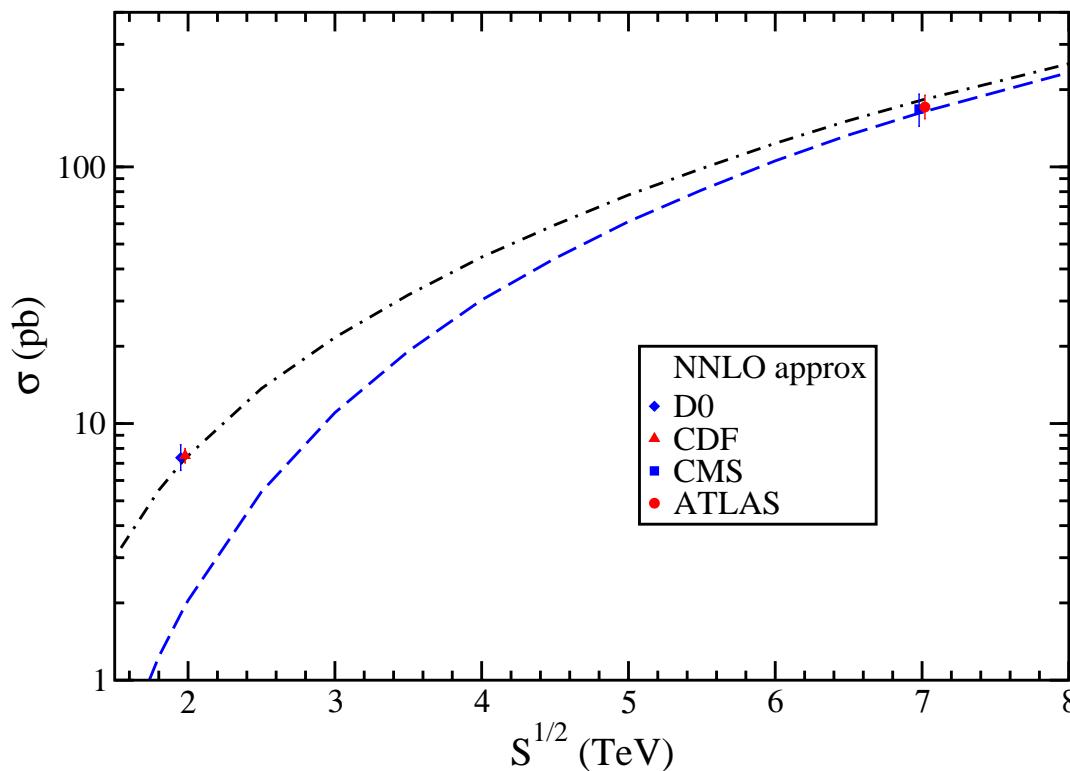
# $t\bar{t}$ cross section at the LHC



pp -> t̄t  at  LHC      $S^{1/2}$=7 TeV      μ=m

pp -> t̄t  at  LHC      $S^{1/2}$=7 TeV

$$\sigma_{t\bar{t}}^{\mathrm{NNLOapprox}}(m_t = 173\,\mathrm{GeV},\, 7\,\mathrm{TeV}) \quad = \quad 163^{+7+9}_{-5-9}\,\mathrm{pb}$$

$$\sigma_{t\bar{t}}^{\mathrm{NNLOapprox}}(m_t = 173\,\mathrm{GeV},\, 14\,\mathrm{TeV}) \quad = \quad 920^{+50+33}_{-39-35}\,\mathrm{pb}$$

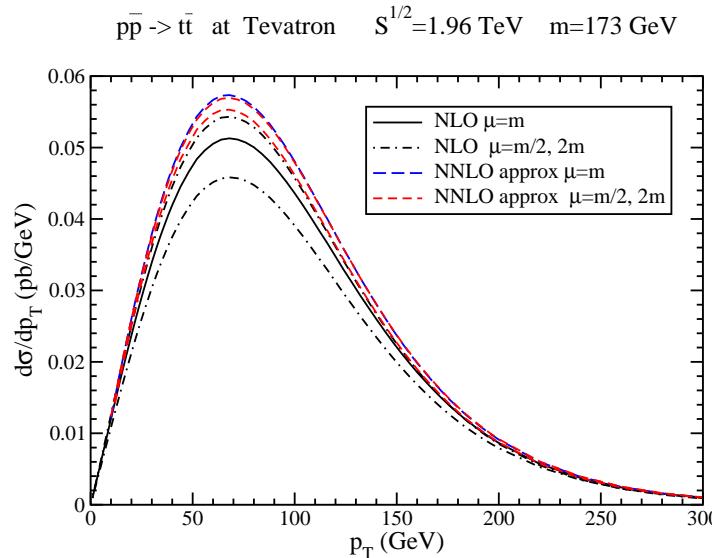**NNLO approx: enhancement over NLO is 7.6% at 7 TeV; 8.0% at 14 TeV**

# $t\bar{t}$ cross section versus energy
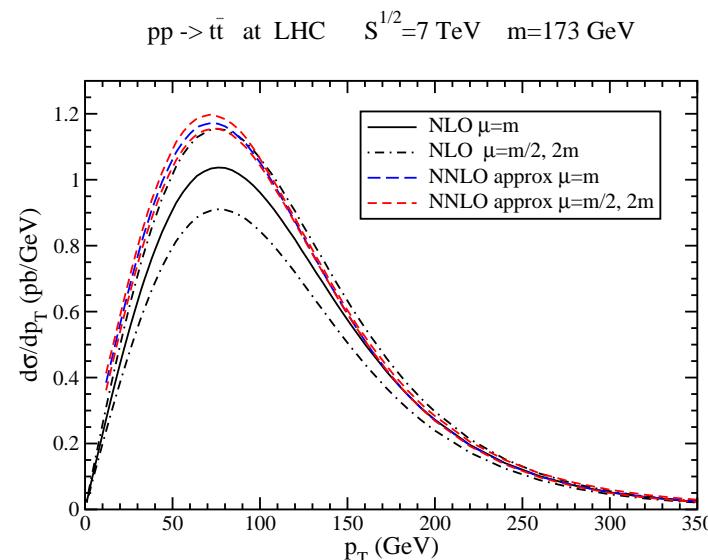
$t\bar{t}$  cross  section  at  p$\bar{\text{p}}$  and  pp  colliders



**Tevatron and LHC data are consistent with theory**

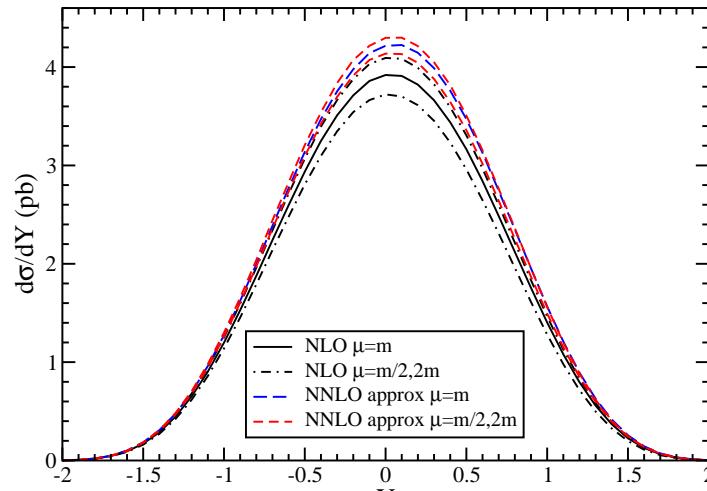# Top quark $p_T$ distribution at Tevatron and LHC

$p\bar{p} \rightarrow t\bar{t}$ at Tevatron $\quad$ S$^{1/2}$=1.96 TeV $\quad$ m=173 GeV



Good agreement

with Tevatron data

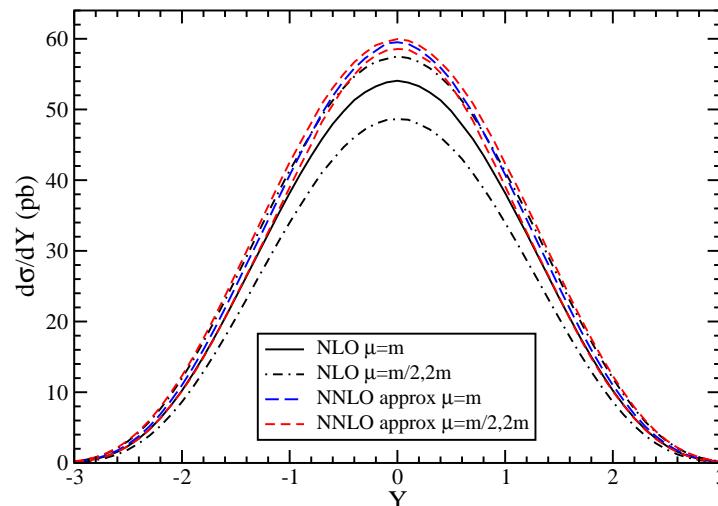$pp \rightarrow t\bar{t}$ at LHC $\quad$ S$^{1/2}$=7 TeV $\quad$ m=173 GeV

# Top quark rapidity distribution at Tevatron and LHC

Top quark rapidity at Tevatron $S^{1/2}$=1.96 TeV  m=173 GeV



Top quark rapidity at LHC  $S^{1/2}$=7 TeV  m=173 GeV

# Top Forward-backward asymmetry

$gg$ **channel is symmetric at all orders**

$q\bar{q}$ **channel is asymmetric starting at NLO**

$$A_{\text{FB}} = \frac{\sigma(Y > 0) - \sigma(Y < 0)}{\sigma(Y > 0) + \sigma(Y < 0)}$$
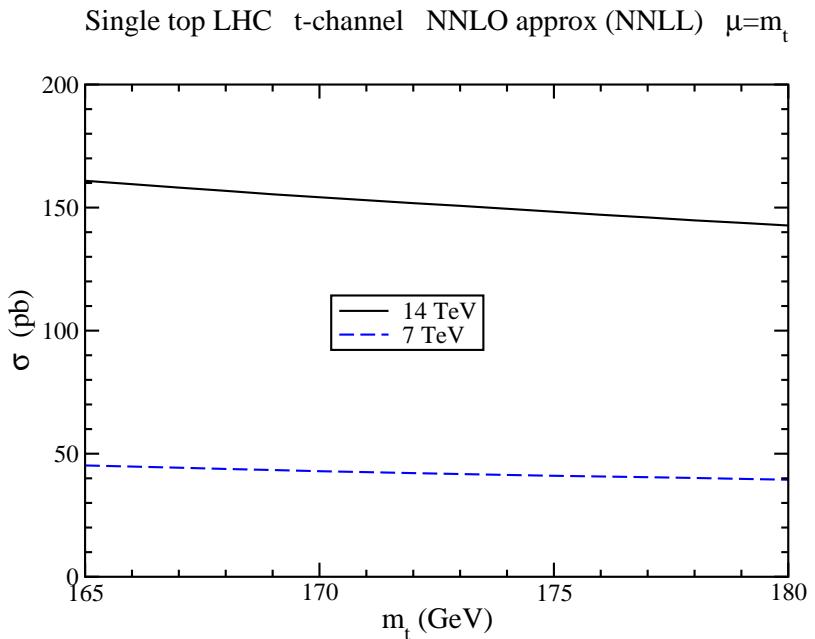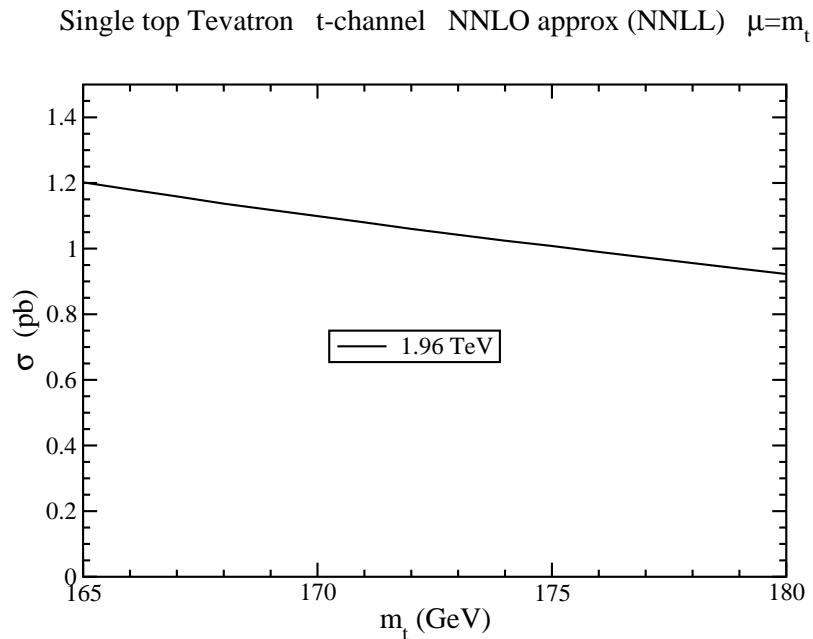
**Asymmetry significant at the Tevatron**

**Theoretical result at Tevatron:** $A_{\text{FB}} = 0.052^{+0.000}_{-0.006}$

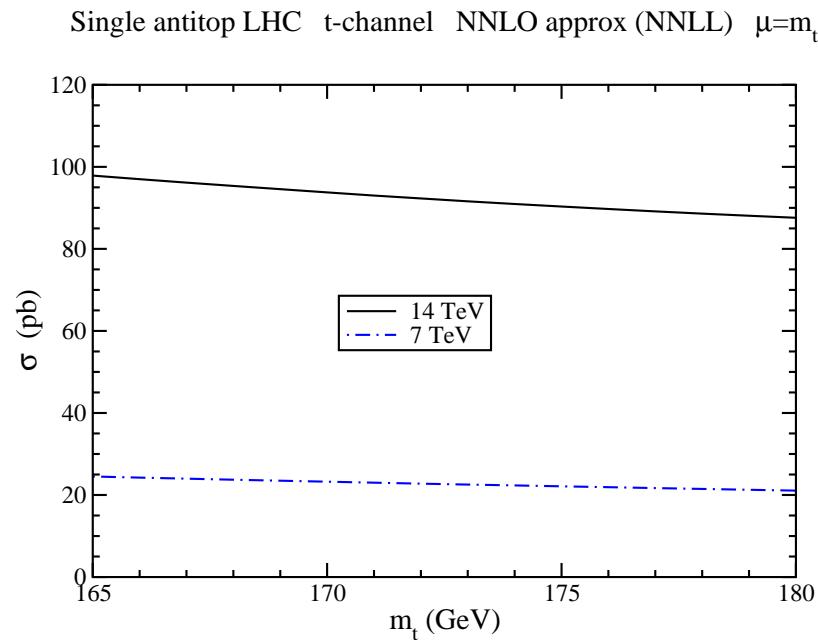NK, Phys. Rev. D 84, 011504(R) (2011), arXiv:1105.5167 [hep-ph]

**Much smaller than current experimental values**

# Single top quark production at Tevatron and LHC - $t$ channel

Single top Tevatron   t-channel   NNLO approx (NNLL)   $\mu=m_t$

Single top LHC   t-channel   NNLO approx (NNLL)   $\mu=m_t$



$$\sigma^{\mathrm{NNLOapprox,\,top}}_{t-\mathrm{channel}}(m_t = 173\,\mathrm{GeV},\, 1.96\,\mathrm{TeV}) = 1.04^{+0.00}_{-0.02} \pm 0.06 \text{ pb}$$

$$\sigma^{\mathrm{NNLOapprox,\,top}}_{t-\mathrm{channel}}(m_t = 173\,\mathrm{GeV},\, 7\,\mathrm{TeV}) = 41.7^{+1.6}_{-0.2} \pm 0.8 \text{ pb}$$

$$\sigma^{\mathrm{NNLOapprox,\,top}}_{t-\mathrm{channel}}(m_t = 173\,\mathrm{GeV},\, 14\,\mathrm{TeV}) = 151^{+4}_{-1} \pm 3 \text{ pb}$$

# Single antitop production at LHC - $t$ channel



Single antitop LHC   t-channel   NNLO approx (NNLL)   $\mu = m_t$
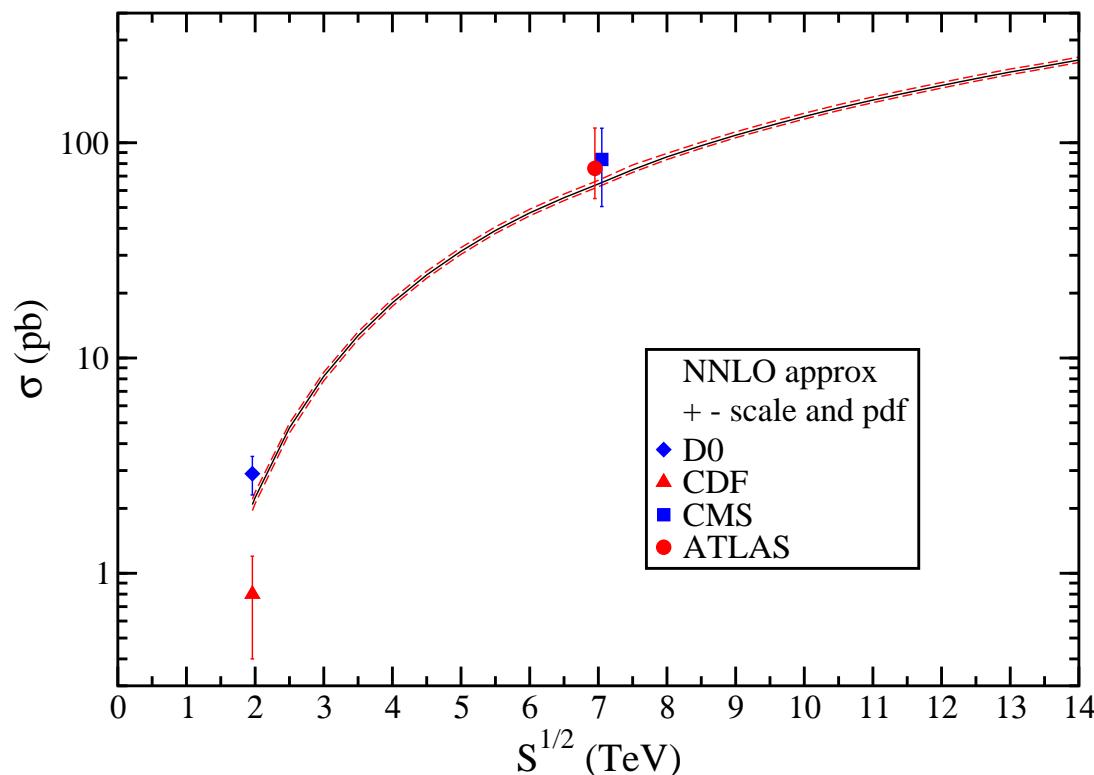
$$\sigma_{t-\text{channel}}^{\text{NNLOapprox, antitop}}(m_t = 173\,\text{GeV},\,7\,\text{TeV}) \quad = \quad 22.5 \pm 0.5^{+0.7}_{-0.9}\,\text{pb}$$

$$\sigma_{t-\text{channel}}^{\text{NNLOapprox, antitop}}(m_t = 173\,\text{GeV},\,14\,\text{TeV}) \quad = \quad 92^{+2+2}_{-1-3}\,\text{pb}$$

**NK, Phys. Rev. D 83, 091503(R) (2011), arXiv:1103.2792 [hep-ph]**

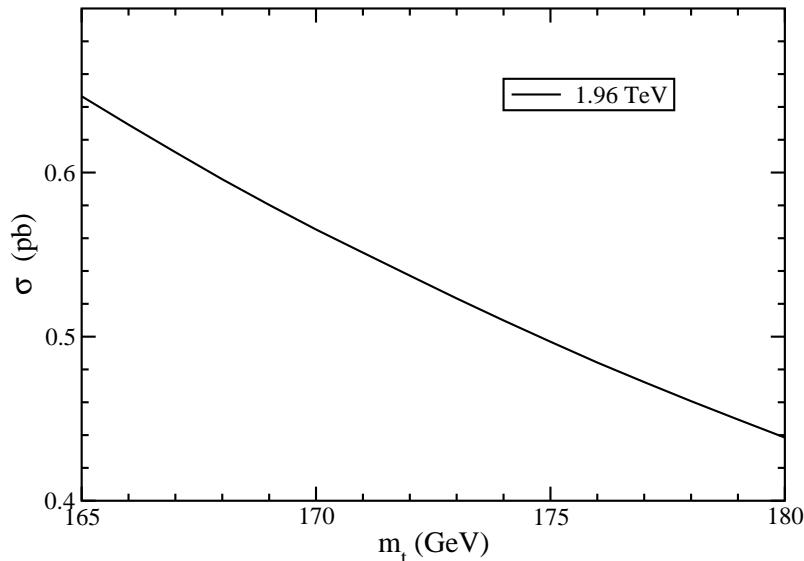# *t*-channel combined cross section versus energy

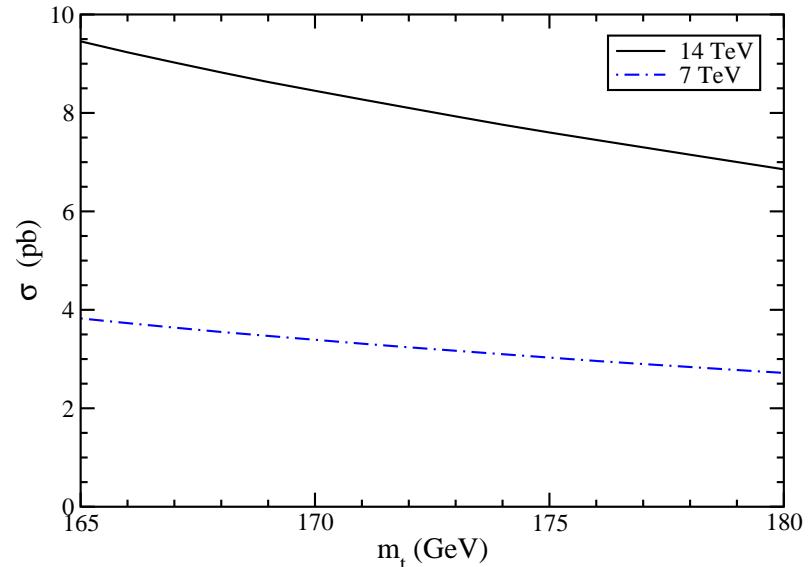t-channel single top + single antitop cross section



**Tevatron and LHC results are consistent with theory**

# Single top quark production at Tevatron and LHC - $s$ channel

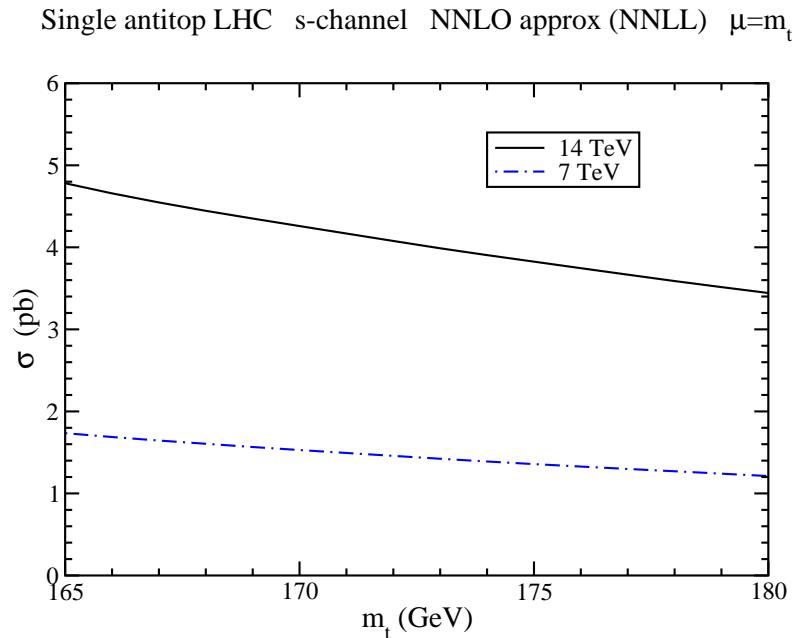Single top Tevatron  s-channel  NNLO approx (NNLL)  $\mu = m_t$

Single top LHC  s-channel  NNLO approx (NNLL)  $\mu = m_t$



$$\sigma^{\text{NNLOapprox, top}}_{s-\text{channel}}(m_t = 173\,\text{GeV},\, 1.96\,\text{TeV}) \;=\; 0.523^{+0.001\,+0.030}_{-0.005\,-0.028}\ \text{pb}$$

$$\sigma^{\text{NNLOapprox, top}}_{s-\text{channel}}(m_t = 173\,\text{GeV},\, 7\,\text{TeV}) \;=\; 3.17 \pm 0.06^{+0.13}_{-0.10}\ \text{pb}$$

$$\sigma^{\text{NNLOapprox, top}}_{s-\text{channel}}(m_t = 173\,\text{GeV},\, 14\,\text{TeV}) \;=\; 7.93 \pm 0.14^{+0.31}_{-0.28}\ \text{pb}$$

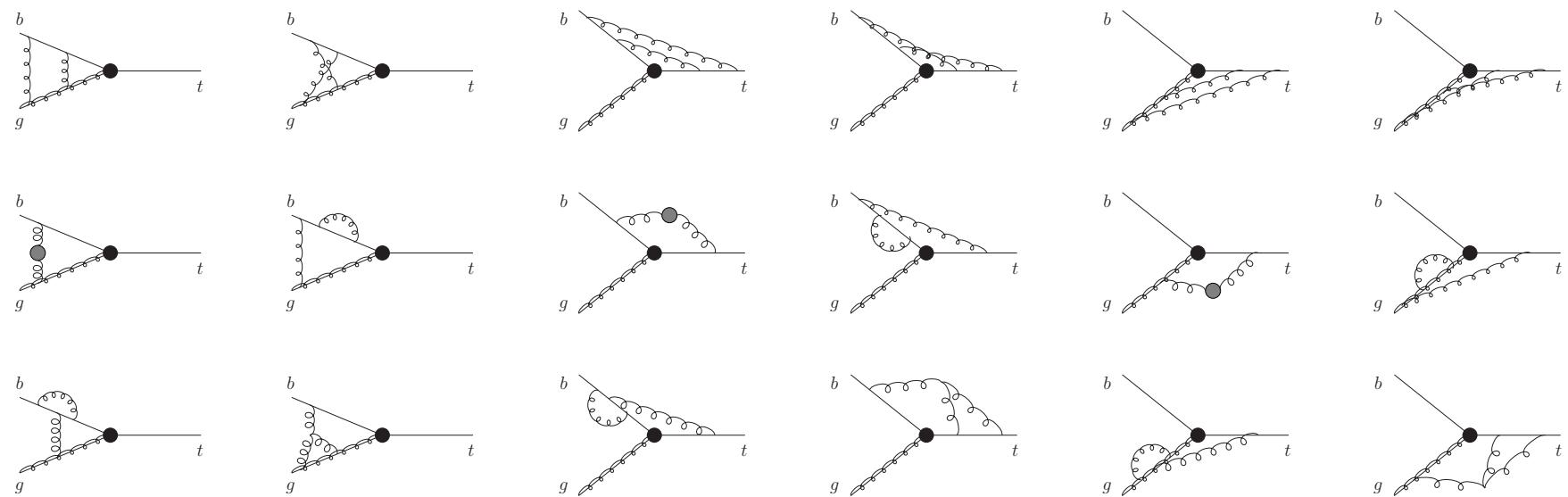**NNLO approx: enhancement over NLO is 15% at Tevatron; 13% at LHC**

# Single antitop production at LHC - $s$ channel

Single antitop LHC   s-channel   NNLO approx (NNLL)   $\mu = m_t$



$$\sigma^{\text{NNLOapprox, antitop}}_{s-\text{channel}}(m_t = 173\,\text{GeV}, 7\,\text{TeV}) \quad = \quad 1.42 \pm 0.01^{+0.06}_{-0.07}\,\text{pb}$$

$$\sigma^{\text{NNLOapprox, antitop}}_{s-\text{channel}}(m_t = 173\,\text{GeV}, 14\,\text{TeV}) \quad = \quad 3.99 \pm 0.05^{+0.14}_{-0.21}\,\text{pb}$$

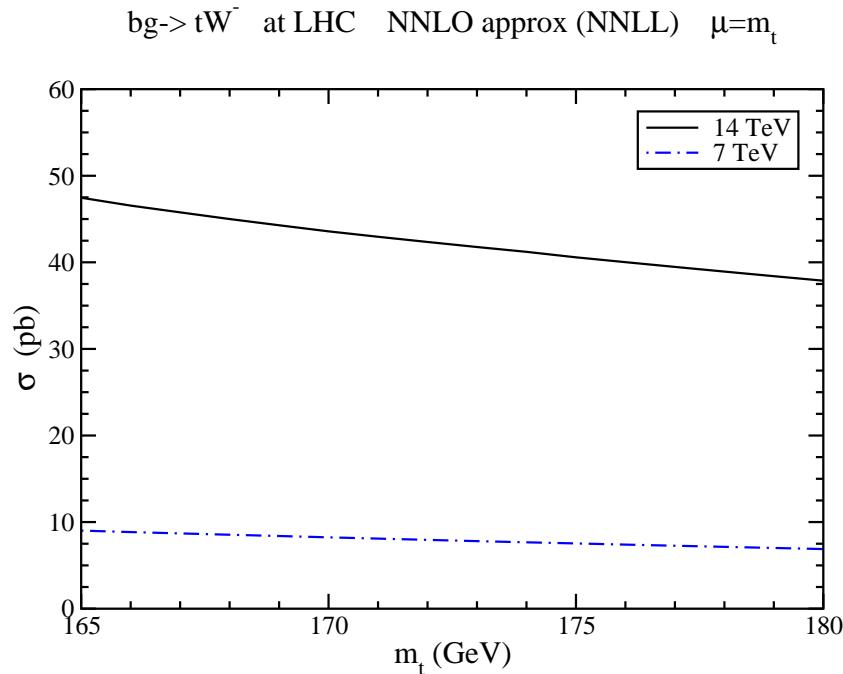**NK, Phys. Rev. D 81, 054028 (2010), arXiv:1001.5034 [hep-ph]**

# Associated production of a top quark with a $W^-$
## Two-loop eikonal diagrams (+ top-quark self-energy graphs)



## Soft anomalous dimension for $bg \to tW^-$ (and for $bg \to tH^-$) from UV poles in dimensional regularization

# Associated $tW^-$ production at the LHC

bg-> tW$^-$   at LHC   NNLO approx (NNLL)   $\mu$=m$_t$
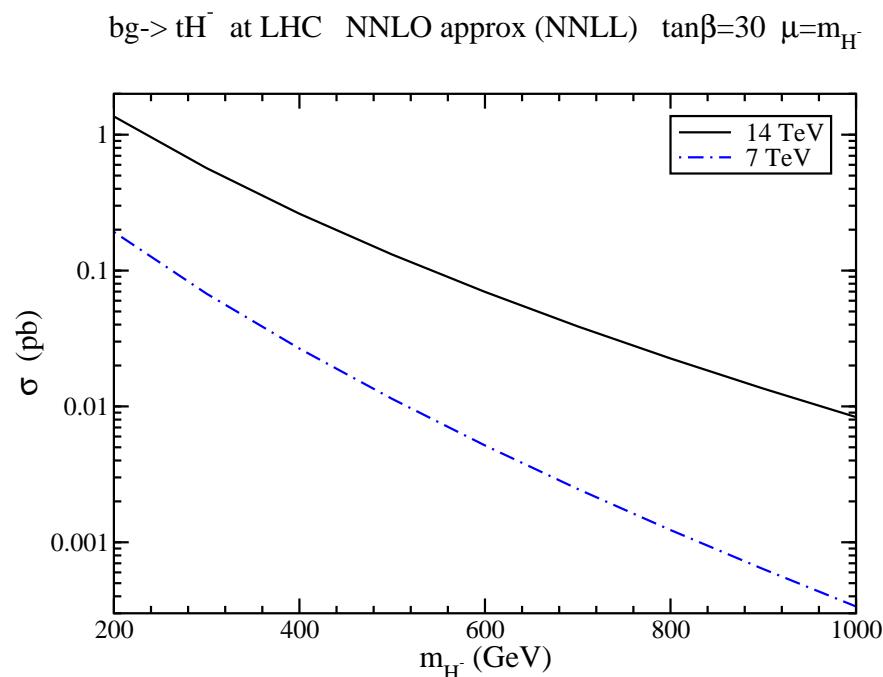


$$\sigma_{tW}^{\mathrm{NNLOapprox}}(m_t = 173\,\mathrm{GeV}, 7\,\mathrm{TeV}) \quad = \quad 7.8 \pm 0.2^{+0.5}_{-0.6}\ \mathrm{pb}$$

$$\sigma_{tW}^{\mathrm{NNLOapprox}}(m_t = 173\,\mathrm{GeV}, 14\,\mathrm{TeV}) \quad = \quad 41.8 \pm 1.0^{+1.5}_{-2.4}\ \mathrm{pb}$$

**NNLO approx corrections increase NLO cross section by $\sim 8\%$**

**Cross section for $\bar{t}W^+$ production is identical**

# Associated production of a top quark with a charged Higgs



bg-> tH⁻ at LHC   NNLO approx (NNLL)   tanβ=30   μ=m$_{H^-}$

**NNLO approx corrections increase NLO cross section by $\sim 15$ to $\sim 20\%$**

# Summary

- **NNLL resummation for top quark pair and single top production**

- $t\bar{t}$ **production cross section**

- **top quark** $p_T$ **and rapidity distributions**

- **theoretical top quark forward-backward asymmetry smaller than observed at the Tevatron**

- $t$**-channel and** $s$**-channel single top production cross section**

- $bg \to tW^-$ **and** $bg \to tH^-$ **at LHC**

- **NNLO approx corrections for top pair and single top production are significant at Tevatron and LHC**