Flavour Physics and CP Violation II

AEPSHEP 2012 at Fukuoka

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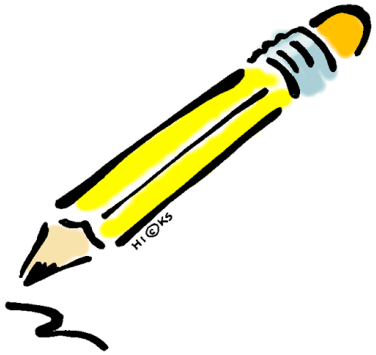
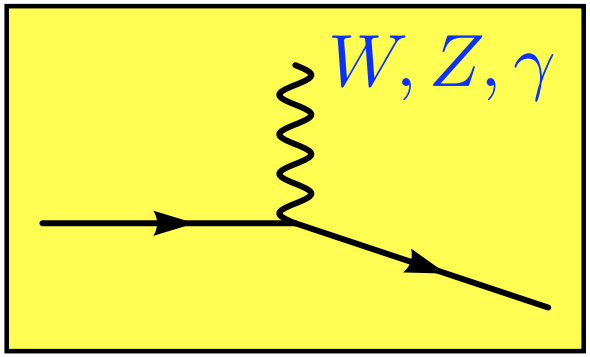
19/10/2012

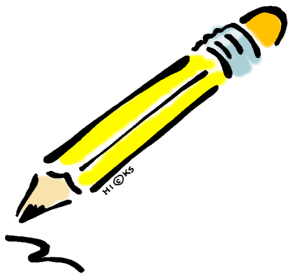
Plan

- 1st lecture: Introduction to flavour physics
  - ★ Weak interaction processes: historical review
  - ★ Discovery of CP violation in the K system
  - ★ Charged/Neutral processes and GIM mechanism
- 2nd lecture: Describing flavour physics and CP violation within SM
  - ★ Charged/Neutral current and CP violation in SM
  - ★ Measuring CP violating phase in B factories
  - ★ Testing the unitarity of the CKM matrix

Theoretical description of
Electroweak Interaction of SM







Yukawa Interaction (quark)

Mass eigen-basis

$K^{U/D}_{L/R}$: 3x3 matrix, "^" is for 3 vector

$$\hat{u}_L = K^u_L \hat{u}'_L, \quad \hat{u}_R = K^u_R \hat{u}'_R \qquad \hat{d}_L = K^d_L \hat{d}'_L, \quad \hat{d}_R = K^d_R \hat{d}'_R$$

$$m^{u\prime}_{\rm diag} = K^{u\dagger}_L m^u K^u_R, \quad m^{d\prime}_{\rm diag} = K^{d\dagger}_L m^d K^d_R$$

ex. down-type

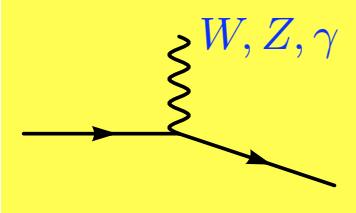
$$(\bar{d}\ ,\bar{s}\ ,\bar{b}\ )\begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & m_{\square} & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix}\begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Inserting the unit matrix

$$= \underbrace{(\bar{d}\ ,\bar{s}\ ,\bar{b}\ )K^{\square}}_{\bar{\hat{d}}'_L}\ \underbrace{(K^{\square})^{\dagger}\begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & m_{\square} & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix}K^{\square}}_{\begin{pmatrix} m_{\square} & 0 & 0 \\ 0 & m_{\square} & 0 \\ 0 & 0 & m \end{pmatrix}}\ \underbrace{(K^{\square})^{\dagger}\begin{pmatrix} d \\ s \\ b \end{pmatrix}}_{\hat{d}'_R}$$

Charged and Neutral Currents on the mass basis

$$
\begin{aligned}
J_W^{\mu+} &= \frac{1}{\sqrt{2}}(\hat{\bar{u}}_L \gamma^\mu \hat{d}_L) \\
J_W^{\mu-} &= \frac{1}{\sqrt{2}}(\hat{\bar{d}}_L \gamma^\mu \hat{u}_L) \\
J_3^\mu &= \frac{1}{2}\hat{\bar{u}}_L \gamma^\mu \hat{u}_L - \frac{1}{2}\hat{\bar{d}}_L \gamma^\mu \hat{d}_L
\end{aligned}
$$



$$
\hat{u}_L = K_L^u \hat{u}'_L, \quad \hat{u}_R = K_R^u \hat{u}'_R
$$
$$
\hat{d}_L = K_L^d \hat{d}'_L, \quad \hat{d}_R = K_R^d \hat{d}'_R
$$

mass basis

$$
\begin{aligned}
(K_L^u)^\dagger K_L^d &\equiv V_{\rm CKM} \\
(K_L^d)^\dagger K_L^u &\equiv V_{\rm CKM}^\dagger
\end{aligned}
$$

$$
\begin{aligned}
J_W^{\mu+} &= \frac{1}{\sqrt{2}}(\hat{\bar{u}}'_L (K_L^u)^\dagger \gamma^\mu K_L^d \hat{d}'_L) \\
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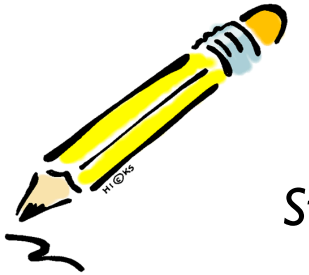
\end{aligned}
$$

$$
\begin{aligned}
(K_L^u)^\dagger K_L^u &= \mathbf{1} \\
(K_L^d)^\dagger K_L^d &= \mathbf{1}
\end{aligned}
$$

No tree FCNC in SM!

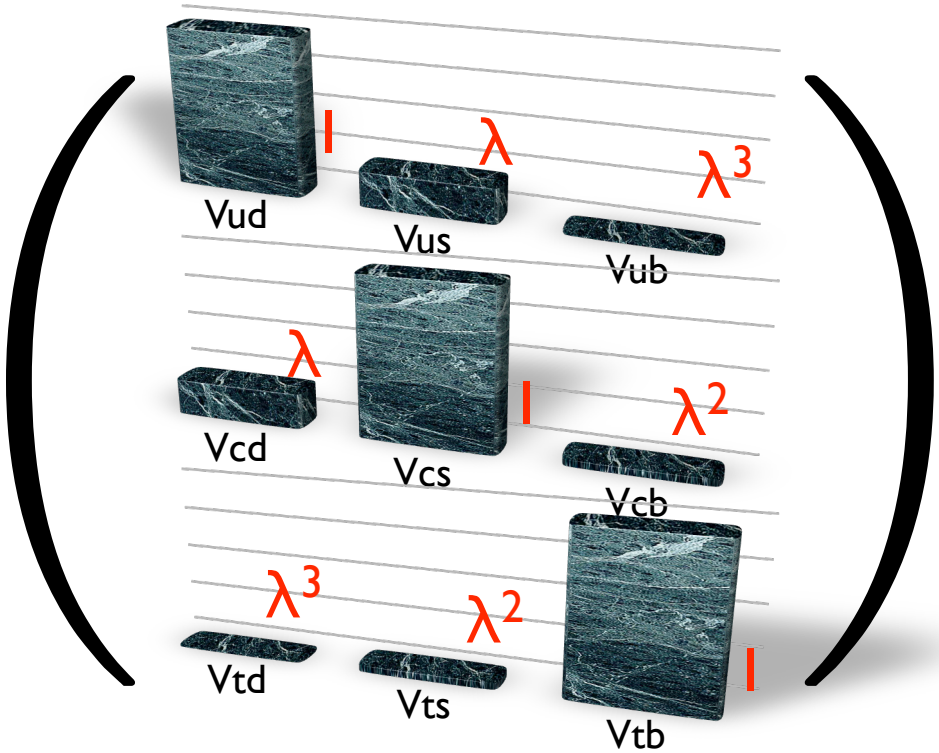


CKM matrix elements

Strength of the weak couplings between different flavours

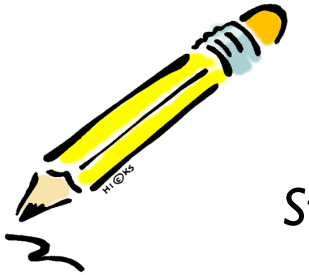
$$J_W^{\mu+} = (\overline{u_L}, \overline{c_L}, \overline{t_L}) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

It turned out that the CKM matrix is close to diagonal!



It contains many information.
Ex) the life time difference of K, D, B:

$\tau_D \ll \tau_K$, $\tau_D < \tau_B$



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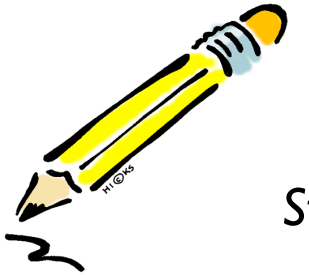
It provides a source of CP violation!

It took nearly 10 years to find the solution for this complex coupling since discovery of CP violation...



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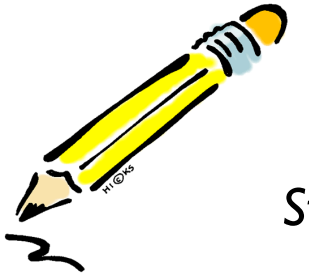
Parameter counting of the unitary matrix to go to diagonalize the Yukawa coupling

Unitarity condition $UU^\dagger = 1 \longrightarrow 2n^2 - n^2 = n^2$

↓

Phase convention $n^2 - (2n-1) = (n-1)^2$



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For two generation, only 1 rotation remains while for three generation, 3 rotations plus 1 phase remains (prediction of the 3rd generation).

Kobayashi, Maskawa
(1973)

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3 mixings and 1 phase

phase. The rotation is defined as follows:

$$\omega(\theta_{12},0) = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

$$\omega(\theta_{13},\delta_1) = \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{-i\delta_1} \\ 0 & 1 & 0 \\ -\sin\theta_{13}se^{-i\delta_1} & 0 & \cos\theta_{13} \end{pmatrix} \tag{2}$$

$$\omega(\theta_{23},0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \tag{3}$$

Then, the standard CKM matrix is obtained by choosing to multiply these matrices in the following order:

$$V_{\mathrm{CKM}}^{3\times3} = \omega(\theta_{23},0)\omega(\theta_{13},\delta_1)\omega(\theta_{12},0). \tag{4}$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

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We need experimental verifications that all 9 complex elements can be explained by the 4 input parameters.

A new parameterization

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We re-parametrize in terms of λ, A, ρ and η:

$$\sin\theta_{12} = \lambda, \sin\theta_{13} = A(\rho - i\eta)\lambda^3, \sin\theta_{23} = A\lambda^2$$

Realizing the hierarchy in the matrix, we expand in terms of λ~=0.22:

$$\sin\theta_{12} = \mathcal{O}(\lambda), \sin\theta_{23} = \mathcal{O}(\lambda^2), \sin\theta_{13} = \mathcal{O}(\lambda^3)$$

Then,
order:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

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Wolfenstein's parameterization

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

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$$= \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

New parameters:
λ, A, ρ and η

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Phases appear at 13, 31 elements

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Expansion in order λ^3

$$= \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$= \begin{pmatrix} 1-\lambda^2/2-\lambda^4/8 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2+(-1/8-A^2/2)\lambda^4 & A\lambda^2 \\ A\lambda^3(1-\rho - i\eta) & -A\lambda^2 + A\lambda^4(1/2-\rho - i\eta) & 1 \end{pmatrix}$$

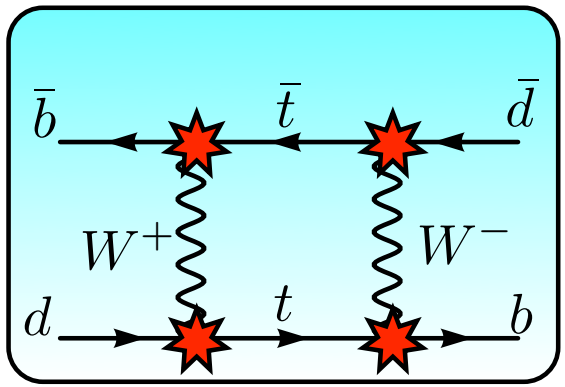
$$+ \quad \mathcal{O}(\lambda^5)$$

Expansion in order λ^4

Computing q/p for B system

In the B system, we have $M_{12} >> \Gamma_{12}$, thus

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \simeq \sqrt{\frac{M_{12}^*}{M_{12}}} \equiv e^{i\phi}$$



$$M_{12} = \frac{G_F^2 m_W^2}{16\pi^2}(V_{tb}V_{td}^*)^2 S_0\left(\frac{m_t^2}{m_W^2}\right) \times \eta_{\rm QCD} \frac{\langle B^0|(\bar{d}b)_{\rm V-A}(\bar{d}b)_{\rm V_A}|\overline{B}^0\rangle}{m_B}$$

Loop function dominant=top quark

Strong interaction part

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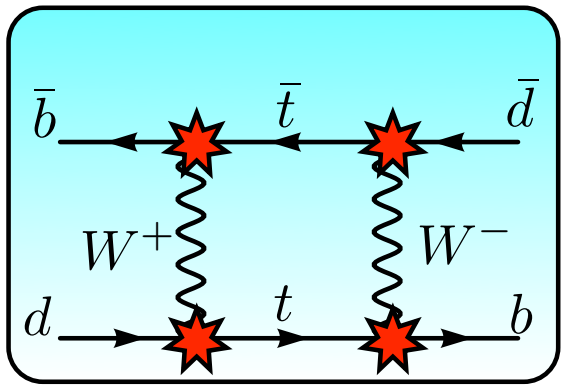

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Loop function dominant=top quark

Strong interaction part

$$V_{td} = A\lambda^3(1-\rho-i\eta)$$
$$V_{tb} = 1$$

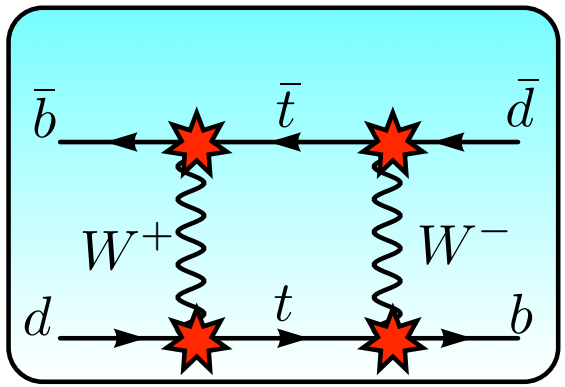
$$\Rightarrow \boxed{\frac{q}{p} = e^{-2i\ arg(V_{tb}^* V_{td})}}$$

DONE!

Computing q/p for B system

In the B system, we have $M_{12} >> \Gamma_{12}$, thus

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \simeq \sqrt{\frac{M_{12}^*}{M_{12}}} =$$



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$$\times \eta_{\rm QCD} \frac{\langle B^0|(\bar{d}$$

rk

Strong interaction part

If the Kobayashi-Maskawa ansatz is correct, the CP violation in B system should be very large!!!

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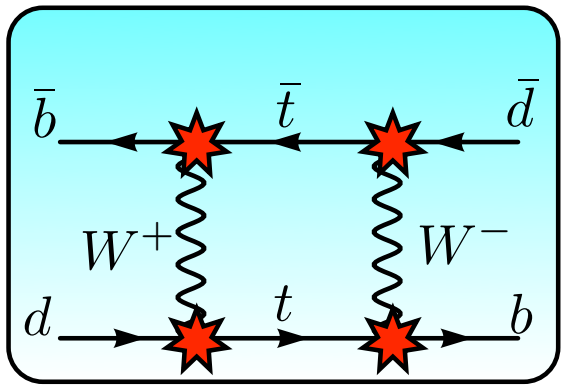

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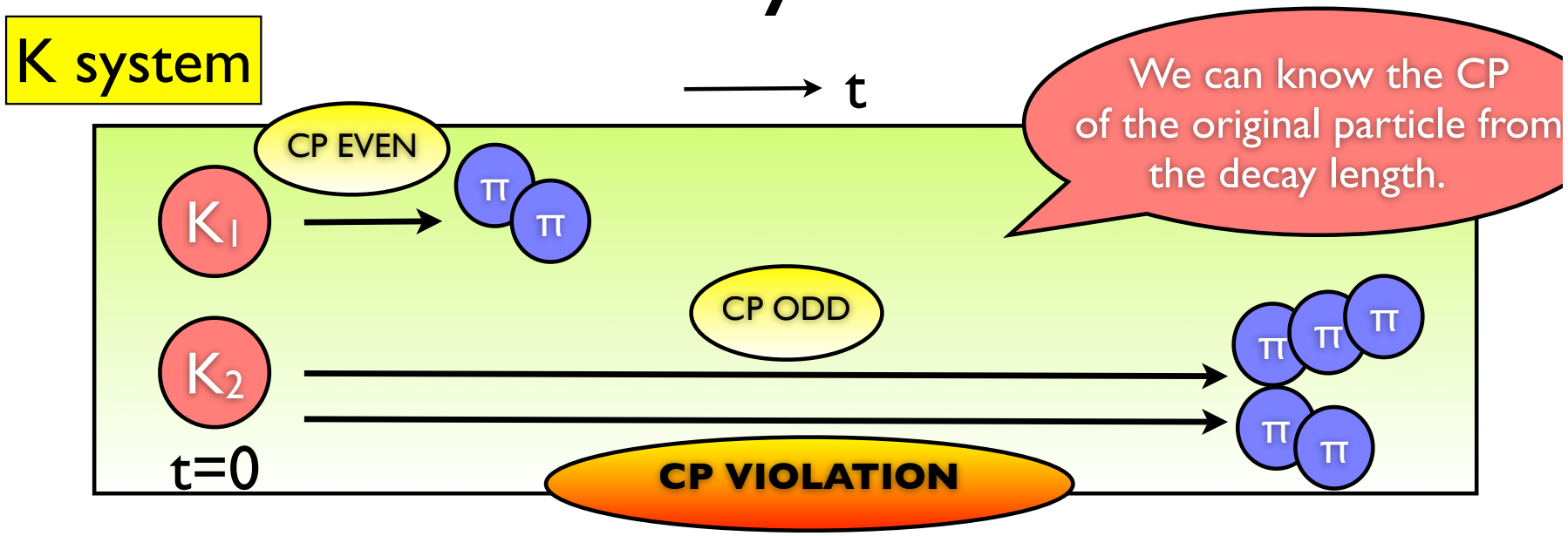
If the Kobayashi-Maskawa ansatz is correct, the CP violation in B system should be very large!!!

B factories!

CP violation in K system vs B system



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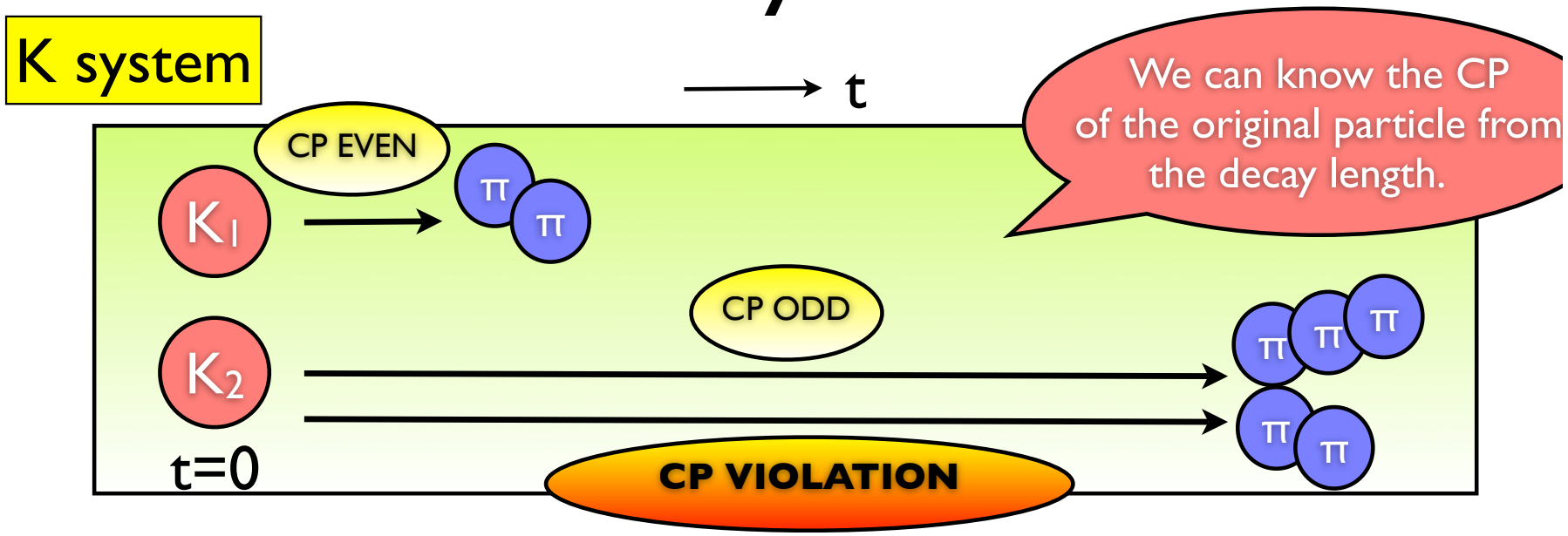


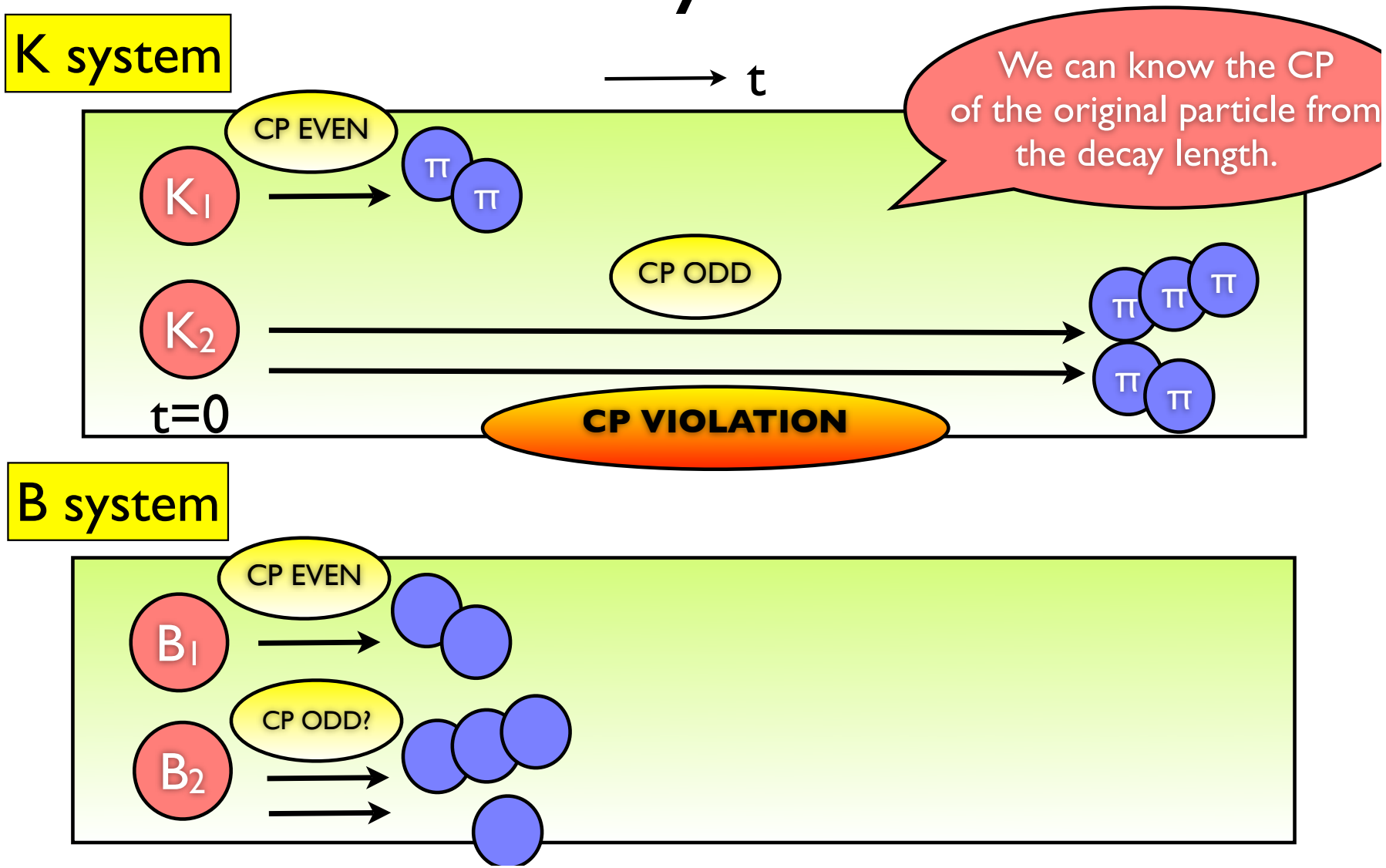
$$\mathcal{CP}|K_1\rangle = +\frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) = |K_1\rangle \quad \boxed{\textbf{CP EVEN}}$$

$$\mathcal{CP}|K_2\rangle = -\frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) = -|K_2\rangle \quad \boxed{\textbf{CP ODD}}$$

$$|K_S\rangle = \frac{1}{\sqrt{2}}\left(p|K^0\rangle + q|\overline{K}^0\rangle\right) = \frac{p}{2}\left[(1+\frac{q}{p})|K_1\rangle + (1-\frac{q}{p})|K_2\rangle)\right]$$

$$|K_L\rangle = \frac{1}{\sqrt{2}}\left(p|K^0\rangle - q|\overline{K}^0\rangle\right) = \frac{p}{2}\left[(1-\frac{q}{p})|K_1\rangle + (1+\frac{q}{p})|K_2\rangle)\right]$$

CP violation in K system vs B system



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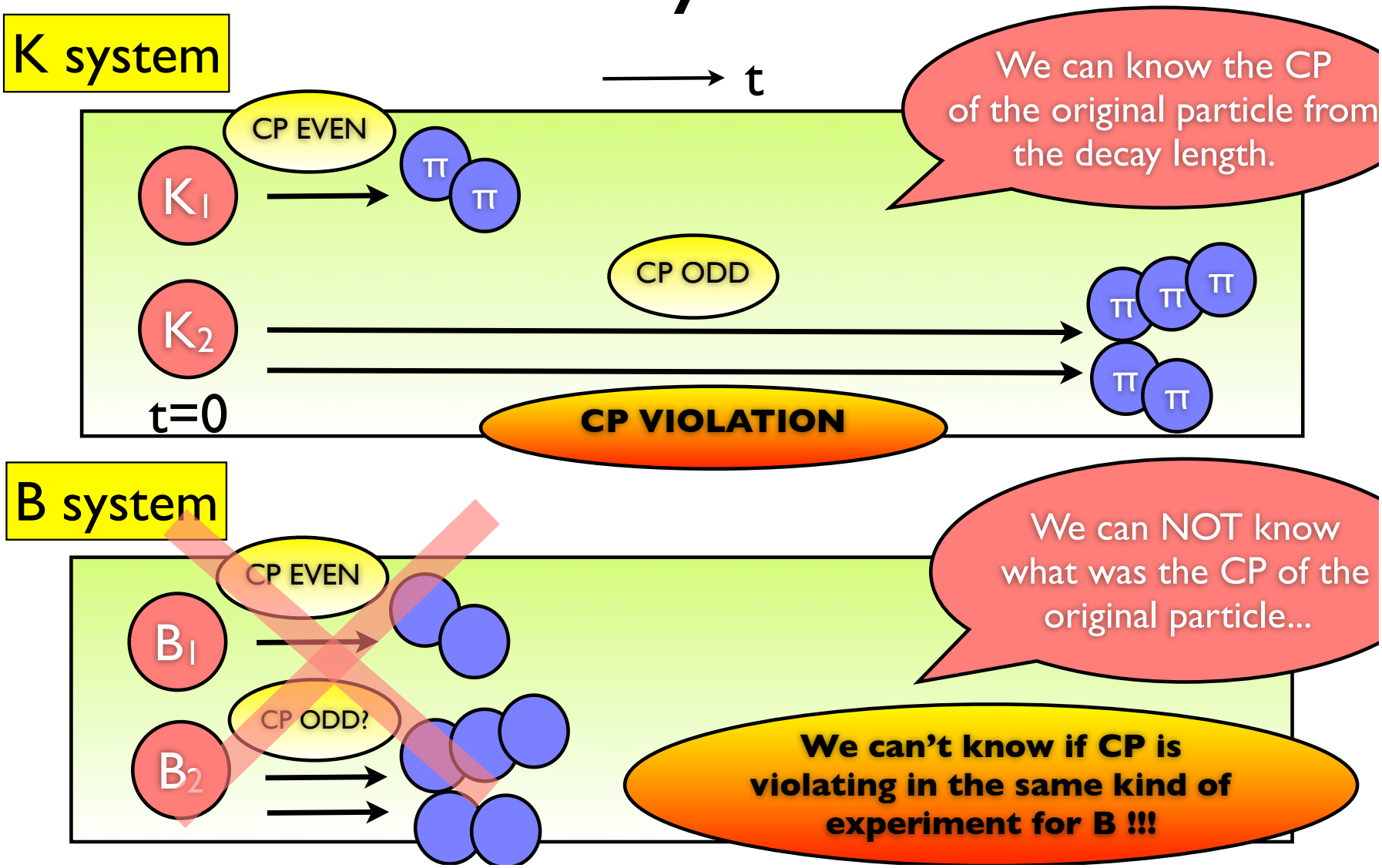


Time evolution formula

Let us describe the time evolution, in terms of the Hilbert space:

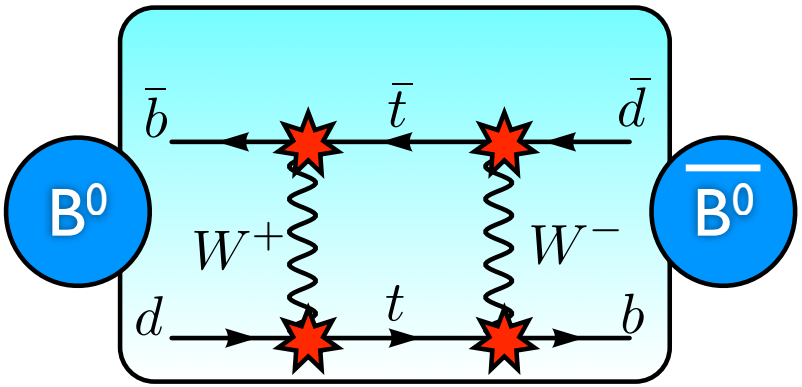
$$|\Psi(t)\rangle = a(t)|B\rangle + b(t)|\overline{B}\rangle$$

The time dependence can be described by the Schroedinger equation:

$$i\hbar\frac{\partial}{\partial t}\Psi(t) = \mathcal{H}\Psi(t); \qquad \Psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

where the Hamiltonian is given as

$$\mathcal{H} = \mathbf{M} - \tfrac{i}{2}\mathbf{\Gamma} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$



Time evolution formula

$$\mathcal{H} = \mathbf{M} - \tfrac{i}{2}\mathbf{\Gamma} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$

Using CPT invariance (M_{11}=M_{22}, Γ_{11}=Γ_{22}) and ***M*** *and* **Γ** *being Hermitian, we find the eigensystem of this matrix:*

$$M_1 + \frac{i}{2}\Gamma_1 \equiv M_{11} - \frac{i}{2}\Gamma_{11} + \frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right); \quad \begin{pmatrix} p \\ q \end{pmatrix}$$

$$M_2 + \frac{i}{2}\Gamma_2 \equiv M_{11} - \frac{i}{2}\Gamma_{11} - \frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right); \quad \begin{pmatrix} p \\ -q \end{pmatrix}$$

Thus, the mass eigenstate of K is obtained as:

$$\begin{aligned} |B_1\rangle &= p|B\rangle + q|\overline{B}\rangle \\ |B_2\rangle &= p|B\rangle - q|\overline{B}\rangle \end{aligned} \quad \textit{with} \quad \frac{q}{p} = \pm\sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

Time evolution formula

Now, we obtain the time evolution of the B states:

These states were B or Bbar at t=0.

$$|B(t)\rangle = f_+(t)|B\rangle + \frac{q}{p} f_-(t)|\overline{B}\rangle$$

$$|\overline{B}(t)\rangle = f_+(t)|\overline{B}\rangle + \frac{p}{q} f_-(t)|B\rangle$$

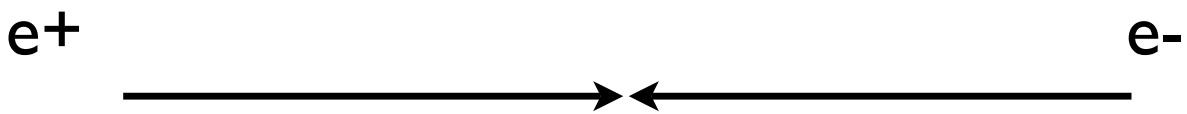
where $$f_\pm = \frac{1}{2} e^{-iM_1 t} e^{-\frac{1}{2}\Gamma_1 t} \left[1 \pm e^{-i\Delta M t} e^{\frac{1}{2}\Delta\Gamma t}\right]$$

with $$\Delta M \equiv M_2 - M_1, \quad \Delta\Gamma \equiv \Gamma_1 - \Gamma_2$$ *+ sign for q/p*

If p/q≠ 1, B and Bbar states behave differently.

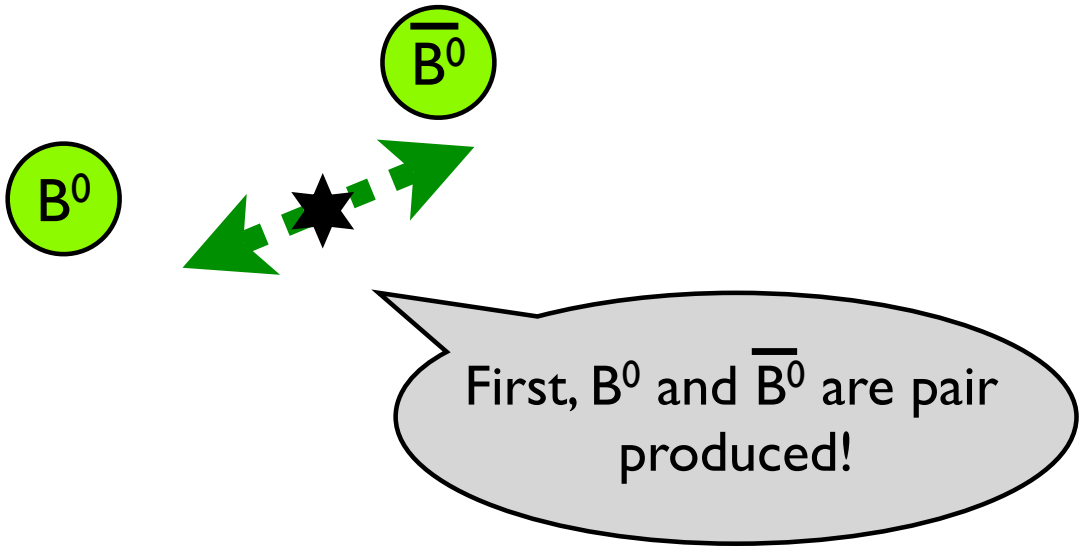
Flavour specific mixing CP violation

(CP Violation in oscillation)



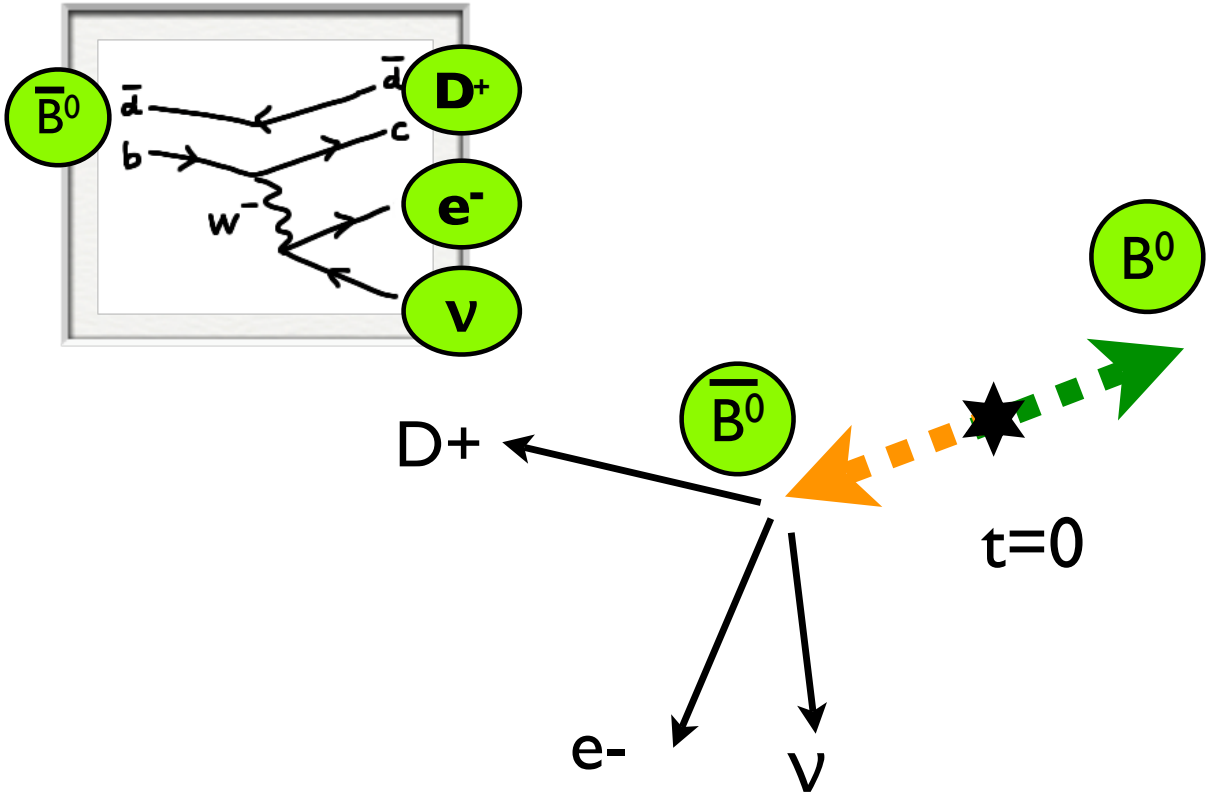
Flavour specific mixing CP violation

(CP Violation in oscillation)



Flavour specific mixing CP violation

(CP Violation in oscillation)

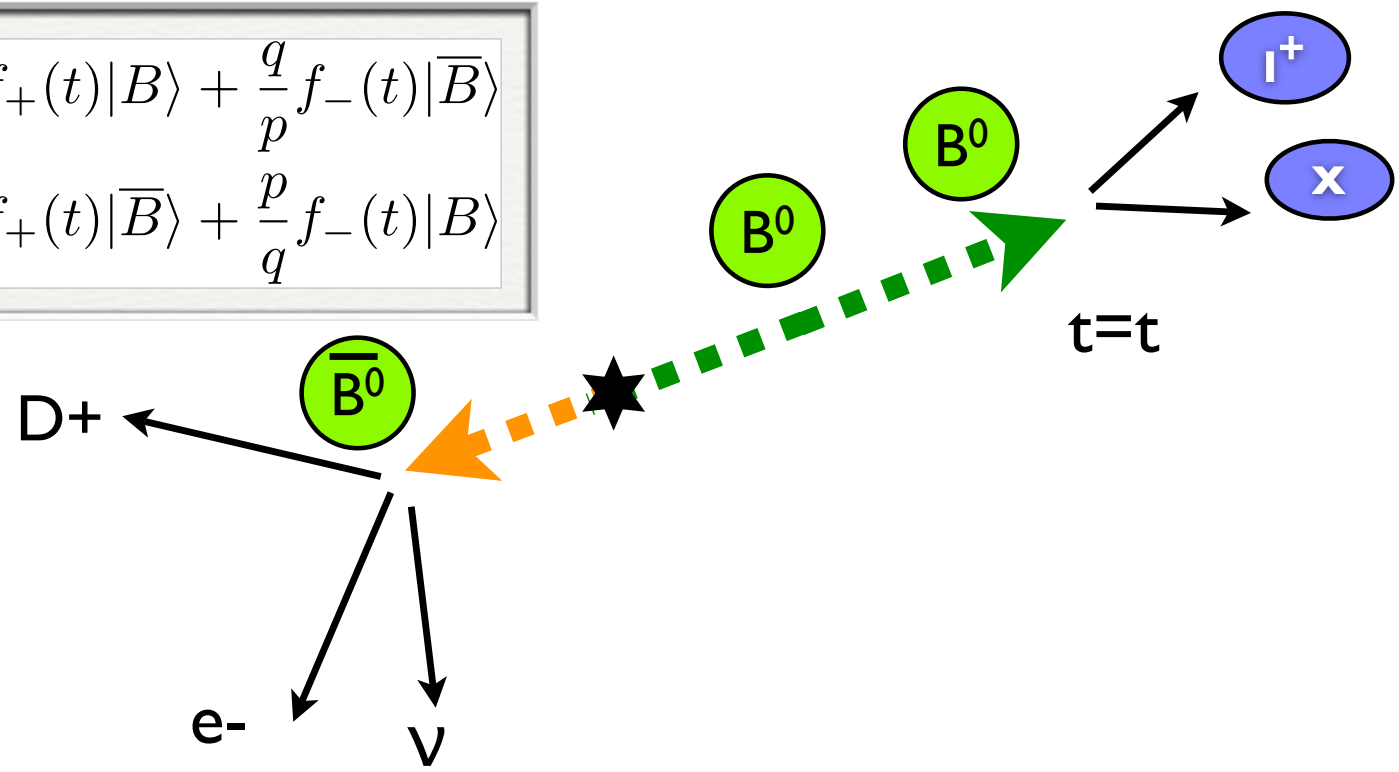


If one of them decays semi-leptonically, we can tell if it was B^0 or $\overline{B}^0$ on one side at given time, which allows us to tell about the other side.

Flavour specific mixing CP violation

(CP Violation in oscillation)

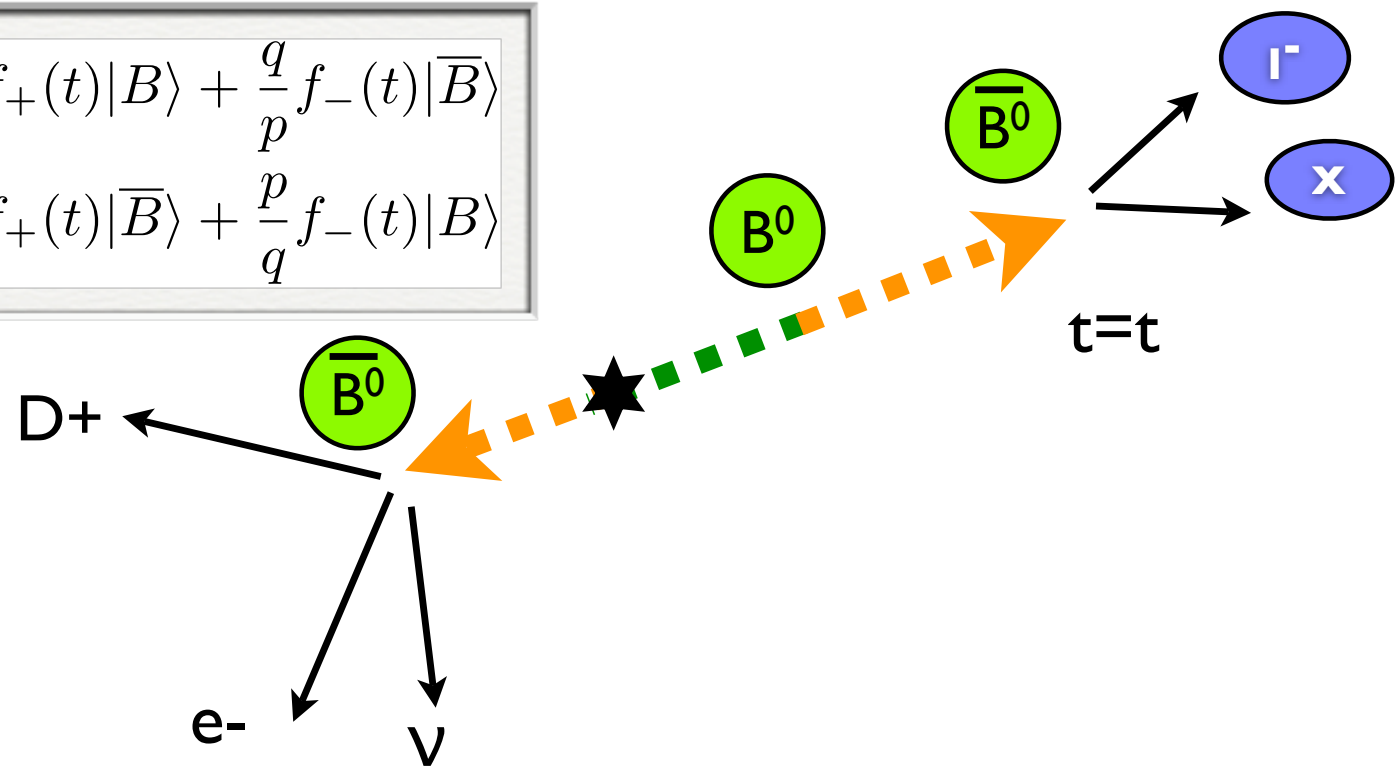
$$
\begin{aligned}
|B(t)\rangle &= f_+(t)|B\rangle + \frac{q}{p} f_-(t)|\overline{B}\rangle \\
|\overline{B}(t)\rangle &= f_+(t)|\overline{B}\rangle + \frac{p}{q} f_-(t)|B\rangle
\end{aligned}
$$



Flavour specific mixing CP violation

(CP Violation in oscillation)

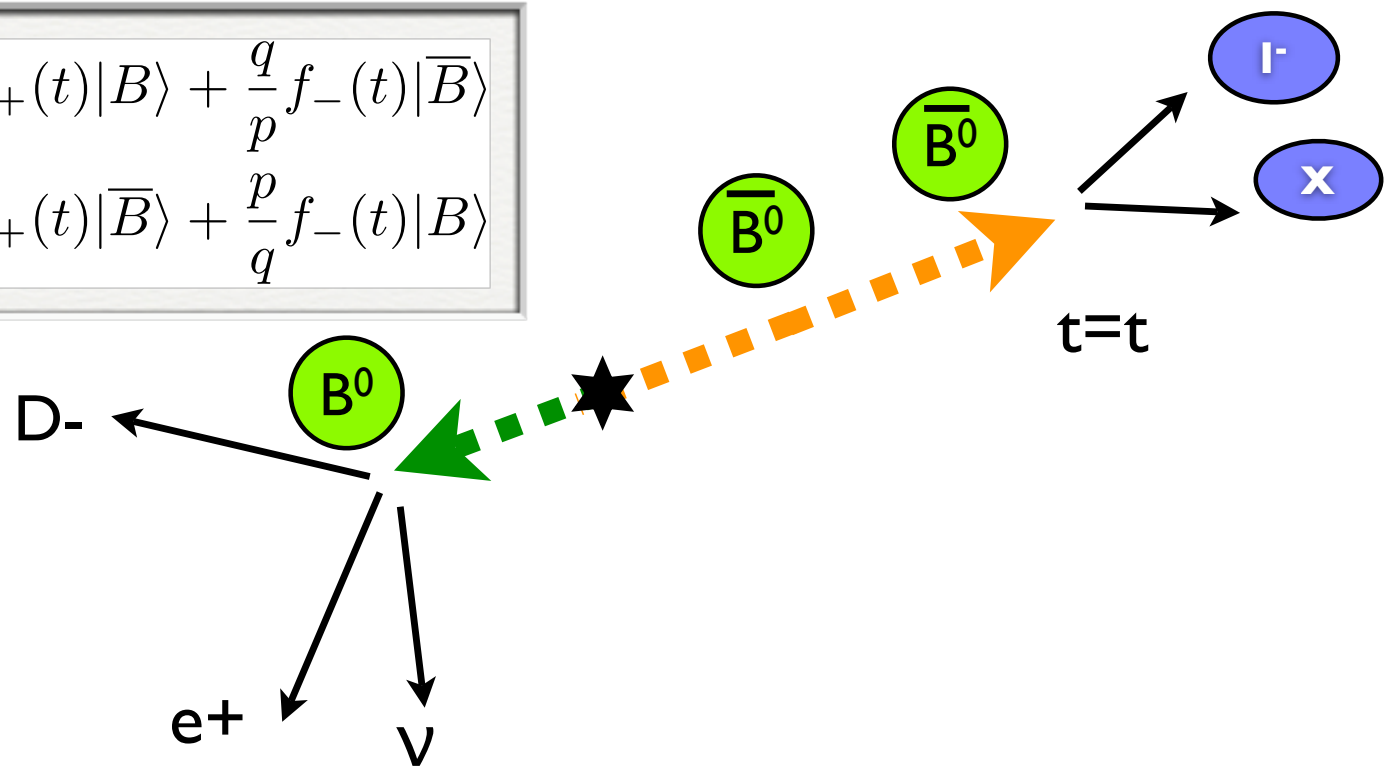
$$
\begin{aligned}
|B(t)\rangle &= f_+(t)|B\rangle + \frac{q}{p} f_-(t)|\overline{B}\rangle \\
|\overline{B}(t)\rangle &= f_+(t)|\overline{B}\rangle + \frac{p}{q} f_-(t)|B\rangle
\end{aligned}
$$



Flavour specific mixing CP violation

(CP Violation in oscillation)

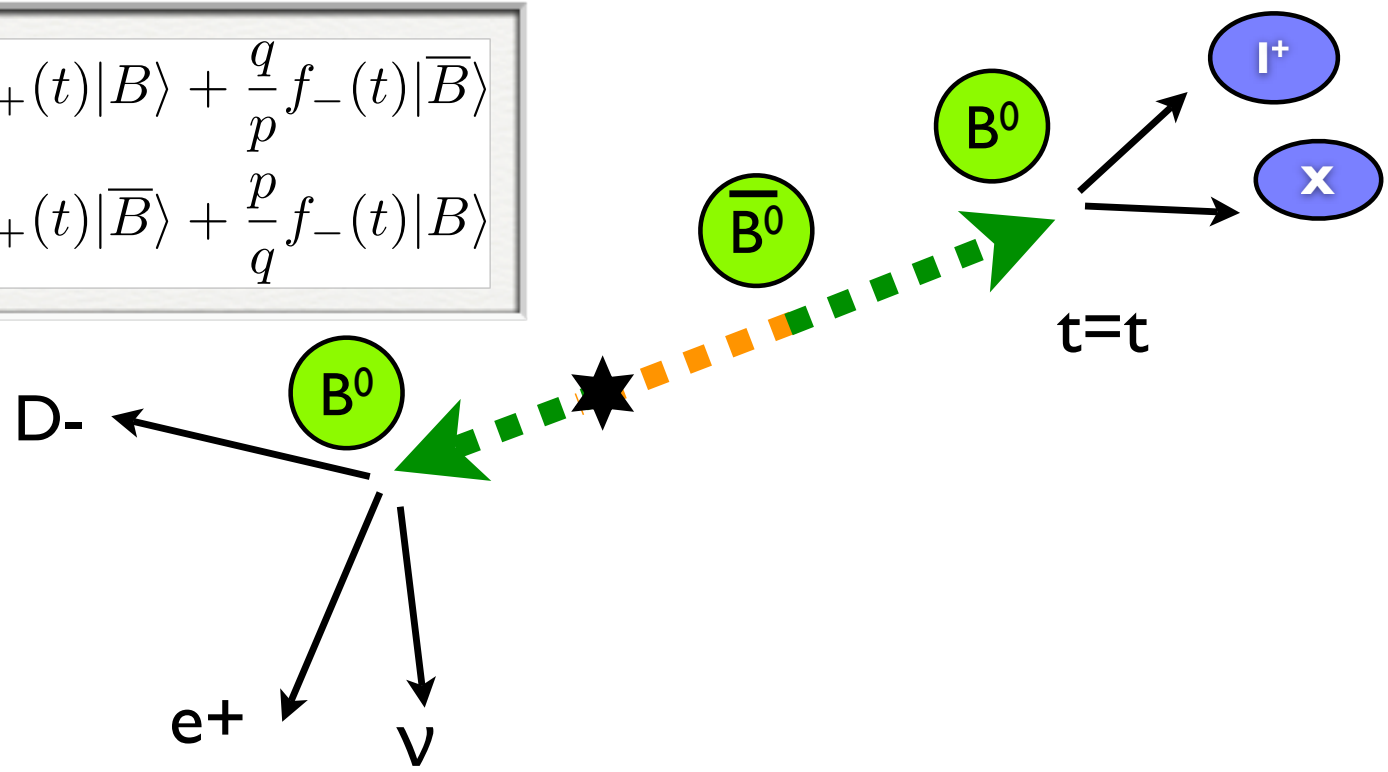
$$
\begin{aligned}
|B(t)\rangle &= f_+(t)|B\rangle + \frac{q}{p} f_-(t)|\overline{B}\rangle \\
|\overline{B}(t)\rangle &= f_+(t)|\overline{B}\rangle + \frac{p}{q} f_-(t)|B\rangle
\end{aligned}
$$



Flavour specific mixing CP violation

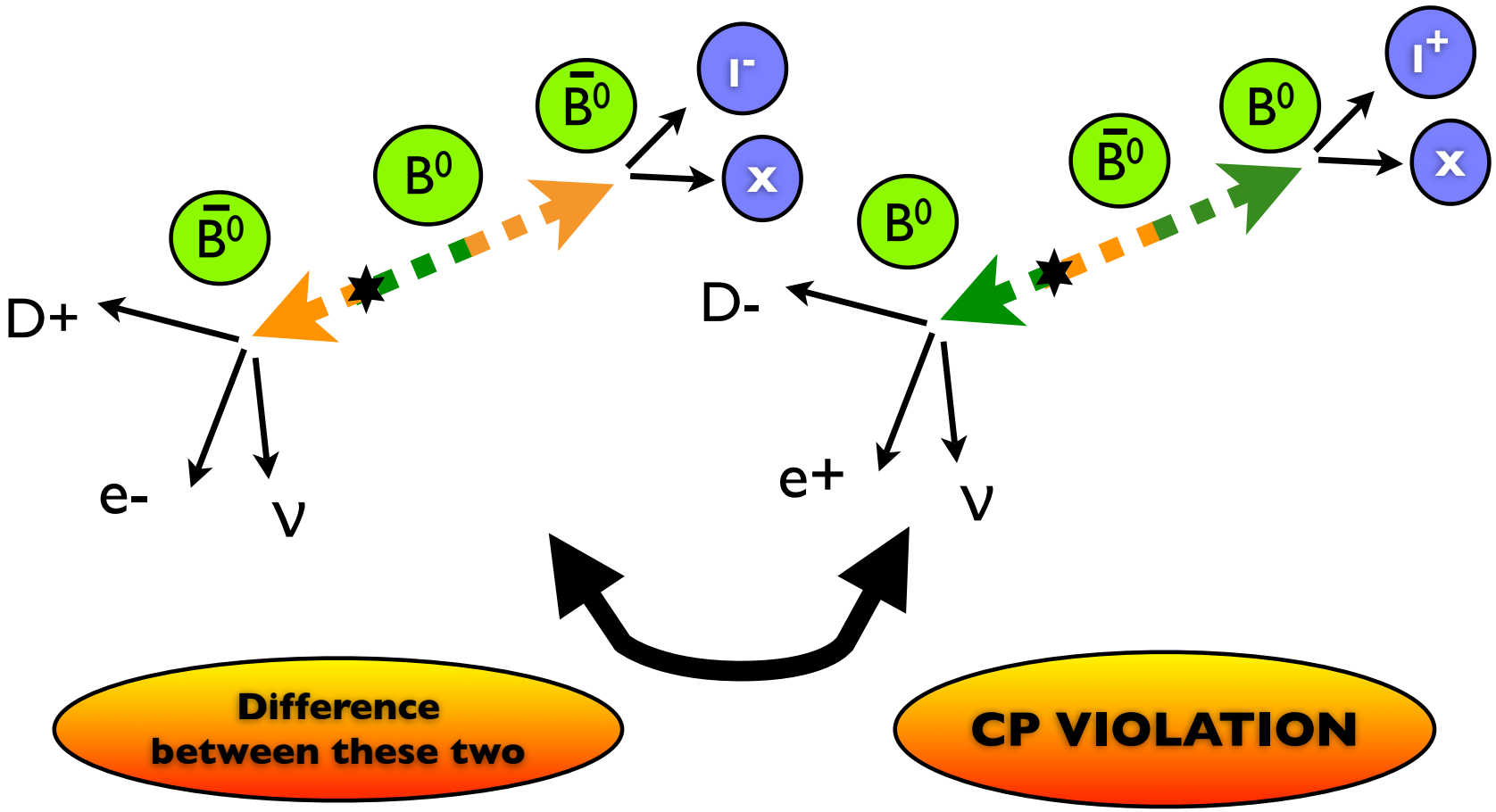
(CP Violation in oscillation)

$$
\begin{aligned}
|B(t)\rangle &= f_+(t)|B\rangle + \frac{q}{p} f_-(t)|\overline{B}\rangle \\
|\overline{B}(t)\rangle &= f_+(t)|\overline{B}\rangle + \frac{p}{q} f_-(t)|B\rangle
\end{aligned}
$$



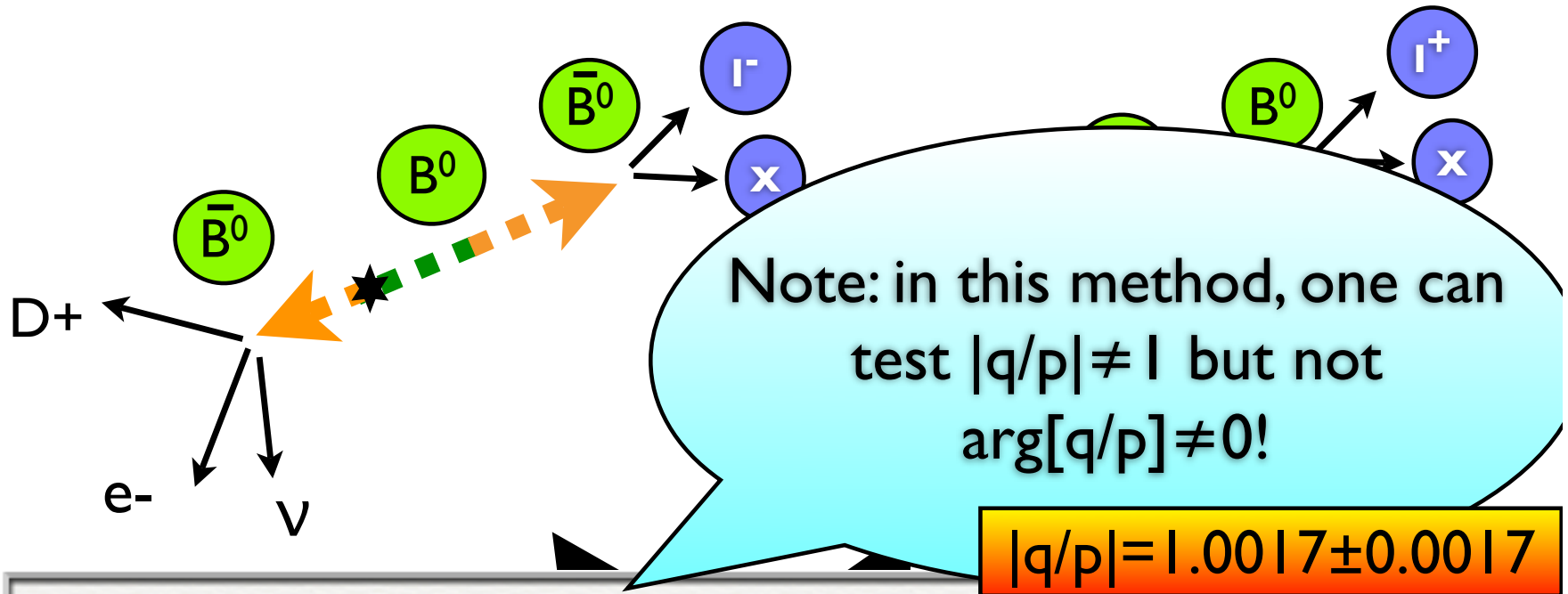
Flavour specific mixing CP violation

(CP Violation in oscillation)



Flavour specific mixing CP violation

(CP Violation in oscillation)

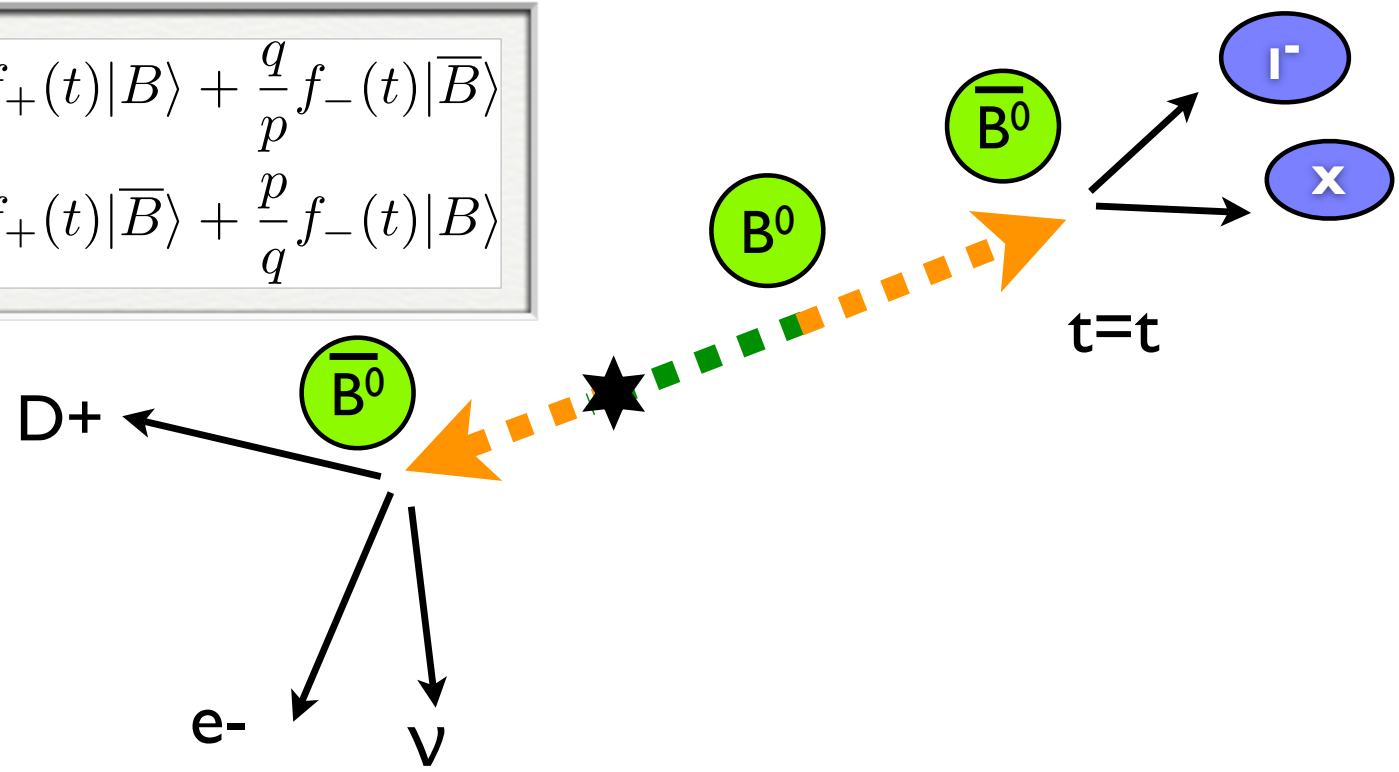


$$\mathcal{A} = \frac{\Gamma(\overline{B}^0 B^0 \to X l^+ l^+) - \Gamma(\overline{B}^0 B^0 \to X l^- l^-)}{\Gamma(\overline{B}^0 B^0 \to X l^+ l^+) + \Gamma(\overline{B}^0 B^0 \to X l^- l^-)} = \frac{|p/q|^4 - 1}{|p/q|^4 + 1}$$

Flavour specific mixing CP violation

(CP Violation in oscillation)

$$
\begin{aligned}
|B(t)\rangle &= f_+(t)|B\rangle + \frac{q}{p} f_-(t)|\overline{B}\rangle \\
|\overline{B}(t)\rangle &= f_+(t)|\overline{B}\rangle + \frac{p}{q} f_-(t)|B\rangle
\end{aligned}
$$

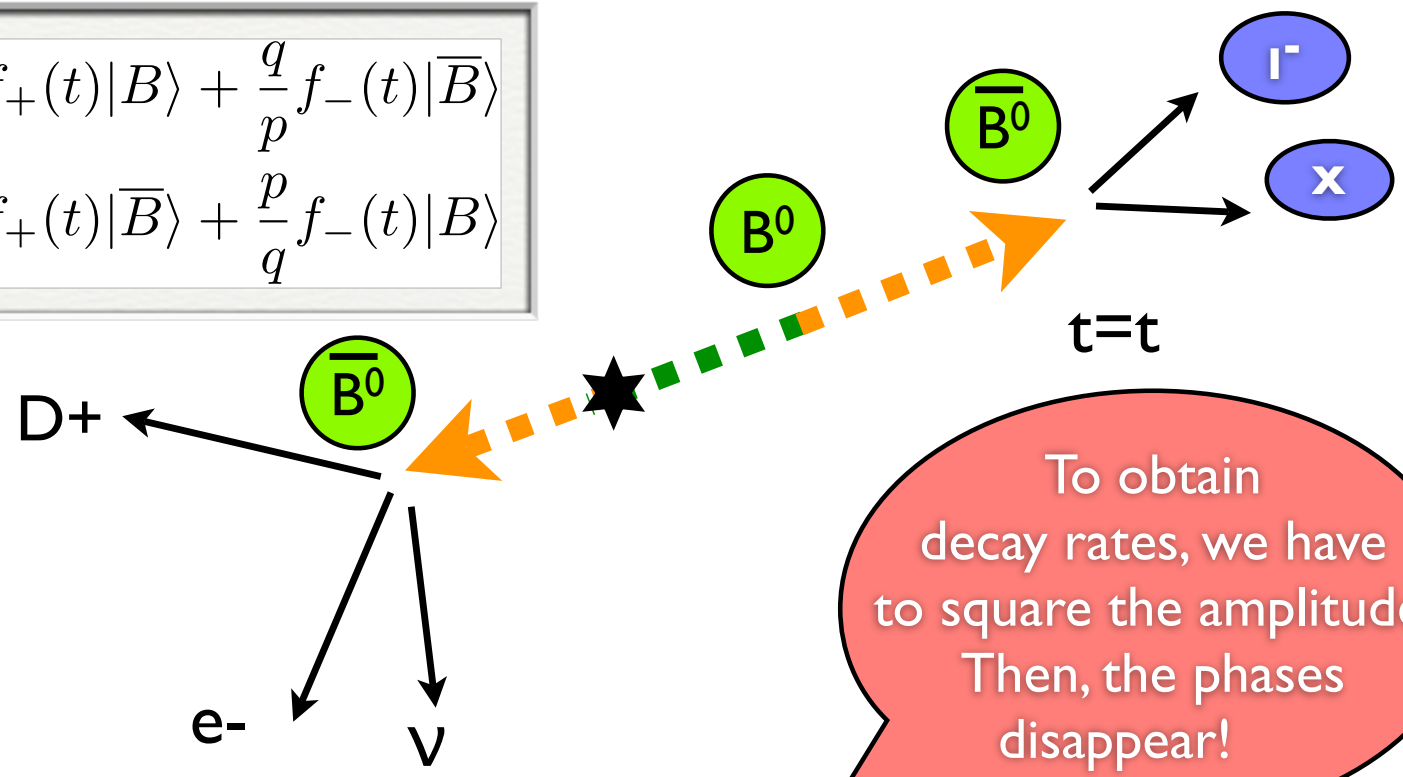


$$\langle l^- X|\mathcal{H}^{\Delta B=1}|B(t)\rangle = f_+(t)\langle l^- X|\mathcal{H}^{\Delta B=1}|B\rangle + \frac{q}{p} f_-(t)\langle l^- X|\mathcal{H}^{\Delta B=1}|\overline{B}\rangle$$

Flavour specific mixing CP violation

(CP Violation in oscillation)

$$
\begin{aligned}
|B(t)\rangle &= f_+(t)|B\rangle + \frac{q}{p} f_-(t)|\overline{B}\rangle \\
|\overline{B}(t)\rangle &= f_+(t)|\overline{B}\rangle + \frac{p}{q} f_-(t)|B\rangle
\end{aligned}
$$



To obtain decay rates, we have to square the amplitude. Then, the phases disappear!

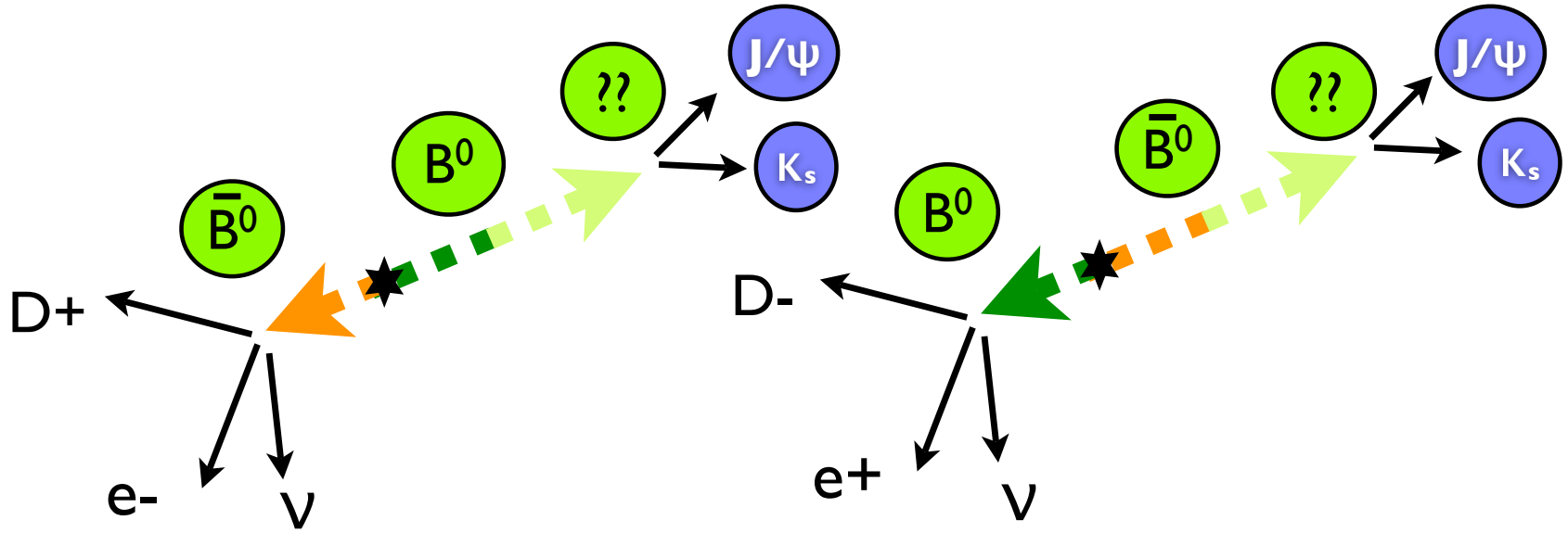
$$\langle l^- X|\mathcal{H}^{\Delta B=1}|B(t)\rangle = f_+(t)\langle l^- X|\mathcal{H}^{\Delta B=1}|B\rangle + \frac{q}{p} f_-(t)\langle l^- X|\mathcal{H}^{\Delta B=1}|\overline{B}\rangle$$

Flavour Non-specific mixing CPV

(CP Violation in oscillation)

Choose a final state which could come both B and Bbar!

ex: $J/\psi K_s$ final state



$$|B(t)\rangle = f_+(t)|B\rangle + \frac{q}{p} f_-(t)|\overline{B}\rangle$$

$$|\overline{B}(t)\rangle = f_+(t)|\overline{B}\rangle + \frac{p}{q} f_-(t)|B\rangle$$

Flavour Non-specific mixing CPV

(CP Violation in oscillation)

Choose a final state which could come both B and Bbar!

ex: J/ψK_s final state

$$
\begin{aligned}
|B(t)\rangle &= f_+(t)|B\rangle + \frac{q}{p} f_-(t)|\overline{B}\rangle \\
|\overline{B}(t)\rangle &= f_+(t)|\overline{B}\rangle + \frac{p}{q} f_-(t)|B\rangle
\end{aligned}
$$

$$
\begin{aligned}
\langle J/\psi K_s|\mathcal{H}^{\Delta B=1}|B(t)\rangle &= f_+(t)\langle J/\psi K_s|\mathcal{H}^{\Delta B=1}|B\rangle + \frac{q}{p} f_-(t)\langle J/\psi K_s|\mathcal{H}^{\Delta B=1}|\overline{B}\rangle \\
\langle J/\psi K_s|\mathcal{H}^{\Delta B=1}|\overline{B}(t)\rangle &= f_+(t)\langle J/\psi K_s|\mathcal{H}^{\Delta B=1}|\overline{B}\rangle + \frac{p}{q} f_-(t)\langle J/\psi K_s|\mathcal{H}^{\Delta B=1}|B\rangle
\end{aligned}
$$

We assume...

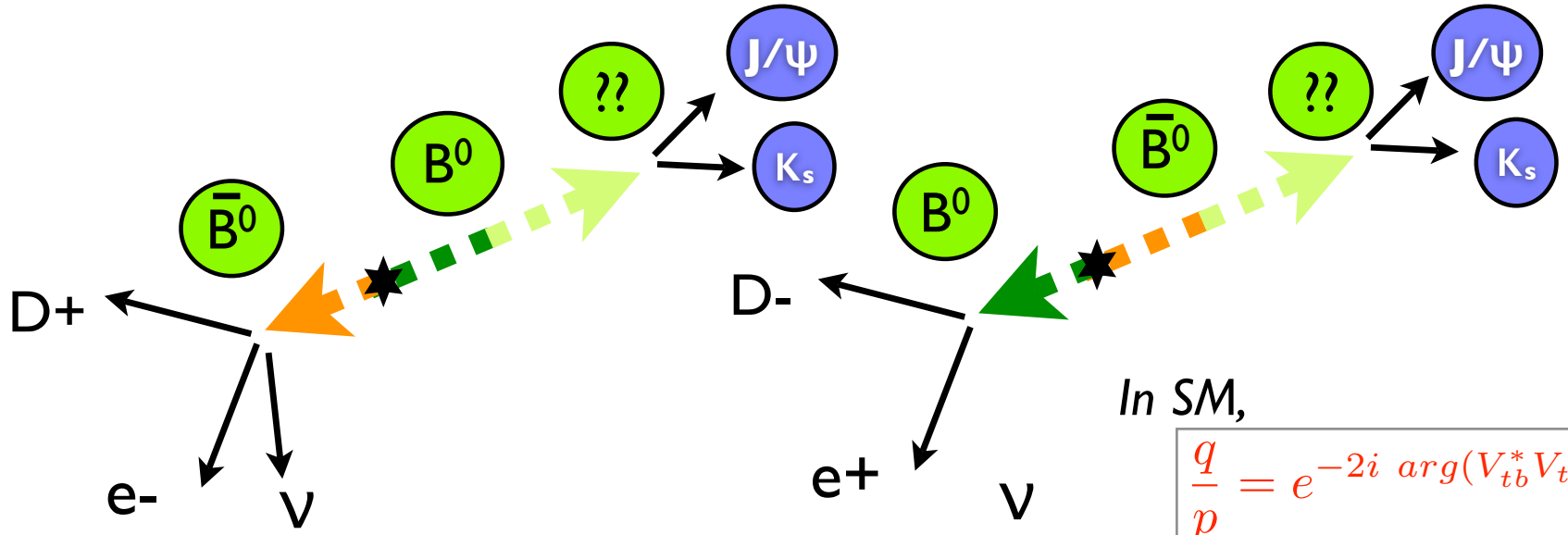
$$\langle J/\psi K_s|\mathcal{H}^{\Delta B=1}|\overline{B}\rangle = \langle J/\psi K_s|\mathcal{H}^{\Delta B=1}|B\rangle \qquad {}_{12} << M_{12}$$

Flavour Non-specific mixing CPV

(CP Violation in oscillation)

Choose a final state which could come both B and Bbar!

ex: $J/\psi K_s$ final state



In SM,

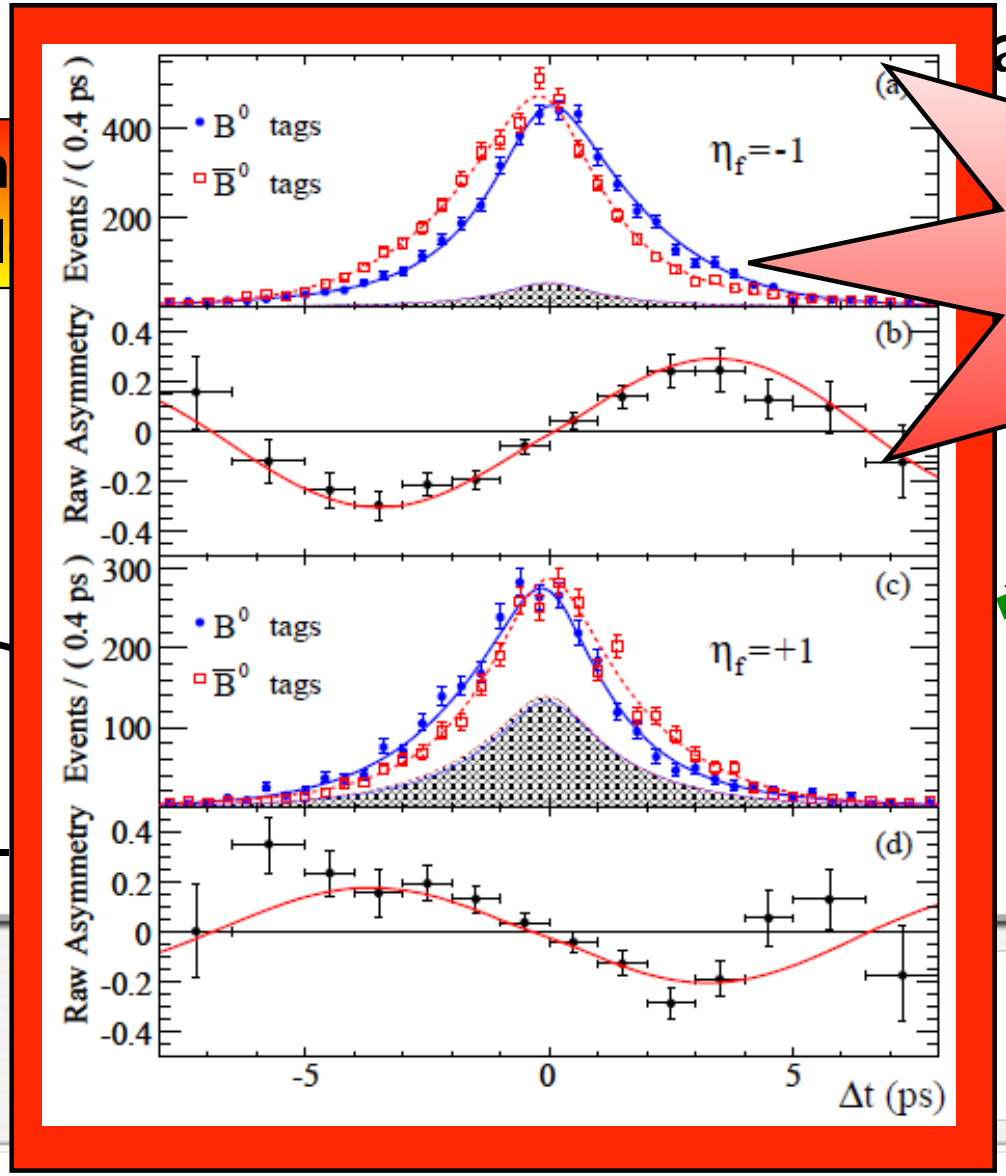
$$\frac{q}{p} = e^{-2i\ arg(V_{tb}^* V_{td})}$$

$$\mathcal{A} = \frac{\Gamma(\overline{B}^0 \to J/\psi K_s) - \Gamma(B^0 \to J/\psi K_s)}{\Gamma(\overline{B}^0 \to J/\psi K_s) - \Gamma(B^0 \to J/\psi K_s)} = \sin(\Delta M t) Im\left(\frac{q}{p}\right)$$

Flavour Non-specific mixing CPV

ation)

Ch
coul



B factory measurments!

state

$\bar{B}^0$

K_s

D+

e-

In SM,

$$\frac{q}{p} = e^{-2i\ arg(V_{tb}^* V_{td})}$$

$$= \sin(\Delta M t) Im\left(\frac{q}{p}\right)$$

Bs oscillation and CP violation measurements at hadron machines

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Expansion in order λ^3

$$= \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

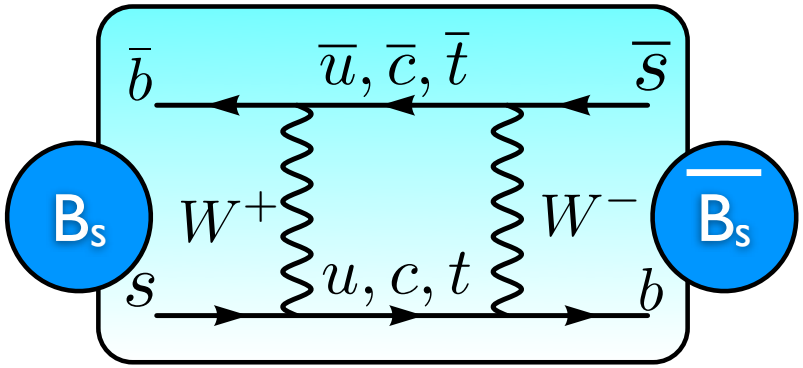
B_d mixing

$$= \begin{pmatrix} 1-\lambda^2/2-\lambda^4/8 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2+(-1/8-A^2/2)\lambda^4 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 + A\lambda^4(1/2-\rho-i\eta) & 1 \end{pmatrix}$$

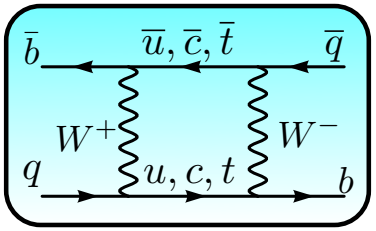
$$+ \quad \mathcal{O}(\lambda^5)$$

B_s mixing

Expansion in order λ^4



Oscillation in B_q System



Mass matrix of B_q system:

$$\mathcal{H} = \mathbf{M} - \frac{i}{2}\mathbf{\Gamma} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$

$$\begin{aligned} \phi_q &\equiv arg[M_{12}] \\ \zeta_q &\equiv arg[\Gamma_{12}] - arg[M_{12}] \end{aligned}$$

When Γ_{12} is real, $\zeta_q = -\Phi_q$

Using CPT invariance, we find the mass eigenstate P_1 and P_2

$$\begin{aligned} |P_1\rangle &= p|P^0\rangle + q|\overline{P}^0\rangle \\ |P_2\rangle &= p|P^0\rangle - q|\overline{P}^0\rangle \end{aligned} \; ; \qquad \frac{q}{p} = \pm\sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

CP violation $q/p \neq 1$

Experimental measurements are carried out for the observables

HFAG11

$$\begin{aligned} \Delta M_q &\equiv M_2 - M_1 = -2|M_{12}| \\ \Delta\Gamma_q &\equiv \Gamma_1 - \Gamma_2 = 2|\Gamma_{12}|\cos\zeta_q \\ \frac{q}{p} &\simeq e^{-i\phi_q}\left(1 + \frac{\Delta\Gamma_q}{2\Delta M_q}\tan\zeta_q\right) \\ \left|\frac{q}{p}\right| &\simeq 1 + \frac{\Delta\Gamma_q}{2\Delta M_q}\tan\zeta_q \end{aligned}$$

ΔM_d=(0.507±0.004) ps^{-1},
ΔM_s=(17.69±0.08)ps^{-1}

Golden-Channels to measure q/p from time-dependent CP asymmery
$B_d \to c\bar{c}K_s$ ($\Phi_d=2\beta$): sin2β=0.676±0.020
$B_s \to J/\psi\Phi$ ($\Phi_d=2\beta_s$): Tevatron, LHCb
*ΔΓ/ΔM is non-negligible for B_s

Di-lepton charge asymmetry
A_{SL}^d=-0.0033±0.0033
A_{SL}^s=-0.0105±0.0064

Recent measurements of B_s Oscillation

$$\Delta M_q \equiv M_2 - M_1 = -2|M_{12}|$$
$$\Delta\Gamma_q \equiv \Gamma_1 - \Gamma_2 = 2|\Gamma_{12}|\cos\zeta_q$$
$$\frac{q}{p} \simeq e^{-i\phi_q}\left(1 + \frac{\Delta\Gamma_q}{2\Delta M_q}\tan\zeta_q\right)$$
$$\left|\frac{q}{p}\right| \simeq 1 + \frac{\Delta\Gamma_q}{2\Delta M_q}\tan\zeta_q$$

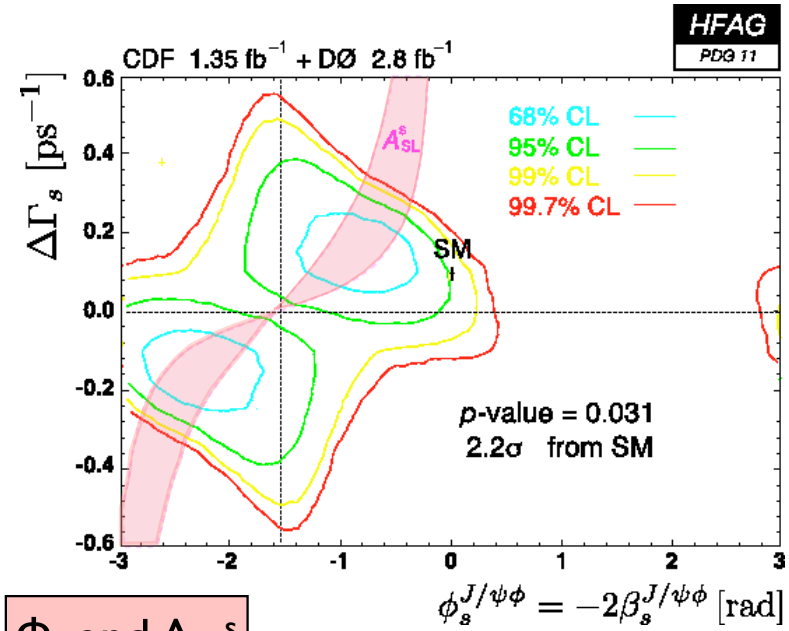
Golden-Channels to measure q/p from time-dependent CP asymmery

$B_s \to J/\psi\Phi$ ($\Phi_d = -2\beta_s$): Tevatron, LHCb

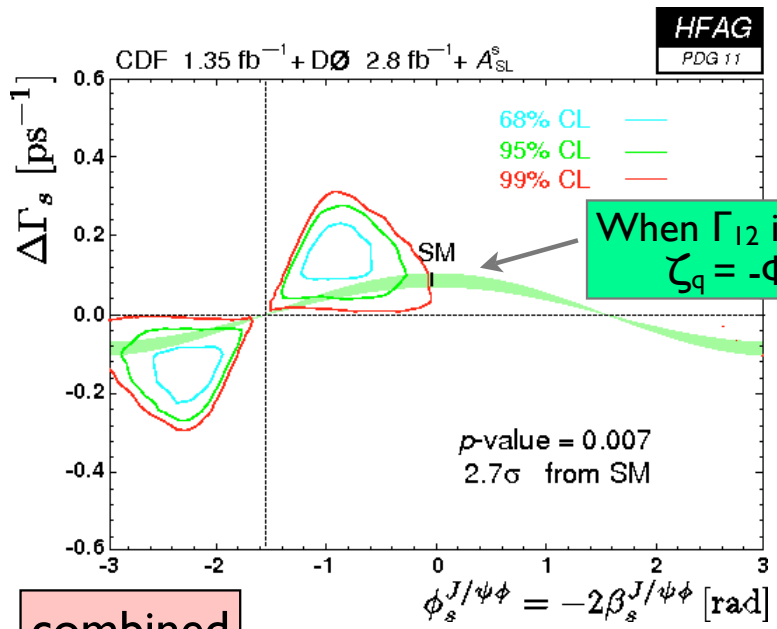
*$\Delta\Gamma/\Delta M$ is non-negligible for B_s

Di-lepton charge asymmetry

$A_{SL}^s = -0.0089 \pm 0.0062$



Φ_s and A_{SL}^s



When Γ_{12} is real, $\zeta_q = -\Phi_q$

combined

Recent measurements of B_s Oscillation

$$\Delta M_q \equiv M_2 - M_1 = -2|M_{12}|$$
$$\Delta\Gamma_q \equiv \Gamma_1 - \Gamma_2 = 2|\Gamma_{12}|\cos\zeta_q$$
$$\frac{q}{p} \simeq e^{-i\phi_q}\left(1 + \frac{\Delta\Gamma_q}{2\Delta M_q}\tan\zeta_q\right)$$
$$\left|\frac{q}{p}\right| \simeq 1 + \frac{\Delta\Gamma_q}{2\Delta M_q}\tan\zeta_q$$

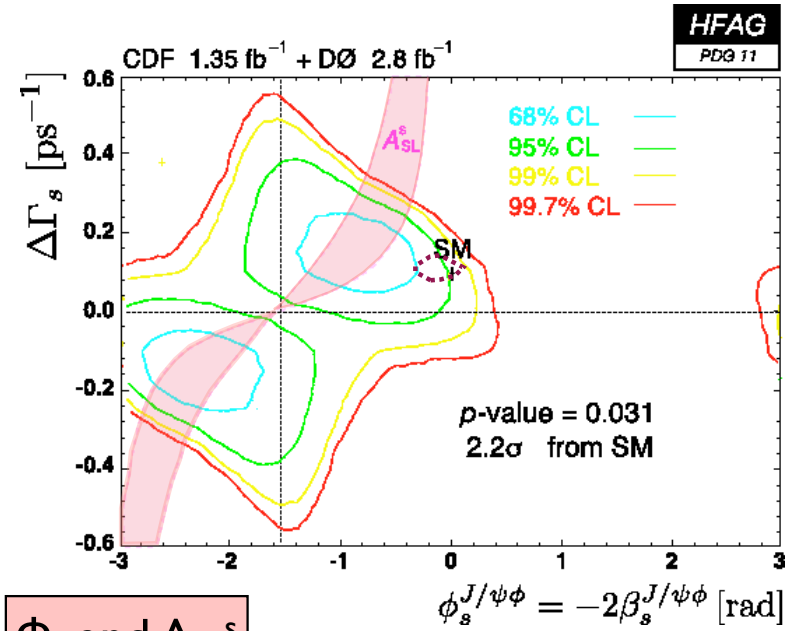
Golden-Channels to measure q/p from time-dependent CP asymmery

$B_s \to J/\psi\Phi/f$ ($\Phi_d = -2\beta_s$): LHCb

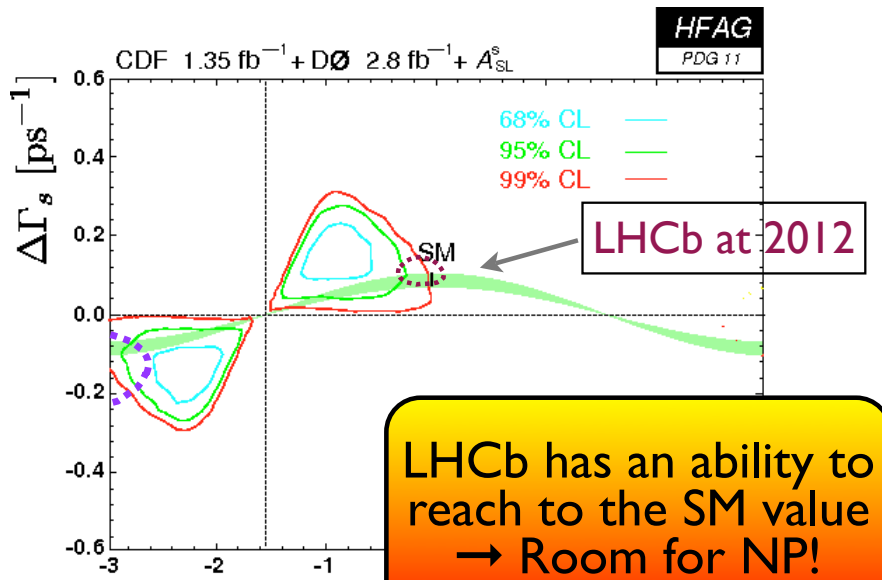
$\Phi_s = -0.14^{+0.16}_{-0.11}$

Di-lepton charge asymmetry

$A_{SL}^s = -0.0089 \pm 0.0062$



Φ_s and A_{SL}^s



LHCb at 2012

combined

LHCb has an ability to reach to the SM value → Room for NP!

Three types of CP violation

- Flavour specific mixing CP violation (CPV in oscillation)

$$\mathcal{A} = \frac{|p/q|^2 - |q/p|^2}{|p/q|^2 + |q/p|^2} = \frac{|p/q|^4 - 1}{|p/q|^4 + 1}$$

- Flavour non-specific mixing CP violation (CPV in oscillation, requiring time-dependent analysis)

$$\mathcal{A} = \frac{2\sin(\arg q/p + \arg\overline{\rho})e^{\frac{1}{2}\Delta\Gamma t}\sin\Delta M t}{1 + e^{\Delta\Gamma t} + \cos(\arg q/p + \arg\overline{\rho})[1 - e^{\Delta\Gamma t}]}$$

- Direct CP violation (CPV in decay process)

$$\mathcal{A} = \frac{|\overline{A}(\overline{f})|^2 - |A(f)|^2}{|\overline{A}(\overline{f})|^2 + |A(f)|^2} = \frac{|\overline{\rho}(\overline{f})|^2 - 1}{|\overline{\rho}(\overline{f})|^2 + 1}$$

Direct CP violation (no-oscillation)

This type of CP violation occurs due to the CP violation in the decay part

$$\overline{\rho}(f) \equiv \frac{\overline{A}(f)}{A(f)} \equiv \frac{1}{\rho(f)} \neq 1$$

Thus, the non-zero CP violation can be measured without time-dependent analysis, nor without CP eigenstate final state.

examples:

$$A(B^0 \to K^+ \pi^-) \longleftrightarrow A(\overline{B}^0 \to K^- \pi^+)$$

$$A(B^+ \to K^+ \pi^0) \longleftrightarrow A(B^- \to K^- \pi^0)$$

$$A(B^+ \to \pi^+\pi^-\pi^+) \longleftrightarrow A(B^- \to \pi^-\pi^+\pi^-)$$

CP asymmetry

$$\mathcal{A} = \frac{\Gamma(\overline{B}^0 \to K^-\pi^+) - \Gamma(B^0 \to K^+\pi^-)}{\Gamma(\overline{B}^0 \to K^-\pi^+) + \Gamma(B^0 \to K^+\pi^-)} \neq 0$$

Direct CP violation (no-oscillation)

This type of CP violation occurs due to the CP violation in the decay part

$$\overline{\rho}(f) \equiv \frac{\overline{A}(f)}{A(f)} \equiv \frac{1}{\rho(f)} \neq 1$$

Thus, the non-zero CP violation can be measured without time-dependent analysis, nor without CP eigenstate final state.

examples:

$A(B^0 \to K^+ \pi^-$

$A(B^+ \to K^+$

$A(B^+ \to \pi^+\pi^-\pi$

Do you know, in how many channels, the direct CP asymmetry is measured?

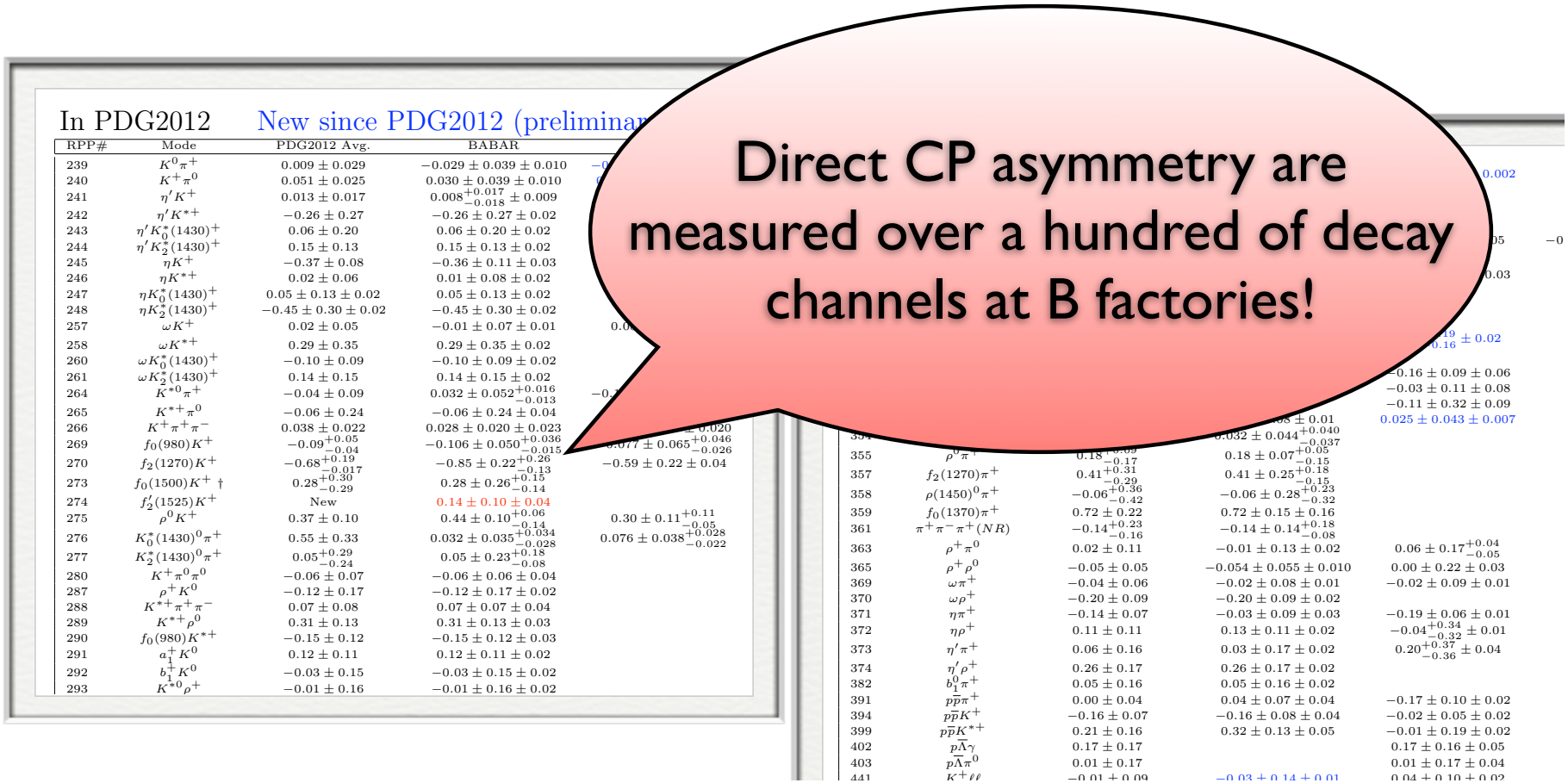
CP asymmetry

$$\mathcal{A} = \frac{\Gamma(\overline{B}^0 \to K^-\pi^+) - \Gamma(B^0 \to K^+\pi^-)}{\Gamma(\overline{B}^0 \to K^-\pi^+) + \Gamma(B^0 \to K^+\pi^-)} \neq 0$$

Direct CP violation (no-oscillation)

This type of CP violation occurs due to the CP violation in the decay part

$$\overline{\rho}(f) \equiv \frac{\overline{A}(f)}{A(f)} \equiv \frac{1}{\rho(f)} \neq 1$$



In PDG2012 | New since PDG2012 (preliminar

RPP#	Mode	PDG2012 Avg.	BABAR	
239	$K^0\pi^+$	0.009 ± 0.029	$-0.029 \pm 0.039 \pm 0.010$	
240	$K^+\pi^0$	0.051 ± 0.025	$0.030 \pm 0.039 \pm 0.010$	
241	$\eta' K^+$	0.013 ± 0.017	$0.008^{+0.017}_{-0.018} \pm 0.009$	
242	$\eta' K^{*+}$	-0.26 ± 0.27	$-0.26 \pm 0.27 \pm 0.02$	
243	$\eta' K_0^*(1430)^+$	0.06 ± 0.20	$0.06 \pm 0.20 \pm 0.02$	
244	$\eta' K_2^*(1430)^+$	0.15 ± 0.13	$0.15 \pm 0.13 \pm 0.02$	
245	ηK^+	-0.37 ± 0.08	$-0.36 \pm 0.11 \pm 0.03$	
246	ηK^{*+}	0.02 ± 0.06	$0.01 \pm 0.08 \pm 0.02$	
247	$\eta K_0^*(1430)^+$	$0.05 \pm 0.13 \pm 0.02$	$0.05 \pm 0.13 \pm 0.02$	
248	$\eta K_2^*(1430)^+$	$-0.45 \pm 0.30 \pm 0.02$	$-0.45 \pm 0.30 \pm 0.02$	
257	ωK^+	0.02 ± 0.05	$-0.01 \pm 0.07 \pm 0.01$	
258	ωK^{*+}	0.29 ± 0.35	$0.29 \pm 0.35 \pm 0.02$	
260	$\omega K_0^*(1430)^+$	-0.10 ± 0.09	$-0.10 \pm 0.09 \pm 0.02$	
261	$\omega K_2^*(1430)^+$	0.14 ± 0.15	$0.14 \pm 0.15 \pm 0.02$	
264	$K^{*0}\pi^+$	-0.04 ± 0.09	$0.032 \pm 0.052^{+0.016}_{-0.013}$	
265	$K^{*+}\pi^0$	-0.06 ± 0.24	$-0.06 \pm 0.24 \pm 0.04$	
266	$K^+\pi^+\pi^-$	0.038 ± 0.022	$0.028 \pm 0.020 \pm 0.023$	
269	$f_0(980)K^+$	$-0.09^{+0.05}_{-0.04}$	$-0.106 \pm 0.050^{+0.036}_{-0.015}$	
270	$f_2(1270)K^+$	$-0.68^{+0.19}_{-0.017}$	$-0.85 \pm 0.22^{+0.26}_{-0.13}$	$-0.59 \pm 0.22 \pm 0.04$
273	$f_0(1500)K^+$ †	$0.28^{+0.30}_{-0.29}$	$0.28 \pm 0.26^{+0.15}_{-0.14}$	
274	$f_2'(1525)K^+$	New	$0.14 \pm 0.10 \pm 0.04$	
275	$\rho^0 K^+$	0.37 ± 0.10	$0.44 \pm 0.10^{+0.06}_{-0.14}$	$0.30 \pm 0.11^{+0.11}_{-0.05}$
276	$K_0^*(1430)^0\pi^+$	0.55 ± 0.33	$0.032 \pm 0.035^{+0.034}_{-0.028}$	$0.076 \pm 0.038^{+0.028}_{-0.022}$
277	$K_2^*(1430)^0\pi^+$	$0.05^{+0.29}_{-0.24}$	$0.05 \pm 0.23^{+0.18}_{-0.08}$	
280	$K^+\pi^0\pi^0$	-0.06 ± 0.07	$-0.06 \pm 0.06 \pm 0.04$	
287	$\rho^+ K^0$	-0.12 ± 0.17	$-0.12 \pm 0.17 \pm 0.02$	
288	$K^{*+}\pi^+\pi^-$	0.07 ± 0.08	$0.07 \pm 0.07 \pm 0.04$	
289	$K^{*+}\rho^0$	0.31 ± 0.13	$0.31 \pm 0.13 \pm 0.03$	
290	$f_0(980)K^{*+}$	-0.15 ± 0.12	$-0.15 \pm 0.12 \pm 0.03$	
291	$a_1^+ K^0$	0.12 ± 0.11	$0.12 \pm 0.11 \pm 0.02$	
292	$b_1^+ K^0$	-0.03 ± 0.15	$-0.03 \pm 0.15 \pm 0.02$	
293	$K^{*0}\rho^+$	-0.01 ± 0.16	$-0.01 \pm 0.16 \pm 0.02$	

355	$\rho^0\pi^+$	$0.18^{+0.09}_{-0.17}$	$0.18 \pm 0.07^{+0.05}_{-0.15}$	
357	$f_2(1270)\pi^+$	$0.41^{+0.31}_{-0.29}$	$0.41 \pm 0.25^{+0.18}_{-0.15}$	
358	$\rho(1450)^0\pi^+$	$-0.06^{+0.36}_{-0.42}$	$-0.06 \pm 0.28^{+0.23}_{-0.32}$	
359	$f_0(1370)\pi^+$	0.72 ± 0.22	$0.72 \pm 0.15 \pm 0.16$	
361	$\pi^+\pi^-\pi^+(NR)$	$-0.14^{+0.23}_{-0.16}$	$-0.14 \pm 0.14^{+0.18}_{-0.08}$	
363	$\rho^+\pi^0$	0.02 ± 0.11	$-0.01 \pm 0.13 \pm 0.02$	$0.06 \pm 0.17^{+0.04}_{-0.05}$
365	$\rho^+\rho^0$	-0.05 ± 0.05	$-0.054 \pm 0.055 \pm 0.010$	$0.00 \pm 0.22 \pm 0.03$
369	$\omega\pi^+$	-0.04 ± 0.06	$-0.02 \pm 0.08 \pm 0.01$	$-0.02 \pm 0.09 \pm 0.01$
370	$\omega\rho^+$	-0.20 ± 0.09	$-0.20 \pm 0.09 \pm 0.02$	
371	$\eta\pi^+$	-0.14 ± 0.07	$-0.03 \pm 0.09 \pm 0.03$	$-0.19 \pm 0.06 \pm 0.01$
372	$\eta\rho^+$	0.11 ± 0.11	$0.13 \pm 0.11 \pm 0.02$	$-0.04^{+0.34}_{-0.32} \pm 0.01$
373	$\eta'\pi^+$	0.06 ± 0.16	$0.03 \pm 0.17 \pm 0.02$	$0.20^{+0.37}_{-0.36} \pm 0.04$
374	$\eta'\rho^+$	0.26 ± 0.17	$0.26 \pm 0.17 \pm 0.02$	
382	$b_1^0\pi^+$	0.05 ± 0.16	$0.05 \pm 0.16 \pm 0.02$	
391	$p\overline{p}\pi^+$	0.00 ± 0.04	$0.04 \pm 0.07 \pm 0.04$	$-0.17 \pm 0.10 \pm 0.02$
394	$p\overline{p}K^+$	-0.16 ± 0.07	$-0.16 \pm 0.08 \pm 0.04$	$-0.02 \pm 0.05 \pm 0.02$
399	$p\overline{p}K^{*+}$	0.21 ± 0.16	$0.32 \pm 0.13 \pm 0.05$	$-0.01 \pm 0.19 \pm 0.02$
402	$p\overline{\Lambda}\gamma$	0.17 ± 0.17		$0.17 \pm 0.16 \pm 0.05$
403	$p\overline{\Lambda}\pi^0$	0.01 ± 0.17		$0.01 \pm 0.17 \pm 0.04$
441	$K^+\ell\ell$	-0.01 ± 0.09	$-0.03 \pm 0.14 \pm 0.01$	$0.04 \pm 0.10 \pm 0.02$

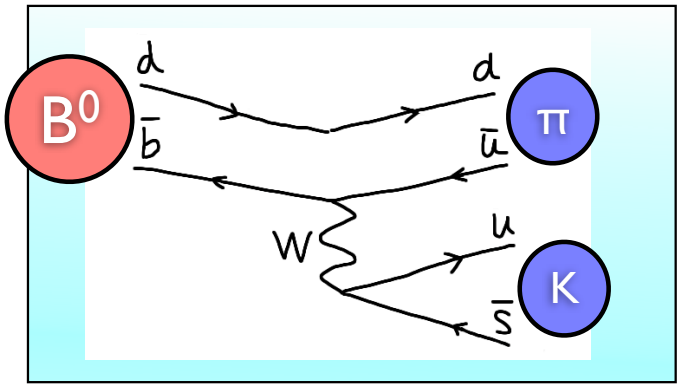
Direct CP asymmetry are measured over a hundred of decay channels at B factories!

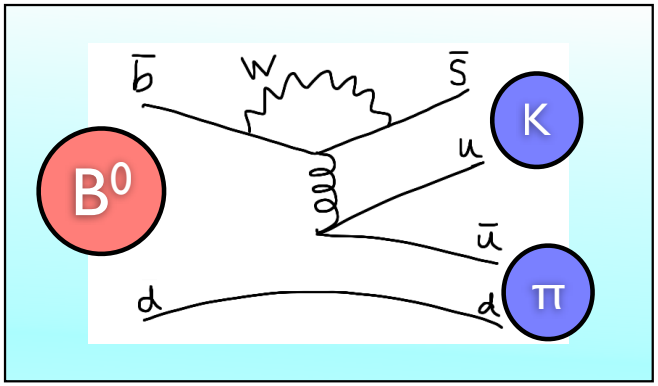
Direct CP violation (no-oscillation)

Example: $B^0 \to K^+ \pi^-$ mode

CP asymmetry

$$\mathcal{A} = \frac{\Gamma(\overline{B}^0 \to K^-\pi^+) - \Gamma(B^0 \to K^+\pi^-)}{\Gamma(\overline{B}^0 \to K^-\pi^+) + \Gamma(B^0 \to K^+\pi^-)} = -0.086 \pm 0.007$$





$A_1(\overline{B}^0 \to K^- \pi^+) = V_{ub} V_{us}^* H_1(BKpi)$

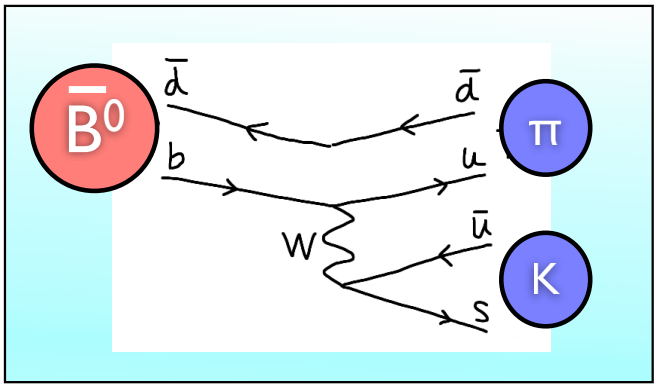
$A_2(\overline{B}^0 \to K^- \pi^+) = V_{tb} V_{ts}^* H_2(BKpi)$

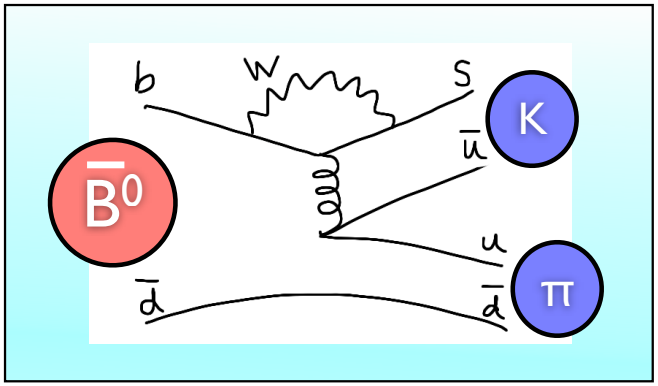
Direct CP violation (no-oscillation)

Example: $B^0 \to K^+ \pi^-$ mode

CP asymmetry

$$\mathcal{A} = \frac{\Gamma(\overline{B}^0 \to K^-\pi^+) - \Gamma(B^0 \to K^+\pi^-)}{\Gamma(\overline{B}^0 \to K^-\pi^+) + \Gamma(B^0 \to K^+\pi^-)} = -0.086 \pm 0.007$$





$A_1(B^0 \to K^+ \pi^-) = V_{ub} V_{us}^* H_1(BKpi)$

$A_2(B^0 \to K^+ \pi^-) = V_{tb} V_{ts}^* H_2(BKpi)$

Direct CP violation (no-oscillation)

Example: $B^0 \to K^+ \pi^-$ mode

CP asymmetry

$$\mathcal{A} = \frac{\Gamma(\overline{B}^0 \to K^-\pi^+) - \Gamma(B^0 \to K^+\pi^-)}{\Gamma(\overline{B}^0 \to K^-\pi^+) + \Gamma(B^0 \to K^+\pi^-)} = -0.086 \pm 0.007$$

$$A(B^0 \to K^+ \pi^-) = A_1 + A_2 = V_{ub}V_{us}^* H_1(K^+\pi^-) + V_{tb}V_{ts}^* H_2(K^+\pi^-)$$

change *no change*

$$A(\overline{B}^0 \to K^- \pi^+) = \overline{A}_1 + \overline{A}_2 = V_{ub}^* V_{us} H_1(K^-\pi^+) + V_{tb}^* V_{ts} H_2(K^-\pi^+)$$

$$\Gamma(B^0 \to K^+ \pi^-) = |A_1 + A_2|^2 = (A_1 + A_2)\,(A_1^* + A_2^*)$$

$$\Gamma(\overline{B}^0 \to K^- \pi^+) = |\overline{A}_1 + \overline{A}_2|^2 = (\overline{A}_1 + \overline{A}_2)\,(\overline{A}_1^* + \overline{A}_2^*)$$

Direct CP violation (no-oscillation)

This type of CP violation occurs due to the CP violation in the decay part

$$|A(f)| \neq |\overline{A}(\overline{f})|$$

Thus, the non-zero CP violation can be measured without time-dependent analysis, nor without CP eigenstate final state. However, it still requires certain conditions

We can measure ~~CP~~ *only* through an interference of two amplitudes with different CP conserving and CP violating phases.

$$\begin{aligned} A(\overline{B}^0 \to \overline{f}) &= A_1 e^{+i\theta_1} e^{+i\delta_1} + A_2 e^{+i\theta_2} e^{+i\delta_2} \\ A(B^0 \to f) &= A_1 e^{-i\theta_1} e^{+i\delta_1} + A_2 e^{-i\theta_2} e^{+i\delta_2} \end{aligned}$$

$\theta_{1,2}$: CP the violating phase, $\delta_{1,2}$: the CP conserving phase.

$$\frac{\Gamma(\overline{B}^0 \to \overline{f}) - \Gamma(B^0 \to f)}{\Gamma(\overline{B}^0 \to \overline{f}) + \Gamma(B^0 \to f)} = \frac{2(A_2/A_1)\sin(\theta_1 - \theta_2)\sin(\delta_1 - \delta_2)}{1 + 2(A_2/A_1)\cos(\theta_1 - \theta_2)\cos(\delta_1 - \delta_2)}$$

Direct CP violation (no-oscillation)

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We can measure ~~CP~~ *only*
different CP conserving an

$$A(\overline{B}^0 \to \overline{f}) =$$
$$A(B^0 \to f) =$$

$\theta_{1,2}$: CP the violating

Conditions for Non-zero Direct CPV
more than two diagrams overlaps
with different CPV phase
with different CPC phase

$$\frac{\Gamma(\overline{B}^0 \to \overline{f}) - \Gamma(B^0 \to f)}{\Gamma(\overline{B}^0 \to \overline{f}) + \Gamma(B^0 \to f)} = \frac{2(A_2/A_1)\sin(\theta_1 - \theta_2)\sin(\delta_1 - \delta_2)}{1 + 2(A_2/A_1)\cos(\theta_1 - \theta_2)\cos(\delta_1 - \delta_2)}$$

Direct CP violation (no-oscillation)

This type of CP violation occurs due to the CP violation in the decay part

$$|A(f)| \neq |\overline{A}(\overline{f})|$$

Thus, the non-zero CP violation can be measured without time-dependent analysis, nor without CP eigenstate final state. However it still requires certain conditions

We can measure ~~CP~~ *only* t
different CP conserving an

$$A(\overline{B}^0 \to \overline{f}) =$$
$$A(B^0 \to f) =$$

$\theta_{1,2}$: CP the violating r $\delta_{1,2}$.

Resulting direct CP violation contains two informations:
CP violating phase in weak int.
CP conserving phase in strong int.

$$\frac{\Gamma(\overline{B}^0 \to \overline{f}) - \Gamma(B^0 \to f)}{\Gamma(\overline{B}^0 \to \overline{f}) + \Gamma(B^0 \to f)} = \frac{2(A_2/A_1)\sin(\theta_1 - \theta_2)\sin(\delta_1 - \delta_2)}{1 + 2(A_2/A_1)\cos(\theta_1 - \theta_2)\cos(\delta_1 - \delta_2)}$$

Test of Unitarity of CKM

$$V_{CKM}^{\dagger} V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$V_{CKM} V_{CKM}^{\dagger} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Test of Unitarity

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Unitarity: 9 complex numbers can be replaced by the 4 real number parameters

⇩

We must test *at which extent* this is satisfied!

Unitarity triangles

$$\underbrace{V_{ud}V_{us}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{cd}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{td}V_{ts}^*}_{\mathcal{O}(\lambda^5)} = 0$$

ds

$$\underbrace{V_{ud}V_{cd}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{us}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{ub}V_{cb}^*}_{\mathcal{O}(\lambda^5)} = 0$$

uc

$$\underbrace{V_{us}V_{ub}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cs}V_{cb}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{ts}V_{tb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

sb

$$\underbrace{V_{td}V_{cd}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{ts}V_{cs}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{tb}V_{cb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

ct

$$\underbrace{V_{ud}^*V_{ub}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{cd}^*V_{sb}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{td}^*V_{tb}}_{\mathcal{O}(\lambda^3)} = 0$$

db

$$\underbrace{V_{ud}V_{ub}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{cd}V_{cb}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{td}V_{tb}^*}_{\mathcal{O}(\lambda^3)} = 0$$

ut

Unitarity triangles

$$\underbrace{V_{ud}V_{us}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{cd}V_{cs}^*}_{\mathcal{O}} + \underbrace{V_{td}V_{ts}^*}_{} = 0$$

$$\underbrace{V_{ud}V_{cd}^*}_{} + \underbrace{V_{us}V_{cs}^*}_{} + \underbrace{V_{ub}V_{cb}^*}_{\mathcal{O}(\lambda^5)} = 0$$

uc

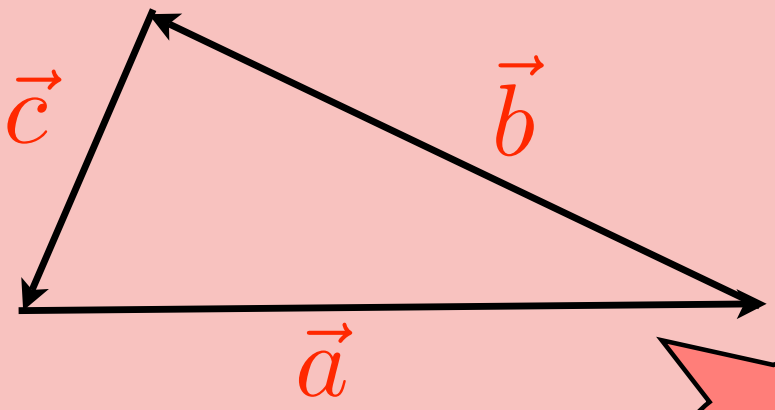
$$\underbrace{V_{us}V_{ub}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{c}}_{\mathcal{O}}$$

$$\underbrace{{}_{tb}V_{cb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

ct

$$\underbrace{V_{ud}^*V_{ub}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{c}}_{\mathcal{O}(\lambda^3)} \quad \underbrace{\phantom{V_{td}V_{ts}^*}}_{\mathcal{O}(\lambda^3)} \quad \underbrace{\phantom{V_{ud}V_{cd}^*}}_{\mathcal{O}(\lambda^3)}$$

$$\underbrace{V_{1i}^*V_{1k}}_{\vec{a}} + \underbrace{V_{2i}^*V_{1k}}_{\vec{b}} + \underbrace{V_{3i}V_{1k}}_{\vec{c}} = 0$$

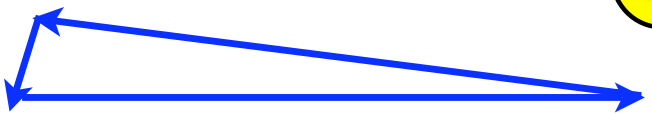


Unitarity Triangle

Unitarity triangles

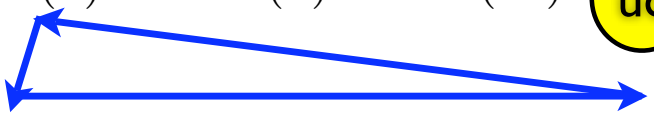
$$\underbrace{V_{ud}V_{us}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{cd}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{td}V_{ts}^*}_{\mathcal{O}(\lambda^5)} = 0$$

ds



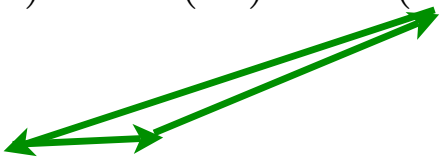
$$\underbrace{V_{ud}V_{cd}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{us}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{ub}V_{cb}^*}_{\mathcal{O}(\lambda^5)} = 0$$

uc



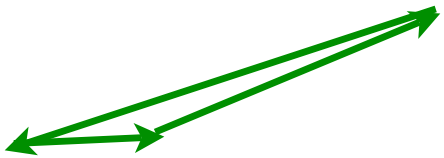
$$\underbrace{V_{us}V_{ub}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cs}V_{cb}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{ts}V_{tb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

sb



$$\underbrace{V_{td}V_{cd}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{ts}V_{cs}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{tb}V_{cb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

ct



$$\underbrace{V_{ud}^*V_{ub}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{cd}^*V_{cb}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{td}^*V_{tb}}_{\mathcal{O}(\lambda^3)} = 0$$

db

$$\underbrace{V_{td}V_{ud}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{ts}V_{us}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{tb}V_{ub}^*}_{\mathcal{O}(\lambda^3)} = 0$$

ut

Unitarity triangles

$$\underbrace{V_{ud}V_{us}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{cd}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{td}V_{ts}^*}_{\mathcal{O}(\lambda^5)} = 0$$

ds

K physics

$$\underbrace{V_{ud}V_{cd}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{us}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{ub}V_{cb}^*}_{\mathcal{O}(\lambda^5)} = 0$$

uc

D physics

$$\underbrace{V_{us}V_{ub}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cs}V_{cb}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{ts}V_{tb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

sb

B_s physics

$$\underbrace{V_{td}V_{cd}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{ts}V_{cs}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{tb}V_{cb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

ct

$$\underbrace{V_{ud}^*V_{ub}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{cd}^*V_{cb}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{td}^*V_{tb}}_{\mathcal{O}(\lambda^3)} = 0$$

db

B_d physics

$$\underbrace{V_{td}V_{ud}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{ts}V_{us}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{tb}V_{ub}^*}_{\mathcal{O}(\lambda^3)} = 0$$

ut

Unitarity triangles

$$\underbrace{V_{ud}V_{us}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{cd}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{td}V_{ts}^*}_{\mathcal{O}(\lambda^5)} = 0$$

ds

K physics

$$\underbrace{V_{ud}V_{cd}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{us}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{ub}V_{cb}^*}_{\mathcal{O}(\lambda^5)} = 0$$

uc

D physics

$$\underbrace{V_{us}V_{ub}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cs}V_{cb}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{ts}V_{tb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

sb

B_s physics

$$\underbrace{V_{td}V_{cd}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{ts}V_{cs}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{tb}V_{cb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

ct

The Unitarity Triangle!

db

B_d physics

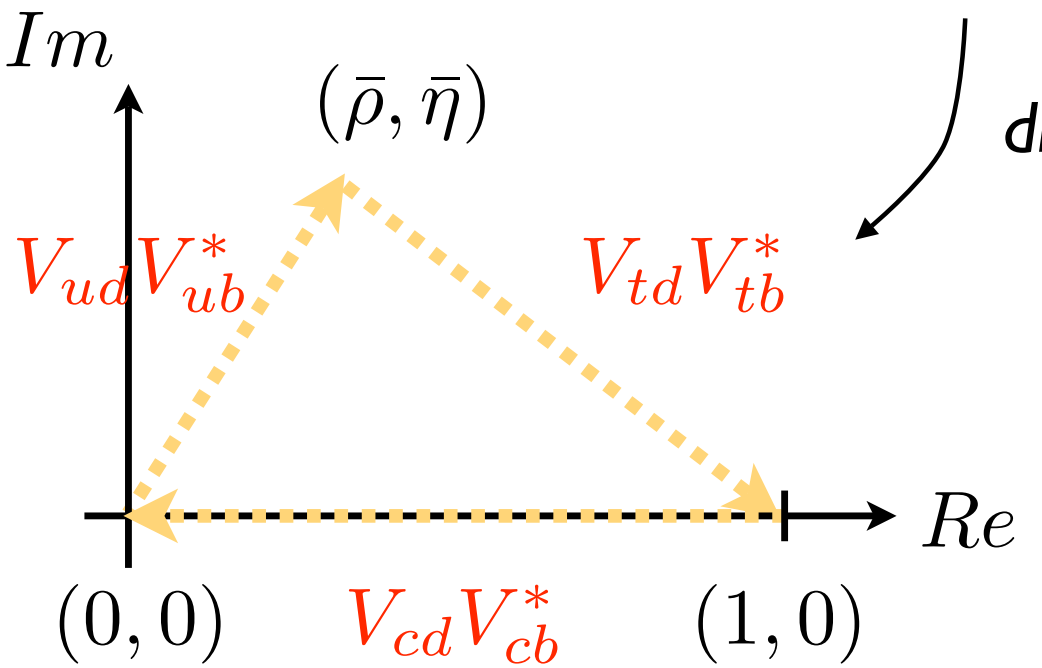
$$\underbrace{V_{td}V_{ud}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{ts}V_{us}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{tb}V_{ub}^*}_{\mathcal{O}(\lambda^3)} = 0$$

ut

The Unitarity Triangle

$$\underbrace{V_{ud}V_{ub}^*}_{A\lambda^3(\rho+i\eta)} + \underbrace{V_{cd}V_{cb}^*}_{-A\lambda^3} + \underbrace{V_{td}V_{tb}^*}_{A\lambda^3(1-\rho-i\eta)} = 0$$

divide by $A\lambda^3$

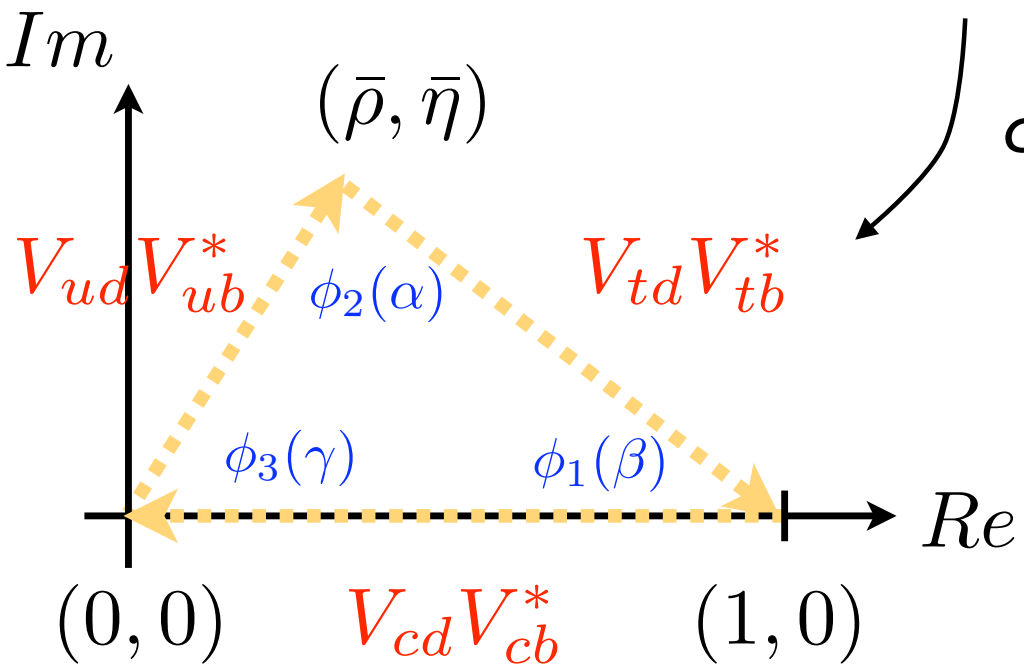


B_d physics

The Unitarity Triangle

$$\underbrace{V_{ud}V_{ub}^*}_{A\lambda^3(\rho+i\eta)} + \underbrace{V_{cd}V_{cb}^*}_{-A\lambda^3} + \underbrace{V_{td}V_{tb}^*}_{A\lambda^3(1-\rho-i\eta)} = 0$$

divide by $A\lambda^3$



$$arg\left(\frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cs}}\right) \equiv -\phi_1$$

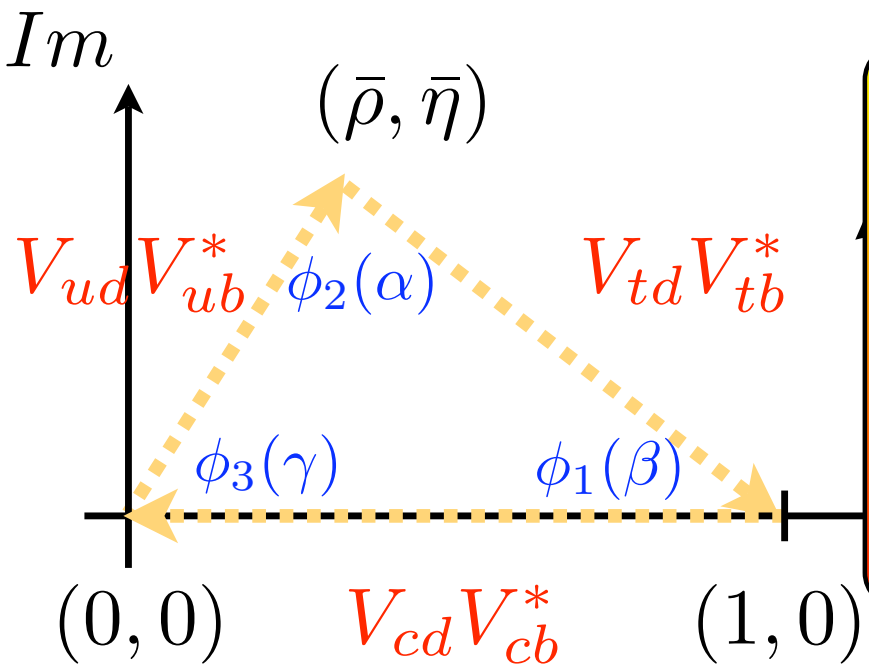
$$arg\left(\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}}\right) \equiv -\phi_2$$

$$arg\left(\frac{V_{cb}^* V_{cs}}{V_{ub}^* V_{ud}}\right) \equiv -\phi_3$$

B_d physics

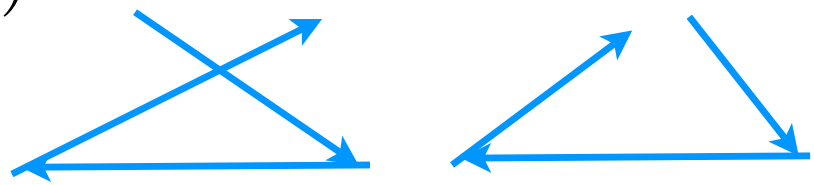
The Unitarity Triangle

$$\underbrace{V_{ud}V_{ub}^*}_{A\lambda^3(\rho+i\eta)} + \underbrace{V_{cd}V_{cb}^*}_{-A\lambda^3} + \underbrace{V_{td}V_{tb}^*}_{A\lambda^3(1-\rho-i\eta)} = 0$$

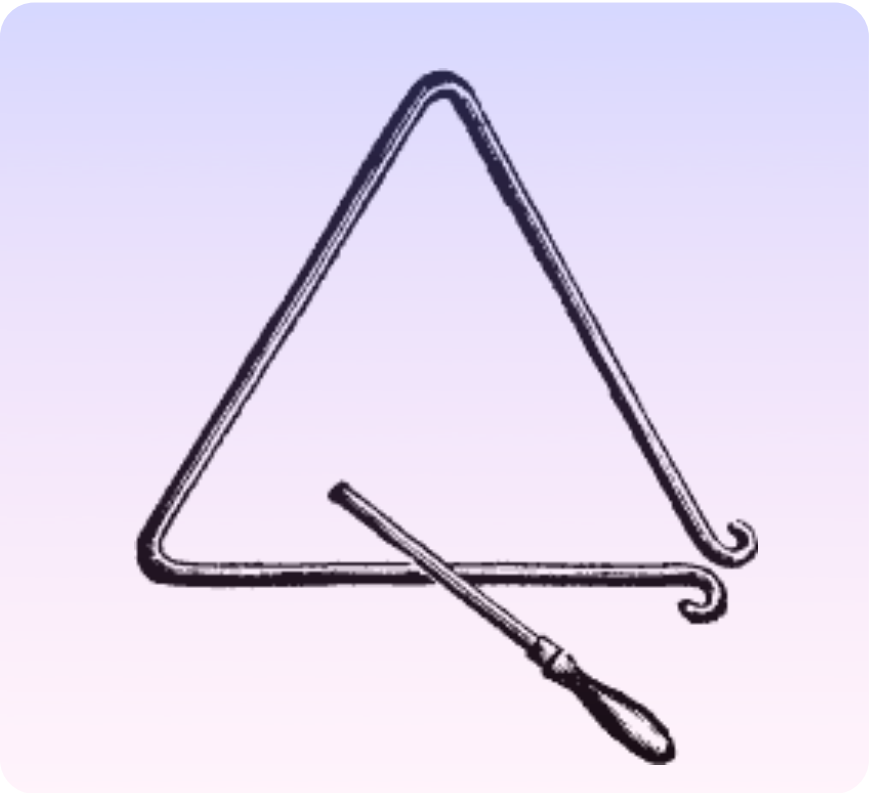


Unitarity test is to verify if the triangle closes at the apex from *independent* measurements for three sides and three angles!!

B_d physics

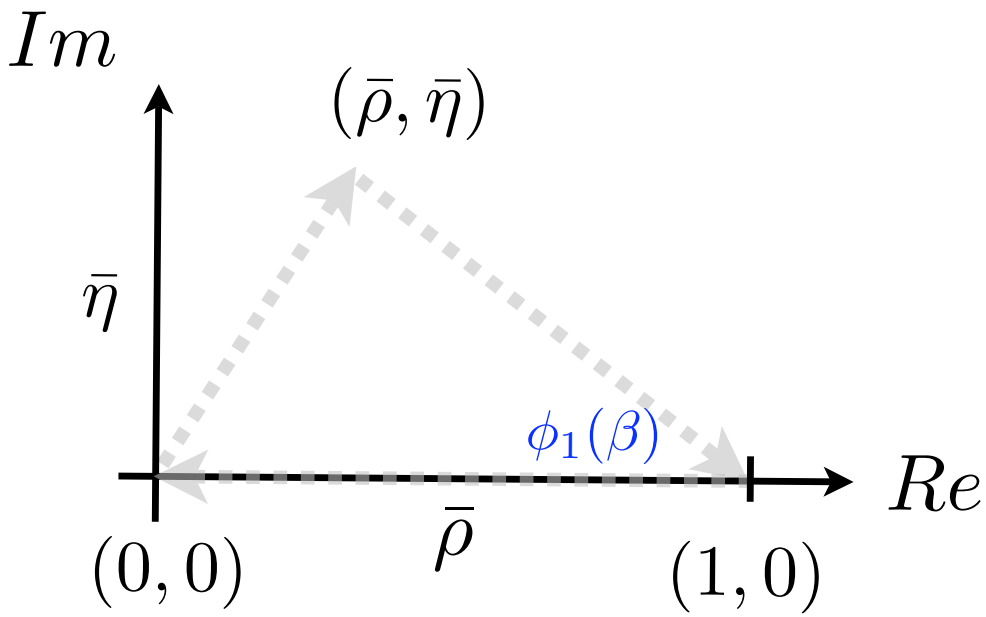


Determination of the CKM matrix



Determination of the CKM matrix: sin2Φ₁ (β) (phase)

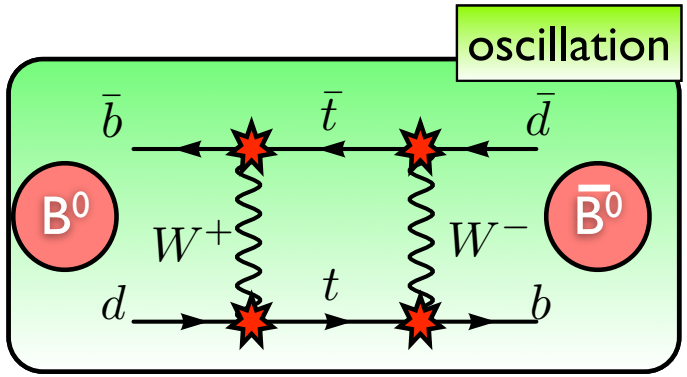
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

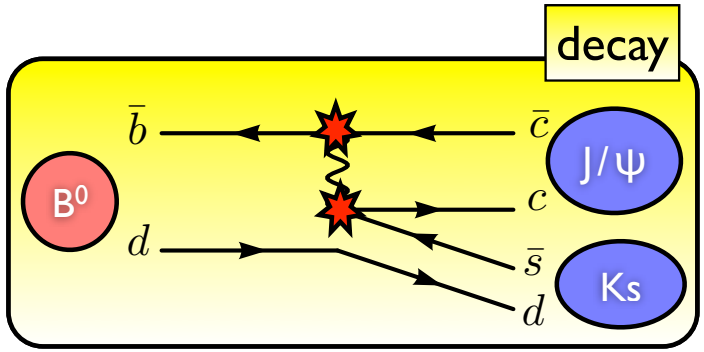


CP asymmetry in B→J/ψK$_S$

$$A_{J/\psi K_S}(t) = \frac{\Gamma(\overline{B}^0(t)\to J/\psi K_S)-\Gamma(B^0(t)\to J/\psi K_S)}{\Gamma(\overline{B}^0(t)\to J/\psi K_S)+\Gamma(B^0(t)\to J/\psi K_S)} = S_{J/\psi K_s}\sin\Delta M_B\ t$$

$$\begin{aligned} S_{J/\psi K_s} &= Im\left[\underbrace{\frac{M_{12}}{M_{12}^*}}_{oscill.}\underbrace{\frac{A(\overline{B}\to J/\psi K_S)}{A(B\to J/\psi K_S)}}_{decay}\right] \\ &= Im\left[\underbrace{\frac{V_{tb}V_{td}^*}{V_{tb}^*V_{td}}}_{oscill.}\underbrace{\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}}_{decay}\right] \\ &= \sin 2\phi_1 \end{aligned}$$



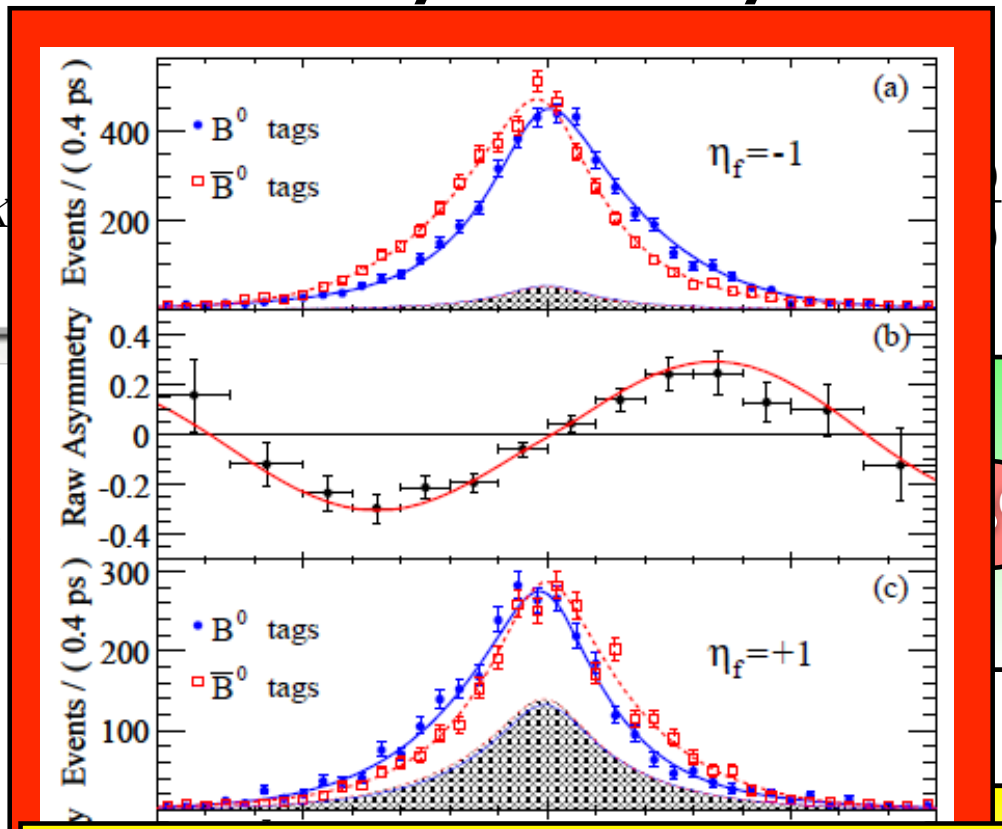


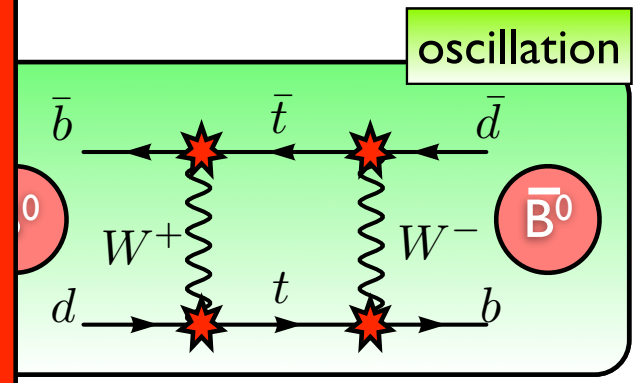
CP asymmetry in B→J/ψK$_S$

$A_{J/\psi K}$

$S_{J/\psi K_s}$

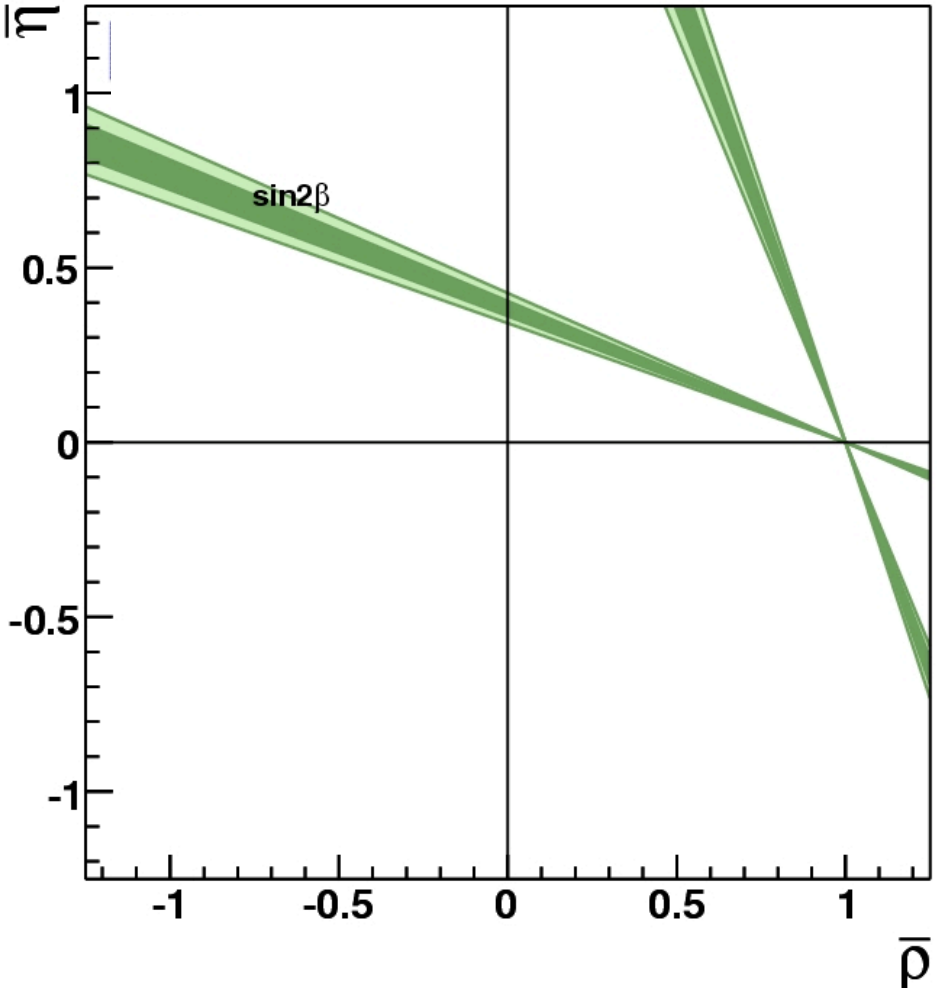
$= S_{J/\psi K_s} \sin \Delta M_B \; t$





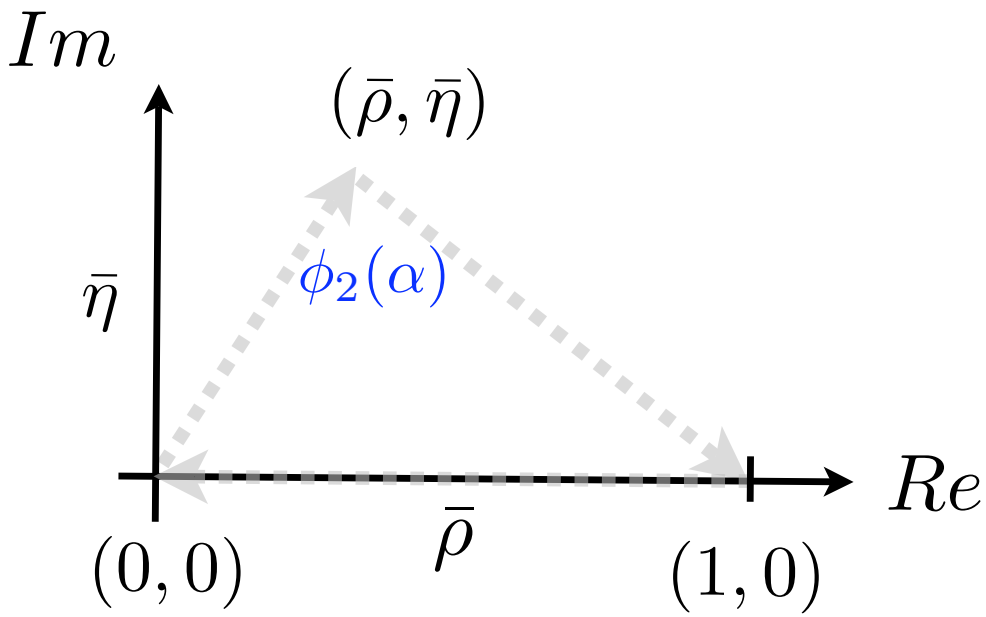
Including various b->cc$_{bar}$s the measurements, we find Φ_I=(21.1±0.9) °

Determination of the CKM matrix:



Determination of the CKM matrix: sin2Φ$_2$ (α) (phase)

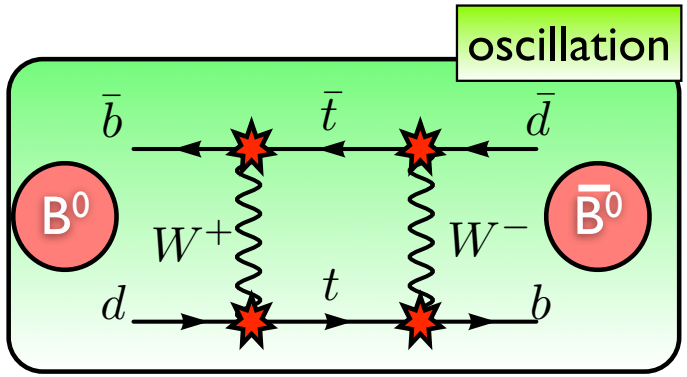
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

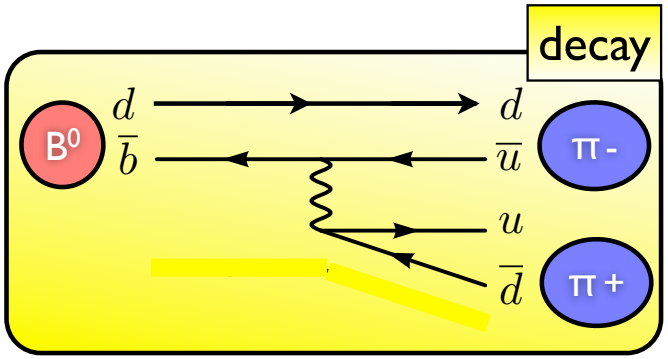


CP asymmetry in $B \to \pi^+\pi^-$

$$A_{\pi^+\pi^-}(t) = \frac{\Gamma(\overline{B}^0(t) \to \pi^+\pi^-) - \Gamma(B^0(t) \to \pi^+\pi^-)}{\Gamma(\overline{B}^0(t) \to \pi^+\pi^-) + \Gamma(B^0(t) \to \pi^+\pi^-)} = S_{\pi^+\pi^-} \sin \Delta M_B \; t$$

$$\begin{aligned} S_{\pi^+\pi^-} &= Im \left[\underbrace{\frac{M_{12}}{M_{12}^*}}_{oscill.} \underbrace{\frac{A(\overline{B} \to \pi^+\pi^-)}{A(B \to \pi^+\pi^-)}}_{decay} \right] \\ &= Im \left[\underbrace{\frac{V_{tb}V_{td}^*}{V_{tb}^*V_{td}}}_{oscill.} \underbrace{\frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}}}_{decay} \right] \\ &= \sin 2\phi_2(\alpha) \end{aligned}$$



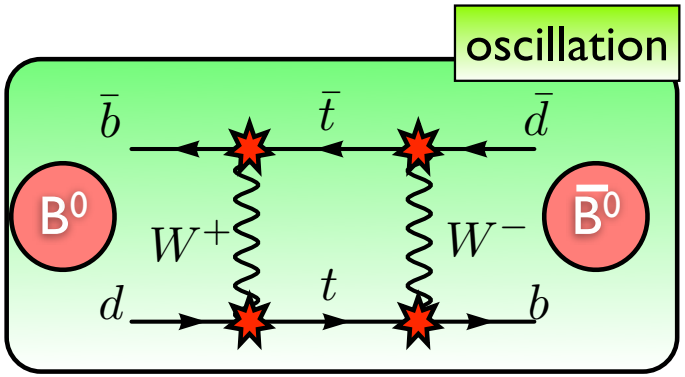


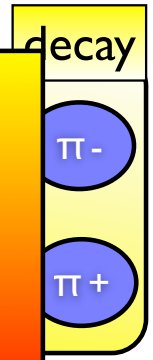
*Assuming tree-dominant...

CP asymmetry in $B \to \pi^+\pi^-$

$$A_{\pi^+\pi^-}(t) = \frac{\Gamma(\overline{B}^0(t) \to \pi^+\pi^-) - \Gamma(B^0(t) \to \pi^+\pi^-)}{\Gamma(\overline{B}^0(t) \to \pi^+\pi^-) + \Gamma(B^0(t) \to \pi^+\pi^-)} = S_{\pi^+\pi^-} \sin \Delta M_B \, t$$

Penguin pollution prevents a precise measurement of Φ_2 (α)





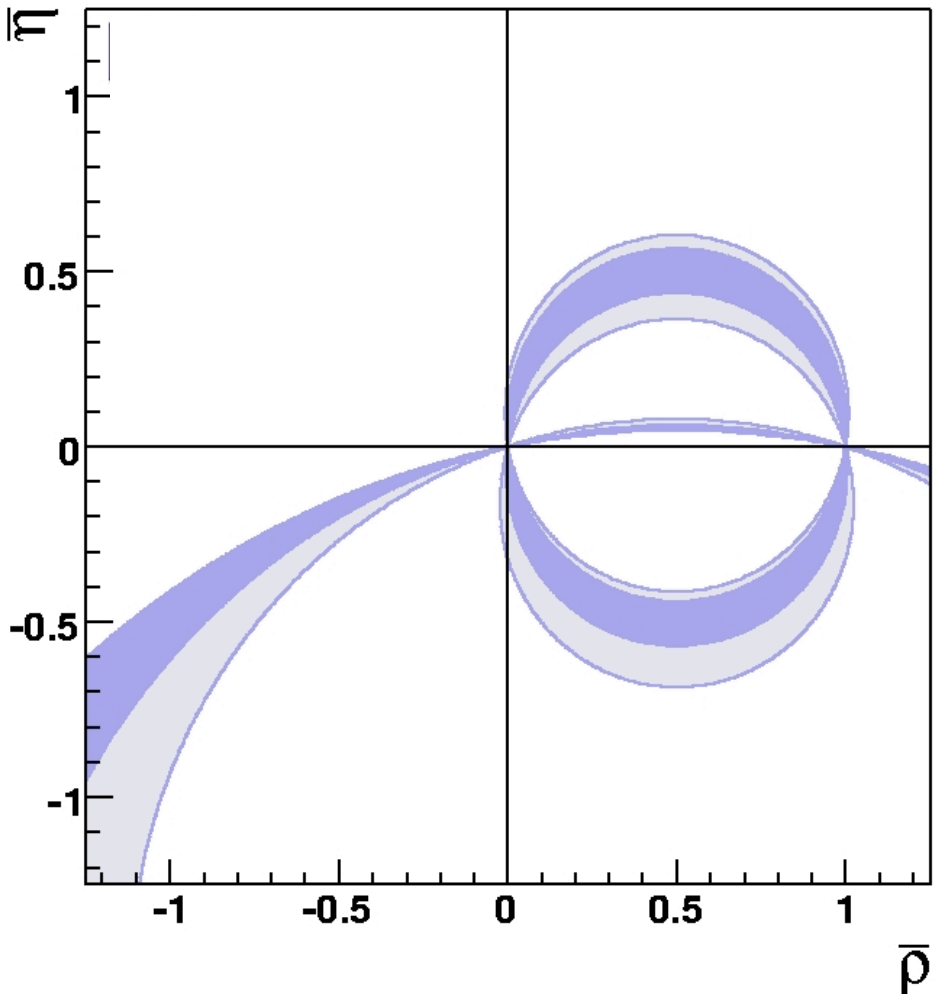
Including the measurements with

$\pi\pi$, $\rho\rho$, $\pi\rho$,

$\alpha = (89.0^{+4.4}_{-4.2})$ °

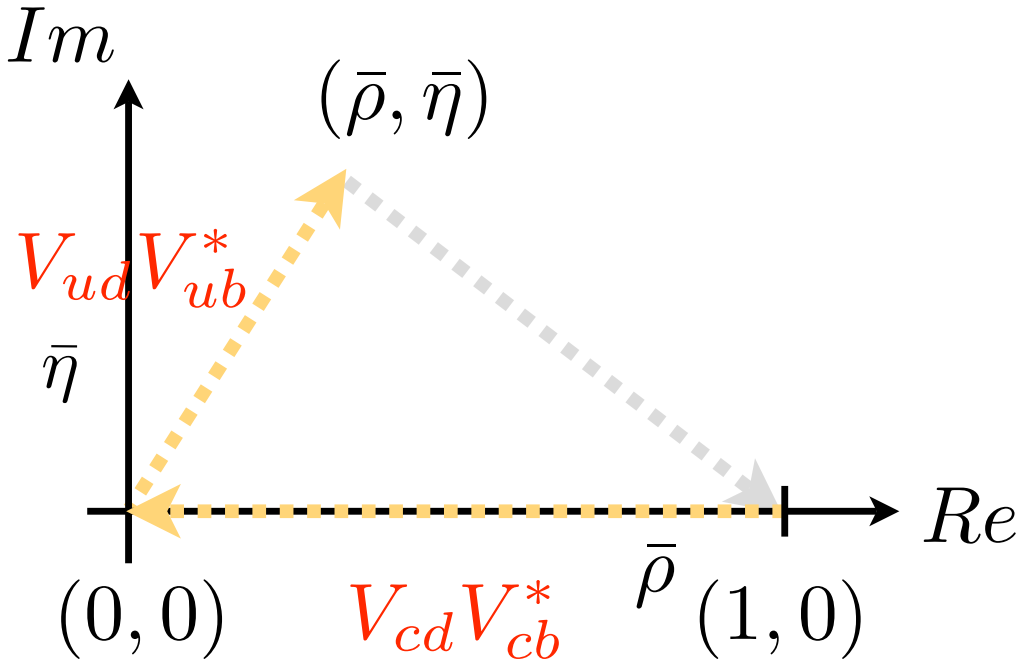
* Assuming tree-dominant...

Determination of the CKM matrix:



Determination of the CKM matrix: $|V_{ub}|$ and $|V_{cb}|$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

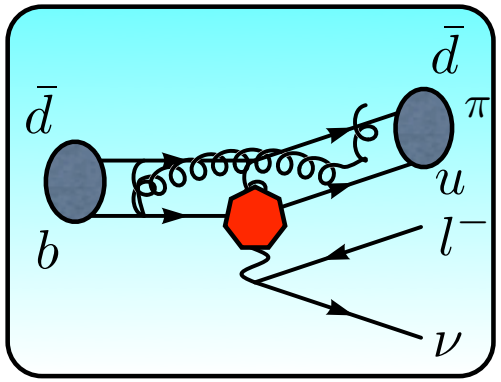


Determination of the CKM matrix: $|V_{ub}|$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

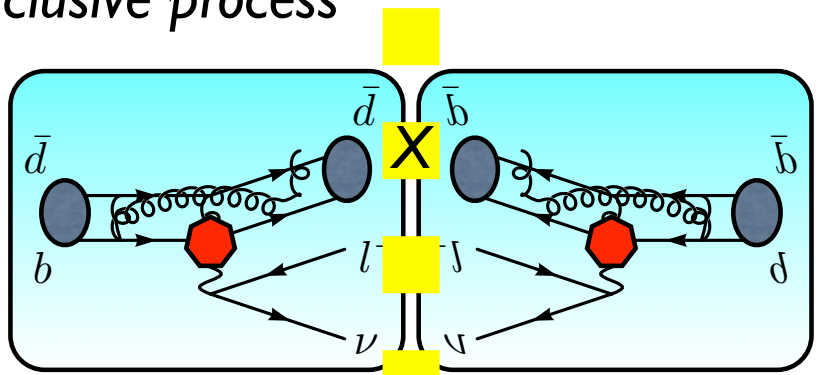
** Similar in $|V_{cb}|$*

Exclusive process



$$\mathcal{A}(B \to \pi l \nu) \propto |V_{ub}| F^{B\to\pi}(q^2)$$

Inclusive process



Optical theorem

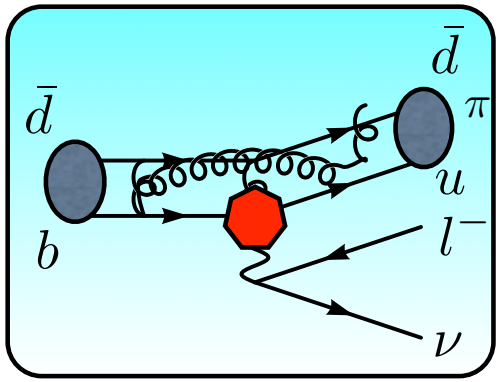
$$\sum |\mathcal{A}(B \to X_u l \nu)|^2 \propto |V_{ub}|^2 f(q^2, \mu_\pi, ...)$$

Determination of the CKM matrix: $|V_{ub}|$

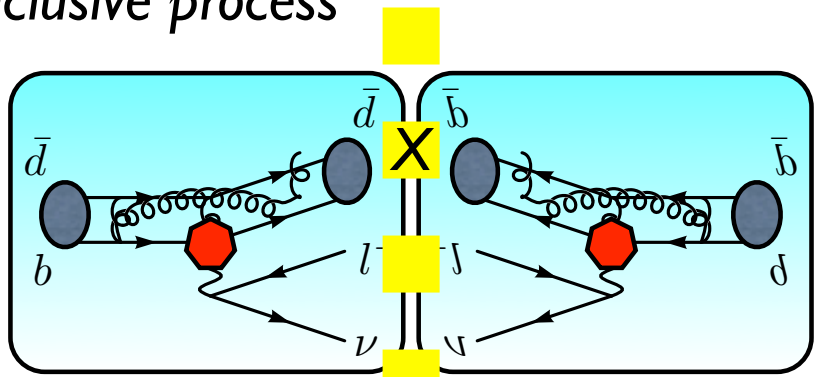
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

** Similar in $|V_{cb}|$*

Exclusive process



Inclusive process



Optical theorem

$$\mathcal{A}(B \to \pi l \nu) \propto |V_{ub}| F^{B\to\pi}(q^2)$$

$$\sum |\mathcal{A}(B \to X_u l \nu)|^2 \propto |V_{ub}|^2 f(q^2, \mu_\pi, \ldots)$$

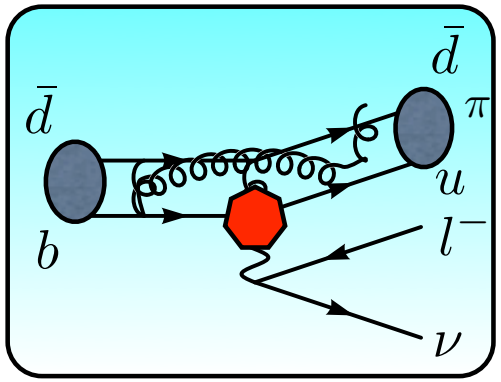
Hadronic uncertainties!

Determination of the CKM matrix: $|V_{ub}|$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

** Similar in $|V_{cb}|$*

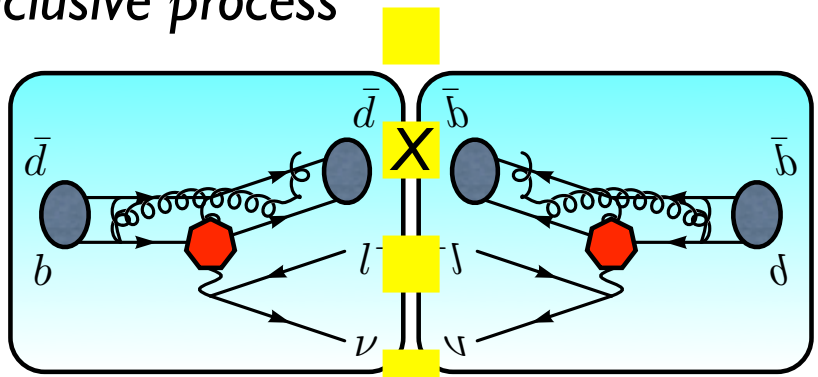
Exclusive process



$$\mathcal{A}(B \to \pi l \nu) \propto |V_{ub}| F^{B\to\pi}(q^2)$$

$|V_{ub}|=(3.89\pm0.44)10^{-3}$

Inclusive process



Optical theorem

$$\sum |\mathcal{A}(B \to X_u l \nu)|^2 \propto |V_{ub}|^2 f(q^2, \mu_\pi, ...)$$

$|V_{ub}|=(4.27\pm0.38)10^{-3}$

Determination of the CKM matrix: $|V_{ub}|$

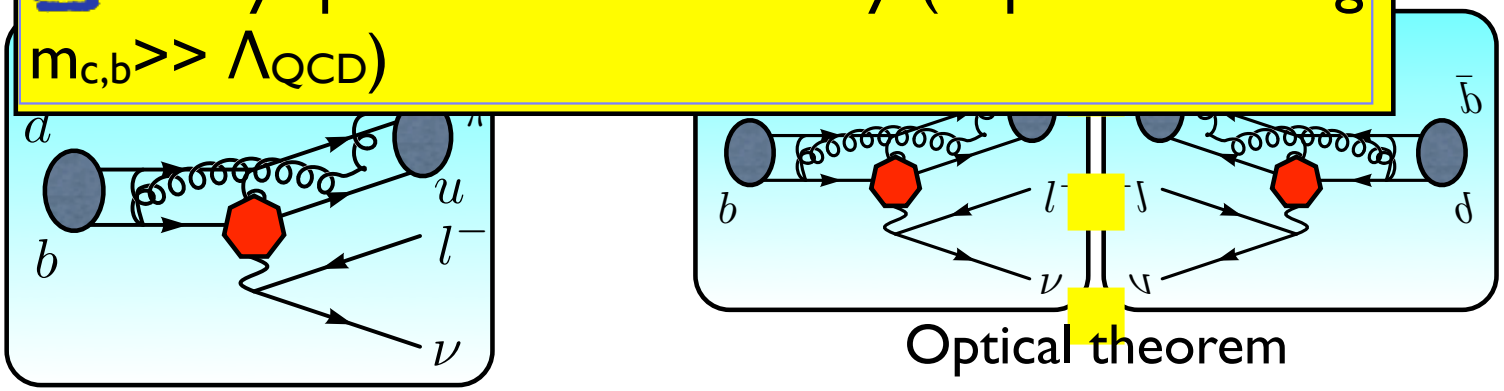
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \end{pmatrix}$$

* *Similar in $|V_{cb}|$*

Further improvements in hadronic uncertainties:

- ☑ Many efforts in Lattice QCD
- ☑ Heavy quark effective theory (expansion using $m_{c,b} >> \Lambda_{QCD}$)

Ex



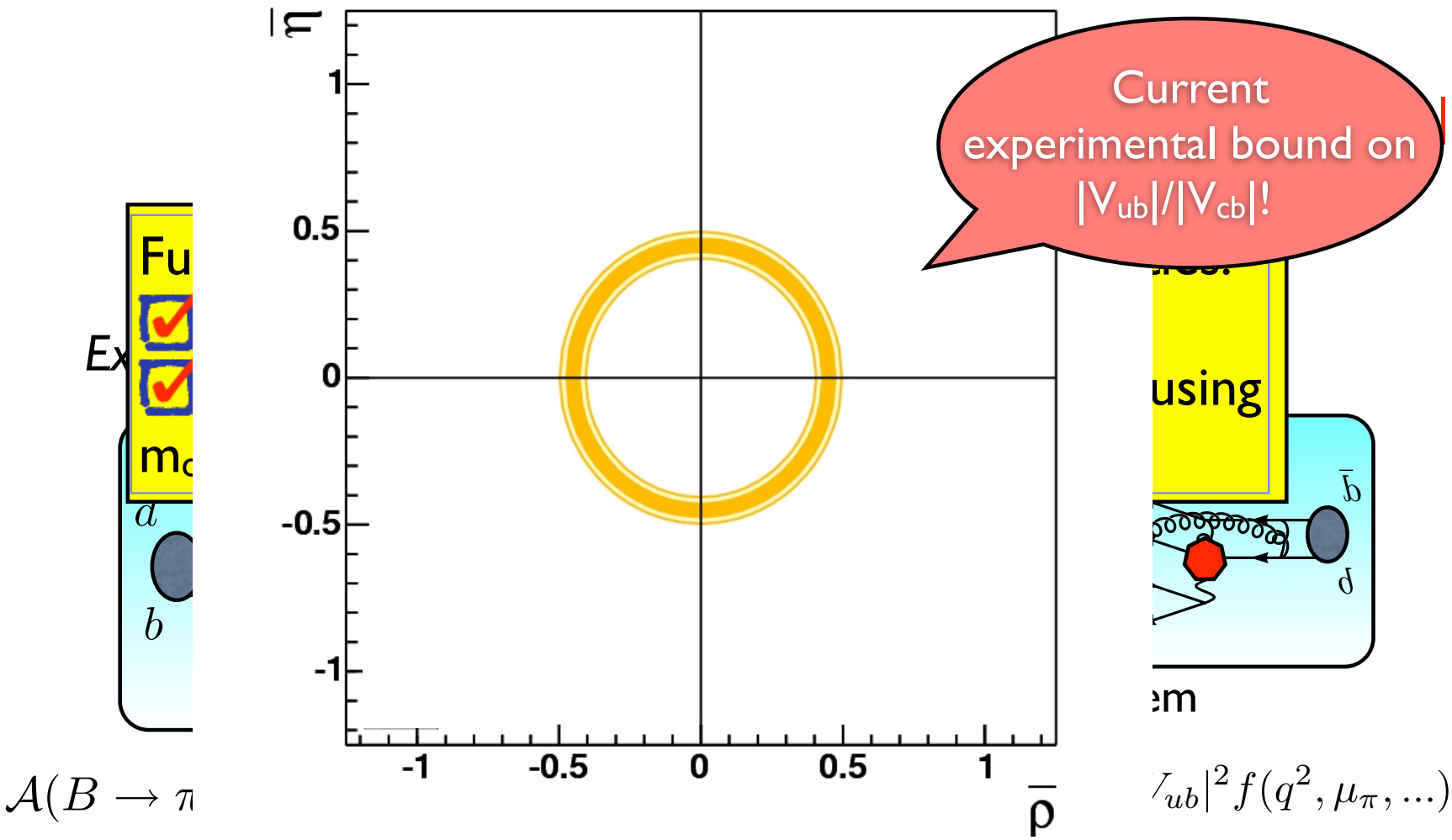
$$\mathcal{A}(B \to \pi l \nu) \propto |V_{ub}| F^{B\to\pi}(q^2)$$

$$\sum |\mathcal{A}(B \to X_u l \nu)|^2 \propto |V_{ub}|^2 f(q^2, \mu_\pi, ...)$$

$|V_{ub}|=(3.89\pm0.44)10^{-3}$

$|V_{ub}|=(4.27\pm0.38)10^{-3}$

Determination of the CKM matrix:

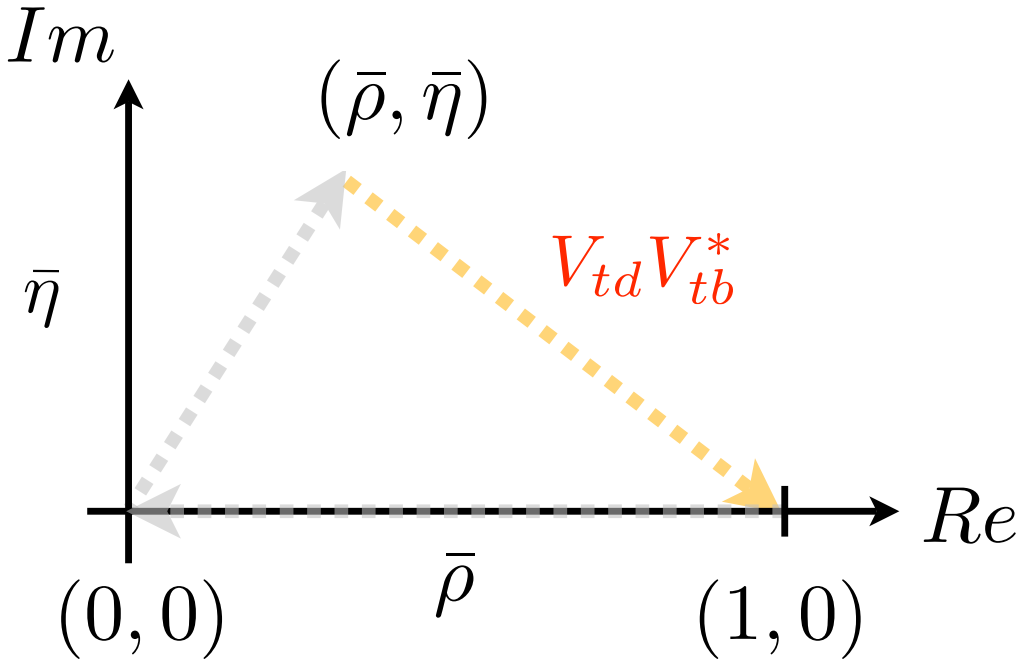


$|V_{ub}|=(3.89\pm0.44)10^{-3}$

$|V_{ub}|=(4.27\pm0.38)10^{-3}$

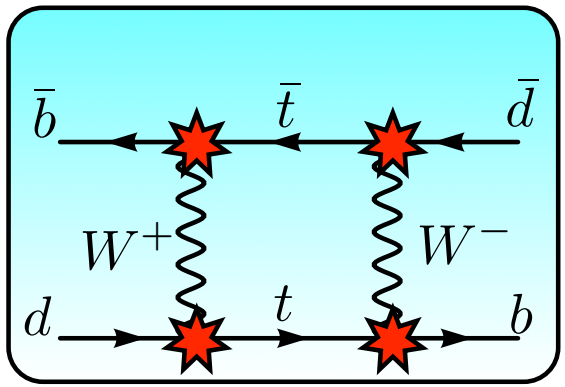
Determination of the CKM matrix: $|V_{td}V_{tb}^*|$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



Determination of the CKM matrix: $|V_{td}V_{tb}^*|$

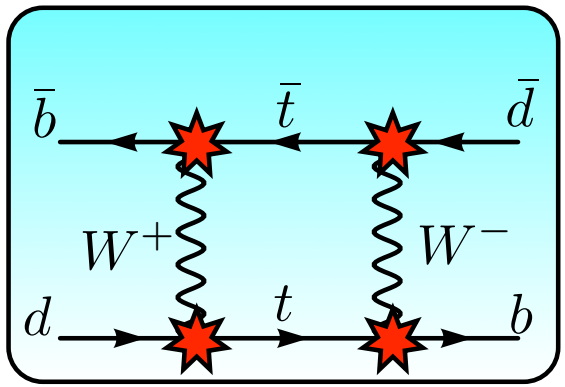
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



$\Delta M_d \Leftrightarrow |V_{td}V_{tb}^*|$

Determination of the CKM matrix: $|V_{td}V_{tb}^*|$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



$$\Delta M_d (\propto |M_{12}|) = \frac{G_F^2 m_W^2}{16\pi^2} |V_{tb}V_{td}^*|^2 S_0 \left(\frac{m_t^2}{m_W^2}\right) \eta_{\rm QCD} \times \frac{1}{m_B} \langle B^0 | (\bar{d}b)_{V-A} (\bar{d}b)_{V-A} | \overline{B}^0 \rangle$$

$\Delta M_d \Leftrightarrow |V_{td}V_{tb}^*|$

Determination of the CKM matrix: $|V_{td}V_{tb}^*|$

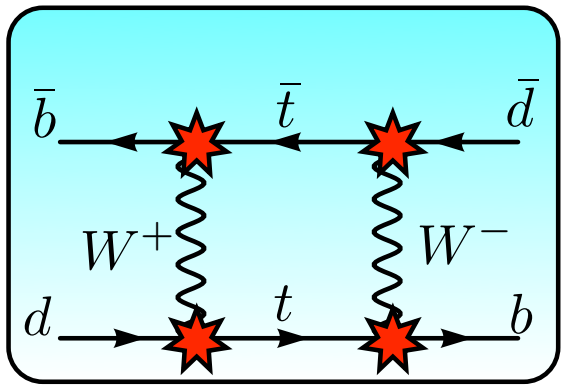
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Perturbative QCD correction

Loop function dominant=top quark

$$\Delta M_d (\propto |M_{12}|) = \frac{G_F^2 m_W^2}{16\pi^2} |V_{tb}V_{td}^*|^2 S_0\left(\frac{m_t^2}{m_W^2}\right) \eta_{\rm QCD} \times \frac{1}{m_B} \langle B^0 | (\bar{d}b)_{V-A} (\bar{d}b)_{V-A} | \overline{B}^0 \rangle$$

$$\equiv \frac{8}{3} B_B f_B^2 m_B^2$$



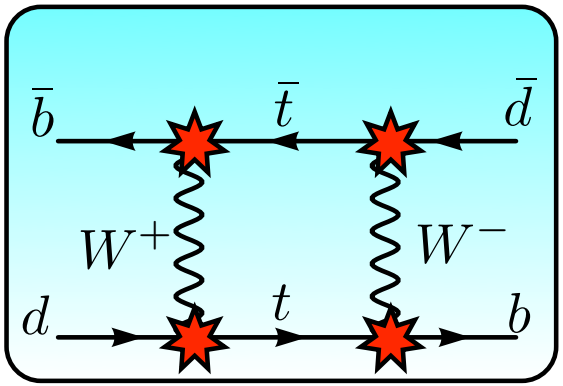
$\Delta M_d \Leftrightarrow |V_{td}V_{tb}^*|$

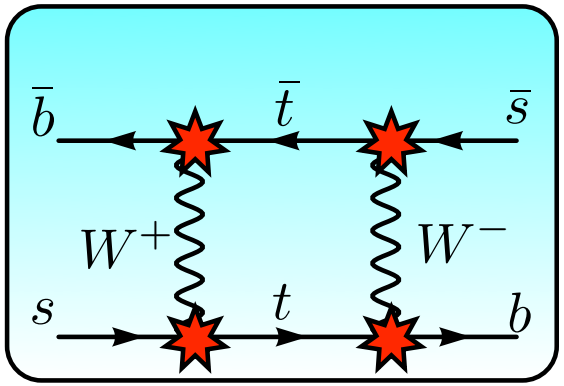
Main source of the hadronic uncertainties in determining |Vtb|: Lattice QCD computation very important!

Determination of the CKM matrix: $|V_{td}V_{tb}^*|$ and $|V_{ts}V_{tb}^*|$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Some hadronic uncertainties cancel in the ratio between ΔM_d and ΔM_s

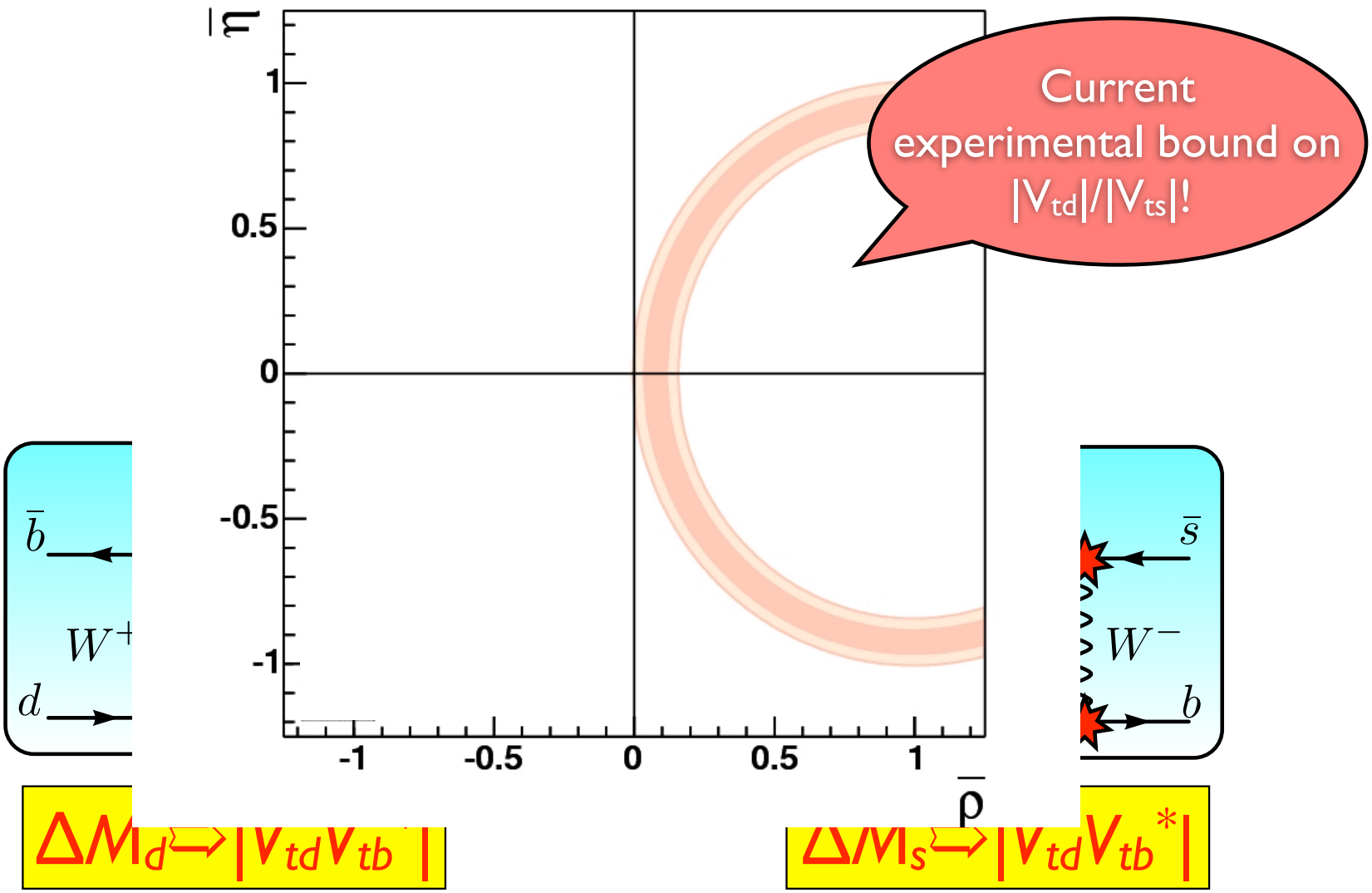




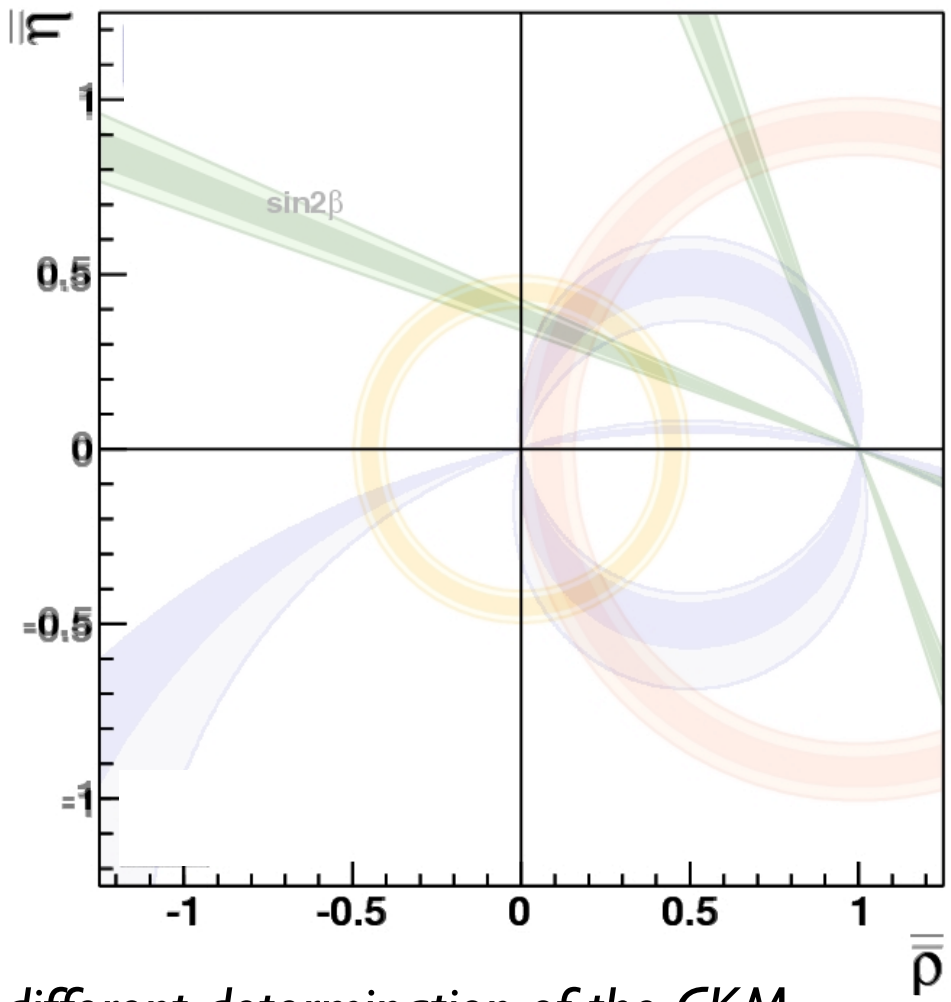
$\Delta M_d \Rightarrow |V_{td}V_{tb}^*|$

$\Delta M_s \Rightarrow |V_{td}V_{tb}^*|$

Determination of the CKM matrix:

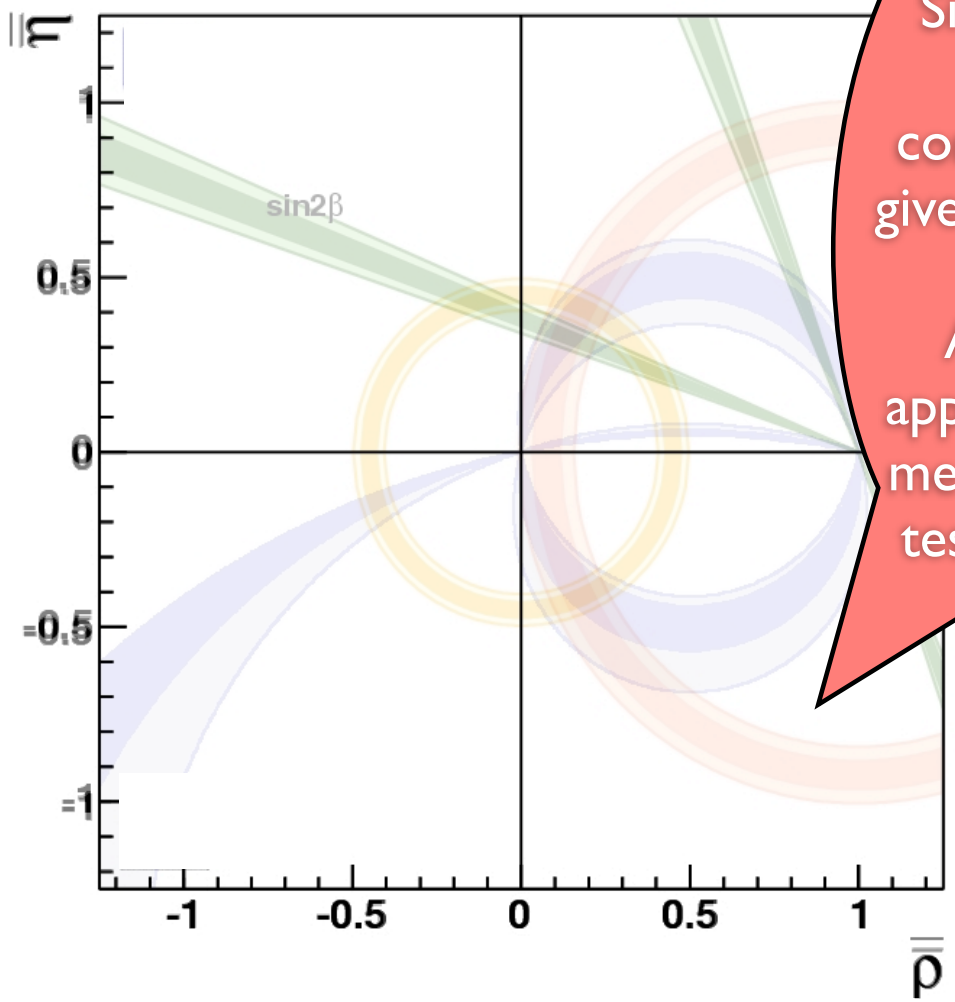


Combining the constraints...

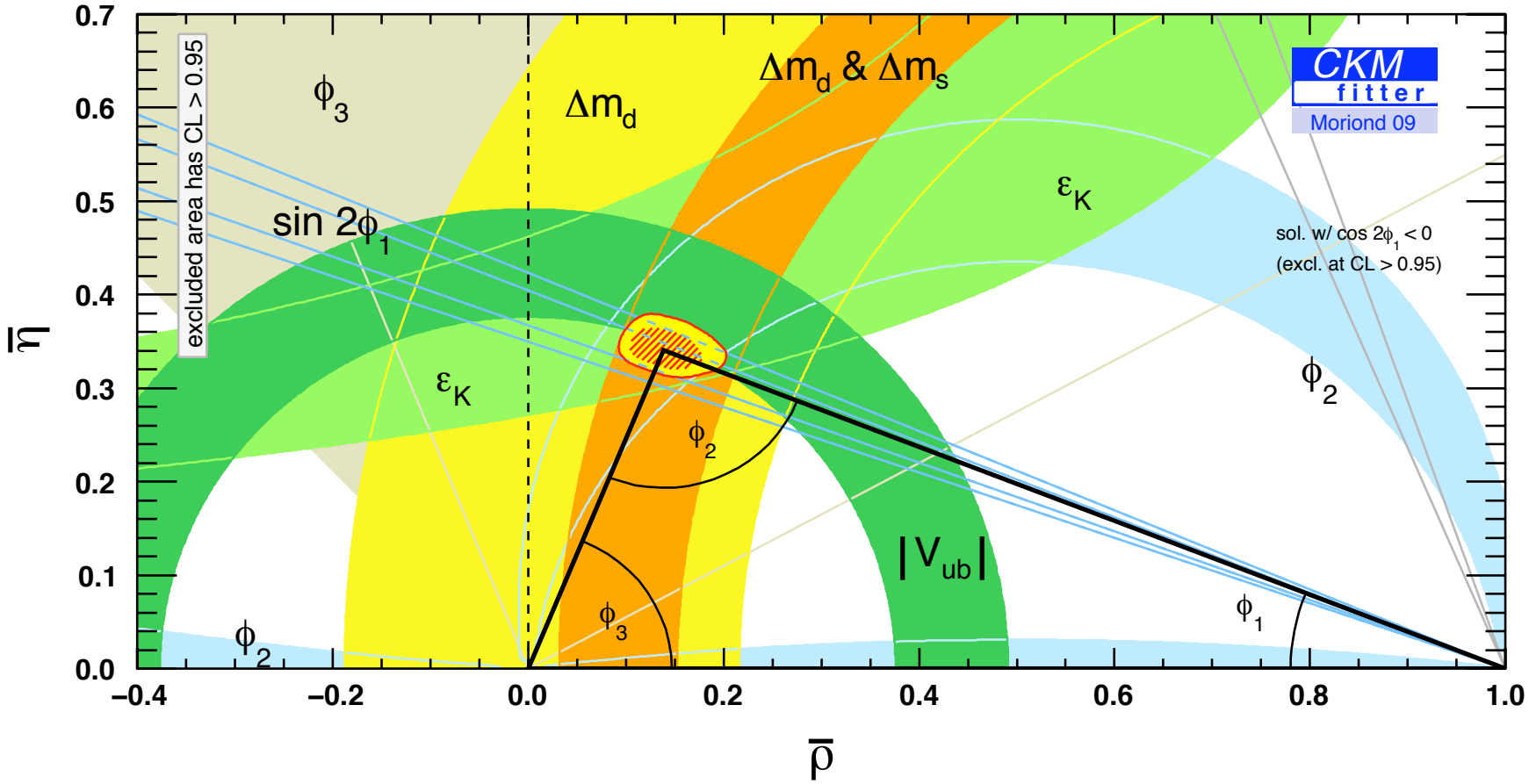


Indeed the different determination of the CKM parameters are "relatively" consistent to each other...

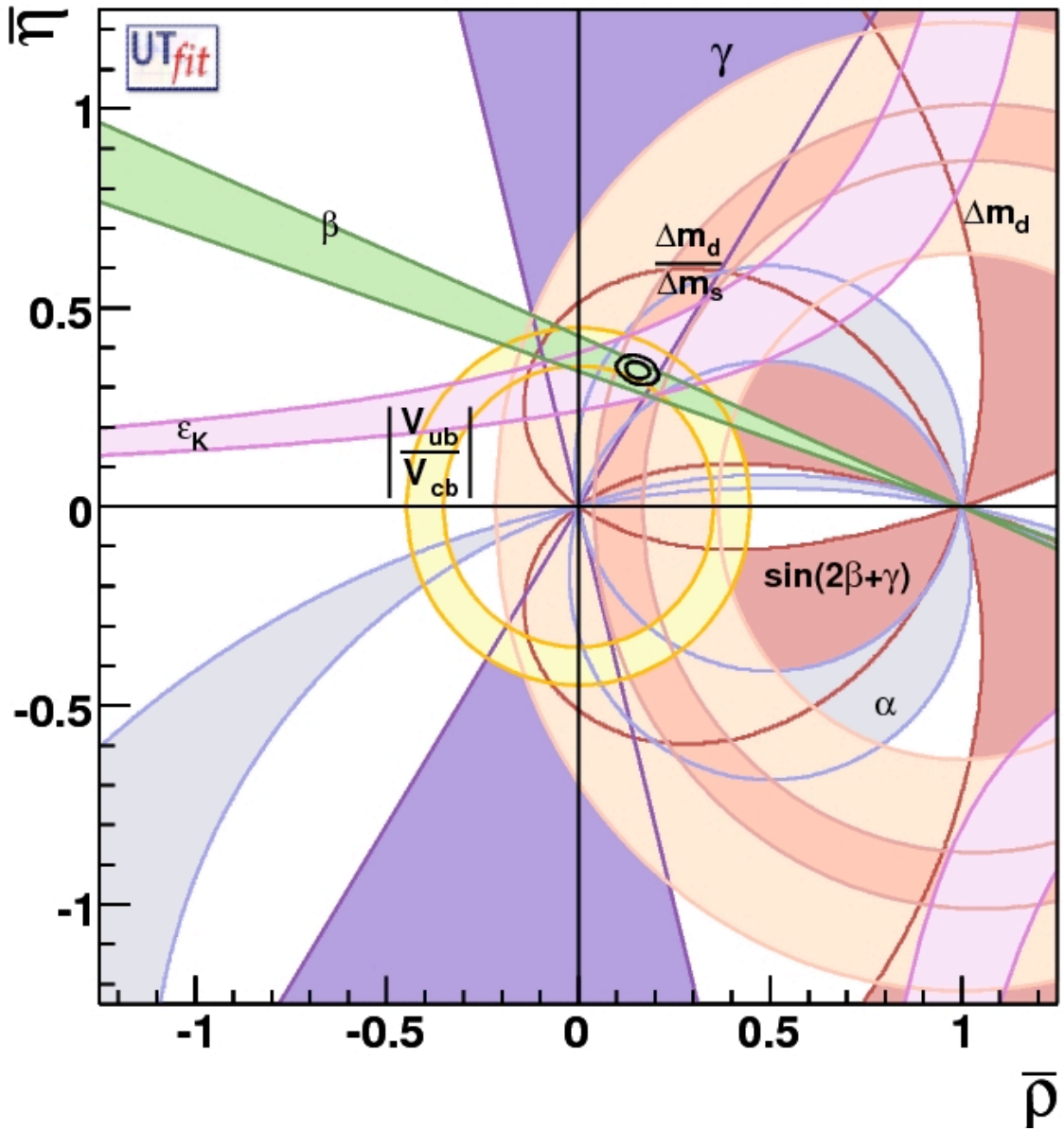
Combining the constra



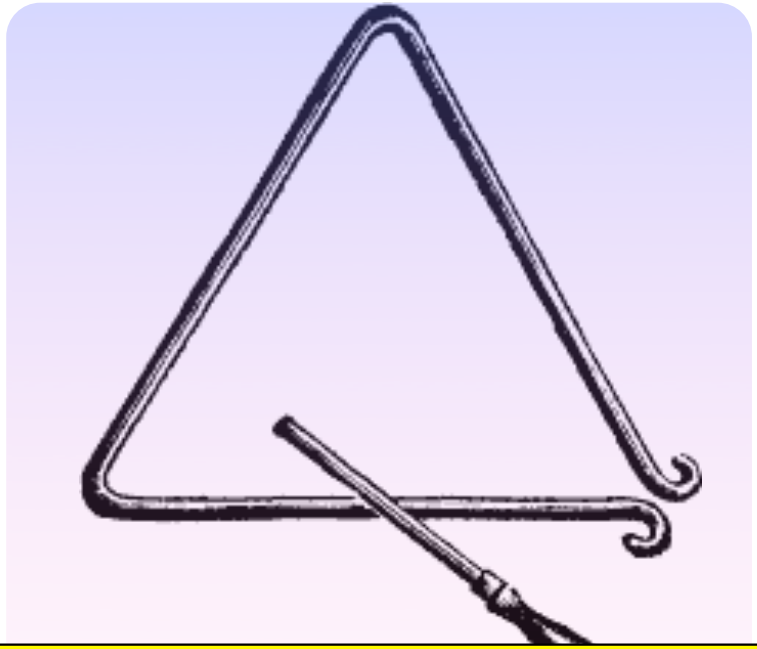
Warning:
Simply overlapping the different constraints does not give a right confidence level.
An applicaton of appropriate statistical methods is crucial to test the unitarity!!



http://ckmfitter.in2p3.fr/



http://www.utfit.org/UTfit/



We can say that the main part of the CP violation comes from the complex phase in the CKM matrix. However, there is still a possibility that the unitarity is not exact for a certin extent. Our challenges for more precise experiemental data as well as improvements in the theoretical predcitions continue!!!

Determination of CKM matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Determination of the CKM matrix: Cabibbo angle

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta_c & \sin\theta_c & V_{ub} \\ -\sin\theta_c & \cos\theta_c & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

You remember the Cabibbo angle.

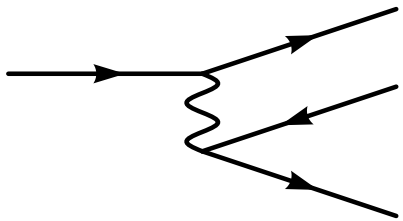
Determination of the CKM matrix: Cabibbo angle

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{ud} = 0.9746 \underbrace{(4)}_{\tau_n} \underbrace{(18)}_{G_A/G_V} \underbrace{(2)}_{\text{RCorr.}}$$

Hadronic uncertainties!

Nucleus	ft (sec)	V_{ud}
^{10}C	3039.5(47)	0.97370(80)(14)(19)
^{14}O	3042.5(27)	0.97411(51)(14)(19)
^{26}Al	3037.0(11)	0.97400(24)(14)(19)
^{34}Cl	3050.0(11)	0.97417(34)(14)(19)
^{38}K	3051.1(10)	0.97413(39)(14)(19)
^{42}Sc	3046.4(14)	0.97423(44)(14)(19)
^{46}V	3049.6(16)	0.97386(49)(14)(19)
^{50}Mn	3044.4(12)	0.97487(45)(14)(19)
^{54}Co	3047.6(15)	0.97490(54)(14)(19)
Weighted Ave.		0.97418(13)(14)(19)



Determination of the CKM matrix: Cabibbo angle

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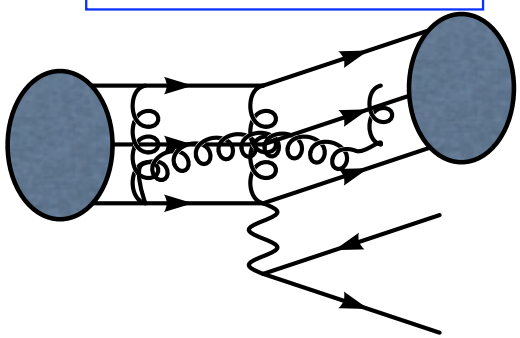
$$V_{ud} = 0.9746 \underbrace{(4)}_{\tau_n} \underbrace{(18)}_{G_A/G_V} \underbrace{(2)}_{\text{RCorr.}}$$

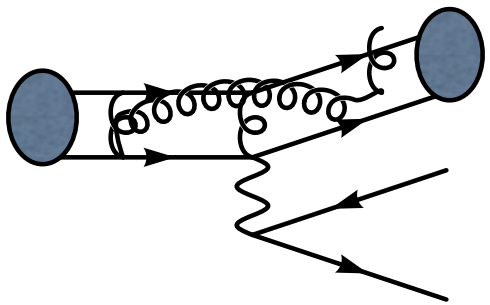
Nucleus	ft (sec)	V_{ud}
^{10}C	3039.5(47)	0.97370(80)(14)(19)
^{14}O	3042.5(27)	0.97411(51)(14)(19)
^{26}Al	3037.0(11)	0.97400(24)(14)(19)
^{34}Cl	3050.0(11)	0.97417(34)(14)(19)
^{38}K	3051.1(10)	0.97413(39)(14)(19)
^{42}Sc	3046.4(14)	0.97423(44)(14)(19)
^{46}V	3049.6(16)	0.97386(49)(14)(19)
^{50}Mn	3044.4(12)	0.97487(45)(14)(19)
^{54}Co	3047.6(15)	0.97490(54)(14)(19)
Weighted Ave.		0.97418(13)(14)(19)

Hadronic uncertainties!



Determination of the CKM matrix: Cabibbo angle

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



$$f_+(0)|V_{us}| = 0.21668(45)$$

Hadronic uncertainties!

Chiral Perturbation Theory

$$f_+(0) = 0.961 \pm 0.08 \ (2\%)$$

Lattice

$$f_+(0) = 0.9609(51)$$

Decay Mode	$\lvert V_{us}\rvert f_+(0)$
$K^{\pm}e3$	0.21746 ± 0.00085
$K^{\pm}\mu 3$	0.21810 ± 0.00114
$K_L e3$	0.21638 ± 0.00055
$K_L \mu 3$	0.21678 ± 0.00067
$K_S e3$	0.21554 ± 0.00142
Average	0.21668 ± 0.00045

Determination of the CKM matrix: Cabibbo angle

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ \end{pmatrix}$$

Vud and Vus:
All in all, it is close to Vud=cosθ_c, Vus=sinθ_c.
BUT many efforts continue!

Hadr
uncertainties:

	0.21746 ± 0.00085
$K^-\mu 3$	0.21810 ± 0.00114
$K_L e3$	0.21638 ± 0.00055
$K_L \mu 3$	0.21678 ± 0.00067
$K_S e3$	0.21554 ± 0.00142
Average	0.21668 ± 0.00045

Chiral Perturbation Theory

$$f_+(0) = 0.961 \pm 0.08 \ (2\%)$$

Lattice

$$f_+(0) = 0.9609(51)$$

Determination of the CKM matrix: Charm meson decays

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Hadronic uncertainties for heavy mesons are more difficult to control.

- ☑ No chiral perturbation
- ☑ Lattice possible (but much more computing power needed)
- ☑ Heavy quark effective theory (expansion using $m_{c,b} >> \Lambda_{QCD}$)

$$\begin{aligned} V_{cd} &= 0.230 \pm 0.011 \\ V_{cs} &= 1.04 \pm 0.06 \\ V_{cb} &= (41.2 \pm 1.1) \times 10^{-3} \\ V_{ub} &= (3.93 \pm 0.36) \times 10^{-3} \end{aligned}$$

Determination of the CKM matrix: Charm meson decays

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ \end{pmatrix}$$

The unitarity is more or less fine. For the most of decay channels, hadronic uncertainties are larger than the experimental ones...

Theoretical challenges!!!

- ☑ No chi
- ☑ Lattice possib
more computer power needed)
- ☑ Heavy quark effective theory (expansion using $m_{c,b} >> \Lambda_{QCD}$)

± 0.06

$$V_{cb} = (41.2 \pm 1.1) \times 10^{-3}$$

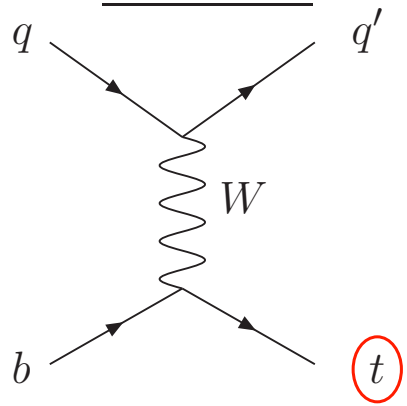
$$V_{ub} = (3.93 \pm 0.36) \times 10^{-3}$$

Determination of the CKM matrix: |Vtb| from single top production

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

The tree level determination becomes only possible by the top physics.

t-channel



Tev.	$\sim$ 0.9 pb
LHC	$\sim$ 240 pb