Lecture 3
PQCD for jet physics

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Oct. 21, 2012

Outlines

- Jet in experiment
- Jet in theory
- Jet substructures

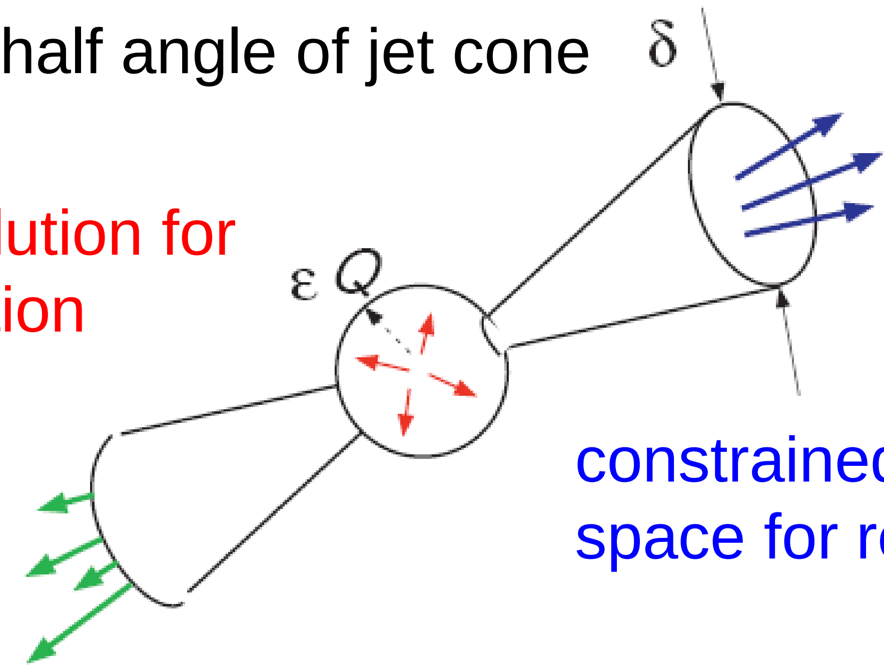
Introduction

- Jets are abundantly produced at colliders
- Jets carry information of hard scattering and parent particles
- Study of jets is crucial
- Usually use event generators
- How much can be done in PQCD
- A lot!

Dijet in e+e- annihiation

- Dijet production is part of total cross section
- Born cross section is the same as total cross section

$$\sigma_{2j}{}^{(0)}(Q,\epsilon,\delta) = N\left(\sum_f Q_f{}^2\right)\frac{4\pi\alpha^2}{3Q^2}$$



NLO corrections

- Isotropic soft gluons within energy resolution

$$[2\ln^2(2\epsilon E/\mu) - \pi^2/6]$$

- Collinear gluons in cone with energy higher than resolution

$$[-3\ln(E\delta/\mu) - 2\ln^2 2\epsilon - 4\ln(E\delta/\mu)\ln(2\epsilon) + \tfrac{17}{4} - \pi^2/3]$$

- Virtual corrections

$$[-2\ln^2(E/\mu) + 3\ln(E/\mu) - \tfrac{7}{4} + \pi^2/6]$$

- Dijet cross section is infrared finite, but logarithmically enhanced

$$(3\ln\delta + 4\ln\delta\ln 2\epsilon + \pi^2/3 - \tfrac{5}{2})$$

overlap of collinear and soft logs

Jet phenomenology

VOLUME 39, NUMBER 23 — PHYSICAL REVIEW LETTERS — 5 DECEMBER 1977

Jets from Quantum Chromodynamics

George Sterman

Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11790

and

Steven Weinberg

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

(Received 26 July 1977)

double-log enhancement

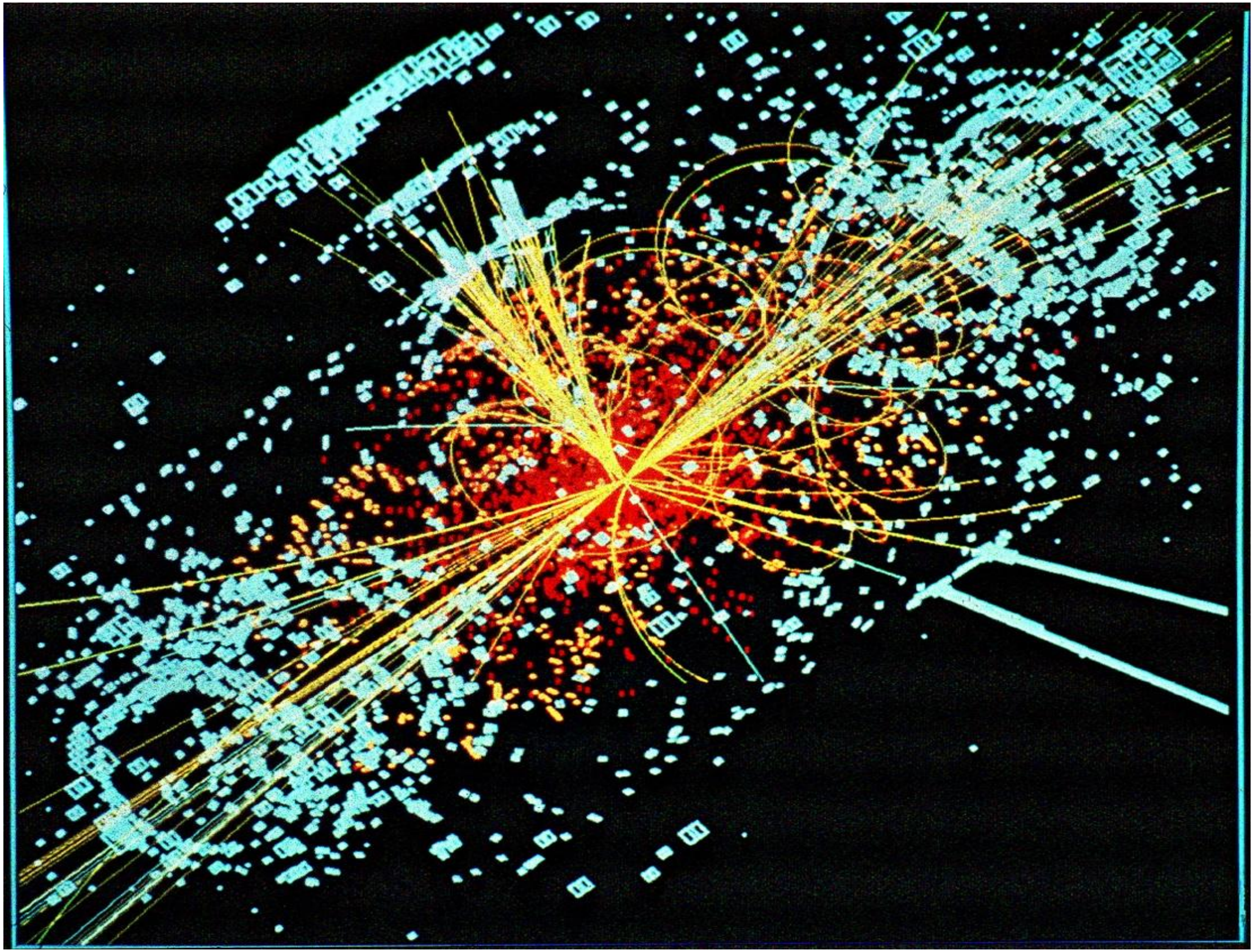
The properties of hadronic jets in e^+e^- annihilation are examined in quantum chromodynamics, without using the assumptions of the parton model. We find that two-jet events dominate the cross section at high energy, and have the experimentally observed angular distribution. Estimates are given for the jet angular radius and its energy dependence. We argue that the detailed results of perturbation theory for production of arbitrary numbers of quarks and gluons can be reinterpreted in quantum chromodynamics as predictions for the production of jets.

jet as an observable (jet physics)
not quarks and gluons

jet substructures

Jet in experiment

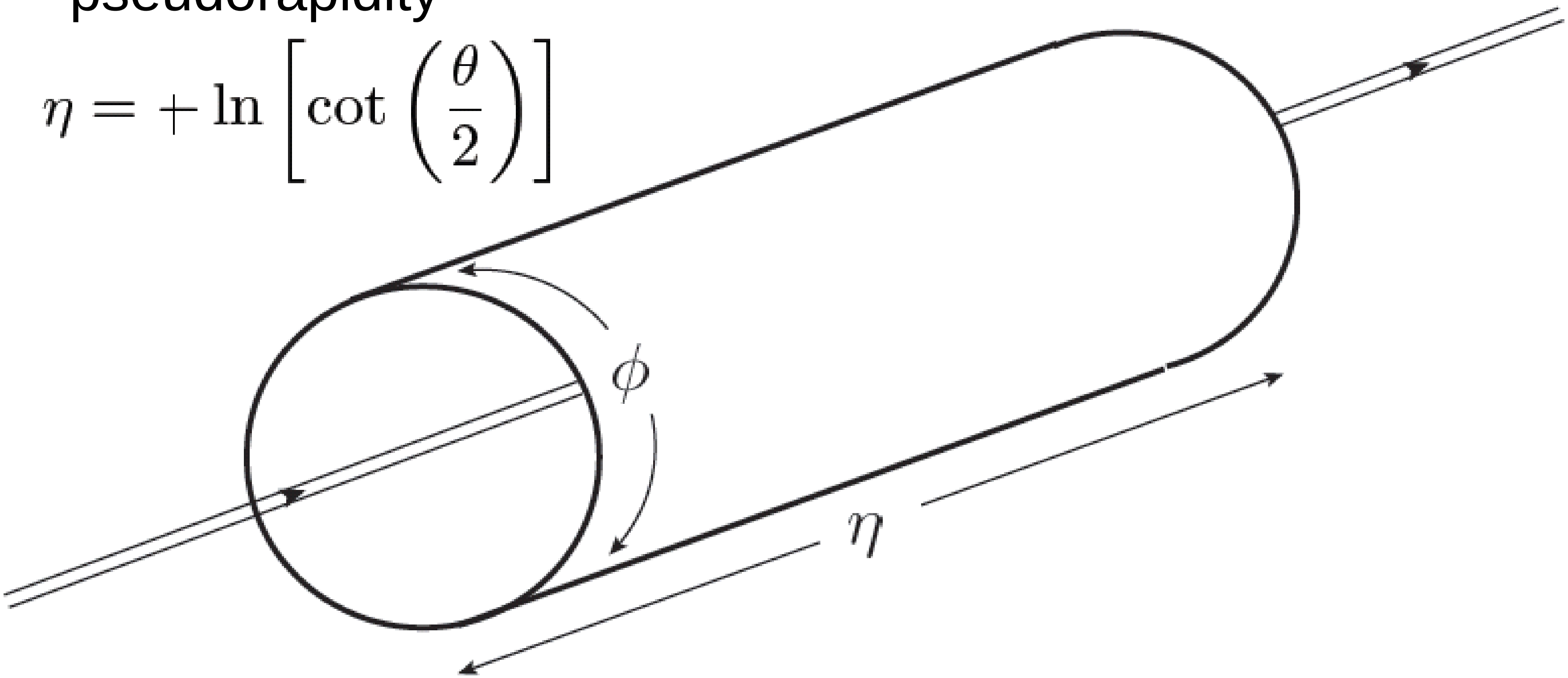
Jets



Coordinates for jets

pseudorapidity

$$\eta = +\ln\left[\cot\left(\frac{\theta}{2}\right)\right]$$



$$\theta = 0 \Rightarrow \eta = \infty, \quad \theta = 90^o \Rightarrow \eta = 0, \quad \theta = 180^o \Rightarrow \eta = -\infty$$

Jet algorithms

- Comparison of theory with experiment is nontrivial
- Need jet algorithms
- Algorithms should be well-defined so that they map experimental measurements with theoretical calculations as close as possible
- Infrared safety is important guideline, because Sterman-Weinberg jet is infrared finite

Types of algorithms

- Two main classes of jet algorithms
- Cone algorithms: stamp out jets as with a cookie cutter

  Geometrical method
- Sequential algorithms: combine parton four-momenta one by one

  Depend on particle kinematics

Seeded cone algorithm

- Find stable cones via iterative-cone procedure
- Start from seed particle i and consider set of particles j with separations smaller than jet cone

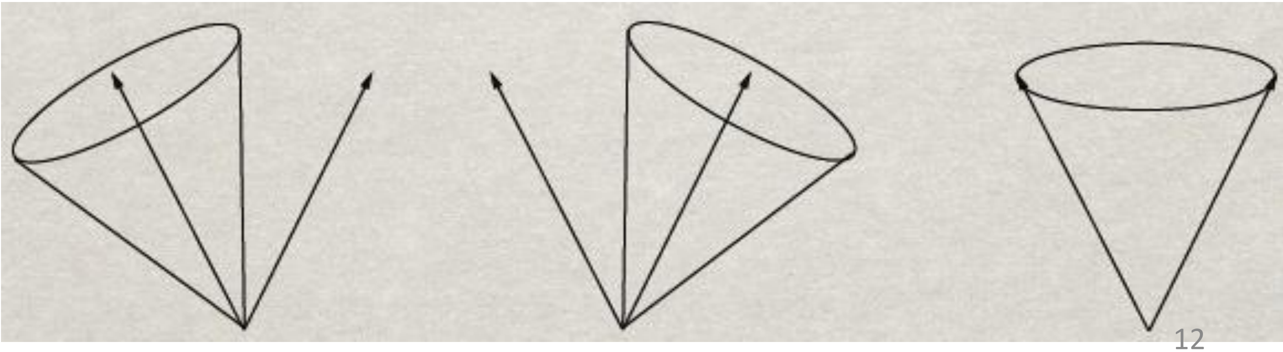
$$\Delta R_{ij} \equiv (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2 < R$$

- If the cone is stable, procedure stops. Otherwise the cone center J is taken as a new seed, and repeat the above procedure
- A stable cone is a set of particles i satisfying

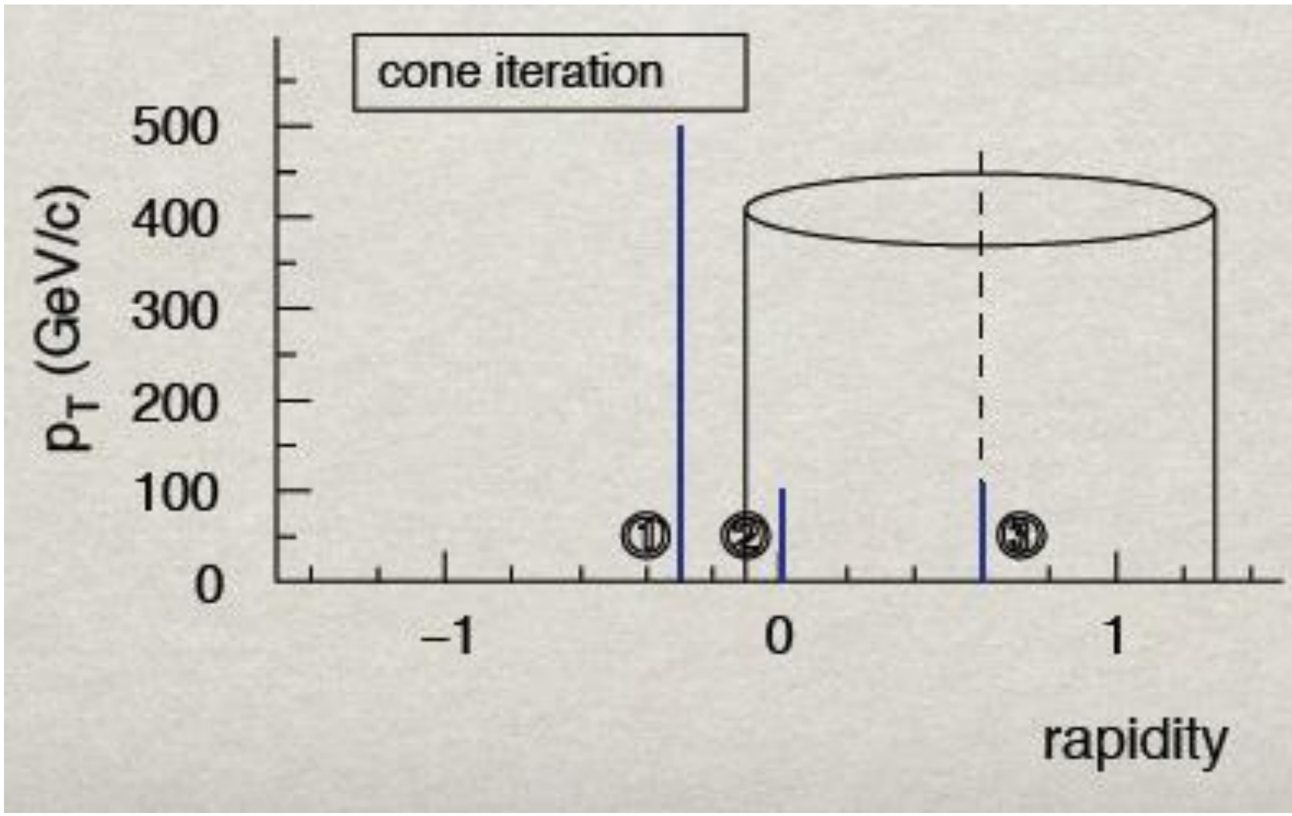
$\Delta R_{iJ} < R$

- Examples:

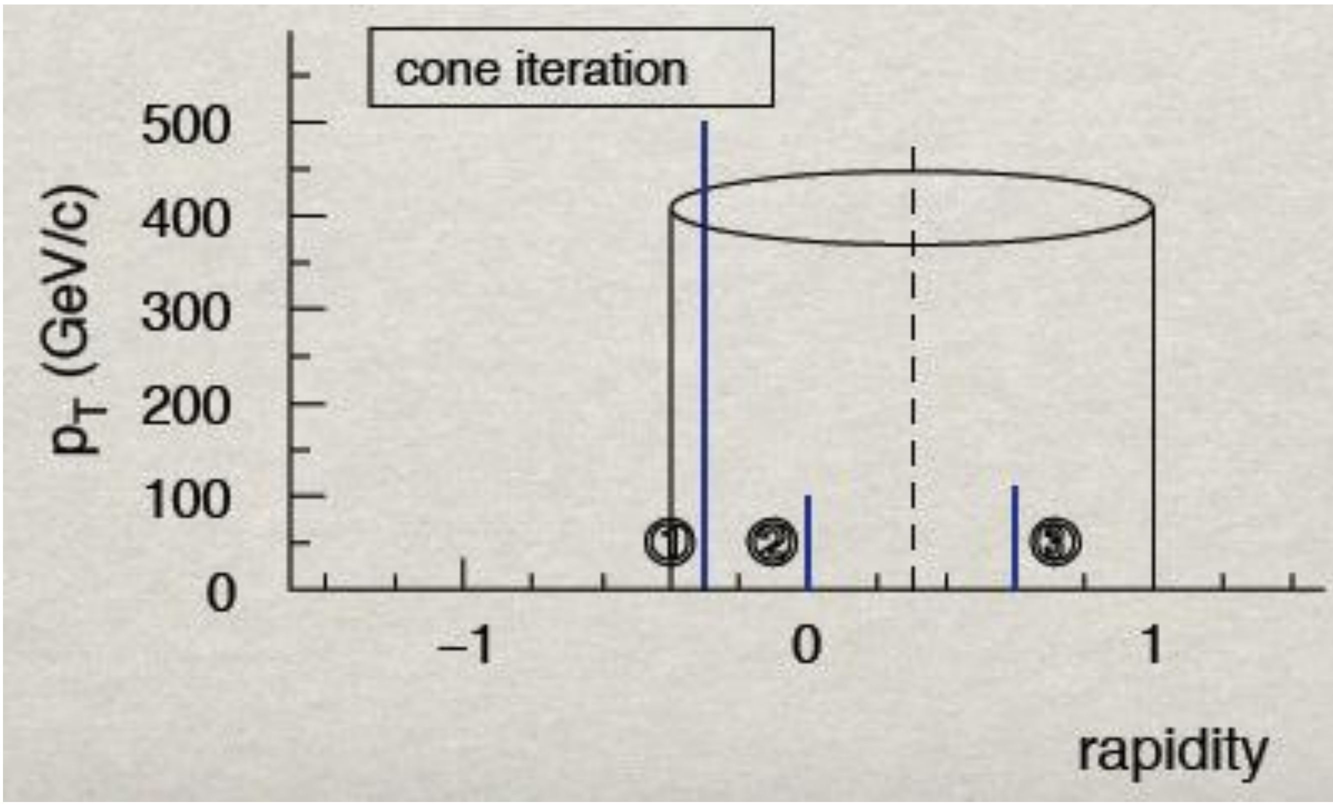
$R < R_{12} < 2R$



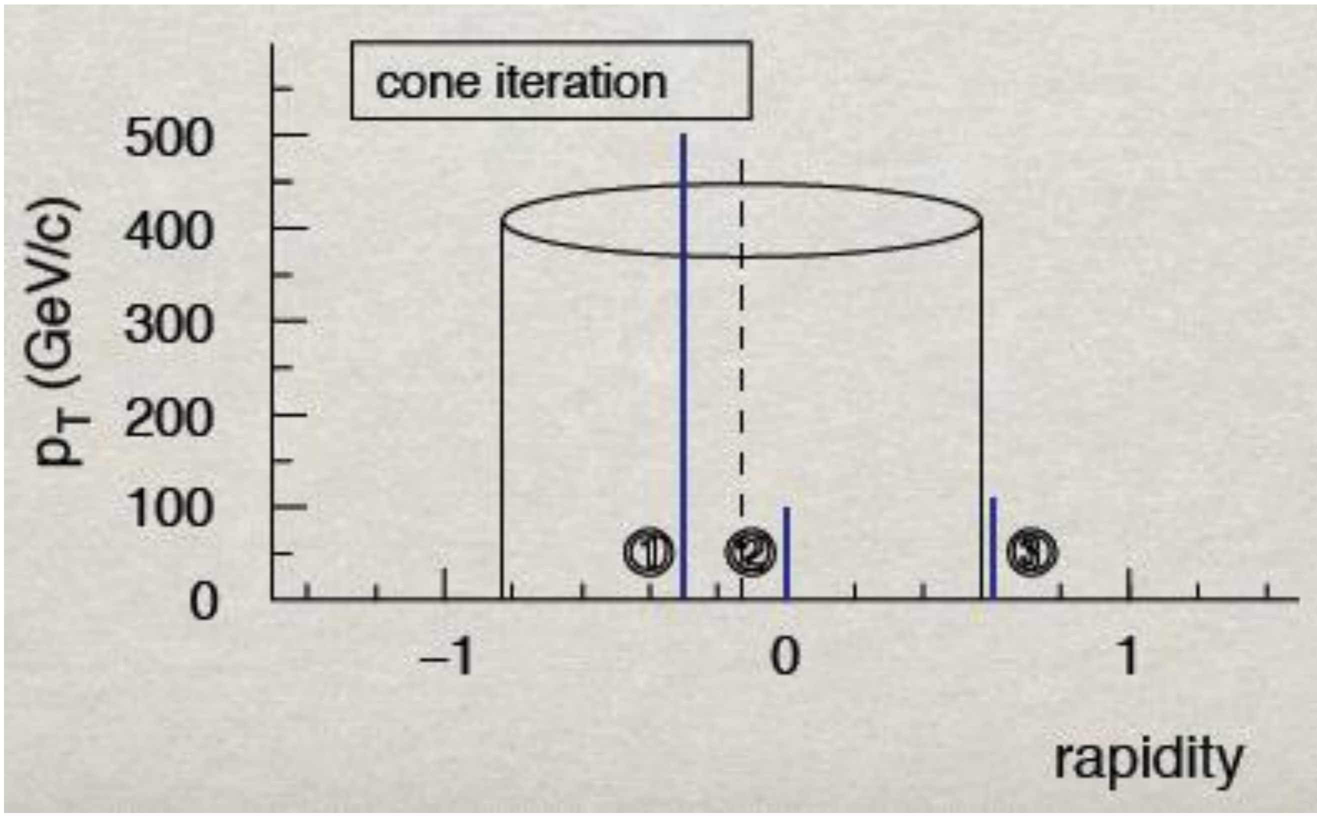
Iterative step 1



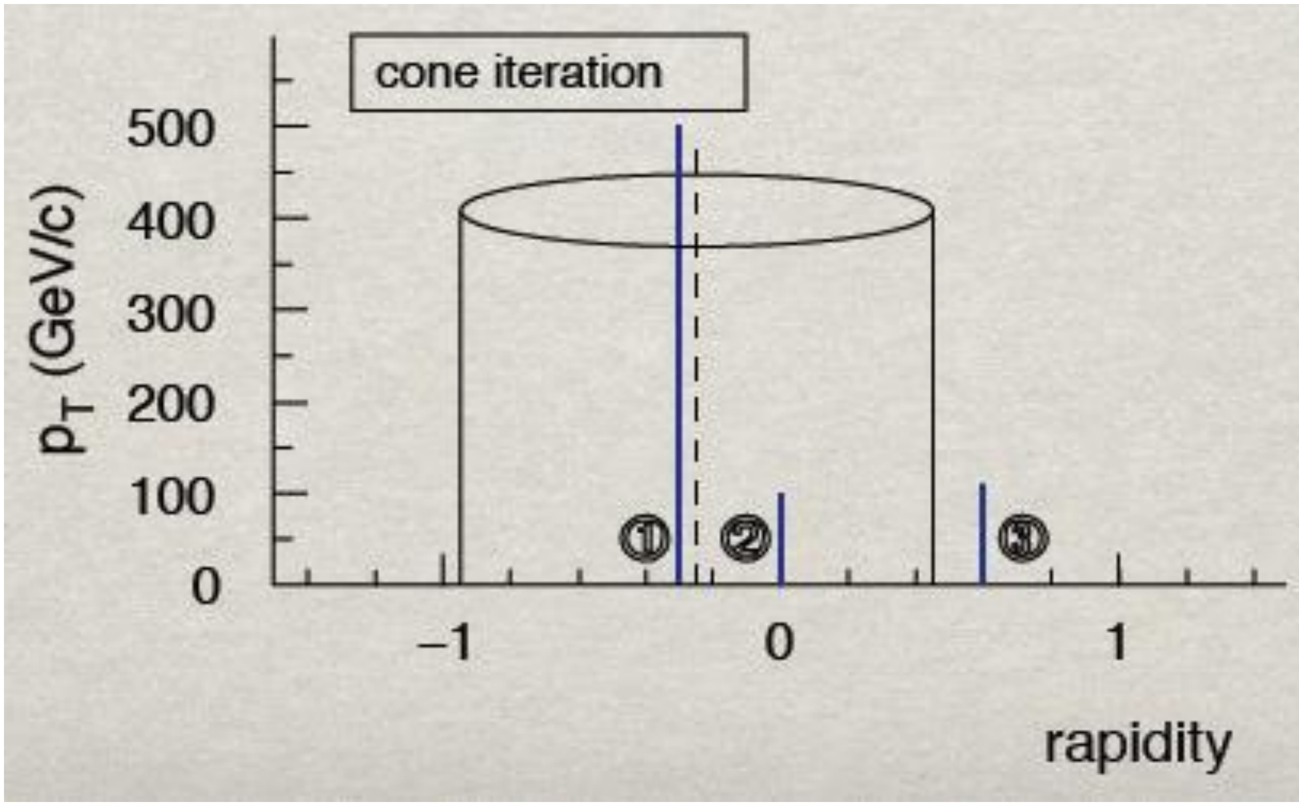
Iterative step 2



Iterative step 3

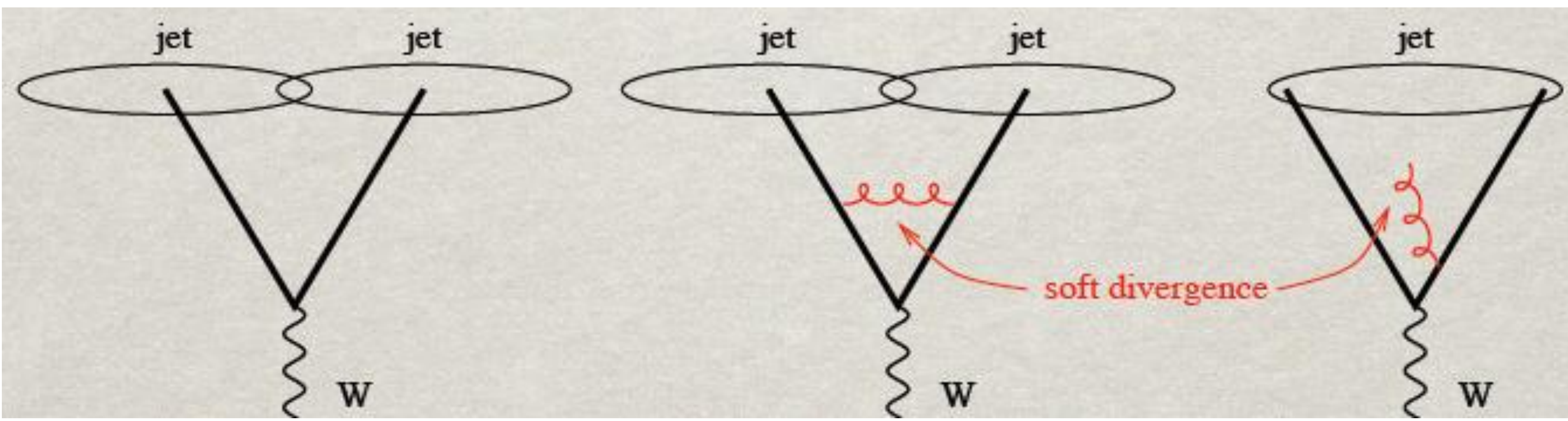


Iterative step 4



Problem of seeded cone

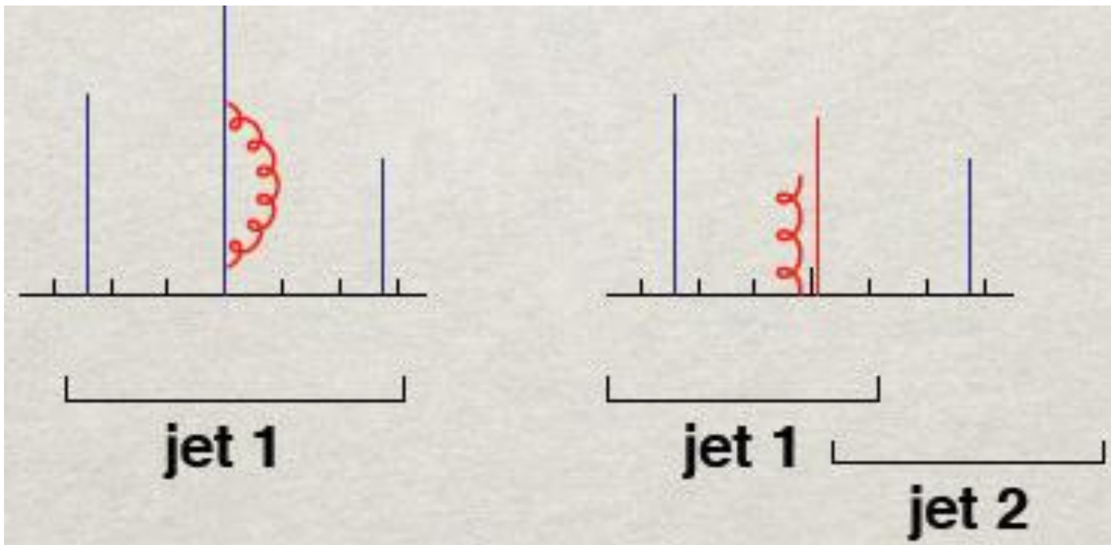
- Geometrical algorithm does not differentiate infrared gluons from ordinary gluons
- Final results (split-merge) depend on soft radiation and collinear splitting



- Virtual (real) soft gluon contributes to two (single) jet cross section, no cancellation

Not infrared safe

- How abut starting from the hardest particle?
- Collinear splitting change final results



- Virtual (real) gluon contributes to single (two) jet cross section, no cancellation
- Seeded cone algorithm is not infrared safe

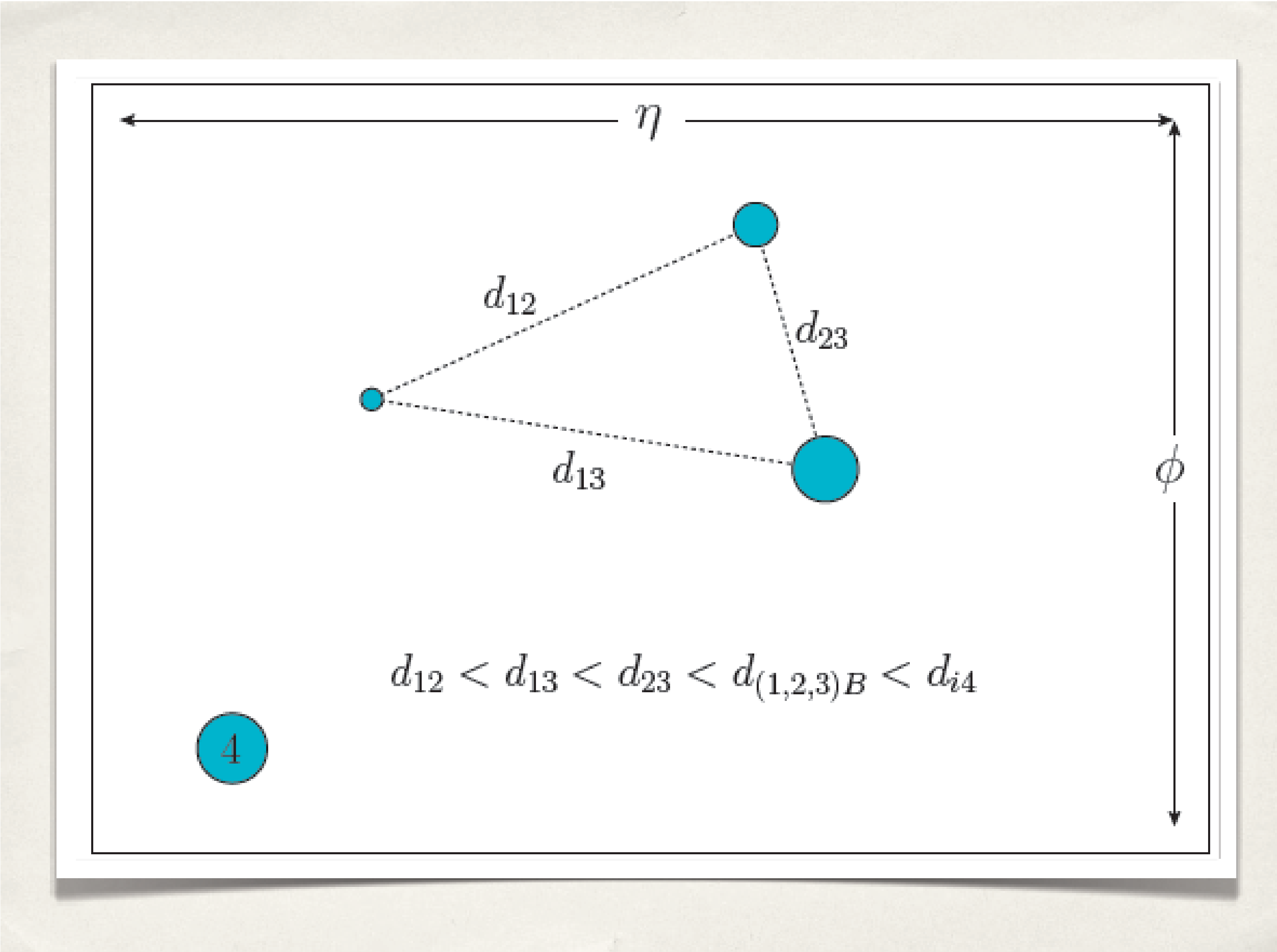
Sequential algorithms

- Take kT algorithm as an example.
- For any pair of particles i and j, find the minimum of

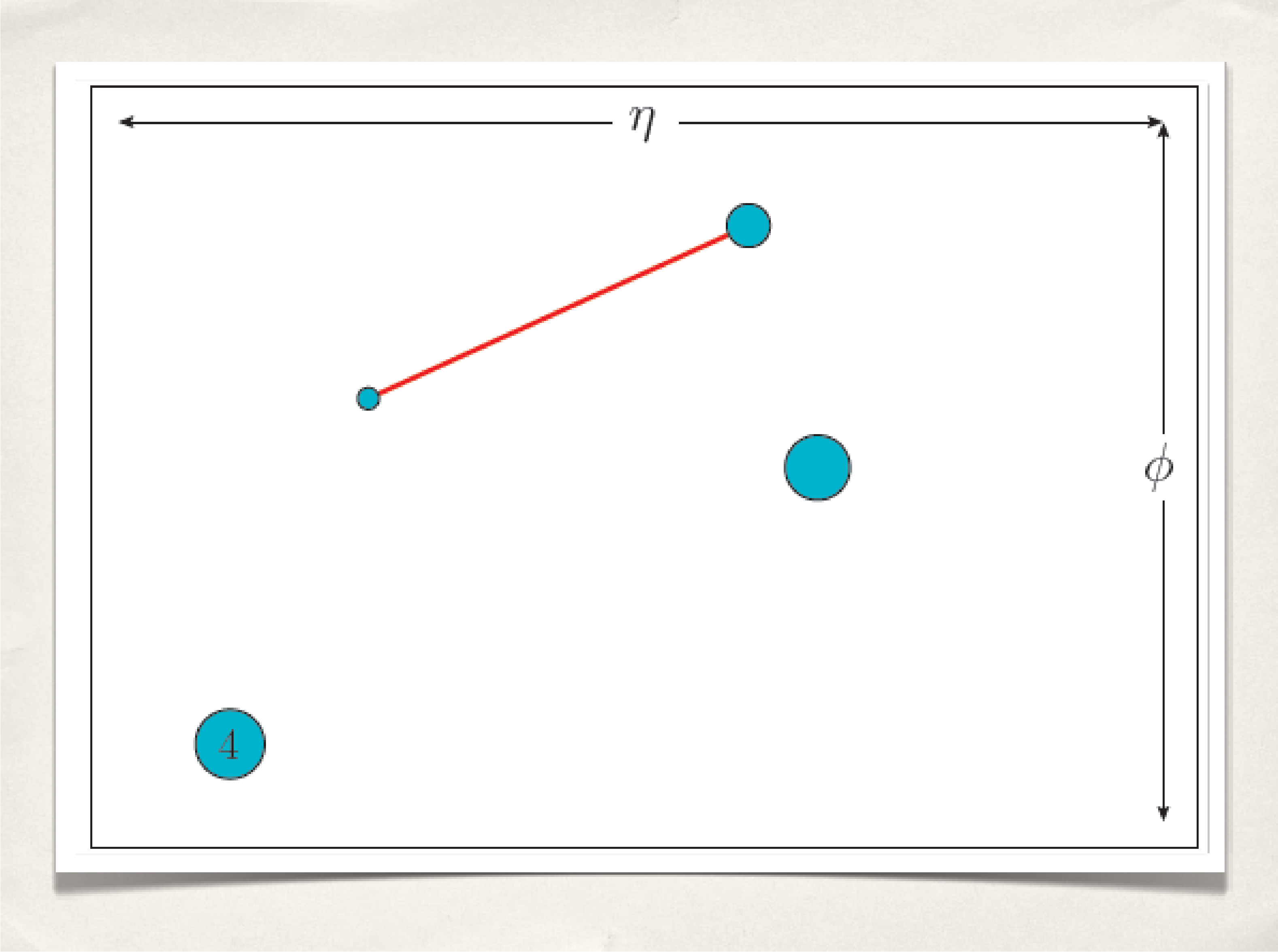
$$d_{ij} = \frac{\min\{k_{ti}^2, k_{tj}^2\}}{R^2} \Delta R_{ij}^2 \simeq k_{t,ij}^2, \quad d_{iB} = k_{ti}^2, \quad d_{jB} = k_{tj}^2$$

- If it is diB or djB, i or j is a jet, removed from the list of particles. Otherwise, i and j merged
- Repeat procedure until no particles are left
- Differentiate infrared and ordinary gluons

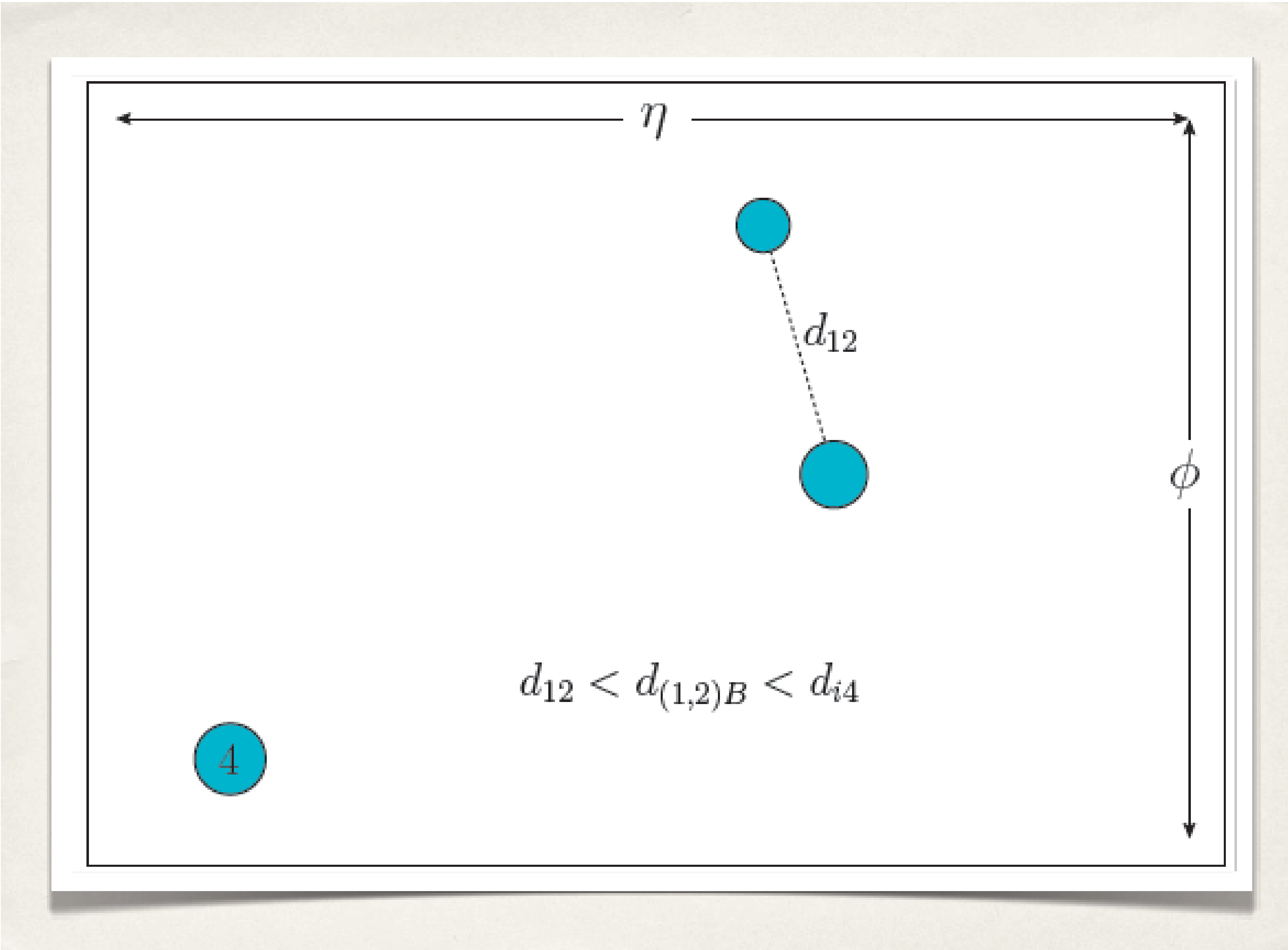
Step 1



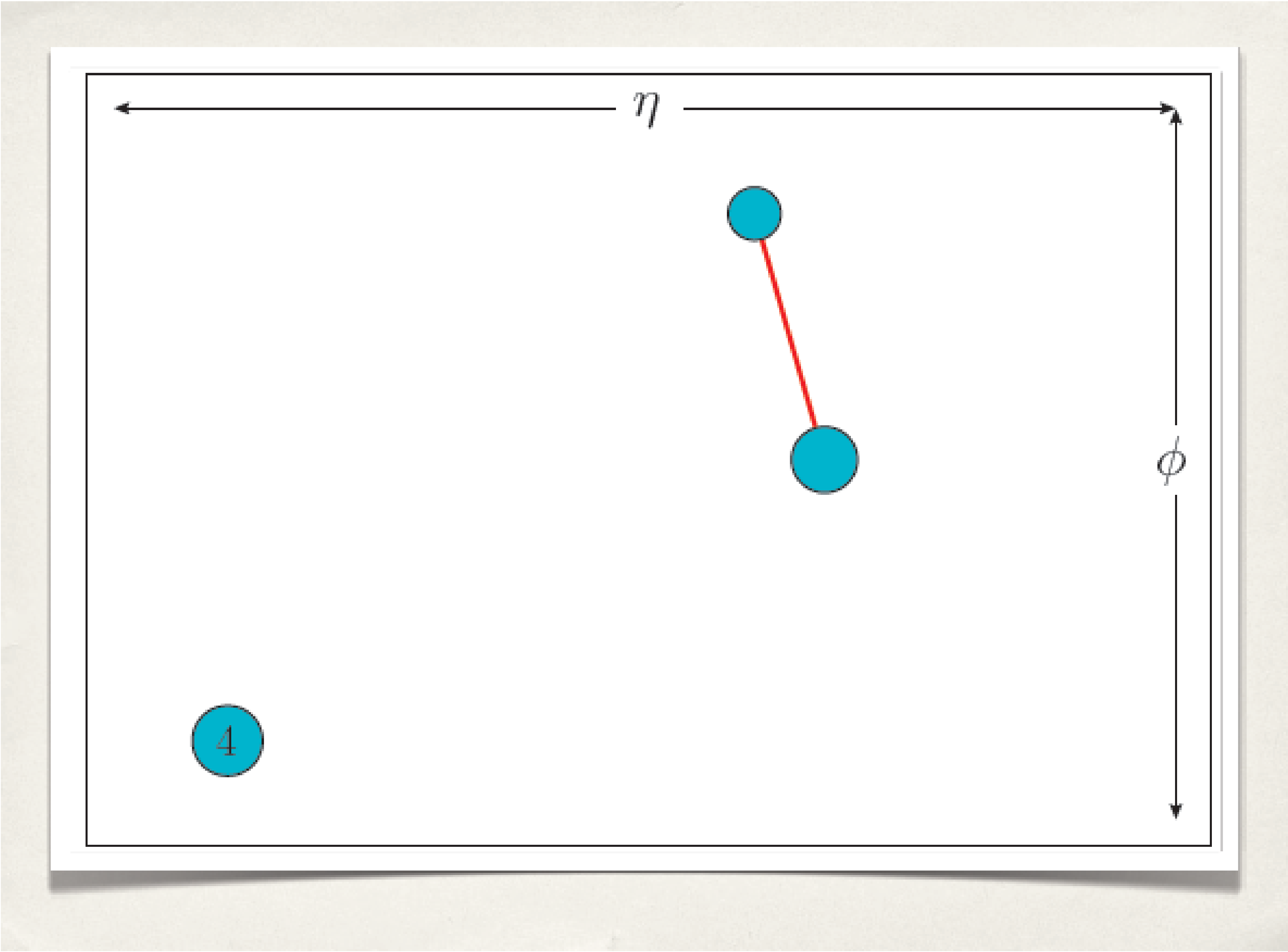
Step 2



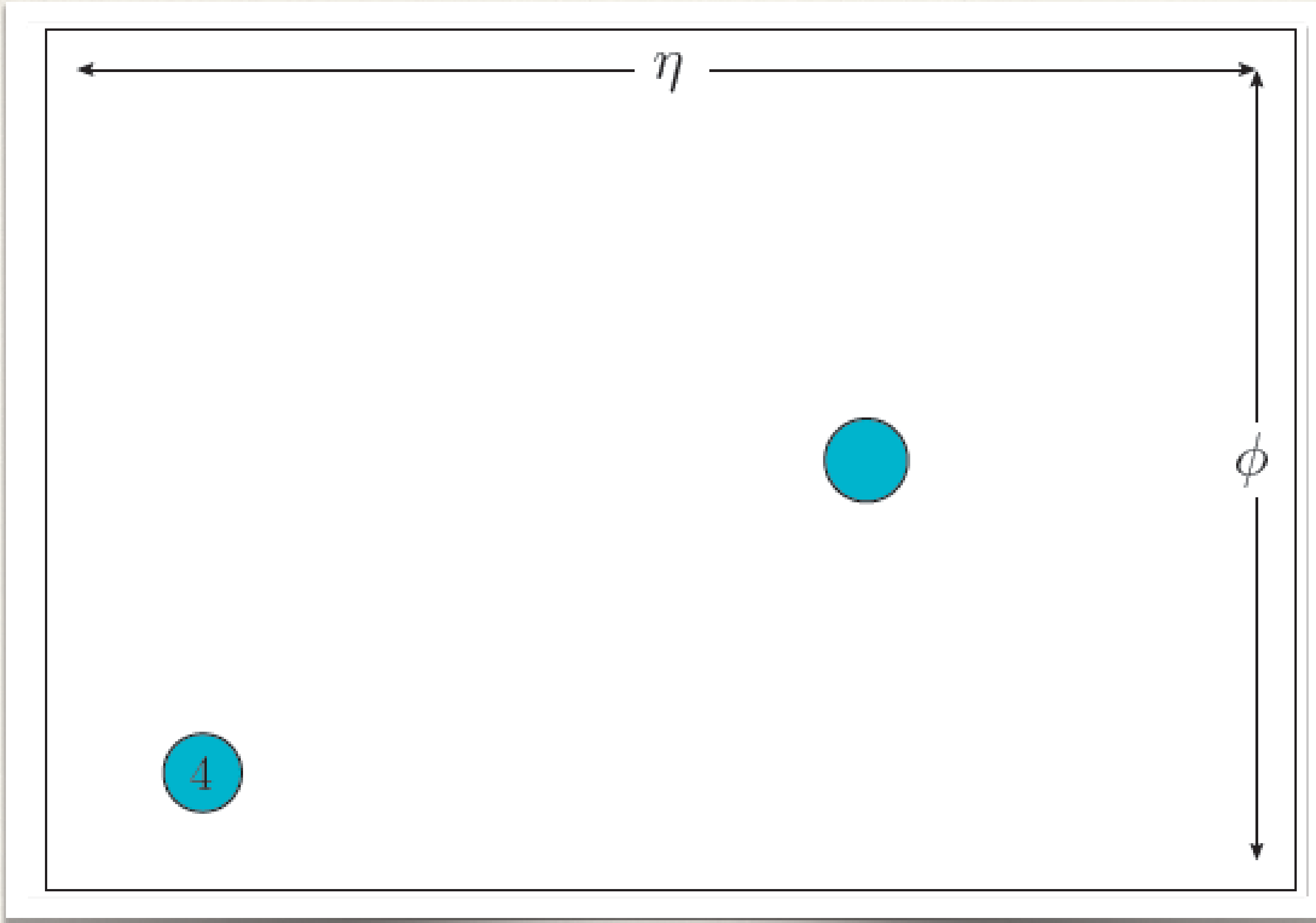
Step 3



Step 4



Step 5



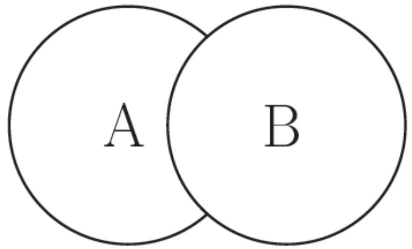
Done!

Recombination Algorithms

$p_{TA} > p_{TB}$

- k_T algorithm start with softer particles

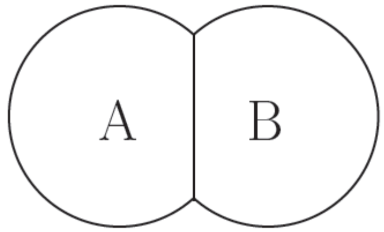
$$d_{ij} = \min(p_{Ti}^2, p_{Tj}^2)\left(\frac{\Delta R}{R_0}\right)^2, \; d_{iB} = p_{Ti}^2$$



k_T

- C/A algorithm

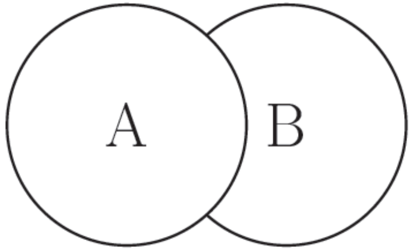
$$d_{ij} = \left(\frac{\Delta R}{R_0}\right)^2, \; d_{iB} = 1$$



C/A

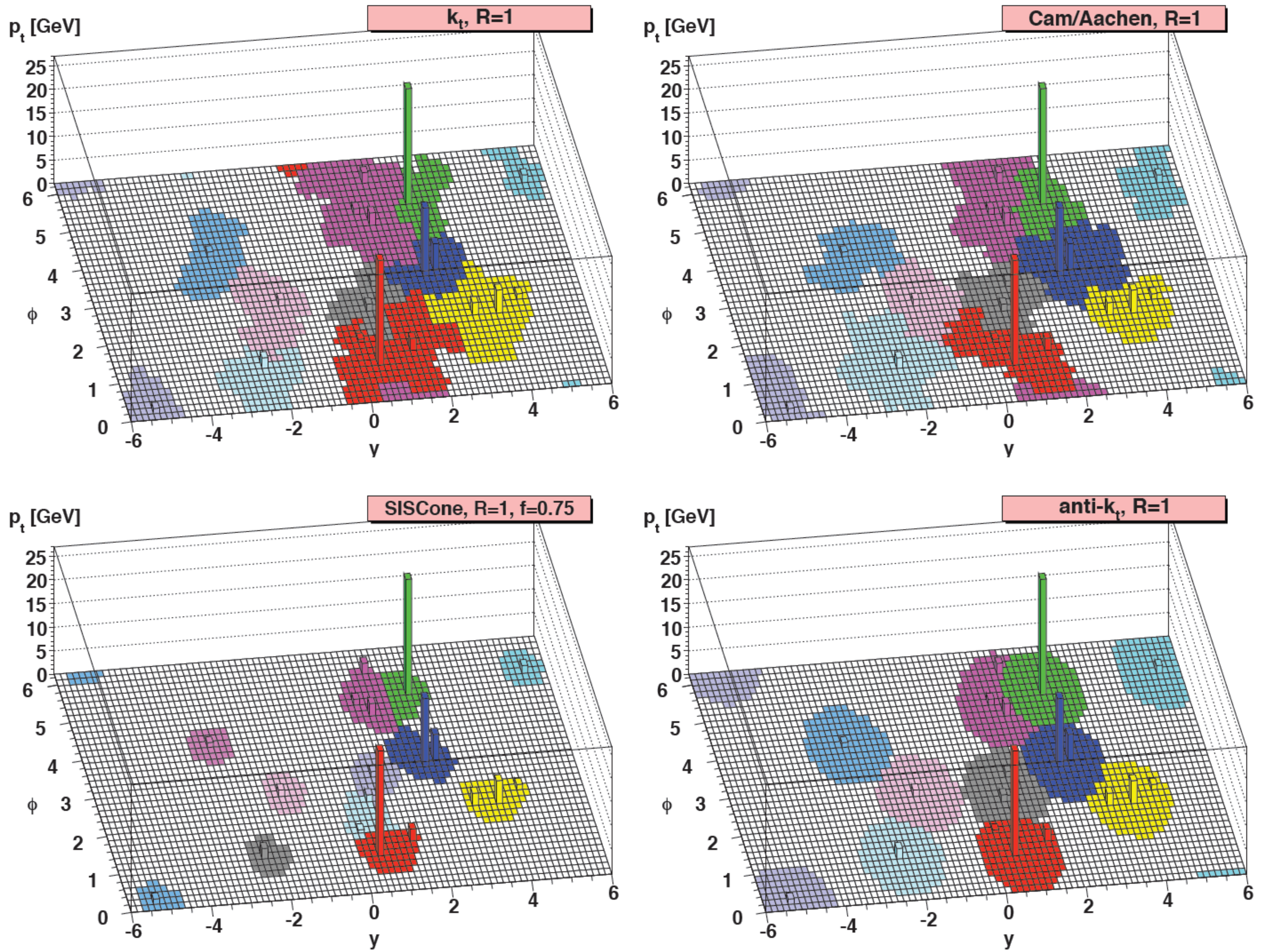
- anti-k_T algorithm start with harder particles

$$d_{ij} = \min(p_{Ti}^{-2}, p_{Tj}^{-2})\left(\frac{\Delta R}{R_0}\right)^2, \; d_{iB} = p_{Ti}^{-2}$$



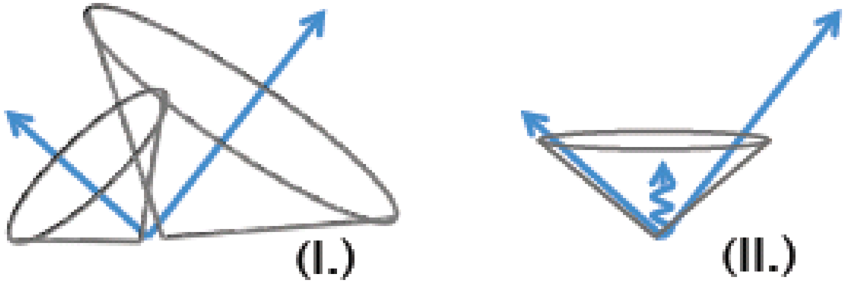
anti$-k_T$

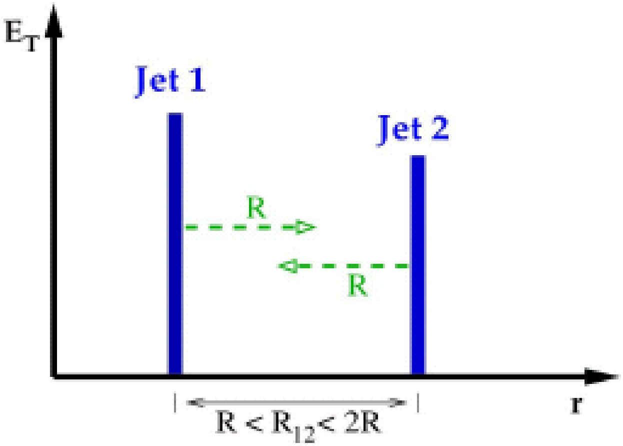
$$(\Delta R)^2 \equiv (\Delta\eta)^2 + (\Delta\phi)^2$$


kt, R=1
Cam/Aachen, R=1
SISCone, R=1, f=0.75
anti-kt, R=1
pt [GeV]
φ
y

Infrared safety

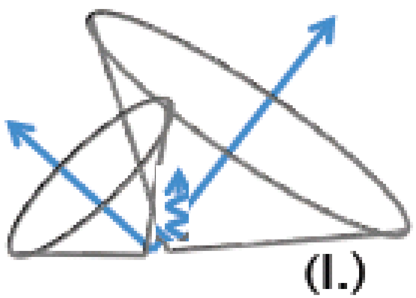
- In seeded cone algorithm

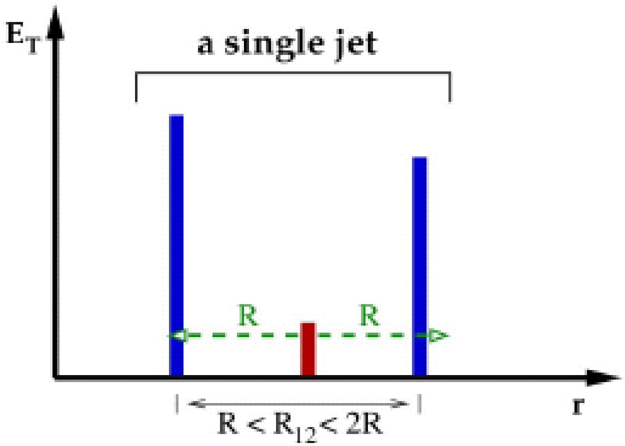




(Identified as 2 jets)

- In kt algorithm, remain two jets---infrared safety





(Identified as 1 jet)

Jet in theory

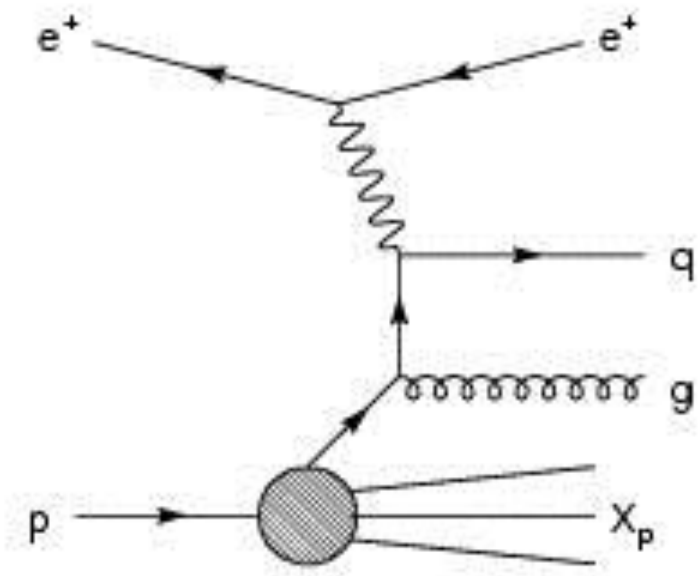
Factorization of DIS

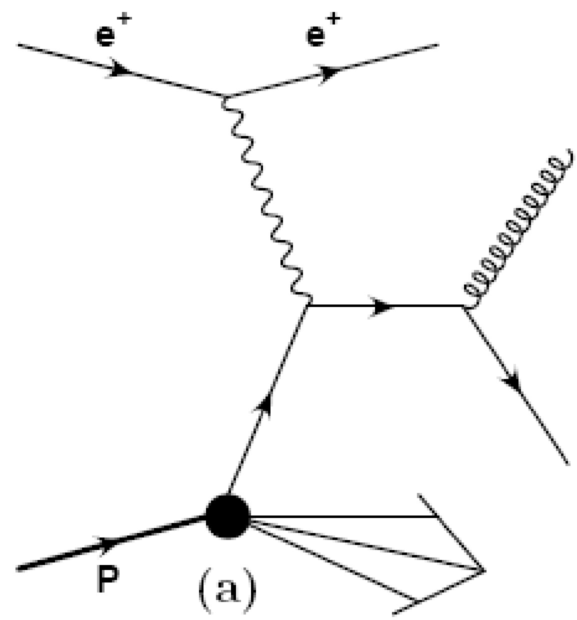
- More sophisticated factorization is needed for jet production in DIS
- Cross section = H convoluted with PDF and Jet
- H is defined as contribution with collinear piece for initial state and collinear piece for final state being subtracted
- Basis for applying PQCD to jet physics

Jet production in DIS

- Restrict phase space of final-state quark and gluon in small angular separation
- Jet production enhanced by collinear dynamics

$$I_{3,\mathrm{IR}} = (2\pi) \int_0 \frac{dk}{k} \int_0 \frac{d\theta}{\theta}$$





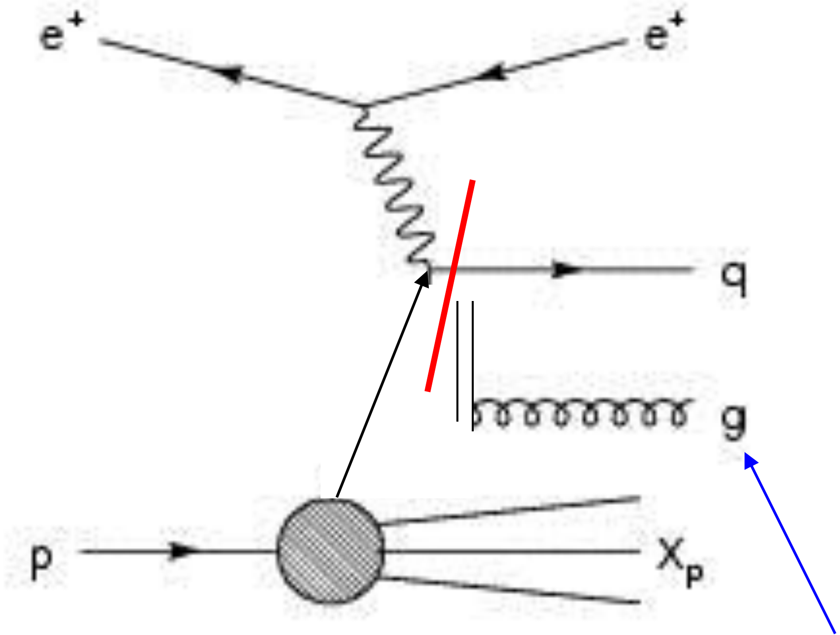
(a)

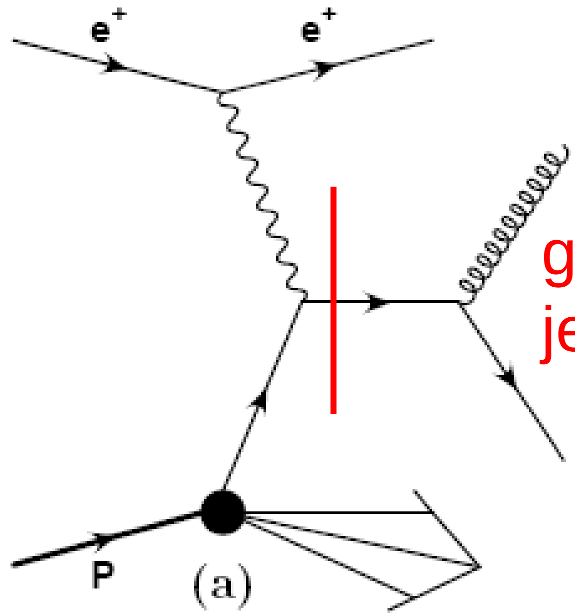
Wilson link

- Feynman rules with ξ are from Wilson link

$$\Phi_\xi^{(f)}(\infty,0;0) = \mathcal{P}\left\{ e^{-ig\int_0^\infty d\eta\, \xi\cdot A^{(f)}(\eta\xi^\mu)} \right\}$$

- Represented by double lines





go into
jet function

collinear gluon detached and factorized

Quark Jet function

- Eikonalization leads to factorization

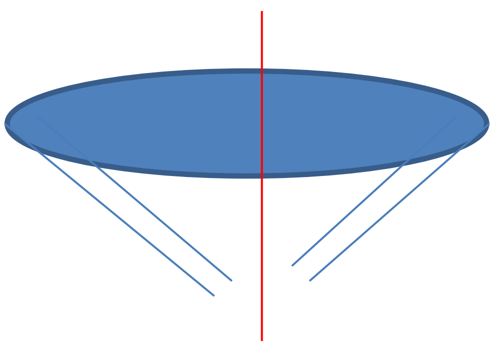
$$J_i^q(m_J^2, p_{0,J_i}, R) = \frac{(2\pi)^3}{2\sqrt{2}(p_{0,J_i})^2}\frac{\xi_\mu}{N_c}\sum_{N_{J_i}} Tr\left\{\gamma^\mu \langle 0|q(0)\Phi_\xi^{(\bar{q})\dagger}(\infty,0)|N_{J_i}\rangle\right.$$

$$\left.\times\langle N_{J_i}|\Phi_\xi^{(\bar{q})}(\infty,0)\bar{q}(0)|0\rangle\right\}\delta(m_J^2-\tilde{m}_J^2(N_{J_i},R))$$

$$\times\delta^{(2)}(\hat{n}-\tilde{n}(N_{J_i}))\delta(p_{0,J_i}-\omega(N_{J_c}))$$

projector

- Define jet axis, jet energy, jet invariant mass
- Wilson links are needed for gauge invariance of nonlocal matrix elements
- LO jet $J_i^{(0)}(m_{J_i}^2, p_{0,J_i}, R) = \delta(m_{J_i}^2)$



Almeida et al. 08

Gluon jet function

- Similar definition for gluon jet function

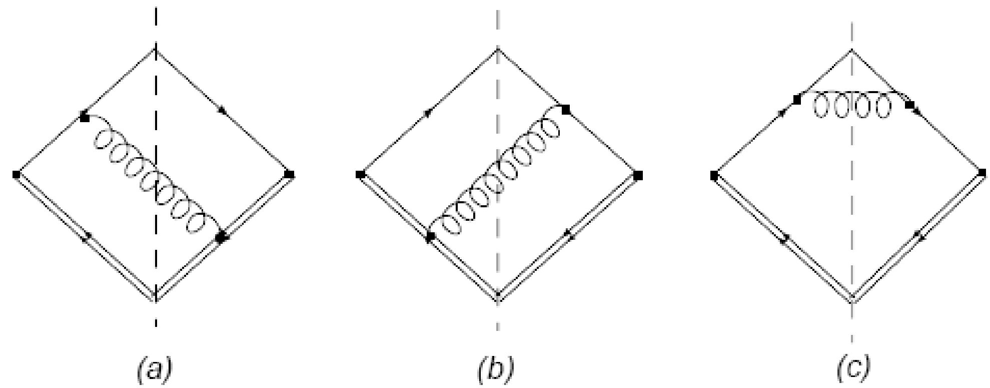
$$J_i^g(m_J^2, p_{0,J_i}, R) = \frac{(2\pi)^3}{2(p_{0,J_i})^3} \sum_{N_{J_i}} \langle 0|\xi_\sigma F^{\sigma\nu}(0)\Phi_\xi^{(g)\dagger}(0,\infty)|N_{J_i}\rangle$$

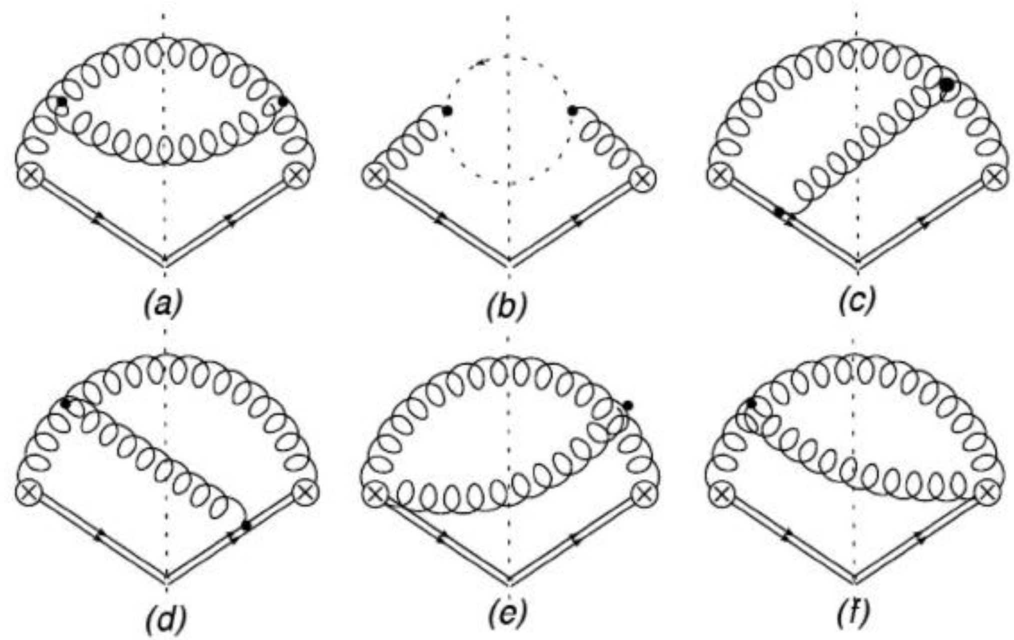
$$\times \langle N_{J_i}|\Phi_\xi^{(g)}(0,\infty)F_\nu^\rho(0)\xi_\rho|0\rangle \delta(m_J^2 - \tilde{m}_J^2(N_{J_i}, R))$$

$$\times \delta^{(2)}(\hat{n} - \tilde{n}(N_{J_i}))\delta(p_{0,J_i} - \omega(N_{J_c}))$$

NLO diagrams

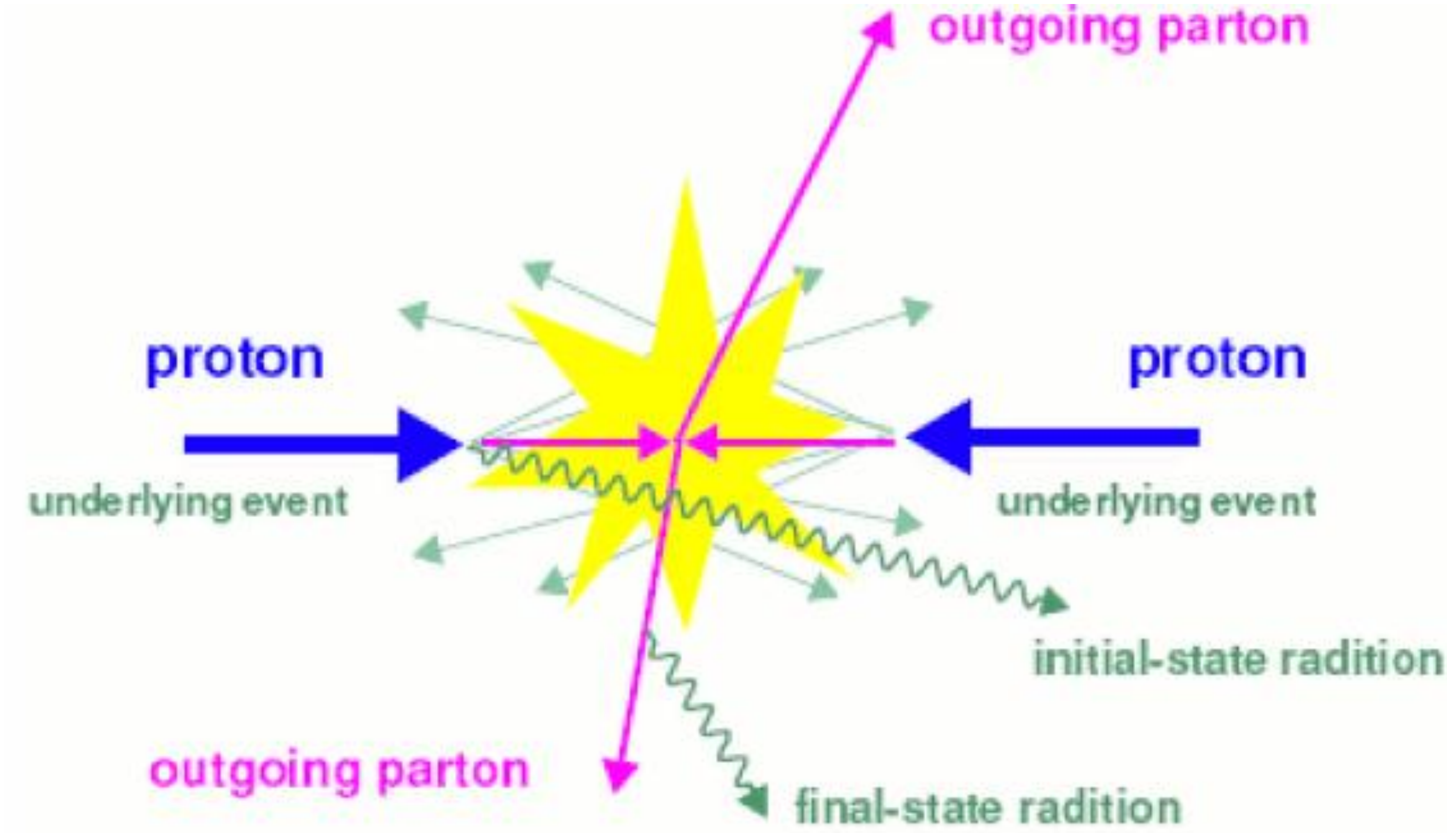
- quark jet
- gluon jet





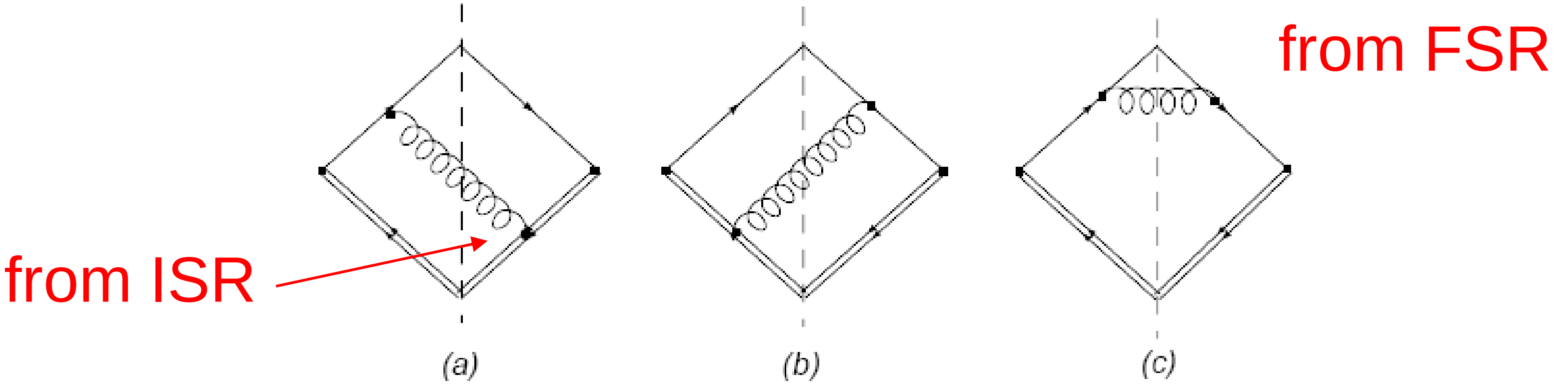
Underlying events

- Everything but hard scattering
- Initial-state radiation, final-state radiation, multi-parton interaction



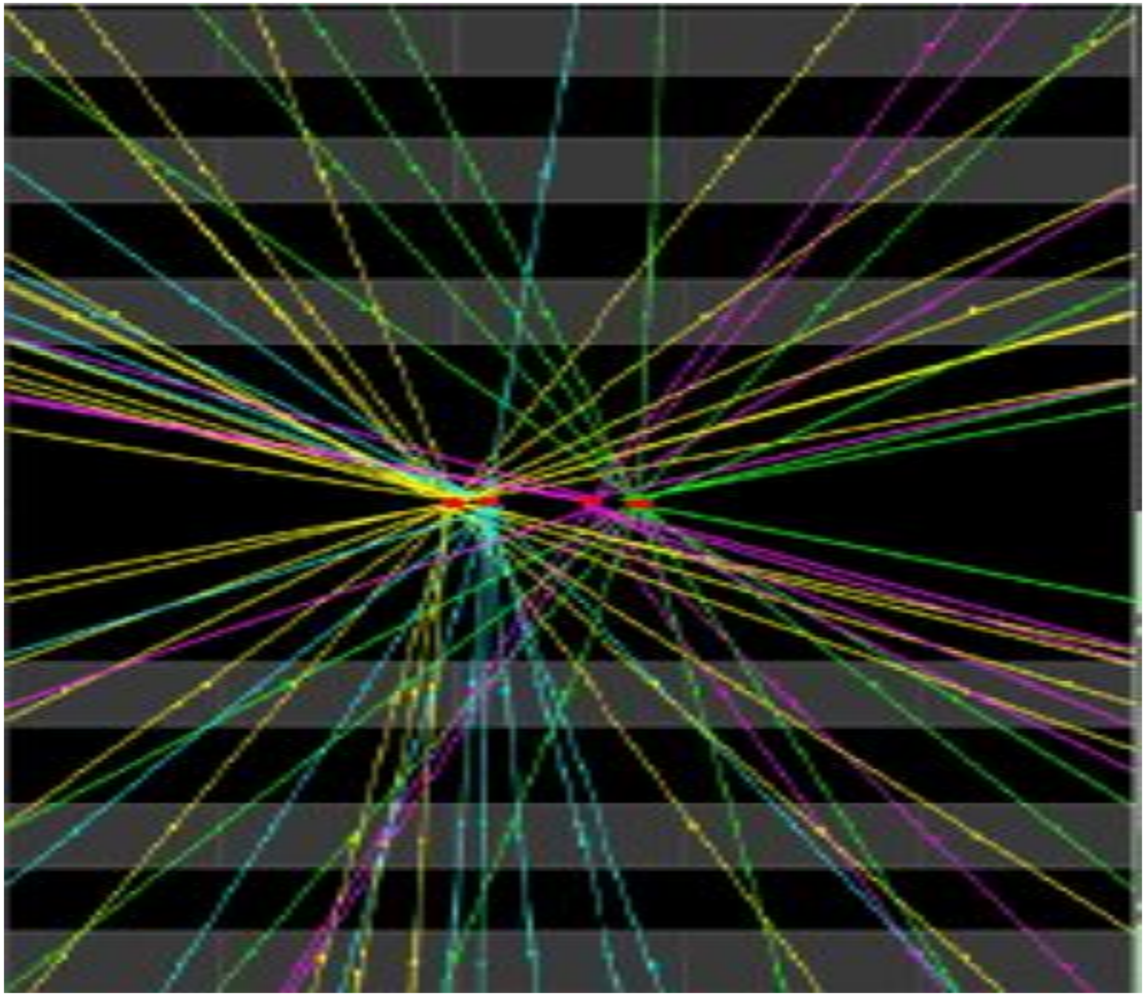
Power counting

- ISR, FSR are leading power, and should be included in jet definition



- MPI are sub-leading power: chance of involving more partons in scattering is low. They should be excluded

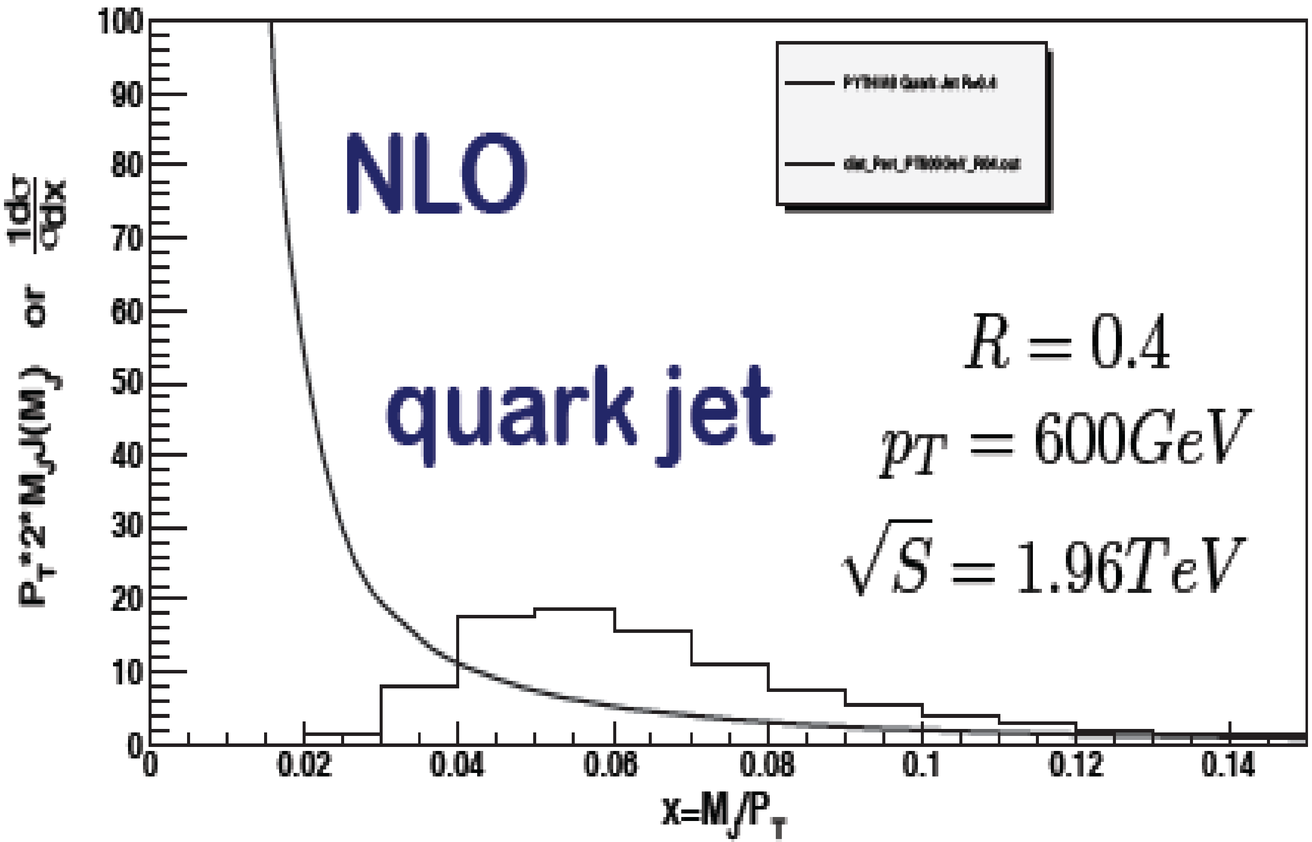
Pile-up events should be excluded



4 pile-up vertices

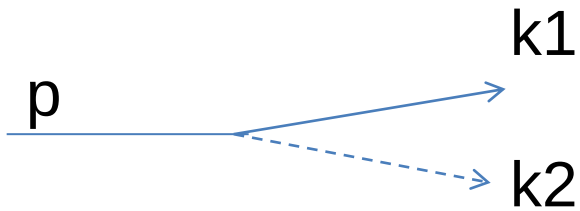
NLO jet distribution

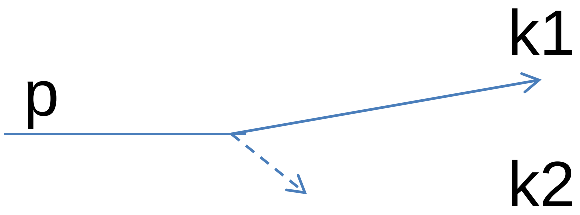
- Divergence of NLO quark jet distribution at small MJ



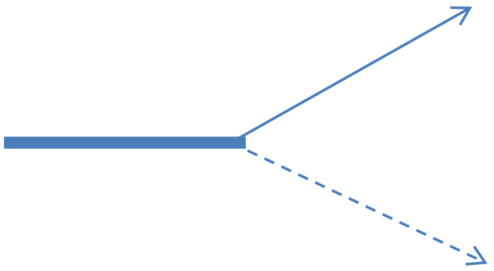
Soft/collinear gluons vs jet mass

- Small jet mass





large jet mass



Double logarithm

- Total NLO in Mellin space.

$$\int_0^1 dx(1-x)^{N-1} J_q^{(1)} = \frac{\alpha_s(\mu^2)C_F}{\pi R^2 P_T^2}\left[-\frac{1}{2}\ln^2\bar{N} - \left(\ln\nu^2 - \frac{3}{4}\right)\ln\bar{N}\right.$$

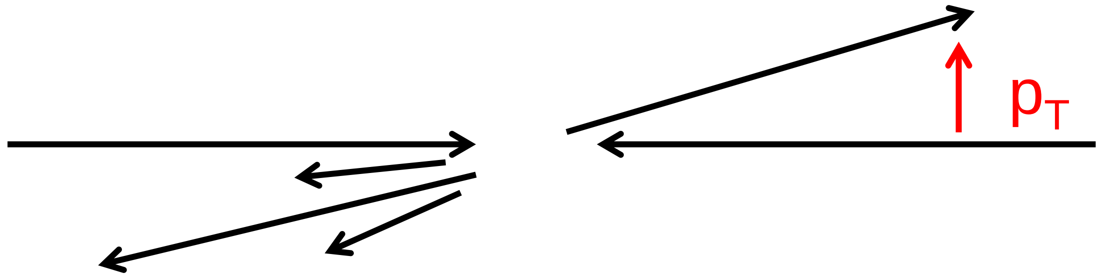
$$x \equiv M_J^2/(RP_T)^2 \qquad \ln M_J \to \ln N$$

Wilson line vector

- Double log hints resummation
- Angular resolution is related to jet mass. When M_J is not zero, particles in a jet can not be completely collimated.
- Energy resolution is also related to jet mass. When M_J is not zero, the jet must have finite minimal energy

Resummation

- Recall low pT spectra of direct photon dominated by soft/collinear radiations



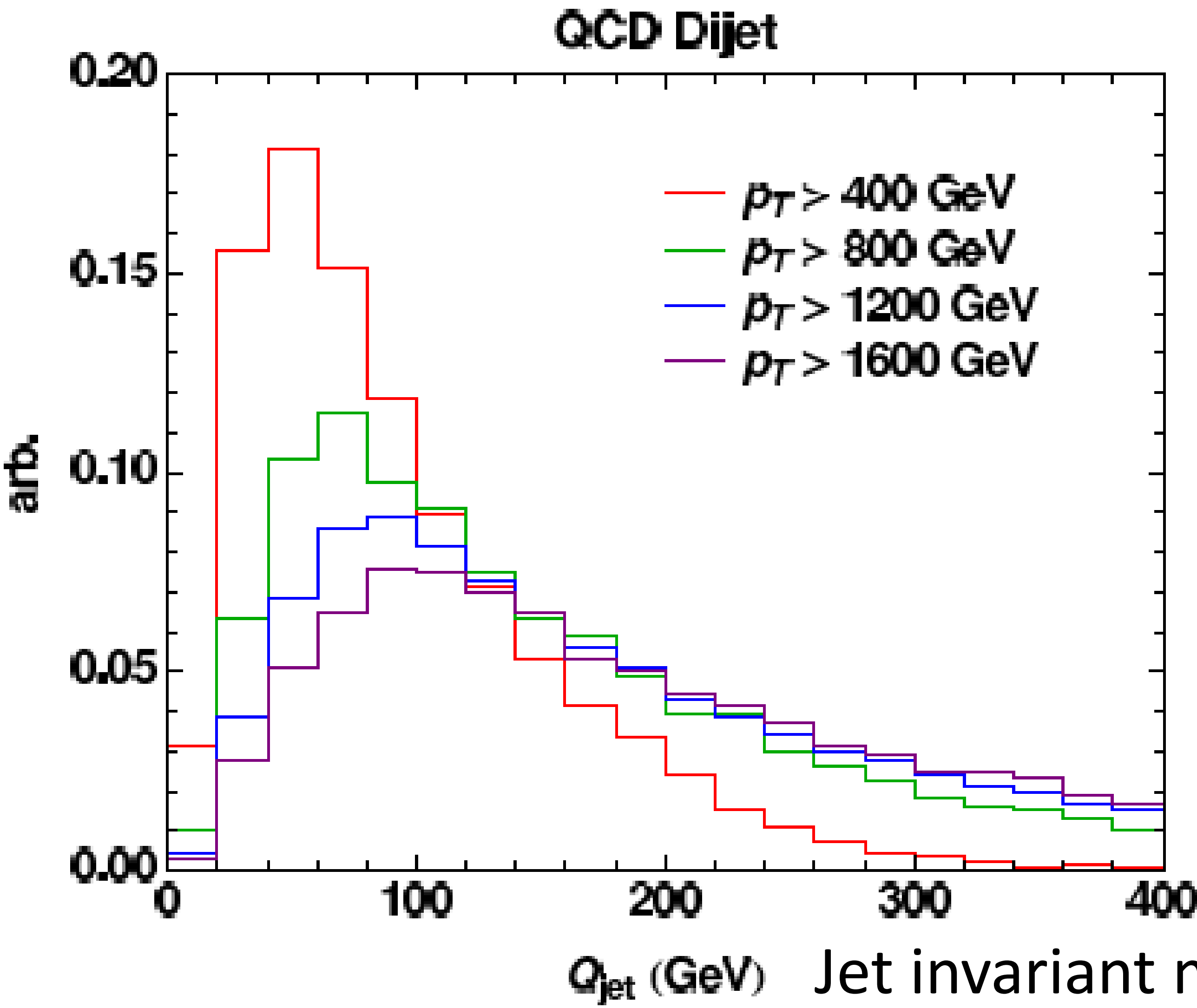
- Require kT resummation
- Jet mass arises from soft/collinear radiations
- Can be described by resummation!
- Anti-kT algorithm is preferred in view point of resummation

Jet substructures

Boosted heavy particles

- Large Hadron Collider (LHC) provide a chance to search new physics
- New physics involve heavy particles decaying possibly through cascade to SM light particles
- New particles, if not too heavy, may be produced with sufficient boost -> a single jet
- How to differentiate heavy-particle jets from ordinary QCD jets?
- Similar challenge of identifying energetic top quark at LHC

Fat QCD jet fakes top jet at high pT

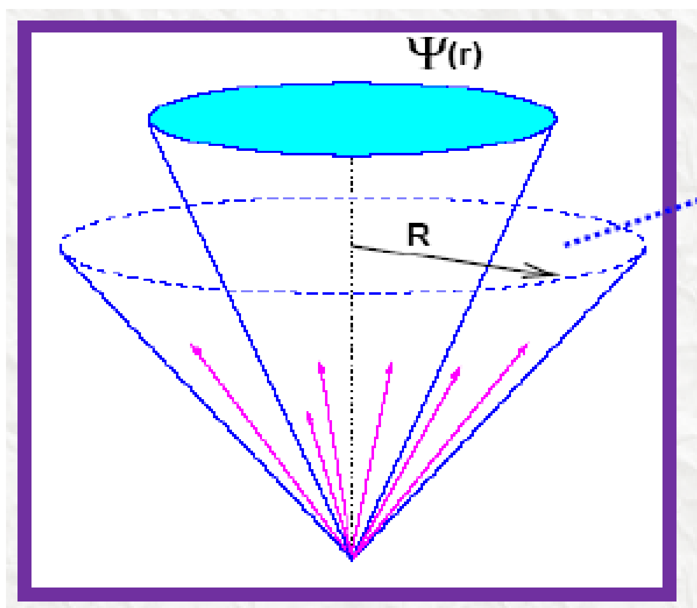


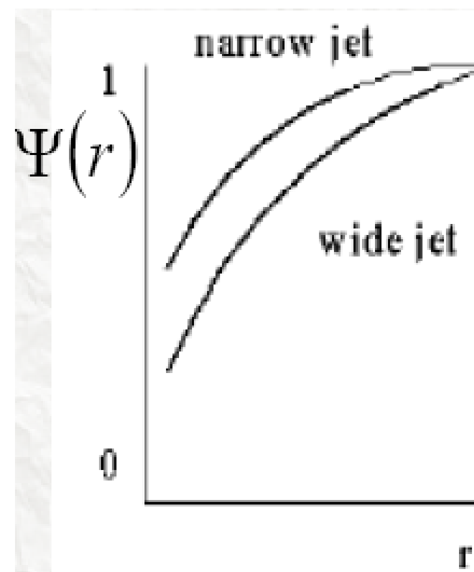
Jet invariant mass

Thaler
& Wang
0806.0023
Pythia 8.108

Jet substructure

- Make use of jet internal structure in addition to standard event selection criteria
- Energy fraction in cone size of r, $\Psi(r), \Psi(R)=1$
- Quark jet is narrower than gluon jet
- Heavy quark jet energy profile should be different

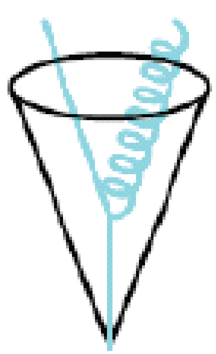


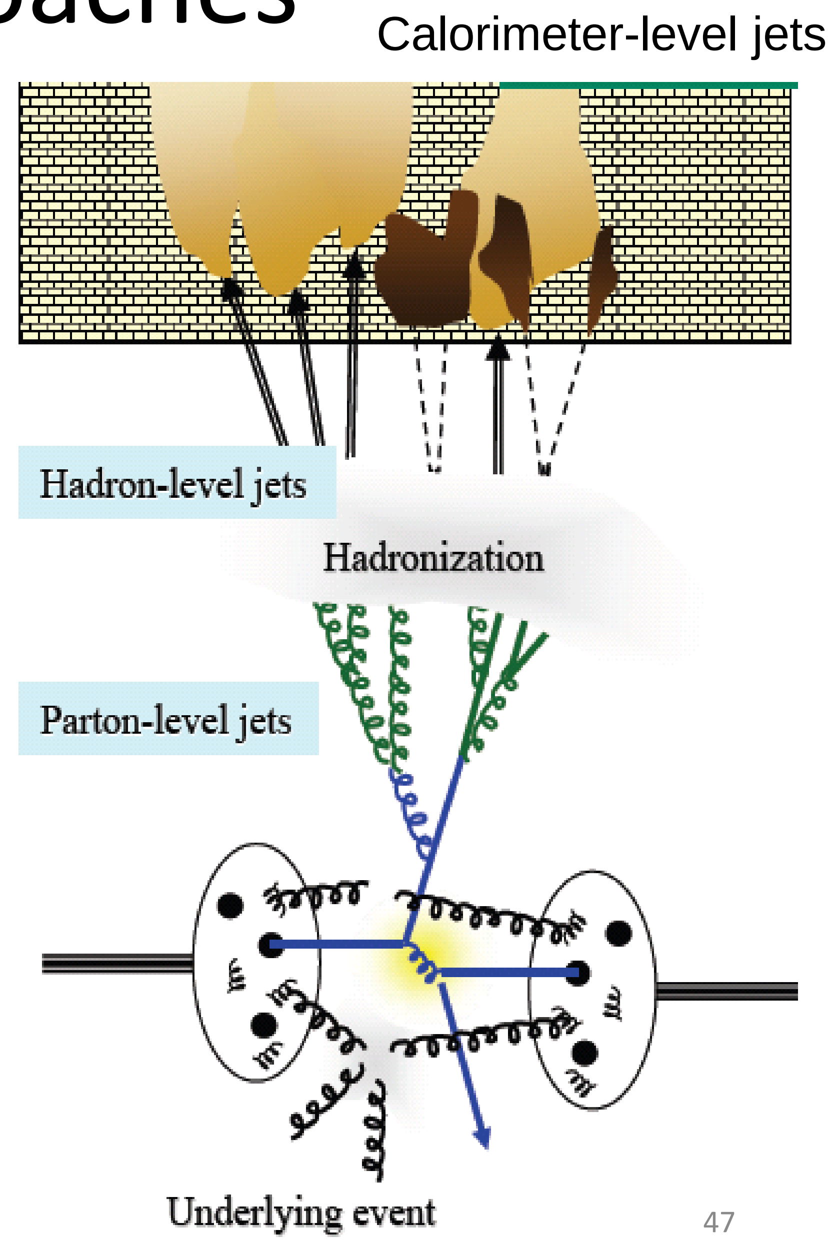


Jet substructures are finger
prints of particles
crucial for particle identification

Various approaches

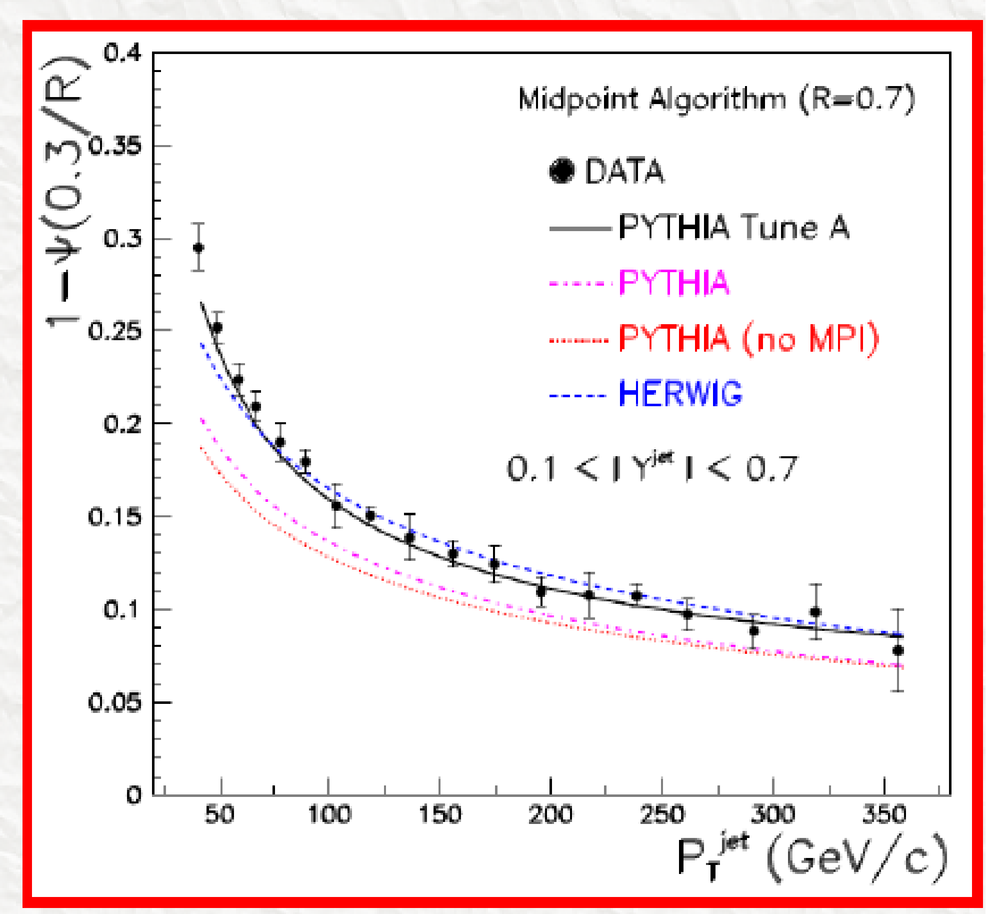
- Monte Carlo: leading log radiation, hadronization, underlying events
- Fixed order: finite number of collinear/soft radiations
- Resummation: all-order collinear/soft radiations





Why resummation?

- Monte Carlo may have ambiguities from tuning scales for coupling constant
- NLO is not reliable at small jet mass
- Predictions from QCD resummation are necessary



Tevatron data vs MC predictions

N. Varelas 2009

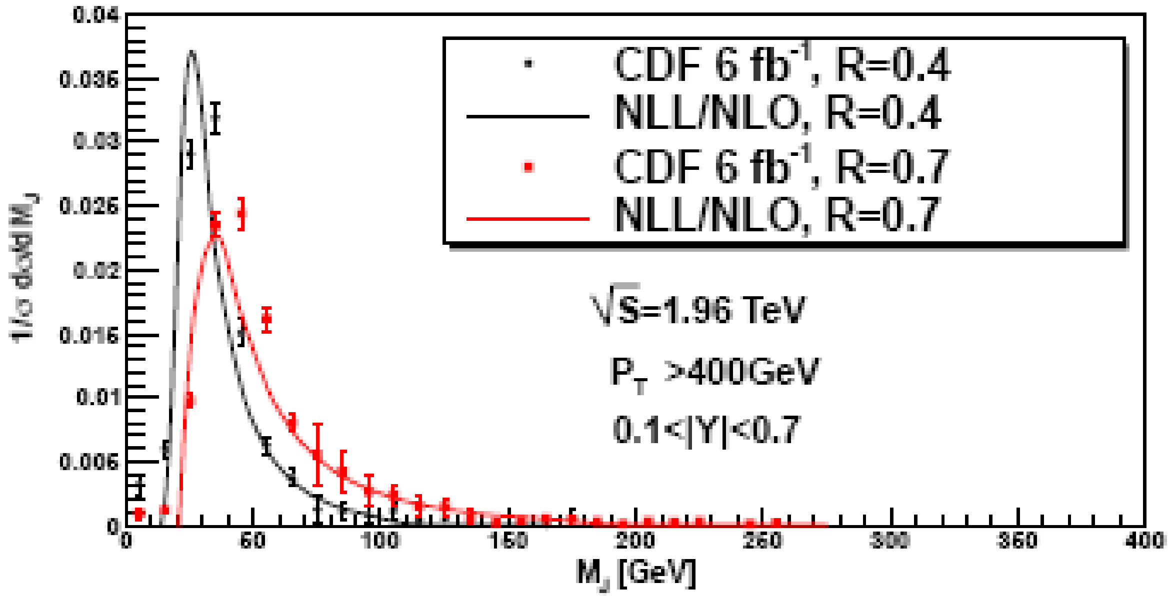
Resummation equation

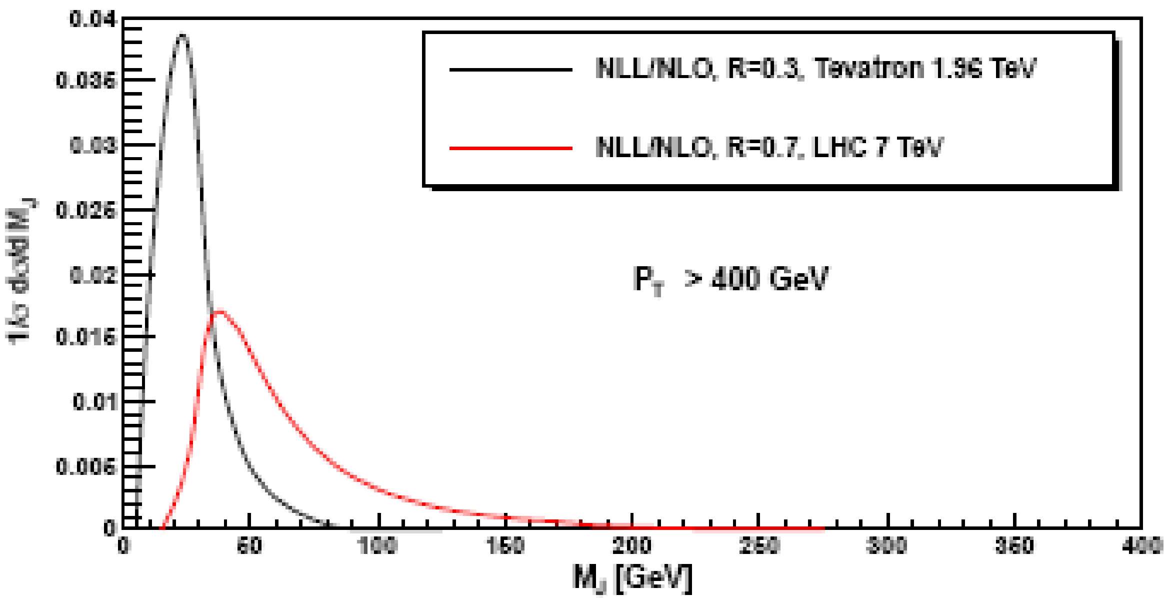
- Up to leading logs, resummation equation

$$-\frac{n^2}{P_J\cdot n}P_J^\alpha\frac{d}{dn^\alpha}J=[G^{(1)}+K_v^{(1)}+K_r^{(1)}]\otimes J$$

- See Lecture 2
- Regarded as associating soft gluon in Kr in single-log kernel into jet function J
- This is anti-kT algorithm!

Predictions for jet mass distribution





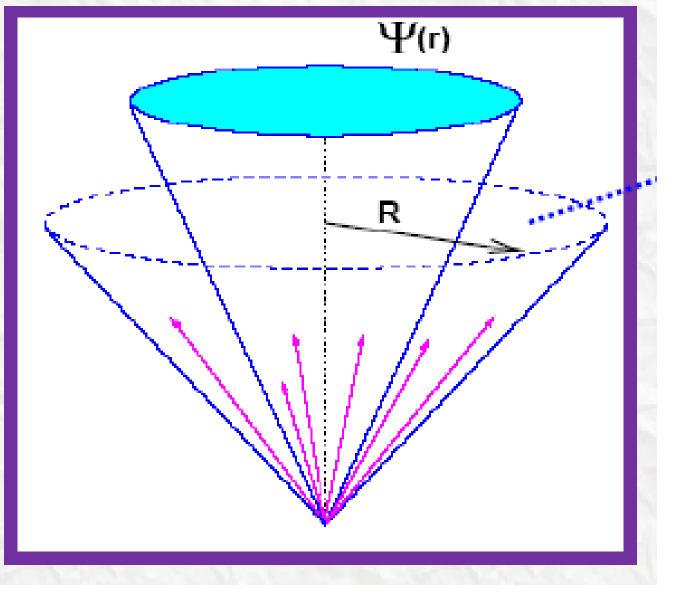
NLL in resummation
NLO in initial condition

CTEQ6L PDFs

Li, Li, Yuan, 2011

Energy profiles

- If can calculate jet mass in arbitrary jet cone size R, can certainly calculate jet energy in arbitrary jet cone $\Psi(r)$
- It is still attributed to soft/collinear radiations
- Resummation applies



Jet energy functions

- Jet energy function for quark

$$\frac{(2\pi)^3}{2\sqrt{2}(P_J^0)^2 N_c}\sum_{\sigma,\lambda}\int\frac{d^3p}{(2\pi)^3 2p^0}\frac{d^3k}{(2\pi)^3 2k^0}[p^0\Theta(r-\theta_p)+k^0\Theta(r-\theta_k)]$$
$$\times\mathrm{Tr}\left\{\not\xi\langle 0|q(0)W_\xi^{(\bar{q})\dagger}(\infty,0)|p,\sigma;k,\lambda\rangle\langle k,\lambda;p,\sigma|W_\xi^{(\bar{q})}(\infty,0)\bar{q}(0)|0\rangle\right\}$$
$$\times\delta(M_J^2-(p+k)^2)\delta(\hat{n}-\hat{n}_{\mathbf{p}+\mathbf{k}})\delta(P_J^0-p^0-k^0),$$

insert step functions

- Jet energy function for gluon

$$\frac{(2\pi)^3}{2(P_J^0)^3 N_c}\sum_{\sigma,\lambda}\int\frac{d^3p}{(2\pi)^3 2p^0}\frac{d^3k}{(2\pi)^3 2k^0}[p^0\Theta(r-\theta_p)+k^0\Theta(r-\theta_k)]$$
$$\times\langle 0|\xi_\sigma F^{\sigma\nu}(0)W_\xi^{(g)\dagger}(\infty,0)|p,\sigma;k,\lambda\rangle\langle k,\lambda;p,\sigma|W_\xi^{(g)}(\infty,0)F_\nu^\rho(0)\xi_\rho|0\rangle$$
$$\times\delta(M_J^2-(p+k)^2)\delta(\hat{n}-\hat{n}_{\mathbf{p}+\mathbf{k}})\delta(P_J^0-p^0-k^0),$$

Resummation equation

- Resummation equation for jet profile

$$\bar{K}_r^{(1)}(1) = g^2 C_F \int \frac{d^4 l}{(2\pi)^3} \frac{n^2}{(n\cdot l + i\epsilon)^2} \delta(l^2 - a^2) \Theta\left(r - \frac{|\mathbf{l}|\sin\theta}{P_J^0}\right)$$

$$-\frac{n^2}{v\cdot n} v_\alpha \frac{d}{dn_\alpha} \bar{J}_q^E(1, P_T, \nu^2, R, r)$$

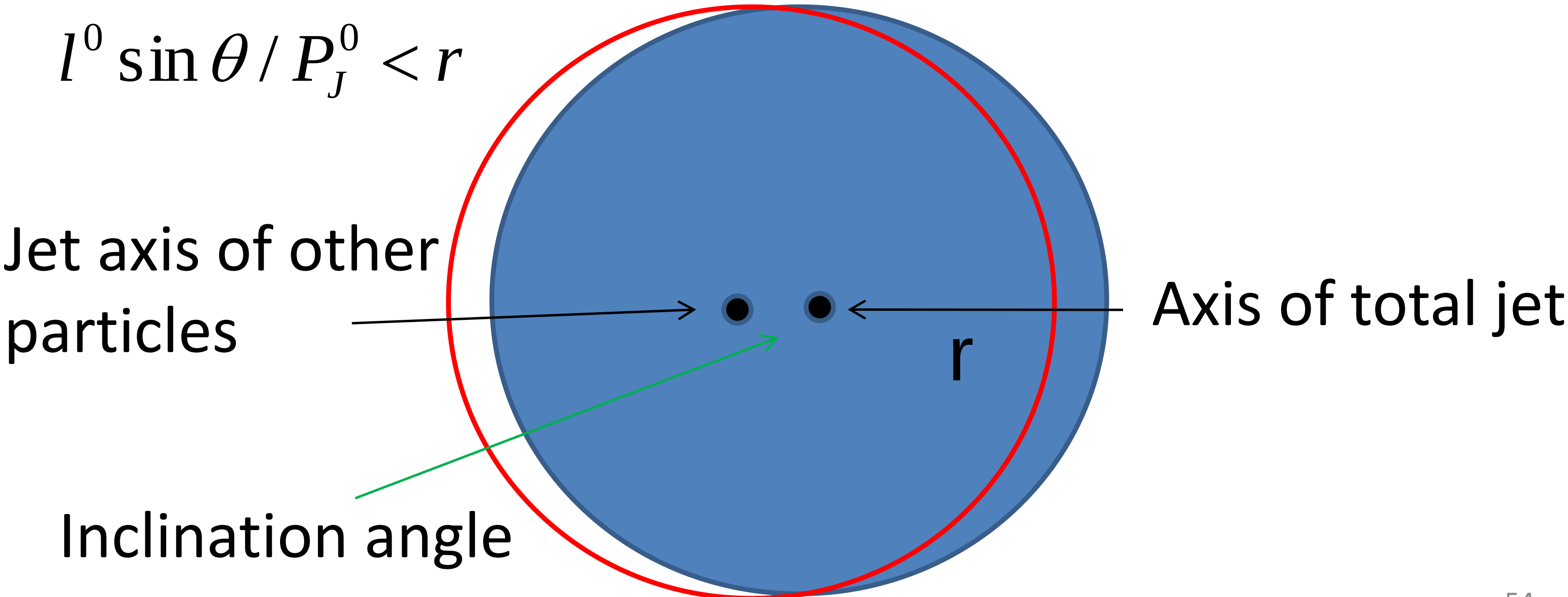
$$= 2[G^{(1)} + K^{(1)}(1)] \bar{J}_q^E(1, P_T, \nu^2, R, r)$$

- Have considered N=1 here, corresponding to integration over jet mass (insensitive to nonperturbative physics)
- Resum $\alpha_s \ln^2 r$, $\alpha_s \ln r$ from phase space constraint for real gluons

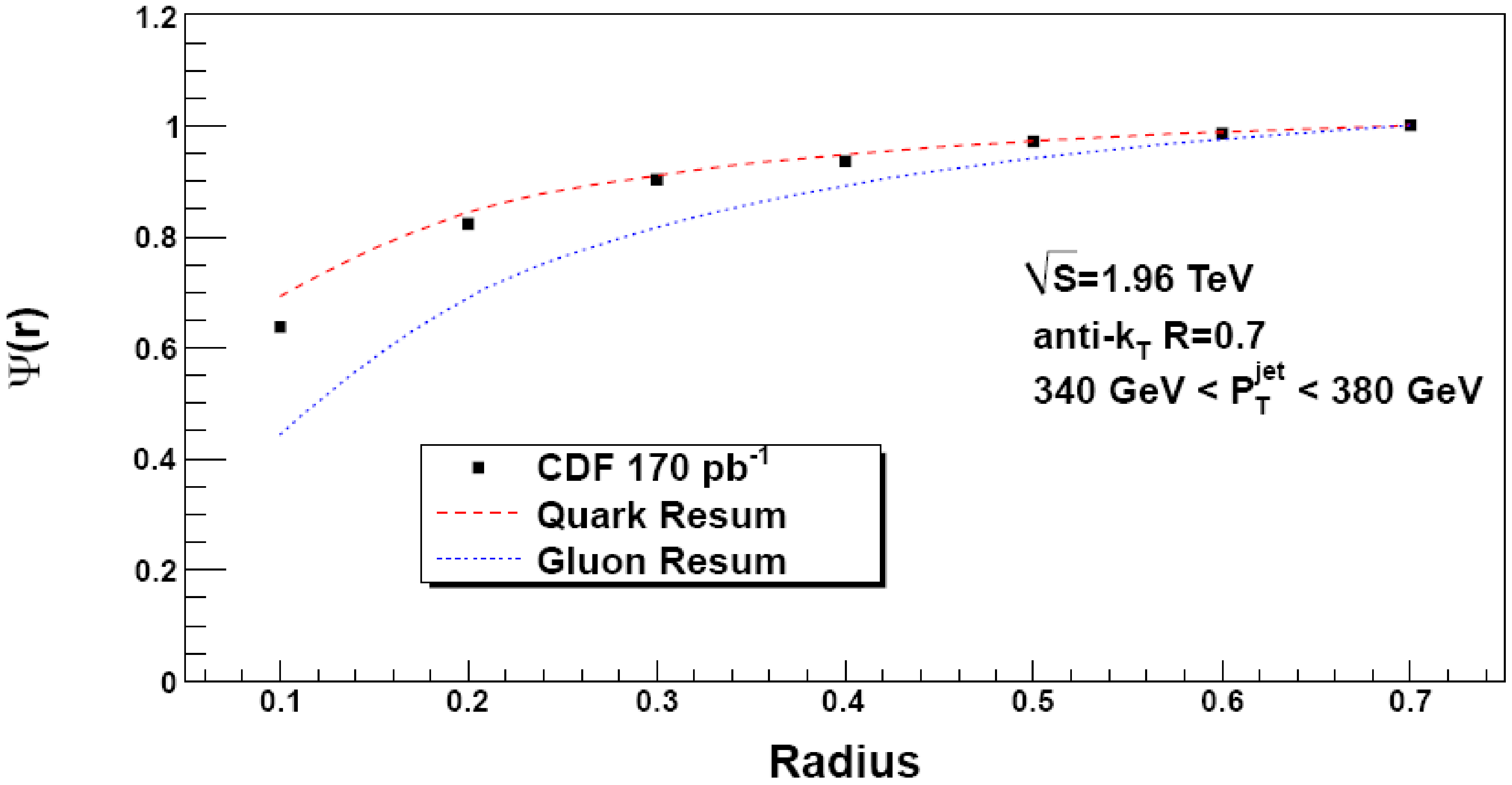
Soft gluon effect

- Soft real gluon in Kr renders jet axis of other particles inclined by small angle $l^0 \sin\theta / P_J^0$
- This jet axis can not go outside of the subcone
- This is how real gluons affect r dependence

$$l^0 \sin\theta / P_J^0 < r$$

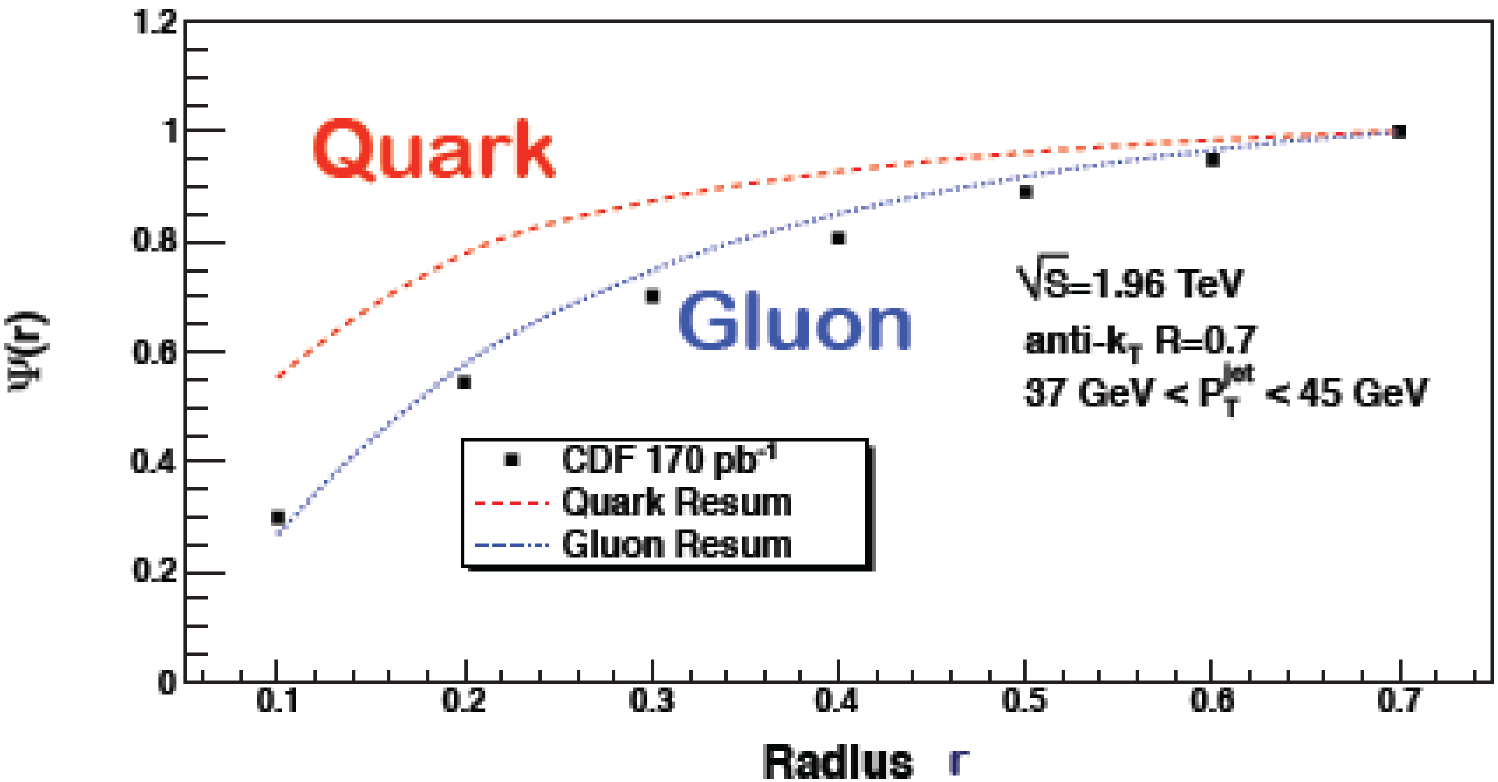


Quark jet or gluon jet?



- It is a quark jet!

Opportunities at LHC

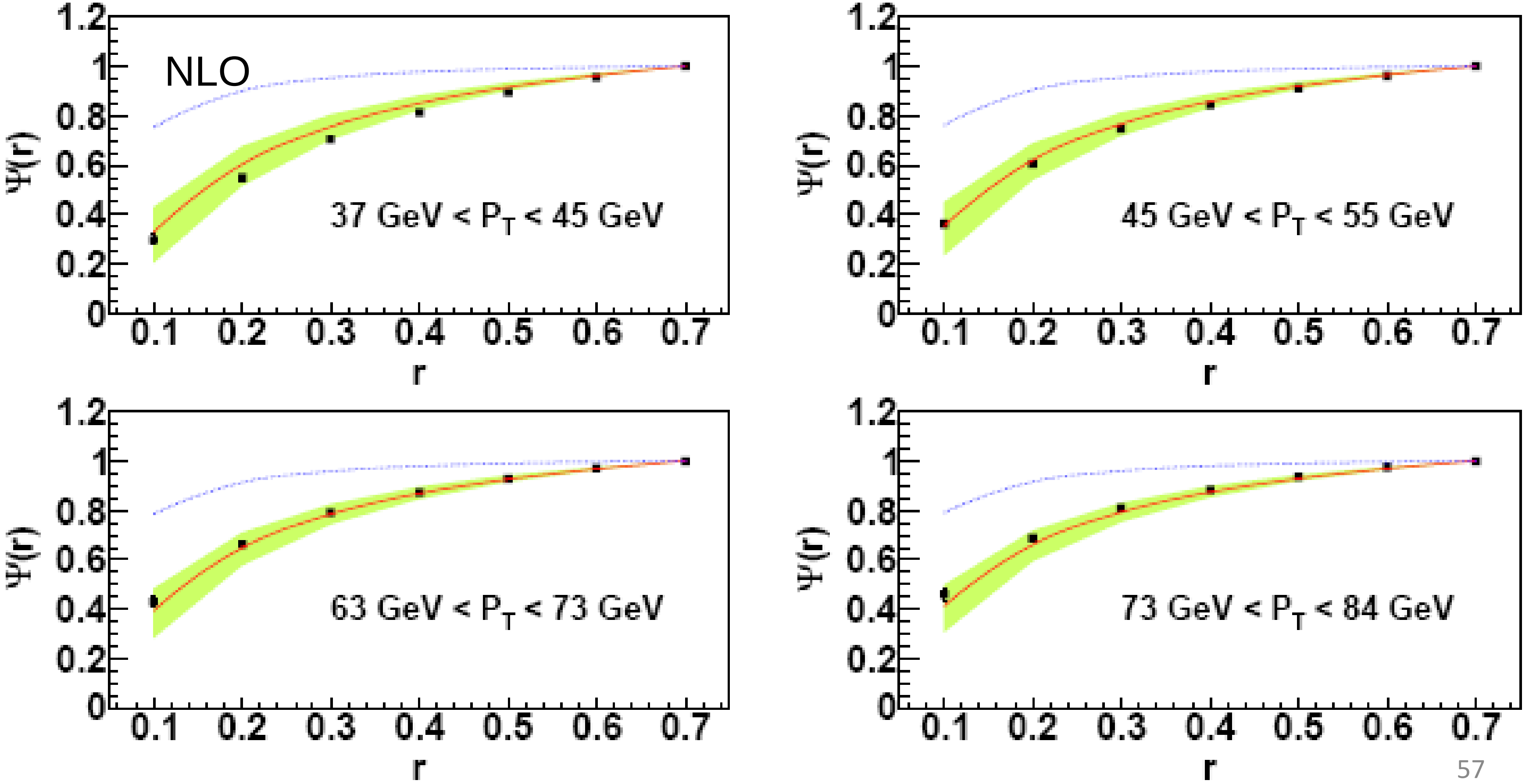


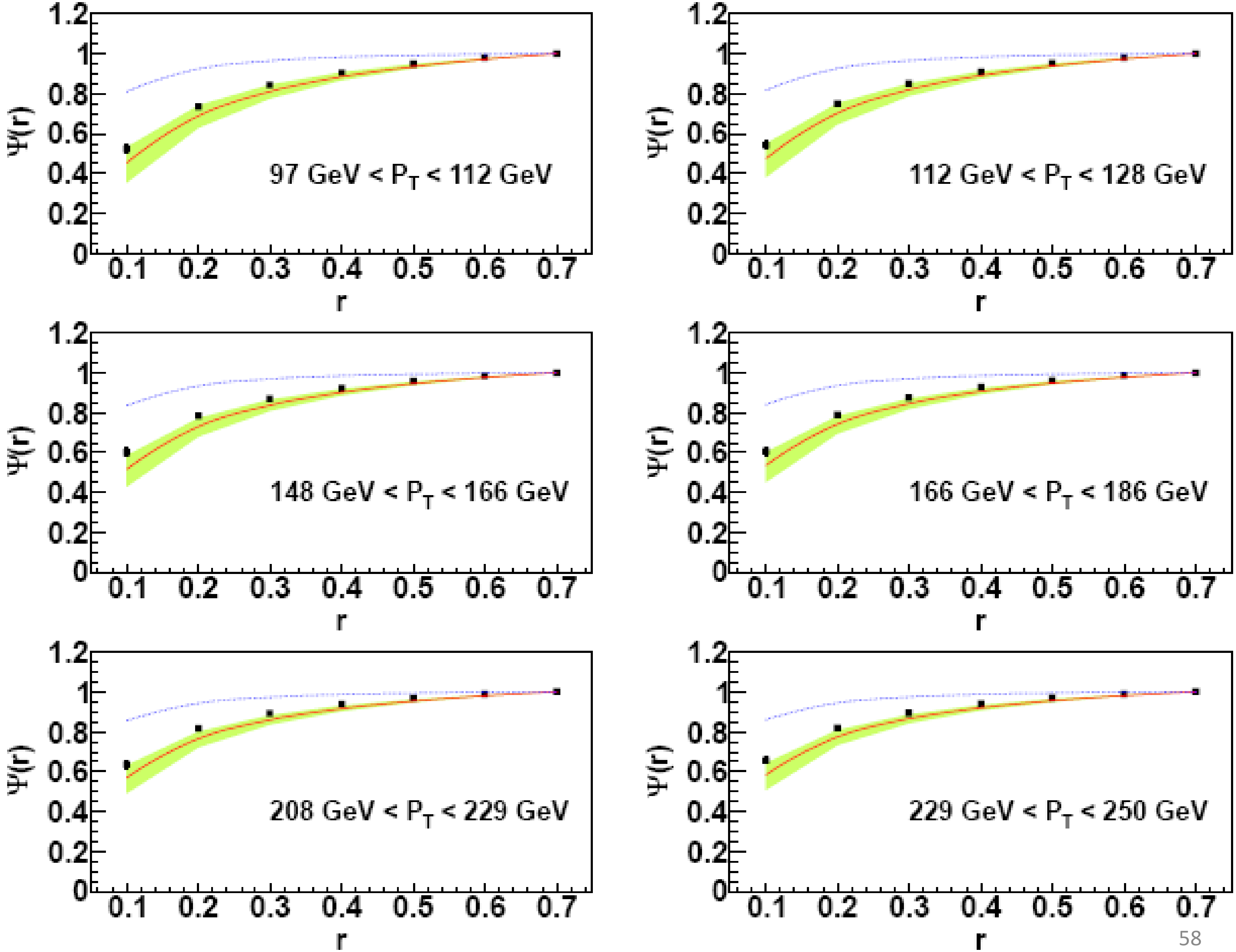
- It is a gluon jet!
- Test new physics models from composition of observed jets, e.g., CDF "W+jj" anomaly

Comparison with CDF data

$$\Psi(r) = \frac{1}{\mathrm{N_{jet}}} \sum_{\mathrm{jets}} \frac{P_T(0,r)}{P_T(0,R)}, \qquad 0 \le r \le R$$

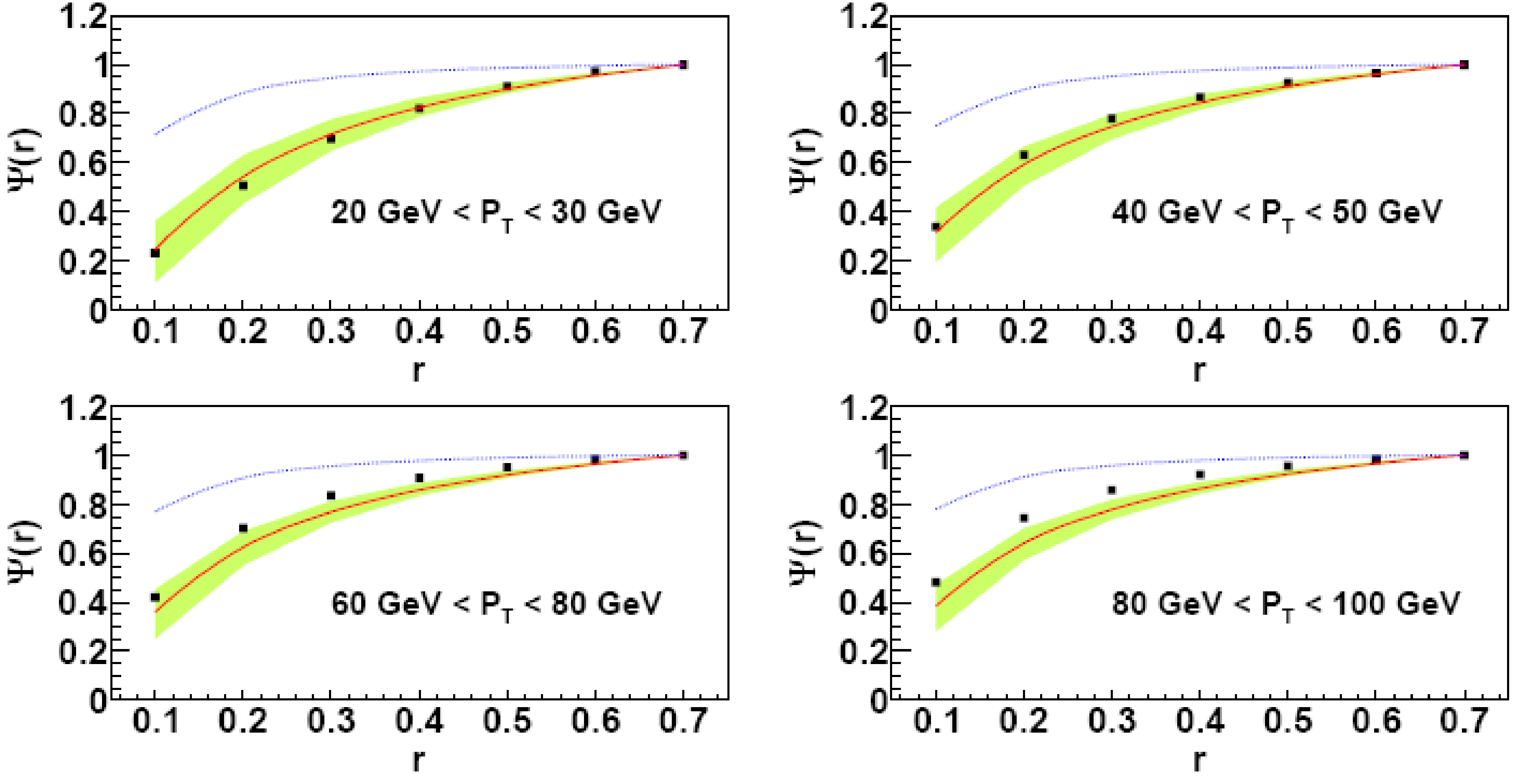
quark, gluon jets, convoluted with LO hard scattering, PDFs





Ψ(r)
r
1.2
1
0.8
0.6
0.4
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
97 GeV < P_T < 112 GeV
112 GeV < P_T < 128 GeV
148 GeV < P_T < 166 GeV
166 GeV < P_T < 186 GeV
208 GeV < P_T < 229 GeV
229 GeV < P_T < 250 GeV

Compasion with CMS data



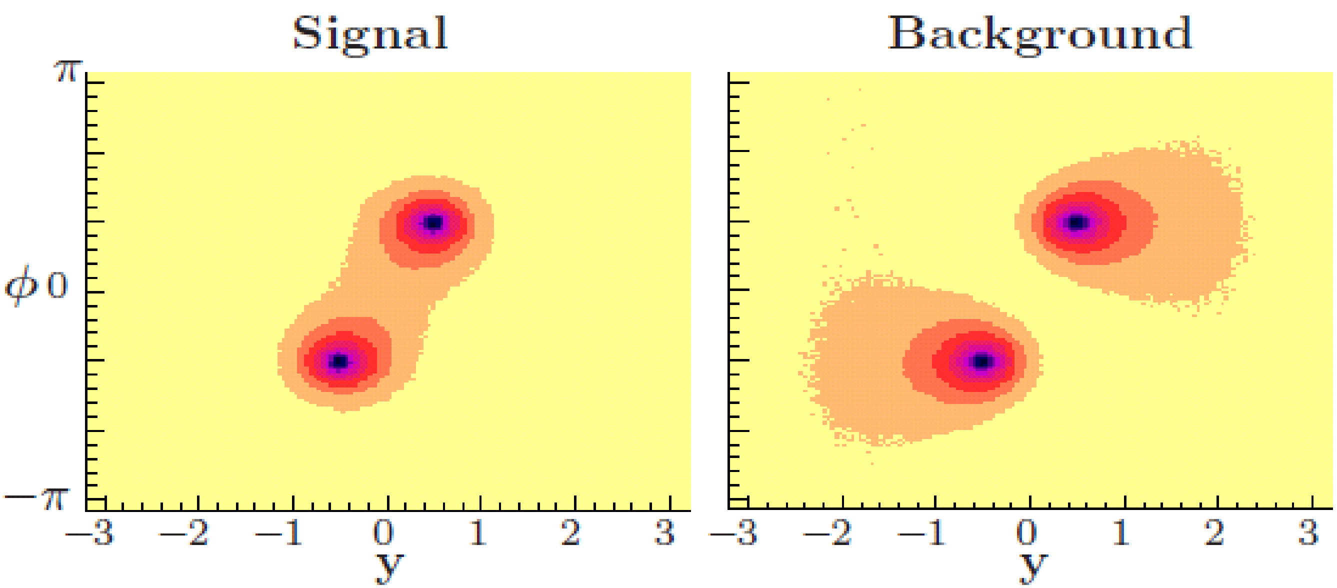
Higgs jet

- One of major Higgs decay modes H -> bb with Higgs mass ~ 125 GeV
- Important background g -> bb
- Analyze substructure of Higgs jet improves its identification
- For instance, color pull made of soft gluons

Gallicchio, Schwartz, 2010

Color pull

- Higgs is colorless, bb forms a color dipole
- Soft gluons exchanged between them
- Gluon has color, b forms color dipole with other particles, such as beam particles



Summary

- Jet substructures can be studied in PQCD
- Start with Sterman-Weinberg definition, apply factorization and resummation, predict observables consistent with data
- Fixed-order calculation not reliable at small MJ
- Event generators have ambiguities
- Can improve jet identification and new particle search