Quarkonium spectra from Lattice QCD

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Myths: that lattice QCD can ...

- ... only study hadronic ground states with precision
- ... not study states with high spin
- ... not determine isoscalar mesons (with precision)
- ... not compute scattering properties

- where do these myths come from?
- can we address them?

Origins of the myths

- Difficulties stem mostly from restrictions with standard techniques used in numerical simulations, particularly those to study quarks.
- Physics focus of LQCD has long been matrix elements and new approaches were needed for precision spectroscopy of high-spin states and exotics.

Are there solutions?

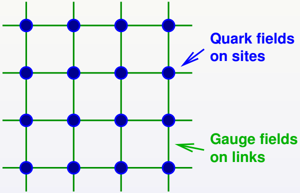
- New methods are debunking the myths
- Can study excited and high-spin states reliably
- Isoscalar mesons now almost as precise as isovectors!
- Many groups have results on scattering & resonances (light sector).

Outline

- Spectroscopy from Lattice QCD
  - Methods: old and new
  - Heavy quarks and lattice QCD
- Recent results $(N_f = 3, 2, 2+1, 2+1+1)$
  - charm and bottom states below threshold
  - above threshold
- Future work: what to expect & outstanding challenges

Lattice regularisation

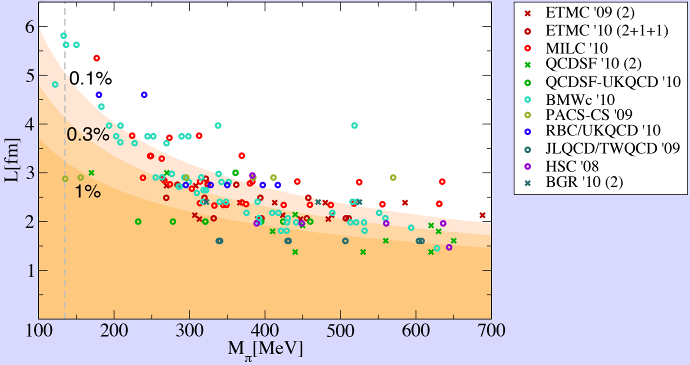
- Lattice provides a **non-perturbative, gauge-invariant** regulator for QCD
- Quarks live on sites
- Gluons live on links
- a - lattice spacing
- $a \sim 0.1$ fm



- Nielson-Ninomiya theorem: chirally symmetric quarks are missing, but can discretise quarks by trading-off some symmetry
- Wick rotation, $t \to i\tau$, $\frac{-i}{\hbar}S \to \frac{i}{\hbar}S$
- Enables importance sampling ie Monte Carlo
- Lose direct access to dynamical properties of the theory like decay widths.
- Finite V: path-integral is ordinary (but large). Predictions from the QCD lagrangian by **Monte Carlo**

The current landscape

C. Hoelbling, Lattice 2010 arXiv:1102.0410



- Dynamical simulations with $N_f = 2$ or $2+1$
- Large volumes, $L \geq 3\text{fm} \Rightarrow \mathcal{O}(1\%)$ on m_π.
- Light quark masses, now close to or at m_π.

Spectroscopy

Spectroscopy in lattice QCD

- Energies of colourless QCD states can be extracted from **two-point functions** in Euclidean time

$$C(t) = \langle 0 | \, \Phi(t)\Phi^\dagger(0) \, | 0 \rangle$$

- Euclidean time: $\Phi(t) = e^{Ht}\Phi e^{-Ht}$ so $C(t) = \langle \Phi | e^{-Ht} | \Phi \rangle$. Insert a complete set of energy eigenstate and:

$$C(t) = \sum_{k=0}^{\infty} |\langle \Phi | k \rangle|^2 e^{-E_k t}$$

- $\lim_{t\to\infty} C(t) = Z e^{-E_0 t}$, so if observe large-t fall-off, then **energy of ground-state** is measured.

Euclidean metric very useful for spectroscopy; it provides a way of isolating and examining low-lying states

Excited states

- **Excited-state** energies can be measured by correlating between operators in a bigger set, $\{\Phi_1, \Phi_2, \ldots, \Phi_N\}$

$$C_{ij}(t) = \langle 0| \, \Phi_i(t)\Phi_j^\dagger(0) \, |0\rangle$$

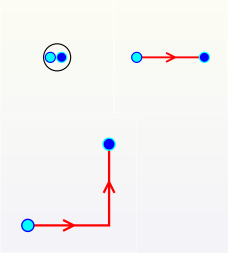
- Solve generalised eigenvalue problem:

$$C(t_1) \, \underline{v} = \lambda \, C(t_0) \, \underline{v}$$

  for different t_0 and t_1 [Lüscher & Wolff, C. Michael]
- Then $\lim_{(t_1-t_0)\to\infty} \lambda_n = e^{-E_n(t_1-t_0)}$
- Method constructs optimal ground-state creation operator, then builds orthogonal states.

Excited states accessed if basis of creation operators is used and the matrix of correlators can be computed

Spectroscopy - making measurements



- ϕ are bilinears with path-ordered products of quark and anti-quark fields; different offsets, paths and spin contractions give different projections into lattice symmetry channels.

- Design ops with good overlap onto states of interest
- Good idea to smooth fields spatially before measuring: smearing

  Distillation [from Hadron Spectrum Collab.]
  - Reduce the size of space of fields (on a time-slice) preserving important features.
  - All elements of the (reduced) quark propagator can be computed: allows for many operators, disconnected diagrams, multi-hadron operators.
  - With stochastic methods to improve volume scaling.

The spectroscopy of “single-particle” states:
As well as control of usual lattice systematics ($a \to 0$,$L \to \infty$, $m_q \sim m_\pi$) requires methods for

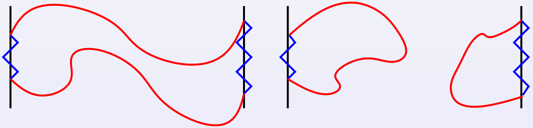
- statistical precision at % percent level
- reliable spin identification
- heavy quark methods for charm and bottom

The spectroscopy of “single-particle” states:
As well as control of usual lattice systematics ($a \to 0$,$L \to \infty$, $m_q \sim m_\pi$) requires methods for

- statistical precision at percent level
  - to include multi-hadrons and study resonances
- reliable spin identification
- heavy quark methods for charm and bottom

statistical precision at percent level

- “distillation” (0905.2160) - a new approach to simulating correlators. Particularly good for spectroscopy.
- enables precision determination of disconnected diagrams, crucial for isoscalar spectroscopy and multi-hadron operators.

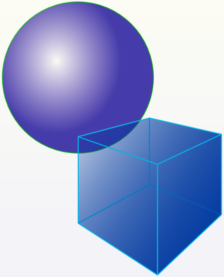


- large bases of interpolating operators now feasible, for better determination of excited states

The spectroscopy of “single-particle” states:
As well as control of usual lattice systematics ($a \to 0$,$L \to \infty$, $m_q \sim m_\pi$) requires methods for

- statistical precision at % percent level
- reliable spin identification
  - understanding symmetries and connection between lattice and continuum
  - designing operators with overlap onto J^{PC} of interest.
- heavy quark methods for charm and bottom

Spin on the lattice

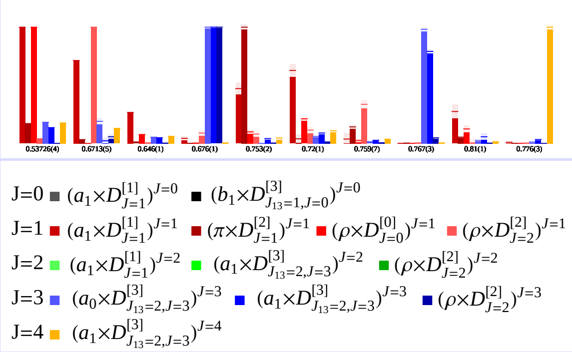


- Lattice breaks $O(3) \to O_h$
- Lattice states classified by quantum letter, $R \in \{A_1, A_2, E, T_1, T_2\}$.
- Continuum: subduce $O(3)$ irreps $\to O_h$. $J > 1$ split over irreps
- Look for degeneracies. Problem: spin-4 has same pattern as $0 \oplus 1 \oplus 2$.
- Better spin assignment by constructing operators from lattice representation of covariant derivative.
- Start in continuum with operator of definite J, subduce into O_h and replace derivatives with lattice equivalent. Measure $\langle 0|\Phi|J^{PC}\rangle$ and look for remnants of continuum symmetries.

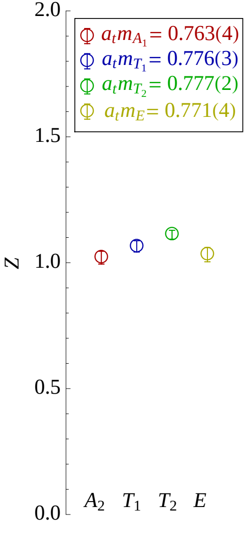
Remnants of continuum spin found on the lattice if we build operators more carefully and can measure their correlators

Reliable spin identification - overlaps

Hadron Spectrum Collaboration, 2012



Charmonium, T_1^{--}



The ground state 4^{--}.

The spectroscopy of "single-particle" states:
As well as control of usual lattice systematics
($a \to 0$,$L \to \infty$, $m_q \sim m_\pi$) requires methods for

- statistical precision at % percent level
- reliable spin identification
- heavy quark methods for charm and bottom

$\mathcal{O}(am_Q)$ are significant for charm and large for bottom

Relativistic actions

- Isotropic ($a_s = a_t$): needs very fine lattices. Working well for charm, extended to (nearly) bottom [arXiv:1010.3848].
- Anisotropic ($a_s \neq a_t$): reduce relevant temporal $a_t m_Q$ errors. Works well for charm.

Effective Theories

- NRQCD: m_c not heavy enough? Good for bottomonium.
- Fermilab: works well for charm and bottom.

Convergence through Universality: $\mathcal{O}(am_Q)$ can be controlled and methods agree.

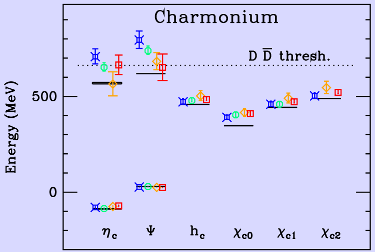
Results

Results

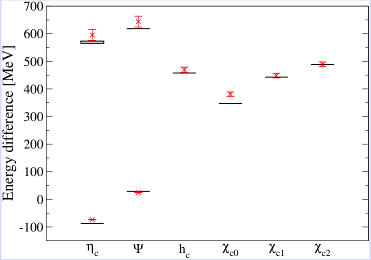
Precision spectroscopy of low-lying states

Results: charmonium below threshold

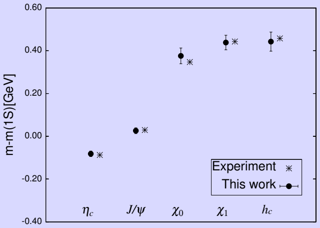
FNAL+MILC: 0912.2701



Mohler: 1110.6457



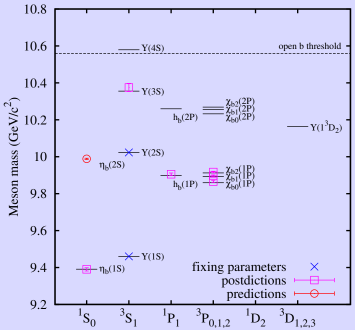
PACS-CS: 1111.0142



- improved actions
- multiple a
- light/physical m_π
- different treatments of sea & valence quarks

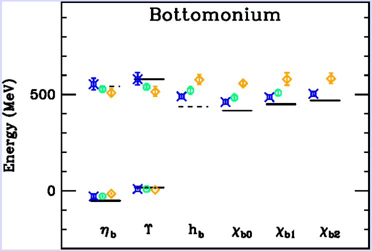
Results: bottomonium below threshold

HPQCD: 1110.6887

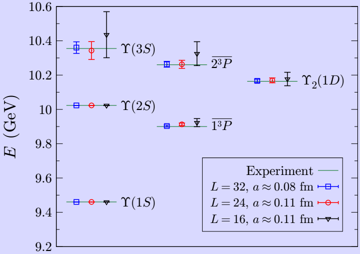


NRQCD

FNAL+MILC: 0912.2701



Meinel: 1007.3966

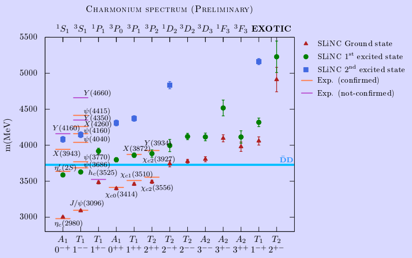


Results

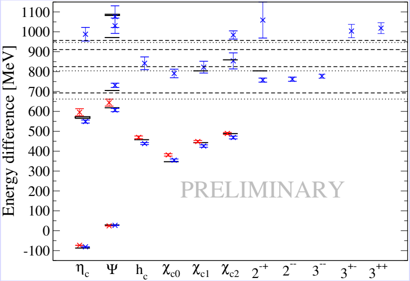
Spectroscopy above threshold

Results: charmonium above threshold

Bali et al: 1108.6147

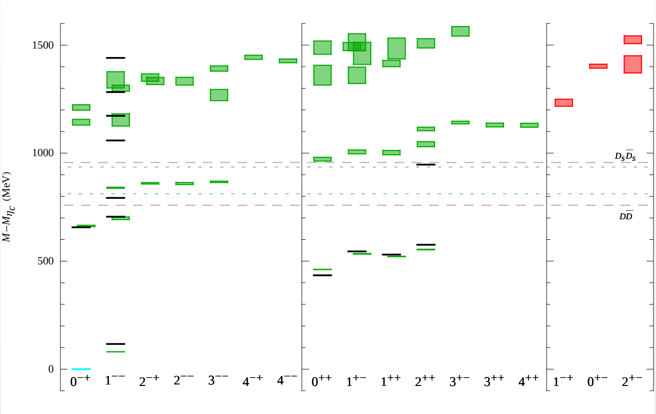


Mohler et al: unpublished



- light m_π, multiple a
- still preliminary but using variational analysis
- spin identification for excited/high-spin/exotics not robust?

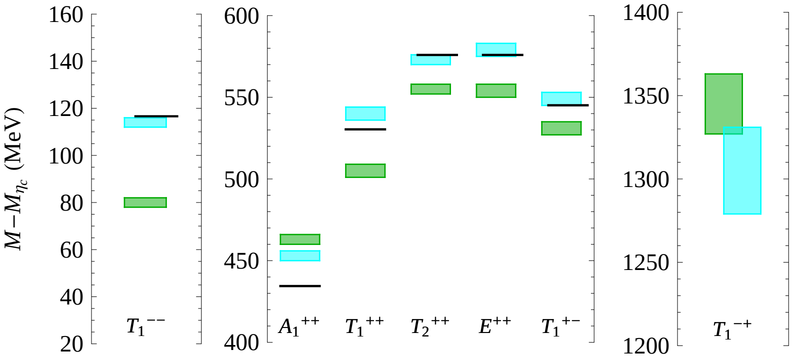
Charmonium from the Hadron Spectrum 2012



- $m_\pi \sim$ 400MeV; single a (on anisotropic lattice)
- using distillation, variation, spin id via overlaps: resolve all states up to $J = 4$
- error on S waves ~ 1 MeV; error on $1^{-+} \sim 15$ MeV

the hyper-fine splitting in S waves

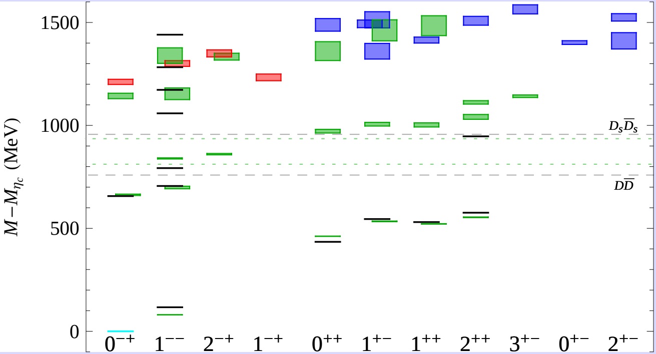
HFS and O(a) effects



- HFS sensitive to discretisation
- test the effect of c_s: tree-level; 2.0
- assign ~ 40MeV discretistion error

Results: charm hybrids

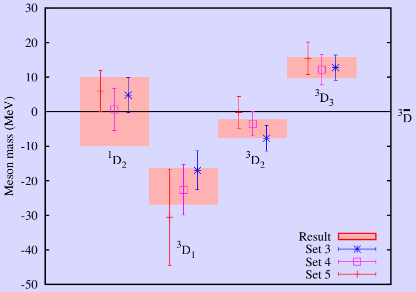
Exotic and non-exotic hybrids



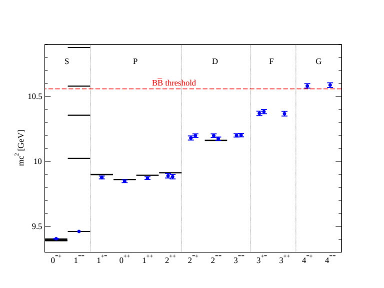
- the lightest hybrid supermulitplet; excited supermultiplet
- pattern as in light sector (Dudek 1106.5515)
- model implications - appears to disagree with flux-tube models

Results: bottomonium above threshold

HPQCD (nrqcd): 1112.2590



Lewis+Woloshyn (nrqcd): 1204.4675



- a first prediction from LQCD

- no variational analysis
- spin identification at $J > 1$?

Summary and Prospects

Spectroscopy

- Technology for LQCD spectroscopy dramatically improved: precision determinations of excited states, exotics with reliable spin identification.
- Charm more mature than bottom.
- Bottomonium spectrum underway by HadSpec.
- Many questions remain:
  - are there intrinsic excitations of gluons in hadrons?
  - can we understand the states above open thresholds?

Resonances

- scattering and resonances: Early results from Bali for charm (0911.1238).
- Can we move beyond the elastic region?

Expect a lot more progress in the next 5 years.