CP Violation
Symmetry Aspect of SM

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FPCP2012

2012.5.21-25 at USTC

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Important Issue:
Origin/Mechanism of CP Violation

- 1964：Indirect CP violation was discovered in Kaon decays （Cronin & Fitch)

  $K \rightarrow \pi\pi, \; \pi\pi\pi$

  which involves only three flavors: u, d, s
- 1964: Superweak Hypothesis (Wolfenstein)

  $\boldsymbol{\varepsilon \neq 0}$，$\boldsymbol{\varepsilon'/\varepsilon = 0}$
- The Question: CP violation is via weak-type interaction or superweak-type interaction.

CP Violation
From 3 Flavors to 3 Families

- 1973: CP violation can occur in the weak interaction with three families of SM (Kobayashi-Maskawa)
- which must be tested via the direct CP violation
- **$\varepsilon'/\varepsilon \approx 0$** (superweak hypothesis)
- **$\varepsilon'/\varepsilon \neq 0$** (weak interaction)

CP Violation Mechanism of SM

- Explicit CP violation with complex Yukawa couplings.
- There are in general 3 x 2x(3x3)=54 real (or 27 complex) Yukawa coupling constants (for massless neutrinos), i.e. Up-, Down-,Electron-type (3) complex (2) mass matrices 3x3 , but only 3x3 (masses) + 3 (angles) +1 (phase) =13 are physical observables, namely 41 can be rotated away.
- Origin of CP violation remains unknown
- It is not big enough to explain matter-antimatter asymmetry in the Universe.

CP Violation
Via Spontaneous Symmetry Breaking

1973: Spontaneous CP Violation, T.D. Lee

Scalars are responsible to CP violation

CP originates from vacuum !!!

1975: SU(2)_L x SU(2)_R Model,

Mohapatra & Pati, Senjanovic and Mohapatra

Spontaneous CP violation via Higgs bi-doublet

(One Higgs bidoublet model with spontaneous CP violation is likely excluded from the combining constraints of K and B systems and CP violations)

Multi-Higgs Doublet Models

1976: 3HDM with Discrete Symmetry S.Weinberg

CP violation arise solely from Higgs Potential

(ruled out by CP, b→ s γ decay, EDM of neutron)

1977: NFC hypothesis, Glashow & Weinberg

Discrete symmetry: no CP violation in 2HDM

Type I & Type II 2HDM

1978: 2HDM with soft breaking of CP, H. Georgi

strong CP problem, baryogenesis

1987: 2HDM with approximate discrete symmetry

Smallness of FCNC , Liu & Wolfenstein

(superweak type interaction for CP violation)

2HDM with Spontaneous CP

1993: 2HDM with approximate flavor symmetry,

Superweak type CP violation by input Hall & Weinberg

1994: 2HDM with approximate U(1) family symmetries & spontaneous CP violation, Wu & Wolfenstein

(Type III 2HDM) CP violation originates from a single relative phase of vacuum expectation values

Induced four types of CP violations after symmetry breaking:

(1) Induced KM phase in CKM matrix of SM
(2) New sources of CP violation via charged Higgs
(3) FCNC via neutral Higgs (superweak type)
(4) scalar-pseudoscalar mixing (boson interactions)

Atwood,Rena, Soni, et al. General 2HDM (explicit CP violation)

……

Statements

- One Higgs Doublet needed for Understanding Mass Generation.

- Two Higgs Doublets or some others enable us to understand the origin of CP violation via Spontaneous Symmetry Breaking.

- (i) Each quark and lepton get not only mass but also CP phase which cannot be rotated away (new sources of CP violation)

  (ii) Rich physics phenomena (g-2, **$\mu \to e\gamma$, $\tau \to \mu\gamma$,** Rare decays of bottom mesons,……)

  (consistent with current experiments)

Wolfenstein & YLW, PRL 73, 2809 (1994).
YLW, Chin. Phys. Lett. 16 339 (1999),
YLW & Y.F. Zhou, Phys.Rev. D61 096001 (2000),
YLW & Y.F. Zhou, Phys.Rev. D64 115018 (2001).
YLW & C. Zhuang, Phys.Rev. D75 115006 (2007).

Origin of Both P & CP Violation

Spontaneous P & CP Violation →
New Physics in Gauge/Higgs/Fermion Sectors

- SU(2)_L x SU(2)_R x U(1)_B-L Gauge Symmetry for Spontaneous P Violation
- **Two Higgs Bi-doublets (2HBDM) Necessary for Spontaneous CP Violation Being Consistent with current experiments**
- **Rich CP violating Sources**
- **Smallness of FCNC via Mechanism of Approximate U(1) Family Symmetries**
- **Existence of Light Higgs Particles due to decoupling of 2HBDM to 2HDM in the limit** $\boxed{v_R \gg \kappa_1,\ \kappa_2,\ w_1,\ w_2,\ v_L}$
- **Gauge Bosons at TeV scale**

YLW & Y.F. Zhou, Sciences in China 81G (2008); arXiv:0709.0042, 2007
J.Y.Liu, L.M. Wang, YLW, Y.F. Zhou, to be published in PRD, 2012

CP Violation in Light Flavors

&

Approximate Chiral Symmetry

&

Dynamically Generated
Spontaneous Symmetry Breaking

&

Chiral Perturbation Theory (ChPT)

Direct CP Violation

- Caused by the interferences among different decay amplitudes
- Nonzero relative strong phases among the decay amplitudes
- Nonzero relative weak CP-violating phase among different decay amplitudes

$$\boxed{\mathbf{a}_{\epsilon''} = \frac{|<\mathrm{f}|\mathrm{H_{eff}}|\mathrm{M}>|^2 - |<\bar{\mathrm{f}}|\mathrm{H_{eff}}|\bar{\mathrm{M}}>|^2}{|<\mathrm{f}|\mathrm{H_{eff}}|\mathrm{M}>|^2 + |<\bar{\mathrm{f}}|\mathrm{H_{eff}}|\bar{\mathrm{M}}>|^2}}$$

$$M \to f \qquad \bar{M} \to \bar{f}$$

Direct CP Violation in Kaon Decays

It arises from both nonzero relative weak and strong phases via the KM mechanism

$$\frac{\varepsilon'}{\varepsilon} = \frac{1}{\sqrt{2}|\varepsilon|} Im\left(\frac{A_2}{A_0}\right) = \frac{\omega}{\sqrt{2}|\varepsilon|}\left(\frac{ImA_2}{ReA_2} - \frac{ImA_0}{ReA_0}\right)$$

Isospin Amplitudes **& ΔI = ½ Rule**

$$A_I e^{i\delta_I} = \langle \pi\pi | \mathcal{H}_{eff}^{\Delta S=1} | K \rangle \equiv \frac{G_F}{\sqrt{2}} \lambda_u \sum_{i=1}^{8} c_i(\mu) \langle Q_i(\mu) \rangle_I$$

Wilson coefficient functions & CKM matrix

$$c_i(\mu) = z_i(\mu) + \tau y_i(\mu)$$

$$\tau = -\lambda_t/\lambda_u \ , \ \lambda_q = V_{qs}^* V_{qd}$$

Hadronic matrix element at low energy

$$\langle Q_i(\mu)\rangle_I \equiv \langle(\pi\pi)_I|Q_i(\mu)|K\rangle$$

$$\mu < \Lambda_\chi = 1GeV$$

$$Q_1 = 4\,\bar{s}_L\gamma^\mu d_L\,\bar{u}_L\gamma_\mu u_L\,,\qquad Q_2 = 4\,\bar{s}_L\gamma^\mu u_L\,\bar{u}_L\gamma_\mu d_L\,,$$

$$Q_3 = 4\sum_q \bar{s}_L\gamma^\mu d_L\,\bar{q}_L\gamma_\mu q_L\,,\qquad Q_4 = 4\sum_q \bar{s}_L\gamma^\mu q_L\,\bar{q}_L\gamma_\mu d_L\,,$$

$$Q_5 = 4\sum_q \bar{s}_L\gamma^\mu d_L\,\bar{q}_R\gamma_\mu q_R\,,\qquad Q_6 = -8\sum_q \bar{s}_L q_R\,\bar{q}_R d_L\,,$$

$$Q_7 = 4\sum_q \frac{3}{2}e_q\,\bar{s}_L\gamma^\mu d_L\,\bar{q}_R\gamma_\mu q_R\qquad Q_8 = -8\sum_q \frac{3}{2}e_q\,\bar{s}_L q_R\,\bar{q}_R d_L$$

THE HARD TASK IS TO CALCULATE THE HADRONIC MATRIX ELEMENTS AT THE LOW ENERGY DUE TO NONPERTUBATIVE STRONG INTERACTION OF QCD

Strong Interaction of QCD & Approximate Chiral Symmetry

Chiral limit: Taking vanishing quark masses $m_q \to 0$.

QCD Lagrangian

$$L_{QCD}^{(o)} = \bar{q}_L \gamma_\mu i D_\mu q_L + \bar{q}_R \gamma_\mu i D_\mu q_R - \frac{1}{4} G_{\mu\nu}^{\alpha} G^{\alpha\mu\nu}$$

$$D_\mu = \partial_\mu - g_s \lambda_\alpha / 2 G_\mu^\alpha$$

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \qquad q_{R,L} = \frac{1}{2}(1 \pm \gamma_5) q$$

has maximum global Chiral symmetry :

$$SU_L(3) \times SU_R(3) \times U_A(1) \times U_B(1)$$

Strong Interaction of QCD & Approximate Chiral Symmetry

- QCD Lagrangian with massive light quarks

$$\mathcal{L}_{\rm QCD} = \bar{q}\gamma^{\mu}(i\partial_{\mu} + g_s G^a_{\mu} T^a)q - \bar{q}Mq - \frac{1}{2}{\rm tr}G_{\mu\nu}G^{\mu\nu}$$

$$q = (u, d, s), \qquad M = {\rm diag.}(m_1, m_2, m_3) \equiv {\rm diag.}(m_u, m_d, m_s)$$

Approximate Global Chiral Symmetry

$$U(3)_L \times U(3)_R, \qquad m_i << \Lambda_{\rm QCD}(i = 1, 2, 3)$$

Instanton Effects via t'Hooft Determination

$$\mathcal{L}^{\rm inst} = \kappa_{\rm inst} e^{i\theta_{\rm inst}} \det(-\bar{q}_R q_L) + h.c., \qquad \kappa_{\rm inst} \sim e^{-8\pi^2/g^2}$$

$$\boxed{U(1)_L \times U(1)_R \to U(1)_V}$$

Effective Lagrangian Based on Loop Regularization

Y.B. Dai and Y-L. Wu, EPJC

Effective Four Quark Interactions-NJL at low energy

$$\mathcal{L}^{4q} = \frac{1}{\mu_f^2}(\bar{q}_{Li} q_{Rj})(\bar{q}_{Rj} q_{Li}) + h.c.$$

Effective Lagrangian for Quarks and Bound States

Integrating over the gluon field and considering the bound state solution

$$\begin{aligned}\mathcal{L}_{\text{eff}}(q, \bar{q}, \Phi) &= \bar{q}\gamma^\mu i\partial_\mu q + \bar{q}_L \gamma_\mu \mathcal{A}_L^\mu q_L + \bar{q}_R \gamma_\mu \mathcal{A}_R^\mu q_R - [\ \bar{q}_L(\Phi - M) q_R + h.c.\ ] \\ &+ 2\mu_f^2 \text{tr}\left(\Phi M^\dagger + M\Phi^\dagger\right) - \mu_f^2 \text{tr}\Phi\Phi^\dagger + \mu_{\text{inst}}(\det\Phi + h.c.)\end{aligned}$$

After integrating out quark fields by the LORE method

Dynamically Generated Spontaneous Symmetry Breaking

Dynamically Generated Effective Potential

$$\begin{aligned} \mathrm{V_{eff}}(\Phi) &= -\mathrm{tr}\hat{\mu}^2_{\mathrm{m}}\left(\Phi \mathrm{M}^\dagger + \mathrm{M}\Phi^\dagger\right) + \frac{1}{2}\mathrm{tr}\hat{\mu}^2_{\mathrm{f}}(\Phi\Phi^\dagger + \Phi^\dagger\Phi) \\ &+ \frac{1}{2}\mathrm{tr}\lambda[\,(\hat{\Phi}\hat{\Phi}^\dagger)^2 + (\hat{\Phi}^\dagger\hat{\Phi})^2\,] - \mu_{\mathrm{inst}}\left(\det\Phi + \mathrm{h.c.}\right) \end{aligned}$$

with $\hat{\mu}^2_f$, $\hat{\mu}^2_m$ and λ the three diagonal matrices

$$\begin{aligned} \hat{\mu}^2_{\mathrm{f}} &= \mu^2_{\mathrm{f}} - \frac{\mathrm{N_c}}{8\pi^2}\left(\mathrm{M_c^2 T_2} + \bar{\mathrm{M}}^2\mathrm{T_0}\right) \\ \hat{\mu}^2_{\mathrm{m}} &= \mu^2_{\mathrm{m}} - \frac{\mathrm{N_c}}{8\pi^2}\left(\mathrm{M_c^2 T_2} + \bar{\mathrm{M}}^2\mathrm{T_0}\right), \quad \lambda = \frac{\mathrm{N_c}}{16\pi^2}\mathrm{T_0} \end{aligned}$$

$$T_0^{(i)}\left(\frac{\mu_i^2}{M_c^2}\right) = \ln\frac{M_c^2}{\mu_i^2} - \gamma_w + y_0\left(\frac{\mu_i^2}{M_c^2}\right)$$

$$T_2^{(i)}\left(\frac{\mu_i^2}{M_c^2}\right) = 1 - \frac{\mu_i^2}{M_c^2}\left[\,\ln\frac{M_c^2}{\mu_i^2} - \gamma_w + 1 + y_2\left(\frac{\mu_i^2}{M_c^2}\right)\,\right]$$

Dynamically Generated Spontaneous Symmetry Breaking

Vacuum Expectation Values (VEVs)

$$\Phi(x) = \xi_L(x)\phi(x)\xi_R^\dagger(x), \quad \phi(x) = V + \varphi(x), \quad V = <\phi> = \mathrm{diag.}(v_1, v_2, v_3)$$

Minimal Conditions/Generalized Gap Equations

$$-(\hat{\mu}_f^2)_i v_i + (\hat{\mu}_m^2)_i m_i - 2\lambda_i \bar{m}_i^3 + \mu_{inst}\bar{v}^3/v_i = 0, \quad i = 1, 2, 3, \quad \bar{v}^3 = v_1 v_2 v_3$$

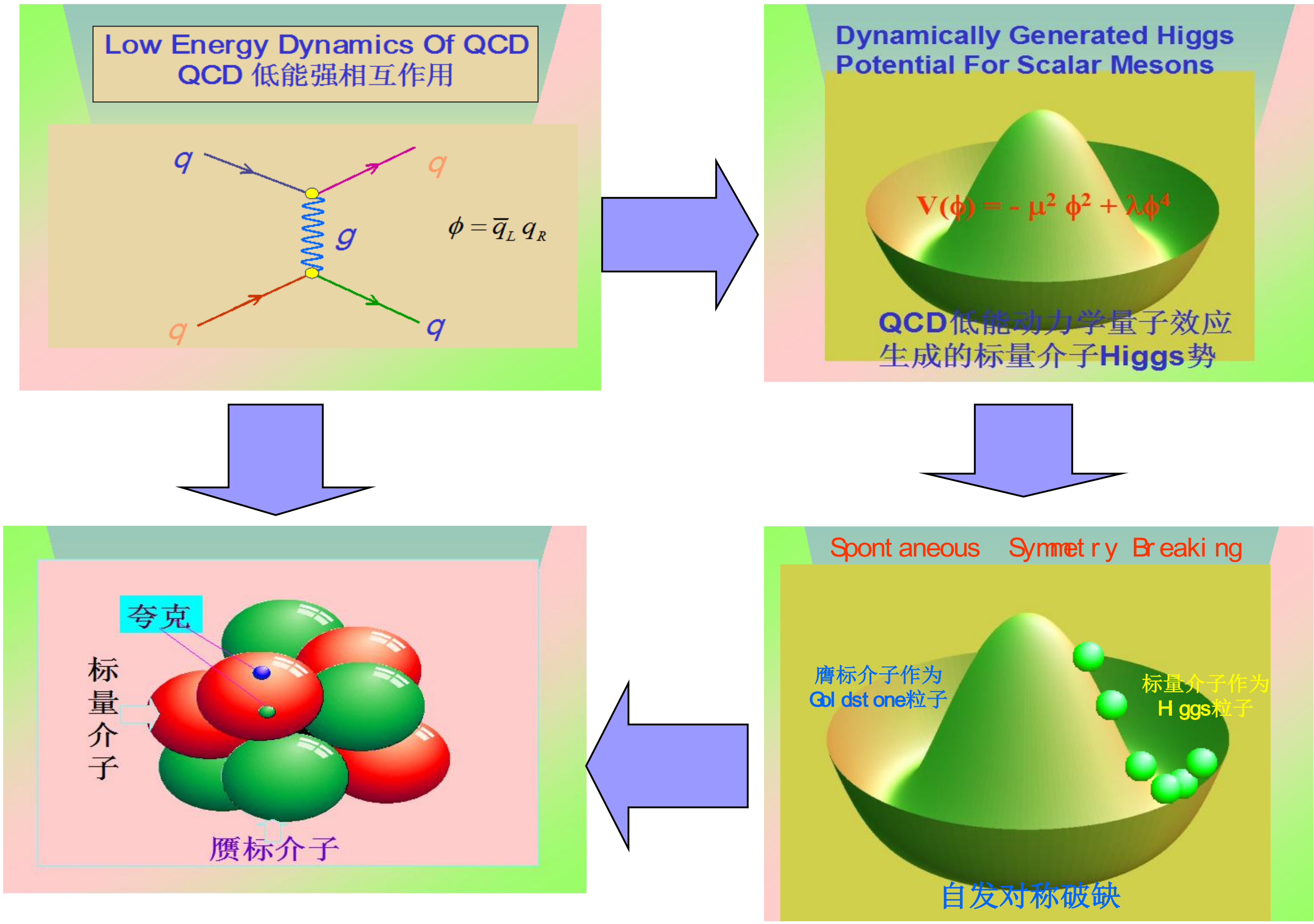
Gap Equation without Instanton ($v_{inst} = 0$)

$$\frac{N_c}{8\pi^2\mu_f^2}\left[\, M_c^2 - \mu_o^2\left(\ln\frac{M_c^2}{\mu_o^2} - \gamma_w + 1 + y_2(\frac{\mu_o^2}{M_c^2})\right)\,\right] = 1$$

Quadratic Term by the LORE method

$$T_2^{(i)}(\frac{\mu_i^2}{M_c^2}) = 1 - \frac{\mu_i^2}{M_c^2}[\, \ln\frac{M_c^2}{\mu_i^2} - \gamma_w + 1 + y_2(\frac{\mu_i^2}{M_c^2})\,]$$

Composite Higgs Fields



Scalars as Partner of Pseudoscalars & The Lightest Composite Higgs Bosons

Scalar mesons:

$$\sqrt{\mathbf{2}}\varphi = \begin{pmatrix} \frac{a_0^0}{\sqrt{2}} + \frac{1}{\sqrt{6}} f_8 + \sqrt{\frac{1}{3}} f_s & a_0^+ & \kappa_0^+ \\ a_0^- & -\frac{a_0^0}{\sqrt{2}} + \frac{1}{\sqrt{6}} f_8 + \sqrt{\frac{1}{3}} f_s & \kappa_0^0 \\ \kappa_0^- & \bar{\kappa}_0^0 & -\frac{2}{\sqrt{6}} f_8 + \sqrt{\frac{1}{3}} f_s \end{pmatrix}$$

Pseudoscalar mesons :

$$\sqrt{\mathbf{2}}\mathbf{\Pi} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{1}{\sqrt{6}} \eta_8 + \sqrt{\frac{1}{3}} \eta_0 & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{1}{\sqrt{6}} \eta_8 + \sqrt{\frac{1}{3}} \eta_0 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta_8 + \sqrt{\frac{1}{3}} \eta_0 \end{pmatrix}$$

Mass Formula

Pseudoscalar mesons :

$$\mathrm{m}^2_{\pi^\pm} \simeq \frac{2\mu_{\mathrm{P}}^3}{\mathrm{f}^2}(\mathrm{m_u}+\mathrm{m_d})$$

$$\mathrm{m}^2_{\mathrm{K}^\pm} \simeq \frac{2\mu_{\mathrm{P}}^3}{\mathrm{f}^2}(\mathrm{m_u}+\mathrm{m_s})$$

$$\mathrm{m}^2_{\mathrm{K}^0} \simeq \frac{2\mu_{\mathrm{P}}^3}{\mathrm{f}^2}(\mathrm{m_d}+\mathrm{m_s})$$

$$\mathrm{m}^2_{\eta_8} \simeq \frac{2\mu_{\mathrm{P}}^3}{\mathrm{f}^2}[\ \frac{1}{3}(\mathrm{m_u}+\mathrm{m_d})+\frac{4}{3}\mathrm{m_s}\ ] = \frac{1}{3}(4\mathrm{m}^2_{\mathrm{K}}-\mathrm{m}^2_{\pi})$$

$$\mathrm{m}^2_{\eta_8\eta_0} \simeq -\frac{2\mu_{\mathrm{P}}^3}{\mathrm{f}^2}\frac{\sqrt{2}}{3}[\ 2\mathrm{m_s}-(\mathrm{m_u}+\mathrm{m_d})\ ] = -\frac{2\sqrt{2}}{3}(\mathrm{m}^2_{\mathrm{K}}-\mathrm{m}^2_{\pi})$$

$$\mathrm{m}^2_{\eta_0} \simeq \frac{2\mu_{\mathrm{P}}^3}{\mathrm{f}^2}\frac{2}{3}(\mathrm{m_u}+\mathrm{m_d}+\mathrm{m_s})+\frac{12\bar{\mathrm{v}}^3}{\mathrm{f}^2}\mu_{\mathrm{inst}} = \frac{1}{3}(2\mathrm{m}^2_{\mathrm{K}}+\mathrm{m}^2_{\pi})+\frac{24\bar{\mathrm{v}}^3}{\mathrm{f}^2}\bar{\lambda}\mathrm{v}_{\mathrm{inst}}$$

$$\mu_{\mathrm{P}}^3 = (\bar{\mu}^2_{\mathrm{m}}+2\bar{\lambda}\mathrm{v}_{\mathrm{o}}^2)\mathrm{v_o} \simeq 12\bar{\lambda}\mathrm{v}_{\mathrm{o}}^3 \simeq 3\mathrm{v_o}\mathrm{f}^2$$

Mixing Angles

$$\tan 2\theta_{\mathrm{P}} = 2\sqrt{2}[1-\frac{9\mathrm{v}_{\mathrm{inst}}\mathrm{v}_3}{\mathrm{m}^2_{\mathrm{K}}-\mathrm{m}_{\pi^2}}]^{-1}$$

Mass Formula

Scalar Mesons - Lightest Composite Higgs Bosons

$$m^2_{a_0^\pm} \simeq m^2_{a_0^0} \simeq 2(2\bar{m}_u + \bar{m}_d)\bar{m}_u + 2v_{inst}v_3 \sim 8v_o^2$$

$$m^2_{k_0^\pm} \simeq 2(2\bar{m}_u + \bar{m}_s)\bar{m}_u + 2v_{inst}v_2 \sim 8v_o^2$$

$$m^2_{k_0^0} \simeq 2(2\bar{m}_d + \bar{m}_s)\bar{m}_d + 2v_{inst}v_1 \sim 8v_o^2$$

$$m^2_{f_8} \simeq \bar{m}_u^2 + \bar{m}_d^2 + 4\bar{m}_s^2 + \frac{2}{3}v_{inst}(2v_1 + 2v_2 - v_3) \sim 8v_o^2$$

$$m^2_{f_s} \simeq 2(\bar{m}_u^2 + \bar{m}_d^2 + \bar{m}_s^2) - \frac{4}{3}v_{inst}(v_1 + v_2 + v_3) \sim 2v_o^2$$

$$m^2_{f_sf_8} \simeq \sqrt{2}(2\bar{m}_s^2 - \bar{m}_u^2 - \bar{m}_d^2) - \frac{\sqrt{2}}{3}v_{inst}(2v_3 - v_1 - v_2) \sim 0$$

Mixing Angles

$$\tan 2\theta_S = \frac{2m^2_{f_sf_8}}{m^2_{f_s} - m^2_{f_8}}$$

Predictions for Mass Spectra & Mixings

Input Parameters

$f_\pi = 94\mathrm{MeV} \qquad v_o = 340\mathrm{MeV}$

$m_u \simeq 3.8\mathrm{MeV} \qquad m_d \simeq 5.7\mathrm{MeV} \qquad m_s/m_d \simeq 20.5$

Output Predictions

$\mu_f \simeq 144\mathrm{MeV}, \qquad \mu_{inst} \simeq 8.0\mathrm{MeV}$

$M_c \simeq 922\mathrm{MeV}, \qquad \mu_s \simeq 333\mathrm{MeV}$

$< \bar{u}u > \simeq < \bar{d}d > \simeq < \bar{s}s > = -(242\mathrm{MeV})^3$

$m_\pi \simeq 139\mathrm{MeV}, \qquad m_\pi|_{exp} \simeq 139\mathrm{MeV}$

$m_{K^0} \simeq 500\mathrm{MeV}, \qquad m_{K^0}|_{exp} \simeq 500\mathrm{MeV}$

$m_{K^\pm} \simeq 496\mathrm{MeV} \qquad m_{K^\pm}|_{exp} \simeq 496\mathrm{MeV}$

$m_\eta \simeq 503\mathrm{MeV}, \qquad m_\eta|_{exp} \simeq 548\mathrm{MeV}$

$m_{\eta'} \simeq 986\mathrm{MeV}, \qquad m_{\eta'}|_{exp} \simeq 958\mathrm{MeV}$

Predictions

$$m_{a_0} \simeq 978 \text{ MeV}, \qquad m_{a_0}^{\text{exp.}} = 984.8 \pm 1.4 \text{ MeV} \quad \text{PDG}$$

$$m_{\kappa_0} \simeq 970 \text{ MeV}, \qquad m_{\kappa_0}^{\text{exp.}} = 797 \pm 19 \pm 43 \text{ MeV} \quad \text{E7912}$$

$$m_{f_0} \simeq 1126 \text{ MeV}, \qquad m_{f_0}^{\text{epx.}} = 980 \pm 10 \text{ MeV} \quad \text{PDG}$$

$$m_{\sigma} \simeq 677 \text{ MeV}, \qquad m_{\sigma}^{\text{exp.}} = (400 - 1200) \text{ MeV} \quad \text{PDG}$$

$$\theta_P \simeq -18^o, \qquad \theta_S \simeq -18^o$$

$$\eta_8 = \cos\theta_P \ \eta + \sin\theta_P \ \eta'$$

$$\eta_0 = \cos\theta_P \ \eta' - \sin\theta_P \ \eta$$

$$f_8 = \cos\theta_S \ f_0 + \sin\theta_S \ \sigma$$

$$f_s = \cos\theta_S \ \sigma - \sin\theta_S \ f_0$$

The Chiral Lagrangian and Chiral Perturbation Theory for characterizing the QCD nonperturbative effects

$$
\begin{aligned}
\mathcal{L}_{\rm eff} = \frac{f^2}{4}\{ & \; tr(D_\mu U^\dagger D^\mu U) + \frac{m_\alpha^2}{4N_c} tr(\ln U^\dagger - \ln U)^2 \\
& + r\frac{\chi_5}{\Lambda_\chi^2} tr\left(D_\mu U^\dagger D^\mu U(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right) + r\; tr(\mathcal{M}U^\dagger + U\mathcal{M}^\dagger) \\
& + r^2\frac{\chi_8}{\Lambda_\chi^2} tr\left(\mathcal{M}^\dagger U \mathcal{M}^\dagger U + \mathcal{M}U^\dagger \mathcal{M}U^\dagger\right) + r^2\frac{\kappa_2}{\Lambda_\chi^2} tr(\mathcal{M}^\dagger\mathcal{M}) \; \}
\end{aligned}
$$

$$\frac{\mathbf{p^2}}{\mathbf{\Lambda_f^2}} \sim \frac{\mathbf{1}}{\mathbf{N_c}} - \textbf{Large } N_c \textbf{ Expansion}$$

$$\frac{\mathbf{p^2}}{\mathbf{\Lambda_\chi^2}} \;\&\; \frac{\mathbf{m_q^2}}{\mathbf{\Lambda_\chi^2}} - \textbf{Momentum/Mass-Expansion}$$

The chiral representation of four quark operators

$$Q_1^\chi + H.c. = -f^4 \; tr\left(\lambda_6 U^\dagger \partial_\mu U\right) tr\left(\lambda^{(1)} U^\dagger \partial^\mu U\right) + O(1/\Lambda_\chi^2)$$

$$Q_2^\chi + H.c. = -f^4 \; tr\left(\lambda_6 U^\dagger \partial_\mu U \lambda^{(1)} U^\dagger \partial^\mu U\right) + O(1/\Lambda_\chi^2) \; ,$$

$$Q_3^\chi + H.c. = -f^4 \; tr\left(\lambda_6 U^\dagger \partial_\mu U\right) tr\left(U^\dagger \partial^\mu U\right) + O(1/\Lambda_\chi^2) \; ,$$

$$Q_4^\chi + H.c. = -f^4 \; tr\left(\lambda_6 \partial_\mu U^\dagger \partial^\mu U\right) + O(1/\Lambda_\chi^2) \; ,$$

$$Q_5^\chi + H.c. = -f^4 \; tr\left(\lambda_6 U^\dagger \partial_\mu U\right) tr\left(U \partial^\mu U^\dagger\right) + O(1/\Lambda_\chi^2) \; ,$$

$$Q_6^\chi + H.c. = +f^4 \; \left(\frac{r^2 \chi_5}{\Lambda_\chi^2}\right) tr\left(\lambda_6 \partial_\mu U^\dagger \partial^\mu U\right) + O(1/\Lambda_\chi^4) \; ,$$

$$Q_7^\chi + H.c. = -\frac{1}{2} Q_5^\chi - \frac{3}{2} f^4 \; tr\left(\lambda_6 U^\dagger \partial_\mu U\right) tr\left(\lambda^{(1)} U \partial^\mu U^\dagger\right) + O(1/\Lambda_\chi^2)$$

$$\begin{aligned} Q_8^\chi + H.c. = & -\frac{1}{2} Q_6^\chi + f^4 r^2 \frac{3}{4} tr\left(\lambda_6 U^\dagger \lambda^{(1)} U\right) \\ & + f^4 r^2 \frac{3}{4} \frac{\chi_5}{\Lambda_\chi^2} tr \lambda_6 \left(U^\dagger \lambda^{(1)} U \partial_\mu U^\dagger \partial^\mu U + \partial_\mu U^\dagger \partial^\mu U U^\dagger \lambda^{(1)} U\right) \\ & + f^4 r^2 \frac{3}{4} \frac{\chi_8}{\Lambda_\chi^2} 2r \; tr \lambda_6 \left(U^\dagger \lambda^{(1)} U \mathcal{M}^\dagger U + U^\dagger \mathcal{M} U^\dagger \lambda^{(1)} U\right) + O(1/\Lambda_\chi^4). \end{aligned}$$

The chiral loop contribution of QCD nonperturbative effects was found to be significant. It is important to keep quadratic terms as proposed by BBG (1986)

$$Q_1(\mu) \to Q_1^\chi(M(\mu)) = Q_1^\chi(M(\mu')) - \frac{2(M^2(\mu) - M^2(\mu'))}{\Lambda_F^2} Q_2^\chi(M(\mu')) \,,$$

$$Q_2(\mu) \to Q_2^\chi(M(\mu)) = Q_2^\chi(M(\mu')) - \frac{2(M^2(\mu) - M^2(\mu'))}{\Lambda_F^2} Q_1^\chi(M(\mu'))$$
$$+ \frac{M^2(\mu) - M^2(\mu')}{\Lambda_F^2} (Q_2^\chi - Q_1^\chi)(M(\mu')) \,,$$

$$Q_4(\mu) \to Q_4^\chi(M(\mu)) = (Q_2^\chi - Q_1^\chi)(M(\mu))$$

$$Q_6(\mu) \to Q_6^\chi(\mu, M(\mu)) = \left(1 + 3(N_c - 1/N_c)\frac{\alpha_s}{4\pi}\ln(\frac{\mu^2}{\mu_\chi^2})\right) Q_6^\chi(\mu_\chi, M(\mu))$$

$$Q_8(\mu) \to Q_8^\chi(\mu, M(\mu)) = \left(1 + 3(N_c - 1/N_c)\frac{\alpha_s}{4\pi}\ln(\frac{\mu^2}{\mu_\chi^2})\right) Q_8^\chi(\mu_\chi, M(\mu))$$

Importance of matching between ChPT & QCD

Scale independence condition

$$\frac{\partial}{\partial \mu}\mathbf{Q}_1(\mu_{\mathrm{Q}}) = \mathbf{0} \quad \Rightarrow \quad \mu\frac{\partial}{\partial \mu}\left(\frac{\mathbf{2M}^2(\mu)}{\mathbf{\Lambda}_{\mathrm{F}}^2}\right) = \frac{\mathbf{3}\alpha_{\mathrm{S}}}{\mathbf{2}\pi}$$

Matching between anomalous dimensions

$$\gamma_i^{Meson} \equiv \mu\frac{\partial}{\partial \mu}Q_i^{\chi}(M(\mu)) = \gamma_i^{Quark} \equiv \mu\frac{\partial}{\partial \mu}Q_i(\mu)$$

$$\Downarrow$$

$$Q_6^{\chi}(\mu_{\chi}, M(\mu)) = -\frac{11}{2}(Q_2^{\chi} - Q_1^{\chi})(M(\mu))$$

Algebraic Relations of Chiral Operators

$$Q_4^{\chi} = \left(Q_2^{\chi} - Q_1^{\chi}\right) \qquad Q_6^{\chi} = -\frac{r^2}{\Lambda_{\chi}^2}\chi_5\left(Q_2^{\chi} - Q_1^{\chi}\right)$$

$$\boxed{\frac{2\mathrm{M}^2(\mu)}{\Lambda_{\mathrm{F}}^2} \simeq \frac{3\alpha_{\mathrm{s}}}{4\pi} + \frac{3\alpha_{\mathrm{s}}}{4\pi}\ln\left(\frac{\mu^2}{\mu_0^2}\right)} \qquad \frac{r^2}{\Lambda_{\chi}^2}\chi_5 = \frac{11}{2}$$

Inputs and Theoretical Uncertainties

$$\boxed{\Lambda_{QCD} = 325 \pm 80 \ \mathrm{MeV}} \quad \boxed{\alpha_{\mathrm{s}}(\mu_0)/2\pi \simeq 0.19^{+0.06}_{-0.05}}$$

$$\boxed{\mathrm{M}_{\chi} \equiv \mathrm{M}(\mu = \Lambda_{\chi} \simeq 1\mathrm{GeV}) \simeq 0.71^{+0.11}_{-0.12}\mathrm{GeV}} \quad \boxed{M(\mu_0) \simeq \mu_0}$$

Theoretical Prediction & Experimental Measurements

Theoretical Prediction:

- ε'/ε ＝（20±4±5）×10^{-4}

(Y.L. Wu Phys. Rev. D64: 016001,2001)

Experimental Results:

- ε'/ε ＝（19.2±2.1）×10^{-4}

(KTeV Collab. Phys. Rev. D 83, 092001, 2011)

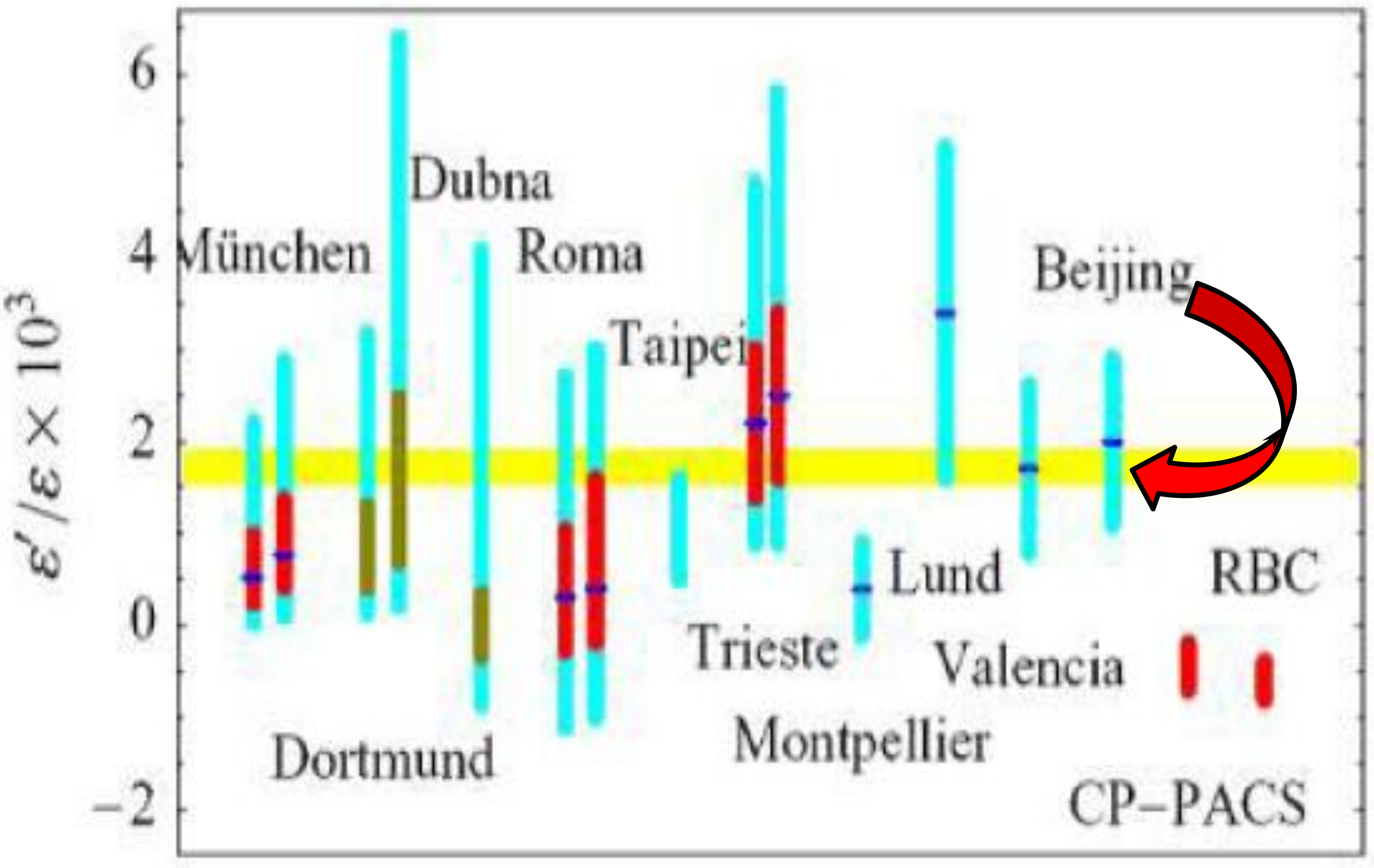
- ε'/ε ＝（20.7±2.8）×10^{-4}

(KTeV Collab. Phys. Rev. D67: 012005,2003)

- ε'/ε ＝（14.7±2.2）×10^{-4}

(NA48 Collab. Phys. Lett. B544: 97,2002)

Direct CP violation ε'/ε in kaon decays can be well explained by the KM CP-violating mechanism in SM



S. Bertolini, Theory Status of ε'/ε FrascatiPhys.Ser.28 275 (2002)

Consistency of Prediction

The consistency of our theoretical prediction is strongly supported from a simultaneous prediction for the ΔI = ½ isospin selection rule of decay amplitudes ($|A_0/A_2|$= 22.5 (exp.) $|A_0/A_2| \approx$ 1.4 (naïve fac.), differs by a factor of 16)

Theoretical Prediction

$$\mathrm{Re}\,A_0 = 3.10^{+0.94}_{-0.61} \times 10^{-4} \qquad \mathrm{Re}\,A_2 = 0.12 \pm 0.02 \times 10^{-4}$$

Experimental Results

$$\mathrm{Re}\,A_0 = 3.33 \times 10^{-4} \qquad \mathrm{Re}\,A_2 = 0.15 \times 10^{-4}$$

CP Violation in Heavy Flavors

&

Approx. Spin-Flavor Symmetry

&

Heavy Quark Effective Field Theory

Effective Hamiltonian of Six Quark Operators

&

Approx. SU(3) Flavor Symmetry

F.Su, YLW, C. Zhuang, Y.B. Yang, Eur. Phys. J. C72 (2012) 1914 ;
J.Phys.G38:015006,2011; Int.J.Mod.Phys.A25:69-111,2010;

Heavy Quark Symmetry

$$m_Q \gg \Lambda_{QCD} \gg m_q \quad (Q = b, c; \quad q = u, d, s)$$

$$P_Q^\mu = m_Q v^\mu + k^\mu, \quad v^2 = 1 \qquad m_Q \to \infty$$

$$\frac{i}{\not{p}_Q - m_Q} \to \frac{i}{v \cdot k} \frac{1 + \not{v}}{2}$$

$$\boxed{\mathcal{L}_{m_Q \to \infty} = \bar{h}_v i v \cdot D h_v}$$

SU(2N) heavy quark spin-flavor symmetry

With N-the number of heavy flavor

Motivation of Effective Six Quark Operators

1. Meson:
   Quark-antiquark bound state;

2. B decays to two light mesons:
   Three quark-antiquark pairs;

3. Leading order:
   One W boson and one gluon exchange;

4. The four quarks via W-boson exchange can be regarded as a local four quark interaction at the energy scale much below the W-boson mass, while two QCD vertexes due to gluon exchange are at the independent space-time points, the resulting effective six quark operators are hence in general nonlocal;

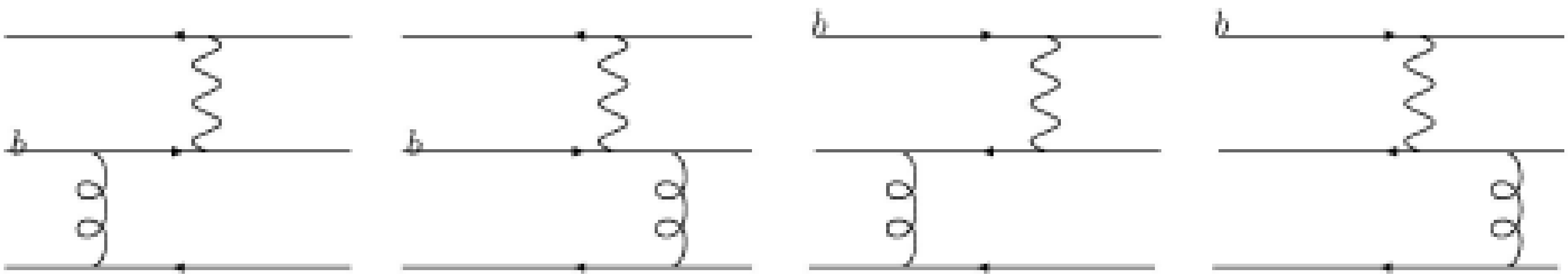
Effective Hamiltonian of Four Quark Operator

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} \lambda_q^s \left[C_1(\mu) O_1^{(q)}(\mu) + C_2(\mu) O_2^{(q)}(\mu) + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right] + \text{h.c.}$$

where $\lambda_q^s = V_{qb} V_{qs}^*$ are products of the CKM matrix elements, $C_i(\mu)$ the Wilson coefficient functions, and $O_i(\mu)$ the four-quark operators

$$\begin{aligned}
O_1^{(q)} &= (\bar{q}_i b_i)_{V-A} (\bar{s}_j q_j)_{V-A} , & O_2^{(q)} &= (\bar{s}_i b_i)_{V-A} (\bar{q}_j q_j)_{V-A} , \\
O_3 &= (\bar{s}_i b_i)_{V-A} \sum_{q'} (\bar{q}'_j q'_j)_{V-A} , & O_4 &= \sum_{q'} (\bar{q}'_i b_i)_{V-A} (\bar{s}_j q'_j)_{V-A} , \\
O_5 &= (\bar{s}_i b_i)_{V-A} \sum_{q'} (\bar{q}'_j q'_j)_{V+A} , & O_6 &= -2 \sum_{q'} (\bar{q}'_i b_i)_{S-P} (\bar{s}_j q'_j)_{S+P} , \\
O_7 &= \frac{3}{2} (\bar{s}_i b_i)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_j q'_j)_{V+A} , & O_8 &= -3 \sum_{q'} e_{q'} (\bar{q}'_i b_i)_{S-P} (\bar{s}_j q'_j)_{S+P} , \\
O_9 &= \frac{3}{2} (\bar{s}_i b_i)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_j q'_j)_{V-A} , & O_{10} &= \frac{3}{2} \sum_{q'} e_{q'} (\bar{q}'_i b_i)_{V-A} (\bar{s}_j q'_j)_{V-A}
\end{aligned}$$

Six Quark Diagrams and Operators



Four different six-quark diagrams with a single W-boson exchange and a single gluon exchange

$$O^{(6)}_{q2} = 4\pi\alpha_s \int\int \frac{d^4k}{(2\pi)^4}\frac{d^4p}{(2\pi)^4} e^{-i((x_1-x_2)p+(x_2-x_3)k)}(\bar{q}'(x_3)\gamma_\nu T^a q'(x_3))\frac{1}{k^2+i\epsilon}$$
$$(\bar{q}_2(x_2)\frac{\not{p}+m_{q_1}}{p^2-m^2_{q_1}+i\epsilon}\gamma^\nu T^a\Gamma_1 q_1(x_1)) * (\bar{q}_4(x_1)\Gamma_2 q_3(x_1)),$$

gluon exchange

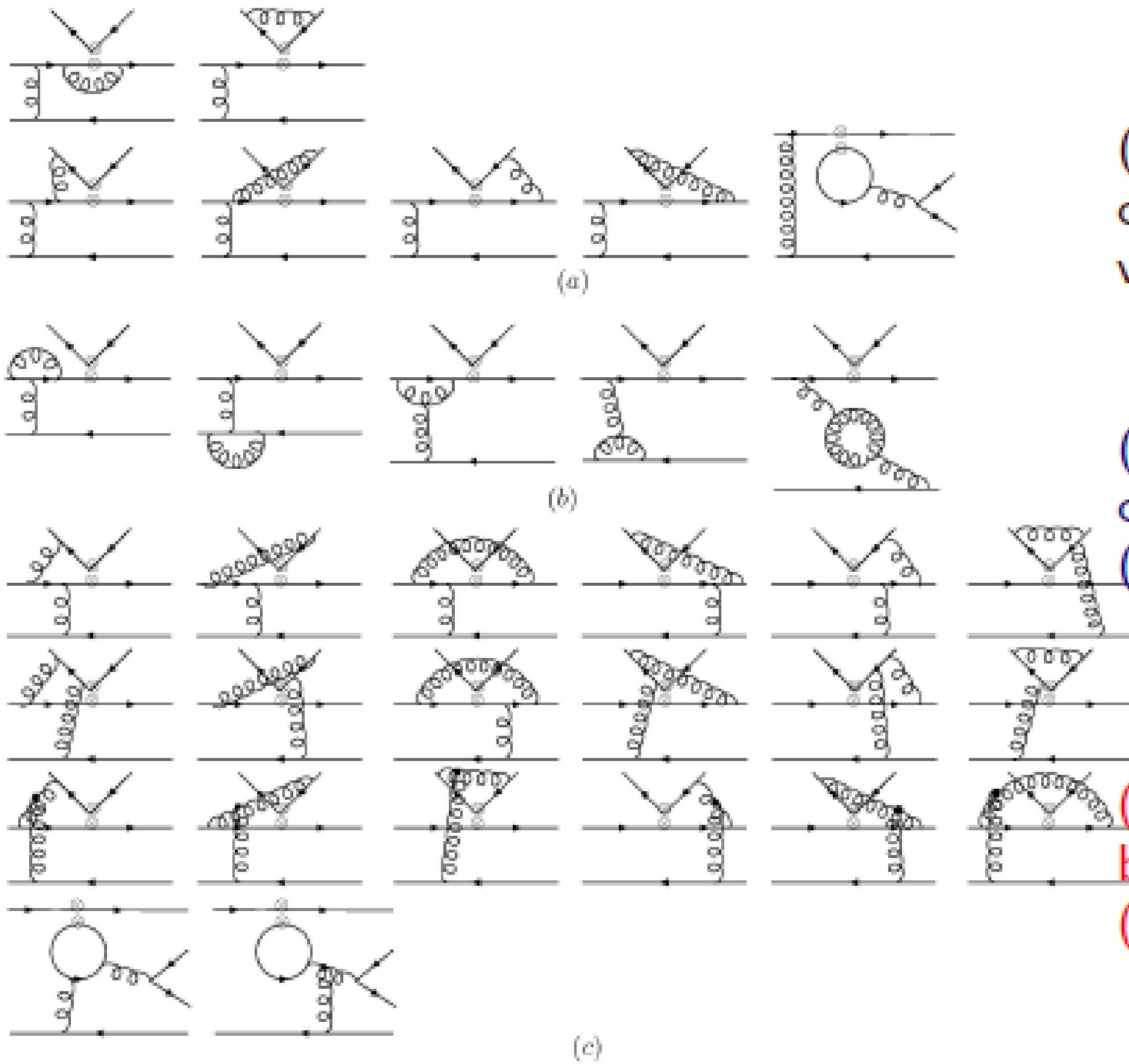
Nambu-Jona-Lasino

$$O \equiv (\bar{q}_2\Gamma_1 q_1) * (\bar{q}_4\Gamma_2 q_3)$$

$$O^{(6)} = \sum_{j=1}^{4} O^{(6)}_{q_j}$$

$$L = \frac{\alpha_s}{\mu^2}\bar{\psi}\psi\bar{\psi}\psi$$

Possible Six Quark Diagrams at One Loop



(a)One-Loop contributions only to the effective weak vertex(type I).

(b)One-Loop contributions only to the gluon vertexes (type II).

(c)One-Loop contributions for both weak and strong vertexes (type III).

Effective Hamiltonian of Six Quark Operators

$$\boxed{\begin{aligned} H_{\rm eff}^{(6)} &= \frac{G_F}{\sqrt{2}}\sum_{j=1}^{4}\{\sum_{q=u,c}\lambda_q^{s(d)}[C_1(\mu)O_{1q_j}^{(q)(6)}(\mu)+C_2(\mu)O_{2q_j}^{(q)(6)}(\mu)] \\ &\quad +\sum_{i=3}^{10}\lambda_t^{s(d)}C_i(\mu)O_{i\ q_j}^{(6)}(\mu)\}+h.c.+\ldots, \end{aligned}}$$

$\boxed{O_{i\ q_j}^{(6)}(\mu)(j=1,2,3,4)}$ are six quark operators which can effectively be obtained from the corresponding four quark operators

$\boxed{O_i(\mu)(i=1-10)}$ are four quark operators at the scale μ, from which nonlocal six quark operators are obtained via the effective gluon exchanging interactions between one of the external quark lines of four quark operators and a spectator quark line at the same scale in the leading approximation.

Hadronic Matrix Element of Six Quark Operators

For illustration, consider the hadronic matrix elements of a typical six-quark operator for $\boxed{B \to \pi^0\pi^0}$

$$\boxed{\begin{aligned} O_{LL}^{(6)} \quad = \quad & \int\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\mathrm{d}^4p}{(2\pi)^4}\, e^{-i((x_1-x_2)p+(x_2-x_3)k)}\frac{1}{k^2}\,\frac{1}{p^2-m_d^2} \\ & [\bar{d}_k(x_2)(\not{p}+m_d)\gamma^\nu T^a_{ki}\gamma^\mu(1-\gamma^5)b_i(x_1)] \\ & [\bar{d}_j(x_1)\gamma_\mu(1-\gamma^5)d_j(x_1)][\bar{d}_m(x_3)\gamma_\nu T^a_{mn}d_n(x_3)], \end{aligned}}$$

$$\boxed{\begin{aligned} & M_{LL}^{O}(B\pi\pi) \equiv <\pi^0\pi^0 \mid O_{LL}^{(6)} \mid \bar{B}_0> \\ & = \int\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\mathrm{d}^4p}{(2\pi)^4}\, e^{-i((x_1-x_2)p+(x_2-x_3)k)}\frac{1}{k^2}\,\frac{1}{p^2-m_d^2} \\ & <\pi^0\pi^0 \mid [\bar{d}_k(x_2)(\not{p}+m_d)\gamma^\nu T^a_{ki}\gamma^\mu(1-\gamma^5)b_i(x_1)] \\ & [\bar{d}_j(x_1)\gamma_\mu(1-\gamma^5)d_j(x_1)][\bar{d}_m(x_3)\gamma_\nu T^a_{mn}d_n(x_3)] \mid \bar{B}_0> \\ & \equiv M_{LL}^{O(1)} + M_{LL}^{O(2)} + M_{LL}^{O(3)} + M_{LL}^{O(4)}, \end{aligned}}$$

The advantage of Six Quark Operator for the evaluation of hadronic matrix elements of two body decays is: Applicable of Naïve QCD Factorization

$$M_{LL}^{O(1)} = \int\int \frac{d^4k}{(2\pi)^4}\frac{d^4p}{(2\pi)^4} e^{-i((x_1-x_2)p+(x_2-x_3)k)} \frac{1}{k^2(p^2-m_d^2)} T^a_{ki} T^a_{mn}$$

$$[(\not p + m_d)\gamma^\nu\gamma^\mu(1-\gamma^5)]_{\rho\sigma}[\gamma_\mu(1-\gamma^5)]_{\alpha\beta}[\gamma_\nu]_{\gamma\delta} M_{Bim}^{\sigma\gamma}(x_1,x_3) M_{\pi nk}^{\delta\rho}(x_3,x_2) M_{\pi jj}^{\beta\alpha}(x_1,x_1)$$

$$M_{Bnm}^{\beta\alpha}(x_i,x_j) = \langle 0 \mid \bar d_m^\alpha(x_j) b_n^\beta(x_i) \mid \bar B^0(P_B) \rangle$$

$$= -\frac{iF_B}{4}\frac{\delta_{mn}}{N_c}\int_0^1 du\, e^{-i(u P_B^+ x_j + (P_B - u P_B^+) x_i)} M_B^{\beta\alpha}(u, P_B)$$

$$M_{\pi nm}^{\beta\alpha}(x_i,x_j) = \langle \pi^0(P) \mid \bar d_m^\alpha(x_j) d_m^\beta(x_i) \mid 0 \rangle$$

$$= \frac{iF_\pi}{4}\frac{\delta_{mn}}{N_c}\int_0^1 dx\, e^{-i(x P x_j + (1-x) P x_i)} M_\pi^{\beta\alpha}(x, P),$$

F_M $(M = B, \pi)$ the decay constants.

$M_B^{\beta\alpha}(u, P_B)$ and $M_\pi^{\beta\alpha}(x, P)$

spin structures for the bottom meson and light meson π

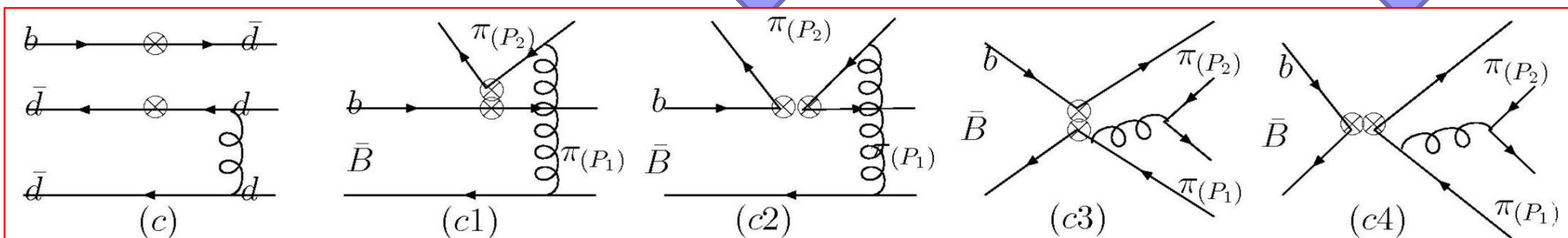
Four Kinds of Hadronic Matrix Elements for each Six Quark Operator with Naïve QCD Factorization

Four different ways of reducing the hadronic matrix element of six quark operator via the naïve QCDF

$M_{LL}^{O(i)}\,(i=1,2,3,4)$

Non-factorizable color suppressed

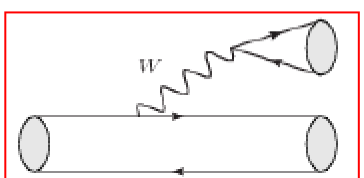
Factorizable annihilation

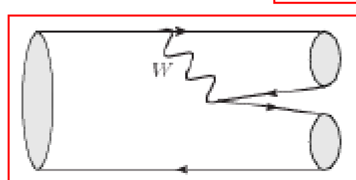


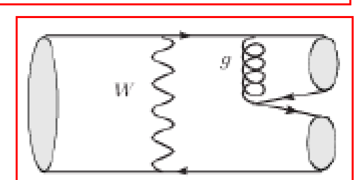
Six Quark Operator

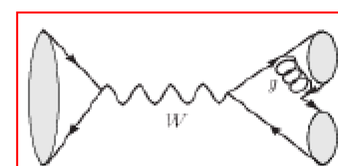
Factorizable

Non-factorizable Annihilation or W-exchange









Four Kinds of Hadronic Matrix Elements for each Six Quark Operator with Naïve QCD Factorization

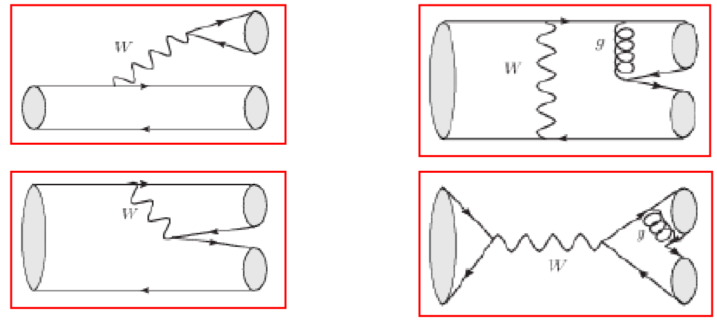
$$M_{LL}^{O}(B\pi\pi) = < \pi^0\pi^0 \mid O_{LL}^{(6)} \mid \bar{B}_0 > = \int_0^1\int_0^1\int_0^1 du\, dx\, dy$$
$$\{\frac{1}{(u P_B^{+} - (1-x)P_1)^2}[\frac{M_{LL}^{(1)}}{(P_1 - u P_B^{+})^2 - m_d^2} + \frac{M_{LL}^{(2)}}{((1-x)P_1 + y P_2 - u P_B^{+})^2 - m_d^2}]$$
$$+ \quad \frac{1}{(xP_1 + (1-y)P_2)^2}[\frac{M_{LL}^{(3)}}{(x P_1 + P_2)^2 - m_d^2} + \frac{M_{LL}^{(4)}}{(x P_1 + (1-y)P_2 - u P_B^{+})^2 - m_d^2}]\},$$

$\boxed{M_{LL}^{(i)}(i=1,2,3,4)}$ are obtained by performing the trace of matrices and determined by the distribution amplitudes.

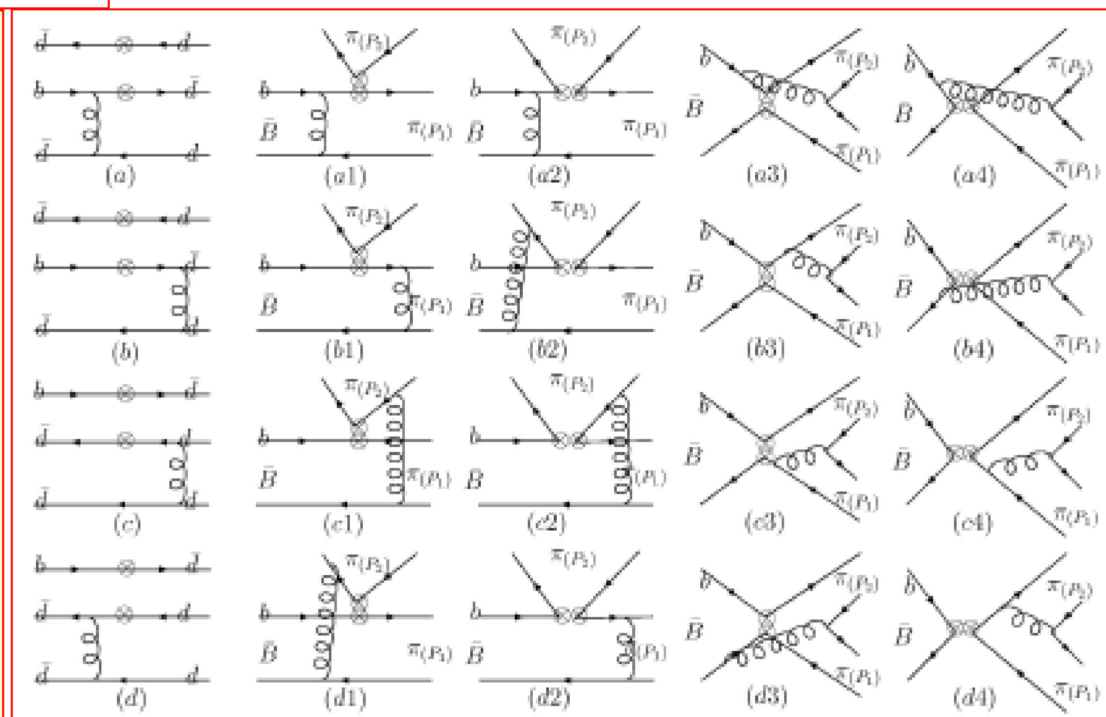
$$M_{LL}^{(1)} = \frac{C_F}{N_C} * F_B\ F_\pi^2 \mathrm{Tr}[M_{\mathrm{B}}(u, P_B)\gamma_\nu M_\pi(x, P_1)\gamma^\nu(\not{P}_1 - u \not{P}_B^{+} + m_d)\gamma_\mu(1-\gamma^5)]$$
$$\mathrm{Tr}[M_\pi(y, P_2)\gamma^\mu(1-\gamma^5)] = i\frac{C_F}{4N_C}F_B\ F_\pi^2\phi_B(u)m_B^3\mu_\pi\phi_\pi(y)\phi_\pi^p(x),$$

Hadronic Matrix Elements
Four Quark Operator vs Six Quark Operator

There are 4 types of diagrams corresponding to each four quark operator, which contribute to the hadronic matrix element of two body hadronic decays



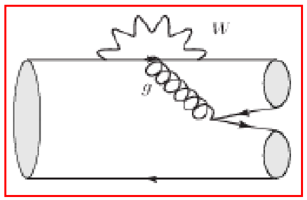
For each four quark operator, it induces **4** kinds of effective six quark operators which lead to **16** types of diagrams for hadronic two body decays of heavy meson via naïve QCD factorization

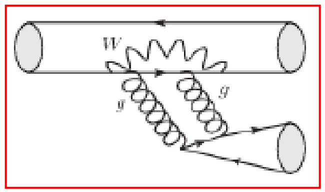


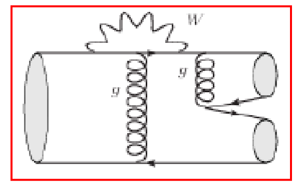
Sixteen Types of Hadronic Matrix Elements Induced From Each four Quark Operator

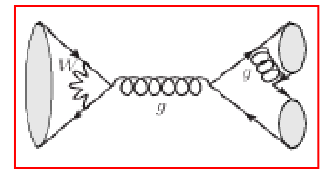
There are two additional vertex operators due to pinguin

$$(V-A)\times(V-A) \text{ or } (LL),\ (V-A)\times(V+A) \text{ or } (LR),\ (S-P)\times(S+P) \text{ or } (SP)$$









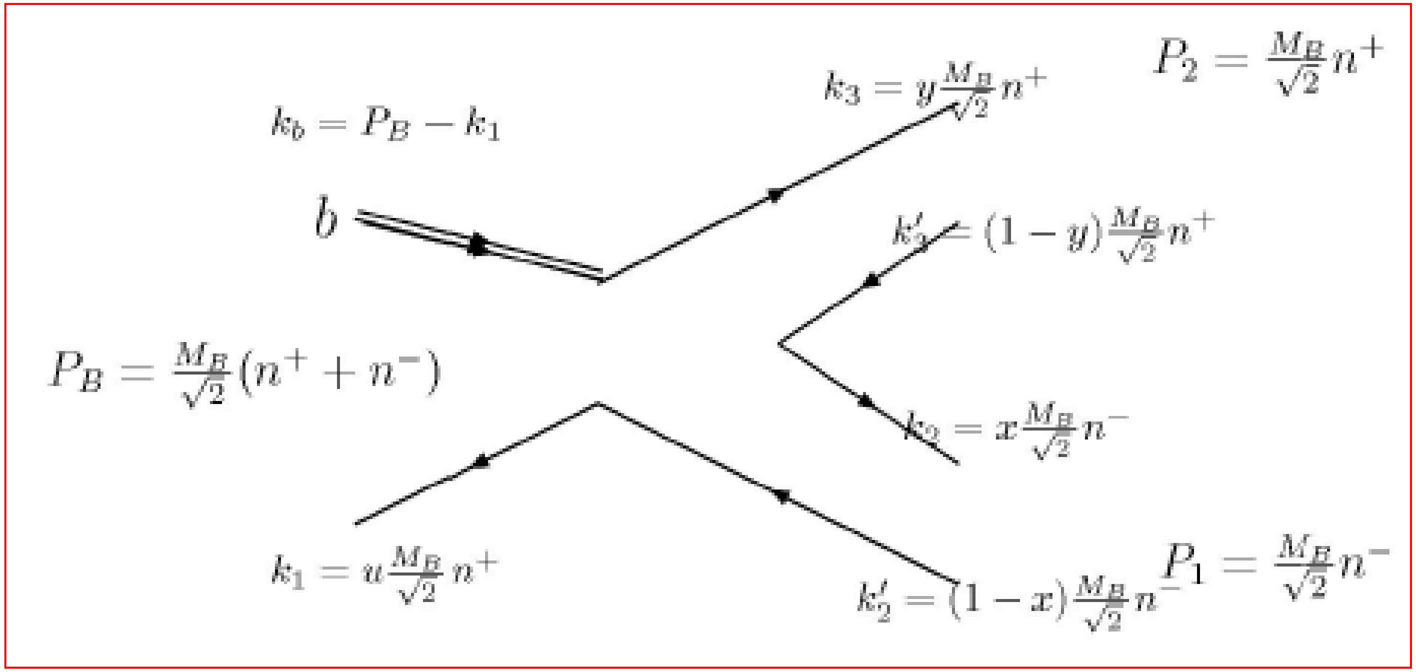
3x16=48 kinds of hadronic matrix elements with relations

$$\begin{array}{llllllllllll}
M^{a1}_{LL} & = & T^F_{LLa}; & M^{a2}_{LL} & = & T^F_{LLa}/N_C; & M^{a3}_{LL} & = & A^N_{LLa}/N_C; & M^{a4}_{LL} & = & 0; \\
M^{a1}_{LR} & = & T^F_{LRa}; & M^{a2}_{LR} & = & T^F_{SPa}/N_C; & M^{a3}_{LR} & = & A^N_{SPa}/N_C; & M^{a4}_{LR} & = & 0; \\
M^{a1}_{SP} & = & T^F_{SPa}; & M^{a2}_{SP} & = & T^F_{LRa}/N_C; & M^{a3}_{SP} & = & A^N_{LRa}/N_C; & M^{a4}_{SP} & = & 0; \\
M^{b1}_{LL} & = & T^F_{LLb}; & M^{b2}_{LL} & = & T^N_{LLb}/N_C; & M^{b3}_{LL} & = & A^F_{LLb}/N_C; & M^{b4}_{LL} & = & 0; \\
M^{b1}_{LR} & = & T^F_{LRb}; & M^{b2}_{LR} & = & T^N_{SPb}/N_C; & M^{b3}_{LR} & = & A^F_{SPb}/N_C; & M^{b4}_{LR} & = & 0; \\
M^{b1}_{SP} & = & T^F_{LLb}; & M^{b2}_{SP} & = & T^N_{LRb}/N_C; & M^{b3}_{SP} & = & A^F_{LRb}/N_C; & M^{b4}_{SP} & = & 0; \\
M^{c1}_{LL} & = & 0; & M^{c2}_{LL} & = & T^N_{LLa}/N_C; & M^{c3}_{LL} & = & A^F_{LLa}/N_C; & M^{c4}_{LL} & = & A^F_{LLa}; \\
M^{c1}_{LR} & = & 0; & M^{c2}_{LR} & = & T^N_{SPa}/N_C; & M^{c3}_{LR} & = & A^F_{SPa}/N_C; & M^{c4}_{LR} & = & A^F_{LRa}; \\
M^{c1}_{SP} & = & 0; & M^{c2}_{SP} & = & T^N_{LRa}/N_C; & M^{c3}_{SP} & = & A^F_{LRa}/N_C; & M^{c4}_{SP} & = & A^F_{SPa}; \\
M^{d1}_{LL} & = & 0; & M^{d2}_{LL} & = & T^F_{LLb}/N_C; & M^{d3}_{LL} & = & A^N_{LLb}/N_C; & M^{d4}_{LL} & = & A^F_{LLb}; \\
M^{d1}_{LR} & = & 0; & M^{d2}_{LR} & = & T^F_{SPb}/N_C; & M^{d3}_{LL} & = & A^N_{SPb}/N_C; & M^{d4}_{LL} & = & A^F_{LRb}; \\
M^{d1}_{SP} & = & 0; & M^{d2}_{SP} & = & T^F_{LRb}/N_C; & M^{d3}_{LL} & = & A^N_{LRb}/N_C; & M^{d4}_{LL} & = & A^F_{SPb}.
\end{array}$$

Treatment of Singularities

- Infrared divergence of gluon exchanging interaction
- On mass-shell divergence of internal quark propagator

Definition of momentum in $B \to M_1 M_2$ decay.



$\boxed{(n^+, n^-, \vec{k}_\perp)}$ *The light-cone coordinate*

Treatment of Singularities

Use dynamics masses as infrared cut-off for both gluon and light quark to treat the infrared divergence and investigate the cut-off dependence.

$$\boxed{\frac{1}{k^2}\frac{\not{p}+m_q}{(p^2-m_q^2)} \quad \rightarrow \quad \frac{1}{(k^2-\mu_g^2+i\epsilon)}\frac{\not{p}+\mu_q}{(p^2-\mu_q^2+i\epsilon)}}$$

Applying the Cutkosky rule to deal with a physical-region singularity for the propagators with following formula:

$$\boxed{\frac{1}{p^2-m_q^2+i\epsilon}=P\left[\frac{1}{p^2-m_q^2}\right]-i\pi\delta[p^2-m_q^2],}$$

It is known as the principal integration method, and the integration with the notation of capital letter P is the so-called principal integration

$\boxed{\mu_g,\mu_q}$ energy scale play the role of infrared cut-off with preserving gauge symmetry and translational symmetry of original theory. Use of this effective gluon propagator is supported by the lattice and the field theoretical studies.

For illustration, applying the prescription to the previously examined amplitude, we have

$$M^O_{LL}(B\pi\pi) =< \pi^0\pi^0 \mid O^{(6)}_{LL} \mid \bar{B}_0 >= \int_0^1 \int_0^1 \int_0^1 du\, dx\, dy$$

$$\{ \frac{1}{(u P_B^+ - (1-x)P_1)^2 - \mu_g^2 + i\epsilon} [\frac{M^{(1)}_{LL}}{(P_1 - u P_B^+)^2 - m_q^2 + i\epsilon}$$

$$+ \frac{M^{(2)}_{LL}}{((1-x)P_1 + y P_2 - u P_B^+)^2 - m_q^2 + i\epsilon}]$$

$$+ \frac{1}{(xP_1 + (1-y)P_2)^2 - \mu_g^2 + i\epsilon} [\frac{M^{(3)}_{LL}}{(x P_1 + P_2)^2 - m_q^2 + i\epsilon}$$

$$+ \frac{M^{(4)}_{LL}}{(x P_1 + (1-y)P_2 - u P_B^+)^2 - m_q^2 + i\epsilon}]\}.$$

For example, the hadronic matrix element of factorizable emission diagram with (V-A) x (V-A) vertex is given by

$$\begin{aligned} T_{LLa}^{FM_1M_2}(M) \quad = \quad & i\frac{1}{4}\frac{C_F}{N_C}\; F_M\; F_{M_1}\; F_{M_2} \int_0^1 \int_0^1 \int_0^1 du\, dx\, dy\; m_B^2 \phi_M(u) \\ & \{m_B(2m_b - m_B x)\phi_{M_1}(x) + \mu_{M_1}(2m_B x - m_b) \\ & [\phi_{M_1}^p(x) - \phi_{M_1}^T(x)]\}\phi_{M_2}(y) h_{Ta}^F(u,x), \end{aligned}$$

The nonlocal effect from the propagators of gluon and quark

$$h_{Ta}^F(u,x) = \frac{1}{(-u(1-x)m_B^2 - \mu_g^2 + i\epsilon)(x m_B^2 - m_b^2 + i\epsilon)},$$

Vertex Corrections

$$C_{2n}(\mu) + \frac{C_{2n-1}}{N_C}(\mu) \to C_{2n}(\mu) + \frac{C_{2n-1}}{N_C}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_F \frac{C_{2n-1}(\mu)}{N_c} V_{2n}(M_2)$$

$$C_{2n-1}(\mu) + \frac{C_{2n}}{N_C}(\mu) \to C_{2n-1}(\mu) + \frac{C_{2n}}{N_C}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_F \frac{C_{2n}(\mu)}{N_c} V_{2n-1}(M_2)$$

The vertex corrections are proposed to improve the scale dependence of Wilson coecient functions of factorizable emission amplitudes in QCDF

$$V_i(M) = \begin{cases} 12\ln(\frac{m_b}{\mu}) - 18 + \int_0^1 dx\, \phi_a(x)\, g(x)\,, & \text{for } i = 1-4, 9, 10 \\ -12\ln(\frac{m_b}{\mu}) + 6 - \int_0^1 dx\, \phi_a(1-x)\, g(1-x)\,, & \text{for } i = 5, 7\,, \\ -6 + \int_0^1 dx\, \phi_b(x)\, h(x)\,, & \text{for } i = 6, 8\,, \end{cases}$$

$\phi_a(x)$	Leading-twist for a pseudoscalar meson
$\phi_b(x)$	**Twist-3 distribution amplitudes** for a pseudoscalar meson

Non-perturbative non-local effects

$$a_i \to a_i^{eff} = C_i(\mu) + \frac{C_{i\pm1}}{N_C}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_F \frac{C_{i\pm1}(\mu)}{N_c}(V_i(M_2) + \widetilde{V}_1(M_2))$$ $(i = 1-4, 9, 10)$

$$a_i \to a_i^{eff} = C_i(\mu) + \frac{C_{i\pm1}}{N_C}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_F \frac{C_{i\pm1}(\mu)}{N_c}(V_i(M_2) + \widetilde{V}_2(M_2))$$ $(i = 5-8)$

$\widetilde{V}_1(M_2)$ **(V-A) x (V-A) structure**

$\widetilde{V}_2(M_2)$ **(V-A) x (V+A) structure**

They depend on properties of mesons and could be caused from the higher order non-perturbative non-local effects.

$\widetilde{V}_1(P) = 26e^{-\frac{\pi}{3}i}$, $\widetilde{V}_2(P) = -26$,

$\widetilde{V}_1(V) = 15e^{\frac{\pi}{8}i}$, $\widetilde{V}_2(V) = -15e^{\frac{\pi}{8}i}$

Both the branching ratios and CP asymmetries of B → PP; PV; VV decay modes are improved.

Strong Phase Effects

When the virtual particles in the Feynman diagram become on mass-shell, it will lead to an imaginary part for the decay amplitudes, which generates the strong phase of the process.

The calculation of strong phase from nonperturbative QCD effects is a hard task, there exist no efficient approaches to evaluate reliably the strong phases caused from nonperturbative QCD effects, so we set strong phases of $\boxed{\tilde{V}_1(M_2)}$ and $\boxed{\tilde{V}_2(M_2)}$ as input parameter.

Taking different strong phases for annihilation processes in B → PP; PV; VV decay modes to get reasonable results.

Nonperturbative Corrections via Annihilation

Annihilation contributions are mainly from factorizable annihilation diagrams with (S- P) x (S + P) effective four-quark vertex

$$A_{SP}^{P_1P_2}(M) \sim \int dxdy \frac{(\mu_{P_1} + \mu_{P_2})y(1-y)}{(x(1-y)m_B^2 - \mu_g^2 + i\epsilon)((1-y)m_B^2 - m_q^2 + i\epsilon)},$$

$$A_{SP}^{P_1V_2}(M) \sim \int dxdy \frac{(\mu_{P_1} - 3(2x-1)m_{V_2})y(1-y)}{(x(1-y)m_B^2 - \mu_g^2 + i\epsilon)((1-y)m_B^2 - m_q^2 + i\epsilon)},$$

$$A_{SP}^{V_1P_2}(M) \sim \int dxdy \frac{(-3(1-2x)m_{V_1} - \mu_{P_2})y(1-y)}{(x(1-y)m_B^2 - \mu_g^2 + i\epsilon)((1-y)m_B^2 - m_q^2 + i\epsilon)},$$

$$A_{SP}^{V_1V_2}(M) \sim \int dxdy \frac{3(1-2x)(-m_{V_1} + 3(2x-1)m_{V_2})y(1-y)}{(x(1-y)m_B^2 - \mu_g^2 + i\epsilon)((1-y)m_B^2 - m_q^2 + i\epsilon)}.$$

1. Since the contributions of these amplitudes are dominated by the area $x \sim 0$ or $y \sim 1$, $A_{SP}^{P_1P_2}(M)$ and $A_{SP}^{P_1V_2}(M)$ have the same sign, while $A_{SP}^{V_1P_2}(M)$ and $A_{SP}^{V_1V_2}(M)$ have a different sign from $A_{SP}^{P_1P_2}(M)$
2. As a result, we use the same strong phase for $A_{SP}^{P_1P_2}(M)$ and $A_{SP}^{P_1V_2}(M)(\theta_1^a \sim 5^o)$, and another one for $A_{SP}^{V_1P_2}(M)$ and $A_{SP}^{V_1V_2}(M)(\theta_1^a \sim 60^o)$.

Input Parameters

H. Y. Cheng and K. C. Yang, Phys. Lett. B 511, 40 (2001); Phys. Rev. D 64, 074004 (2001)

B meson wave function

$$\phi_B(x) = N_B x^2(1-x)^2 \exp\left[-\frac{1}{2}\left(\frac{x m_B}{\omega_B}\right)^2\right]$$

$$\omega_B = 0.25 GeV$$

Light-cone distribution amplitudes for light mesons

$$\phi_P(x,\mu) = 6x(1-x)[1+\sum_{n=1}^{\infty} a_n^P(\mu) C_n^{3/2}(2x-1)]$$

$$\phi_V(x,\mu) = 6x(1-x)[1+\sum_{n=1}^{\infty} a_n^V(\mu) C_n^{3/2}(2x-1)]$$

$$\phi_V^T(x,\mu) = 6x(1-x)[1+\sum_{n=1}^{\infty} a_n^{T,V}(\mu) C_n^{3/2}(2x-1)]$$

Twist-2

Cn(x) the Gegenbauer polynomials

Input Parameters

Light-cone distribution amplitudes for light mesons

$$\phi_p(x,\mu) = 1, \qquad \phi_\sigma(x,\mu) = 6x(1-x)$$

$$\phi_\nu(x,\mu) = 3\left[2x-1+\sum_{n=1}^{\infty} a_n^{T,V}(\mu) P_{n+1}(2x-1)\right]$$

$$\phi_+(x) = 3(1-x)^2, \qquad \phi_-(x) = 3x^2,$$

Twist-3

Pn(x) the Legendre polynomials

Values of Gegenbauer moments at the scale μ=1 GeV and at μ=1.5GeV via a running

	μ	π	K	ρ	K^*	ϕ	ω
a_1	1.0	–	0.06 ± 0.03	–	0.03 ± 0.02	–	–
	1.5	–	0.05 ± 0.03	–	0.03 ± 0.02	–	–
a_2	1.0	0.25 ± 0.15	0.25 ± 0.15	0.15 ± 0.07	0.11 ± 0.09	0.15 ± 0.07	0.18 ± 0.08
	1.5	0.20 ± 0.12	0.20 ± 0.12	0.12 ± 0.05	0.09 ± 0.07	0.12 ± 0.05	0.14 ± 0.06
a_1^T	1.0	–	–	–	0.04 ± 0.03	–	–
	1.5	–	–	–	0.03 ± 0.03	–	–
a_2^T	1.0	–	–	0.14 ± 0.06	0.10 ± 0.08	0.16 ± 0.06	0.14 ± 0.07
	1.5	–	–	0.11 ± 0.05	0.08 ± 0.06	0.13 ± 0.05	0.11 ± 0.05

Input Parameters

CKM matrix elements with Wolfenstein parametrization

$$A = 0.798^{+0.023}_{-0.017},\ \lambda = 0.2252^{+0.00083}_{-0.00082},\ \bar{\rho} = 0.141^{+0.035}_{-0.021},\ \bar{\eta} = 0.340 \pm 0.016$$

Running scale

$$\mu = 1.5 \pm 0.1\mathrm{GeV} \sim \sqrt{2\Lambda_{QCD} m_b}.$$ $\Rightarrow$ $$\alpha_s(\mu)$$

$$\Lambda_{QCD} \simeq 288^{+21}_{-18}\mathrm{MeV}$$ $\Leftarrow$ $$\alpha_s(M_z) = 0.1172 \pm 0.002.$$

$$m_q(\mu) \;=\; \mathcal{R}(\alpha_s(\mu))\hat{m}_q$$

$$\mathcal{R}(\alpha_s) \;=\; \left(\frac{\alpha_s}{\pi}\right)^{\gamma_0/\beta_0}\left[1 + \frac{\alpha_s}{\pi} c_1\right]$$

$$m_b(\mu) \simeq 5.54\,GeV.$$

$$\mu = 1.5\,GeV.$$

Cut-off scale

$$\mu_q = \mu_g = 0.37\,GeV.$$

The infrared cut-off scale for gluon and light quarks are the basic scale to determine annihilation diagram contributions

Input Parameters

The hadronic input parameters:
Life-time, masses and decay constants

$\tau_{B^\pm}$	τ_{B_d}	m_B	m_b	m_t	m_u	m_d
1.638ps	1.525ps	5.28GeV	4.4GeV	173.3GeV	4.2MeV	7.6MeV
m_c	m_s	$m_{\pi^\pm}$	m_{π^0}	m_K	m_{ρ^0}	$m_{\rho^\pm}$
1.5GeV	0.122GeV	0.140GeV	0.135GeV	0.494GeV	0.775GeV	0.775GeV
m_ω	m_ϕ	$m_{K^{*\pm}}$	$m_{K^{*0}}$	μ_π	μ_K	f_ϕ
1.7GeV	1.8GeV	300MeV	0.78GeV	1.02GeV	0.892GeV	0.215GeV
f_B	f_π	f_K	f_ρ	f_ω	f_{K^*}	f_ω^T
0.210GeV	0.130GeV	0.16GeV	0.216GeV	0.187GeV	0.220GeV	0.151GeV
$f_{K^*}^T$	f_ϕ^T	f_ρ^T				
0.185GeV	0.186GeV	0.165GeV				

Numerical Results: Form Factors

The method developed based on the six quark effective Hamiltonian allows us to calculate the relevant transition form factors B→M via a simple factorization approach.

$$F_0^{B\to M_1} = \frac{4\pi\alpha(\mu)C_F}{N_c m_B^2 F_{M_2}} T_{LL}^{FM_1M_2}(B)(M_1, M_2 = P),$$

$$V^{B\to M_1} = \frac{4\pi\alpha(\mu)C_F}{N_c m_B^2 F_{M_2}} T_{LL,\perp}^{FM_1M_2}(B)\frac{m_B^2(m_B + m_{M_1})}{m_{M_2}(m_B^2 - m_{M_1}m_{M_2})}(M_1, M_2 = V),$$

$$A_0^{B\to M_1} = \frac{4\pi\alpha(\mu)C_F}{N_c m_B^2 F_{M_2}} T_{LL}^{FM_1M_2}(B)(M_1 = V, M_2 = P),$$

$$A_1^{B\to M_1} = \frac{4\pi\alpha(\mu)C_F}{N_c m_B^2 F_{M_2}} T_{LL,//}^{FM_1M_2}(B)\frac{m_B^2}{m_{M_2}(m_B + m_{M_1})}(M_1, M_2 = V)$$

$$T_{LL,\perp} = \frac{1}{2}(T_{LL,+} - T_{LL,-}).$$

$$C_F = \frac{N_c^2 - 1}{2N_c},$$

Numerical Results: Form Factors

Relevant transition form factors B_s→M

$$F_0^{B_s\to M_1} = \frac{4\pi\alpha_s(\mu)C_F}{N_c m_{B_s}^2 F_{M_2}} T_{LL}^{FM_1M_2}(B_s)(M_1, M_2 = P),$$

$$V^{B_s\to M_1} = \frac{4\pi\alpha_s(\mu)C_F}{N_c m_{B_s}^2 F_{M_2}} T_{LL,\perp}^{FM_1M_2}(B_s)\frac{m_{B_s}^2(m_{B_s}+m_{M_1})}{m_{M_2}(m_{B_s}^2 - m_{M_1}m_{M_2})}(M_1, M_2 = V),$$

$$A_0^{B_s\to M_1} = \frac{4\pi\alpha_s(\mu)C_F}{N_c m_{B_s}^2 F_{M_2}} T_{LL}^{FM_1M_2}(B_s)(M_1 = V, M_2 = P),$$

$$A_1^{B_s\to M_1} = \frac{4\pi\alpha_s(\mu)C_F}{N_c m_{B_s}^2 F_{M_2}} T_{LL,//}^{FM_1M_2}(B_s)\frac{m_{B_s}^2}{m_{M_2}(m_{B_s}+m_{M_1})}(M_1, M_2 = V),$$

Numerical Results: Form Factors

The B → P, V form factors at q^2 = 0. In comparison with QCD Sum Rules, HQEFT QCD Sum Rules, Light Cone QCD Sum Rules.

Mode	F(0)	QCDSR	LC	LC(HQEFT)	PQCD	This work
$B \to K^*$	V	0.411	0.339	0.331	0.406	0.277
	A_0	0.374	0.283	0.280	0.455	0.328
	A_1	0.292	0.248	0.274	0.30	0.220
$B \to \rho$	V	0.323	0.298	0.289	0.318	0.233
	A_0	0.303	0.260	0.248	0.366	0.280
	A_1	0.242	0.227	0.239	0.25	0.193
$B \to \omega$	V	0.293	0.275	0.268	0.305	0.206
	A_0	0.281	0.240	0.231	0.347	0.251
	A_1	0.219	0.209	0.221	0.30	0.170
$B \to \pi$	F_0	0.258	0.247	0.285	0.292	0.269
$B \to K$	F_0	0.331	0.297	0.345	0.321	0.349

Numerical Results: Form Factors

The B_s → P, V form factors at q^2 = 0. In comparison with QCD Sum Rules,, Light Cone QCD Sum Rules (HQEFT).

Mode	F(0)	QCDSR	LC	LC(HQEFT)	This work
$B_s \to K^*$	V	0.311	0.323	0.285	$0.227^{+0.064+0.003}_{-0.037-0.002}$
	A_0	0.360	0.279	0.222	$0.280^{+0.082+0.013}_{-0.043-0.008}$
	A_1	0.233	0.228	0.227	$0.178^{+0.046+0.002}_{-0.027-0.002}$
$B_s \to \phi$	V	0.434	0.329	0.339	$0.259^{+0.080+0.006}_{-0.036-0.003}$
	A_0	0.474	0.279	0.212	$0.311^{+0.096+0.014}_{-0.047-0.006}$
	A_1	0.311	0.232	0.271	$0.194^{+0.052+0.004}_{-0.028-0.002}$
$B_s \to K$	F_0		0.290	0.296	$0.260^{+0.053+0.007}_{-0.031-0.003}$

The errors stem mainly from the uncertainties in the global parameters

$$\mu_{scale} = 1.5 \pm 0.1\text{GeV} \qquad \mu_g = 0.37 \pm 0.037\text{GeV}$$

Amplitudes of Hadronic Two Body Decays

$$
\begin{aligned}
&A(B \to \boldsymbol{M_1 M_2}) = \\
&\sum_{i=1,4,6,8,10}\sum_{p=u,c}\Big\{C_i(M_i^{a1}+M_i^{b1}+M_i^{c1}+M_i^{d1})B\boldsymbol{M_1}U_i\boldsymbol{M_2}\,\boldsymbol{\Lambda}_p \\
&\qquad +C_i(M_i^{a2}+M_i^{b2}+M_i^{c2}+M_i^{d2})B\boldsymbol{M_1}\boldsymbol{\Lambda}_p\cdot \mathrm{Tr}\,[U_i\boldsymbol{M_2}] \\
&\qquad +C_i(M_i^{a3}+M_i^{b3}+M_i^{c3}+M_i^{d3})BU_i\boldsymbol{M_1}\boldsymbol{M_2}\boldsymbol{\Lambda}_p \\
&\qquad +C_i(M_i^{a4}+M_i^{b4}+M_i^{c4}+M_i^{d4})B\boldsymbol{\Lambda}_p\cdot \mathrm{Tr}\,[U_i\boldsymbol{M_1}\boldsymbol{M_2}]\Big\} + \\
&\sum_{i=2,3,5,7,9}\sum_{p=u,c}\Big\{C_i(M_i^{a1}+M_i^{b1}+M_i^{c1}+M_i^{d1})B\boldsymbol{M_1}\boldsymbol{\Lambda}_p\cdot \mathrm{Tr}\,[U_i\boldsymbol{M_2}] \\
&\qquad +C_i(M_i^{a2}+M_i^{b2}+M_i^{c2}+M_i^{d2})B\boldsymbol{M_1}U_i\boldsymbol{M_2}\,\boldsymbol{\Lambda}_p \\
&\qquad +C_i(M_i^{a3}+M_i^{b3}+M_i^{c3}+M_i^{d3})B\boldsymbol{\Lambda}_p\cdot \mathrm{Tr}\,[U_i\boldsymbol{M_1}\boldsymbol{M_2}] \\
&\qquad +C_i(M_i^{a4}+M_i^{b4}+M_i^{c4}+M_i^{d4})BU_i\boldsymbol{M_1}\boldsymbol{M_2}\boldsymbol{\Lambda}_p\Big\},
\end{aligned}
$$

Amplitudes of Hadronic Two Body Decays

All the mesons are expressed in terms of SU(3) flavor symmetry into a vector or matrix form

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_q}{\sqrt{2}} + \frac{\eta_q'}{\sqrt{2}} & \pi^- & K^- \\ \pi^+ & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_q}{\sqrt{2}} + \frac{\eta_q'}{\sqrt{2}} & \bar{K}^0 \\ K^+ & K^0 & \eta_s + \eta_s' \end{pmatrix}$$

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^- & K^{*-} \\ \rho^+ & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \bar{K}^{*0} \\ K^{*+} & K^{*0} & \phi \end{pmatrix}$$

$$U_i = \begin{cases} U_p & \text{for } i = 1, 2\ , \\ I & \text{for } i = 3 - 6\ , \\ Q & \text{for } i = 7 - 10 \end{cases}$$

$$U_p = \begin{pmatrix} \delta_{pu} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}$$

$$B = (B^+, B^0, B_s)$$

$$\Lambda_p = \begin{pmatrix} 0 \\ \lambda_p^{(d)} \\ \lambda_p^{(s)} \end{pmatrix}$$

$$M_i = \begin{cases} M_{LL} & \text{for } i = 1 - 4, 9, 10 & (V-A) \times (V-A) \\ M_{LR} & \text{for } i = 5, 7 & (V-A) \times (V+A) \\ M_{SP} & \text{for } i = 6, 8 & (S-P) \times (S+P). \end{cases}$$

Amplitudes of Hadronic Two Body Decays

$$
\begin{aligned}
&A(B \to M_1 M_2) = \\
&\sum_{p=u,c} \Big\{ B M_1 \left(T^{M_1 M_2}(B)\, U_p + P^{M_1 M_2}(B)^p + P_{EW}^{C M_1 M_2}(B)^p\, Q \right) M_2\, \Lambda_p \\
&\qquad + B M_1 \Lambda_p \cdot \mathrm{Tr} \left[\left(C^{M_1 M_2}(B)\, U_p + P_C^{M_1 M_2}(B)^p + P_{EW}^{M_1 M_2}(B)\, Q \right) M_2 \right] \\
&\qquad + B \left(A^{M_1 M_2}(B)\, U_p + P_A^{M_1 M_2}(B)^p + P_{EW}^{E M_1 M_2}(B)^p\, Q \right) M_1 M_2 \Lambda_p \\
&\qquad + B \Lambda_p \cdot \mathrm{Tr} \left[\left(E^{M_1 M_2}(B)\, U_p + P_E^{M_1 M_2}(B)^p + P_{EW}^{A M_1 M_2}(B)^p\, Q \right) M_1 M_2 \right] \Big\}
\end{aligned}
$$

There are totally 12 types of amplitudes

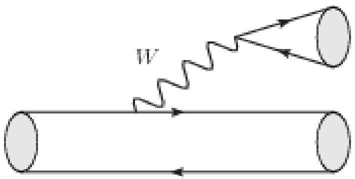
$$T^{M_1 M_2}(M),\ C^{M_1 M_2}(M),\ P^{M_1 M_2}(M),\ P_C^{M_1 M_2}(M),\ P_{EW}^{M_1 M_2}(M),\ A^{M_1 M_2}(M),$$

$$E^{M_1 M_2}(M),\ P_E^{M_1 M_2}(M),\ P_A^{M_1 M_2}(M),\ P_{EW}^{C M_1 M_2}(M),\ P_{EW}^{E M_1 M_2}(M),\ P_{EW}^{A M_1 M_2}(M)$$

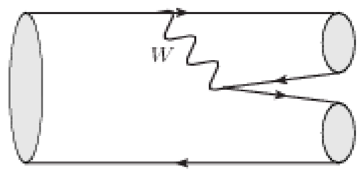
Amplitudes of Hadronic Two Body Decays

The 12 types of two-body hadronic decays of heavy mesons can be expressed in terms of the distinct topological diagrams

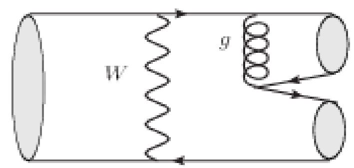
[Chau ('80); Chau, HYC('86)]



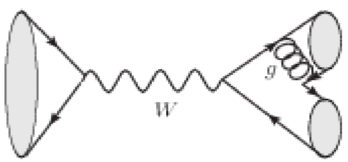
T (tree)



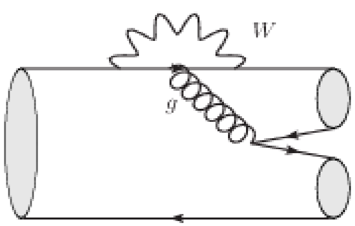
C (color-suppressed)



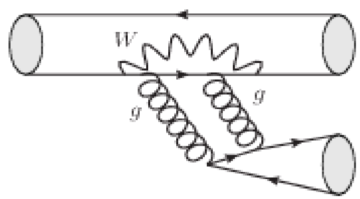
E (W-exchange)



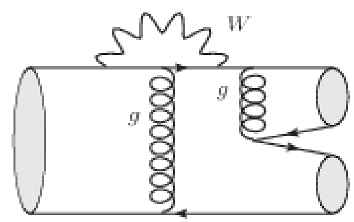
A (W-annihilation)



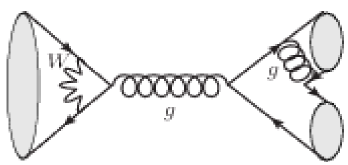
P, P^c_{EW}



S, P_{EW}



PE, PE_{EW}



PA, PA_{EW}

HYC, Oh ('11)

All quark graphs are topological with all strong interactions encoded and the SU(3) flavor symmetry assumed !!!

Amplitudes of Hadronic Two Body Decays

6 amplitudes for the so-called emission diagrams

$$
\begin{aligned}
T^{M_1M_2}(M) &= 4\pi\alpha_s(\mu)\frac{G_F}{\sqrt{2}}\{[C_1(\mu)+\frac{1}{N_C}C_2(\mu)]T_{LL}^{FM_1M_2}(M)+\frac{1}{N_C}C_2(\mu)T_{LL}^{NM_1M_2}(M)\},\\
C^{M_1M_2}(M) &= 4\pi\alpha_s(\mu)\frac{G_F}{\sqrt{2}}\{[C_2(\mu)+\frac{1}{N_C}C_1(\mu)]T_{LL}^{FM_1M_2}(M)+\frac{1}{N_C}C_1(\mu)T_{LL}^{NM_1M_2}(M)\},\\
P^{M_1M_2}(M) &= 4\pi\alpha_s(\mu)\frac{G_F}{\sqrt{2}}\{[C_4(\mu)+\frac{1}{N_C}C_3(\mu)]T_{LL}^{FM_1M_2}(M)+\frac{1}{N_C}C_3(\mu)T_{LL}^{NM_1M_2}(M)\\
&\quad +[C_6(\mu)+\frac{1}{N_C}C_5(\mu)]T_{SP}^{FM_1M_2}(M)+\frac{1}{N_C}C_5(\mu)T_{LR}^{NM_1M_2}(M)\},\\
P_C^{M_1M_2}(M) &= 4\pi\alpha_s(\mu)\frac{G_F}{\sqrt{2}}\{[C_3(\mu)+\frac{1}{N_C}C_4(\mu)]T_{LL}^{FM_1M_2}(M)+\frac{1}{N_C}C_4(\mu)T_{LL}^{NM_1M_2}(M)\\
&\quad +[C_5(\mu)+\frac{1}{N_C}C_6(\mu)]T_{SP}^{FM_1M_2}(M)+\frac{1}{N_C}C_6(\mu)T_{LR}^{NM_1M_2}(M)\},\\
P_{EW}^{M_1M_2}(M) &= 4\pi\alpha_s(\mu)\frac{G_F}{\sqrt{2}}\frac{3}{2}\{[C_9(\mu)+\frac{1}{N_C}C_{10}(\mu)]T_{LL}^{FM_1M_2}(M)+\frac{1}{N_C}C_{10}(\mu)T_{LL}^{NM_1M_2}(M)\\
&\quad +[C_7(\mu)+\frac{1}{N_C}C_8(\mu)]T_{LR}^{FM_1M_2}(M)+\frac{1}{N_C}C_8(\mu)T_{SP}^{NM_1M_2}(M)\},\\
P_{EW}^{CM_1M_2}(M) &= 4\pi\alpha_s(\mu)\frac{G_F}{\sqrt{2}}\frac{3}{2}\{[C_{10}(\mu)+\frac{1}{N_C}C_9(\mu)]T_{LL}^{FM_1M_2}(M)+\frac{1}{N_C}C_9(\mu)T_{LL}^{NM_1M_2}(M)\\
&\quad +[C_8(\mu)+\frac{1}{N_C}C_7(\mu)]T_{SP}^{FM_1M_2}(M)+\frac{1}{N_C}C_7(\mu)T_{LR}^{NM_1M_2}(M)\}, \qquad \text{(A-7)}
\end{aligned}
$$

Amplitudes of Hadronic Two Body Decays

6 amplitudes for the so-called annihilation diagrams

$$
\begin{aligned}
A^{M_1M_2}(M) &= 4\pi\alpha_s(\mu)\frac{G_F}{\sqrt{2}}\{[C_1(\mu)+\frac{1}{N_C}C_2(\mu)]A_{LL}^{FM_1M_2}(M)+\frac{1}{N_C}C_2(\mu)A_{LL}^{NM_1M_2}(M)\}.\\
E^{M_1M_2}(M) &= 4\pi\alpha_s(\mu)\frac{G_F}{\sqrt{2}}\{[C_2(\mu)+\frac{1}{N_C}C_1(\mu)]A_{LL}^{FM_1M_2}(M)+\frac{1}{N_C}C_1(\mu)A_{LL}^{NM_1M_2}(M)\},\\
P_E^{M_1M_2}(M) &= 4\pi\alpha_s(\mu)\frac{G_F}{\sqrt{2}}\{[C_4(\mu)+\frac{1}{N_C}C_3(\mu)]A_{LL}^{FM_1M_2}(M)+\frac{1}{N_C}C_3(\mu)A_{LL}^{NM_1M_2}(M)\\
&\quad +[C_6(\mu)+\frac{1}{N_C}C_5(\mu)]A_{SP}^{FM_1M_2}(M)+\frac{1}{N_C}C_5(\mu)A_{LR}^{NM_1M_2}(M)\},\\
P_A^{M_1M_2}(M) &= 4\pi\alpha_s(\mu)\frac{G_F}{\sqrt{2}}\{[C_3(\mu)+\frac{1}{N_C}C_4(\mu)]A_{LL}^{FM_1M_2}(M)+\frac{1}{N_C}C_4(\mu)A_{LL}^{NM_1M_2}(M)\\
&\quad +[C_5(\mu)+\frac{1}{N_C}C_6(\mu)]A_{LR}^{FM_1M_2}(M)+\frac{1}{N_C}C_6(\mu)A_{SP}^{NM_1M_2}(M)\},\\
P_{EW}^{AM_1M_2}(M) &= 4\pi\alpha_s(\mu)\frac{G_F}{\sqrt{2}}\frac{3}{2}\{[C_9(\mu)+\frac{1}{N_C}C_{10}(\mu)]A_{LL}^{FM_1M_2}(M)+\frac{1}{N_C}C_{10}(\mu)A_{LL}^{NM_1M_2}(M)\\
&\quad +[C_7(\mu)+\frac{1}{N_C}C_8(\mu)]A_{LR}^{FM_1M_2}(M)+\frac{1}{N_C}C_8(\mu)A_{SP}^{NM_1M_2}(M)\},\\
P_{EW}^{EM_1M_2}(M) &= 4\pi\alpha_s(\mu)\frac{G_F}{\sqrt{2}}\frac{3}{2}\{[C_{10}(\mu)+\frac{1}{N_C}C_9(\mu)]A_{LL}^{FM_1M_2}(M)+\frac{1}{N_C}C_9(\mu)A_{LL}^{NM_1M_2}(M)\\
&\quad +[C_8(\mu)+\frac{1}{N_C}C_7(\mu)]A_{SP}^{FM_1M_2}(M)+\frac{1}{N_C}C_7(\mu)A_{LR}^{NM_1M_2}(M)\}, \qquad \text{(A-8)}
\end{aligned}
$$

Amplitudes of Hadronic Two Body Decays

There are in general 24 amplitude contributions. Here for an example

The factorizable emission contributions for the (V-A)x(V-A)

$$
\begin{aligned}
T_{LL}^{FM_1M_2}(M) &= T_{LLa}^{FM_1M_2}(M) + T_{LLb}^{FM_1M_2}(M), \\
T_{LLa}^{FM_1M_2}(M) &= i\frac{1}{4}\frac{C_F}{N_C}\, F_M\;F_{M_1}\;F_{M_2}\int_0^1\int_0^1\int_0^1 du\,dx\,dy\, m_B^2\phi_M(u) \\
&\quad \{m_B(2m_b - m_B x)\phi_{M_1}(x) + \mu_{M_1}(2m_B x - m_b)[\phi_{M_1}^p(x) - \phi_{M_1}^T(x)]\}\phi_{M_2}(y)h_{Ta}^F(u,x), \\
T_{LLb}^{FM_1M_2}(M) &= i\frac{1}{2}\frac{C_F}{N_C}\, F_M\;F_{M_1}\;F_{M_2}\int_0^1\int_0^1\int_0^1 du\,dx\,dy\, m_B^3\mu_{M_1}\phi_M(u)\phi_{M_2}(y)\phi_{M_1}^p(x)h_{Tb}^F(u,x),
\end{aligned}
$$

The factorizable annihilation contributions for the (V-A)x(V-A)

$$
\begin{aligned}
A_{LL}^{FM_1M_2}(M) &= A_{LLa}^{FM_1M_2}(M) + A_{LLb}^{FM_1M_2}(M), \\
A_{LLa}^{FM_1M_2}(M) &= -\frac{1}{4}\frac{C_F}{N_C}\, F_M\;F_{M_1}\;F_{M_2}\int_0^1\int_0^1\int_0^1 du\,dx\,dy\;\; m_B^2\phi_M(u) \\
&\quad \{(1-y)m_B^2\phi_{M_2}(y)\phi_{M_1}(x) + 2\mu_{M_2}\;\mu_{M_1}[(2-y)\phi_{M_2}^p(y) + y\phi_{M_2}^T(y)]\phi_{M_1}^p(x)\}h_{Aa}^F(x,y), \\
A_{LLb}^{FM_1M_2}(M) &= \frac{1}{4}\frac{C_F}{N_C}\, F_M\;F_{M_1}\;F_{M_2}\int_0^1\int_0^1\int_0^1 du\,dx\,dy\;\; m_B^2\phi_M(u) \\
&\quad \{x m_B^2\phi_{M_2}(y)\phi_{M_1}(x) + 2\mu_{M_2}\;\mu_{M_1}[(1+x)\phi_{M_1}^p(x) - (1-x)\phi_{M_1}^T(x)]\phi_{M_2}^p(y)\}h_{Ab}^F(x,y),
\end{aligned}
$$

Amplitudes for B → ππ

Take B → ππ decay as example, the decay amplitudes can be expressed as follows:

$$\begin{aligned} A(B^0 \to \pi^+\pi^-) &= V_{td}V_{tb}^*[P_T^{\pi\pi}(B) + \frac{2}{3}P_{EW}^{C\pi\pi}(B) + P_E^{\pi\pi}(B) + 2P_A^{\pi\pi}(B) \\ &\quad + \frac{1}{3}P_{EW}^{A\pi\pi}(B) - \frac{1}{3}A_{EW}^{E\pi\pi}(B)] - V_{ud}V_{ub}^*[T^{\pi\pi}(B) + E^{\pi\pi}(B)] \end{aligned}$$

$$A(B^+ \to \pi^+\pi^0) = \frac{1}{\sqrt{2}}\{V_{td}V_{tb}^*[P_{EW}^{\pi\pi}(B) + P_{EW}^{C\pi\pi}(B)] - V_{ud}V_{ub}^*[T^{\pi\pi}(B) + C^{\pi\pi}(B)]\}$$

$$\begin{aligned} A(B^0 \to \pi^0\pi^0) &= \frac{1}{\sqrt{2}}\{-V_{td}V_{tb}^*[P_T^{\pi\pi}(B) - P_{EW}^{\pi\pi}(B) - \frac{1}{3}P_{EW}^{C\pi\pi}(B) + P_E^{\pi\pi}(B) \\ &\quad + 2P_A^{\pi\pi}(B) + \frac{1}{3}P_{EW}^{A\pi\pi}(B) - \frac{1}{3}P_{EW}^{E\pi\pi}(B)] \\ &\quad + V_{ud}V_{ub}^*[-C^{\pi\pi}(B) + E^{\pi\pi}(B)]\}, \end{aligned}$$

Amplitudes for B → ππ

Totally, it involves 11 amplitudes, which are defined as follows:

$$\begin{aligned}
T^{M_1M_2}(M) &= 4\pi\alpha_s(\mu)\frac{G_F}{\sqrt{2}}\{[C_1(\mu)+\frac{1}{N_C}C_2(\mu)]T_{LL}^{FM_1M_2}(M) \\
&\quad +\frac{1}{N_C}C_2(\mu)T_{LL}^{NM_1M_2}(M)\}, \\
C^{M_1M_2}(M) &= 4\pi\alpha_s(\mu)\frac{G_F}{\sqrt{2}}\{[C_2(\mu)+\frac{1}{N_C}C_1(\mu)]T_{LL}^{FM_1M_2}(M) \\
&\quad +\frac{1}{N_C}C_1(\mu)T_{LL}^{NM_1M_2}(M)\}, \\
P_{EW}^{M_1M_2}(M) &= 4\pi\alpha_s(\mu)\frac{G_F}{\sqrt{2}}\frac{3}{2}\{[C_9(\mu)+\frac{1}{N_C}C_{10}(\mu)]T_{LL}^{FM_1M_2}(M) \\
&\quad +\frac{1}{N_C}C_{10}(\mu)T_{LL}^{NM_1M_2}(M)+[C_7(\mu)+\frac{1}{N_C}C_8(\mu)]T_{LR}^{FM_1M_2}(M) \\
&\quad +\frac{1}{N_C}C_8(\mu)T_{SP}^{NM_1M_2}(M)\},
\end{aligned}$$

……

Numerical Results:
Branch Ratio & CP Violation for B→ PP

Table: The branching ratios (in units of 10^{-6}) and direct CP asymmetries in $B \to \pi K$ decays. The central values are obtained at $\mu_q = \mu_g$=0.37GeV.*(Penguin dominate)*

Mode	Data[HFAG]	This work				
		NLO+Vertex	NLO^{eff}	$\text{NLO}^{eff}(-10^\circ)$	$\text{NLO}^{eff}(5^\circ)$	$\text{NLO}^{eff}(20^\circ)$
$B^+ \to \pi^+ K^0$	23.1 ± 1.0	22.5	21.4	19.0	22.6	25.9
$B^+ \to \pi^0 K^+$	12.9 ± 0.6	12.8	12.5	11.2	13.1	14.9
$B^0 \to \pi^- K^+$	19.4 ± 0.6	19.2	19.5	17.4	20.5	23.3
$B^0 \to \pi^0 K^0$	9.8 ± 0.6	8.3	8.4	7.4	8.9	10.2
$A_{CP}(\pi^+ K^0)$	0.009 ± 0.025	-0.006	-0.006	-0.006	-0.007	-0.007
$A_{CP}(\pi^0 K^+)$	0.050 ± 0.025	-0.053	0.012	0.003	0.018	0.034
$A_{CP}(\pi^- K^+)$	-0.098 ± 0.012	-0.118	-0.139	-0.158	-0.131	-0.105
$A_{CP}(\pi^0 K^0)$	-0.01 ± 0.10	-0.052	-0.139	-0.143	-0.138	-0.137
$S_{\pi^0 K_S}$	0.58 ± 0.17	0.699	0.760	0.768	0.756	0.745

NLO^{eff} — "NLO correction + effective Wilson coefficients"

$\text{NLO}^{eff}(\theta^a)$ — "NLO correction + effective Wilson coefficients + annihilation with strong phase"

Numerical Results:
Branch Ratio & CP Violation for B→ PP

Table: $B \to \pi\pi, KK$ decay modes *(Tree dominate)*

Mode	Data[HFAG]	This work				
		NLO+Vertex	NLO^{eff}	NLO^{eff} (-40°)	NLO^{eff} (5°)	NLO^{eff} (50°)
$B^0 \to \pi^-\pi^+$	5.16 ± 0.22	7.1	6.5	6.00	6.6	7.6
$B^+ \to \pi^+\pi^0$	5.59 ± 0.40	4.1	5.5	5.51	5.5	5.5
$B^0 \to \pi^0\pi^0$	1.55 ± 0.19	0.3	1.0	1.11	1.0	1.0
$B^+ \to K^+\bar{K}^0$	1.36 ± 0.28	1.7	1.6	1.0	1.7	2.2
$B^0 \to K^0\bar{K}^0$	0.96 ± 0.20	1.5	1.4	0.7	1.5	2.2
$B^0 \to K^+K^-$	0.15 ± 0.10	0.09	0.09	0.09	0.09	0.09
$A_{CP}(\pi^-\pi^+)$	0.38 ± 0.06	0.206	0.266	0.239	0.260	0.141
$A_{CP}(\pi^+\pi^0)$	0.06 ± 0.05	-0.000	-0.001	-0.001	-0.001	-0.001
$A_{CP}(\pi^0\pi^0)$	0.43 ± 0.25	0.382	0.453	0.272	0.485	0.789
$S_{\pi\pi}$	-0.61 ± 0.08	-0.504	-0.506	-0.353	-0.524	-0.638
$A_{CP}(K^+\bar{K}^0)$	0.12 ± 0.17	0.101	0.098	0.041	0.101	0.106
$A_{CP}(K^0\bar{K}^0)$	-0.58 ± 0.7	0.000	0.000	0.000	0.000	0.000
$A_{CP}(K^+K^-)$	-	-0.184	-0.184	-0.184	-0.184	-0.184

Table: Comparisons of predictions between our framework and QCDF, pQCD methods in $B \to \pi\pi, \pi K$ decays.

Mode	Data	QCDF	pQCD	This work	
			NLO+Vertex	NLO+Vertex	NLO(a^{eff}, θ^a)
$B^+ \to \pi^+ K^0$	23.1 ± 1.0	$21.7^{+9.2+9.0}_{-6.0-6.9}$	$24.5^{+13.6\,(+12.9)}_{-\ 8.1\,(-\ 7.8)}$	22.5	$22.6^{+6.1+9.8}_{-3.5-2.8}$
$B^+ \to \pi^0 K^+$	12.9 ± 0.6	$12.5^{+4.7+4.9}_{-3.0-3.8}$	$13.9^{+10.0\,(+\ 7.0)}_{-\ 5.6\,(-\ 4.2)}$	12.8	$13.1^{+3.7+6.0}_{-2.1-1.3}$
$B^0 \to \pi^- K^+$	19.4 ± 0.6	$19.3^{+7.9+8.2}_{-4.8-6.2}$	$20.9^{+15.6\,(+11.0)}_{-\ 8.3\,(-\ 6.5)}$	19.2	$20.5^{+5.2+10.4}_{-3.0-3.0}$
$B^0 \to \pi^0 K^0$	9.8 ± 0.6	$8.6^{+3.8+3.8}_{-2.2-1.4}$	$9.1^{+\ 5.6\,(+\ 5.1)}_{-\ 3.3\,(-\ 2.9)}$	8.3	$8.9^{+2.1+5.0}_{-1.3-1.1}$
$B^0 \to \pi^- \pi^+$	5.16 ± 0.22	$7.0^{+0.4+0.7}_{-0.7-0.7}$	$6.5^{+\ 6.7\,(+\ 2.7)}_{-\ 3.8\,(-\ 1.8)}$	7.1	$6.6^{+3.3+1.1}_{-1.3-0.3}$
$B^+ \to \pi^+ \pi^0$	5.59 ± 0.40	$5.9^{+.2+1.4}_{-1.1-.1.1}$	$4.0^{+\ 3.4\,(+\ 1.7)}_{-\ 1.9\,(-\ 1.2)}$	4.1	$5.5^{+2.3+1.3}_{-1.1-0.4}$
$B^0 \to \pi^0 \pi^0$	1.55 ± 0.19	$1.1^{+1.0+0.7}_{-0.4-0.3}$	$0.29^{+0.50\,(+0.13)}_{-0.20\,(-0.08)}$	0.3	$1.0^{+0.3+0.3}_{-0.1-0.1}$
$A_{CP}(\pi^+ K^0)$	0.009 ± 0.025	$0.0028^{+0.0003+0.0009}_{-0.0003-0.0010}$	$-0.01 \pm 0.00\,(\pm 0.00)$	-0.006	$-0.007^{+0.002+0.003}_{-0.001-0.013}$
$A_{CP}(\pi^0 K^+)$	0.050 ± 0.025	$0.049^{+0.039+0.044}_{-0.021-0.054}$	$-0.01^{+0.03\,(+0.03)}_{-0.05\,(-0.05)}$	-0.053	$0.018^{+0.014+0.022}_{-0.004-0.020}$
$A_{CP}(\pi^- K^+)$	-0.098 ± 0.012	$-0.074^{+0.017+0.043}_{-0.015-0.048}$	$-0.09^{+0.06\,(+0.04)}_{-0.08\,(-0.06)}$	-0.118	$-0.131^{+0.009+0.022}_{-0.003-0.004}$
$A_{CP}(\pi^0 K^0)$	-0.01 ± 0.10	$-0.106^{+0.027+0.056}_{-0.038-0.043}$	$-0.07^{+0.03\,(+0.01)}_{-0.03\,(-0.01)}$	-0.052	$-0.138^{+0.003+0.004}_{-0.006-0.007}$
$S_{\pi^0 K_S}$	0.58 ± 0.17	-	$0.73^{+0.03\,(+0.01)}_{-0.02\,(-0.01)}$	0.699	$0.756^{+0.002+0.002}_{-0.004-0.007}$
$A_{CP}(\pi^- \pi^+)$	0.38 ± 0.06	$0.170^{+0.013+0.043}_{-0.012-0.087}$	$0.18^{+0.20\,(+0.07)}_{-0.12\,(-0.06)}$	0.225	$0.260^{+0.043+0.059}_{-0.032-0.063}$
$A_{CP}(\pi^+ \pi^0)$	0.06 ± 0.05	-0.0002	$0.00 \pm 0.00\,(\pm 0.00)$	-0.000	-0.001^{+0+0}_{-0-0}
$A_{CP}(\pi^0 \pi^0)$	0.43 ± 0.25	$0.572^{+0.148+0.303}_{-0.208-0.346}$	$0.63^{+0.35\,(+0.09)}_{-0.34\,(-0.15)}$	0.382	$0.485^{+0.061+0.169}_{-0.025-0.070}$
$S_{\pi\pi}$	-0.61 ± 0.08	-	$-0.43^{+1.00\,(+0.05)}_{-0.56\,(-0.05)}$	-0.504	$-0.524^{+0.017+0.003}_{-0.004-0.017}$

QCDF: **H. Y. Cheng and C. K. Chua, Phys. Rev. D 80, 114008 (2009).**

pQCD: **H. N. Li, S. Mishima, A. I. Sanda, Phys.Rev. D72, 114005 (2005).**

Numerical Results:
Branch Ratio & CP Violation for B→ PP

From the naive power-counting based on factorization theory predicts

$$Br(\pi^-\pi^+) > Br(\pi^-\pi^0) \gg Br(\pi^0\pi^0).$$

From the experimental data

$$\mathrm{Br}(\pi^-\pi^+) \sim \mathrm{Br}(\pi^-\pi^0)$$

$$\mathrm{Br}(\pi^0\pi^0)$$

Two ratios

$$R_c = \frac{2Br(B^+ \to \pi^0 K^+)}{Br(B^+ \to \pi^+ K^0)}$$

$$R_n = \frac{Br(B^0 \to \pi^- K^+)}{2Br(B^0 \to \pi^0 K^0)}$$

Experimental data

$$R_c = 1.12 \pm 0.07$$

$$R_n = 0.99 \pm 0.07$$

Our Theory Predictions

$$R_c = 1.15$$

$$R_n = 1.13$$

Numerical Results:
Branch Ratio & CP Violation for B→ PV

Table: The branching ratios (in units of 10^{-6}) and direct CP asymmetries in penguin dominated $B \to PV$ decays

Mode	Data[HFAG]	This work				
		NLO+Vertex	NLO^{eff}	NLO^{eff} (-10°)	NLO^{eff} (5°)	NLO^{eff} (20°)
$B^+ \to K^{*0}\pi^+$	9.9 ± 0.8	10.3	9.0	7.6	9.8	11.8
$B^+ \to K^{*+}\pi^0$	6.9 ± 2.3	6.2	5.5	4.8	5.9	7.1
$B^0 \to K^{*-}\pi^+$	8.6 ± 0.9	8.8	8.3	7.2	8.9	10.6
$B^0 \to K^{*0}\pi^0$	2.4 ± 0.7	3.5	3.6	3.1	3.9	4.6
$B^+ \to \phi K^+$	8.30 ± 0.65	9.3	6.9	5.6	7.6	9.6
$B^0 \to \phi K^0$	8.3 ± 1.1	8.9	6.6	5.4	7.3	9.2
$A_{CP}(K^{*0}\pi^+)$	-0.038 ± 0.042	-0.017	-0.018	-0.020	-0.017	-0.015
$A_{CP}(K^{*+}\pi^0)$	0.04 ± 0.29	-0.224	-0.123	-0.164	-0.103	-0.045
$A_{CP}(K^{*-}\pi^+)$	-0.23 ± 0.08	-0.357	-0.355	-0.415	-0.327	-0.251
$A_{CP}(K^{*0}\pi^0)$	-0.15 ± 0.12	-0.067	-0.125	-.114	-0.129	-0.143
$A_{CP}(\phi K^+)$	0.23 ± 0.15	-0.022	-0.025	-0.028	-0.023	-0.020
$A_{CP}(\phi K^0)$	-0.01 ± 0.06	0	0	0	0	0

Numerical Results:
Branch Ratio & CP Violation for B→ PV

Table: The branching ratios (in units of 10^{-6}) and direct CP asymmetries in penguin dominated $B \to PV$ decays

Mode	Data[HFAG]	This work				
		NLO+Vertex	NLO^{eff}	NLO^{eff} (45°)	NLO^{eff} (60°)	NLO^{eff} (75°)
$B^+ \to \rho^+ K^0$	8.0 ± 1.45	5.2	7.1	7.1	6.8	6.3
$B^+ \to \rho^0 K^+$	3.81 ± 0.48	3.0	3.3	2.8	2.6	2.4
$B^0 \to \rho^- K^+$	8.6 ± 1.0	5.4	6.2	7.3	7.3	7.2
$B^0 \to \rho^0 K^0$	4.7 ± 0.7	2.8	3.9	4.9	5.0	5.0
$B^+ \to \omega K^+$	6.7 ± 0.5	2.4	3.6	4.3	5.3	5.2
$B^0 \to \omega K^0$	5.0 ± 0.6	1.9	3.2	4.1	4.8	4.6
$A_{CP}(\rho^+ K^0)$	-0.12 ± 0.17	0.016	0.013	0.014	0.014	0.014
$A_{CP}(\rho^0 K^+)$	0.37 ± 0.11	0.635	0.727	0.594	0.463	0.285
$A_{CP}(\rho^- K^+)$	0.15 ± 0.06	0.605	0.549	0.373	0.290	0.196
$A_{CP}(\rho^0 K^0)$	0.06 ± 0.20	0.056	-0.136	-0.044	-0.015	0.015
$A_{CP}(\omega K^+)$	0.02 ± 0.05	0.453	0.404	0.167	0.091	0.015
$A_{CP}(\omega K^0)$	0.32 ± 0.17	-0.011	0.117	0.048	0.026	0.002

Numerical Results:
Branch Ratio & CP Violation for B→ PV

Table: Tree dominant $B \to PV$ decay modes

Mode	Data[HFAG]	This work					
		NLO+Vertex	NLO^{eff}				
			default	$(-45^{\circ},0^{\circ})$	$(45^{\circ},0^{\circ})$	$(0^{\circ},-45^{\circ})$	$(0^{\circ},45^{\circ})$
$B^+ \to \rho^+\pi^0$	10.9 ± 1.5	12.0	13.9	14.2	13.5	13.7	14.2
$B^+ \to \rho^0\pi^+$	8.3 ± 1.3	5.2	7.4	7.0	7.8	7.2	7.4
$B^0 \to \rho^+\pi^-$	15.7 ± 1.8	19.6	17.4	16.5	19.1	17.5	17.4
$B^0 \to \rho^-\pi^+$	7.3 ± 1.2	6.2	6.5	6.5	6.5	6.1	7.5
$B^0 \to \rho^0\pi^0$	2.0 ± 0.5	0.2	1.3	1.5	1.2	1.5	1.1
$B^+ \to \bar{K}^{*0}K^+$	0.68 ± 0.19	0.6	0.3	0.3	0.3	0.3	0.3
$B^0 \to K^{*0}\bar{K}^0$	< 1.9	0.6	0.4	0.2	0.8	0.8	0.1
$A_{CP}(\rho^+\pi^0)$	0.02 ± 0.11	0.255	0.199	0.196	0.133	0.195	0.131
$A_{CP}(\rho^0\pi^+)$	$0.18^{+0.09}_{-0.17}$	−0.308	-0.344	−0.330	-0.269	-0.309	-0.285
$A_{CP}(\rho^+\pi^-)$	0.11 ± 0.06	0.120	0.126	0.108	0.066	0.121	0.127
$A_{CP}(\rho^-\pi^+)$	-0.18 ± 0.12	−0.281	-0.282	−0.283	-0.281	-0.217	-0.176
$A_{CP}(\rho^0\pi^0)$	-0.30 ± 0.38	0.058	0.187	0.112	0.381	0.258	-0.008
$A_{CP}(\bar{K}^{*0}K^+)$	-	0.191	0.257	−0.342	0.205	0.112	0.837
$A_{CP}(k^{*0}\bar{K}^0)$	-	0.000	0	0	0	0	0

Table: Comparisons of predictions between our framework and QCDF, pQCD methods

$\theta^a_{\pi\rho}=5^\circ$ and $\theta^a_{\rho\pi}=60^\circ$.

Mode	Data[31]	QCDF[11]	pQCD(LO)[33, 34]	this work		
				LO	NLO+Vetex	NLO(a^{eff})
$B^+ \to \rho^+\pi^0$	10.9 ± 1.5	$11.8^{+1.8+1.4}_{-1.1-1.4}$	$6 \sim 9$	12.0	12.0	$13.9^{+5.7+2.8}_{-3.2-0.9}$
$B^+ \to \rho^0\pi^+$	8.3 ± 1.3	$8.7^{+2.7+1.7}_{-1.3-.14}$	$5 \sim 6$	5.4	5.2	$7.4^{+3.7+1.0}_{-1.9-0.2}$
$B^0 \to \rho^+\pi^-$	15.7 ± 1.8	$15.9^{+1.1+0.9}_{-1.5-1.1}$		18.6	19.6	$17.4^{+8.2+3.2}_{-4.2-0.9}$
$B^0 \to \rho^-\pi^+$	7.3 ± 1.2	$9.2^{+0.4+0.5}_{-0.7-0.7}$		6.9	6.2	$6.5^{+4.3+0.1}_{-2.0-0.0}$
$B^0 \to \rho^0\pi^0$	2.0 ± 0.5	$1.3^{+1.7+1.2}_{-0.6-0.6}$	$0.07 \sim 0.11$	0.2	0.2	$1.3^{+0.3+0.4}_{-0.2-0.2}$
$B^+ \to K^{*0}K^+$	0.68 ± 0.19	$0.80^{+0.20+0.31}_{-0.17-0.28}$	$0.32^{+0.12}_{-0.07}$	0.3	0.6	$0.3^{+0.0+0.1}_{-0.0-0.2}$
$B^0 \to K^{*0}\bar{K}^0$	< 1.9	$0.47^{+0.36+0.43}_{-0.17-0.27}$	$0.49^{+0.15}_{-0.09}$	0.3	0.6	$0.4^{+0.1+0.2}_{-0.1-0.2}$
$A_{CP}(\rho^+\pi^0)$	0.02 ± 0.11	$0.097^{+0.021+0.080}_{-0.031-0.103}$	$0 \sim 20$	0.251	0.255	$0.199^{+0.027+0.016}_{-0.044-0.053}$
$A_{CP}(\rho^0\pi^+)$	$0.18^{+0.09}_{-0.17}$	$-0.098^{+0.034+0.114}_{-0.026-0.104}$	$-20 \sim 0$	-0.351	-0.308	$-0.344^{+0.062+0.023}_{-0.41-0.086}$
$A_{CP}(\rho^+\pi^-)$	0.11 ± 0.06	$0.044^{+0.003+0.058}_{-0.003-0.068}$		0.113	0.120	$0.126^{+0.015+0.004}_{-0.023-0.014}$
$A_{CP}(\rho^-\pi^+)$	-0.18 ± 0.12	$-0.227^{+0.009+0.082}_{-0.011-0.044}$		-0.225	-0.281	$-0.284^{+0.064+0.045}_{-0.045-0.047}$
$A_{CP}(\rho^0\pi^0)$	-0.30 ± 0.38	$0.110^{+0.050+0.235}_{-0.057-0.288}$	$-75 \sim 0$	0.048	0.058	$0.187^{+0.004+0.012}_{-0.001-0.007}$
$A_{CP}(K^{*0}K^+)$	-	$-0.089^{+0.011+0.028}_{-0.011-0.024}$	$-0.069^{+0.056+0.010+0.092+0.040}_{-0.053-0.003-0.065-0.0060}$	0.360	0.191	$0.257^{+0.039+0.023}_{-0.042-0.019}$
$A_{CP}(k^{*0}K^0)$	-	$-0.035^{+0.013+0.007}_{-0.017-0.020}$	-	0.000	0.000	0^{+0+0}_{-0-0}

QCDF: **H. Y. Cheng and C. K. Chua, Phys. Rev. D 80, 114008 (2009).**

pQCD: **H. N. Li and S. Mishima, Phys. Rev. D 74, 094020 (2006).**
C. D. Lu and M. Z. Yang, Eur. Phys. J. C 23, 275 (2002).

Table: Comparisons of predictions between our framework and QCDF, pQCD methods

$\theta^a_{\pi K^*} = \theta^a_{K\phi} = 5^\circ$.

Mode	Data[31]	QCDF[11]	pQCD(LO)[33]	this work		
				LO	NLO+Vertex	NLO(a^{eff}, θ^a)
$B^+ \to K^{*0}\pi^+$	9.9 ± 0.8	$10.4^{+1.3+4.3}_{-1.5-3.9}$	$6.0^{+2.8+2.7}_{-1.5-1.4}(5.5)$	7.5	10.3	$9.8^{+2.8+2.5}_{-1.7-0.9}$
$B^+ \to K^{*+}\pi^0$	6.9 ± 2.3	$6.7^{+0.7+2.4}_{-0.7-2.2}$	$4.3^{+5.0+1.7}_{-2.2-1.0}(4.0)$	4.7	6.2	$5.9^{+2.0+1.4}_{-1.1-0.3}$
$B^0 \to K^{*-}\pi^+$	8.6 ± 0.9	$9.2^{+1.0+3.7}_{-1.0-3.3}$	$6.0^{+6.8+2.4}_{-2.6-1.3}(5.1)$	6.5	8.8	$8.9^{+2.4+2.4}_{-1.4-1.0}$
$B^0 \to K^{*0}\pi^0$	2.4 ± 0.7	$3.5^{+0.4+1.6}_{-0.4-1.4}$	$2.0^{+1.2+0.9}_{-0.6-0.4}(1.5)$	2.5	3.5	$3.9^{+0.8+1.1}_{-0.6-0.3}$
$B^+ \to \phi K^+$	8.30 ± 0.65	$8.8^{+2.8+4.4}_{-2.7-3.6}$	$7.8^{+5.9+5.8}_{-1.8-1.7}(13.8)$	10.8	9.3	$7.6^{+1.8+0.6}_{-1.4-0.6}$
$B^0 \to \phi K^0$	8.3 ± 1.1	$8.1^{+2.6+4.4}_{-2.5-3.3}$	$7.3^{+5.4+5.1}_{-1.8-1.5}(12.9)$	10.4	8.9	$7.3^{+1.6+0.5}_{-1.3-0.7}$
$A_{CP}(K^{*0}\pi^+)$	-0.038 ± 0.042	$0.004^{+0.013+0.043}_{-0.016-0.039}$	$-0.01^{+0.01+0.01}_{-0.00-0.00}(-0.03)$	-0.021	-0.017	$-0.017^{+0.003+0.012}_{-0.002-0.003}$
$A_{CP}(K^{*+}\pi^0)$	0.04 ± 0.29	$0.016^{+0.031+0.111}_{-0.017-0.144}$	$-0.32^{+0.21+0.16}_{-0.28-0.19}(-0.38)$	-0.348	-0.224	$-0.103^{+0.055+0.081}_{-0.027-0.016}$
$A_{CP}(K^{*+}\pi^-)$	-0.23 ± 0.08	$-0.121^{+0.005+0.126}_{-0.005-0.160}$	$-0.60^{+0.32+0.20}_{-0.19-0.15}(-0.56)$	-0.443	-0.357	$-0.327^{+0.020+0.070}_{-0.008-0.016}$
$A_{CP}(K^{*0}\pi^0)$	-0.15 ± 0.12	$-0.108^{+0.018+0.091}_{-0.028-0.063}$	$-0.11^{+0.07+0.05}_{-0.05-0.02}(-0.60)$	0.012	-0.067	$-0.130^{+0.016+0.004}_{-0.028-0.019}$
$A_{CP}(\phi K^+)$	0.23 ± 0.15	$0.006^{+0.001+0.001}_{-0.001-0.001}$	$0.01^{+0.00+0.00}_{-0.01-0.01}(-0.02)$	-0.022	-0.022	$-0.023^{+0.004+0.00[illegible]}_{-0.002-0.014}$
$A_{CP}(\phi K^0)$	-0.01 ± 0.06	$0.009^{+0.001+0.001}_{-0.001-0.001}$	$0.03^{+0.01+0.00}_{-0.02-0.01}(0.00)$	0	0	0^{+0+0}_{-0-0}

QCDF: **H. Y. Cheng and C. K. Chua, Phys. Rev. D 80, 114008 (2009).**

pQCD: **H. N. Li and S. Mishima, Phys. Rev. D 74, 094020 (2006).**

Table: Comparisons of predictions between our framework and QCDF, pQCD methods

$\theta^a_{\omega K} = \theta^a_{\rho K} = 60°$

Mode	Data[31]	QCDF[11]	pQCD(LO)[33]	this work		
				LO	NLO+Vertex	NLO(a^{eff}, θ^a)
$B^+ \to \rho^+ K^0$	8.0 ± 1.45	$7.8^{+6.3+7.3}_{-2.9-4.4}$	$8.7^{+6.8+6.4}_{-4.4-4.3}(3.6)$	4.2	5.2	$6.8^{+0.3+1.2}_{-0.2-1.1}$
$B^+ \to \rho^0 K^+$	3.81 ± 0.48	$3.5^{+2.9+2.9}_{-1.2-1.8}$	$5.1^{+4.1+3.6}_{-2.8-2.6}(2.5)$	2.3	3.0	$2.6^{+0.3+0.4}_{-0.2-1.0}$
$B^0 \to \rho^- K^+$	8.6 ± 1.0	$8.6^{+5.7+7.4}_{-2.8-4.5}$	$8.8^{+6.8+6.4}_{-4.4-4.3}(4.7)$	4.9	5.4	$7.3^{+0.8+2.0}_{-0.5-0.4}$
$B^0 \to \rho^0 K^0$	4.7 ± 0.7	$5.4^{+3.4+4.3}_{-1.7-2.8}$	$4.8^{+4.3+3.2}_{-2.3-2.0}(2.5)$	2.7	2.8	$5.0^{+0.6+1.7}_{-0.4-0.3}$
$B^+ \to \omega K^+$	6.7 ± 0.5	$4.8^{+4.4+3.5}_{-1.9-2.3}$	$10.6^{+10.4+7.2}_{-5.8-4.4}(2.1)$	2.4	3.6	$5.3^{+0.6+1.2}_{-0.4-0.5}$
$B^0 \to \omega K^0$	5.0 ± 0.6	$4.1^{+4.2+3.3}_{-1.7-2.2}$	$9.8^{+8.8+8.7}_{-4.9-4.3}(1.9)$	1.9	3.2	$4.8^{+0.1+1.1}_{-0.3-0.5}$
$A_{CP}(\rho^+ K^0)$	-0.12 ± 0.17	$0.003^{+0.002+0.005}_{-0.003-0.002}$	$0.01^{+0.01+0.01}_{-0.01-0.01}(0.02)$	0.019	0.016	$0.014^{+0.001+0.017}_{-0.001-0.015}$
$A_{CP}(\rho^0 K^+)$	0.37 ± 0.11	$0.454^{+0.178+0.314}_{-0.194-0.232}$	$0.71^{+0.25+0.17}_{-0.35-0.14}(0.79)$	0.726	0.635	$0.463^{+0.041+0.025}_{-0.036-0.014}$
$A_{CP}(\rho^- K^+)$	0.15 ± 0.06	$0.319^{+0.115+0.196}_{-0.110-0.127}$	$0.64^{+0.24+0.07}_{-0.30-0.11}(0.83)$	0.593	0.605	$0.290^{+0.021+0.012}_{-0.020-0.007}$
$A_{CP}(\rho^0 K^0)$	0.06 ± 0.20	$0.087^{+0.012+0.087}_{-0.012-0.068}$	$0.07^{+0.08+0.07}_{-0.05-0.04}(0.07)$	-0.040	0.056	$-0.015^{+0.005+0.025}_{-0.002-0.021}$
$A_{CP}(\omega K^+)$	0.02 ± 0.05	$0.221^{+0.137+0.140}_{-0.128-0.130}$	$0.32^{+0.15+0.04}_{-0.17-0.05}(0.32)$	0.688	0.453	$0.091^{+0.031+0.020}_{-0.038-0.073}$
$A_{CP}(\omega K^0)$	0.32 ± 0.17	$-0.047^{+0.018+0.055}_{-0.016-0.058}$	$-0.03^{+0.02+0.02}_{-0.04-0.03}(-0.03)$	0.065	-0.011	$0.026^{+0.001+0.0\text{[illegible]}5}_{-0.001-0.024}$

QCDF: **H. Y. Cheng and C. K. Chua, Phys. Rev. D 80, 114008 (2009).**

pQCD: **H. N. Li and S. Mishima, Phys. Rev. D 74, 094020 (2006).**

Numerical Results:
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Table: Branching ratios for $B \to VV$ decay modes (in unit of 10^{-6}) which includes the contribution of effective Wilson coefficients and effect of different strong phase $\theta^a = 60^\circ \pm 15^\circ$) for annihilation diagram.

Mode	Data[HFAG]	This work				
		NLO+Vertex	NLO^{eff}	NLO^{eff} (45°)	NLO^{eff} (60°)	NLO^{eff} (75°)
$B^+ \to \rho^+\rho^0$	24.0 ± 2.0	13.4	16.8	16.8	16.8	16.8
$B^0 \to \rho^+\rho^-$	24.2 ± 3.1	22.3	19.8	21.7	22.3	22.7
$B^0 \to \rho^0\rho^0$	0.73 ± 0.27	0.4	0.92	0.67	0.61	0.57
$B^+ \to K^{*0}\rho^+$	9.2 ± 1.5	16.2	14.0	9.6	8.3	7.2
$B^+ \to K^{*+}\rho^0$	< 6.1	9.9	9.0	6.4	5.6	5.0
$B^0 \to K^{*+}\rho^-$	< 12	13.9	13.0	9.1	7.9	6.9
$B^0 \to K^{*0}\rho^0$	3.4 ± 1.0	5.6	5.2	3.6	3.1	2.7
$B^+ \to \bar{K}^{*0}K^{*+}$	1.2 ± 0.5	0.9	0.8	0.6	0.5	0.4
$B^0 \to \bar{K}^{*0}K^{*0}$	1.28 ± 0.35	0.8	0.6	0.5	0.5	0.5
$B^0 \to K^{*+}K^{*-}$	< 2	0.07	0.07	0.07	0.007	0.07
$B^+ \to \phi K^{*+}$	10.0 ± 1.1	19.4	15.2	10.9	9.5	8.4
$B^0 \to \phi K^{*0}$	9.8 ± 0.7	18.7	14.8	10.5	9.2	8.1
$B^+ \to \omega K^{*+}$	< 7.4	5.6	4.2	3.3	3.0	2.8
$B^0 \to \omega K^{*0}$	2.0 ± 0.5	6.2	4.1	2.8	2.5	2.2

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Mode	Data[31]	This work				
		NLO+Vertex	NLO^{eff}	$\mathrm{NLO}^{eff}(45^\circ)$	$\mathrm{NLO}^{eff}(60^\circ)$	$\mathrm{NLO}^{eff}(75^\circ)$
$A_{CP}(\rho^+\rho^0)$	-0.051 ± 0.054	0.001	0.001	0.001	0.001	0.001
$A_{CP}(\rho^+\rho^-)$	0.06 ± 0.13	-0.002	-0.029	-0.041	-0.038	-0.033
$A_{CP}(\rho^0\rho^0)$	-	0.702	0.177	0.350	0.417	0.475
$A_{CP}(K^{*0}\rho^+)$	-0.01 ± 0.16	-0.005	-0.006	-0.009	-0.009	-0.008
$A_{CP}(K^{*+}\rho^0)$	$0.20^{+0.32}_{-0.29}$	0.184	0.182	0.266	0.273	0.257
$A_{CP}(K^{*-}\rho^+)$	-	0.148	0.151	0.231	0.231	0.206
$A_{CP}(K^{*0}\rho^0)$	0.09 ± 0.19	-0.090	-0.101	-0.147	-0.155	-0.154
$A_{CP}(\bar{K}^{*0}K^{*+})$	-	0.081	0.085	0.141	0.143	0.128
$A_{CP}(\bar{K}^{*0}K^{*0})$	-	0	0	0	0	0
$A_{CP}(K^{*+}K^{*-})$	-	-0.261	-0.261	-0.261	-0.261	-0.261
$A_{CP}(\phi K^{*+})$	-0.01 ± 0.08	-0.003	-0.003	-0.0006	-0.007	-0.007
$A_{CP}(\phi K^{*0})$	0.01 ± 0.05	0	0	0	0	0
$A_{CP}(\omega K^{*+})$	0.29 ± 0.35	0.341	0.383	0.534	0.522	0.463
$A_{CP}(\omega K^{*0})$	0.45 ± 0.25	0.078	0.116	0.170	0.182	0.185

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The longitudinal polarization fraction

Mode	Data[31]	This work				
		NLO+Vertex	NLO^{eff}	$\mathrm{NLO}^{eff}(45^\circ)$	$\mathrm{NLO}^{eff}(60^\circ)$	$\mathrm{NLO}^{eff}(75^\circ)$
$f_L(\rho^+\rho^0)$	0.950 ± 0.016	0.94	0.92	0.95	0.95	0.95
$f_L(\rho^+\rho^-)$	0.978 ± 0.023	0.95	0.95	0.95	0.95	0.95
$f_L(\rho^0\rho^0)$	0.75 ± 0.15	0.84	0.86	0.77	0.74	0.71
$f_L(K^{*0}\rho^+)$	0.48 ± 0.08	0.85	0.82	0.57	0.45	0.32
$f_L(K^{*+}\rho^0)$	$0.96^{+0.06}_{-0.16}$	0.86	0.85	0.65	0.56	0.47
$f_L(K^{*-}\rho^+)$	-	0.81	0.80	0.57	0.46	0.34
$f_L(K^{*0}\rho^0)$	0.57 ± 0.12	0.78	0.75	0.48	0.36	0.22
$f_L(\bar{K}^{*0}K^{*+})$	$0.75^{+0.16}_{-0.26}$	0.85	0.81	0.60	0.49	0.37
$f_L(\bar{K}^{*0}K^{*0})$	0.80 ± 0.13	0.83	0.63	0.60	0.53	0.46
$f_L(K^{*+}K^{*-})$		0.99	0.99	0.99	0.99	0.99
$f_L(\phi K^{*+})$	0.50 ± 0.05	0.87	0.83	0.58	0.45	0.31
$f_L(\phi K^{*0})$	0.480 ± 0.030	0.87	0.83	0.58	0.45	0.31
$f_L(\omega K^{*+})$	0.41 ± 0.19	0.90	0.86	0.68	0.58	0.48
$f_L(\omega K^{*0})$	0.70 ± 0.13	0.93	0.89	0.67	0.53	0.37

Table: Comparisons of predictions between our framework and QCDF, pQCD methods

with $\tilde{\mu}_g$=0.52GeV and $\theta^a = 60°$. The first error arises from the varying for $\mu_{scale} = 1.4 \sim 1.6$ GeV, the second one stems from the shape parameters of light mesons.

Mode	Data[31]	QCDF[11]	pQCD [47, 48]	this work			
				LO	NLO+Vertex	NLO(θ^a)	NLO(a^{eff}, θ^a)
$B^+ \to \rho^+\rho^0$	24.0 ± 2.0	$20.0^{+4.0+2.0}_{-1.9-0.9}$	$17 \pm 2 \pm 1$	13.7	13.4	13.4	$16.8^{+9.9+1.2}_{-4.4-0.7}$
$B^0 \to \rho^+\rho^-$	24.2 ± 3.1	$25.5^{+1.5+2.4}_{-2.6-1.5}$	$35 \pm 5 \pm 4$	21.1	22.3	24.9	$22.2^{+14.0+1.3}_{-6.2-0.7}$
$B^0 \to \rho^0\rho^0$	0.73 ± 0.27	$0.9^{+1.5+1.1}_{-0.4-0.2}$	$0.9 \pm 0.1 \pm 0.1$	0.3	0.4	0.4	$0.6^{+0.2+0.1}_{-0.1-0.0}$
$B^+ \to K^{*0}\rho^+$	9.2 ± 1.5	$9.2^{+1.2+3.6}_{-1.1-5.4}$	17 (13)	11.4	16.2	9.8	$8.8^{+3.4+3.5}_{-1.2-2.4}$
$B^+ \to K^{*+}\rho^0$	< 6.1	$5.5^{+0.6+1.3}_{-0.5-2.5}$	9.0 (6.4)	7.3	9.9	6.2	$5.9^{+2.1+1.9}_{-1.0-1.2}$
$B^0 \to K^{*+}\rho^-$	< 12	$8.9^{+1.1+4.8}_{-1.0-5.5}$	13 (9.8)	10.2	13.9	8.5	$8.3^{+2.1+3.2}_{-1.0-2.5}$
$B^0 \to K^{*0}\rho^0$	3.4 ± 1.0	$4.6^{+0.6+3.5}_{-0.5-3.5}$	5.9 (4.7)	3.9	5.6	3.4	$3.3^{+0.5+1.7}_{-0.2-1.1}$
$B^+ \to \bar{K}^{*0}K^{*+}$	1.2 ± 0.5	$0.6^{+0.1+0.3}_{-0.1-0.3}$	0.48	0.7	0.9	0.6	$0.5^{+0.2+0.2}_{-0.1-0.1}$
$B^0 \to \bar{K}^{*0}K^{*0}$	1.28 ± 0.35	$0.6^{+0.1+0.2}_{-0.1-0.3}$	0.35	0.5	0.8	0.6	$0.5^{+0.2+0.2}_{-0.1-0.1}$
$B^0 \to K^{*+}K^{*-}$	< 2		$0.1^{+0.0+0.1}_{-0.0-0.1}$	0.07	0.07	0.07	$0.07^{+0.01+0.00}_{-0.01-0.01}$
$B^+ \to \phi K^{*+}$	10.0 ± 1.1	$10.0^{+1.4+12.3}_{-1.3-6.1}$	15.96	15.9	19.4	12.4	$9.6^{+2.5+2.4}_{-0.6-1.6}$
$B^0 \to \phi K^{*0}$	9.8 ± 0.7	$9.5^{+1.3+11.9}_{-1.2-5.9}$	$14.86(10.2^{+2.5}_{-2.1})$	15.4	18.7	11.8	$9.2^{+2.3+2.3}_{-0.5-1.6}$
$B^+ \to \omega K^{*+}$	< 7.4	$3.0^{+0.4+2.5}_{-0.3-1.5}$	7.9(5.5)	5.4	5.6	3.7	$3.0^{+1.0+2.1}_{-0.4-1.0}$
$B^0 \to \omega K^{*0}$	2.0 ± 0.5	$2.5^{+0.4+2.5}_{-0.4-1.5}$	9.6(6.6)	5.8	6.2	3.8	$2.5^{+0.7+1.0}_{-0.3-1.2}$

QCDF: **H. Y. Cheng and C. K. Chua, Phys. Rev. D 80, 114008 (2009).**

pQCD: **H. W. Huang, C. D. Lu, et. al. Phys. Rev. D 73, 014011 (2006).**
H. n. Li and S. Mishima, Phys. Rev. D 73, 114014 (2006).

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Mode	Data[5, 42]	This work				
		NLO	NLO^{eff}	NLO$^{eff}(-10^\circ)$	NLO$^{eff}(5^\circ)$	NLO$^{eff}(20^\circ)$
$B_s \to \pi^+ K^-$	5.0 ± 1.25	7.7	7.0	7.0	7.1	7.2
$B_s \to \pi^0 \bar{K}^0$	-	0.2	1.1	1.1	1.1	1.1
$B_s \to K^+ K^-$	$24.4 \pm 1.4 \pm 3.5$	20.8	20.5	17.5	22.0	26.2
$B_s \to K^0 \bar{K}^0$	-	22.6	20.7	17.5	22.3	26.9
$A_{CP}(\pi^+ K^-)$	39 ± 17	$Br(B_s \to K^- K^+) \approx Br(B_d \to \pi^- K^+)$			27.8	24.7
$A_{CP}(\pi^0 \bar{K}^0)$	-	77.6	61.6	56.0	64.4	72.1
$A_{CP}(K^+ K^-)$	-	-14.4	-15.4	-18.2	-14.2	-10.8
$A_{CP}(K^0 \bar{K}^0)$	-	0	0	0	0	0

$$A(B_s \to K^- \pi^+) \approx A(B_d \to \pi^- \pi^+)$$

$$Br(B_s \to K^- \pi^+) \approx Br(B_d \to \pi^- \pi^+)$$

$$A_{CP}(B_s \to K^- \pi^+) \approx A_{CP}(B_d \to \pi^- \pi^+)$$

SU(3) symmetry

$$Br(B_s \to K^- K^+) \approx Br(B_d \to \pi^- K^+)$$

$$A_{CP}(B_s \to K^- K^+) \approx A_{CP}(B_d \to \pi^- K^+)$$

M. Gronau, PLB 492, 297 (2000). X. G. He, J. Y. Leou and C. Y. Wu, PRD 62, 114015 (2000).

Numerical Results:

Branch Ratio & CP Violation for B_s→ PP

Table: Comparisons of predictions between our framework and other methods in $B_s \to PP$ decays.

Mode	Data	QCDF	pQCD	SCET	This work		
					LO	NLO	NLO(a^{eff}, θ^a)
$B_s \to \pi^+ K^-$	5.0 ± 1.25	$5.3^{+0.4+0.4}_{-0.8-0.5}$	$7.6^{+3.2+0.7+0.5}_{-2.3-0.7-0.5}$	$4.9 \pm 1.2 \pm 1.3 \pm 0.3$	7.2	7.7	$7.1^{+3.2+0.6}_{-1.8-0.2}$
$B_s \to \pi^0 \bar{K}^0$	-	$1.7^{+2.5+1.2}_{-0.8-0.5}$	$0.16^{+0.05+0.10+0.02}_{-0.04-0.05-0.01}$	$0.76 \pm 0.26 \pm 0.27 \pm 0.17$	0.2	0.2	$1.1^{+0.3+0.6}_{-0.2-0.1}$
$B_s \to K^+ K^-$	$24.4 \pm 1.4 \pm 3.5$	$25.2^{+12.7+12.5}_{-7.2-9.1}$	$13.6^{+4.2+7.5+0.7}_{-3.2-4.1-0.2}$	$18.2 \pm 6.7 \pm 1.1 \pm 0.5$	16.6	20.8	$22.0^{+5.0+11.8}_{-3.4-3.0}$
$B_s \to K^0 \bar{K}^0$	-	$26.1^{+13.5+12.9}_{-8.1-9.4}$	$15.6^{+5.0+8.3+0.0}_{-3.8-4.7-0.0}$	$17.7 \pm 6.6 \pm 0.5 \pm 0.6$	18.2	22.6	$22.3^{+4.3+12.2}_{-5.3-3.2}$
$B_s \to \pi^+ \pi^-$	0.5 ± 0.5	$0.26^{+0.00+0.10}_{-0.00-0.09}$	$0.57^{+0.16+0.09+0.01}$ [illegible]		0.18	0.23	$0.23^{+0.01+0.26}_{-0.01-0.01}$
$B_s \to \pi^0 \pi^0$	-	$0.13^{+0.0+0.05}_{-0.0-0.05}$	0.[illegible]		0.09	0.12	$0.12^{+0.01+0.13}_{-0.01-0.0}$[illegible]
$A_{CP}(\pi^+ K^-)$	39 ± 17	$20.7^{+5.0+3.9}_{-3.0-8.8}$	$24.1^{+3.9+3.3+2.3}_{-3.6-3.0-1.2}$	$20 \pm 17 \pm 19 \pm 5$	21.5	24.3	$27.8^{+6.0+6.9}_{-2.0-4.1}$
$A_{CP}(\pi^0 \bar{K}^0)$	-	$36.3^{+17.4+26.6}_{-18.2-24.3}$	$59.4^{+1.8+7.4+2.2}_{-4.0-1.3-3.5}$	$-58 \pm 39 \pm 39 \pm 13$	1.2	77.6	$64.4^{+2.0+6.0}_{-1.8-11.6}$
$A_{CP}(K^+ K^-)$	-	$-7.7^{+1.6+4.0}_{-1.2-5.1}$	$-23.3^{+0.9+4.9+0.8}_{-0.2-4.4-1.1}$	$-6 \pm 5 \pm 6 \pm 2$	-15.7	-14.4	$-14.2^{+0.1+1.7}_{-0.1-0.}$[illegible]
$A_{CP}(K^0 \bar{K}^0)$	-	$0.4^{+0.04+0.10}_{-0.04-0.04}$	0	< 10	0.0	0.0	0.0^{+0+0}_{-0-0}
$A_{CP}(\pi^+ \pi^-)$	-	0	$-1.2^{+0.1+1.2+0.1}_{-0.4-1.2-0.1}$		5.2	4.5	$4.5^{+0.4+1.5}_{-0.2-0.5}$
$A_{CP}(\pi^0 \pi^0)$	-	0	$-1.2^{+0.1+1.2+0.1}_{-0.4-1.2-0.1}$		5.2	4.5	$4.5^{+0.4+1.5}_{-0.2-0.5}$

Pure annihilation processes

QCDF: H. Y. Cheng and C. K. Chua, Phys. Rev. D 80, 114026 (2009).

pQCD: A. Ali, G. Kramer, Y. Li, C. D. Lu, et. Al. Phys. Rev. D76, 074018(2007).

SCET: A. R. Williamson and J. Zupan, Phys. Rev. D 74, 014003 (2006).

Numerical Results:
Branch Ratio & CP Violation for B_s→ PV

$B_s \to \pi K^*, \rho K$

$B_s \to V$ 45, 60, 75

$B_s \to P$ -10, 5, 20

Tree dominated

Color suppressed

Mode	This work				
	NLO	NLO^{eff}	$NLO^{eff}(45^\circ)$	$NLO^{eff}(60^\circ)$	$NLO^{eff}(75^\circ)$
$B_s \to \pi^+ K^{*-}$	8.2	7.3	7.2	7.2	7.3
$B_s \to \pi^0 \bar{K}^{*0}$	0.2	0.3	0.3	0.3	0.3
$A_{CP}(\pi^- K^{*+})$	-27.5	-28.3	-18.7	-12.8	-6.0
$A_{CP}(\pi^0 \bar{K}^{*0})$	-29.3	66.5	15.4	-3.8	-22.7
	NLO	NLO^{eff}	$NLO^{eff}(-10^\circ)$	$NLO^{eff}(5^\circ)$	$NLO^{eff}(20^\circ)$
$B_s \to \rho^+ K^-$	19.7	17.5	17.5	17.6	17.7
$B_s \to \rho^0 \bar{K}^0$	0.4	0.6	0.5	0.6	0.6
$A_{CP}(\rho^- K^+)$	17.9	18.9	19.2	18.5	16.4
$A_{CP}(\rho^0 \bar{K}^0)$	77.0	-29.7	-36.8	-25.8	-12.9

Numerical Results:
Branch Ratio & CP Violation for B_s→ PV

Mode	This work						
	NLO	NLO^{eff}	$(5^\circ, 60^\circ)$	$(-10^\circ, 60^\circ)$	$(20^\circ, 60^\circ)$	$(5^\circ, 45^\circ)$	$(5^\circ, 75^\circ)$
$B_s \to K^{*-}K^+$	5.8	6.6	7.8	7.8	7.8	7.7	7.8
$B_s \to K^-K^{*+}$	8.1	7.6	8.2	6.5	9.7	8.2	8.2
$B_s \to K^{*0}\bar{K}^0$	9.0	7.9	8.5	6.7	10.3	8.5	8.5
$B_s \to K^0\bar{K}^{*0}$	5.6	7.5	7.1	7.1	7.1	7.4	6.6
$B_s \to \rho^-\pi^+$	0.04	0.04	0.008	0.006	0.014	0.014	0.006
$B_s \to \pi^-\rho^+$	0.04	0.04	0.006	0.003	0.011	0.011	0.003
$B_s \to \pi^0\rho^0$	0.04	0.04	0.006	0.003	0.012	0.012	0.005
$A_{CP}(K^+K^{*-})$	54.0	48.8	23.2	23.1	23.2	31.4	14.0
$A_{CP}(K^{*+}K^-)$	-32.6	-32.2	-29.5	-38.2	-22.0	-29.5	-29.5
$A_{CP}(K^0\bar{K}^{*0})$	0	0	0	0	0	0	0
$A_{CP}(K^{*0}\bar{K}^0)$	0	0	0	0	0	0	0
$A_{CP}(\rho^+\pi^-)$	-1.9	-1.9	-0.6	-0.2	-1.1	-1.1	-0.2
$A_{CP}(\pi^+\rho^-)$	-1.6	-1.6	-0.4	-0.3	-0.7	-0.7	-0.3
$A_{CP}(\pi^0\rho^0)$	-1.7	-1.7	-0.6	-0.3	-0.9	-0.9	-0.3

Numerical Results:
Branch Ratio & CP Violation for B_s→ PV

Table: The branching ratios (in units of $\times 10^{-6}$) of $B_s \to PV$ decays. For comparison, we also quote the theoretical estimates of the branching ratios in the QCDF and pQCD frameworks.

Mode	QCDF	pQCD	This work		
			LO	NLO	NLO(a^{eff}, θ^a)
$B_s \to \pi^+ K^{*-}$	$7.8^{+0.4+0.5}_{-0.7-0.7}$	$7.6^{+2.9+0.4+0.5}_{-2.2-0.5-0.3}$			$7.2^{+5.6+0.7}_{-2.2-0.5}$
$B_s \to \pi^0 \bar{K}^{*0}$	$0.89^{+0.80+0.84}_{-0.34-0.35}$	$0.07^{+0.02+0.04+0.01}_{-0.01-0.02-0.01}$			$0.3^{+0.1+0.1}_{-0.1-0.1}$
$B_s \to \rho^+ K^-$	$14.7^{+1.4+0.9}_{-1.9-1.3}$	$17.8^{+7.7+1.3+1.1}_{-5.6-1.6-0.9}$			$17.6^{+8.2+0.1}_{-4.6-0.1}$
$B_s \to \rho^0 \bar{K}^0$	$1.9^{+2.9+1.4}_{-0.9-0.6}$	$0.08^{+0.02+0.07+0.01}_{-0.02-0.03-0.00}$			$0.6^{+0.2+0.1}_{-0.1-0.1}$
$B_s \to K^{*-} K^+$	$11.3^{+7.0+8.1}_{-3.5-5.1}$	$4.7^{+1.1+2.5+0.0}_{-0.8-1.4-0.0}$			$7.8^{+0.3+1.5}_{-0.5-1.1}$
$B_s \to K^- K^{*+}$	$10.3^{+3.0+4.8}_{-2.2-4.2}$	$6.0^{+1.7+1.7+0.7}_{-1.5-1.2-0.3}$			$8.2^{+1.3+2.1}_{-2.3-2.0}$
$B_s \to K^{*0} \bar{K}^0$	$10.5^{+3.4+5.1}_{-2.8-4.5}$	$7.3^{+2.5+2.1+0.0}_{-1.7-1.3-0.0}$			$8.5^{+1.8+1.5}_{-2.1-1.6}$
$B_s \to K^0 \bar{K}^{*0}$	$10.1^{+7.5+7.7}_{-3.6-4.8}$	$4.3^{+0.7+2.2+0.0}_{-0.7-1.4-0.0}$	4.6	5.6	$7.1^{+0.2+1.3}_{-0.4-1.1}$
$B_s \to \rho^- \pi^+$	$0.02^{+0.00+0.01}_{-0.00-0.01}$	$0.22^{+0.05+0.04+0.00}_{-0.05-0.06-0.01}$			$0.01^{+0.00+0.01}_{-0.00-0.00}$
$B_s \to \pi^- \rho^+$	$0.02^{+0.00+0.01}_{-0.00-0.01}$	$0.24^{+0.05+0.05+0.00}_{-0.05-0.06-0.01}$			$0.01^{+0.00+0.01}_{-0.00-0.00}$
$B_s \to \pi^0 \rho^0$	$0.02^{+0.00+0.01}_{-0.00-0.01}$	$0.23^{+0.05+0.05+0.00}_{-0.05-0.06-0.01}$			$0.01^{+0.00+0.01}_{-0.00-0.00}$

Tree dominated

Color suppressed

Penguin dominated,

annihilation contributions

QCDF: H. Y. Cheng and C. K. Chua, Phys. Rev. D 80, 114026 (2009).

pQCD: A. Ali, G. Kramer, Y. Li, C. D. Lu, et. Al. Phys. Rev. D76, 074018(2007).

Numerical Results:
Branch Ratio & CP Violation for B_s→ PV

Table: The direct CP asymmetries (in %) in the $B_s \to PV$ decays.

Mode	QCDF	pQCD	This work		
			LO	NLO	NLO(a^{eff}, θ^a)
$A_{CP}(\pi^- K^{*+})$	$-24.0^{+1.2+7.7}_{-1.5-3.9}$	$-19.0^{+2.5+2.7+0.9}_{-2.6-3.4-1.4}$	-24.9	-27.5	$-12.8^{+7.0+4.9}_{-5.2-3.5}$
$A_{CP}(\pi^0 K^{*0})$	$-26.3^{+10.8+42.2}_{-10.9-36.7}$	$-47.1^{+7.4+35.5+2.9}_{-8.7-29.8-7.0}$	40.1	-29.3	$-3.8^{+6.1+7.5}_{-6.7-7.4}$
$A_{CP}(\rho^- K^+)$	$11.7^{+3.5+10.1}_{-2.1-11.6}$	$14.2^{+2.4+2.3+1.2}_{-2.2-1.6-0.7}$	17.2	17.9	$18.5^{+3.0+2.9}_{-2.6-2.7}$
$A_{CP}(\rho^0 \bar{K}^0)$	$28.9^{+14.6+25.0}_{-14.5-23.7}$	$73.4^{+6.4+16.2+2.2}_{-11.7-47.8-3.9}$	-22.4	77.0	$-25.8^{+4.1+4.5}_{-4.1-4.6}$
$A_{CP}(K^+ K^{*-})$	$25.5^{+9.2+16.3}_{-8.8-11.3}$	$55.3^{+4.4+8.5+5.1}_{-4.9-9.8-2.5}$	52.5	54.0	$23.2^{+1.4+2.7}_{-1.4-2.7}$
$A_{CP}(K^{*+} K^-)$	$-11.0^{+0.5+14.0}_{-0.4-18.8}$	$-36.6^{+2.3+2.8+1.3}_{-2.3-3.5-1.2}$	-40.0	-32.6	$-29.5^{+8.5+4.3}_{-8.5-4.9}$
$A_{CP}(K^0 \bar{K}^{*0})$	$0.10^{+0.08+0.05}_{-0.07-0.02}$	0	0	0	0^{+0+0}_{-0-0}
$A_{CP}(K^{*0} \bar{K}^0)$	$0.49^{+0.08+0.09}_{-0.07-0.12}$	0	0	0	0^{+0+0}_{-0-0}
$A_{CP}(\rho^+ \pi^-)$	$-11.1^{+0.7+13.9}_{-0.8-15.7}$	$-1.3^{+0.9+2.8+0.1}_{-0.4-3.5-0.2}$	-1.8	-1.9	$-0.6^{+0.1+1.1}_{-0.1-2.0}$
$A_{CP}(\pi^+ \rho^-)$	$10.2^{+0.8+12.7}_{-0.7-12.8}$	$4.6^{+0.0+2.9+0.6}_{-0.6-3.5-0.3}$	-1.5	-1.6	$-0.4^{+0.1+1.5}_{-0.1-0.9}$
$A_{CP}(\pi^0 \rho^0)$	0	$1.7^{+0.2+2.8+0.2}_{-0.8-3.6-0.1}$	-1.6	-1.7	$-0.6^{+0.1+1.3}_{-0.1-1.2}$

QCDF: H. Y. Cheng and C. K. Chua, Phys. Rev. D 80, 114026 (2009).

pQCD: A. Ali, G. Kramer, Y. Li, C. D. Lu, et. Al. Phys. Rev. D76, 074018(2007).

Numerical Results:
Branch Ratio & CP Violation for B_s→ VV

Color suppressed

Tree dominated

Penguin dominated

Mode	Exp[35, 46, 47]	This work				
		NLO	NLO^{eff}	NLO$^{eff}(45^\circ)$	NLO$^{eff}(60^\circ)$	NLO$^{eff}(75^\circ)$
$B_s \to \rho^0 \bar{K}^{*0}$	< 767	0.6	0.8	0.7	0.7	0.7
$B_s \to \rho^+ K^{*-}$		23.6	21.0	20.7	20.6	20.6
$B_s \to K^{*-} K^{*+}$		13.4	12.8	11.0	10.4	9.8
$B_s \to K^{*0} \bar{K}^{*0}$	< 1681	15.0	13.1	10.6	9.8	9.1
$B_s \to \phi\phi$	24.0 ± 8.9	22.1	18.7	12.1	10.0	7.9
$A_{CP}(\rho^0 \bar{K}^{*0})$		66.8	56.4	60.8	56.8	50.0
$A_{CP}(\rho^+ K^{*-})$		-10.1	-10.8	-11.3	-9.7	-7.2
$A_{CP}(K^{*-} K^{*+})$		20.1	17.9	26.3	26.4	24.6
$A_{CP}(K^{*0} \bar{K}^{*0})$		0	0	0	0	0
$A_{CP}(\phi\phi)$		0	0	0	0	0
$f_L(\rho^0 \bar{K}^{*0})$		80	84	79	77	76
$f_L(\rho^+ K^{*-})$		96	96	96	95	95
$f_L(K^{*-} K^{*+})$		72	71	54	48	43
$f_L(K^{*0} \bar{K}^{*0})$		76	72	50	41	32
$f_L(\phi\phi)$	$34.8 \pm 4.1 \pm 2.1$	71	65	50	42	31

Numerical Results:
Branch Ratio & CP Violation for B_s→ VV

Table: The comparisons in theoretical methods in $B_s \to VV$ decays. The central values are obtained with μ_g^a=0.52GeV and $\theta^a = 60°$. The first error in our predictions arises from the varying for $\mu_{scale} = 1.4 \sim 1.6$ GeV, the second one stems from the shape parameters of light mesons.

Mode	Exp	QCDF	pQCD	This work		
				LO	NLO	NLO(a^{eff} θ^a)
$B_s \to \rho^0 \bar{K}^{*0}$	< 767	1.3	0.33	0.2	0.6	$1.0^{+0.3+0.3}_{-0.2-0.2}$
$B_s \to \rho^+ K^{*-}$		21.6	20.9	22.3	23.6	$21.0^{+13.4+2.6}_{-6.2-1.8}$
$B_s \to K^{*-} K^{*+}$		7.6	6.7	10.3	13.4	$10.4^{+3.0+2.7}_{-2.5-1.6}$
$B_s \to K^{*0} \bar{K}^{*0}$	< 1681	[illegible]	[illegible]	[illegible]	[illegible]	$9.8^{+3.1+2.5}_{-2.2-2.1}$
$B_s \to \phi\phi$	24.0 ± 8.9	[illegible]	[illegible]	[illegible]	[illegible]	$10.0^{+2.9+3.1}_{-2.0-2.3}$
$B_s \to \rho^+ \rho^-$		[illegible]	[illegible]	[illegible]	[illegible]	$0.70^{+0.01+0.05}_{-0.01-0.05}$
$B_s \to \rho^0 \rho^0$	< 320	0.34	0.51	0.28	0.35	$0.35^{+0.01+0.02}_{-0.01-0.02}$
$A_{CP}(\rho^0 \bar{K}^{*0})$		46	61.8	52.2	66.8	$56.8^{+1.0+3.0}_{-0.5-2.9}$
$A_{CP}(\rho^+ K^{*-})$		-11	-8.2	-10.0	-10.1	$-9.7^{+3.5+1.3}_{-3.0-1.3}$
$A_{CP}(K^{*-} K^{*+})$		21	9.3	16.1	20.1	$26.4^{+2.4+2.1}_{-2.5-2.5}$
$A_{CP}(K^{*0} \bar{K}^{*0})$		0.4	0	0	0	0^{+0+0}_{-0-0}
$A_{CP}(\phi\phi)$		0.2	0	0^{+0+0}_{-0-0}		
$A_{CP}(\rho^+ \rho^-)$		0	-2.1	5.8	5.0	$5.0^{+1.2+0.4}_{-2.5-0.4}$
$A_{CP}(\rho^0 \rho^0)$		0	-2.1	5.8	5.0	$5.0^{+1.2+0.4}_{-2.5-0.4}$

understand from approx.
SU(3) flavor symmetry

Numerical Results:
Branch Ratio & CP Violation for B_s→ VV

Mode	Exp	QCDF	pQCD	This work		
				LO	NLO	NLO(a^{eff}, θ^a)
$f_L(\rho^0 \bar{K}^{*0})$		90	45.5	73	80	77^{+2+0}_{-1-0}
$f_L(\rho^+ K^{*-})$		92	93.7	96	96	95^{+1+0}_{-0-0}
$f_L(K^{*-} K^{*+})$		52	43.8	66	72	48^{+4+2}_{-4-2}
$f_L(K^{*0} \bar{K}^{*0})$		56	49.7	69	76	41^{+3+1}_{-3-1}
$f_L(\phi\phi)$	$34.8 \pm 4.1 \pm 2.1$	36	61.9	65	71	42^{+3+2}_{-3-2}
$f_L(\rho^+ \rho^-)$		100	~ 100	~ 100	~ 100	~ 100
$f_L(\rho^0 \rho^0)$		100	~ 100	~ 100	~ 100	~ 100

QCDF: H. Y. Cheng and C. K. Chua, Phys. Rev. D 80, 114026 (2009).

pQCD: A. Ali, G. Kramer, Y. Li, C. D. Lu, et. Al. Phys. Rev. D76, 074018(2007).

Approximate SU(3) Flavor Symmetry

SU(3) symmetry relations

$$Br(B_s \to K^-\pi^+) \approx Br(B_d \to \pi^-\pi^+)$$

$$A_{CP}(B_s \to K^-\pi^+) \approx A_{CP}(B_d \to \pi^-\pi^+)$$

$$Br(B_s \to K^-K^+) \approx Br(B_d \to \pi^-K^+)$$

$$A_{CP}(B_s \to K^-K^+) \approx A_{CP}(B_d \to \pi^-K^+)$$

M. Gronau, PLB 492, 297 (2000). **X. G. He, J. Y. Leou and C. Y. Wu, PRD 62, 114015 (2000).**

Our Theoretical Predictions

$$R_{Br} = \frac{Br(B_s \to K^-\pi^+)}{Br(B_d \to \pi^-\pi^+)} \approx 1.08,$$

$$R_{A_{CP}} = \frac{A_{CP}(B_s \to K^-\pi^+)}{A_{CP}(B_d \to \pi^-\pi^+)} \approx 1.07,$$

$$R_{Br} = \frac{Br(B_s \to K^-K^+)}{Br(B_d \to \pi^-K^+)} \approx 1.07$$

$$R_{A_{CP}} = \frac{A_{CP}(B_s \to K^-K^+)}{A_{CP}(B_d \to \pi^-K^+)} \approx 1.08,$$

Approximate SU(3) Flavor Symmetry

SU(3) symmetry relations

$$Br(B_s \to K^{*+}K^-) \approx Br(B_d \to K^{*+}\pi^-)$$

$$A_{CP}(B_s \to K^{*+}K^-) \approx A_{CP}(B_d \to K^{*+}\pi^-)$$

$$Br(B_s \to K^{*-}K^+) \approx Br(B_d \to \rho^- K^+)$$

$$A_{CP}(B_s \to K^{*-}K^+) \approx A_{CP}(B_d \to \rho^- K^+)$$

Our Theoretical Predictions

$$R_{Br} = \frac{Br(B_s \to K^{*+}K^-)}{Br(B_d \to K^{*+}\pi^-)} \approx 0.92, \qquad R_{A_{CP}} = \frac{A_{CP}(B_s \to K^{*+}K^-)}{A_{CP}(B_d \to K^{*+}\pi^-)} \approx 0.9,$$

$$R_{Br} = \frac{Br(B_s \to K^{*-}K^+)}{Br(B_d \to \rho^- K^+)} \approx 1.06, \qquad R_{A_{CP}} = \frac{A_{CP}(B_s \to K^{*-}K^+)}{A_{CP}(B_d \to \rho^- K^+)} \approx 0.8.$$

$$R_{Br} = \frac{Br(B_s \to \phi\phi)}{Br(B_d \to \phi K^{*0})} \approx 1.08.$$

Important to be tested

SUMMARY

The six-quark operator effective Hamiltonian approach based on the perturbative QCD, naïve QCD factorization and nonperturbative-nonlocal twist wave functions has been established to compute the amplitudes and CP violations in charmless two body B-meson decays.

With annihilation contribution and extra strong phase, $\boxed{(\theta^a = 5^\circ)}$ for B→ PP and $\boxed{(\theta^a = 60^\circ)}$ for B→ VV, our framework provides a simple way to evaluate the hadronic matrix elements of two body decays.

In the six-quark operator effective Hamiltonian approach, the factorization is a natural result and it is also more clear to see what approximation is made (such as the Type III diagrams), which makes the physics of long-distance contribution much understandable.

CONCLUSION

It is different from the QCDF as our method allows us to calculate the transition form factors from QCD calculation.

It is different from pQCD as our method includes both the perturbative QCD effects via the running coupling constant and the nonperturbative QCD effects via the dynamical infrared mass scale of gluon and quark in the nonlocal effective six quark operators.

It is different from the SCET as our method involves only the well-understood three physical mass scales: The heavy quark mass, the perturbative-nonperturbative QCD matching scale around 1.5 GeV, the basic nonperturnbative QCD scale around 0.4~0.5 GeV.

Our method should be applicable to the Charm meson two body decays by considering higher order mass correction of charm quark in HQEFT

谢谢！

THANKS

Explicit CP Violation in SM

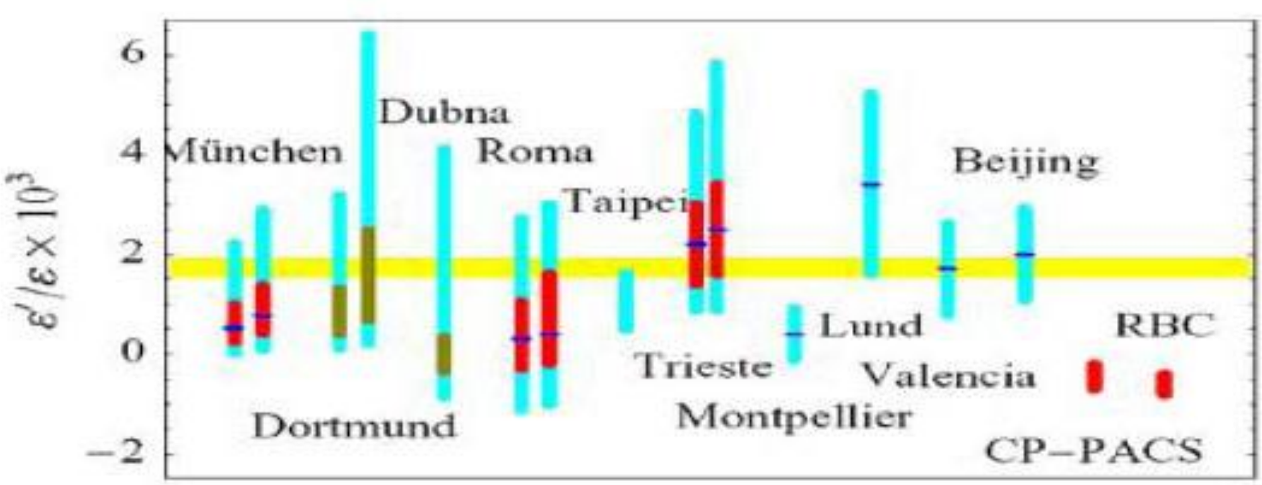
1973: 3 Generation Quarks in SM

Kobayashi-Maskawa Complex Yukawa Couplings

Direct CP violation in kaon decays

ε'/ε ＝（20±4±5）×10-4

(YL. Wu, Phys. Rev. D64: 016001,2001)



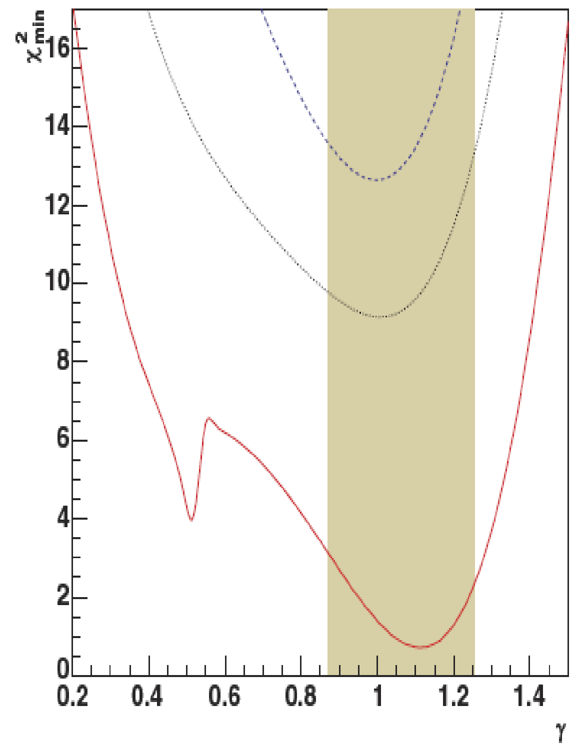
Explicit CP Violation in SM

Direct CP violation in B decays

$$a_{CP}(\pi^+K^-) = -0.11 \pm 0.02$$

$$a_{CP}(\pi^+\pi^-) = 0.46 \pm 0.13$$

Belle Collaboration, K. Abe *et al.*, Phys. Rev. D **68**, 012001 (2003); BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **93**, 131801 (2004); Z. Ligeti, hep-ph/0408267; M. Giorgi, hep-ex/0408113; Y. Sakai, hep-ex/0410006.



CP violation in B decays

YL.Wu & YF. Zhou, PRD71, 021701 (2005)