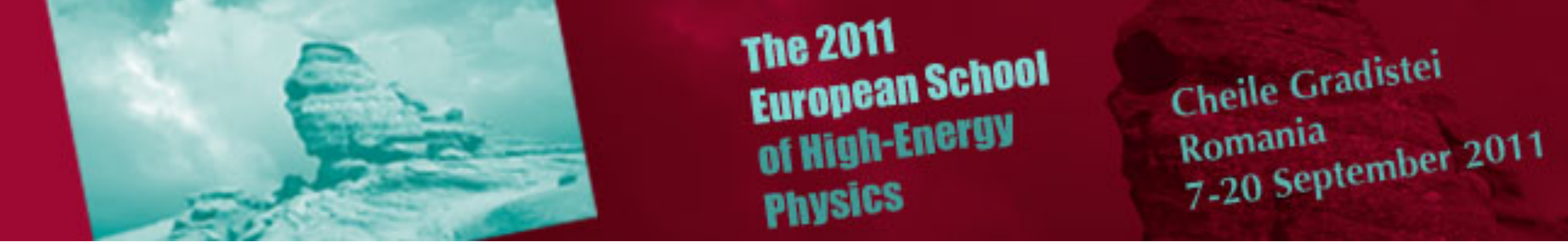

Basics of QCD for the LHC

Lecture IV

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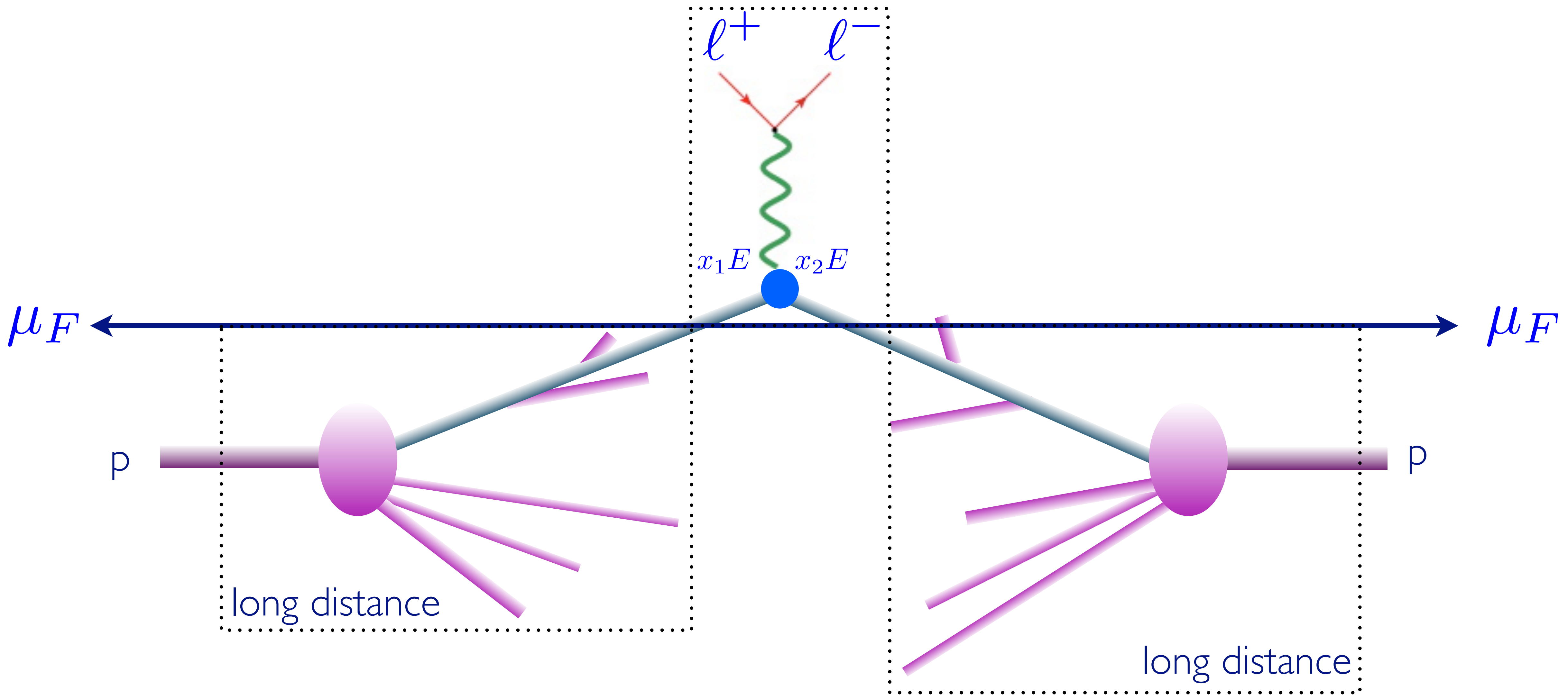
Lectures

1. Intro and QCD fundamentals
2. QCD in the final state
3. QCD in the initial state
4. From accurate QCD to useful QCD
5. Advanced QCD with applications at the LHC

LHC master formula



$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab \to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

How do we make predictions?

1. Fixed order computations: from LO to NNLO   TH-Accurate
2. Parton showers and fully exclusive simulations   EXP-Useful

In practice we use public codes, which are often very-loosely called Monte Carlo's, that implement various results/approaches. In general the predictions of NLO and NNLO calculations are given in terms of **distributions** of infrared safe observables (histograms), while proper Monte Carlo Generators give out **events**. Keep this difference in mind!

LHC master formula

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1,\mu_F^2) f_b(x_2,\mu_F^2) \times \hat{\sigma}_{ab\to X}(x_1,x_2,\alpha_S(\mu_R^2),\frac{Q^2}{\mu_F^2},\frac{Q^2}{\mu_R^2})$$

Two ingredients necessary:

1. Parton Distribution functions (from exp, but evolution from th).

2. Short distance coefficients as an expansion in α_S (from th).

$$\hat{\sigma}_{ab\to X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \dots$$

Leading order

Next-to-leading order

Next-to-next-to-leading order

Predictions at LO

How do we calculate a LO cross section for 3 jets at the LHC?

I. Identify all subprocesses (gg→ggg, qg→qgg....) in:

$$\sigma(pp \to 3j) = \sum_{ijk} \int f_i(x_1) f_j(x_2) \hat{\sigma}(ij \to k_1 k_2 k_3)$$

easy

II. For each one, calculate the amplitude:

$$\mathcal{A}(\{p\},\{h\},\{c\}) = \sum_i D_i$$

difficult

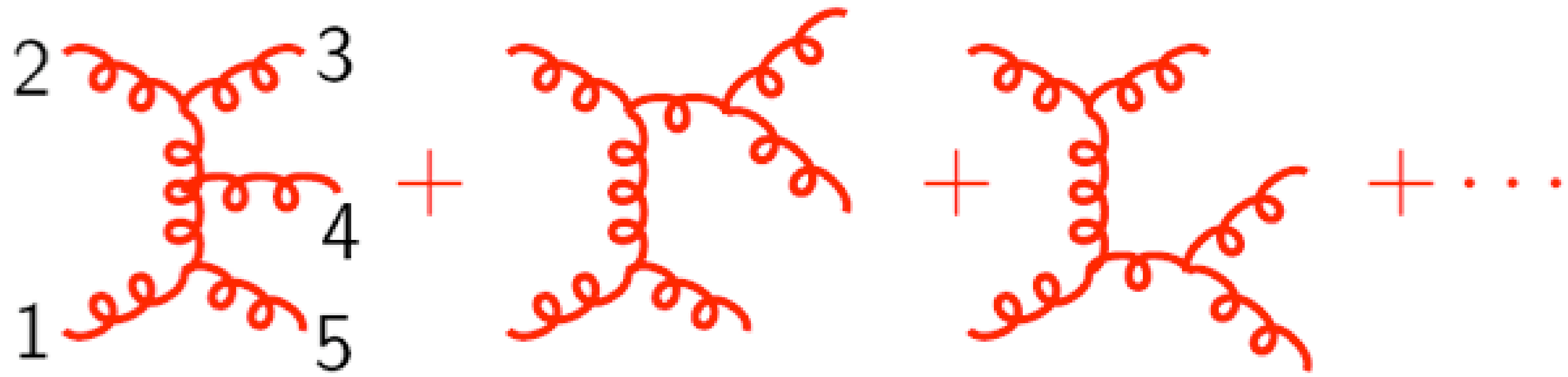
III. Square the amplitude, sum over spins & color, integrate over the phase space (D ~ 3n)

$$\hat{\sigma} = \frac{1}{2\hat{s}} \int d\Phi_p \sum_{h,c} |\mathcal{A}|^2$$

quite hard

How difficult is it to calculate $|A|^2$ for arbitrary processes?

Consider a simple 5 gluon amplitude:



There are 25 diagrams with a complicated tensor structure,
so you get....

```
A(k1,e1,k2,e2,k3,e3,k4,e4,k5,e5) = + Tr(Ta1,Ta2,Ta3,Ta4,Ta5) * ( 1/2*den(2*k1.k2)*k1.e2*e1.e3*e4.e5 - den(2*k1.k2)*k1.e2*e1.e4*e3.e5
+ 1/2*den(2*k1.k2)*k1.e2*e1.e5*e3.e4 - 1/4*den(2*k1.k2)*k1.e3*e1.e2*e4.e5 + 1/2*den(2*k1.k2)*k1.e4*e1.e2*e3.e5 - 1/4*den(2*k1.k2)*k1.e5*e1.e2*e3.e4
- 1/2*den(2*k1.k2)*k2.e1*e2.e3* e4.e5 + den(2*k1.k2)*k2.e1*e2.e4*e3.e5 - 1/2*den(2*k1.k2)*k2.e1*e2.e5 *e3.e4 + 1/4*den(2*k1.k2)*k2.e3*e1.e2*e4.e5
- 1/2*den(2*k1.k2)*k2.e4* e1.e2*e3.e5 + 1/4*den(2*k1.k2)*k2.e5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)* den(2*k3.k4)*k1.k3*k1.e2*e1.e5*e3.e4
- 1/2*den(2*k1.k2)*den(2*k3.k4)* k1.k3*k2.e1*e2.e5*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k3*k2.e5* e1.e2*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k3*k3.e4*e1.e2*e3.e5
+ 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k3*k4.e3*e1.e2*e4.e5 - 1/2*den(2* k1.k2)*den(2*k3.k4)*k1.k3*k4.e5*e1.e2*e3.e4 - 1/2*den(2*k1.k2)*den(2*
k3.k4)*k1.k4*k1.e2*e1.e5*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k4* k2.e1*e2.e5*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k4*k2.e5*e1.e2*
e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k4*k3.e4*e1.e2*e3.e5 + 1/2* den(2*k1.k2)*den(2*k3.k4)*k1.k4*k3.e5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*
```

```
+ Tr(Ta1,Ta2,Ta3,Ta4,Ta5) * ( 1/2*den(2*k1.k2)*k1.e2*e1.e3*e4.e5
```

```
k1.e5*k3.e1*e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*k1.e5*k3.e4*e1.e3 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*k1.e5*k4.e1*e3.e4
- den(2* k1.k2)*den(2*k3.k4)*k1.e2*k1.e5*k4.e3*e1.e4 + 1/2*den(2*k1.k2)*den(2* k3.k4)*k1.e2*k2.k3*e1.e5*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*
k2.k4*e1.e5*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k2.e3*k3.e4*e1.e5 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*k2.e4*k4.e3*e1.e5 - 1/2*den(2*
k1.k2)*den(2*k3.k4)*k1.e2*k2.e5*k3.e1*e3.e4 + den(2*k1.k2)*den(2* k3.k4)*k1.e2*k2.e5*k3.e4*e1.e3 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*
k2.e5*k4.e1*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k2.e5*k4.e3*e1.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*k3.k5*e1.e5*e3.e4 +
den(2* k1.k2)*den(2*k3.k4)*k1.e2*k3.e1*k3.e4*e3.e5 - den(2*k1.k2)*den(2* k3.k4)*k1.e2*k3.e1*k4.e3*e4.e5 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*
k3.e1*k4.e5*e3.e4 - den(2*k                                                                              k1.k2)*
den(2*k3.k4)*k1.e2*k3.e4*                                                                                *k5.e3
*e1.e5 - den(2*k1.k2)*den(2                                                                              )*den(2*
```

Brute force is not an option!

```
k3.k4)*k1.e2*k3.e5*k5.e1*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2* k4.k5*e1.e5*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k4.e1*k4.e3*e4.e5
+ den(2*k1.k2)*den(2*k3.k4)*k1.e2*k4.e3*k4.e5*e1.e4 + den(2*k1.k2)* den(2*k3.k4)*k1.e2*k4.e3*k5.e1*e4.e5 - den(2*k1.k2)*den(2*k3.k4)*
k1.e2*k4.e3*k5.e4*e1.e5 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*k4.e5* k5.e1*e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k1.e3*k2.e1*k3.e4*e2.e5
- den(2*k1.k2)*den(2*k3.k4)*k1.e3*k2.e5*k3.e4*e1.e2 + 1/2*den(2*k1.k2)* den(2*k3.k4)*k1.e3*k3.e4*k3.e5*e1.e2 + 1/2*den(2*k1.k2)*den(2*k3.k4)*
k1.e3*k3.e4*k4.e5*e1.e2 - den(2*k1.k2)*den(2*k3.k4)*k1.e4*k2.e1*k4.e3 *e2.e5 + den(2*k1.k2)*den(2*k3.k4)*k1.e4*k2.e5*k4.e3*e1.e2
- 1/2*den( 2*k1.k2)*den(2*k3.k4)*k1.e4*k3.e5*k4.e3*e1.e2 - 1/2*den(2*k1.k2)*den( 2*k3.k4)*k1.e4*k4.e3*k4.e5*e1.e2 - 1/2*den(2*k1.k2)*den(2*k3.k4)*
k1.e5*k2.k3*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.k4* e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.e1*k3.e2*e3.e4
- den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.e1*k3.e4*e2.e3 - 1/2*den(2* k1.k2)*den(2*k3.k4)*k1.e5*k2.e1*k4.e2*e3.e4 + den(2*k1.k2)*den(2*
k3.k4)*k1.e5*k2.e1*k4.e3*e2.e4 + den(2*k1.k2)*den(2*k3.k4)*k1.e5* k2.e3*k3.e4*e1.e2 - den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.e4*k4.e3*e1.e2
+ 1/4*den(2*k1.k2)*den(2*k3.k4)*k1.e5*k3.k5*e1.e2*e3.e4 - 1/2*den(2* k1.k2)*den(2*k3.k4)*k1.e5*k3.e4*k5.e3*e1.e2 - 1/4*den(2*k1.k2)*den(2*
k3.k4)*k1.e5*k4.k5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e5* k4.e3*k5.e4*e1.e2 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k3*k2.e1*e2.e5*
e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k3*k3.e4*e1.e2*e3.e5 - 1/2* den(2*k1.k2)*den(2*k3.k4)*k2.k3*k4.e3*e1.e2*e4.e5 + 1/2*den(2*k1.k2)*
den(2*k3.k4)*k2.k3*k4.e5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)* k2.k4*k2.e1*e2.e5*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k4*k3.e4*
e1.e2*e3.e5 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k4*k3.e5*e1.e2*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k4*k4.e3*e1.e2*e4.e5 - 1/2*den(2*
k1.k2)*den(2*k3.k4)*k2.k5*k3.e4*e1.e2*e3.e5 + 1/4*den(2*k1.k2)*den(2* k3.k4)*k2.k5*k3.e5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k5*
k4.e3*e1.e2*e4.e5 - 1/4*den(2*k1.k2)*den(2*k3.k4)*k2.k5*k4.e5*e1.e2* e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k2.e1*k2.e3*k3.e4*e2.e5 -
den(2* k1.k2)*den(2*k3.k4)*k2.e1*k2.e4*k4.e3*e2.e5 + 1/2*den(2*k1.k2)*den(2* k3.k4)*k2.e1*k2.e5*k3.e2*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k2.e1*
k2.e5*k3.e4*e2.e3 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.e1*k2.e5*k4.e2* e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k2.e1*k2.e5*k4.e3*e2.e4
```

How difficult is it to calculate $|A|^2$ for arbitrary processes?

Solution

- Work always at the amplitude level (not squared)
- Keep track of all the quantum numbers, (momenta, spin and color)
- Organize them in efficient way, by choosing appropriate basis

How difficult is it to calculate $|A|^2$ for arbitrary processes?

Calculate **helicity amplitudes**, ie amplitudes for gluons and quarks in a definite helicity states. For massless quarks this amounts to condering chirality states:

$$u_\pm(k) = \frac{1}{2}(1 \pm \gamma_5)u(k)$$

External gluons you always think them as attached to a quark-anti-quark pair with a definite (yet arbitrary) polarization vectors:

$$\varepsilon_\mu^+(k;q) = \frac{\langle q^- | \gamma_\mu | k^- \rangle}{\sqrt{2}\,\langle q\, k \rangle}, \qquad \varepsilon_\mu^-(k,q) = \frac{\langle q^+ | \gamma_\mu | k^+ \rangle}{\sqrt{2}\,[k\, q]}$$

It's just a more sophisticated version of the circular polarization. Choosing appropriately the gauge vector, expressions simplify dramatically.

How difficult is it to calculate |A|² for arbitrary processes?

Inspired by the way gauge theories appear as the zero-slope limits of (open) string theories, it has been suggested to decompose the full amplitude as a sum of gauge invariant **subamplitudes** times color coefficients:

$$\mathcal{A}_n(g_1,\ldots,g_n) = g^{n-2} \sum_{\sigma\in S_{n-1}} \mathrm{Tr}(\mathsf{t}^{a_1}\mathsf{t}^{a_{\sigma_2}}\cdots\mathsf{t}^{a_{\sigma_n}}) A_n(1,\sigma_2,\ldots,\sigma_n)$$

where the formula $if^{abc} = \mathrm{Tr}(t^a,[t^b, t^c])$ has been repeatedly used to reduce the f's into traces of lambdas and the Fierz identities to cancel traces of length l<n. Analogously for quarks:

$$\mathcal{A}_n(q_1,g_2,\ldots,g_{n-1},\bar{q}_n) = g^{n-2} \sum_{\sigma\in S_{n-2}} (\mathsf{t}^{a_{\sigma_2}}\cdots\mathsf{t}^{a_{\sigma_{n-1}}})^i_j A_n(1_q,\sigma_2,\ldots,\sigma_{n-2},n_{\bar{q}})$$

The A_n are MUCH simpler objects to calculate, with many less diagrams...

How difficult is it to calculate $|A|^2$ for arbitrary processes?

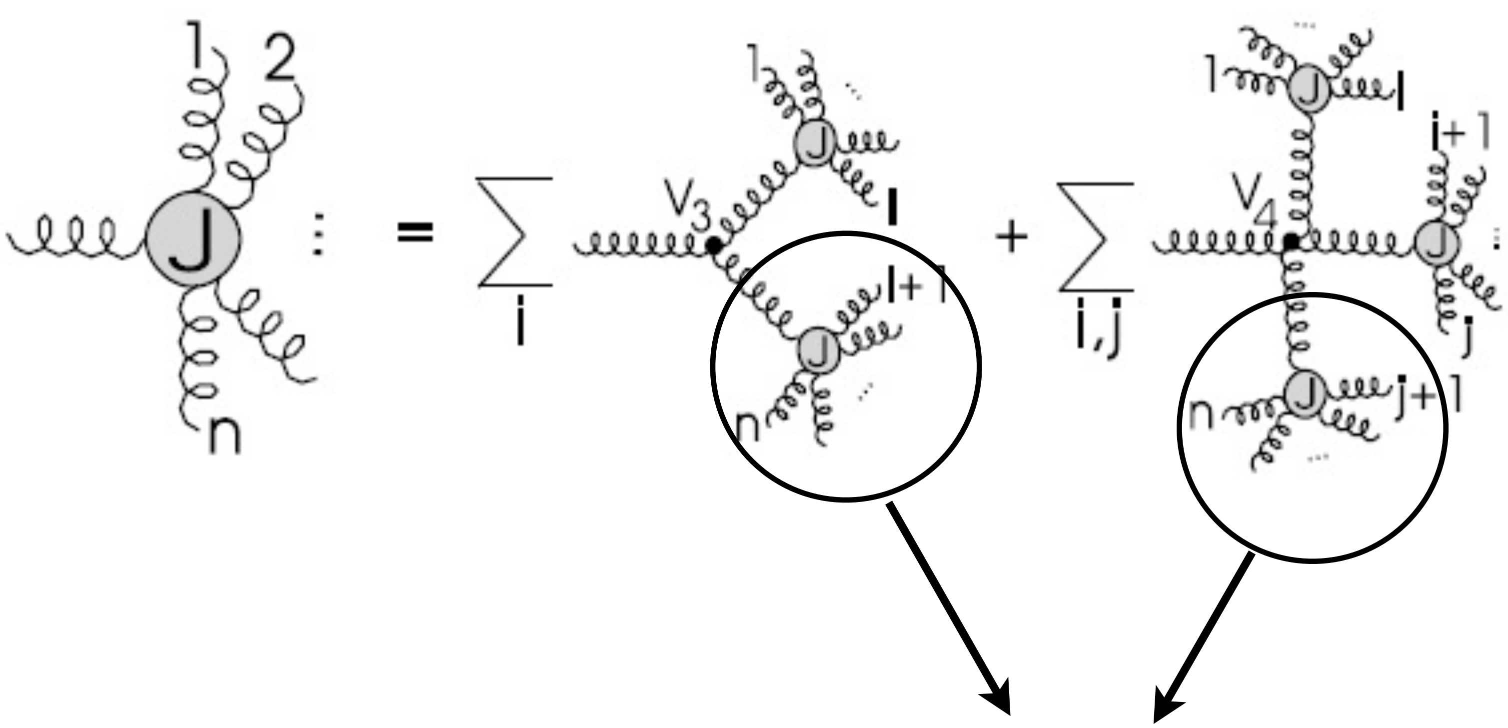
n	full Amp	partial Amp
4	4	3
5	25	10
6	220	36
7	2485	133
8	34300	501
9	559405	1991
10	10525900	7335
11	224449225	28199
12	5348843500	108280

$(2n)!$ $\qquad$ 3.8^n

How difficult is it to calculate $|A|^2$ for arbitrary processes?

Feynman diagrams are not efficient because the same subdiagrams are recomputed over and over.
Solution: cash them! In other words use recursive relations.

For the color-ordered subamplitudes for n gluons, such relations (called Berends-Giele) are very easy:



Off-shell amplitudes with max n-1 number of legs !

How difficult is it to calculate $|A|^2$ for arbitrary processes?

n	full Amp	partial Amp	BG
4	4	3	3
5	25	10	10
6	220	36	35
7	2485	133	70
8	34300	501	126
9	559405	1991	210
10	10525900	7335	330
11	224449225	28199	495
12	5348843500	108280	715
	$(2n)!$	3.8^n	n^4

The factorial growth is tamed to a polynomial one!

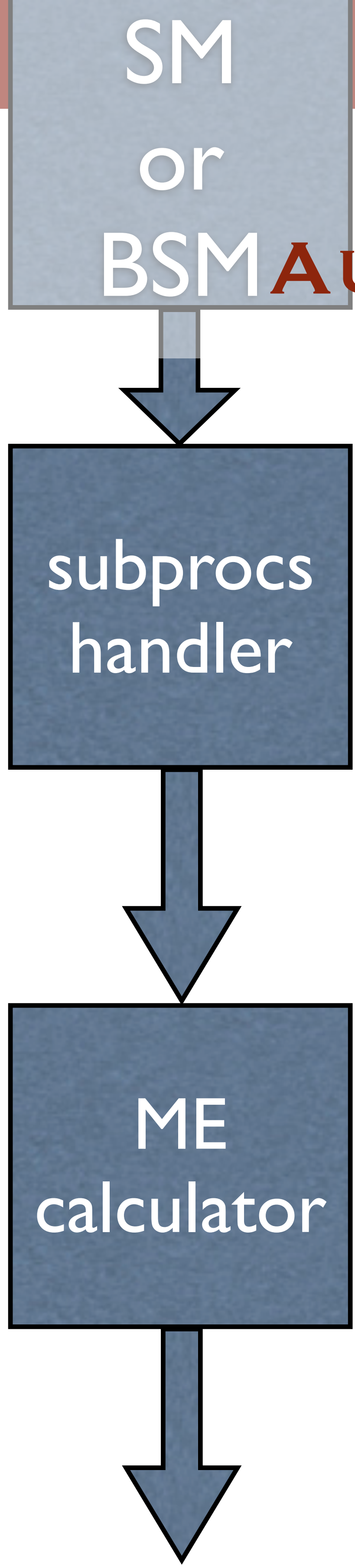
Note, however, one still needs to sum over color, an operation which sets the complexity back to exponential.

How difficult is to calculate $|A|^2$ for arbitrary processes?

Problem of generating the matrix elements for any process of interest has been solved in full generality and it has been automatized!

More than that, also the integration over phase of such matrix elements can be achieved in an automatic way (non-trivial problem not discussed here)!

Several public tools exist:
CompHEP/CalcHEP/MadGraph/SHERPA/Whizard/....



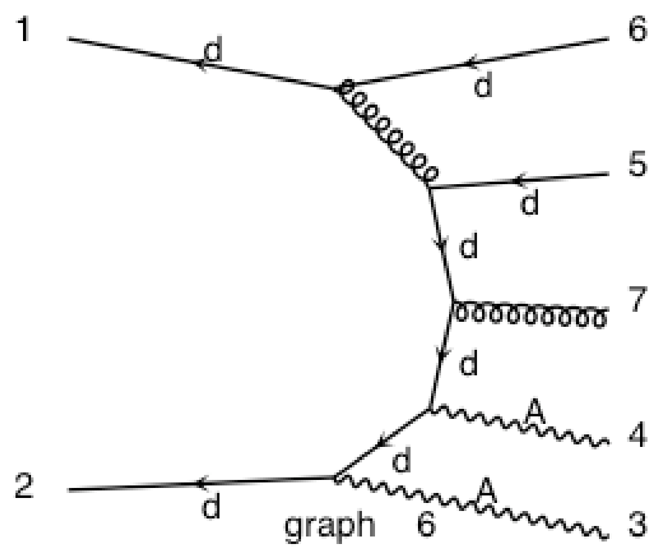
Automatic LO cross sections

Includes all possible subprocess leading to a given multi-jet final state automatically or manually (done once for all)

d~ d -> a a u u~ g
d~ d -> a a c c~ g
s~ s -> a a u u~ g
s~ s -> a a c c~ g

"Automatically" generates a code to calculate |M|^2 for arbitrary processes with many partons in the final state.

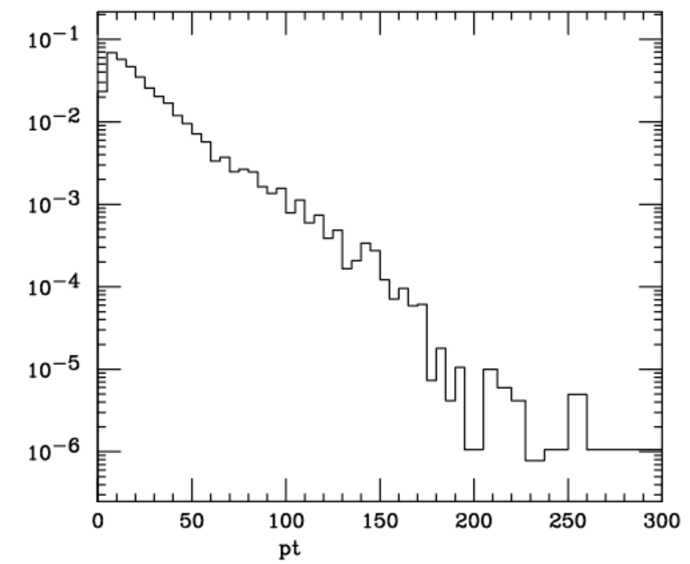
Most use Feynman diagrams w/ tricks to reduce the factorial growth, others have recursive relations to reduce the complexity to exponential. ☺



Automatic LO cross sections

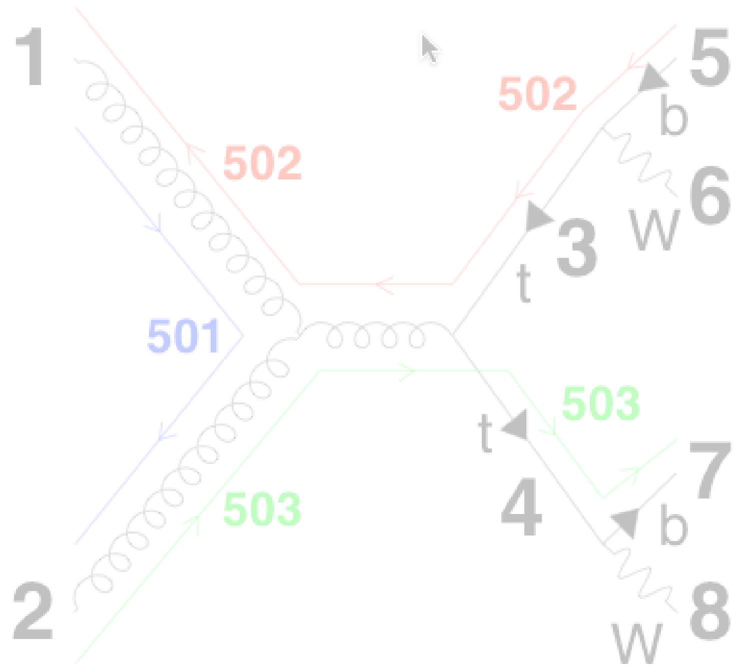
x section

Integrate the matrix element over the phase space using a multi-channel technique and using parton-level cuts.

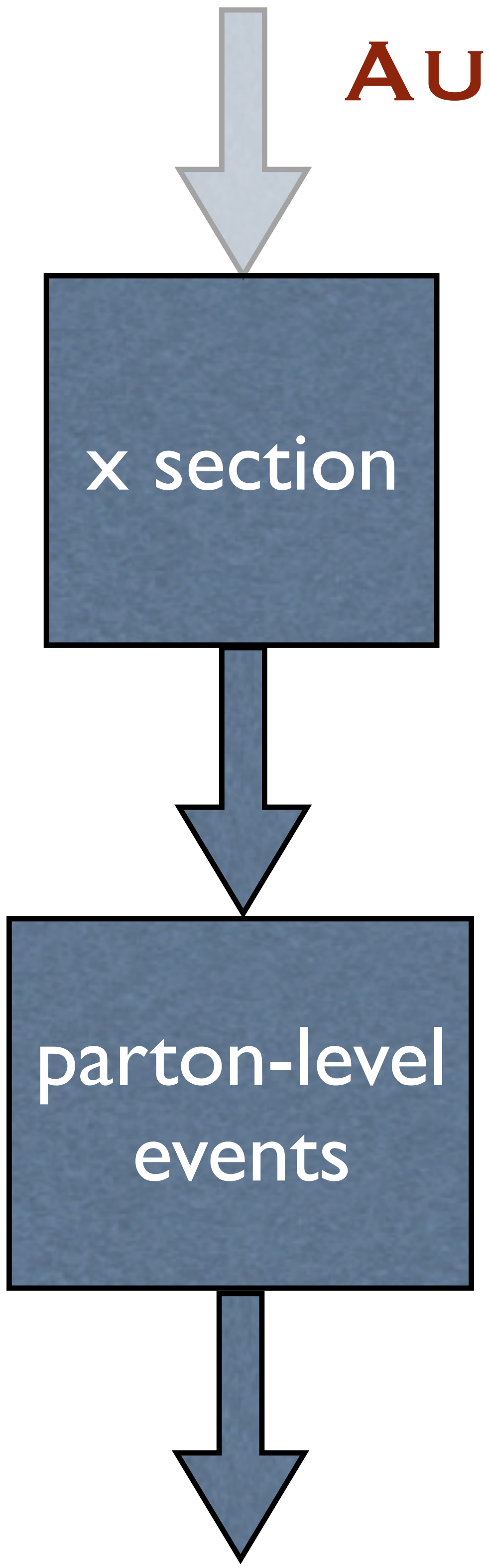


parton-level events

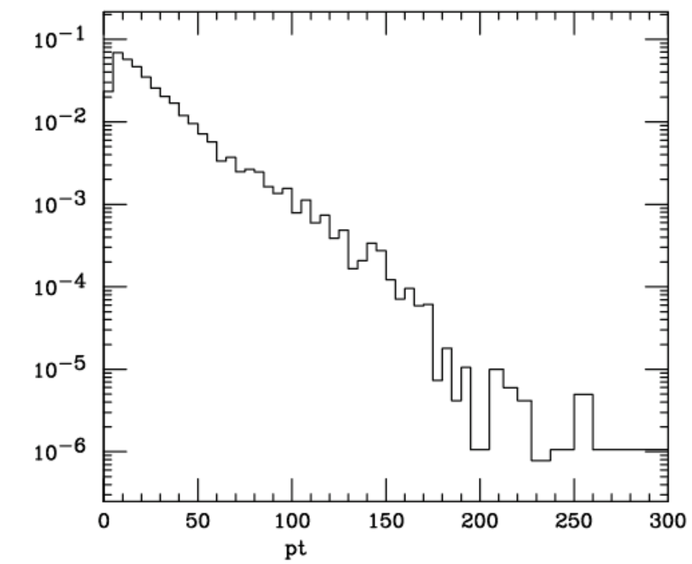
Events are obtained by unweighting. These are at the parton-level. Information on particle id, momenta, spin, color is given in the Les Houches format.



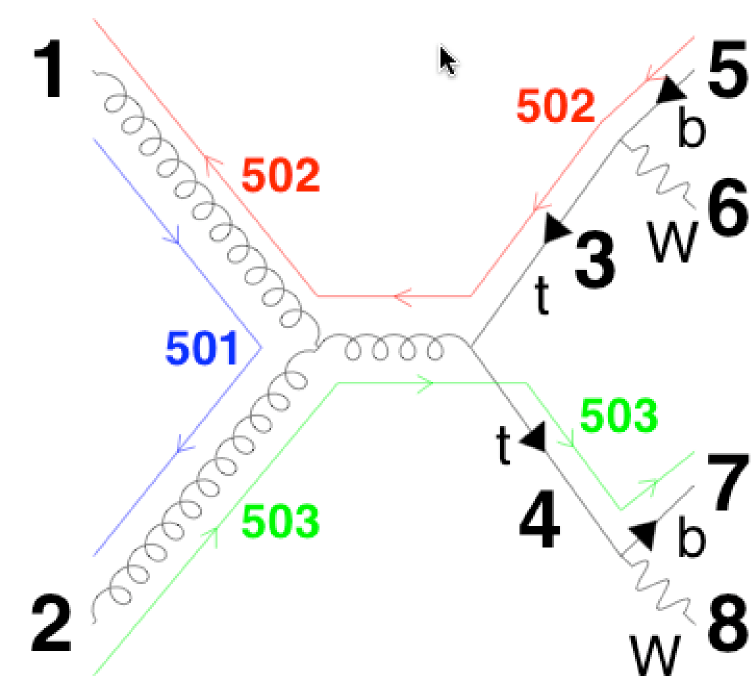
Automatic LO cross sections



Integrate the matrix element over the phase space using a multi-channel technique and using parton-level cuts.



Events are obtained by unweighting. These are at the parton-level. Information on particle id, momenta, spin, color is given in the Les Houches format.



LO predictions : final remarks

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1,\mu_F^2) f_b(x_2,\mu_F^2) \times \hat{\sigma}_{ab\to X}(x_1,x_2,\alpha_S(\mu_R^2),\frac{Q^2}{\mu_F^2},\frac{Q^2}{\mu_R^2})$$

- By calculating the short distance coefficient at tree-level we obtain the first estimate of rates for inclusive final states.

- Even at LO **extra radiation is included:** it is described by the PDF's in the initial state and by the definition of a final state parton, which at LO represents all possible final state evolutions.

- Due to the above approximations a cross section at LO can strongly depend on the factorization and renormalization scales.

- Predictions can be systematically improved, at NLO and NNLO, by including higher order corrections in the short distance and in the evolution of the PDF's.

Predictions at NLO

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab \to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

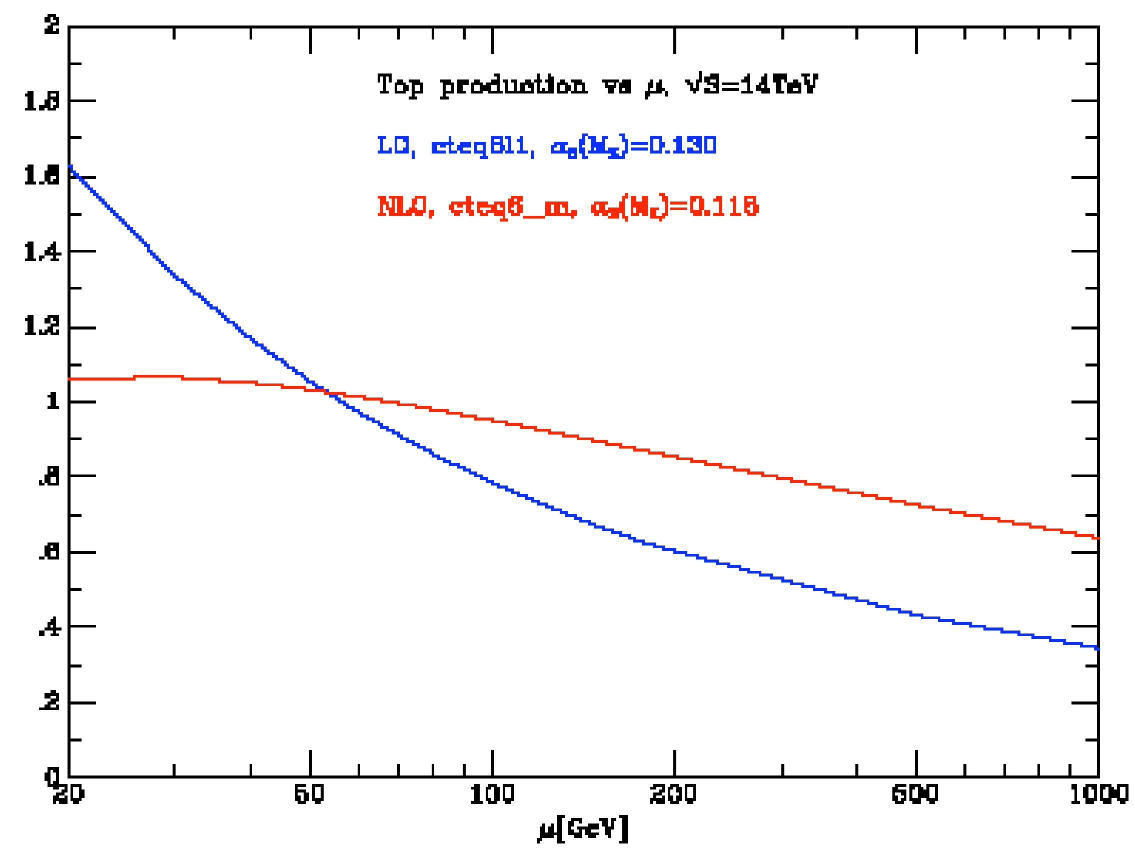
$$\hat{\sigma}_{ab \to X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \dots$$

Why?

1. First order where scale dependences are compensated by the running of α_S and the evolution of the PDF's: FIRST RELIABLE ESTIMATE OF THE TOTAL CROSS SECTION.

2. The impact of extra radiation is included. For example, jets now have a structure.

3. New effects coming up from higher order terms (e.g., opening up of new production channels or phase space dimensions) can be evaluated.

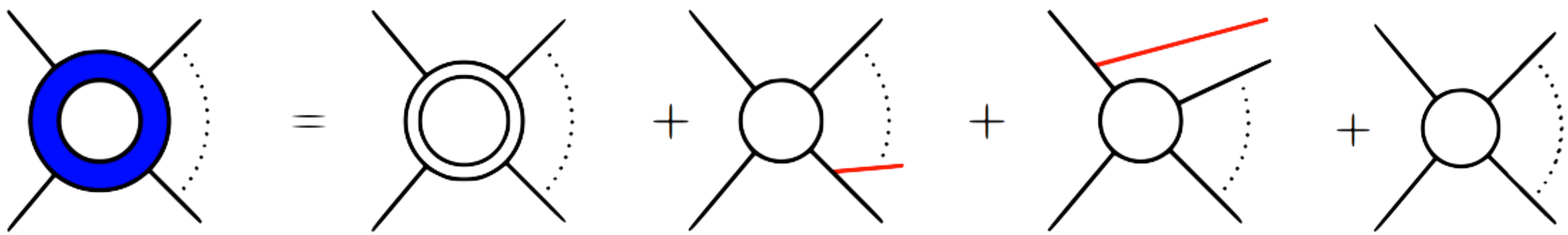


Predictions at NLO

How? 1. Get the "ingredients"

2. "Method" to combine them to calculate infrared observables

Ingredients:



Virtual part Real emission part Born

$$\int d\sigma^{(\mathrm{NLO})} O(\Phi) = \int d\Phi_B V(\Phi_B) O(\Phi_B) + \int d\Phi_R R(\Phi_R)\; O(\Phi_R) \int d\Phi_B B(\Phi_B) O(\Phi_B)$$

Loops have been for long the bottleneck of NLO computations, with their calculations taking years of manual and symbolic work to get the correct results.

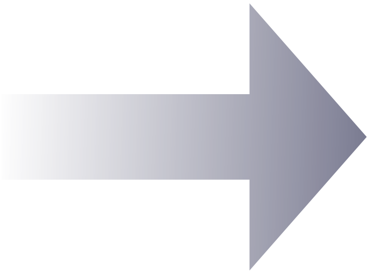
Predictions at NLO

modified by the lecturer

Generalized Unitarity
(ex. BlackHat, Rocket,...)

Integrand Reduction
(ex. CutTools, Samurai)

Tensor Reduction
(ex. Golem)



Thanks to new amazing results, some of them inspired by string theory developments, now the computation of loops has been extended to high-multiplicity processes or/and automated.

t

Predictions at NLO

How?

1. Get the "ingredients"
2. "Method" to combine them to calculate infrared observables

Method: Universal subtraction

$$\int d\sigma^{(\mathrm{NLO})} O(\Phi) = \int d\Phi_B V(\Phi_B) O(\Phi_B) + \int d\Phi_R R(\Phi_R)\, O(\Phi_R) \int d\Phi_B B(\Phi_B) O(\Phi_B)$$

$$= \int d\Phi_B \left[B(\Phi_B) + V(\Phi_B) + \int d\Phi_{R|B} S(\Phi_R) \right] O(\Phi_B)$$
$$+ \int d\Phi_R \left[R(\Phi_R)\, O(\Phi_R) - S(\Phi_R) O(\Phi_B) \right]$$

Local universal counterterms have been identified whose integral on the extra radiation variable is analytically known and that can be used to make reals and virtuals separately finite.

Predictions at NLO

MCFM: downloadable general purpose NLO code [Campbell & Ellis+ collaborators]

Final state	Notes	Reference
W/Z		
diboson (W/Z/γ)	photon fragmentation, anomalous couplings	hep-ph/9905386, arXiv:1105.0020
Wbb	massless b-quark massive b quark	hep-ph/9810489 arXiv:1011.6647
Zbb	massless b-quark	hep-ph/0006304
W/Z+1 jet		
W/Z+2 jets		hep-ph/0202176, hep-ph/0308195
Wc	massive c-quark	hep-ph/0506289
Zb	5-flavour scheme	hep-ph/0312024
Zb+jet	5-flavour scheme	hep-ph/0510362

Final state	Notes	Reference
H (gluon fusion)		
H+1 jet (g.f.)	effective coupling	
H+2 jets (g.f.)	effective coupling	hep-ph/0608194, arXiv:1001.4495
WH/ZH		
H (WBF)		hep-ph/0403194
Hb	5-flavour scheme	hep-ph/0204093
t	s- and t-channel (5F), top decay included	hep-ph/0408158
t	t-channel (4F)	arXiv:0903.0005, arXiv:0907.3933
Wt	5-flavour scheme	hep-ph/0506289
top pairs	top decay included	

☞ ~30 processes

☞ First results implemented in 1998 ...this is 13 years worth of work of several people (~4M$)

☞ Cross sections and parton-level distributions at NLO are provided

☞ One general framework. However, each process implemented by hand

Predictions at NLO

Completely automatically generated NLO codes for a variety of processes via **MadLoop+MadFKS**

Total sample cross sections at the LHC for 26 sample procs

Code generation time: a few hours

Running time: two weeks on a cluster

	Process	μ	n_{lf}	Cross section (pb)	
				LO	NLO
a.1	$pp \to t\bar{t}$	m_{top}	5	123.76 ± 0.05	162.08 ± 0.12
a.2	$pp \to tj$	m_{top}	5	34.78 ± 0.03	41.03 ± 0.07
a.3	$pp \to tjj$	m_{top}	5	11.851 ± 0.006	13.71 ± 0.02
a.4	$pp \to t\bar{b}j$	$m_{top}/4$	4	25.62 ± 0.01	30.96 ± 0.06
a.5	$pp \to t\bar{b}jj$	$m_{top}/4$	4	8.195 ± 0.002	8.91 ± 0.01
b.1	$pp \to (W^+ \to)e^+\nu_e$	m_W	5	5072.5 ± 2.9	6146.2 ± 9.8
b.2	$pp \to (W^+ \to)e^+\nu_e\, j$	m_W	5	828.4 ± 0.8	1065.3 ± 1.8
b.3	$pp \to (W^+ \to)e^+\nu_e\, jj$	m_W	5	298.8 ± 0.4	300.3 ± 0.6
b.4	$pp \to (\gamma^*/Z \to)e^+e^-$	m_Z	5	1007.0 ± 0.1	1170.0 ± 2.4
b.5	$pp \to (\gamma^*/Z \to)e^+e^-\, j$	m_Z	5	156.11 ± 0.03	203.0 ± 0.2
b.6	$pp \to (\gamma^*/Z \to)e^+e^-\, jj$	m_Z	5	54.24 ± 0.02	56.69 ± 0.07
c.1	$pp \to (W^+ \to)e^+\nu_e b\bar{b}$	$m_W + 2m_b$	4	11.557 ± 0.005	22.95 ± 0.07
c.2	$pp \to (W^+ \to)e^+\nu_e t\bar{t}$	$m_W + 2m_{top}$	5	0.009415 ± 0.000003	0.01159 ± 0.00001
c.3	$pp \to (\gamma^*/Z \to)e^+e^- b\bar{b}$	$m_Z + 2m_b$	4	9.459 ± 0.004	15.31 ± 0.03
c.4	$pp \to (\gamma^*/Z \to)e^+e^- t\bar{t}$	$m_Z + 2m_{top}$	5	0.0035131 ± 0.0000004	0.004876 ± 0.000002
c.5	$pp \to \gamma t\bar{t}$	$2m_{top}$	5	0.2906 ± 0.0001	0.4169 ± 0.0003
d.1	$pp \to W^+W^-$	$2m_W$	4	29.976 ± 0.004	43.92 ± 0.03
d.2	$pp \to W^+W^-\, j$	$2m_W$	4	11.613 ± 0.002	15.174 ± 0.008
d.3	$pp \to W^+W^+\, jj$	$2m_W$	4	0.07048 ± 0.00004	0.1377 ± 0.0005
e.1	$pp \to HW^+$	$m_W + m_H$	5	0.3428 ± 0.0003	0.4455 ± 0.0003
e.2	$pp \to HW^+\, j$	$m_W + m_H$	5	0.1223 ± 0.0001	0.1501 ± 0.0002
e.3	$pp \to HZ$	$m_Z + m_H$	5	0.2781 ± 0.0001	0.3659 ± 0.0002
e.4	$pp \to HZ\, j$	$m_Z + m_H$	5	0.0988 ± 0.0001	0.1237 ± 0.0001
e.5	$pp \to Ht\bar{t}$	$m_{top} + m_H$	5	0.08896 ± 0.00001	0.09869 ± 0.00003
e.6	$pp \to Hb\bar{b}$	$m_b + m_H$	4	0.16510 ± 0.00009	0.2099 ± 0.0006
e.7	$pp \to Hjj$	m_H	5	1.104 ± 0.002	1.036 ± 0.002

Predictions at NLO

- NLO calculations have historically presented two types of challenges: the loop calculations and the construction of a numerical code resilient to the cancellation of the divergences.
- Both issues have now basically solved in general and **many NLO calculations can now be done in an automatic way.**
- Several public codes that compute IR-safe quantities (cross sections, jet rates, ...) at the parton level are available.
- Be careful : NLO codes are NOT event generators!!

Predictions at NLO

Warning!

Calling a code "a NLO code" is an abuse of language and can be confusing.
A NLO calculation always refers to an IR-safe observable, when the genuine α_S corrections to this observable on top of the LO estimate are known.

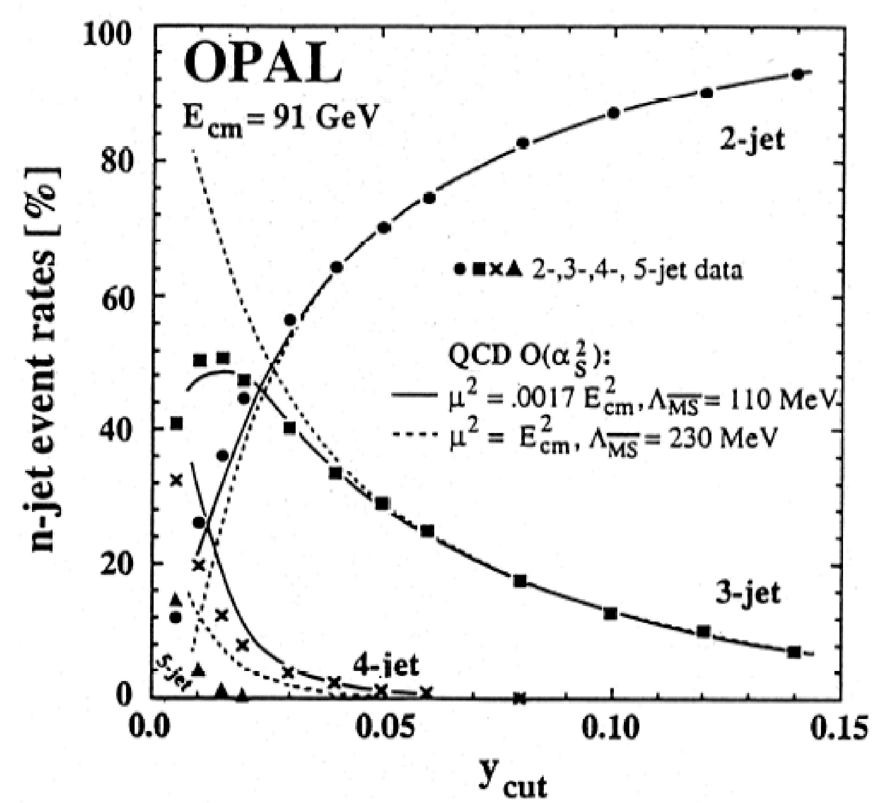
An NLO code will, in general, be able to produce results for several quantities and distributions, only some of which will be at NLO accuracy.

Example: Jet rates in the JADE algorithm:

$$\sigma_{2j} = \sigma^{\rm Born}\left(1 - \frac{\alpha_S C_F}{\pi}\log^2 y + \dots\right)$$

$$\sigma_{3j} = \sigma^{\rm Born}\frac{\alpha_S C_F}{\pi}\log^2 y + \dots$$

σ_{2j} is NLO, while σ_{3j} is just LO !



PREDICTIONS AT NLO

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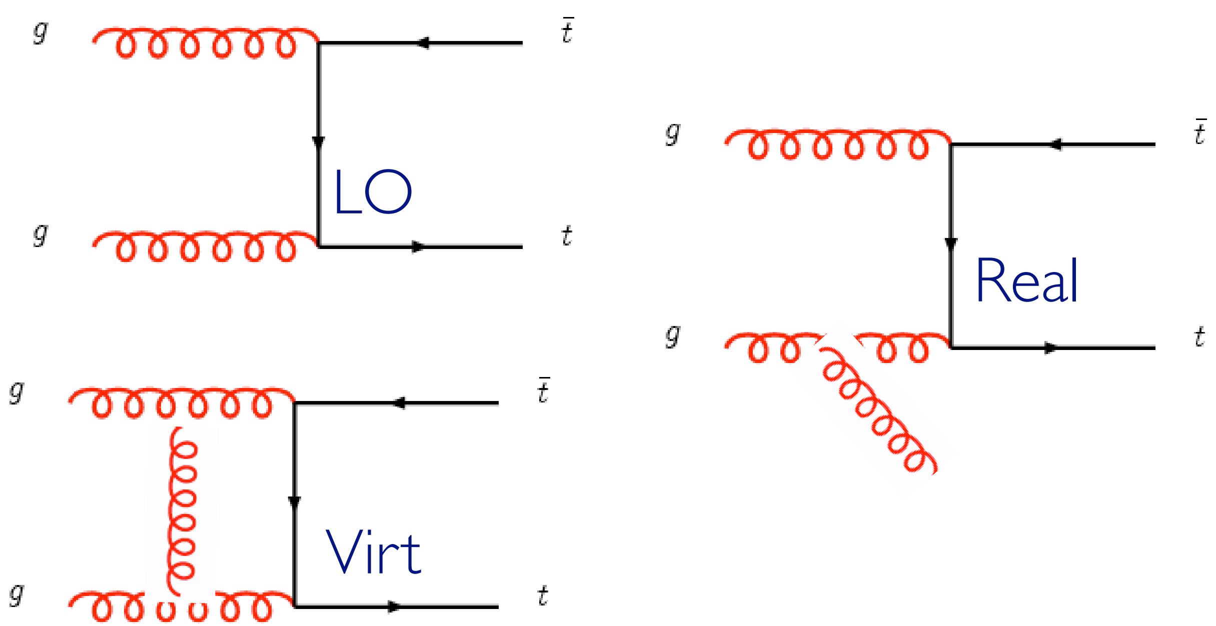
Example: Suppose we use the NLO code for pp → tt



- ☞ Total cross section, σ(tt) .... ✓
- ☞ P_T >0 of one top quark.................. ✓
- ☞ P_T >0 of the tt pair .... ✗
- ☞ P_T >0 of the jet................................ ✗
- ☞ tt invariant mass, m(tt) .... ✓
- ☞ ΔΦ(tt)>0 .... ✗

Experience a "simple" NLO calculation yourself

Hands-on!

pp→Higgs+x at NLO

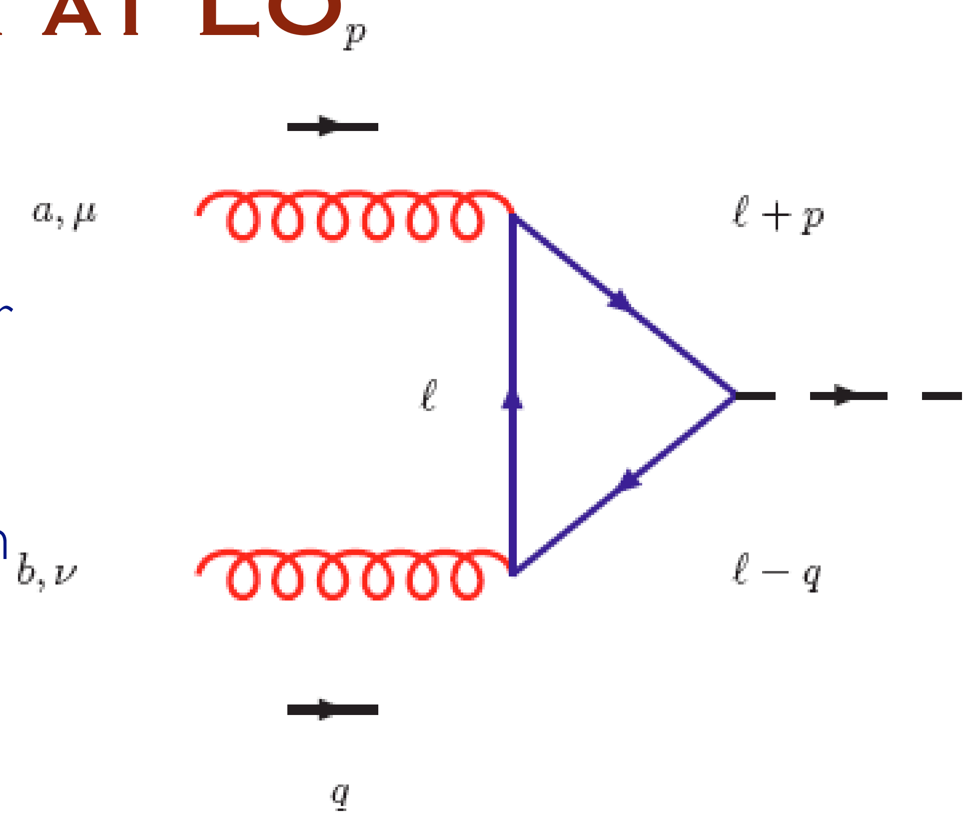
- LO : 1-loop calculation and HEFT
- NLO in the HEFT
  - Virtual corrections and renormalization
  - Real corrections and IS singularities
- Cross sections at the LHC

pp→H+x at LO

This is a "simple" 2→1 process.

However, at variance with pp→W, the LO order process already proceeds through a loop.

In this case, this means that the loop calculation has to give a finite result!
Let's do the calculation!



$$i\mathcal{A} = -(-ig_s)^2 \mathrm{Tr}(t^a t^b)\left(\frac{-im_t}{v}\right)\int \frac{d^d\ell}{(2\pi)^n}\frac{T^{\mu\nu}}{\mathrm{Den}}(i)^3 \epsilon_\mu(p)\epsilon_\nu(q)$$

where

$$\mathrm{Den} = (\ell^2 - m_t^2)[(\ell+p)^2 - m_t^2][(\ell-q)^2 - m_t^2]$$

We combine the denominators into one by using $\frac{1}{ABC} = 2\int_0^1 dx \int_0^{1-x} \frac{dy}{[Ax + By + C(1-x-y)]^3}$

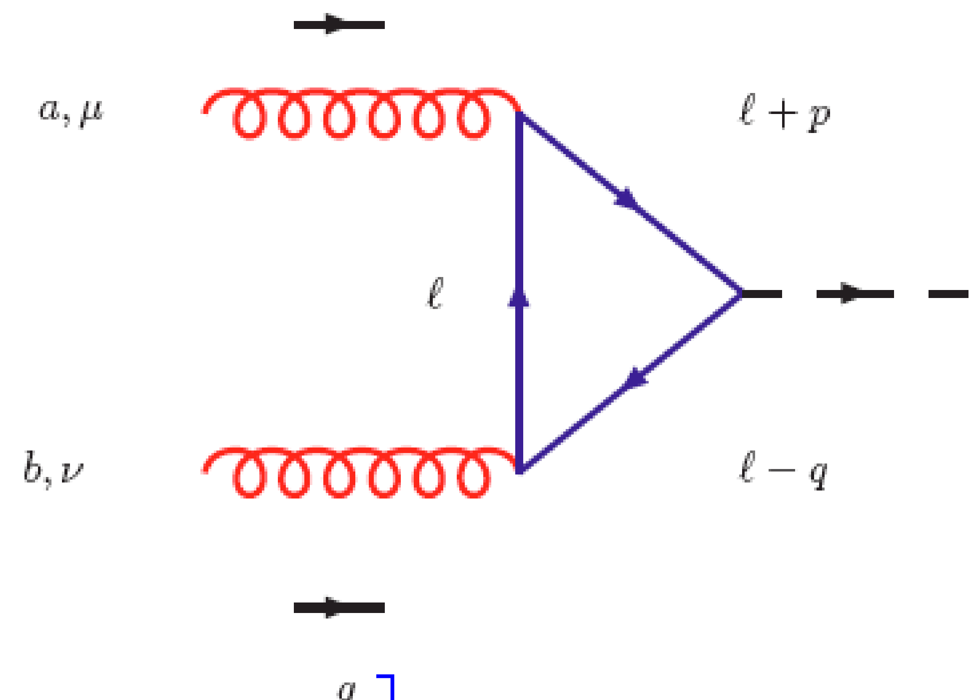
$$\frac{1}{\mathrm{Den}} = 2\int dx\, dy \frac{1}{[\ell^2 - m_t^2 + 2\ell\cdot(px - qy)]^3}.$$

pp→H+X at LO

We shift the momentum:

$$\ell' = \ell + px - qy$$

$$\frac{1}{\mathrm{Den}} \to 2\int dx\, dy \frac{1}{[\ell'^{\,2} - m_t^2 + M_H^2 xy]^3}.$$



And now the tensor in the numerator:

$$T^{\mu\nu} = \mathrm{Tr}\left[(\ell + m_t)\gamma^\mu(\ell + p + m_t)(\ell - q + m_t)\gamma^\nu)\right]$$

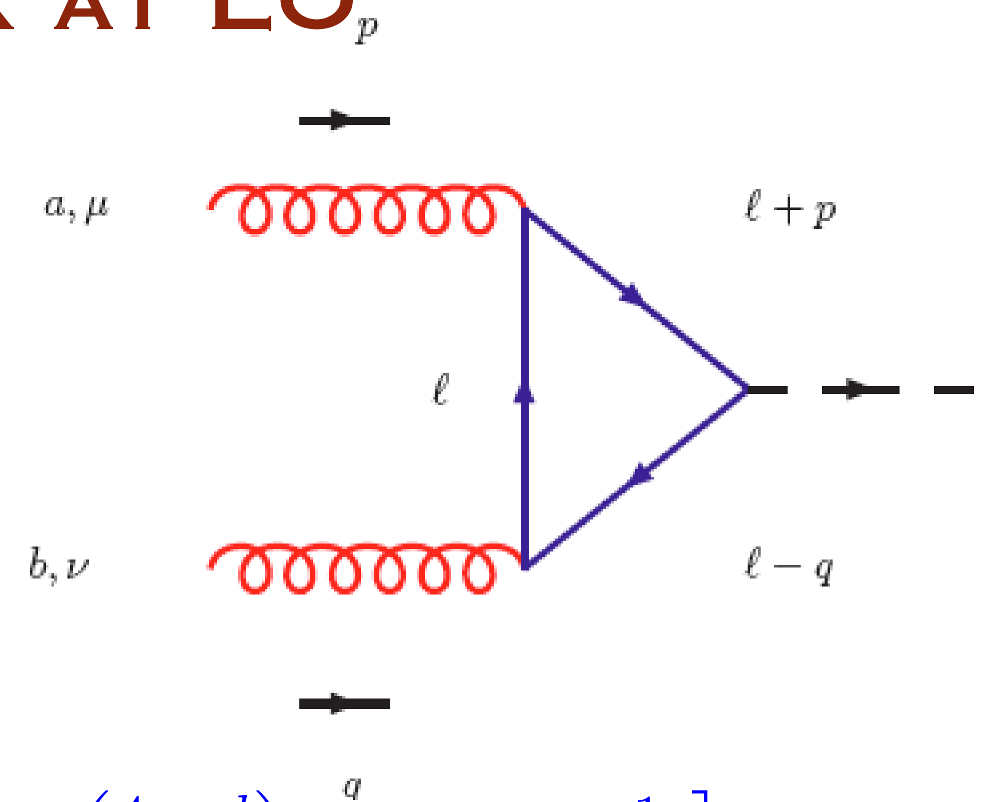
$$= 4m_t\left[g^{\mu\nu}(m_t^2 - \ell^2 - \frac{M_H^2}{2}) + 4\ell^\mu\ell^\nu + p^\nu q^\mu\right]$$

where I used the fact that the external gluons are on-shell. This trace is proportional to mt ! This is due to the spin flip caused by the scalar coupling.

Now we shift the loop momentum also here, we drop terms linear in the loop momentum (they are odd and vanish) and

pp→H+x at LO

We perform the tensor decomposition using:



$$\int d^d k \frac{k^\mu k^\nu}{(k^2 - C)^m} = \frac{1}{d} g^{\mu\nu} \int d^d k \frac{k^2}{(k^2 - C)^m}$$

So I can write an expression which depends only on scalar loop integrals:

$$iA = -\frac{2g_s^2 m_t^2}{v} \delta^{ab} \int \frac{d^d \ell'}{(2\pi)^d} \int dx dy \Big\{ g^{\mu\nu} \left[m^2 + \ell'^2 \left(\frac{4-d}{d} \right) + M_H^2 (xy - \frac{1}{2}) \right]$$

$$+ p^\nu q^\mu (1 - 4xy) \Big\} \frac{2dxdy}{(\ell'^2 - m_t^2 + M_H^2 xy)^3} \epsilon_\mu(p) \epsilon_\nu(q).$$

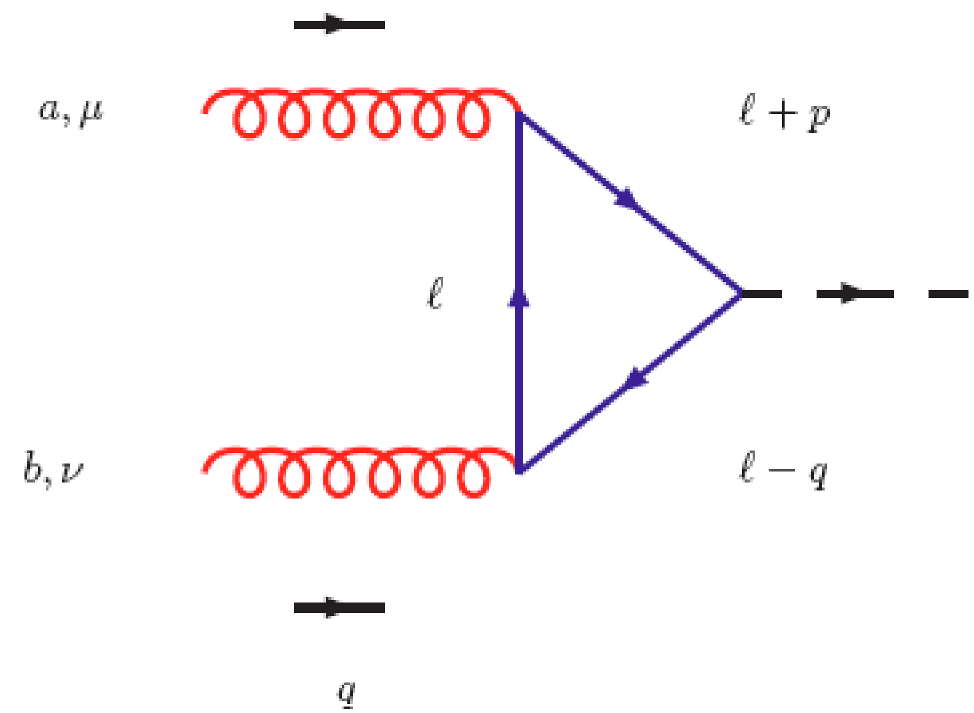
There's a term which apparently diverges....??
Ok, Let's look the scalar integrals up in a table (or calculate them!)

pp→H+x at LO

$$\int \frac{d^dk}{(2\pi)^d}\frac{k^2}{(k^2-C)^3} = \frac{i}{32\pi^2}(4\pi)^\epsilon \frac{\Gamma(1+\epsilon)}{\epsilon}(2-\epsilon)C^{-\epsilon}$$

$$\int \frac{d^dk}{(2\pi)^d}\frac{1}{(k^2-C)^3} = -\frac{i}{32\pi^2}(4\pi)^\epsilon\Gamma(1+\epsilon)C^{-1-\epsilon}.$$

where d=4-2eps. By substituting we arrive at a very simple final result!!



$$\mathcal{A}(gg \to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\int dxdy\left(\frac{1-4xy}{m_t^2 - m_H^2 xy}\right)\epsilon_\mu(p)\epsilon_\nu(q).$$

Comments:

* The final dependence of the result is m_t^2 : one from the Yukawa coupling, one from the spin flip.
* The tensor structure could have been guessed by gauge invariance.
* The integral depends on m_t and m_h.

LO cross section

$$\sigma(pp \to H) = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2\, g(x_1,\mu_f) g(x_2,\mu_f)\, \hat{\sigma}(gg \to H)$$

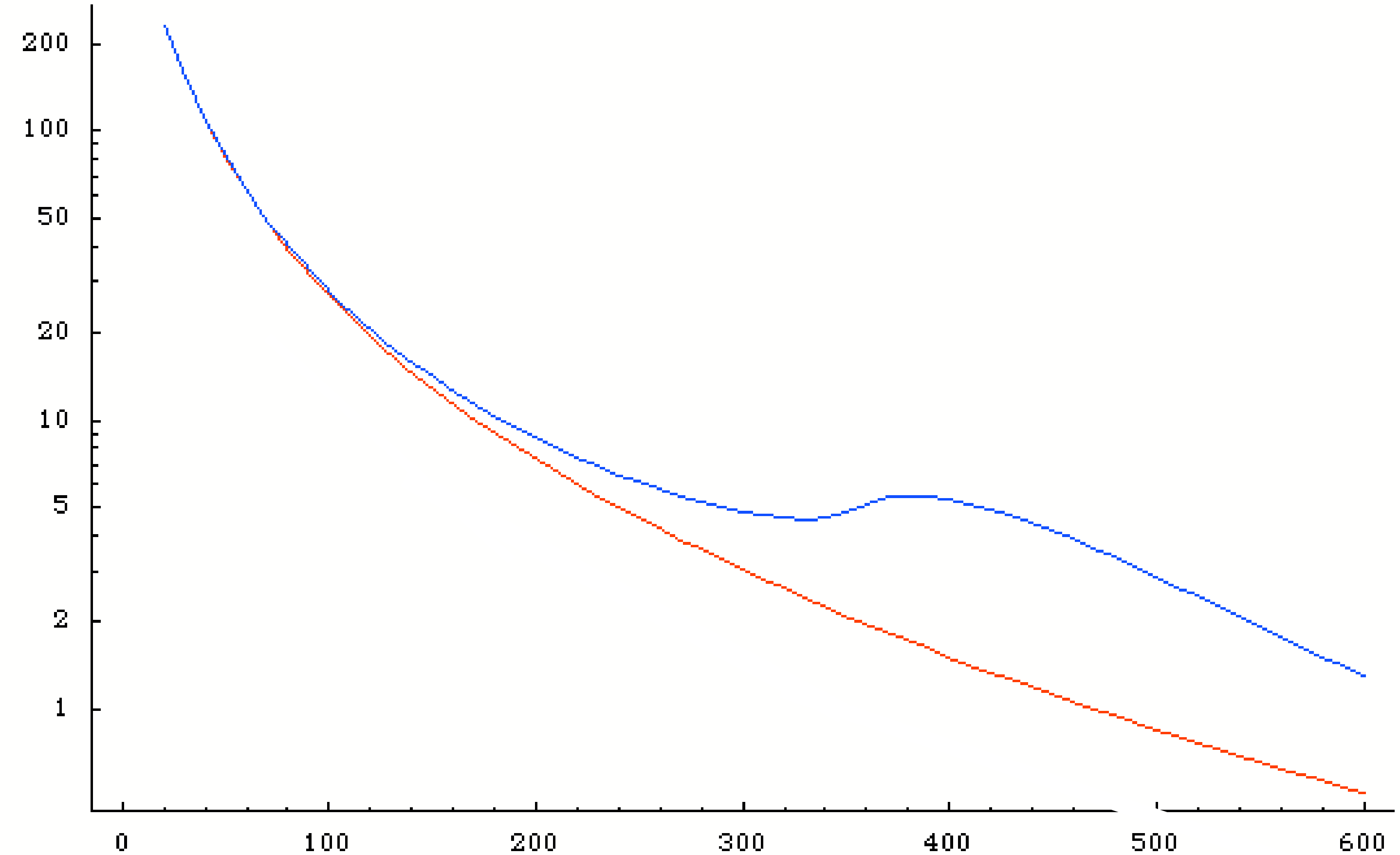
$$x_1 \equiv \sqrt{\tau} e^{y} \quad x_2 \equiv \sqrt{\tau} e^{-y} \quad \tau = x_1 x_2 \qquad \tau_0 = M_H^2/S \quad z = \tau_0/\tau$$

$$= \frac{\alpha_S^2}{64\pi v^2} \left| I\left(\frac{M_H^2}{m^2}\right) \right|^2 \tau_0 \int_{\log\sqrt{\tau_0}}^{-\log\sqrt{\tau_0}} dy g(\sqrt{\tau_0}e^{y}) g(\sqrt{\tau_0}e^{-y})$$

The hadronic cross section can be expressed a function of the gluon-gluon luminosity.

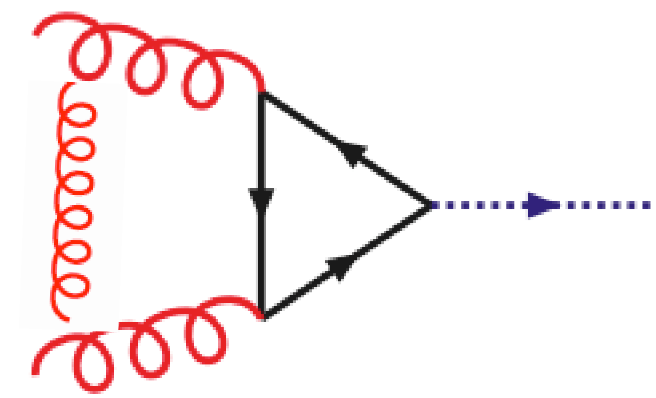
I(x) has both a real and imaginary part, which develops at $m_h = 2m_t$.

This causes a bump in the cross section.



pp →H+x @ NLO

At NLO we have to include an extra parton (virtual or real).



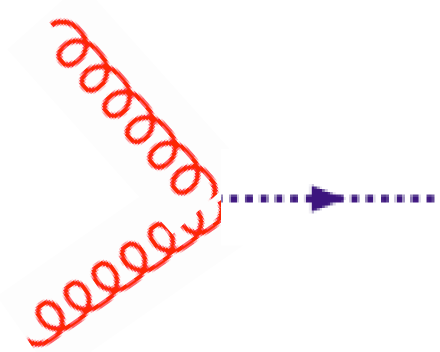
The virtuals will become a two-loop calculation!!

Can we avoid that?

Let's consider the case where the Higgs is light:

$$\mathcal{A}(gg \to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\int dxdy\left(\frac{1-4xy}{m_t^2 - m_H^2 xy}\right)\epsilon_\mu(p)\epsilon_\nu(q).$$

$$\xrightarrow{m \gg M_H} \quad -\frac{\alpha_S}{3\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\epsilon_\mu(p)\epsilon_\nu(q).$$



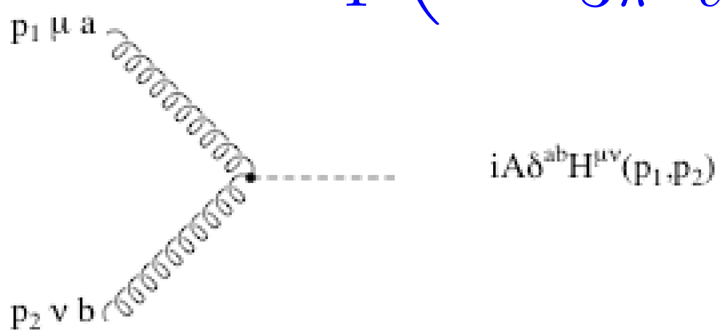
This looks like a local vertex, ggH.

The top quark has disappeared from the low energy theory but it has left something behind (non-decoupling).

Higgs effective field theory

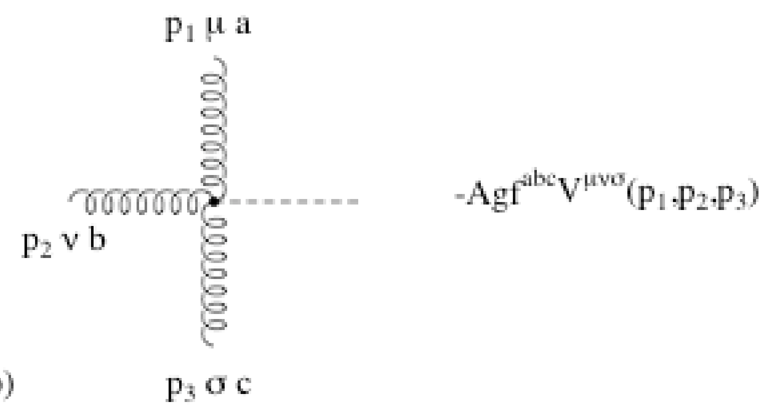
$$\mathcal{L}_{\rm eff} = -\frac{1}{4}\left(1-\frac{\alpha_S}{3\pi}\frac{H}{v}\right)G^{\mu\nu}G_{\mu\nu}$$

This is an effective non-renormalizable theory (no top) which describes the Higgs couplings to QCD.



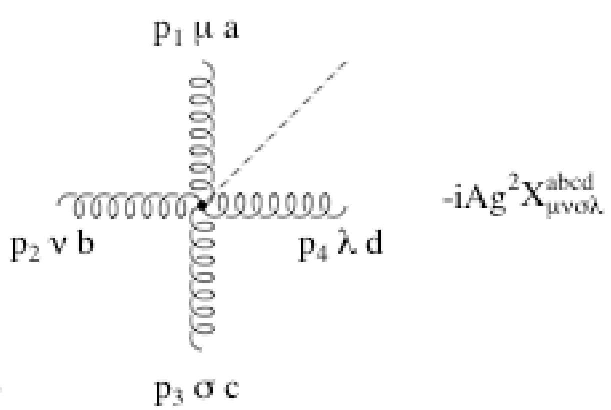
(a)

$$H^{\mu\nu}(p_1,p_2) = g^{\mu\nu}p_1\cdot p_2 - p_1^\nu p_2^\mu.$$



(b)

$$V^{\mu\nu\rho}(p_1,p_2,p_3) = (p_1-p_2)^\rho g^{\mu\nu} + (p_2-p_3)^\mu g^{\nu\rho} + (p_3-p_1)^\nu g^{\rho\mu},$$



(c)

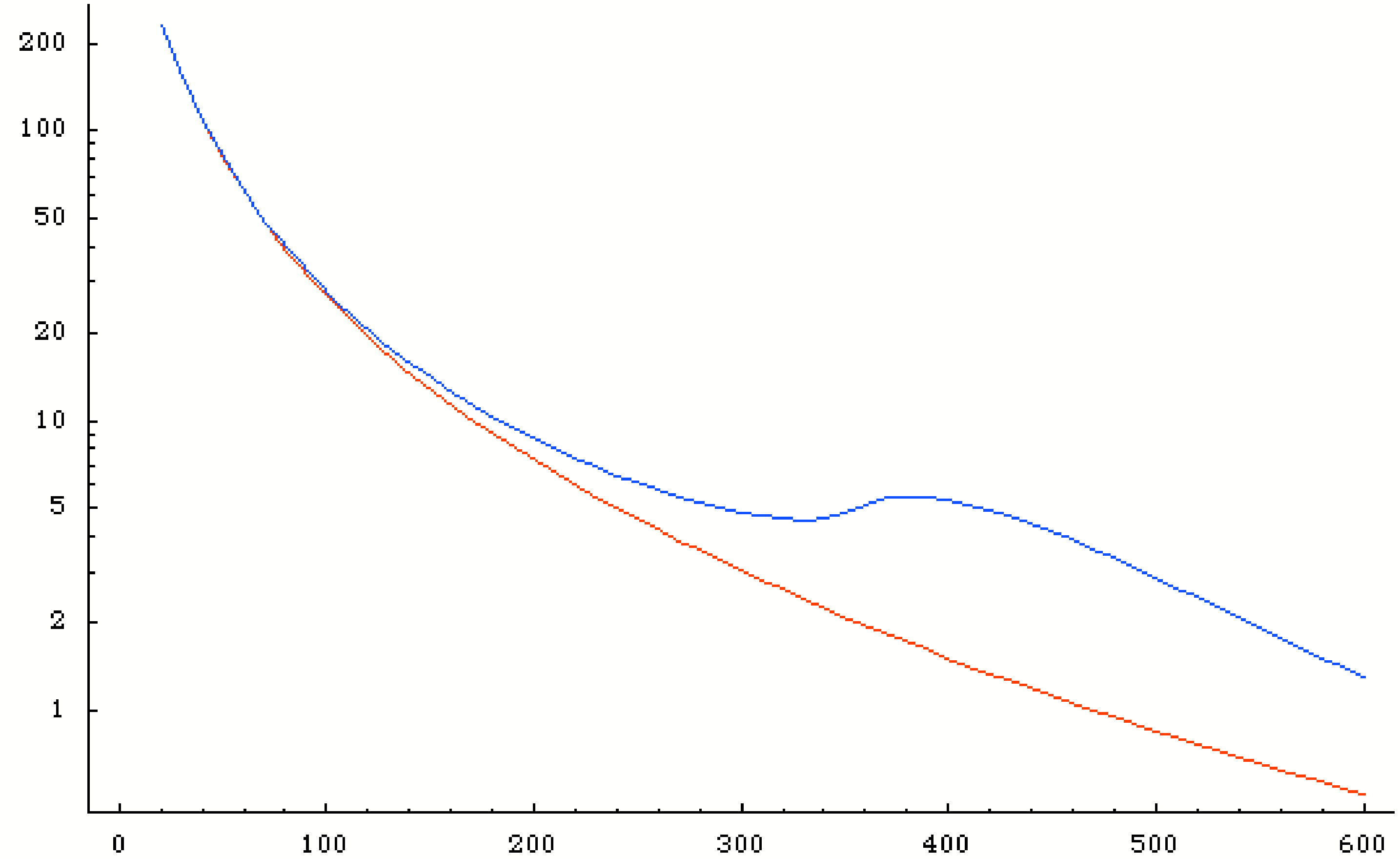
$$\begin{aligned} X^{\mu\nu\rho\sigma}_{abcd} &= f_{abe}f_{cde}(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho}) \\ &+ f_{ace}f_{bde}(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\sigma}g^{\nu\rho}) \\ &+ f_{ade}f_{bce}(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma}). \end{aligned}$$

LO cross section: full vs HEFT

$$\sigma(pp \to H) = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2\, g(x_1, \mu_f) g(x_2, \mu_f)\, \hat{\sigma}(gg \to H)$$

The accuracy of the calculation in the HEFT calculation can be directly assessed by taking the limit m→∞.

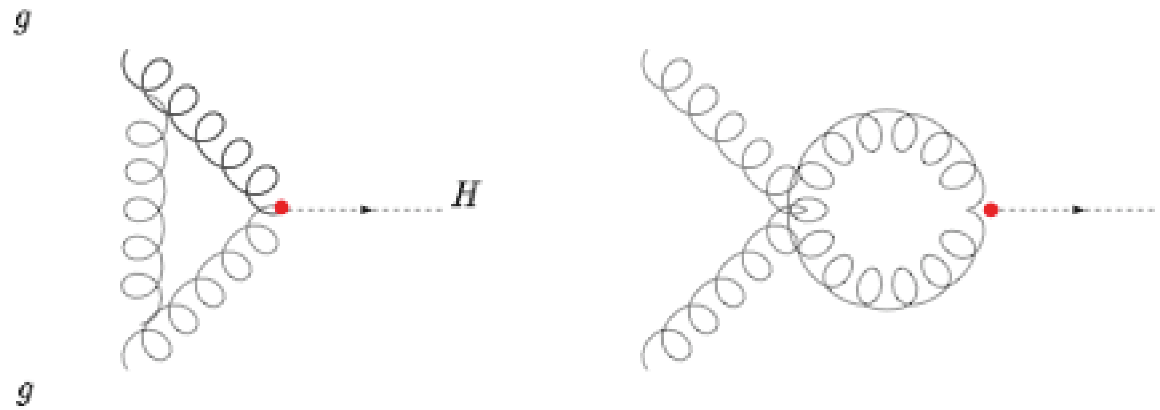
For light Higgs is better than 10%.



So, if we are interested in a light Higgs we use the HEFT and simplify our life. If we do so, the NLO calculation becomes a standard 1-loop calculation, similar to Drell-Yan at NLO.

We can (try to) do it!!

Virtual contributions



Out of 8 diagrams, only two are non-zero (in dimensional regularization), a bubble and a triangle.

They can be easily written down by hand.

Then the integration over the tensor decomposition into scalar integrals and loop integration has to be performed.

$$\mathcal{L}_{\rm eff}^{\rm NLO} = \left(1 + \frac{11}{4}\frac{\alpha_S}{\pi}\right)\frac{\alpha_S}{3\pi}\frac{H}{v}G^{\mu\nu}G_{\mu\nu}$$

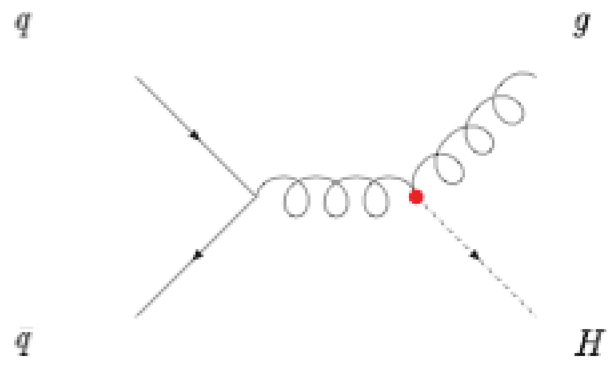
One also have to consider that the coefficient of the HEFT receive corrections which have to be included in the result.

The result is:

$$\sigma_{\rm virt} = \sigma_0\,\delta(1-z)\,\left[1 + \frac{\alpha_S}{2\pi}C_A\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon} c_\Gamma\left(-\frac{2}{\epsilon^2} + \frac{11}{3} + \pi^2\right)\right],$$

$$\sigma_{\rm Born} = \frac{\alpha_S^2}{\pi}\frac{m_H^2}{576 v^2 s}(1+\epsilon+\epsilon^2)\mu^{2\epsilon}\,\delta(1-z) \equiv \sigma_0\,\delta(1-z) \qquad z = m_H^2/s$$

Real contributions I



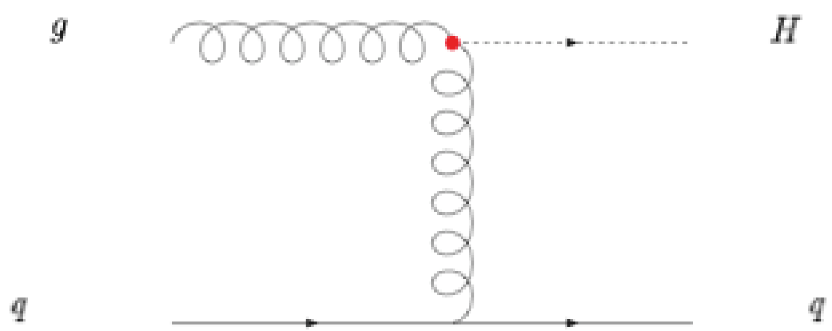
$$\overline{|\mathcal{M}|^2} = \frac{4}{81}\frac{\alpha_S^3}{\pi v^2}\frac{(\hat{u}^2+\hat{t}^2)-\epsilon(\hat{u}+\hat{t})^2}{\hat{s}}$$

Integrating over phase space (cms angle theta)

$$\hat{t} = -\hat{s}(1-z)(1-\cos\theta)/2$$
$$\hat{u} = -\hat{s}(1-z)(1+\cos\theta)/2$$

$$\sigma_{\rm real}(q\bar{q}) = \sigma_0\frac{\alpha_S}{2\pi}\frac{64}{27}\frac{(1-z)^3}{z}$$

finite!



$$\overline{|\mathcal{M}|^2} = -\frac{1}{54(1-\epsilon)}\frac{\alpha_S^3}{\pi v^2}\frac{(\hat{u}^2+\hat{s}^2)-\epsilon(\hat{u}+\hat{s})^2}{\hat{t}}$$

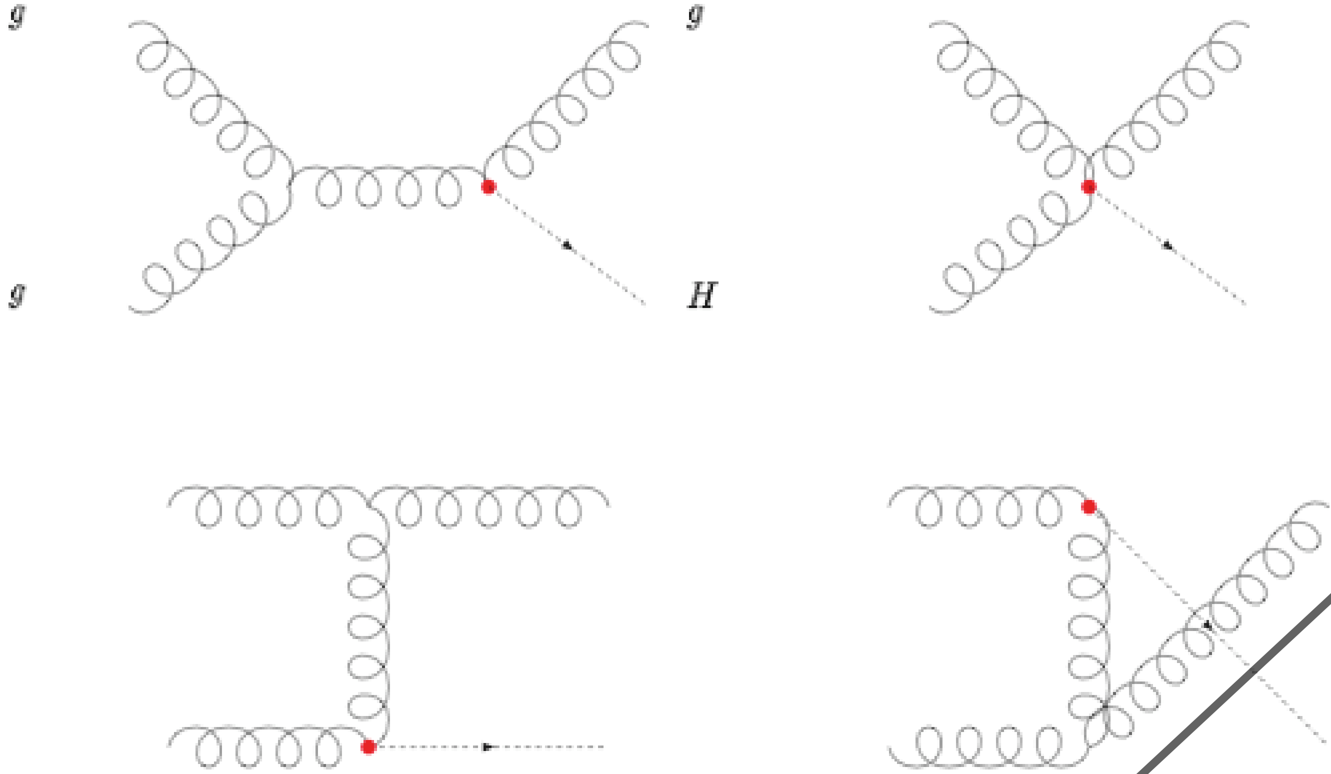
Integrating over the D-dimensional phase space the collinear singularity manifests a pole in 1/eps

$$\sigma_{\rm real} = \sigma_0\frac{\alpha_S}{2\pi}C_F\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon}c_\Gamma\left[-\frac{1}{\epsilon}p_{gq}(z)+\frac{(1-z)(7z-3)}{2z}+p_{gq}(z)\log\frac{(1-z)^2}{z}\right]$$

$$\sigma^{\rm coll.}_{\rm c.t.} = \sigma_0\frac{\alpha_S}{2\pi}\left[\left(\frac{\mu^2}{\mu_F^2}\right)^{\epsilon}\frac{c_\Gamma}{\epsilon}P_{gq}(z)\right]$$

$$\begin{aligned}\sigma^{\overline{\rm MS}}(qg) &= \sigma_{\rm real}+\sigma^{\rm coll.}_{\rm c.t.}\\ &= \sigma_0\frac{\alpha_S}{2\pi}C_F\left[p_{gq}(z)\log\frac{m_H^2}{\mu_F^2}+p_{gq}(z)\log\frac{(1-z)^2}{z}+\frac{(1-z)(7z-3)}{2z}\right]\end{aligned}$$

Real contributions II



This is the last piece: the result at the end must be finite!

2/eps cancels with the virtual contribution ✓

This is the renormalization of the coulping!!

$$\sigma^{\mathrm{UV}}_{\mathrm{c.t.}} = 2\,\sigma_{\mathrm{Born}}\frac{\alpha_S}{2\pi}\left[-\left(\frac{\mu^2}{\mu^2_{\mathrm{UV}}}\right)^{\epsilon} c_\Gamma \frac{b_0}{\epsilon}\right] \checkmark$$

This is an initial-state divergence to be reabsorbed in the pdf

$$\sigma^{\mathrm{coll.}}_{\mathrm{c.t.}} = 2\,\sigma_0\frac{\alpha_S}{2\pi}\left[\left(\frac{\mu^2}{\mu^2_F}\right)^{\epsilon}\frac{c_\Gamma}{\epsilon}P_{gg}(z)\right] \checkmark$$

$$\begin{aligned}\sigma_{\mathrm{real}} = {} & \sigma_0\,\frac{\alpha_S}{2\pi}C_A\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon} c_\Gamma\left[\left(\frac{2}{\epsilon^2}+\frac{2}{\epsilon}\frac{b_0}{C_A}-\frac{\pi^2}{3}\right)\delta(1-z)\right.\\ & -\frac{2}{\epsilon}p_{gg}(z)-\frac{11}{3}\frac{(1-z)^3}{z}-4\frac{(1-z)^2(1+z^2)+z^2}{z(1-z)}\log z\\ & \left.+4\frac{1+z^4+(1-z)^4}{z}\left(\frac{\log(1-z)}{1-z}\right)_+\right].\end{aligned}$$

Final results = you made it!!

$$\sigma(pp \to H) = \sum_{ij} \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 f_i(x_1,\mu_f) f_j(x_2,\mu_f) \hat{\sigma}(ij)[\mu_f/m_h, \mu_r/m_h, \alpha_S(\mu_r)]$$

The final cross section is the sum of three channels: q qbar, q g, and g g.

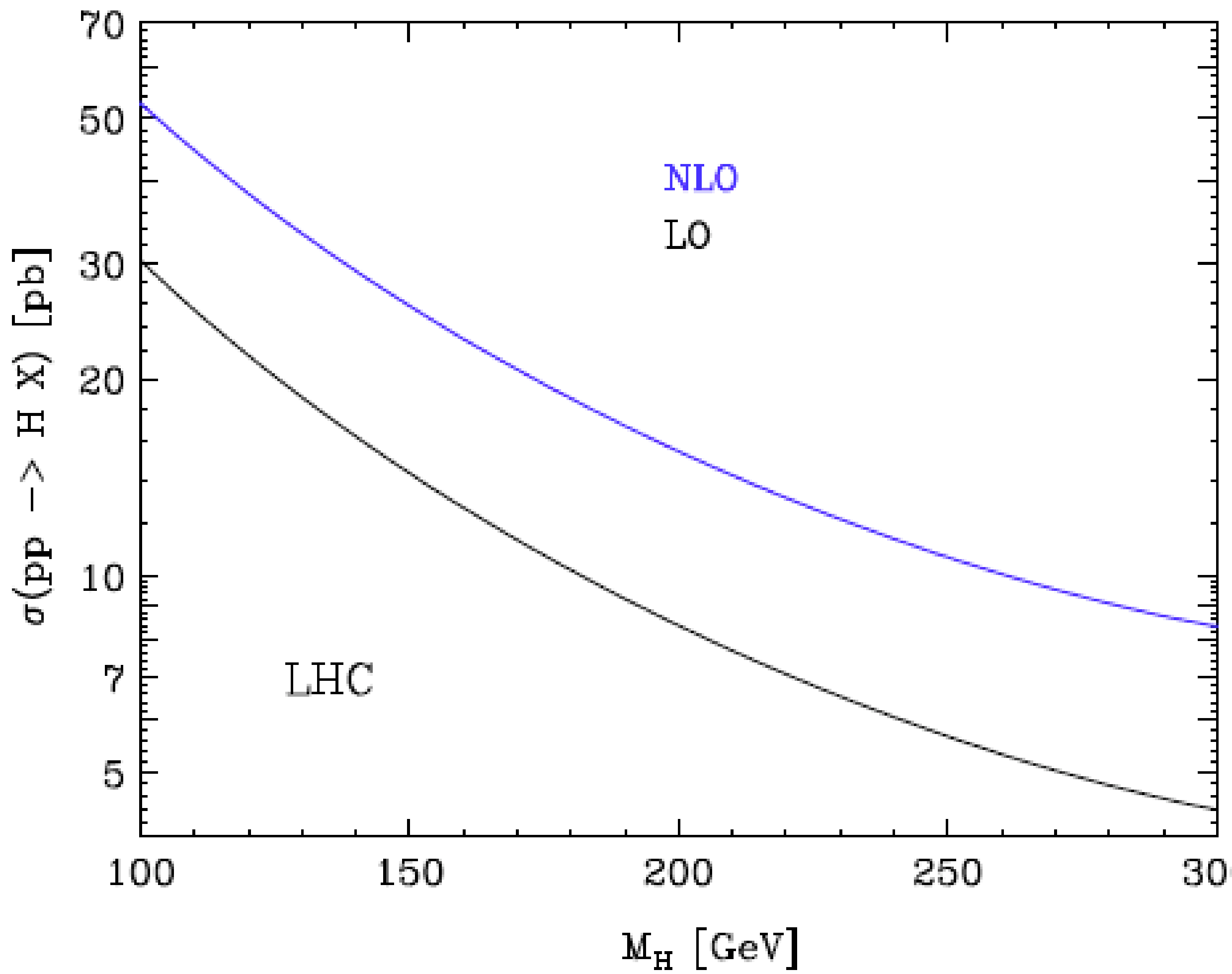
The short distance cross section at NLO depends explicitly on the subtraction scales (renormalization and factorization).

The explicit integration over the pdf's is trivial (just mind the plus distributions).

The result is that the corrections are huge!

K factor is ~2 and scale dependence not really very much improved.

Is perturbation theory valid? NNLO is mandatory...



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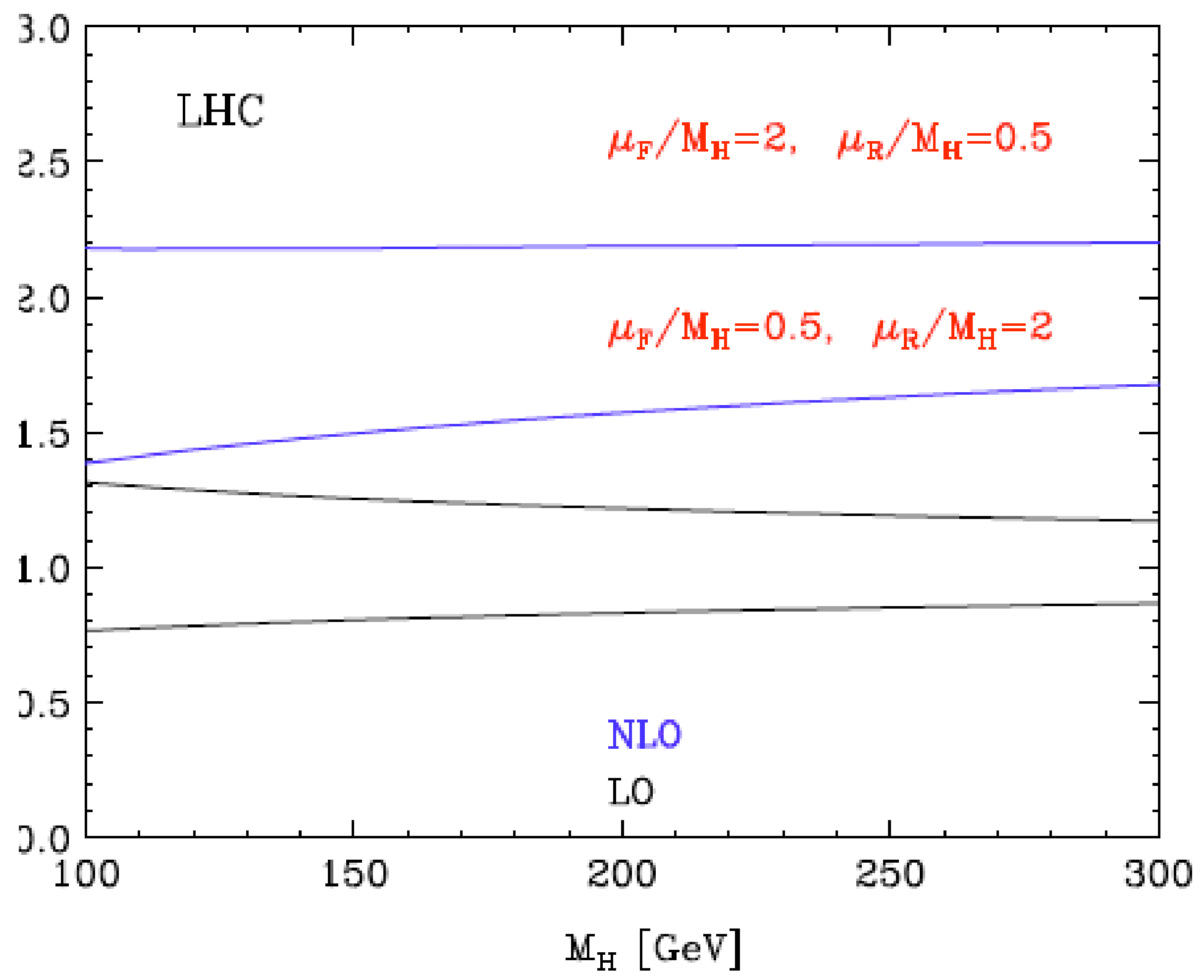


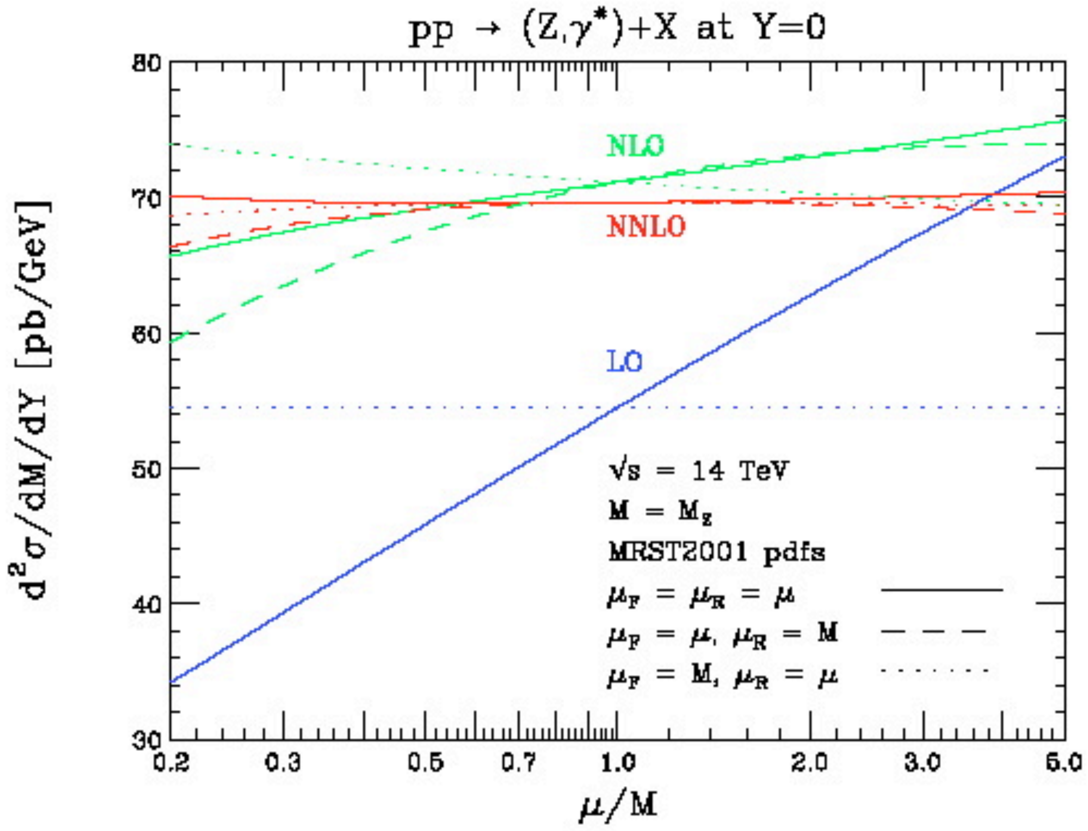
Predictions at NNLO

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1,\mu_F^2) f_b(x_2,\mu_F^2) \times \hat\sigma_{ab\to X}(x_1,x_2,\alpha_S(\mu_R^2),\frac{Q^2}{\mu_F^2},\frac{Q^2}{\mu_R^2})$$

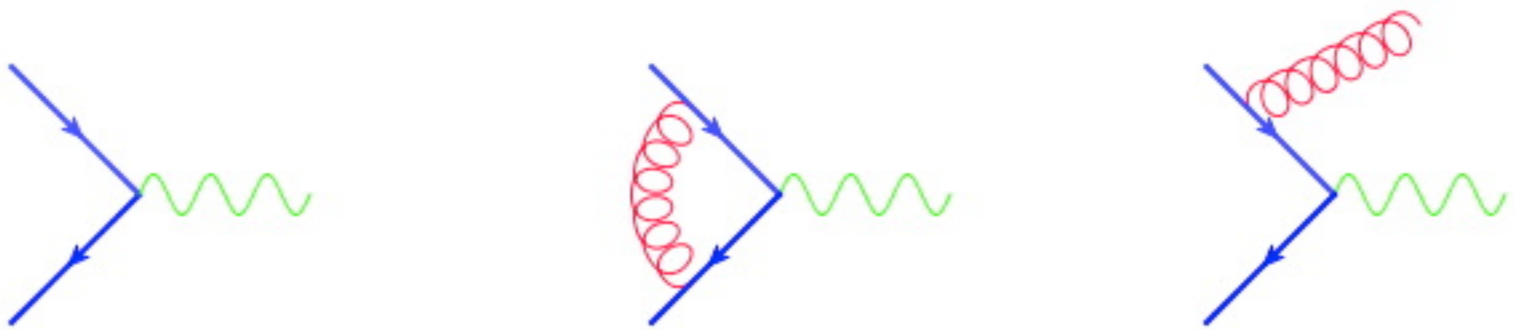
$$\hat\sigma_{ab\to X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \ldots$$

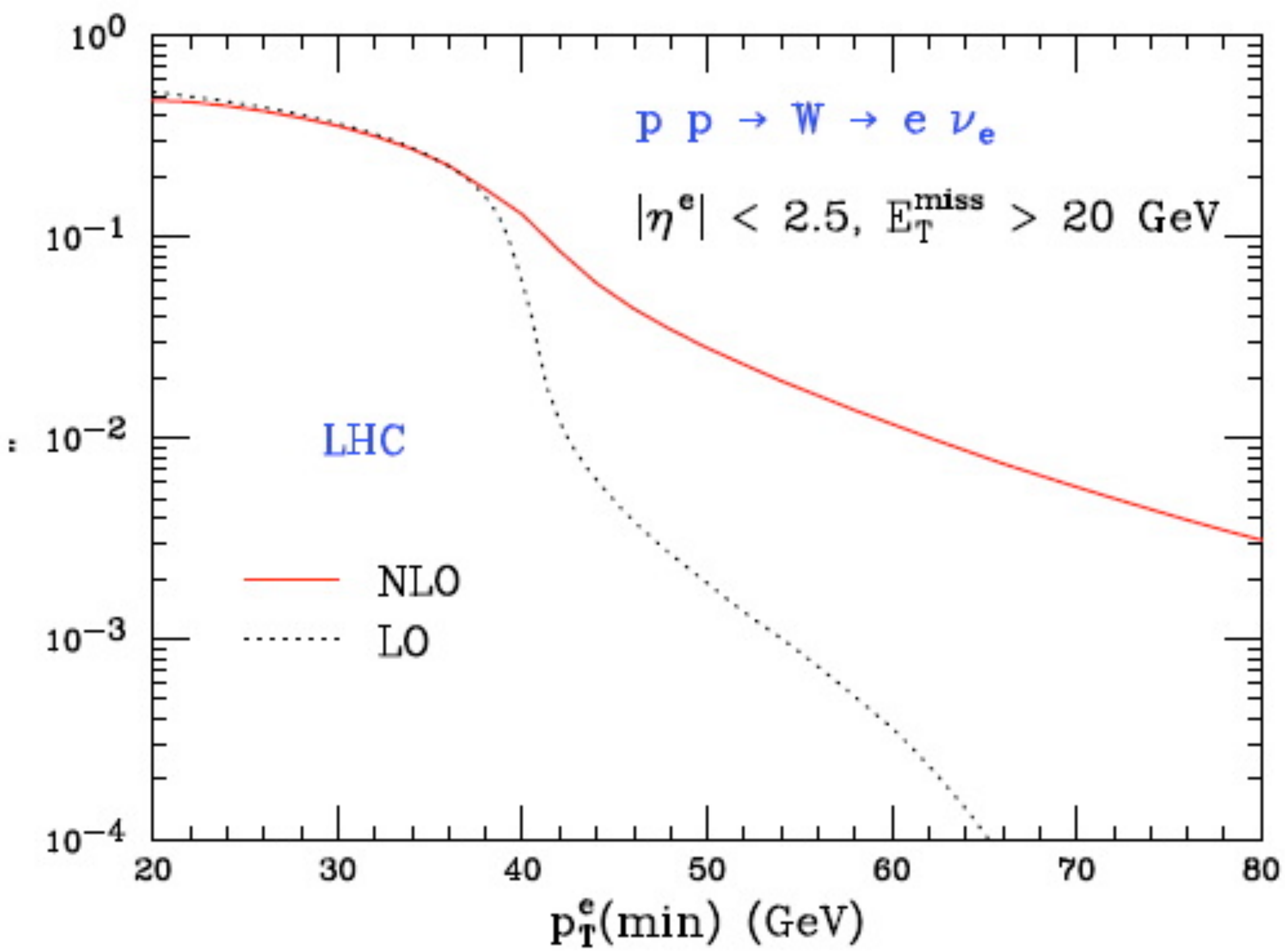
Why?

- A NNLO computation gives control on the uncertainties of a perturbative calculation.
- It's "mandatory" if NLO corrections are very large to check the behaviour of the perturbative series
- It's the best we have! It is needed for Standard Candles and for really exploiting all the available information, for example that of NNLO PDF's.



Drell-Yan predictions at NLO



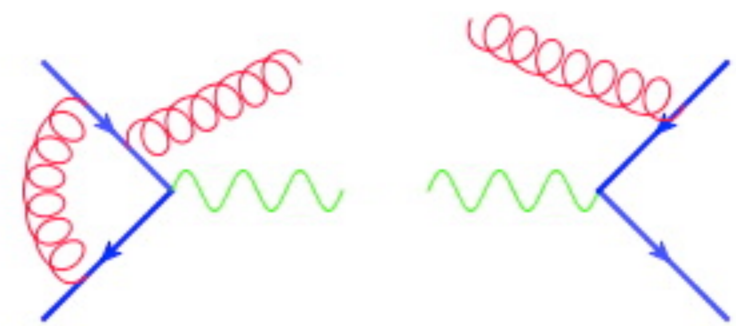


- At LO the W has no p_T, therefore the pt of the lepton has a sharp cutoff.
- The "K-factor" looks like enormous at high p_T. When this happens it means that the observable you are looking at it is actually at LO not at NLO!
- It is important to keep the spin correlations of the lepton in the calculation.

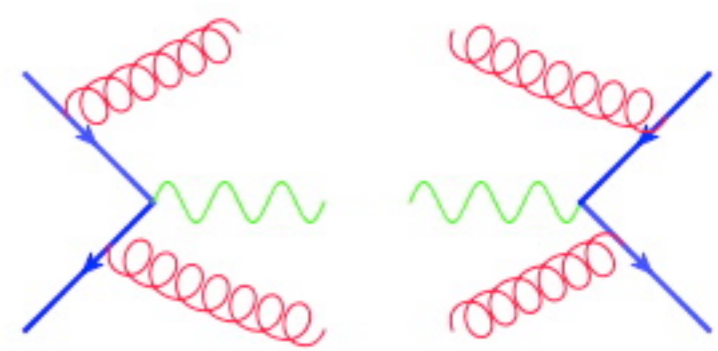
Drell-Yan predictions at NNLO

- Virtual-Virtual : O(100) terms

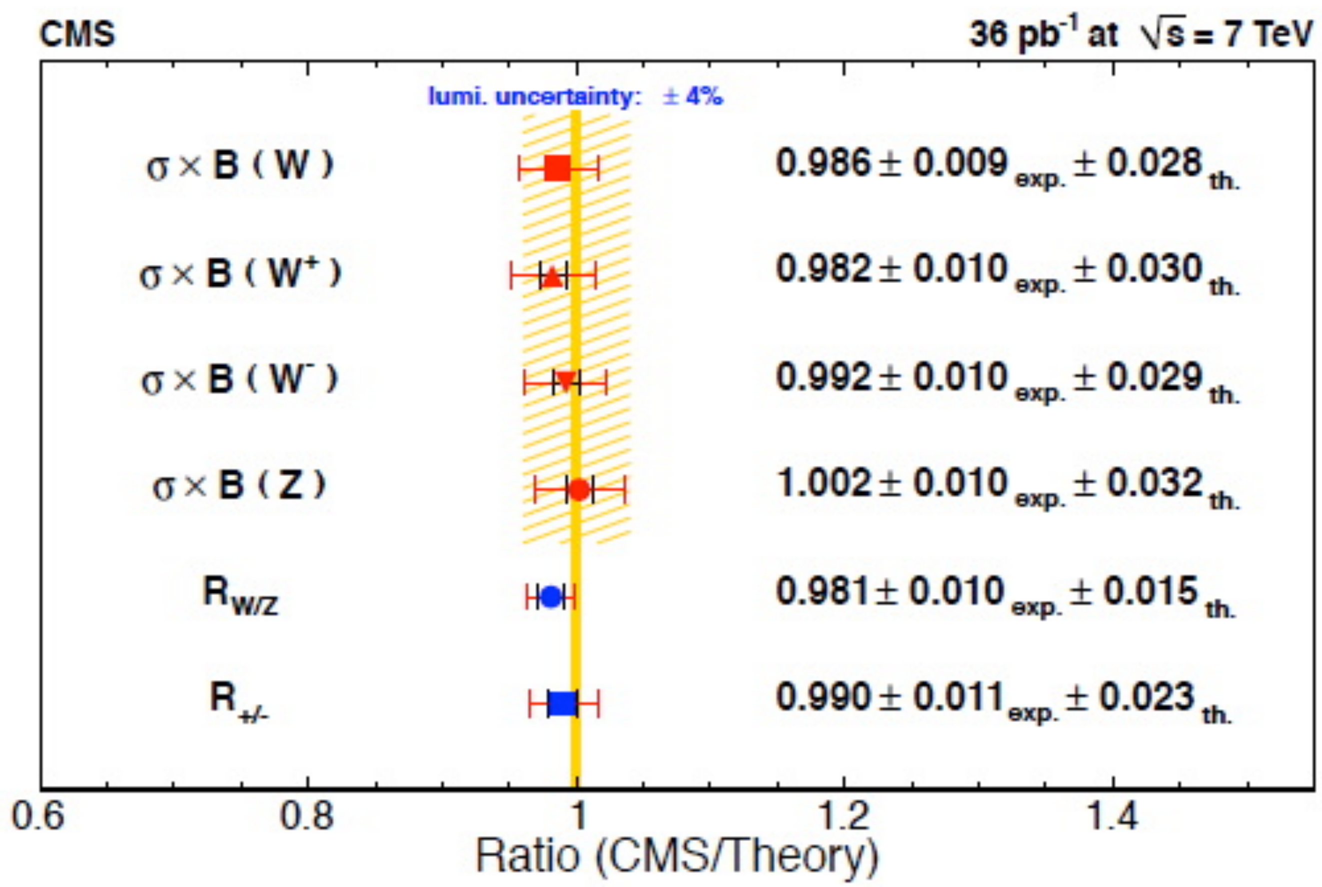
- Real-Virtual : O(300) terms



- Real-Real : O(500) terms



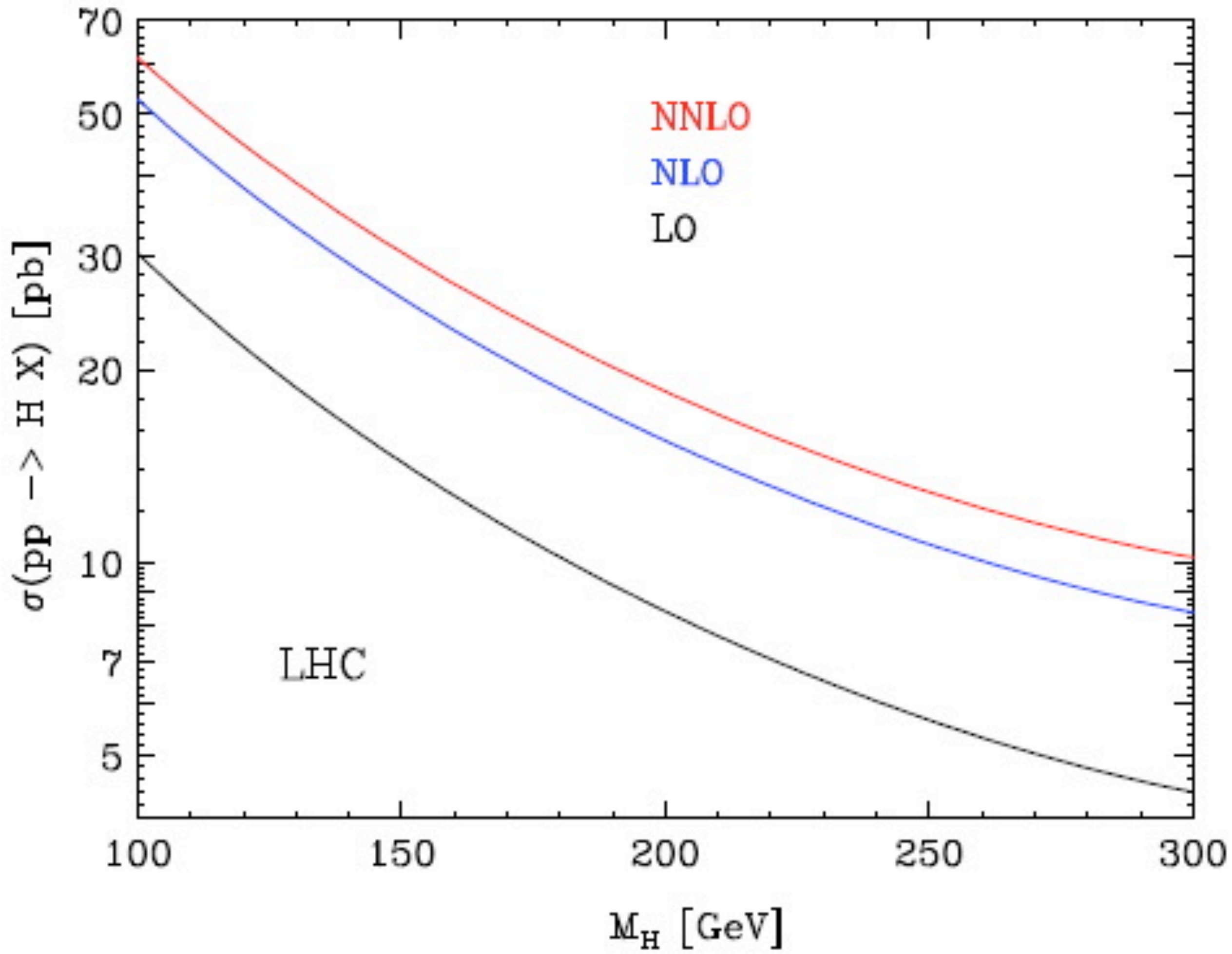
Drell-Yan predictions at NNLO

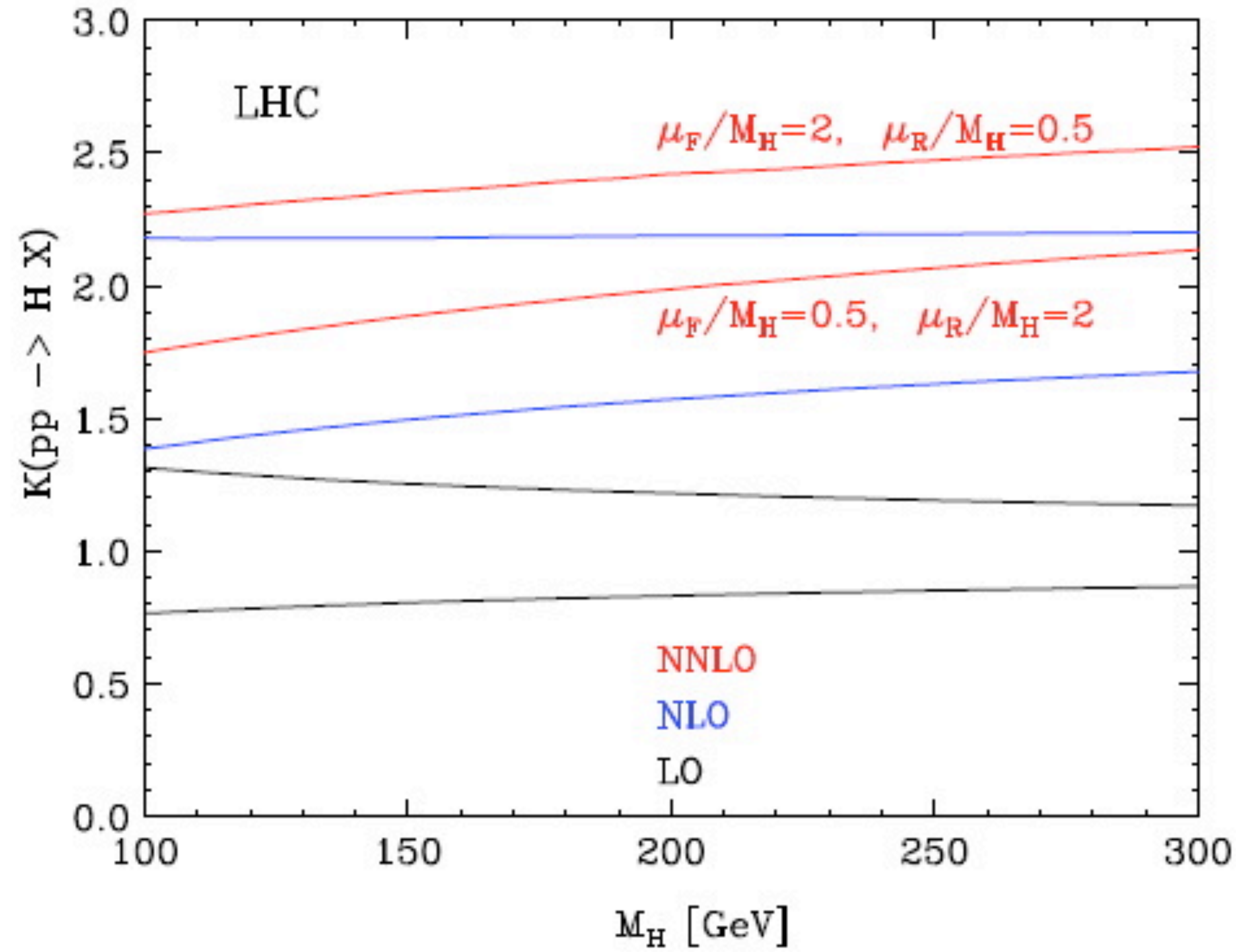


[TH = Anastasiou, Dixon, Melnikov, Petriello. 2004]

- Impressive improvement of the scale dependence.
- High-p_T end of the electron and extra jet known at NLO accuracy

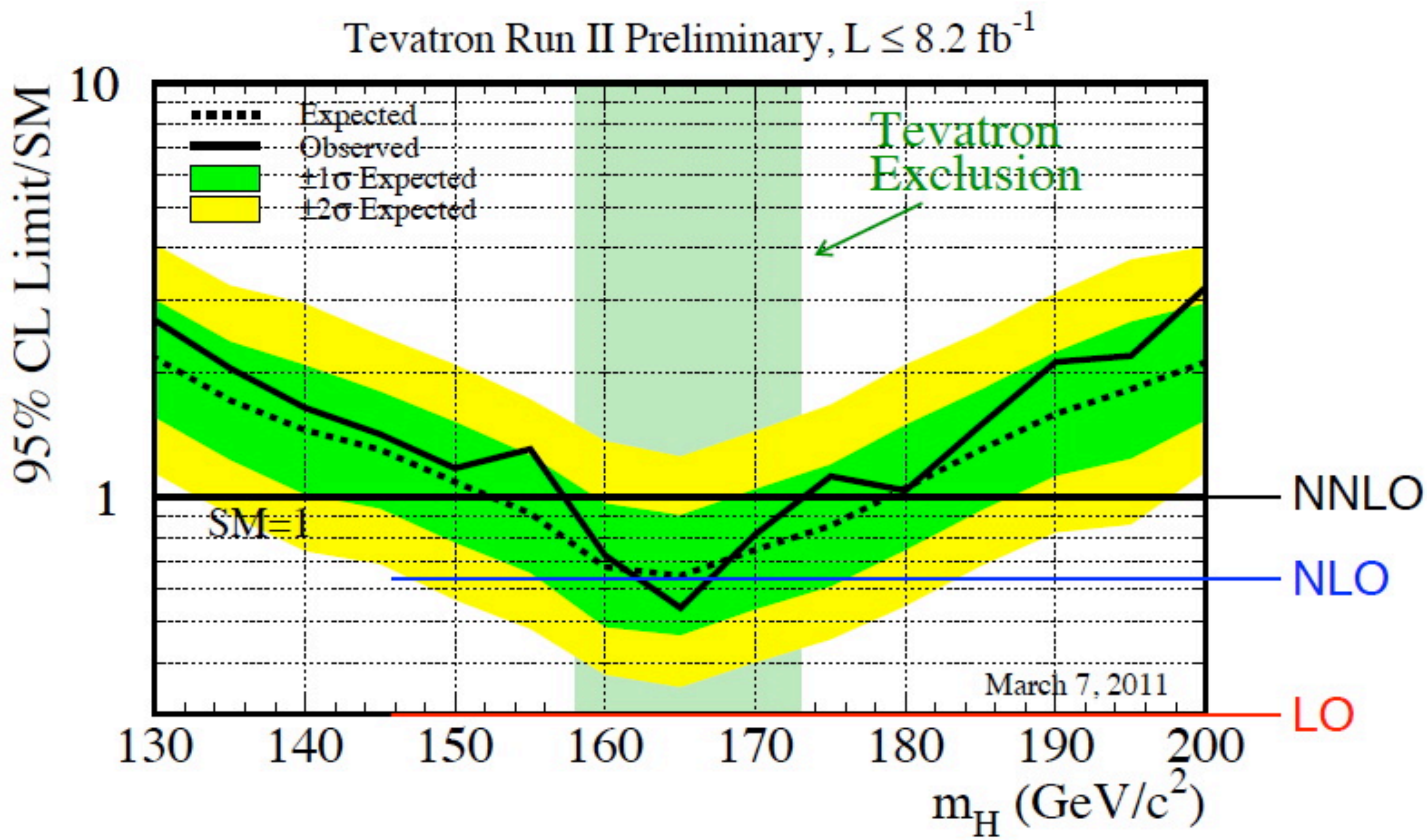
Higgs predictions at NNLO





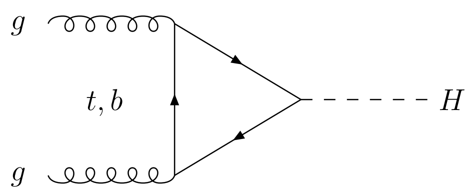
- The perturbative series stabilizes.
- NLO estimation of higher orders effects by scale uncertainty works reasonably well

Higgs predictions at NNLO

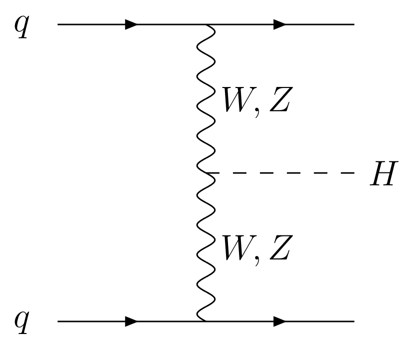


be careful : just illustrative example, not very precise

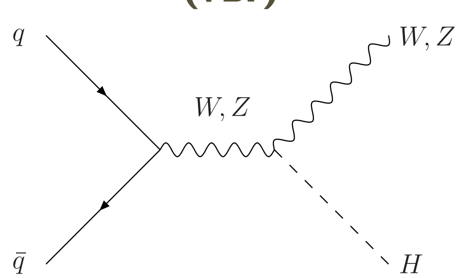
Higgs predictions at 7 TeV



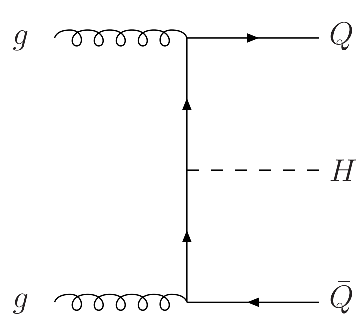
GluonFusion



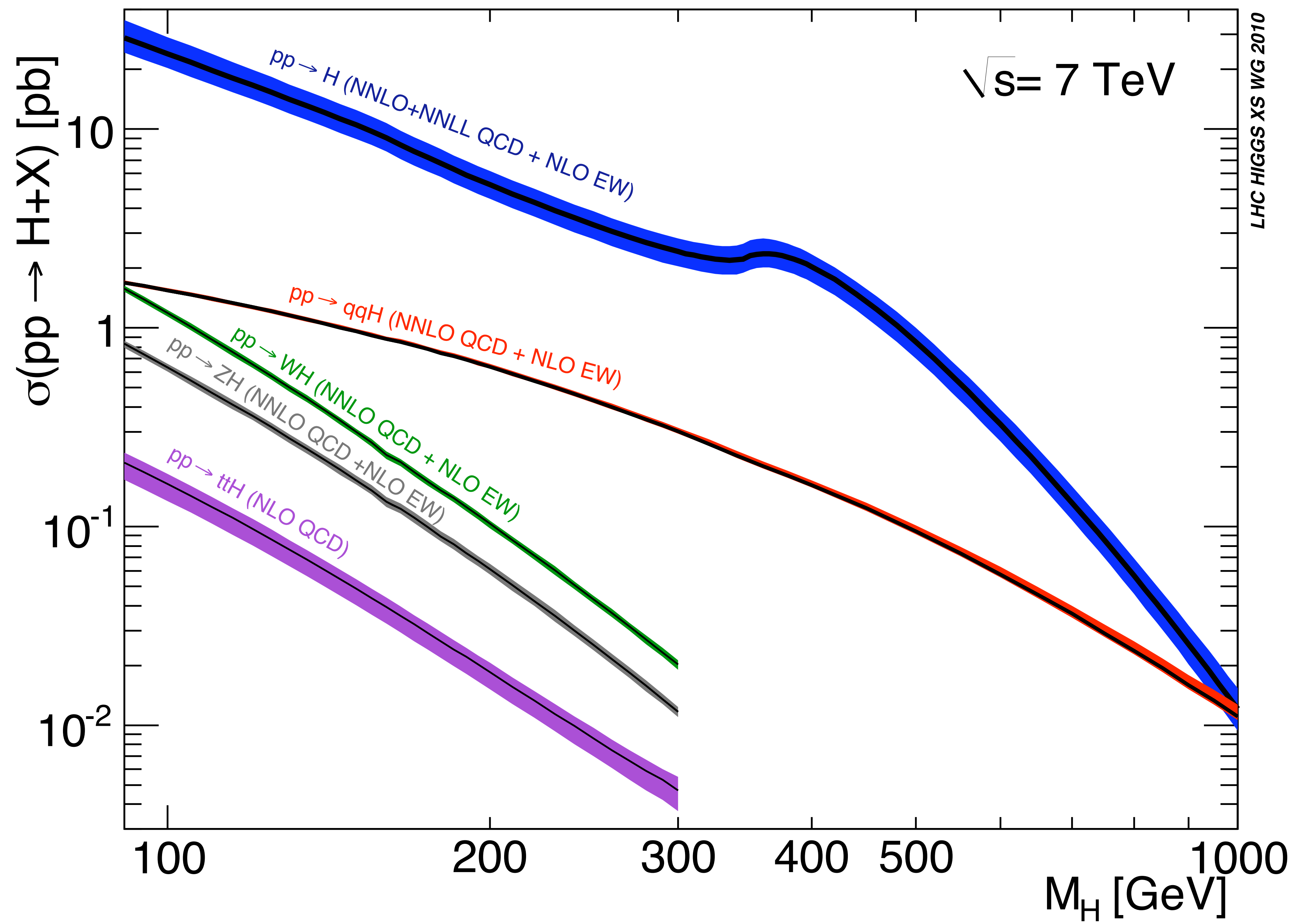
vector boson fusion (VBF)



associated production with vector bosons



associated production with heavy quarks



Predictions at NNLO : final remarks

- Handful of precious predictions at NNLO now available for Higgs and Drell-Yan processes at the parton level for distributions.

- Others (VV, ttbar) in progress and in sight.

NNLO stays to the LHC era
as
NLO stayed to the Tevatron era