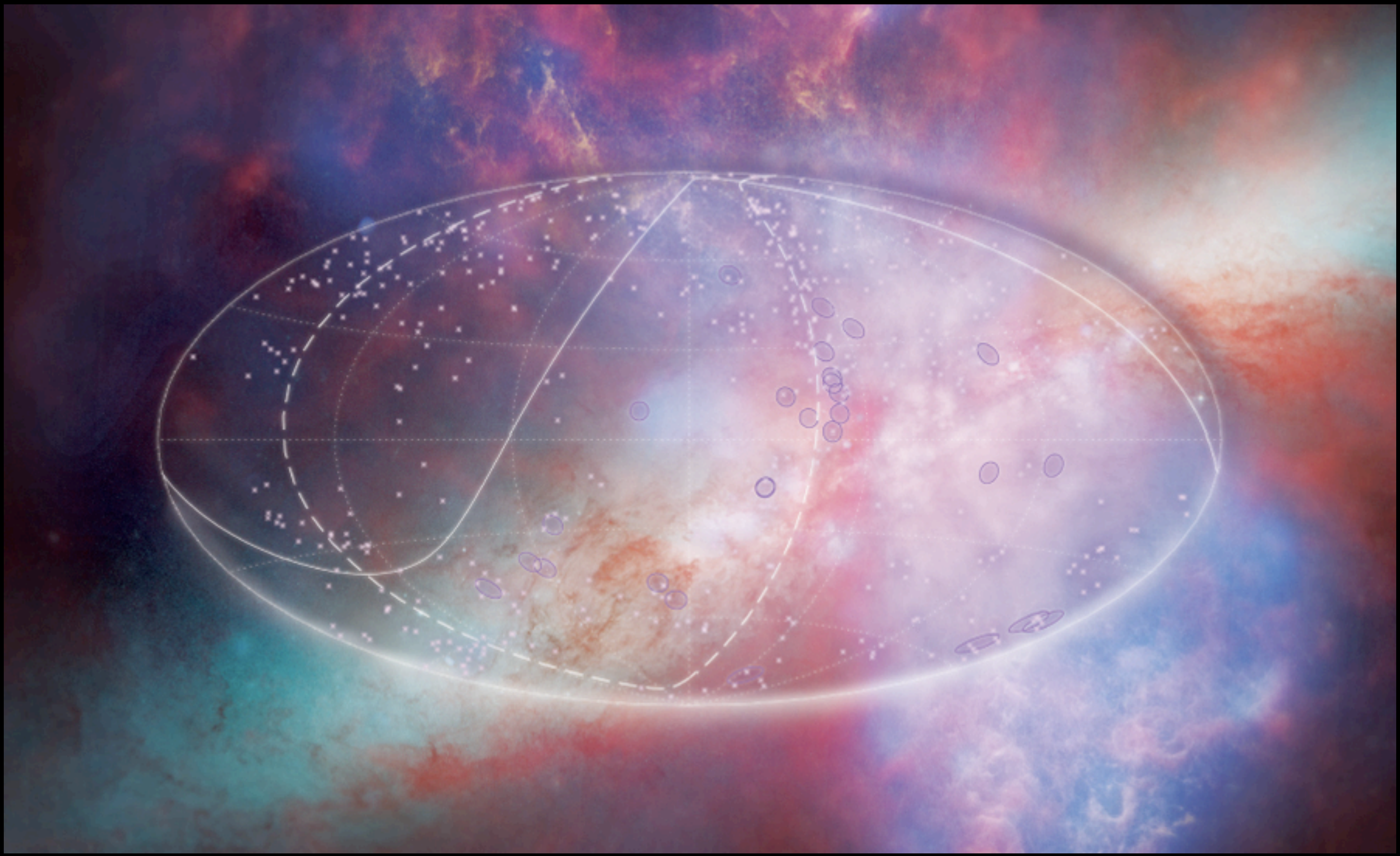
Ultrahigh Energy Cosmic Rays: few more facts and fantasies



Luis Anchordoqui

Myth, Legend, or Fantasy?

Optically thin source

- It is helpful to envision CR engines as machines where protons are accelerated and (possibly) permanently confined by magnetic fields of acceleration region
- Production of neutrons and pions and subsequent decay produces neutrinos, gamma-rays, and CRs
- If the neutrino-emitting source also produces high energy CRs then pion production must be principal agent for high energy cutoff on proton spectrum
- Conversely ☛ since protons must undergo sufficient acceleration inelastic pion production needs to be small below cutoff energy consequently ☛ plasma must be optically thin
- Since interaction time for protons is greatly increased over that of neutrons because of magnetic confinement ☛ neutrons escape before interacting and on decay give rise to observed CR flux

Optically thin source (cont'd)

- 3 conditions on:
  - characteristic nucleon interaction time scale $\tau_{\rm int}$
  - neutron decay lifetime τ_n
  - characteristic cycle time of confinement $\tau_{\rm cycle}$
  - total proton confinement time $\tau_{\rm conf}$

$$(i)\ \tau_{\rm int} \gg \tau_{\rm cycle} \qquad (ii)\ \tau_n > \tau_{\rm cycle} \qquad (iii)\ \tau_{\rm int} \ll \tau_{\rm conf}$$

- (i) ensures that protons attain sufficient energy
- (ii) and (iii) allow neutrons to escape source before decaying
- (iii) permits sufficient interaction to produce n's and nu's

Waxman-Bahcall bound

CR flux above ankle often summarized as

"one 3×10^{10} GeV particle per km square per yr per sr"

translated into energy flux

$$E\,\{EJ_{\rm CR}\} = \frac{3 \times 10^{10}\,{\rm GeV}}{(10^{10}\,{\rm cm}^2)(3 \times 10^7\,{\rm s})\,{\rm sr}} = 10^{-7}\,{\rm GeV\ cm^{-2}\,s^{-1}\,sr^{-1}}$$

Derive energy density in UHECRs using flux = velocity $\times$ density

$$4\pi \int dE\,\{EJ_{\rm CR}\} = c\epsilon_{\rm CR}$$

taking $E_{\rm min} \simeq 10^{10}$ GeV and $E_{\rm max} = 10^{12}$ GeV

$$\epsilon_{\rm CR} = \frac{4\pi}{c} \int_{E_{\rm min}}^{E_{\rm max}} \frac{10^{-7}}{E} dE\,\frac{\rm GeV}{{\rm cm}^2\,{\rm s}} \simeq 10^{-19}\,{\rm TeV\,cm^{-3}}$$

Power required to generate this energy density over Hubble time

$$\mathcal{T} \approx 10^{10}\ {\rm yr}$$

Waxman-Bahcall bound (cont'd)

$$\dot{\epsilon}_{\rm CR}^{[10^{10},10^{12}]} \sim 5 \times 10^{44}\ {\rm TeV\,Mpc^{-3}\,yr^{-1}} \simeq 3 \times 10^{37}\ {\rm erg\,Mpc^{-3}\,s^{-1}}$$

Energy-dependent generation rate of CRs is therefore

$$E^2 \frac{d\dot{n}}{dE} = \frac{\dot{\epsilon}_{\rm CR}^{[10^{10},10^{12}]}}{\ln(10^{12}/10^{10})} \approx 10^{44}\,{\rm erg\,Mpc^{-3} yr^{-1}}$$

Energy density of neutrinos ☛ $E_\nu^2 \dfrac{dn_\nu}{dE_\nu} \approx \dfrac{3}{8} \epsilon_\pi\, \mathcal{T}\, E^2 \dfrac{d\dot{n}}{dE}$

"Waxman-Bahcall bound" is defined by condition $\epsilon_\pi = 1$

$$E_\nu^2\ \Phi_{\rm WB}^{\nu_{\rm all}} \approx (3/8)\ \xi_z\, \epsilon_\pi\, \mathcal{T}\, \frac{c}{4\pi}\, E^2 \frac{d\dot{n}}{dE} \approx 2.3 \times 10^{-8}\, \epsilon_\pi\, \xi_z\, {\rm GeV\, cm^{-2}\, s^{-1}\, sr^{-1}}$$

$\xi_z \sim 3$ accounts for effects of source evolution with redshift

Waxman & Bahcall, Phys. Rev. D 59 (1999) 023002

Ultrahigh energy neutrinos from Cen A

Upper bound on directional flux from Cen A

$$E^2 F_{\nu_{\rm all}} = \frac{1}{4\pi d^2}\, L_{\rm CR}\, \frac{3}{8}\, \epsilon_\pi$$
$$\approx 5 \times 10^{-9}\ {\rm GeV\, cm^{-2}\, s^{-1}}$$

Preliminary upper bound from Auger (ICRC 2009)

$$E^2 \Phi_{\nu_{\rm all}} = 3 \times 10^{-6}\ {\rm GeV\, cm^{-2}\, s^{-1}}$$

Diffuse flux assuming Cen A typifies the FRI population

$\mathcal{R} \simeq 1$ horizon $\simeq 3$ Gpc $\qquad n_{\rm FRI} \sim 8 \times 10^4 {\rm Gpc}^{-3}$

$$E^2 J_{\nu_{\rm all}} = \frac{1}{4\pi}\, \mathcal{R}\, n_{\rm FRI}\, L_{\rm CR}\, \frac{3}{8}\, \epsilon_\pi$$
$$\approx 1.5 \times 10^{-8}\ {\rm GeV\, cm^{-2}\, s^{-1}\, sr^{-1}}$$

LAA, Goldberg, Halzen, Weiler, Phys. Lett. B 600 (2004) 202

Ultrahigh energy neutrinos (cont'd)

- Fit to CR flux + assumption of transparent sources implies WB bound

  Waxman & Bahcall Phys. Rev. D 59 (1999) 023002

- Similar argument for Cen A implies directional neutrino bound

- Additional transparent sources hidden by Xtragalactic B-field should contribute to diffuse neutrino flux

- If Cen A typifies source population maximum emission energy of CRs and neutrinos is reduced

- Reduction of maximum luminosity roughly compensates for presence of far away neutrino sources not visible in CRs no enhancement of WB bound do to hidden sources

Exercise 3

The assumption that GRBs are the sources of the observed UHECRs generates a calculable flux of neutrinos produced when the protons interact with the fireball photons

In the observer's frame, the spectral photon density $(\mathrm{GeV}^{-1}\,\mathrm{cm}^{-3})$ can be adequately parametrized by a broken power-law spectrum

$$n_\gamma^{\mathrm{GRB}}(\epsilon_\gamma) \propto \epsilon_\gamma^{-\beta} \text{ where } \beta \simeq 1,\ 2$$

respectively at energies below and above $\epsilon_\gamma^{\mathrm{break}} \simeq 1\ \mathrm{MeV}$

Show that $\boxed{\Phi_{\mathrm{GRB}}^{\nu_{\mathrm{all}}}(E_\nu > E_\nu^{\mathrm{break}}) \sim 10^{-13}\ \left(\frac{E_\nu^{\mathrm{break}}}{10^5\ \mathrm{GeV}}\right)^{-1}\ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}}$

where ☛ $\boxed{E_\nu^{\mathrm{break}} \sim 5\times 10^5\,\Gamma_{2.5}^2(\epsilon_\gamma^{\mathrm{break}}/\mathrm{MeV})^{-1}\ \mathrm{GeV}}$

Recall that ☛ $\epsilon_\gamma^{\mathrm{lab}} = \Gamma\,\epsilon_\gamma^{\mathrm{fireball}}$

Convince yourself that the non-observation of extraterrestrial neutrinos from sources other than the Sun and SN1987a puts the GRB model of UHECR acceleration on probation

GZK neutrinos

Diffuse neutrino flux has additional component originating in energy losses of UHECRs en route to Earth

Accumulation of these neutrinos over cosmological time is known as cosmogenic neutrino flux

For spatially homogeneous distribution of sources emitting UHECRs of type i ☛ comoving number density Y_i is governed by Boltzman equation

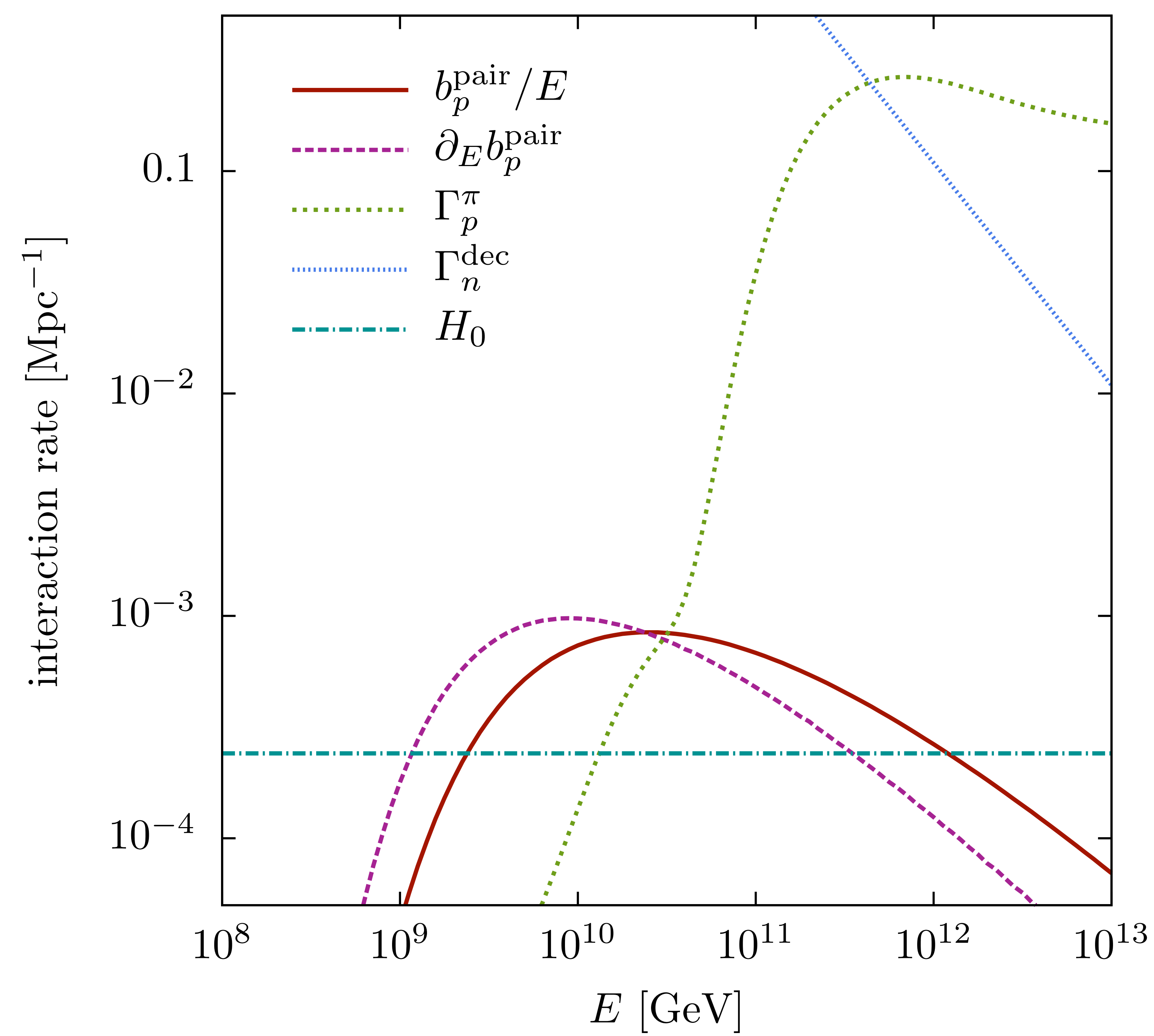
$$\dot{Y}_i = \partial_E(HEY_i) + \partial_E(b_iY_i) - \Gamma_i\,Y_i + \sum_j \int \mathrm{d}E_j\,\gamma_{ji}Y_j + Q_i$$

together with Friedman equation describing cosmic expansion rate $H(z)$ as function of redshift z

$$n_i(z,E) \equiv (1+z)^3\,Y_i(z,E)$$

For CMB only first term rhs contribute (adiabatic scaling) number density per comoving volume is constant number density per volume gets diluted with expanding universe

Fractional energy losses at $z = 0$



Ahlers, LAA, Gonzalez-Garcia, Halzen, and Sarkar, Astropart. Phys. 34 (2010) 106

Universal source population

Emission rate of CR protons per co-moving volume is assumed to follow power-law

$$\mathcal{Q}_p(0,E) \propto (E/E_0)^{-\gamma} \times \begin{cases} f_-(E/E_{\rm min}) & E < E_{\rm min}\,, \\ 1 & E_{\rm min} < E < E_{\rm max}\,, \\ f_+(E/E_{\rm max}) & E_{\rm max} < E \end{cases}$$

consider spectral indices γ in range $2 \div 3$

functions $f_\pm(x) \equiv x^{\pm 2} \exp(1 - x^{\pm 2})$ smoothly turn off contribution below $E_{\rm min}$ and above $E_{\rm max}$

take $E_{\rm max} = 10^{12}$ GeV vary $E_{\rm min}$ in range $10^{8.5} \div 10^{10}$ GeV

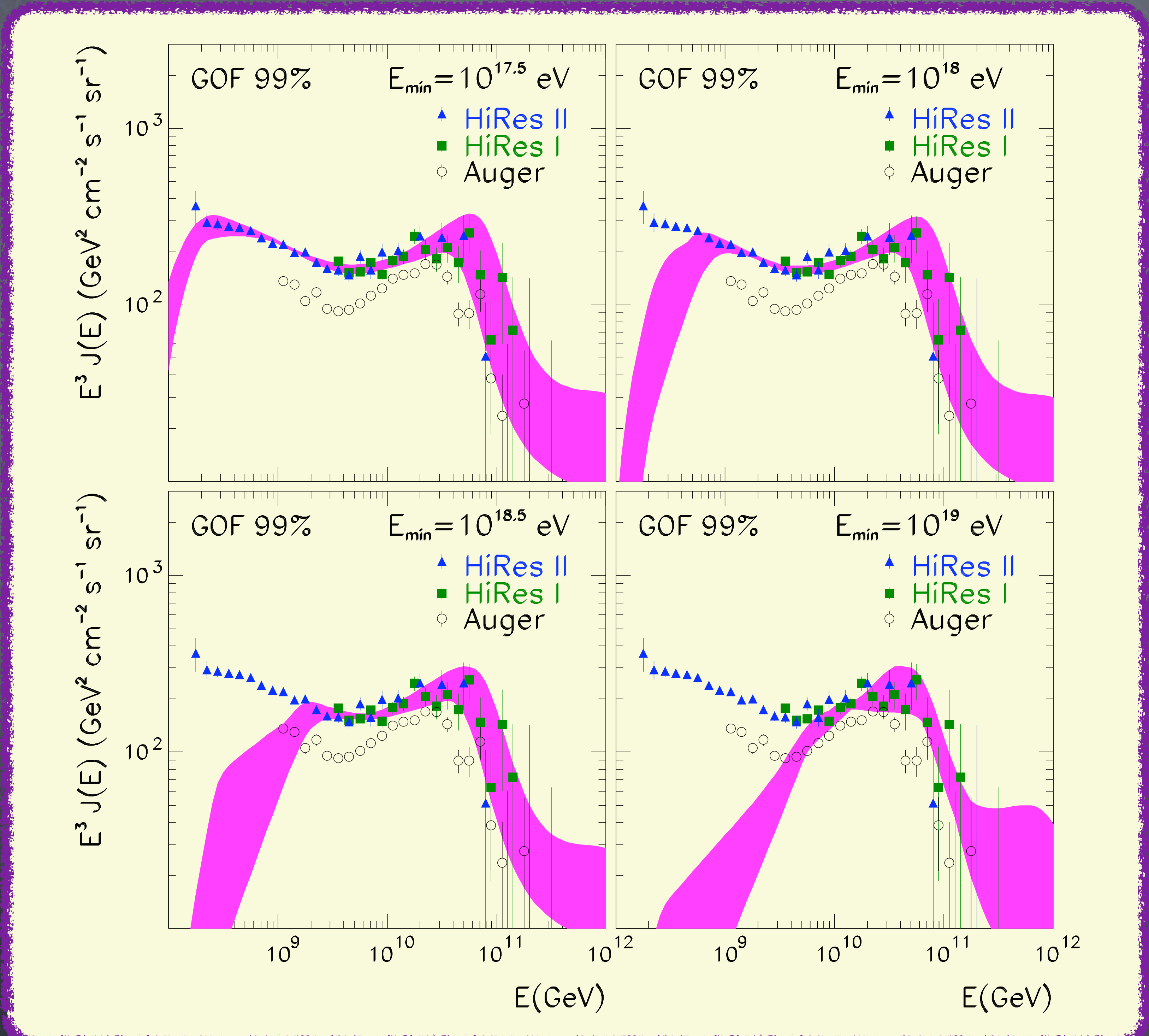
cosmic evolution of spectral emission rate per comoving volume parameterized by ☛ $\mathcal{Q}_p(z,E) = \mathcal{H}(z)\mathcal{Q}_p(0,E)$

For simplicity ☛ we use standard approximation

$$\mathcal{H}(z) \equiv (1+z)^n \Theta(z_{\rm max} - z) \qquad z_{\rm max} = 2$$

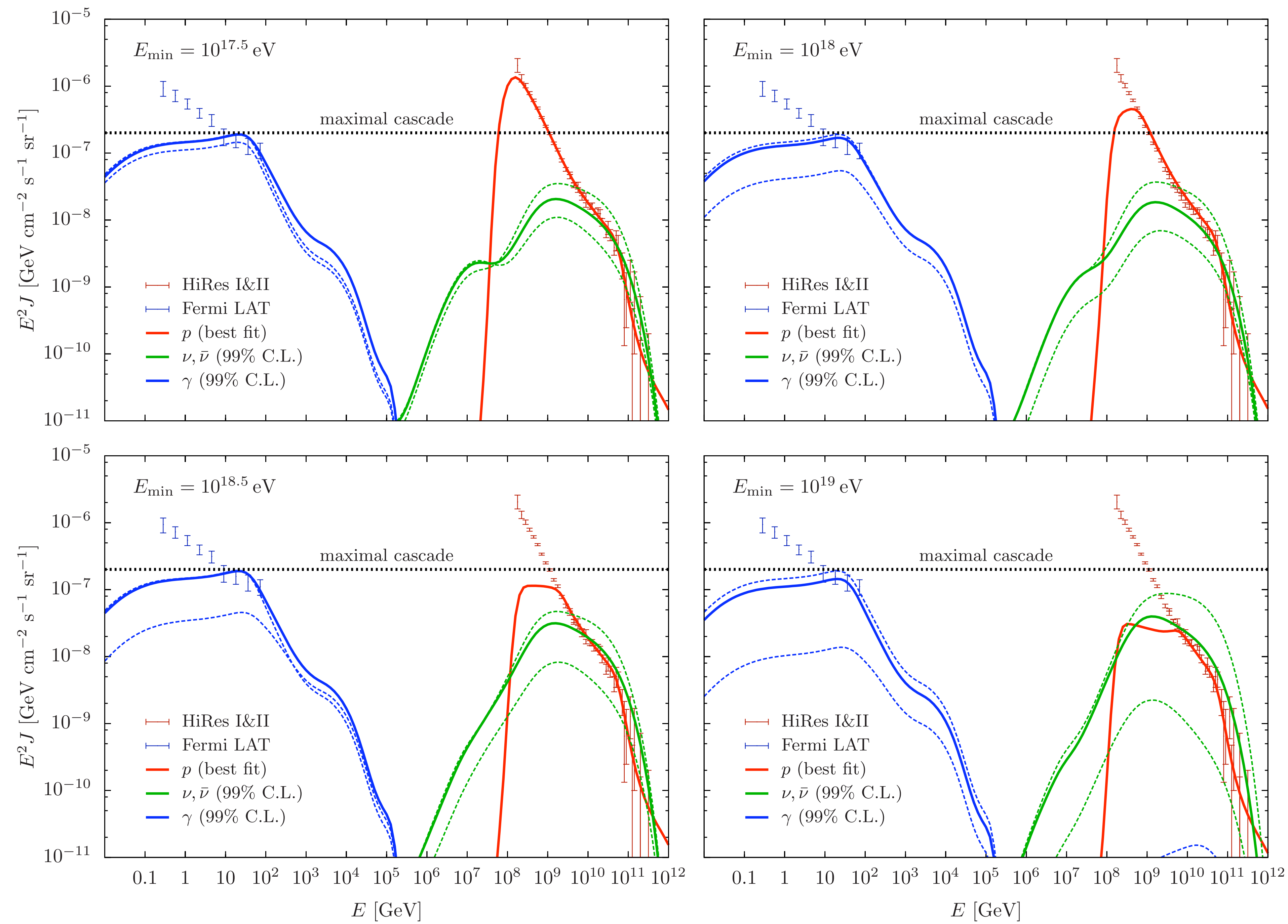
Goodness-of-fit test

Allowed proton flux (@ 99% CL) for increasing crossover energy



Ahlers, LAA, Gonzalez-Garcia, Halzen, and Sarkar, Astropart. Phys. 34 (2010) 106

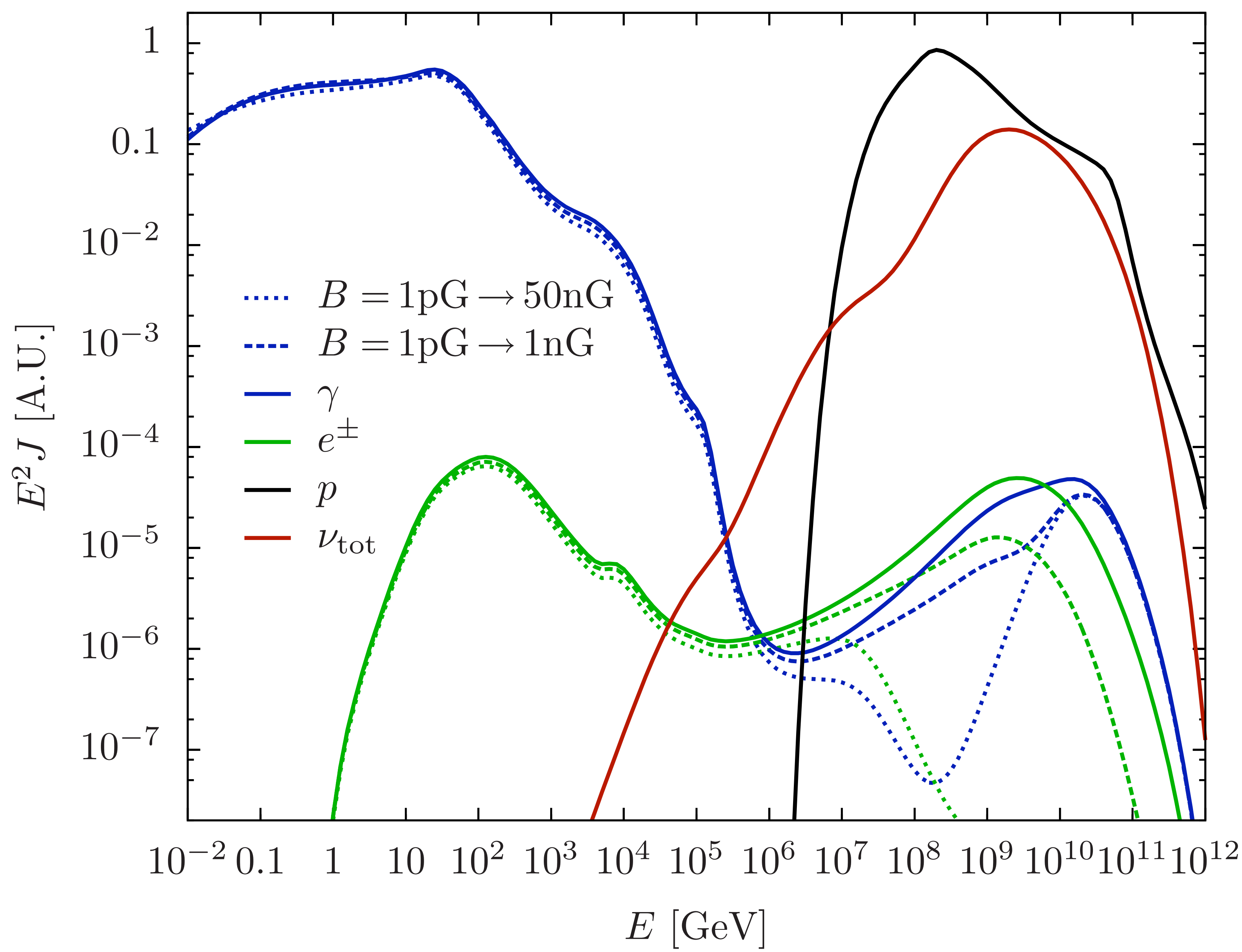
Limits on cosmogenic neutrino flux from Fermi-LAT data



$$B = 10^{-12}\ \mathrm{G}$$

Ahlers, LAA, Gonzalez-Garcia, Halzen, and Sarkar, Astropart. Phys. 34 (2010) 106

Influence of extragalactic $\vec{B}$-field



$$E_\gamma^{\text{syn}} = \left(\frac{3}{2}\right)^{1/2} \frac{h\, e\, E_e^2\, B}{2\,\pi\, m_e^3\, c^5} \sim 5.4 \times 10^{-2}\, B_{\mu\text{G}}\, (E_e/\text{EeV})^2\, \text{TeV}$$

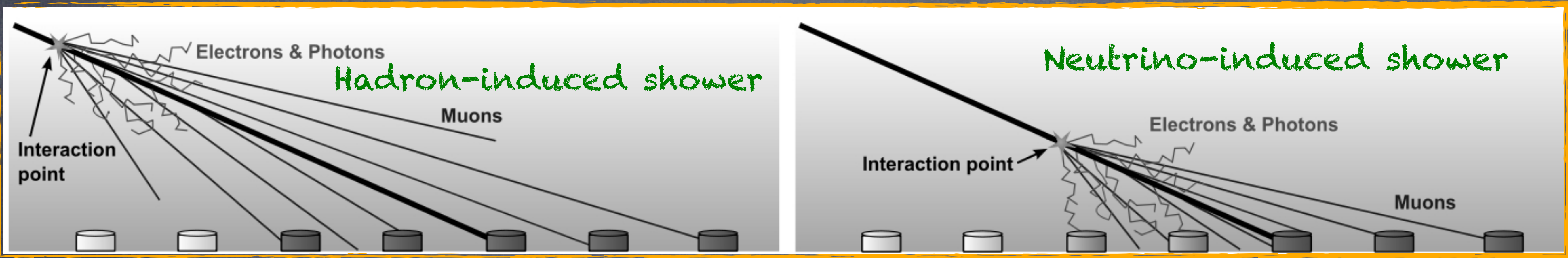
$$\langle E_\gamma^{\text{syn}} \rangle \simeq 0.29\ E_\gamma^{\text{syn}}$$

Detecting neutrinos at Auger

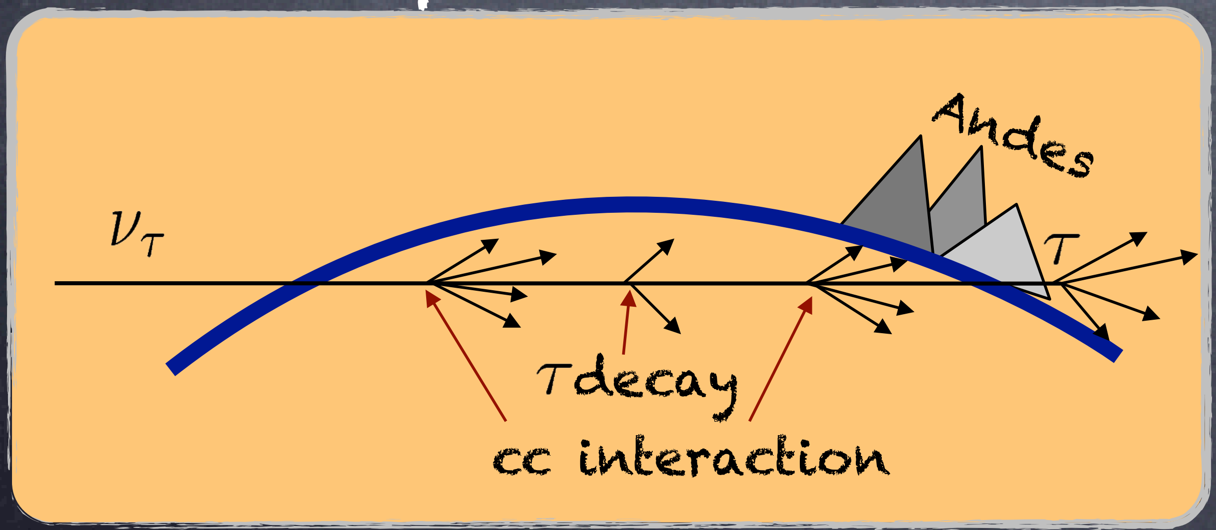
Hadronic background:

- At large zenith angles showers traverse several vertical atmospheres
- Beyond 2 vertical atmospheres most EM component is extinguished
- Hadron shower front is relatively flat only very high muons survive

Downgoing ν_e, ν_μ, ν_τ



Upgoing ν_τ



Signal:

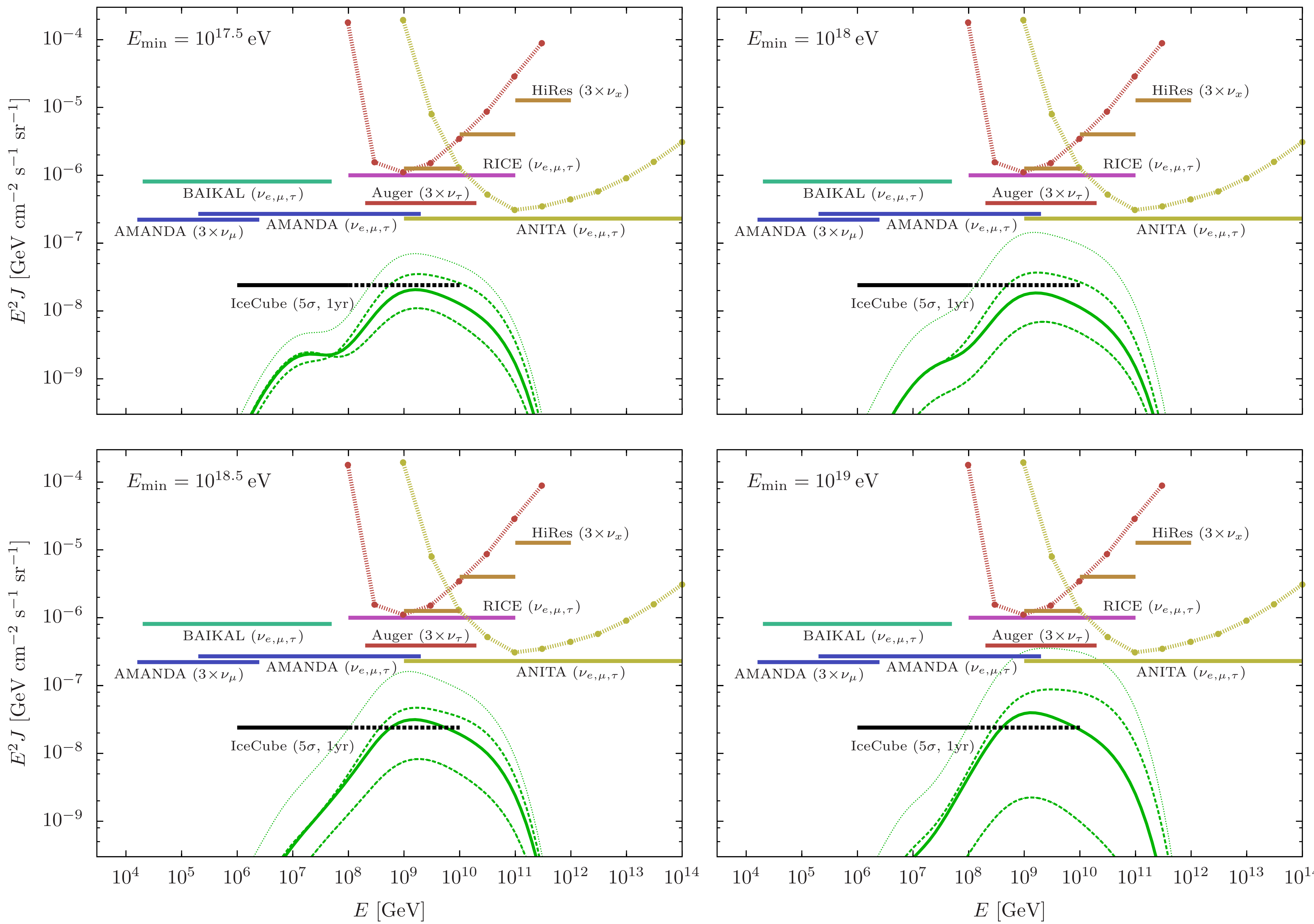
- curved front
- Large time over threshold (ToT)
- For downgoing

  forward-backward assymmetry

  early tanks large ToT (EM)

  late tanks smaller ToT (μ)

Limits on UHCR ν



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Model-independent bounds on neutrino flux

Event rate for quasi-horizontal deep showers

$$N = \sum_{i,X} \int dE_i \, N_A \, \Phi^i(E_i) \, \sigma_{iN \to X}(E_i) \, \mathcal{E}(E_i)$$

Pierre Auger Collaboration has searched for quasi-horizontal showers that are deeply-penetrating

There are no events that unambiguously passes all experimental cuts ☛ with zero events expected from hadronic background

This implies an upper bound of 2.4 events at 90% CL from neutrino fluxes

if number of events integrated over energy is bounded by 2.4
☛ also true bin by bin in energy

$$\sum_{i,X} \int_\Delta dE_i \, N_A \, \Phi^i(E_i) \, \sigma_{iN \to X}(E_i) \, \mathcal{E}(E_i) \; < 2.4$$

Model-independent bounds on neutrino flux (cont'd)

In logarithmic interval Δ where single power law approximation $\Phi^i(E_i)\,\sigma_{iN\to X}(E_i)\,\mathcal{E}(E_i) \sim E_i^\alpha$ is valid

$$\boxed{\int_{\langle E\rangle e^{-\Delta/2}}^{\langle E\rangle e^{\Delta/2}} \frac{dE_i}{E_i}\, E_i\,\Phi^i\,\sigma_{iN\to X}\,\mathcal{E} = \langle \sigma_{iN\to X}\,\mathcal{E}\,E_i\,\Phi^i\rangle\,\frac{\sinh\delta}{\delta}\,\Delta}$$

$$\boxed{\delta = (\alpha+1)\Delta/2}$$

$\langle A\rangle$ ☛ A evaluated at center of logarithmic interval

Since $\sinh\delta/\delta > 1$ ☛ conservative bound

$$\boxed{N_A \sum_{i,X} \langle \sigma_{iN\to X}(E_i)\rangle\,\langle \mathcal{E}(E_i)\rangle\,\langle E_i\Phi^i\rangle < 2.4/\Delta}$$

By taking $\Delta = 1$ as likely interval in which single power law is valid (corresponding to one e-folding of energy)

☛ upper limits on neutrino flux

Model-independent upper limits on diffuse neutrino flux from Auger

E_ν (GeV)	$\langle E_\nu \, \Phi^{\nu_{\rm all}} \rangle$ (cm^{-2} sr^{-1} s^{-1})
1×10^{8}	4.3×10^{-14}
3×10^{8}	5.3×10^{-15}
1×10^{9}	1.2×10^{-15}
3×10^{9}	4.7×10^{-16}
1×10^{10}	2.2×10^{-16}
3×10^{10}	1.3×10^{-16}
1×10^{11}	7.2×10^{-17}
3×10^{11}	4.3×10^{-17}

integrated time $\equiv 0.8$ yr of full Auger exposure

Thanks to Yann Guardincerri

Hadronic Interactions

- Uncertainties in hadronic interactions at UHE constitute one of most problematic sources of systematic error in analysis of air showers

- Below CERN ISR $\sqrt{s} = 62$ GeV soft processes

- Soft interactions are no longer described by single particle exchange

  but by highly complicated modes known Reggeons
  ☛ Pomeron dominant contribution

- Measured minijet cross sections indicates onset of SH interactions has just occured by CERN SPS $\sqrt{s} = 200$ GeV

Semihard Interactions

SH interactions are mediated by minijets ☛ jets with transverse energy $(E_T = |p_T|)$ much smaller than total c.m. energy

- ☛ cannot be identified by jet finding algorithms
- ☛ still they can be calculated using perturbative QCD

$$\sigma_{\rm QCD}(s, p_T^{\rm cutoff}) = \sum_{i,j} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \int_{Q^2_{\rm min}}^{\hat{s}/2} d|\hat{t}| \; \frac{d\hat{\sigma}_{ij}}{d|\hat{t}|} \; x_1 f_i(x_1, |\hat{t}|) \; x_2 f_j(x_2, |\hat{t}|)$$

Mandelstam variables $\hat{s} = x_1 \, x_2 s$ and $\hat{t} = \hat{s}\,(1 - \cos\vartheta^*)/2 = Q^2$

transverse and longitudinal momenta

$$p_T = E_{\rm jet}^{\rm lab} \; \sin\vartheta_{\rm jet} = \frac{\sqrt{\hat{s}}}{2} \; \sin\vartheta^*$$

$$p_{\parallel} = E_{\rm jet}^{\rm lab} \; \cos\vartheta_{\rm jet}$$

for small ϑ^* ☛ $p_T^2 \approx Q^2$

integration limits satisfy ☛ $Q^2_{\rm min} < |\hat{t}| < \hat{s}/2$

DGLAP evolution

First source of uncertainty in modeling UHECR interactions

☛ extrapolation of measured parton densities

several orders of magnitude down to low x

For large Q^2 and not too small x DGLAP equations successfully predict Q^2 dependence of quark and gluon densities

$$\boxed{\frac{\partial}{\partial \ln Q^2}\begin{pmatrix} q(x,Q^2) \\ g(x,Q^2)\end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi}\begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg}\end{pmatrix} \otimes \begin{pmatrix} q(x,Q^2) \\ g(x,Q^2)\end{pmatrix}}$$

P_{ij} ☛ splitting functions indicate probability of finding a daughter parton i in parent parton j with given fraction of parton j momentum

depends on number of splittings allowed in approximation

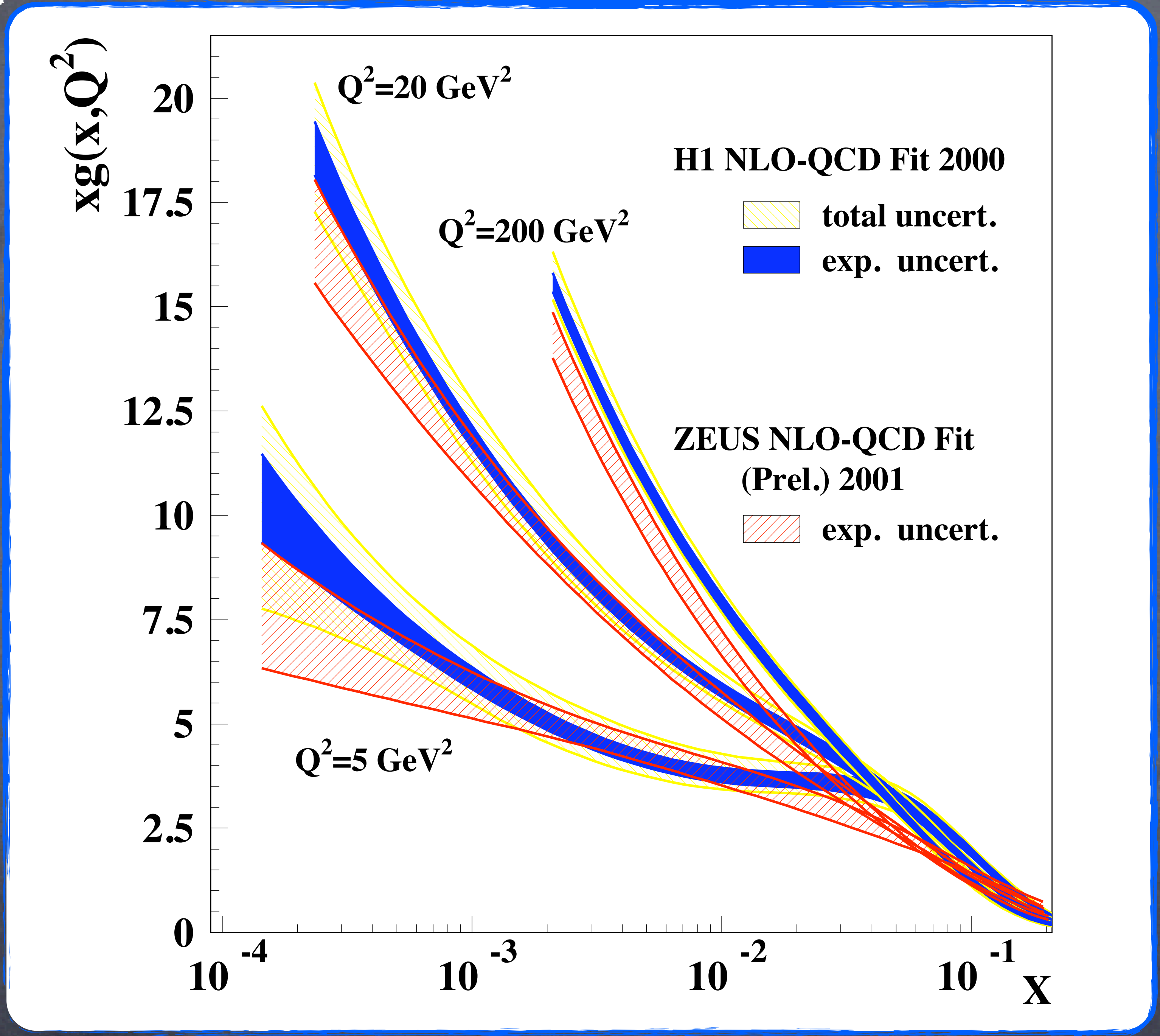
Double-leading-logarithmic approximation $\nearrow \lim_{x\to 0} \ln(1/x)$ $\searrow \lim_{Q^2\to\infty} \ln(Q^2/\Lambda_{\rm QCD})$

DGLAP equations predict a steeply rising gluon density

$xg \sim x^{-0.4}$ which dominates quark density at low x

Gluon momentum distribution

DGLAP prediction in agreement with HERA results



HERA data are found to be consistent with power law

$$xg(x,Q^2) \sim x^{-\Delta_{\mathrm{H}}}$$

$$0.3 < \Delta_{\mathrm{H}} 0.4$$

Minijet cross section

minijet cross section is determined by dominant g distribution

$$\sigma_{\rm QCD}(s, p_T^{\rm cutoff}) \approx \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \int_{Q^2_{\rm min}}^{\hat{s}/2} d|\hat{t}| \; \frac{d\hat{\sigma}}{d|\hat{t}|} \; x_1 g(x_1, |\hat{t}|) \; x_2 g(x_2, |\hat{t}|)$$

integration limits satisfy ☛ $x_1 x_2 s > 2|\hat{t}| > 2Q^2_{\rm min}$

$d\hat{\sigma}/d|\hat{t}|$ is peaked at low end of the $|\hat{t}|$ integration ☛
high energy behavior of $\sigma_{\rm QCD}$ is controlled
(via lower limits of x_1, x_2 integrations)
by small-x behavior of gluons

$$\sigma_{\rm QCD}(s) \propto \int_{2\,Q^2_{\rm min}/s}^{1} \frac{dx_1}{x_1} \; x_1^{-\Delta_{\rm H}} \int_{2\,Q^2_{\rm min}/x_1 s}^{1} \frac{dx_2}{x_2} \; x_2^{-\Delta_{\rm H}} \sim s^{\Delta_{\rm H}} \ln s \underset{s\to\infty}{\simeq} s^{\Delta_{\rm H}}$$

This estimate is too simplistic ☛ at sufficiently small x
g shadowing corrections suppress singular $x^{-\Delta_H}$ behavior of xg
and hence suppress power growth of $\sigma_{\rm QCD}$ with increasing s

Breakdown of Geometrical Scaling

onset of SH processes is an unambiguous prediction of QCD
however in practice
difficult to isolate SH contributions from soft interactions

Reasonable approach ☛ base extrapolation of soft interactions
on assumption of geometrical scaling
which is observed to be true throughout the ISR energy range

$$f(s,b) = f_{\rm GS}\left(\beta = b/R(s)\right)$$

opaqueness of the proton remains constant with rising energy

(i) partial wave at $b=0$ should be energy independent

Immediate consequence of GS

(ii) $\sigma_{\rm el}(s)/\sigma_{\rm tot}(s)$ should be energy independent

Breakdown of Geometrical Scaling (cont'd)

(i) ☛

$$f(s, b = 0) = f_{\rm GS}(\beta = 0)$$

(ii)

$$\sigma_{\rm tot} = 8\pi \int {\rm Im} f(s,b)\, b\, db \underset{\rm GS}{=} 8\pi R^2(s) \int {\rm Im} f_{\rm GS}(\beta)\, \beta\, d\beta$$

$$\sigma_{\rm el} = 8\pi \int |f(s,b)|^2\, b\, db \underset{\rm GS}{=} 8\pi R^2(s) \int |f_{\rm GS}(\beta)|^2\, \beta\, d\beta$$

Breakdown of Geometrical Scaling (cont'd)

At ISR energies ☛ elastic amplitude has a simple form

$$F(s,t) = i\,\sigma_{\rm tot}(s)\;\; e^{Bt/2}$$

Fourier transform of elastic amplitude
has Gaussian shape in impact parameter space

$$f(s,b) = \frac{i\sigma_{\rm tot}(s)}{8\pi B} e^{-b^2/2B}$$

and it follows that

$$\mathrm{Im} f(s, b=0) = \frac{\sigma_{\rm tot}}{8\pi B} = \frac{2\sigma_{\rm el}}{\sigma_{\rm tot}}$$

It is easily seen breakdown of GS and to identify SH interactions

Unitarity ☛ black disk

Unitarity requires $\mathrm{Im}\, f(s,b) \leq \frac{1}{2}$ ☛ in turn implies $\sigma_{\mathrm{el}}/\sigma_{\mathrm{tot}} \leq \frac{1}{2}$

This seems to indicate that Gaussian form may not longer be applicable at ultrahigh energies ☛ but rather it is expected that proton will approximate a "black disk" of radius b_0

$\boxed{f(s,b) = \frac{i}{2}}$ for $\boxed{0 < b \lesssim b_0}$ and zero for $\boxed{b \gtrsim b_0}$

Then $\boxed{\sigma_{\mathrm{el}} \simeq \frac{1}{2}\sigma_{\mathrm{tot}} \simeq \pi b_0^2}$

In order to satisfy unitarity constraints ☛ convenient to introduce

$$\boxed{f(s,b) = \frac{i}{2}\left\{1 - \exp\left[i\chi(s,b)\right]\right\}}$$

$$\Downarrow$$

$$\boxed{\mathrm{Im}\, \chi \geq 0}$$

Unitarity ☛ black disk (cont'd)

If we neglect shadowing corrections to PDFs and take $xg \propto x^{-\Delta_{\rm H}}$

$\boxed{\sigma_{\rm QCD} \sim s^{\Delta_{\rm H}}}$ and $\boxed{{\rm Im}\chi(s, b=0) \gg 1 \quad \text{as} \quad s \to \infty}$

$$
\boxed{
\begin{aligned}
\sigma_{\rm tot} &= 4\pi \int_0^\infty b\; db\, \Theta(b_0 - b) \\
&\simeq 4\pi \int_0^{b_0(s)} b\, db = 2\pi b_0^2
\end{aligned}
}
$$

$\boxed{\chi \simeq \chi_{\rm SH}}$ and $\boxed{b_0(s)}$ is defined by ☛ $\boxed{{\rm Im}\, \chi_{\rm SH}(s, b_0(s)) \simeq 1}$

Unitarized elastic, inelastic, and total cross sections

Hereafter we ignore small real part of scattering amplitude
(good approximation at high energies)
considering (now) a real eikonal function

$$\sigma_{\rm el} = 2\pi \int db\, b\ \{1 - \exp\left[-\chi_{\rm soft}(s,b) - \chi_{\rm SH}(s,b)\right]\}^2$$

$$\sigma_{\rm inel} = 2\pi \int db\, b\ \{1 - \exp\left[-2\chi_{\rm soft}(s,b) - 2\chi_{\rm SH}(s,b)\right]\}$$

$$\sigma_{\rm tot} = 4\pi \int db\, b\ \{1 - \exp\left[-\chi_{\rm soft}(s,b) - \chi_{\rm SH}(s,b)\right]\}$$

$$\chi_{\rm SH} = \frac{1}{2}\,\sigma_{\rm QCD}(s, p_T^{\rm cutoff})\ A(s,\vec{b})$$

$A(b,s)$ ☛ parton distribution in plane transverse to collision axis

QGSJET

Gaussian profile function $A(s,b) = \dfrac{e^{-b^2/R^2(s)}}{\pi R^2(s)}$

For QCD cross section dependence $\sigma_{\rm QCD} \sim s^{\Delta_{\rm H}}$

☛ one gets for Gaussian profile $b_0^2(s) \sim R^2 \Delta_{\rm H} \ln s$

therefore $\sigma_{\rm inel} = 2\pi \int_0^{b_0(s)} db\, b \sim \pi R^2 \Delta_{\rm H} \ln s$

parameter R itself depends on collision energy through convolution with parton momentum fractions

$R^2(s) \sim R_0^2 + 4\,\alpha'_{\rm eff} \ln^2 s$

$\alpha'_{\rm eff} \approx 0.11~{\rm GeV}^{-2}$

cross section saturates the Froissart bound

$\sigma_{\rm inel} \sim 4\pi\,\alpha'_{\rm eff}\,\Delta_{\rm H}\,\ln^2 s$

SIBYLL

Transverse density distribution is taken as Fourier transform of proton electric form factor ☛ resulting in energy-independent exponential (rather than Gaussian) fall-off of density profile for large b

$$A(b) = \frac{\mu^2}{96\pi}(\mu b)^3\, K_3(\mu b) \sim e^{-\mu b}$$

normalization condition is satisfied when ☛ $b_0(s) = \dfrac{\Delta_{\rm H}}{\mu}\,\ln\, s$

$$\chi_{\rm SH} \sim e^{\mu b}\; s^{\Delta_{\rm H}}$$

$$\downarrow$$

$$\sigma_{\rm inel} \sim \pi c\, \frac{\Delta_{\rm H}^2}{\mu^2}\, \ln^2 s$$

transverse momentum cutoff

SIBYLL uses parametrization based on DGLAP

$$p_{_T}^{\rm cutoff}(\sqrt{s}) = p_{_T}^0 + 0.065\ {\rm GeV}\ \exp[0.9\,\sqrt{\ln s}]$$

DPMJET uses ad hoc parametrization

$$p_{_T}^{\rm cutoff}(\sqrt{s}) = p_{_T}^0 + 0.12\ {\rm GeV}\ [\log_{10}(\sqrt{s}/50{\rm GeV})]^3$$

Proton-air production cross section

Glauber Model

$$\sigma_{\text{inel}}^{p-\text{air}} \approx 2\pi \int db\, b\, \{1 - \exp[\sigma_{\text{tot}}\ AT_N(b)]\}$$

$$\sigma_{\text{prod}}^{p-\text{air}} \approx 2\pi \int db\, b\, \{1 - \exp[\sigma_{\text{inel}}\ AT_N(b)]\}$$

$T_N(b)$ ☛ transverse distribution function of nucleon inside a nucleus

proton-air inelastic cross section is sum of:

"quasi-elastic" cross section ☛ target nucleus breaks up without production of any new particle

production cross section ☛ at least one new particle is generated

Development of EAS is mainly sensitive to production cross section

Overall ☛ geometrically large size of nitrogen and oxygen nuclei dominates inclusive proton-target cross section and as a result disagreement from model-dependent extrapolation is not more than about 15%

Exercise 4

Consider a typical air nuclei of average $\langle A \rangle = 14.5$ and calculate the proton-air cross section using approximated expressions for proton-proton cross section together with the z-integrated Woods-Saxon profile

$$T_N(b) = \frac{1}{Z} \int_{-\infty}^{\infty} dz \left\{ 1 + \exp\left[(\sqrt{b^2 + z^2} - R_N)/\alpha \right] \right\}^{-1}$$

where

$$Z = \frac{4\pi}{3} R_N^3 \left[1 + \pi^2 \left(\frac{\alpha}{R_N} \right)^2 \right]$$

$\alpha = 0.5$ fm and $R_N = 1.1 A^{1/3}$ fm

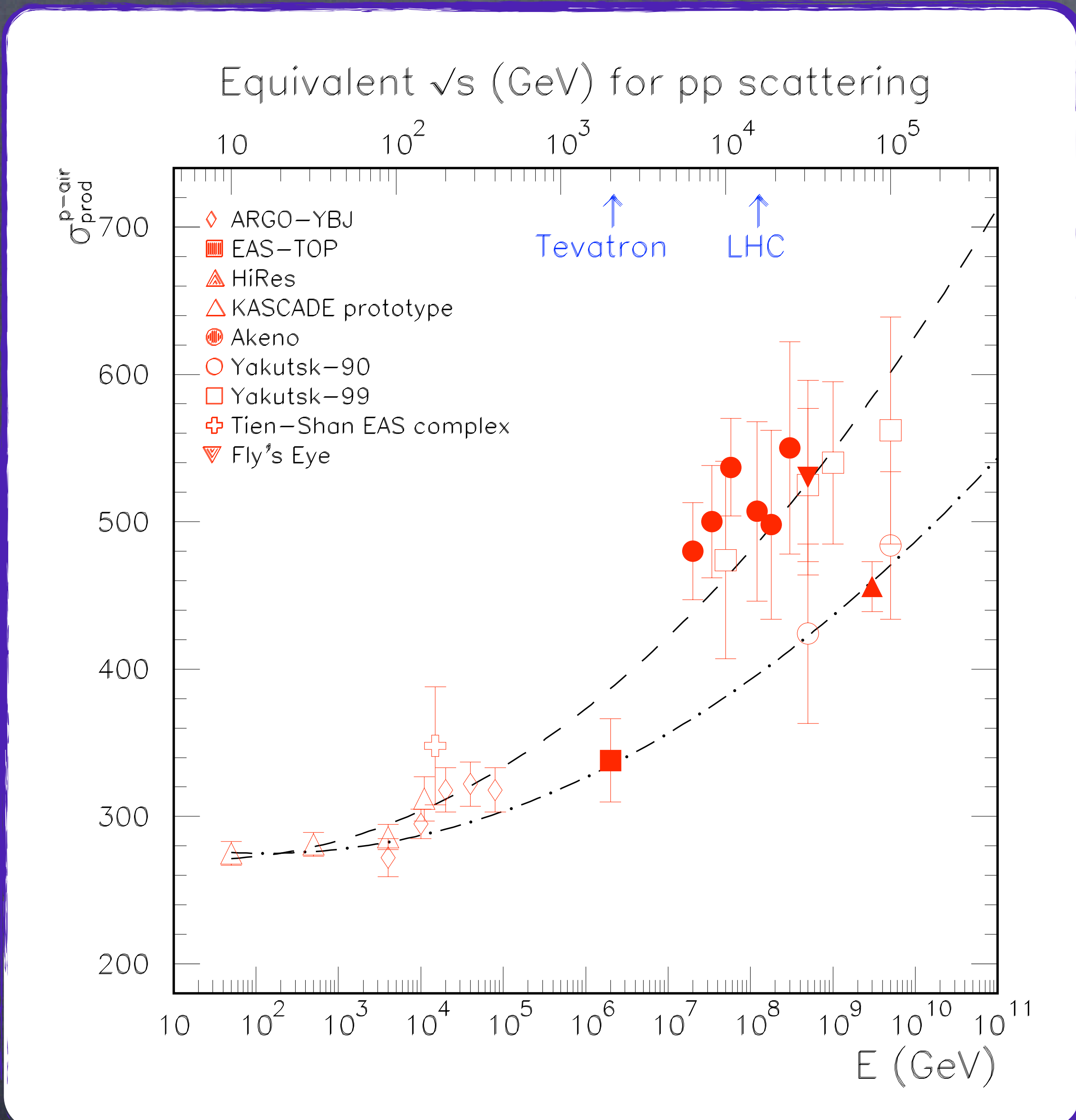
k

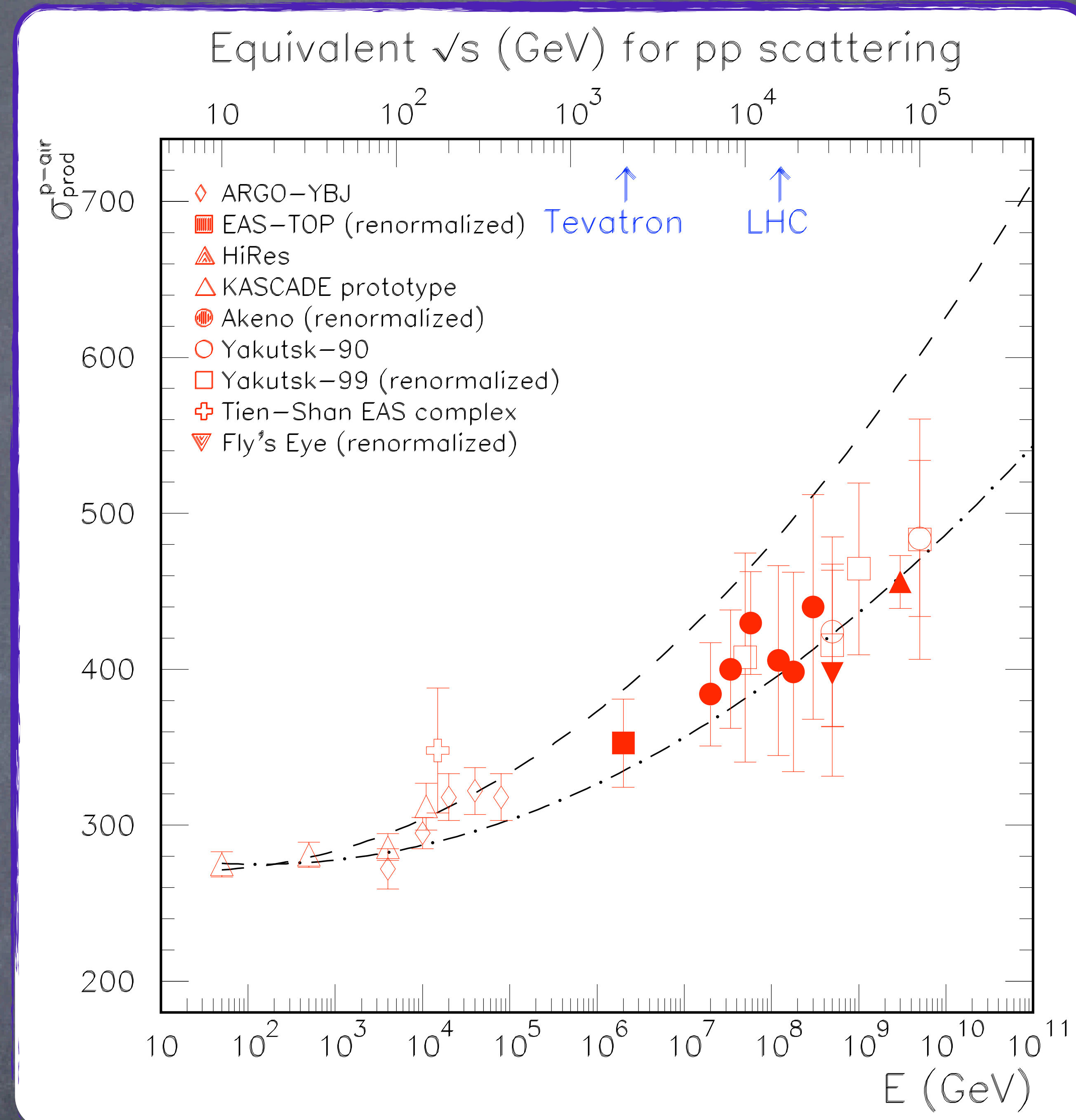
- Adding a greater challenge to determination of UHE proton-air cross section is lack of direct measurements in a controlled laboratory environment
  - Measured shower attenuation length Λ_m is not only sensitive to interaction length of protons in atmosphere $\lambda_{p-\mathrm{air}}$ with

$$\boxed{\Lambda_m = k\lambda_{p-\mathrm{air}} = k\frac{14.4\ m_p}{\sigma_{\mathrm{prod}}^{p-\mathrm{air}}}}$$

but also depends on rate at which energy of primary proton is dissipated into EM shower energy
there is a large range of k values
(from 1.6 for very old model based on Feynman scaling to 1.15 for modern models with large scaling violations)
this makes published values of $\sigma_{p-\mathrm{air}}$ unreliable

Measurements of p-air production cross section





$$\sigma_{\rm prod}^{p-{\rm air}} = \mathcal{A} - \mathcal{B}\,\ln(E/{\rm GeV}) + \mathcal{C}\ln^2(E/{\rm GeV})\ {\rm mb}$$

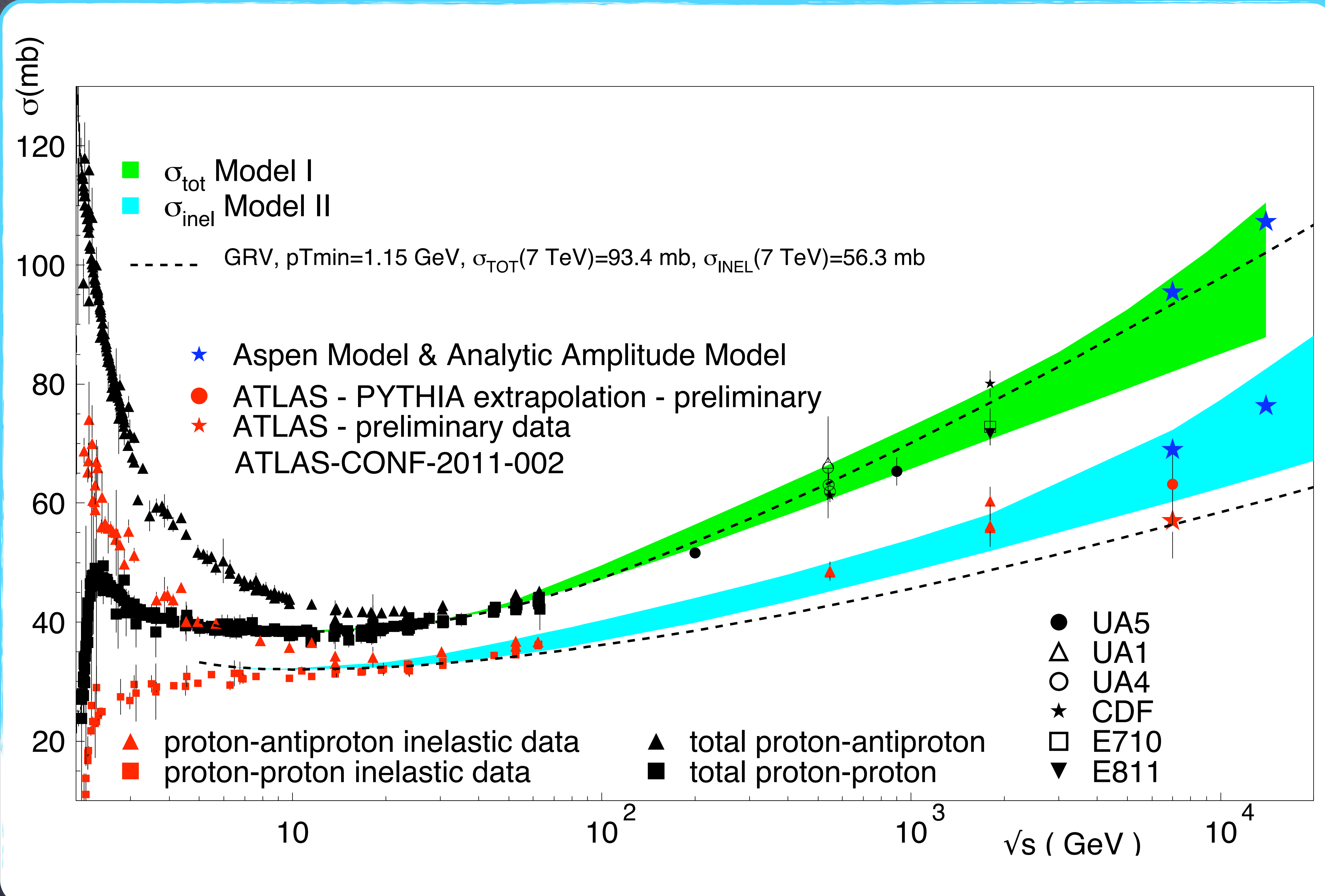
dot-dashed line

$\mathcal{A} = 290 \quad \mathcal{B} = 6.2 \quad \mathcal{C} = 0.64$

dashed line

$\mathcal{A} = 280 \quad \mathcal{B} = 5.7 \quad \mathcal{C} = 0.9$

Clues from LHC data



Achilli, Godbole, Grau, Pancheri, Shekhovtsova and Srivastava, arXiv:1102.1949

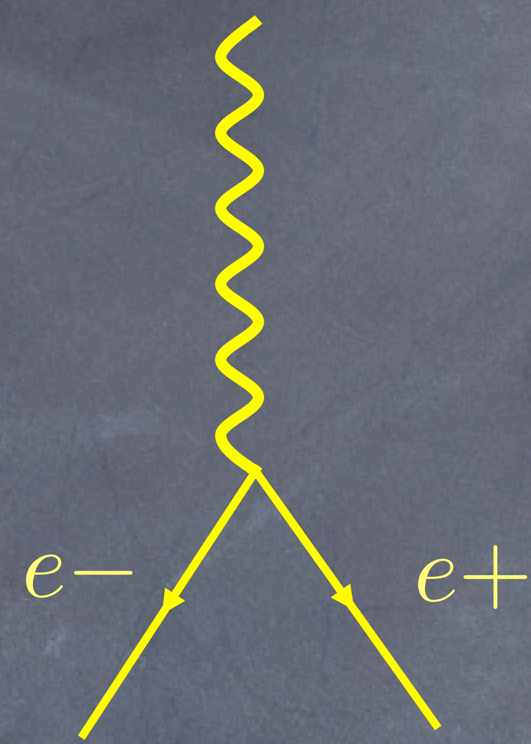
Block and Halzen, arXiv:1102.3163

Heitler model of (EM) shower

Shower is imagined to developed exclusively via bremsstrahlung and pair production each of which results in conversion of 1 particle into 2

Heitler, The Quantum Theory of Radiation ,3rd Ed., (1954), p.386.

Heitler model of (EM) shower



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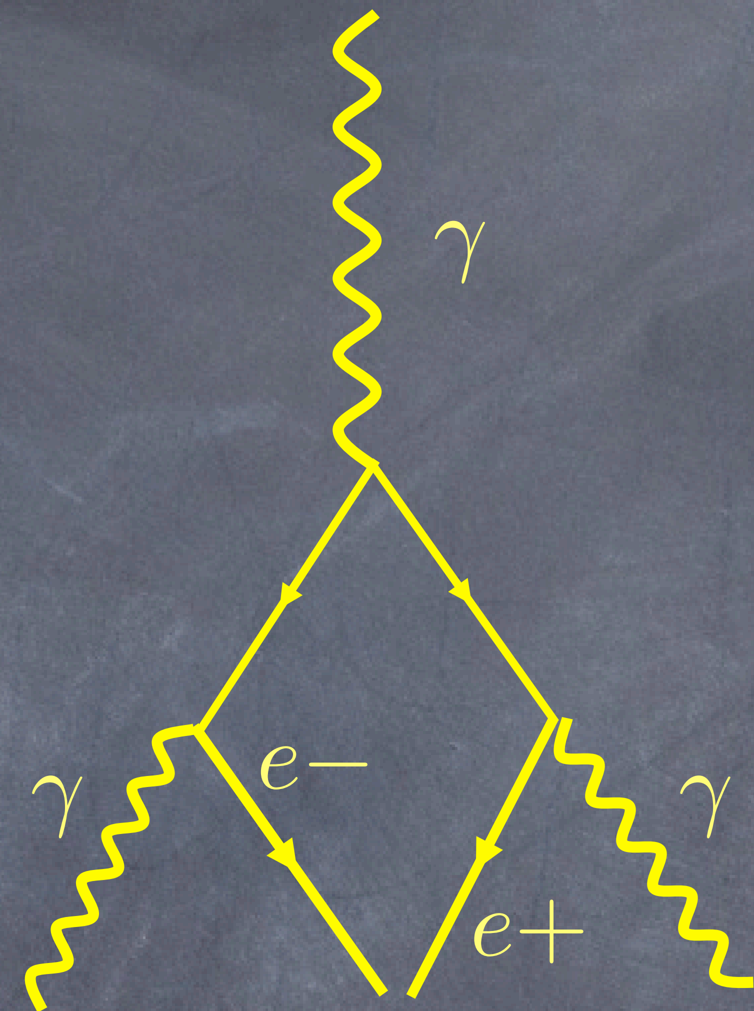

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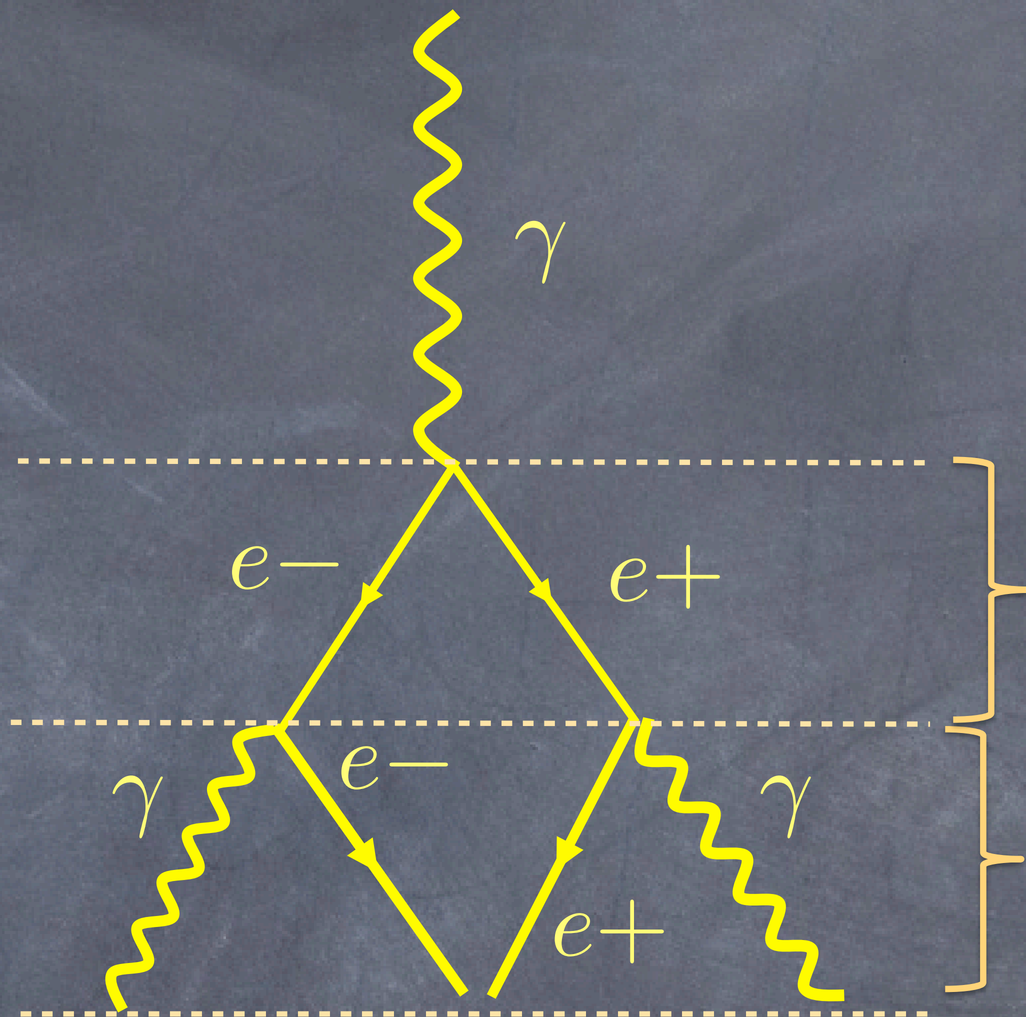
Heitler model of (EM) shower



$$d = X_r \ln 2$$

$$\mathrm{X_r} = 37g/cm^2$$

(radiation length)

Shower is imagined to developed exclusively via Bremsstrhlung and pair production each of which results in conversion of 1 particle into 2

Heitler, The Quantum Theory of Radiation ,3rd Ed., (1954), p.386.

Heitler model of (EM) shower (cont'd)

After n generations:

$$X = nX_r$$

$$N_{\text{part}} = 2^n = 2^{X/X_r}$$

$$E_{\text{part}} = \frac{E_0}{N_{\text{part}}} = \frac{E_0}{2^{X/X_r}}$$

Cascade stops when:

$$E_{\text{part}} < E_{\text{crit}} = \epsilon_0$$

$$N_{\max} = E_0/\epsilon_0$$

$$X_{\max} \sim X_r \frac{\ln(E_0/\epsilon_0)}{\ln 2}$$

Elongation rate

- Changes in mean mass composition of CR flux as function of E will manifest as changes in $\langle X_{\max} \rangle$
- Change of $X_{\max}$ with E is commonly known as elongation rate

$$\boxed{D_e = \frac{\delta X_{\max}}{\delta \ln E}}$$

- For pure EM showers $X_{\max} \approx X_{\rm r} \, \dfrac{\ln(E_0/\epsilon_0)}{\ln 2}$ ☛ $D_e \sim X_{\rm r}$

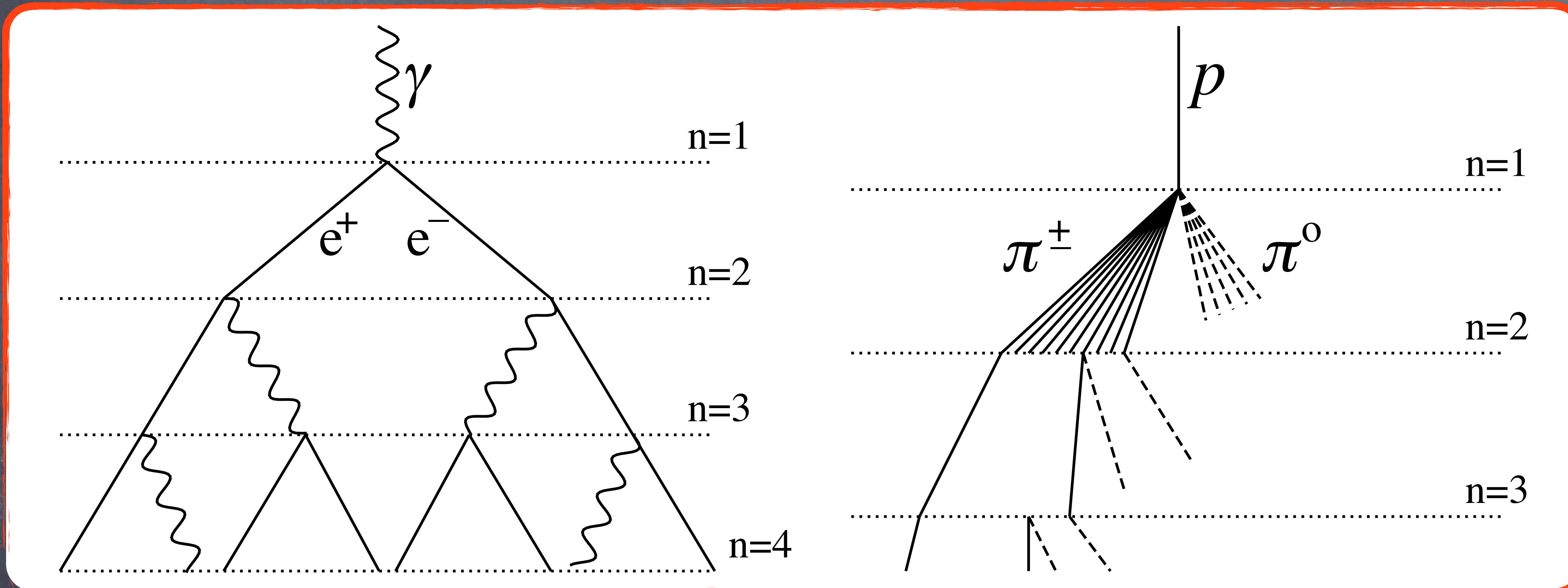
- For convenience elongation rate is often written in terms of energy decades

$$\boxed{D_{10} = \frac{\partial \langle X_{\max} \rangle}{\partial {\rm Log} E}}$$

$$\boxed{D_{10} = 2.3 D_e}$$

Heitler-Matthews model

- Baryon-induced showers are also dominated by electromagnetic processes



Matthews, Astropart. Phys. 22 (2005) 387

- For proton primaries ☛ multiplicity rises with energy and resulting elongation rate becomes smaller

- On average ☛ first interaction is determined by proton mean free path in atmosphere ☛ $\lambda_{p-\mathrm{air}} = X_0$

Heitler-Matthews model (cont'd)

- Incoming proton splits into $\langle n(E)\rangle$ secondary particles each carrying an average energy $E/\langle n(E)\rangle$
- Assuming that $X_{\max}(E)$ depends dominantly on first generation of gamma subshowers

$$X_{\max}(E) \approx X_0 + X_{\mathrm{r}} \ln[E/\langle n(E)\rangle]$$

- Further assume a multiplicity dependence $\langle n(E)\rangle \approx n_0 E^{\Delta}$

$$\frac{\delta X_{\max}}{\delta \ln E} = X_{\mathrm{r}} \left[1 - \frac{\delta \ln\langle n(E)\rangle}{\delta \ln E}\right] + \frac{\delta X_0}{\delta \ln E}$$

or equivalently

$$D_e = X_{\mathrm{r}} \left[1 - \frac{\delta \ln\langle n(E)\rangle}{\delta \ln E} + \frac{X_0}{X_{\mathrm{r}}} \frac{\delta \ln(X_0)}{\delta \ln E}\right] = X_{\mathrm{r}}\,(1 - B)$$

$$B \equiv \Delta - \frac{X_0}{X_{\mathrm{r}}} \frac{\delta \ln X_0}{\delta \ln E}$$

Heitler-Matthews model (cont'd)

First interaction yields $N_\gamma = 2N_{\pi^0} = N_{\pi^\pm}$

Each photon initiates EM shower of energy $E_0/(3N_{\pi^\pm}) = E_0/(6N_\pi)$

Using pp data we parametrized charged particle production in first interaction as $N_{\pi^\pm} = 41.2(E_0/1\ \mathrm{PeV})^{1/5}$

Based on sole evolution of EM cascade of 1st interaction

$$\begin{aligned} X^p_{\max} &= X_0 + X_\mathrm{r}\ \ln[E_0/(6N_\pi\epsilon_0)] \\ &= (470 + 58\log_{10}[E_0/1\ \mathrm{PeV}])\ \mathrm{g/cm}^2 \end{aligned}$$

this falls short of full simulation value by about $100\ \mathrm{g/cm}^2$

Matthews, Astropart. Phys. 22 (2005) 387

Exercise 5

The depth of shower maximum obtained in previous slide is only approximate since it considers just first interaction as hadronic in nature

- Extend the approximation to include hadronic interactions in second generation of particles

- Try the generalization to include all the generations of hadronic collisions until charged pions cool down below the critical energy

Elongation rates for protons

A good approximation of elongation rate can be obtained when introducing cross section and multiplicity $\sqrt{s}$ dependence

Using p-air cross section of 550 mb at 10^9GeV
and a rate of change of about 50 mb per decade of energy

$$X_0 \simeq 90 - 9 \log\left(E_0/\text{EeV}\right) \text{ g/cm}^2$$

Assuming that first interaction initiates 2N_π EM cascades
each of energy $E_0/6N_\pi$

$$D_{10}^p = \frac{dX_{\text{max}}}{d\log E_0} = \frac{d(X_0 \ln 2 + X_{\text{r}} \ln[E_0/(6N_\pi \epsilon_0)]}{d\log E_0}$$

☛ $$D_{10}^p = \frac{4}{5} D_{10}^\gamma - 9 \ln 2 \simeq 62 \text{ g/cm}^2$$

It is in good agreement with Monte Carlo simulation

Elongation rates for mixed primary composition

We apply superposition principle

We pretend that nucleus comprises unbound nucleons

point of 1st interaction of 1-nucleon independent of all others

Shower produced by nucleus with energy E_A and mass A

is modeled by collection of A proton showers

each with A^{-1} of the nucleus energy

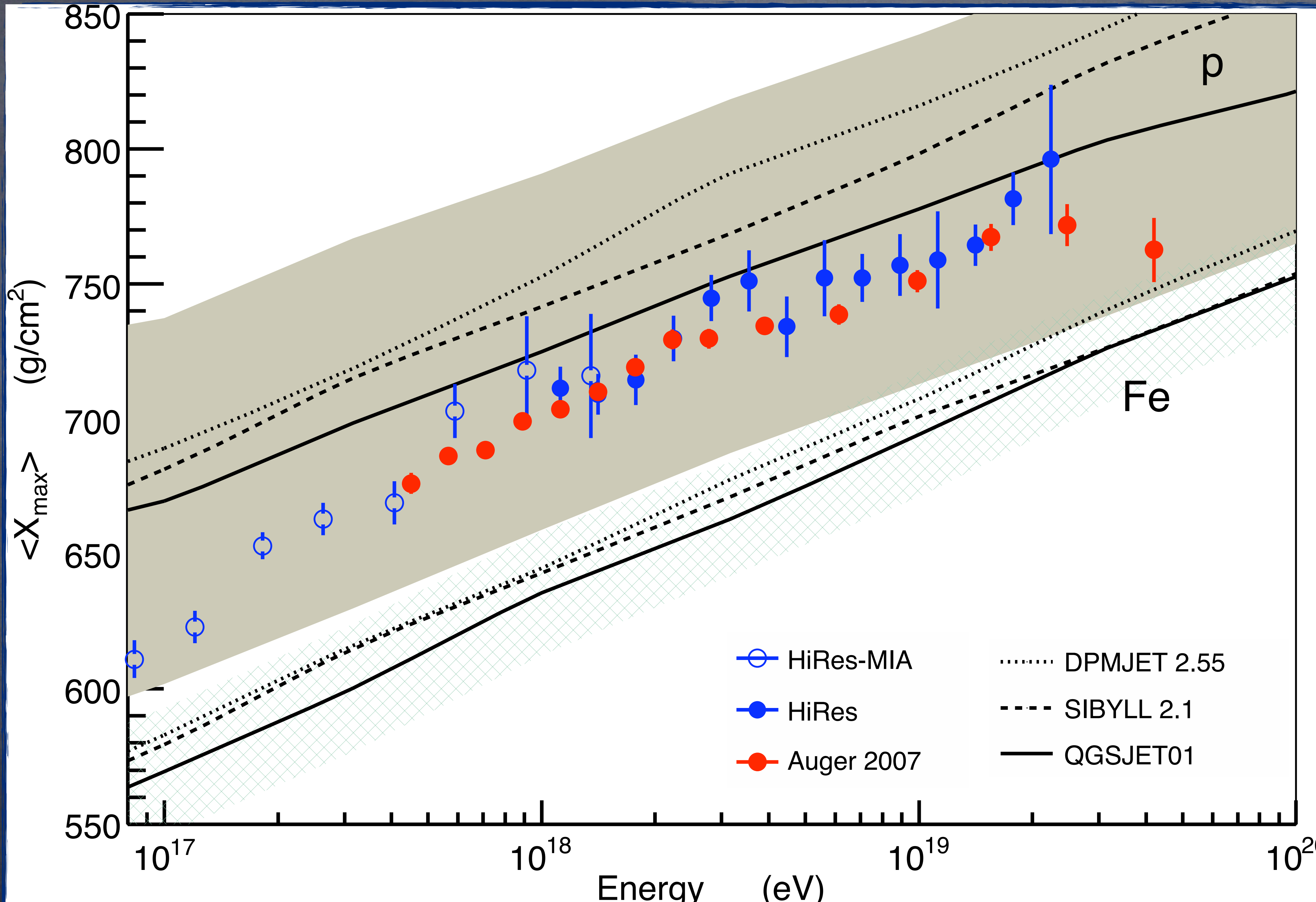
Modifying previous analysis accordingly

$$X_{\max} \propto \ln(E_0/A)$$

Assuming that B is not changing with energy

$$D_e = X_0 \, (1 - B) \left[1 - \frac{\partial \langle \ln A \rangle}{\partial \ln E} \right]$$

Variation of $X_{\max}$ with energy



Bluemer, Engel & Hoerandel, Prog. Part. Nucl. Phys. 63 (2009) 293

Using cosmic rays to search for new physics

- Challenging to search for new physics with CR:

$$\mathcal{L} \sim 7 \times 10^{-10} (E\ /\mathrm{PeV})^{-2}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$$

Almost 50 orders of magnitude smaller than LHC lumi

- But it may be possible anyway (one approach: use ν's)

- Neutrino flux should accompany CR flux

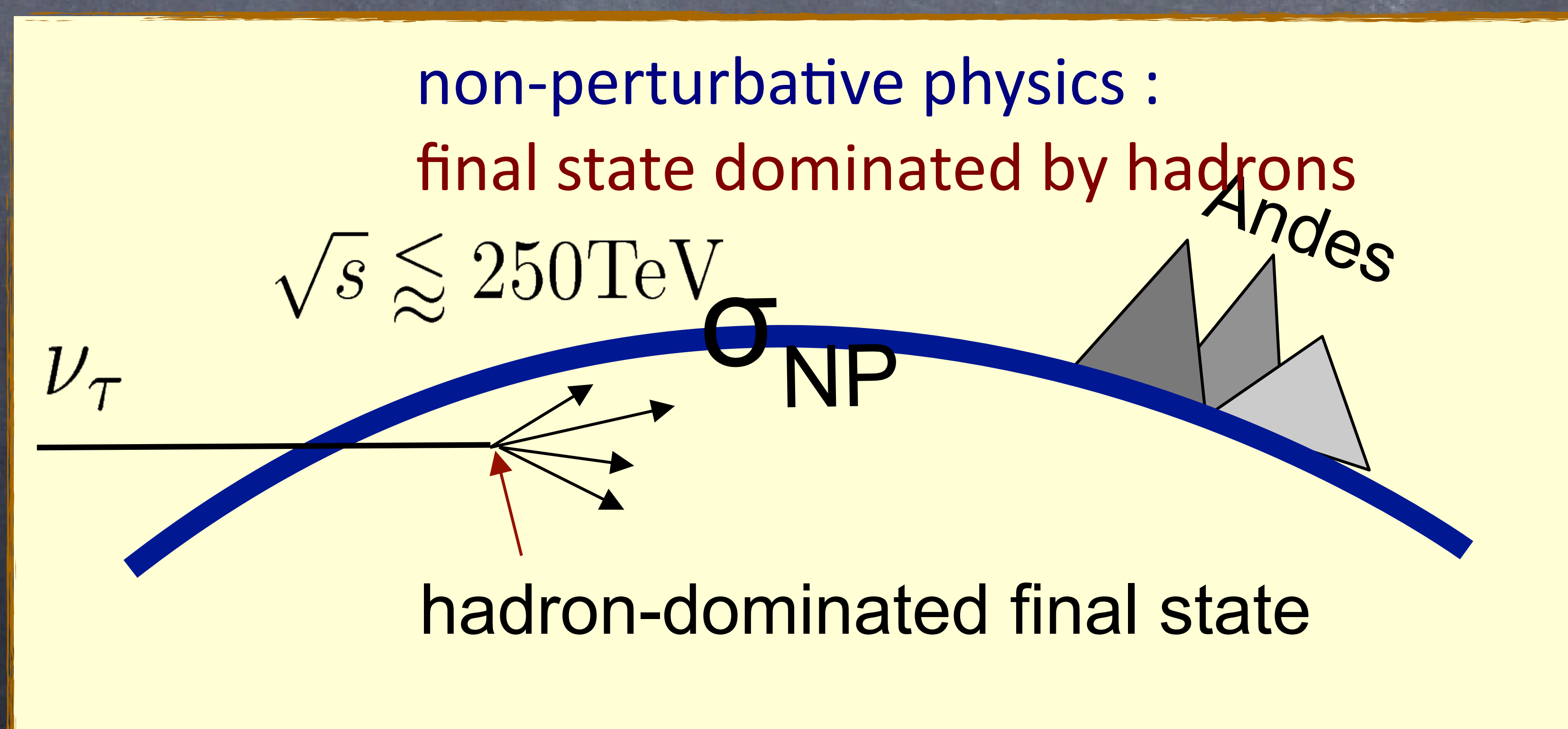
- Take usual benchmark ☛ "Waxman-Bahcall bound"

$$\Phi_0^{\nu_\alpha} = 2.3 \times 10^{-8}\ E_\nu^{-2}\ \mathrm{GeV}^{-1}\,\mathrm{s}^{-1}\,\mathrm{cm}^{-2}\,\mathrm{sr}^{-1}$$

Waxman & Bahcall PRD59 (1999)

Possible effect of non-perturbative physics

- Increase in rate of ν showers could be due to
  - higher flux than expected
  - new physics
- Disentangling unknown physics from unknown flux may be possible by checking the ratios of ES to QH



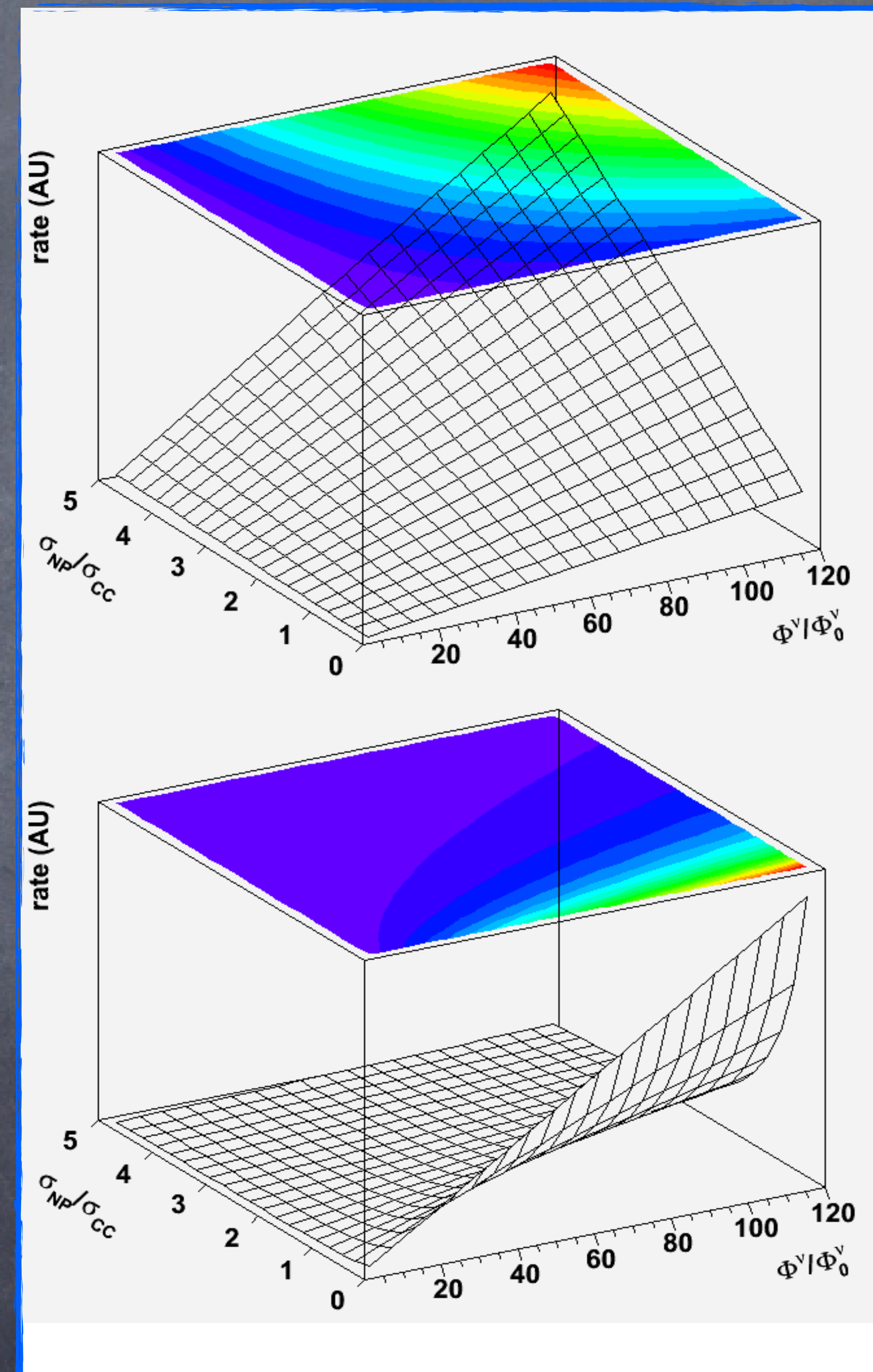
Rates for earth-skimmers vs. horizontals

$$N_{\rm QH} = C_{\rm QH} \frac{\Phi^\nu}{\Phi_0^\nu} \frac{\sigma_{\rm CC}^\nu + \sigma_{\rm NP}^\nu}{\sigma_{\rm CC}^\nu}$$

$$N_{\rm ES} \approx C_{\rm ES} \frac{\Phi^\nu}{\Phi_0^\nu} \frac{\sigma_{\rm CC}^{\nu 2}}{(\sigma_{\rm CC}^\nu + \sigma_{\rm NP}^\nu)^2}$$

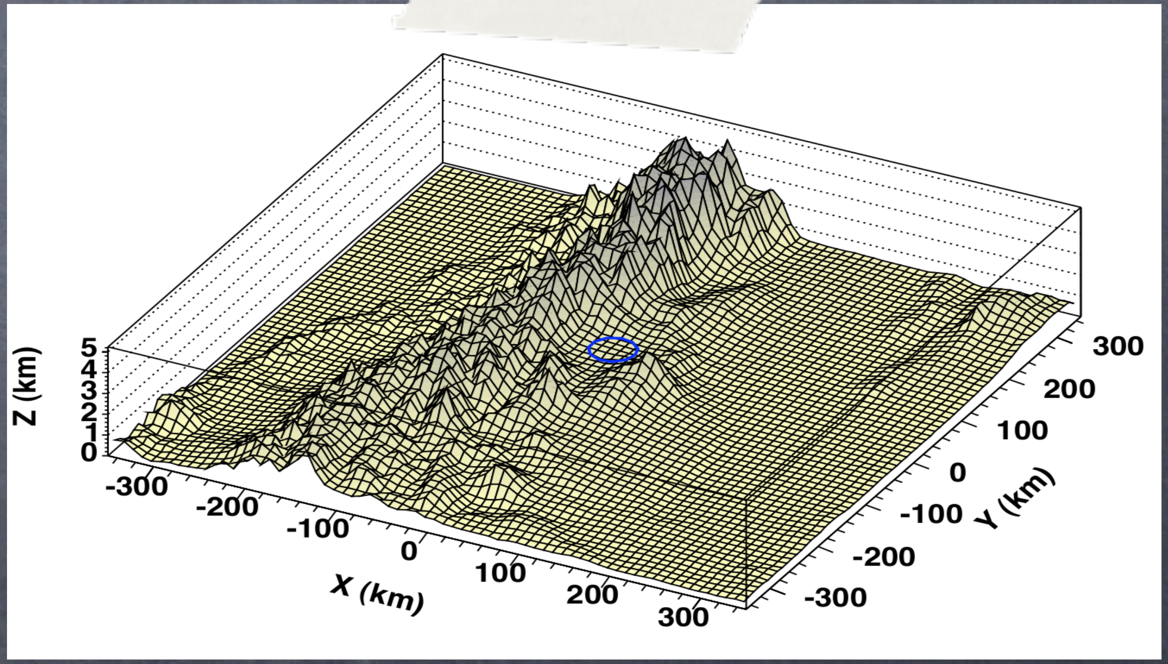
$C_{\rm ES}$ $C_{\rm QH}$ depend on acceptance for those types of events

LAA, Feng, Goldberg, Shapere, PRD65, (2002)



Estimating C_{ES} and C_{QH}

- Monte Carlo-ed "all the way"
- Incoming neutrinos propagated through Earth using ANIS
  Gora, Roth, Tamburro Astropart. Phys. 26 (2007) 402
- τ decays handled by TAUOLA
- Downgoing νN interaction simulated with PYTHIA
- Shower development with AIRES
- Andes modeled with data from Consortium for Spatial Info
  http://srtm.csi.cgiar.org



- Response of surface array simulated in detail using Auger Offline simulation/reconstruction package

Expected number of SM events/year

- Assume isotropic ν flux
- Assume $\nu_e : \nu_\mu : \nu_\tau = 1:1:1$ (at Earth)
- Bracket range of plausible fluxes to estimate systematics

$$1. \Phi_0^{\nu_\alpha}(E_\nu) = (\mathcal{C}/E_0)E_\nu^{-1}$$

$$\mathcal{C} = 2.3 \times 10^{-8}\ \mathrm{GeV}^{-1}\,\mathrm{s}^{-1}\,\mathrm{cm}^{-2}\,\mathrm{sr}^{-1}$$

$$2. \Phi_0^{\nu_\alpha}(E_\nu) = \mathcal{C}E_\nu^{-2}$$

$$E_0 = 10^{10}\mathrm{GeV}$$

$$\sigma = 0.5\mathrm{GeV}$$

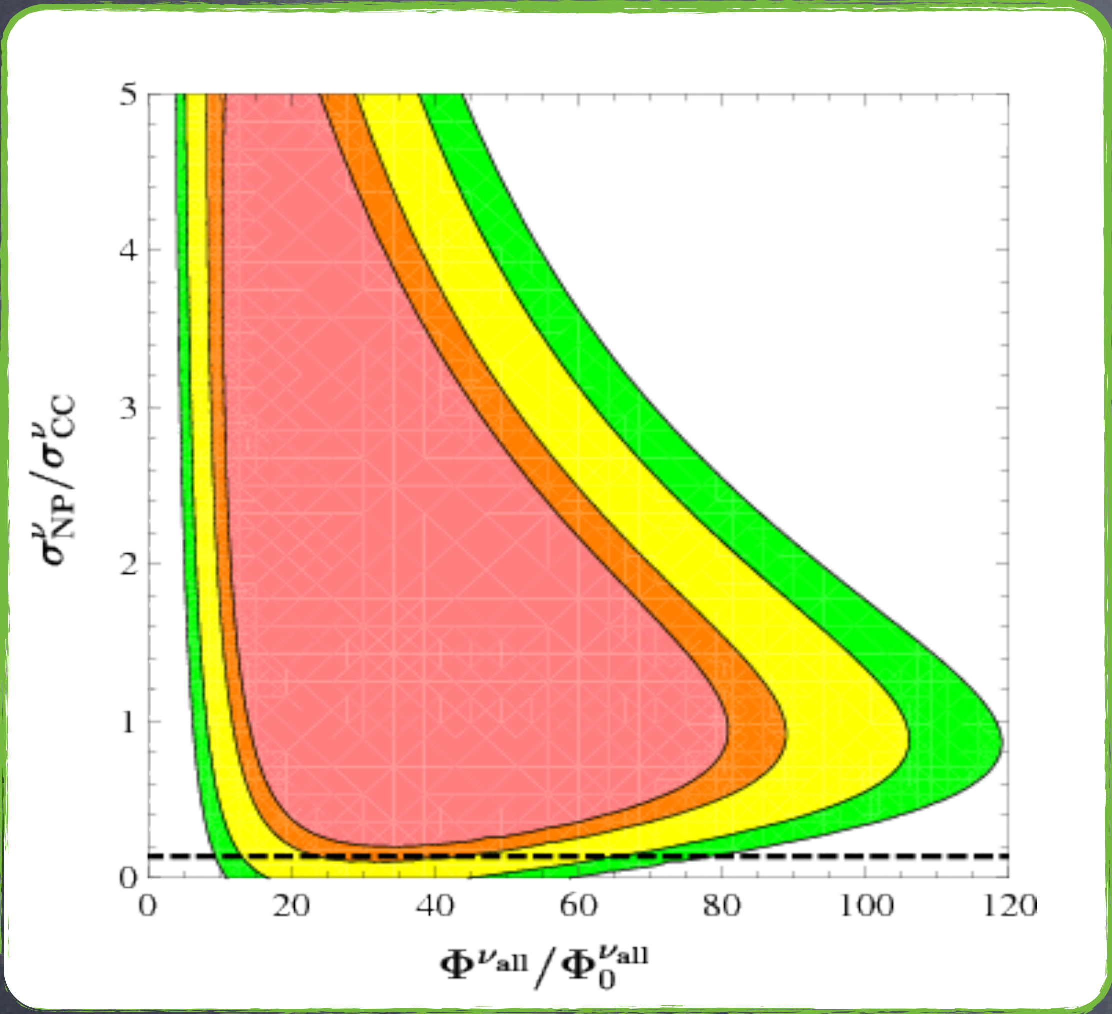
$$3. \Phi_0^{\nu_\alpha}(E_\nu) = (\mathcal{C}E_0)E_\nu^{-3}$$

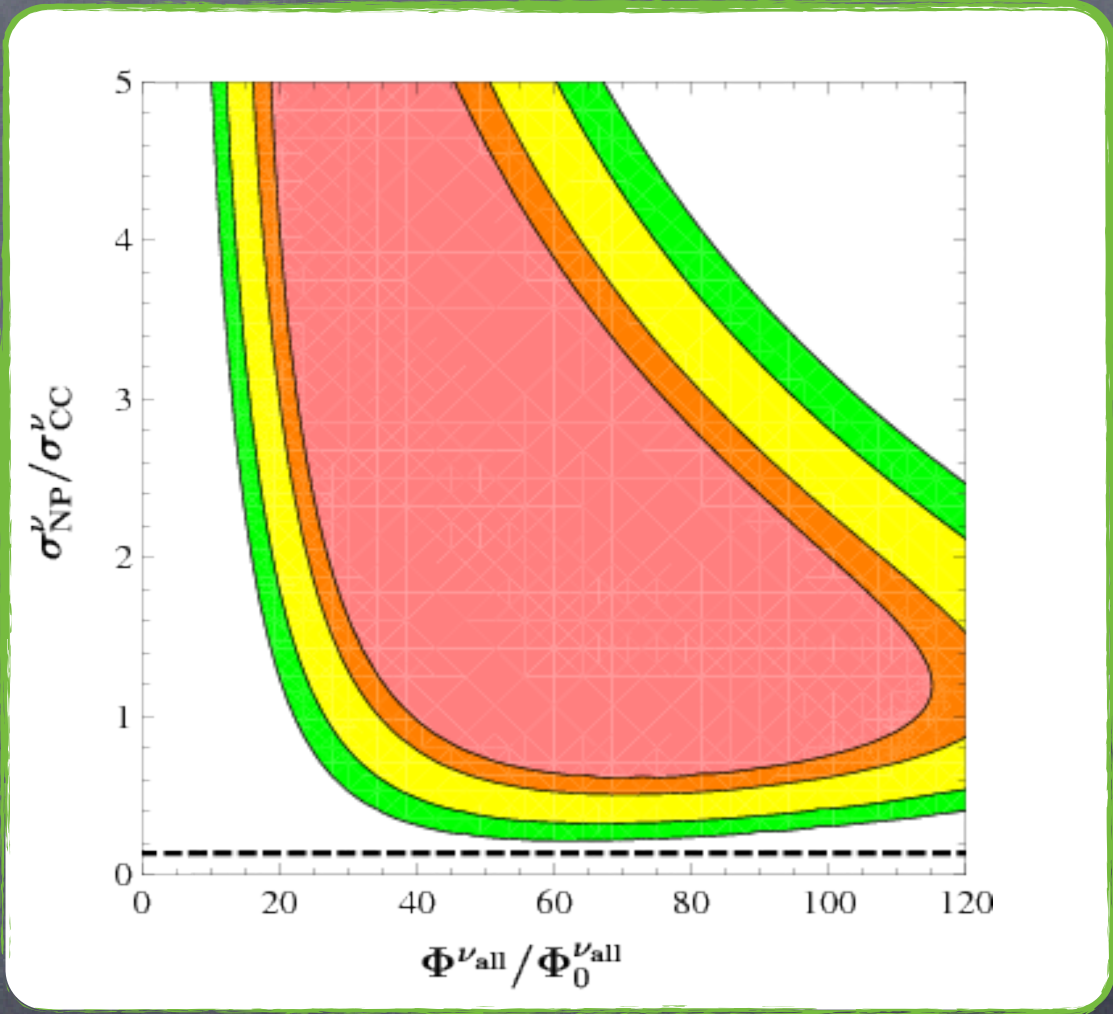
$$4. \Phi_0^{\nu_\alpha}(E_\nu) = \mathcal{C}E_\nu^{-2}\exp[-\log_{10}(\mathrm{E}_\nu/\mathrm{E}_0)^2/(2\sigma^2)]$$

flux	up-going		down-going					ratio
	θ	N_{ν_τ}	θ	N_{ν_e}	N_{ν_τ}	N_{ν_μ}	$N_{\nu_{\rm all}}$	$N_\tau/N_{\nu_{\rm all}}$
(1)	90-95	0.14	75-90	0.027	0.031	0.0056	0.06	2.14
(2)	90-95	0.15	75-90	0.026	0.029	0.0048	0.06	2.47
(3)	90-95	0.23	75-90	0.036	0.041	0.0062	0.08	2.75
(4)	90-95	0.12	75-90	0.021	0.024	0.0040	0.05	2.45

Sensitivity to non-perturbative physics

$$N_{\rm QH} = C_{\rm QH} \frac{\Phi^\nu}{\Phi^\nu_0} \frac{\sigma^\nu_{\rm CC} + \sigma^\nu_{\rm NP}}{\sigma^\nu_{\rm CC}} \qquad N_{\rm ES} \approx C_{\rm ES} \frac{\Phi^\nu}{\Phi^\nu_0} \frac{\sigma^{\nu 2}_{\rm CC}}{(\sigma^\nu_{\rm CC} + \sigma^\nu_{\rm NP})^2}$$





$N^{\rm obs}_{\rm ES} = 1 \quad N^{\rm obs}_{\rm QH} = 5$

$N^{\rm obs}_{\rm ES} = 1 \quad N^{\rm obs}_{\rm QH} = 10$

Systematics from NLO QCD CC neutrino-nucleon cross section

LAA, Goldberg, Gora, Paul, Roth, Sarkar, Winders, Phys. Rev. D 82 (2010) 043001


That's all