

# The Infrared

Parton Distributions

Confinement

Lattice QCD

Hadronization

“Intrinsic  $k_T$ ”

Underlying Event

& Min-Bias physics

Constraints and “Tuning”

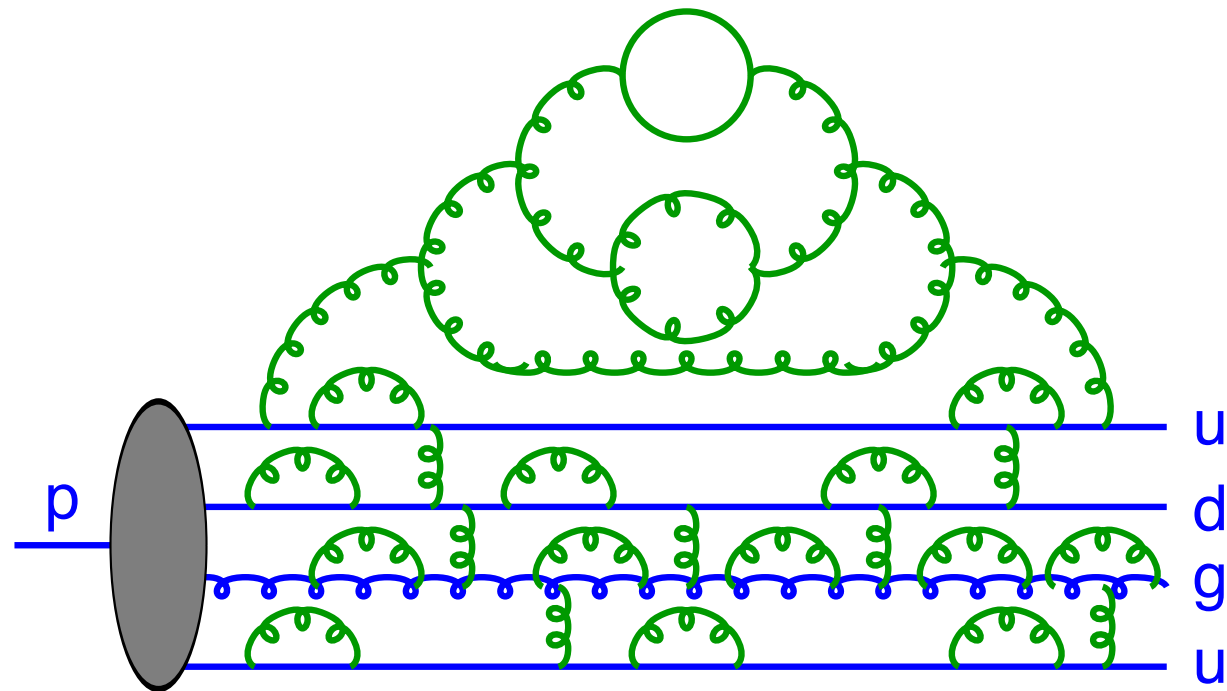


# Parton Densities

A 3D visualization of parton densities within a nucleon. The background is a complex, translucent cyan surface representing the spatial distribution of partons. A prominent yellow-orange region is visible in the upper left. Several colored spheres are placed on the surface: a red sphere at the bottom center, a blue sphere to the right of the center, a green sphere at the bottom left, and a magenta sphere to the right of the blue one. A white line starts from the top left, passes through the text, and ends at the red sphere, with a wavy section near the sphere.

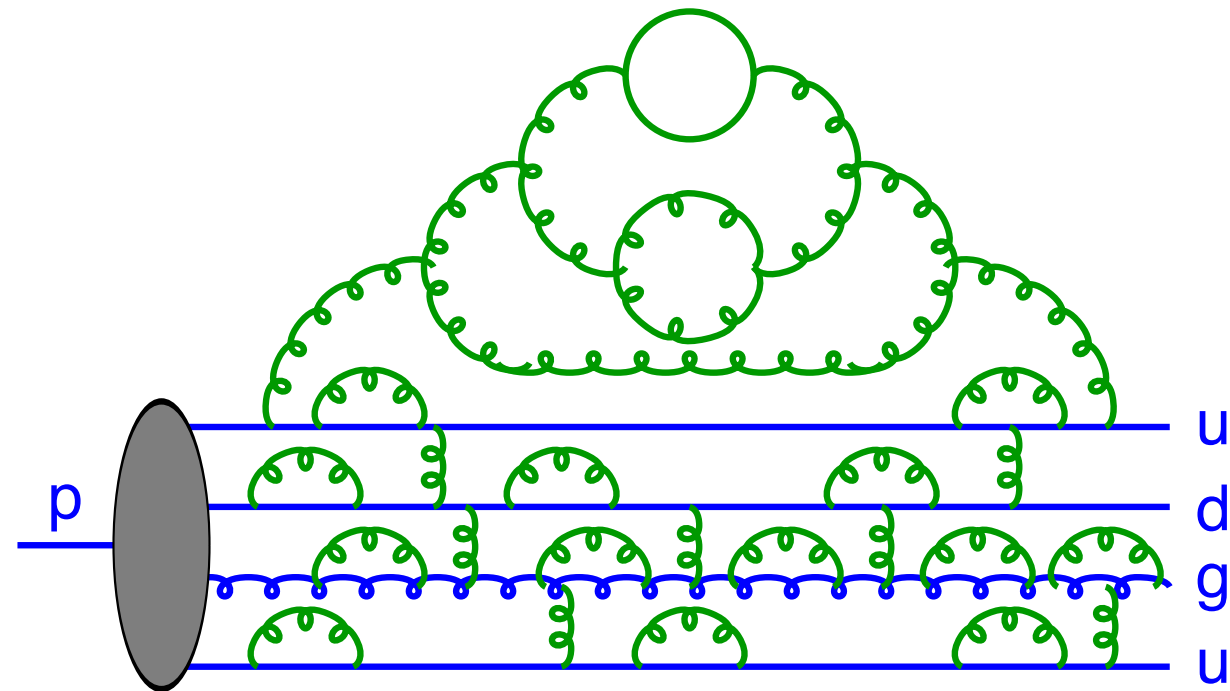
# Parton Densities

Hadrons are composite, with time-dependent structure:



# Parton Densities

Hadrons are composite, with time-dependent structure:



$f_i(x, Q^2)$  = number density of partons  $i$   
at momentum fraction  $x$  and probing scale  $Q^2$ .

Linguistics (example):

$$F_2(x, Q^2) = \sum_i e_i^2 x f_i(x, Q^2)$$

structure function

parton distributions



# Parton Densities

$$\vec{p}_j = x \vec{P}_{proton}$$

$f_a(x_a, Q_i^2)$  Parton distribution functions (PDF)

- sum over long-wavelength histories leading to  $a$  with  $x_a$  at the scale  $Q_i^2$  (ISR)

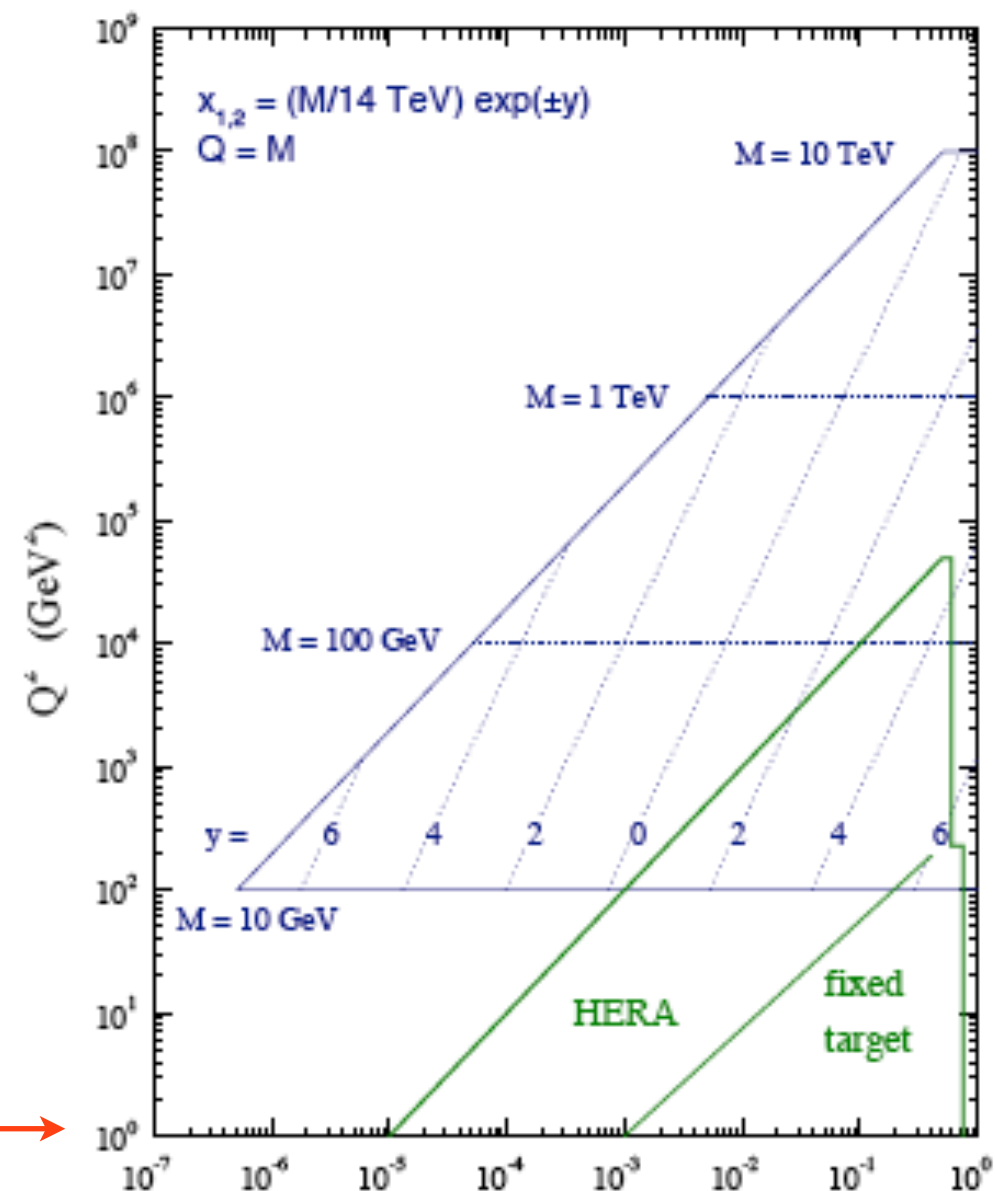
Absolute normalization unknown (non-pert)

→ fit to measurements at small  $Q^2 \approx m_{proton}$

$m_{proton}$  →

LHC kinematics

**LHC parton kinematics**



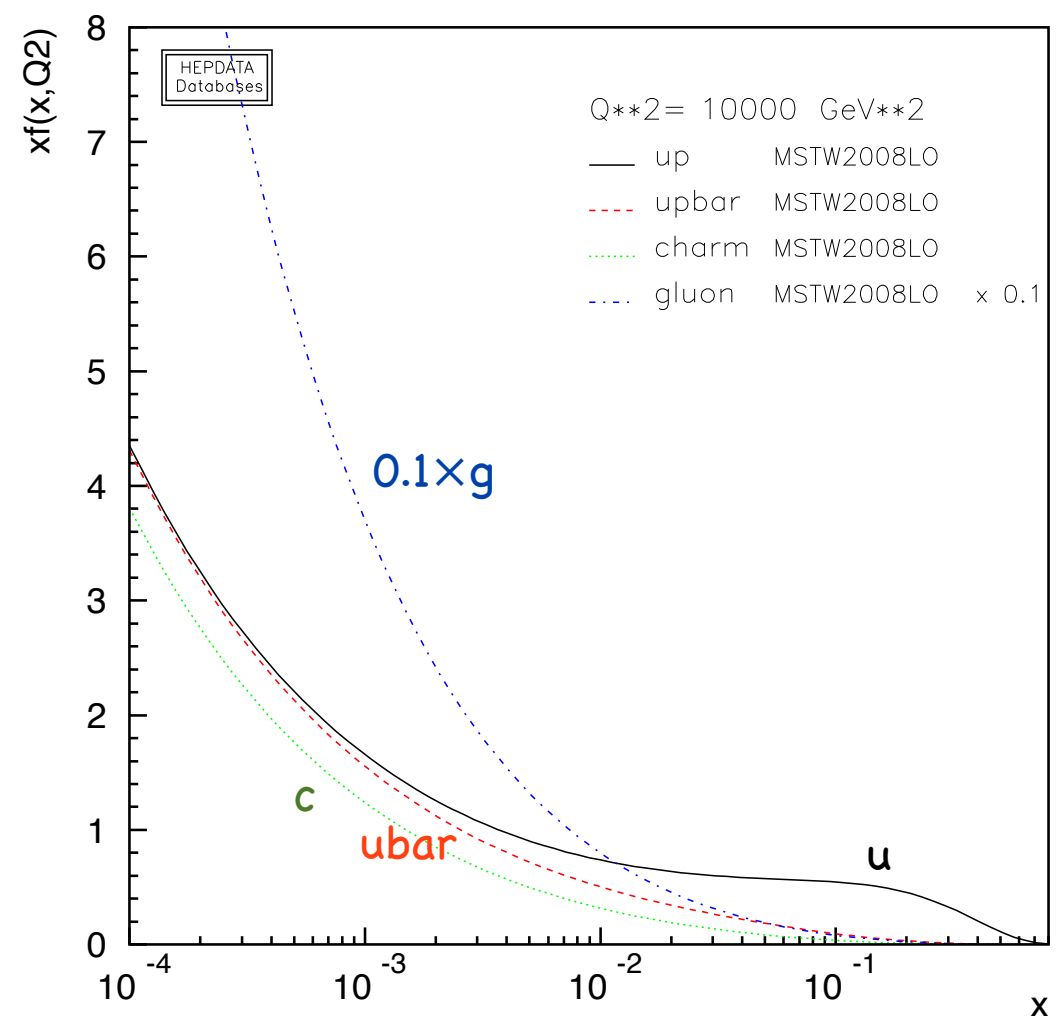
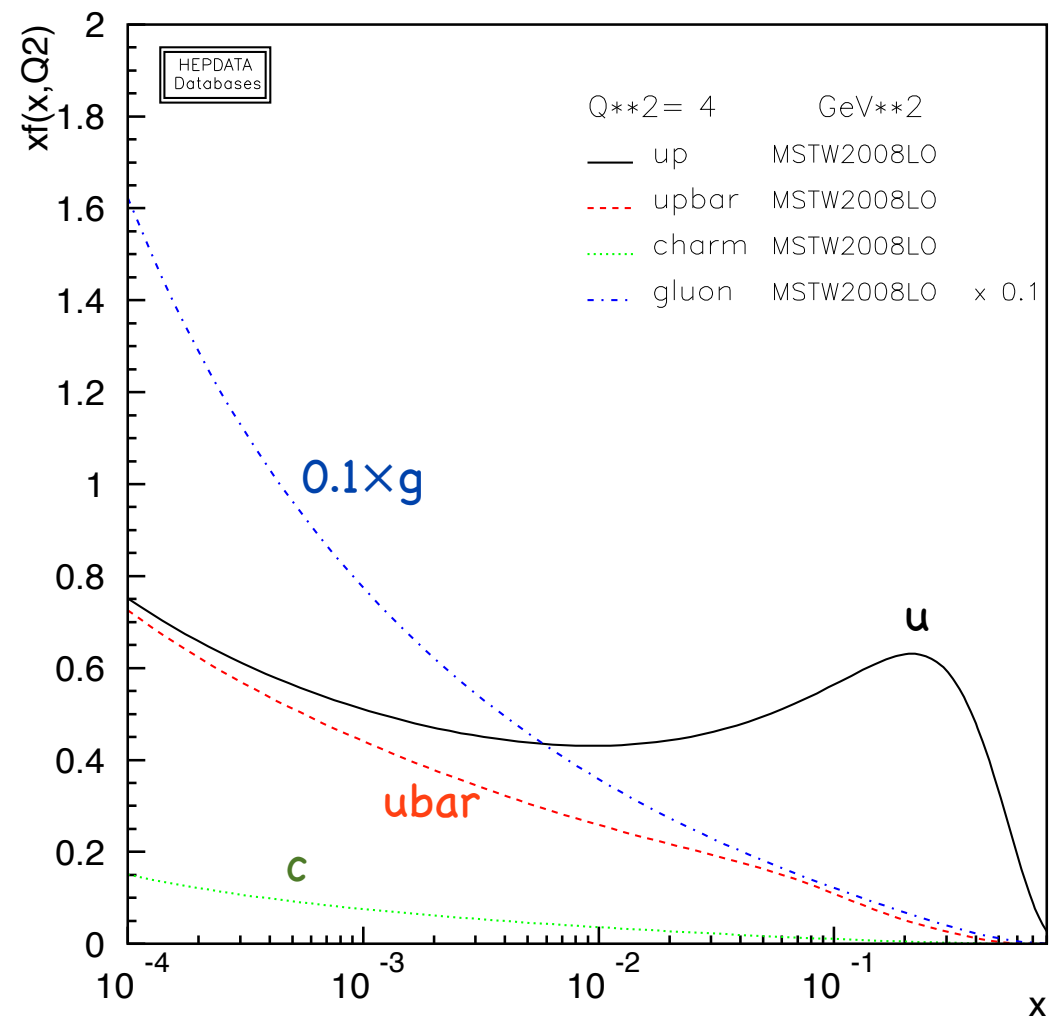
# Evolution in $Q^2$ by DGLAP

(Dokshitzer-Gribov-Lipatov-Altarelli-Parisi)

$$\frac{df_b(x, Q^2)}{d(\ln Q^2)} = \sum_a \int_x^1 \frac{dz}{z} f_a(x', Q^2) \frac{\alpha_s}{2\pi} P_{a \rightarrow bc} \left( z = \frac{x}{x'} \right)$$

$$Q^2 = (2 \text{ GeV})^2$$

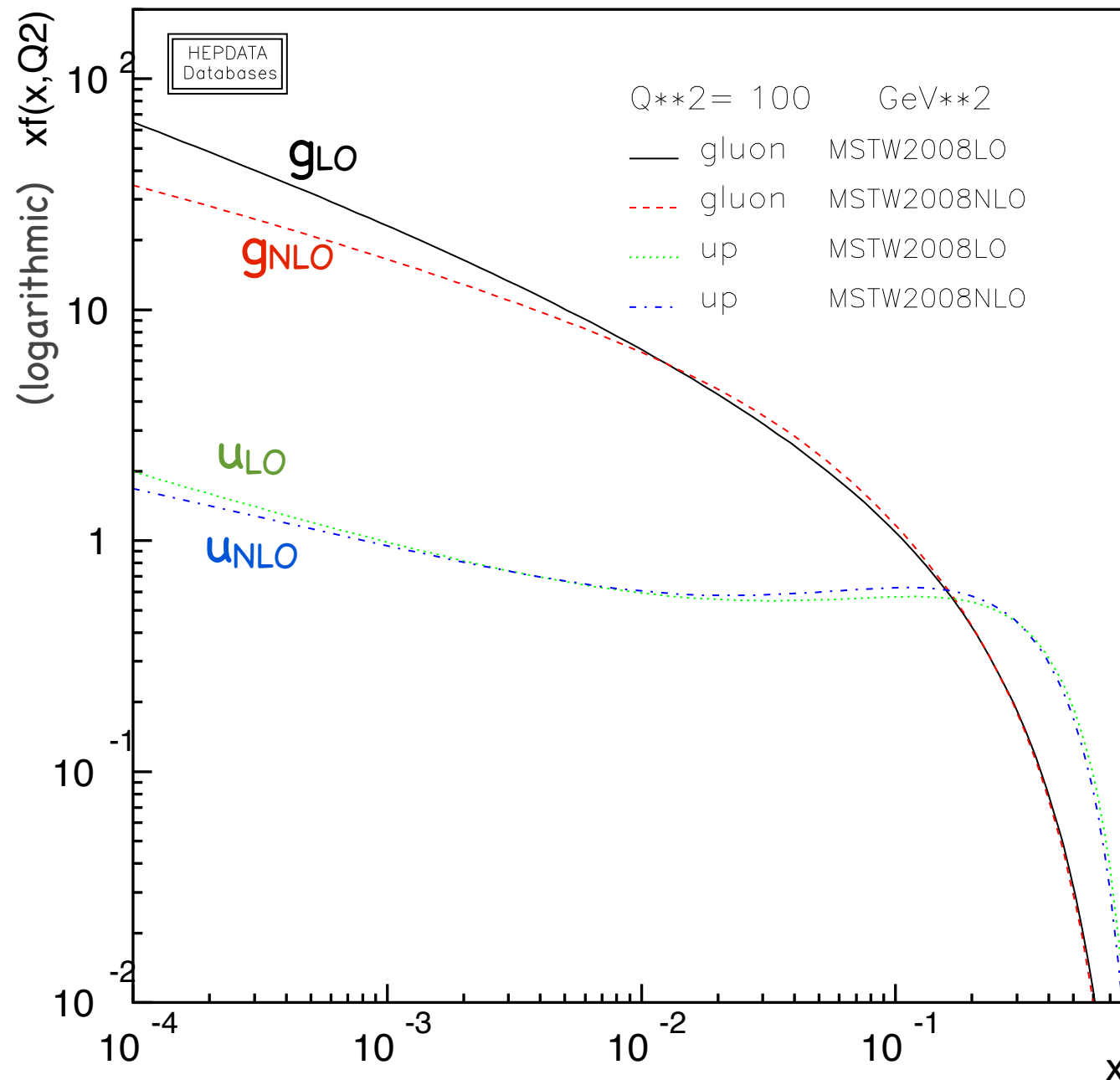
$$Q^2 = (100 \text{ GeV})^2$$





# LO vs NLO

$$Q^2 = (10 \text{ GeV})^2$$



NLO matrix elements  
contain low- $x$   
singularities beyond  
DGLAP ( $\rightarrow$  enhancement)  
 $\rightarrow$  need less low- $x$  PDFs

(+ momentum conservation  
 $\rightarrow$  more partons at high  $x$   
 $\rightarrow$  larger cross sections)

Important to use the  
right PDFs with the  
right Matrix Elements

# PDF Uncertainties

Much debate recently on PDF errors

Attempt to propagate  
experimental errors  
properly  $\rightarrow$  68% CL

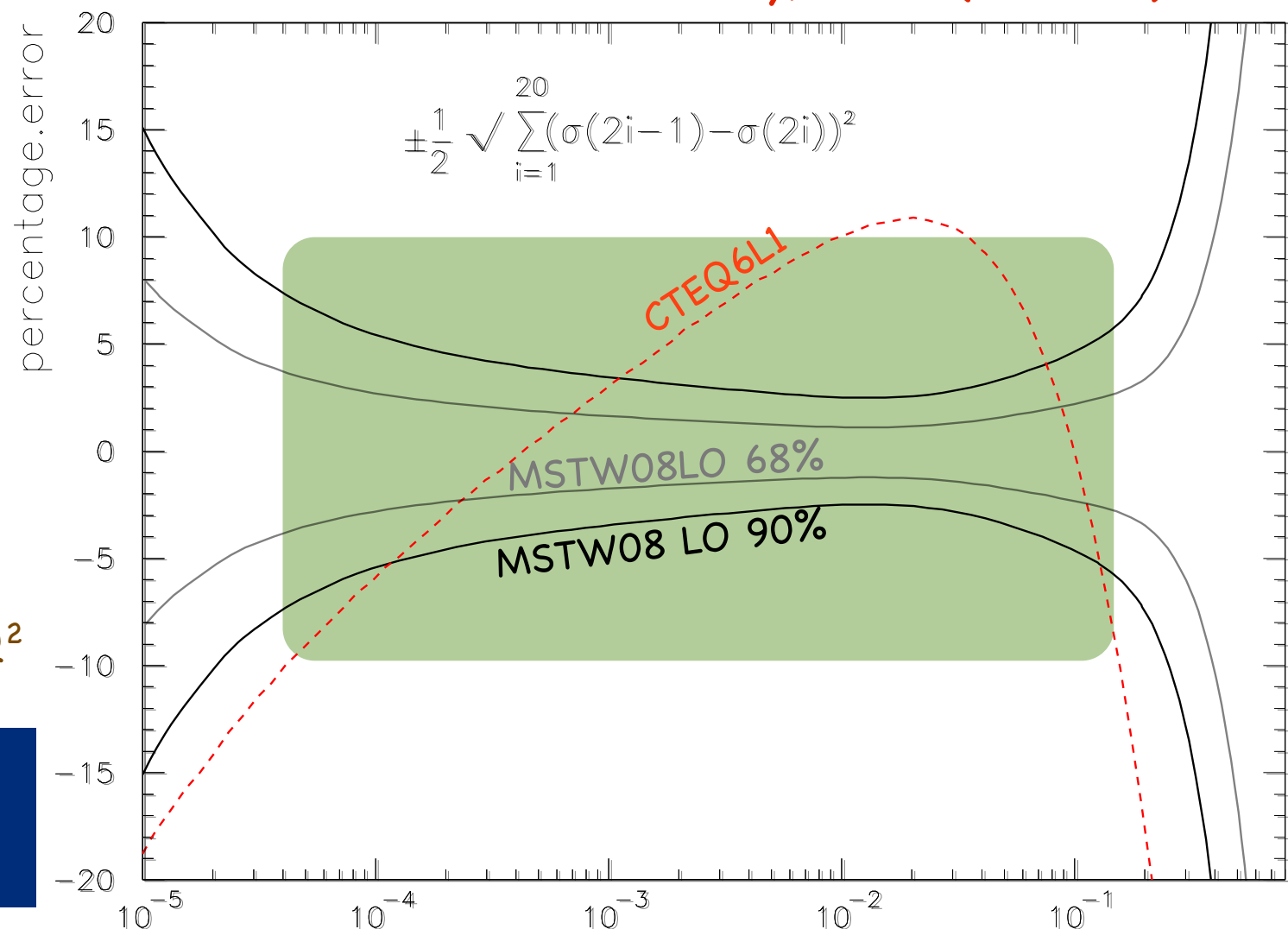
But “tensions” between  
different badly compatible  
data sets  $\rightarrow$  ... ?

$\rightarrow$  90%, something else?

+ unknown uncertainty from  
starting parametrization at low  $Q^2$

Still, good to  $\approx 10\%$  even for LO  
gluon in  $10^{-4} < x < 10^{-1}$   
(bigger errors at lower  $Q^2$ )

Gluon PDF uncertainty,  $Q^2 = (10 \text{ GeV})^2$





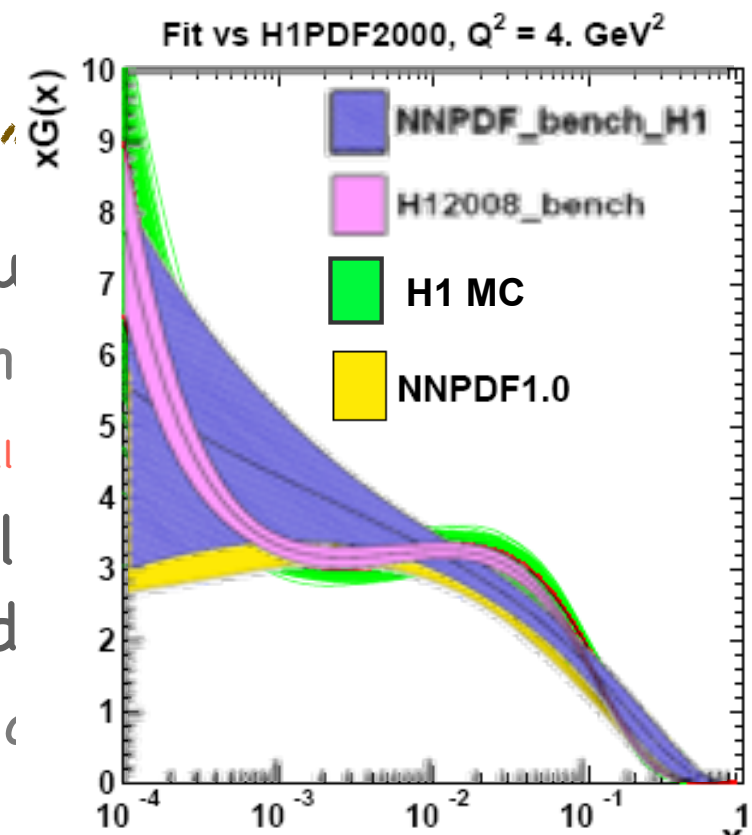
# Neural Nets, LO\*, MC PDFs, ...

## Neural Net PDFs

Attempt to use an unbiased starting parametrization represented by a neural net fitting function

## LO\*, LO\*\*, MC PDFs, ... : "Optimize"

- LO\* allows  $\approx 10\%$  violation of momentum
- Accommodate more low- $x$  glue while main
- Cross sections "closer" to NLO [but still
- MC PDFs, like LO\*\*, attempt to parallel actual evolution equations as implemented
- E.g., using the  $\alpha_s$  choices, physical phase space



PDFs is a rapidly evolving field → important to keep up to date  
→ Reliability of your results and uncertainties

(more in MC lecture ... )



# Confinement

Local Parton-Hadron Duality  
Hadronization / Fragmentation  
“Intrinsic  $k_{\perp}$ ”



# QCD in the Infrared

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## What we know

Asymptotic Freedom ✓

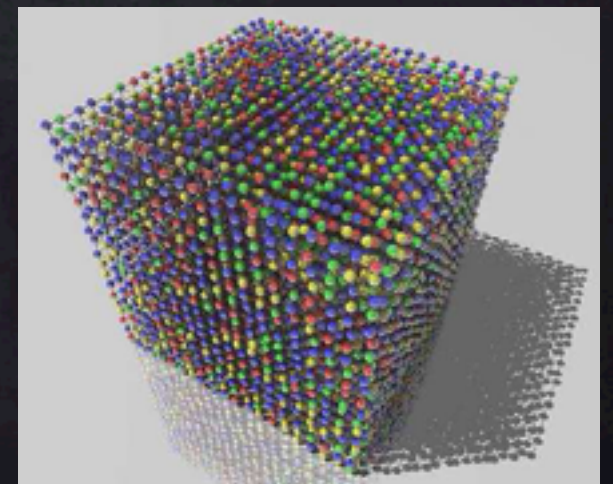
Gauge invariance ✓

C, P, T invariance ✓

Lorentz invariance ✓

Causality ✓

Lattice QCD ...



# Lattice QCD

## Spacetime

Approximated by  
4D (Euclidean) box of points

Similar to crystal lattice (with  
imaginary time)  $3\text{fm}/c \approx 10 \text{ yoctoseconds}$

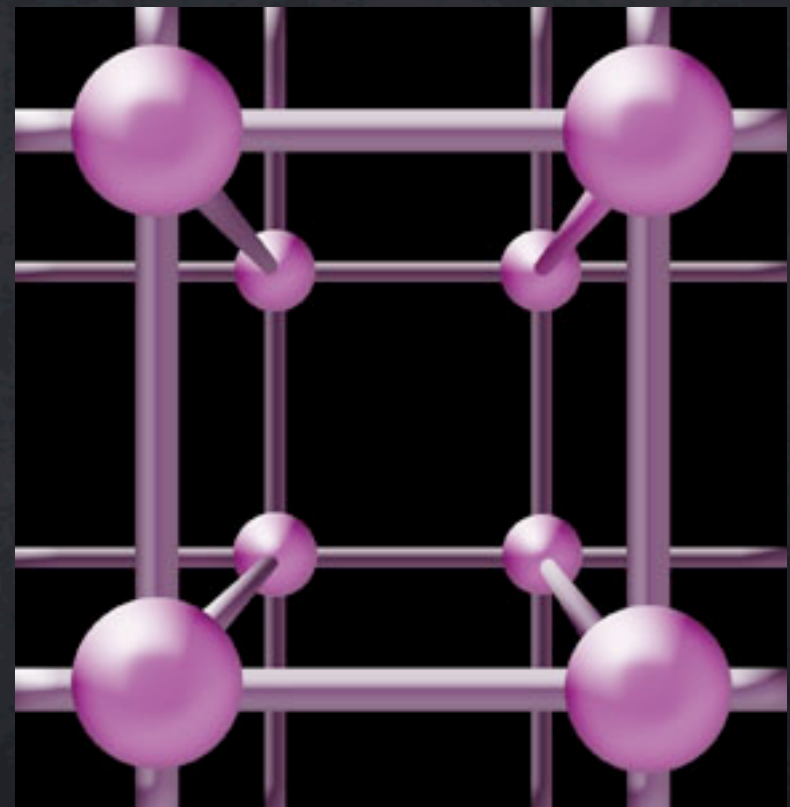
## Symmetries

Full Lorentz  $\rightarrow$  Hypercubic

But gauge invariance ok  $\checkmark$

“Discretization Errors”  $\rightarrow 0$

in limit of infinitely small lattice  
spacing,  $a$



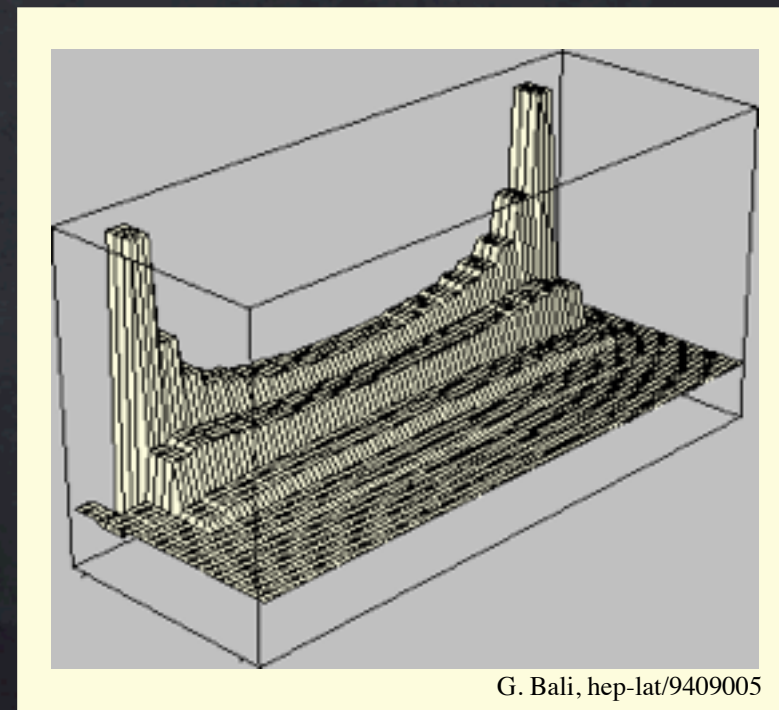
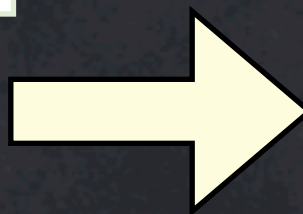
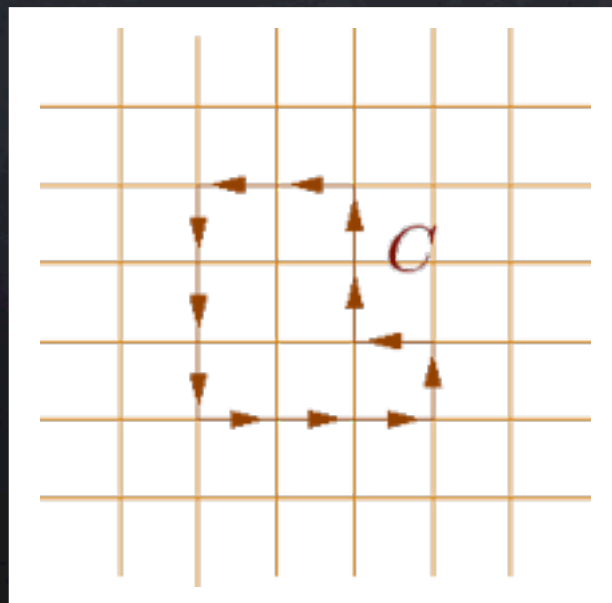


# Solve QCD

## Direct computation of Path Integral

Probability of field configuration  $\{U\}$

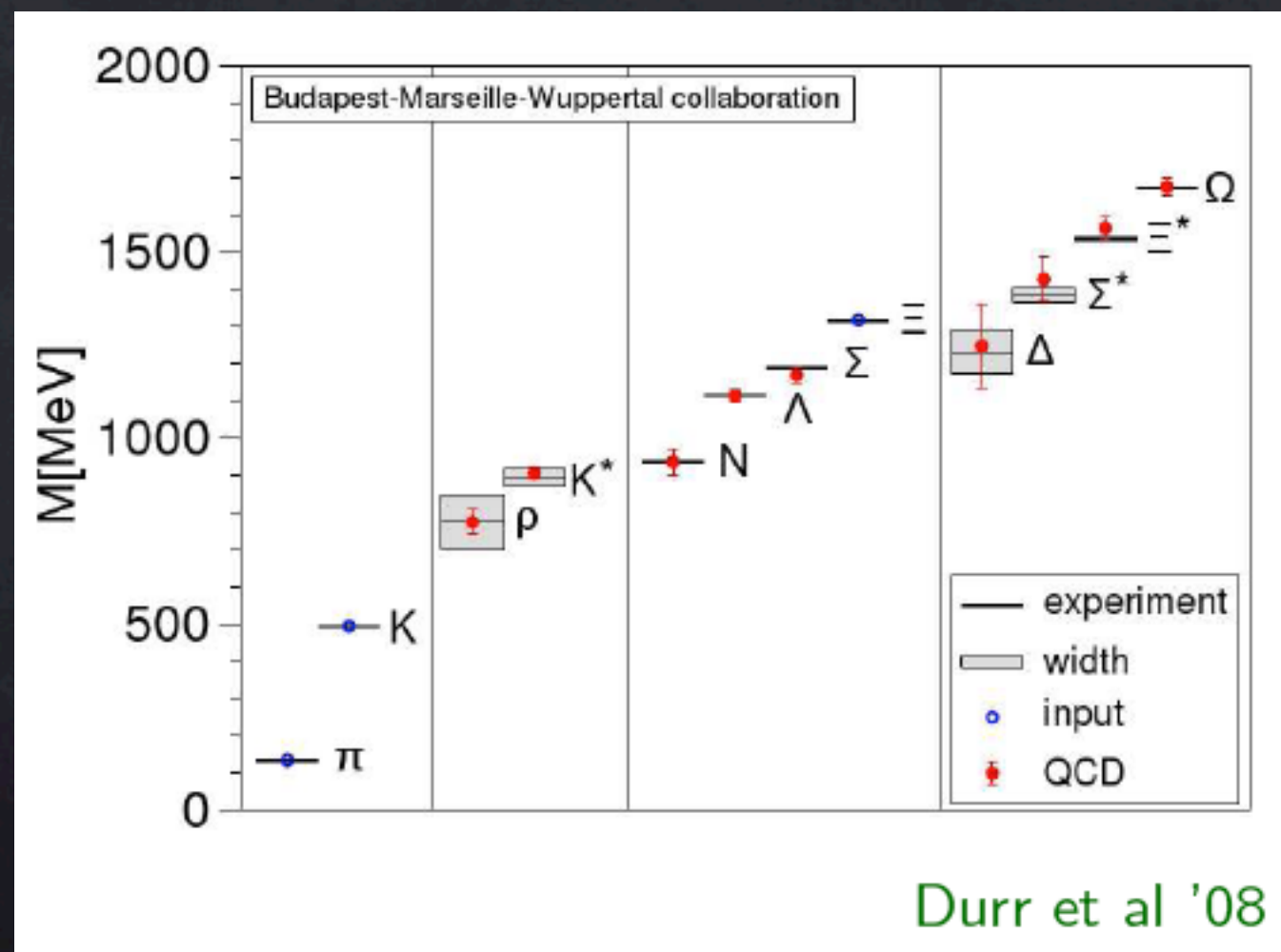
$$\{U\} : e^{-S[U]}$$



Separation of two static color charges by 22 lattice spacings

# Example: Lattice Hadron Masses

Compute Hadron Spectrum (Given  $m_\pi$  and  $m_K$  as inputs)



Durr et al '08



# Why not Lattice for LHC?

To “resolve” a hard LHC collision

$$\text{Lattice spacing: } \frac{1}{14 \text{ TeV}} \sim 10^{-5} \text{ fm}$$

To include hadronization

$$\text{Proper time } t \sim \frac{1}{0.5 \text{ GeV}} \sim 0.4 \text{ fm}/c \times \text{Lorentz Boost Factor}$$

Boost factor at LHC  $\approx 10^4$

→ would need  $\approx 4000 \text{ fm}$  to fit entire collision

→  $10^{34}$  lattice points in total

Biggest lattices today are  $64 \times 64 \times 64 \times 128 \approx 10^7$

→ one or a few hadrons at a time

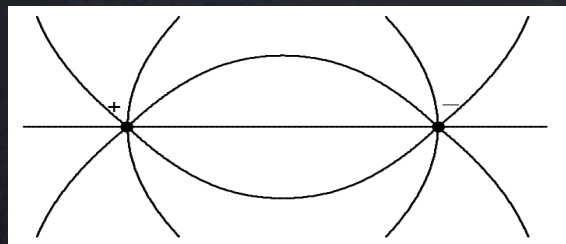


# Linear Confinement

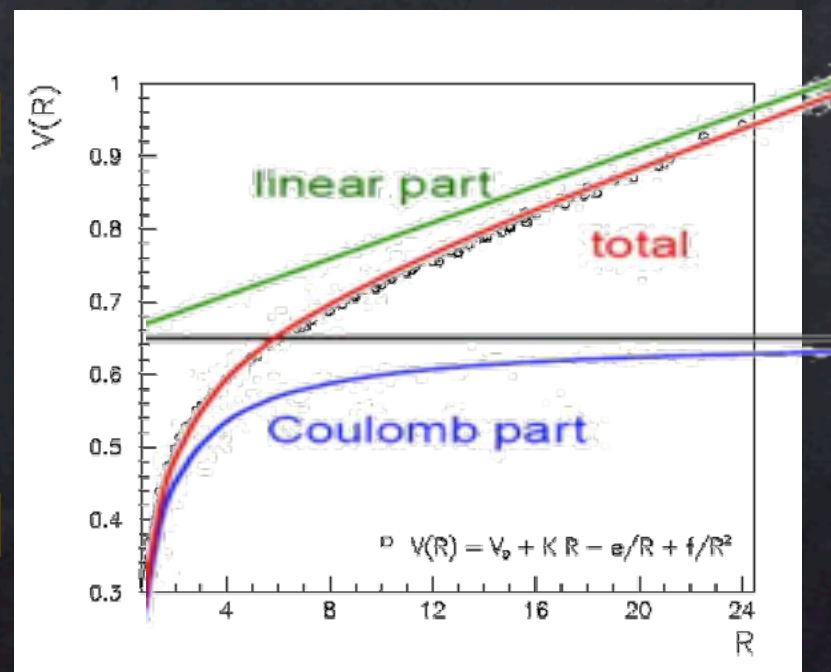
## "Quenched" QCD

Look at the gluon field between two quarks  
= Static quark sources plus dynamic gluon field (no  $g \rightarrow qq$ )

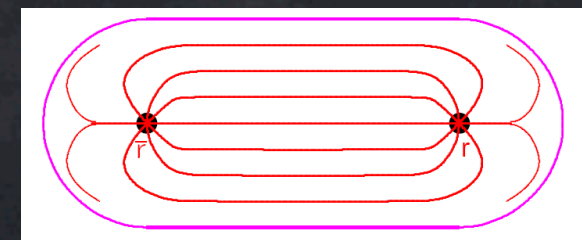
Short Distances ~ pQCD



Partons



Long Distances ~ Linear Confinement



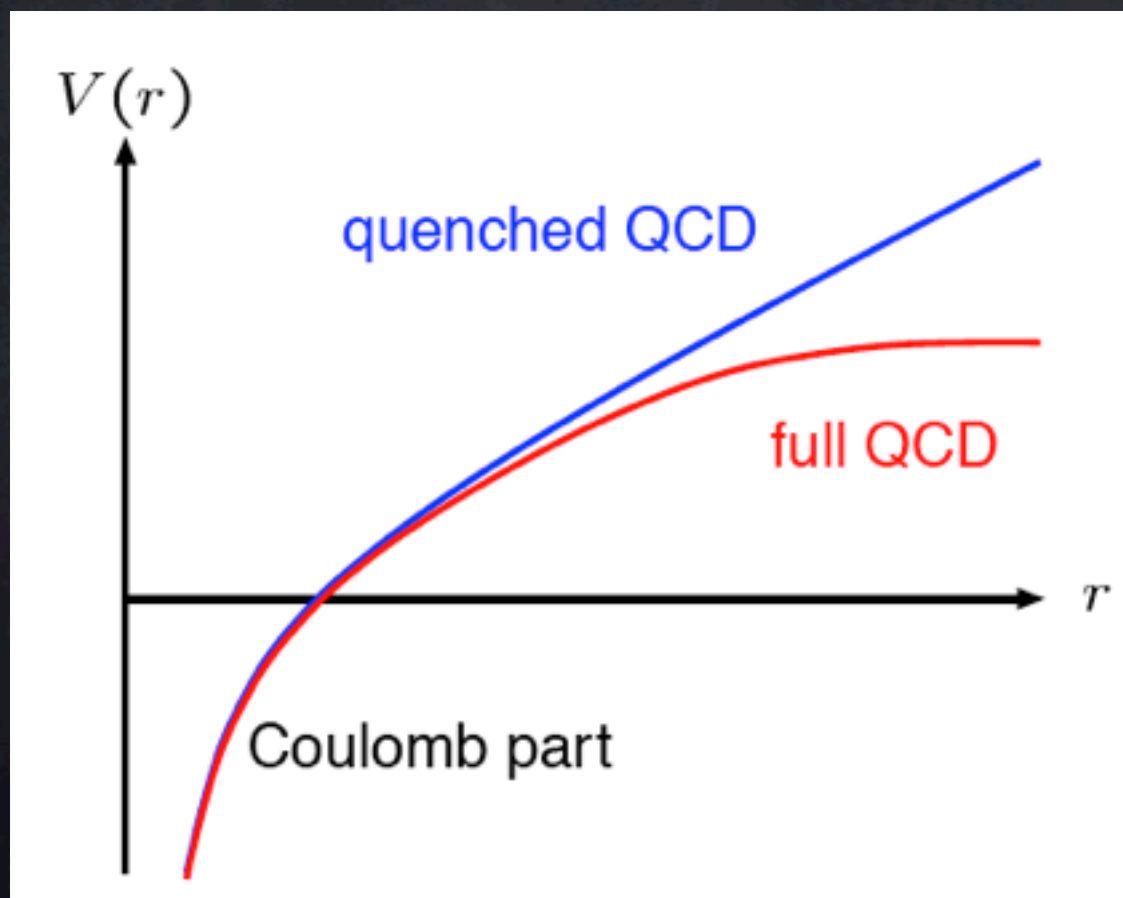
Strings (Flux Tubes), Hadrons

$$F(r) \approx \text{const} = \kappa \approx 1 \text{ GeV/fm} \iff V(r) \approx \kappa r$$

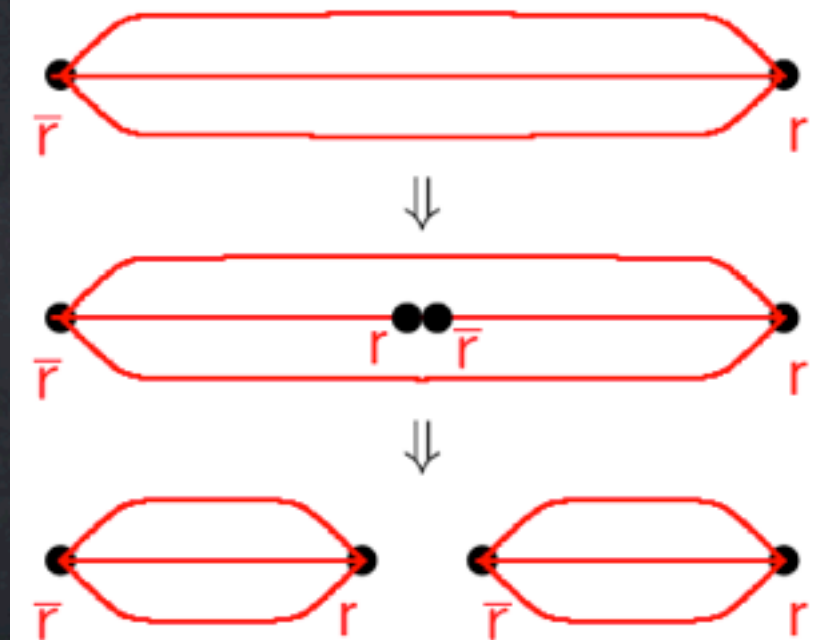
# Real World?

In unquenched QCD

$g \rightarrow qq \rightarrow$  The strings would break



simplified colour representation:



From here  
on: Models



# Local Parton-Hadron Duality

## Hard Line

Each perturbative parton (at very low  $Q^2$ )  
 $\approx$  one hadron in full picture

**THIS IS AWFULLY WRONG!**

(although some success describing incl spectra)

And yet you still find  
both of these  
pictures in modern  
papers

## Soft Line (closer to the truth?)

Partons in perturbative calculations  
 $\approx$  hadronic jets in full picture

**THIS IS STILL PRETTY WRONG!**

(although corrections power-suppressed if jets IR safe)

Today,  
Hard Line  $\rightarrow$   
pQCD  $\times$  FFs  
Soft line  $\rightarrow$   
IR safety

# What's wrong?

LPHD  $\approx$  Independent Fragmentation (I.F.)

Universal fragmentation of a parton into hadrons



But duh!

The point of confinement is that partons are colored

Hadronization = the process of color neutralization

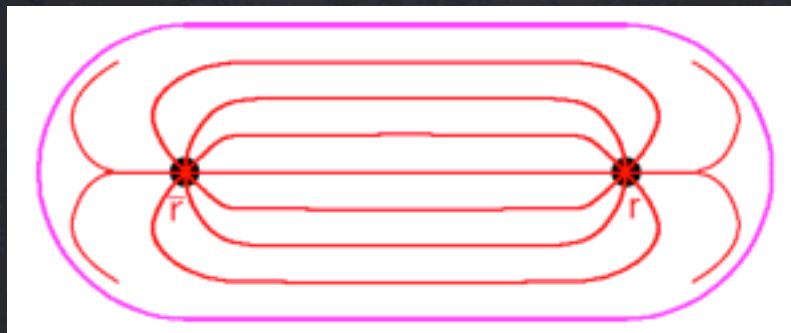
I.e, the one question NOT addressed by LPHD or I.F.

→ fundamentally misguided to think about independent fragmentation of individual partons



# The String Model

## Linear Confinement



Describe as classical  
(1+1 dimensional) string  
(i.e., ignore Coulomb)

$$F(r) \approx \text{const} = \kappa \approx 1 \text{ GeV/fm} \iff V(r) \approx \kappa r$$

## → The (Lund) String Model

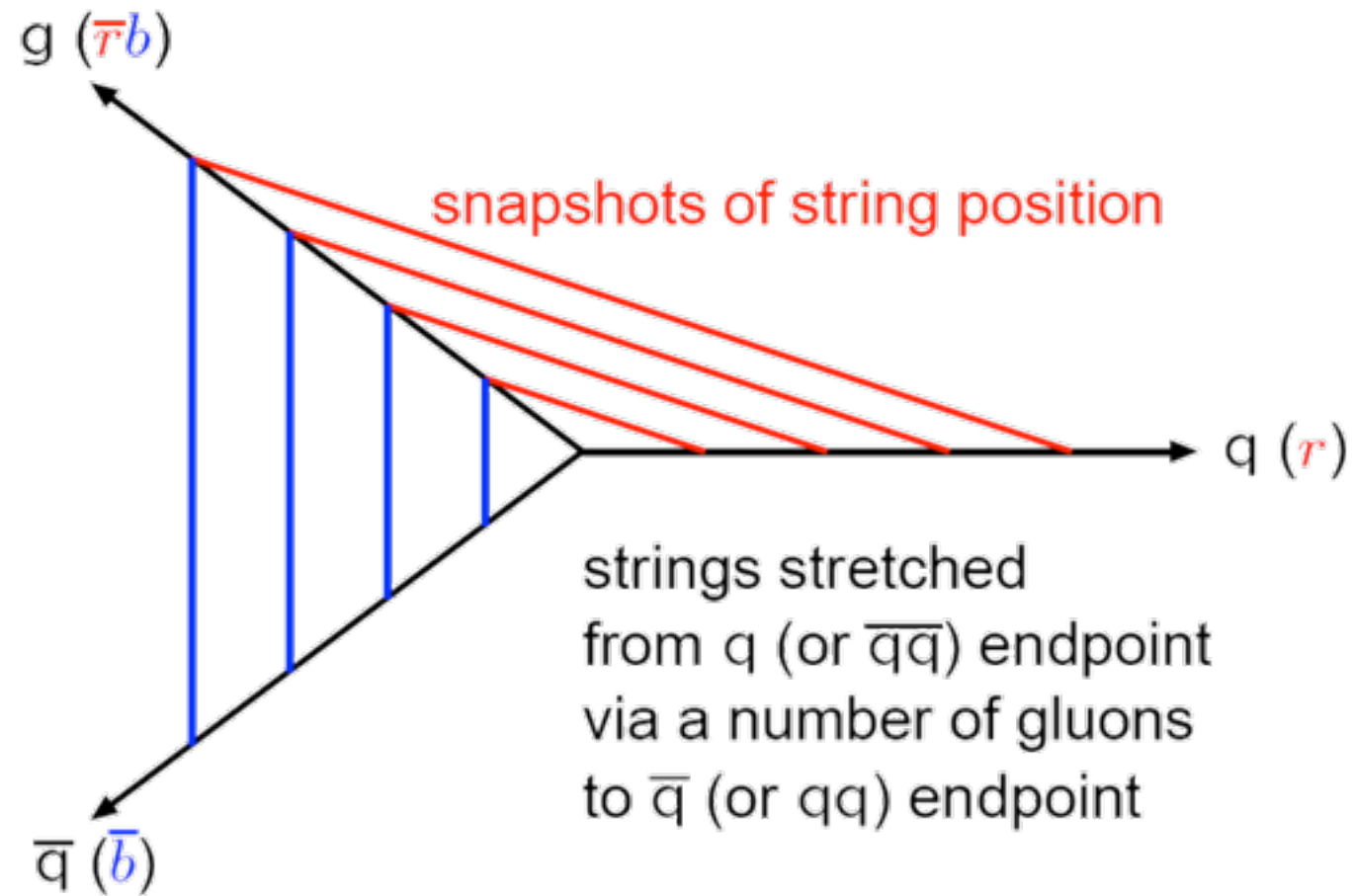
Change degrees of freedom: **two opposite charges moving apart** → **one Lorentz invariant string (piece)**

Classical string theory → **string motion in spacetime**

# The Lund String

Map:

- **Quarks** → String Endpoints
- **Gluons** → Transverse Excitations (kinks)
- Physics then in terms of string worldsheet evolving in spacetime
- Probability of string break constant per unit area → **AREA LAW**



Gluon = kink on string, carrying energy and momentum

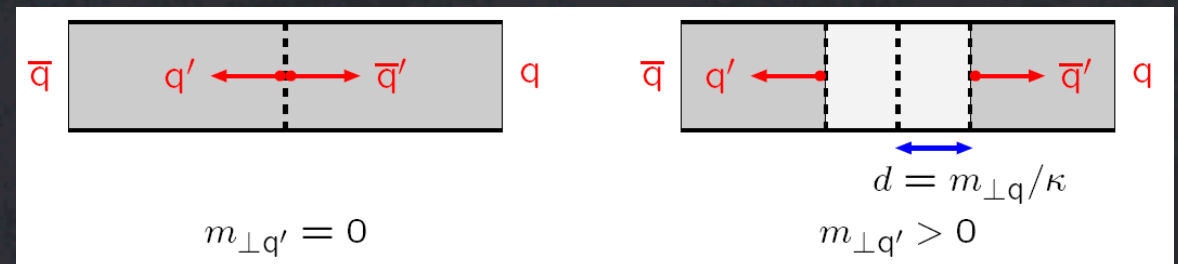
Simple space-time picture + no separate params for  $g$  jets  
Details of string breaks more complicated ...



# String Breaks

## String Breaks

Modeled by tunneling



$$\mathcal{P} \propto \exp\left(-\frac{\pi m_{\perp q}^2}{\kappa}\right) = \exp\left(-\frac{\pi p_{\perp q}^2}{\kappa}\right) \exp\left(-\frac{\pi m_q^2}{\kappa}\right)$$

1) common Gaussian  $p_{\perp}$  spectrum

2) suppression of heavy quarks  $u\bar{u} : d\bar{d} : s\bar{s} : c\bar{c} \approx 1 : 1 : 0.3 : 10^{-11}$

3) diquark  $\sim$  antiquark  $\Rightarrow$  simple model for baryon production

Also depends on:

spins, hadron multiplets, hadronic wave functions, phase space, ...  $\rightarrow$  (much) more complicated  $\rightarrow$  many parameters

$\rightarrow$  Not calculable, must be constrained by data  $\rightarrow$  'tuning'

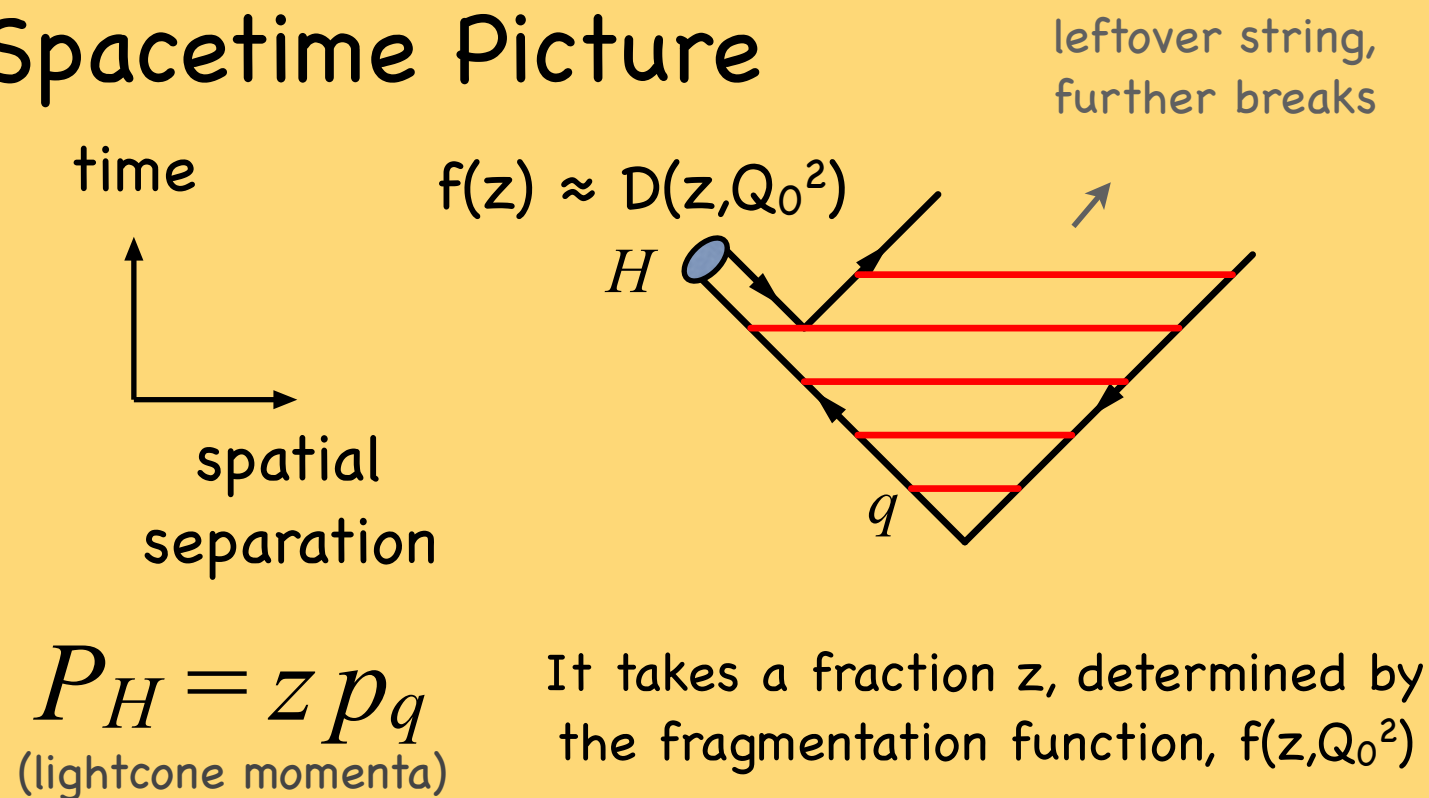


# String Breaks $\rightarrow$ Hadrons

Having selected a hadron flavor

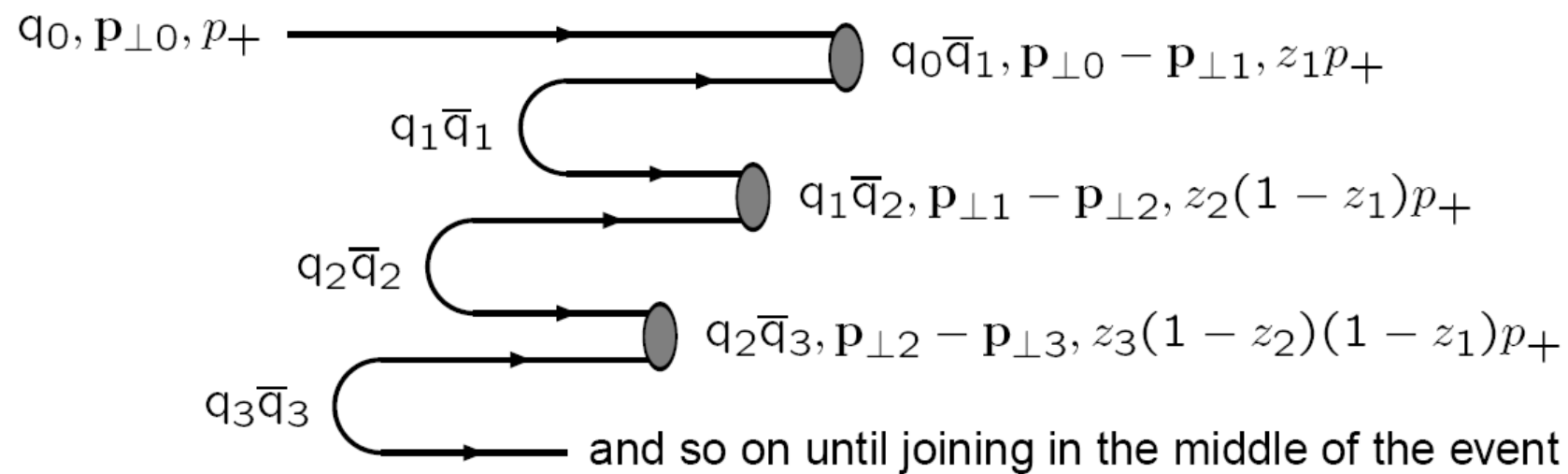
How much momentum does it take?

## Spacetime Picture

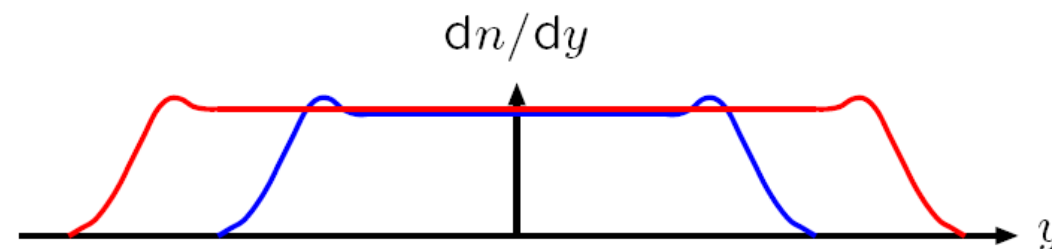


# More String Breaks

## Iterative Ansatz



Scaling in lightcone  $p_{\pm} = E \pm p_z$  (for  $q\bar{q}$  system along  $z$  axis)  
 implies flat central rapidity plateau + some endpoint effects:

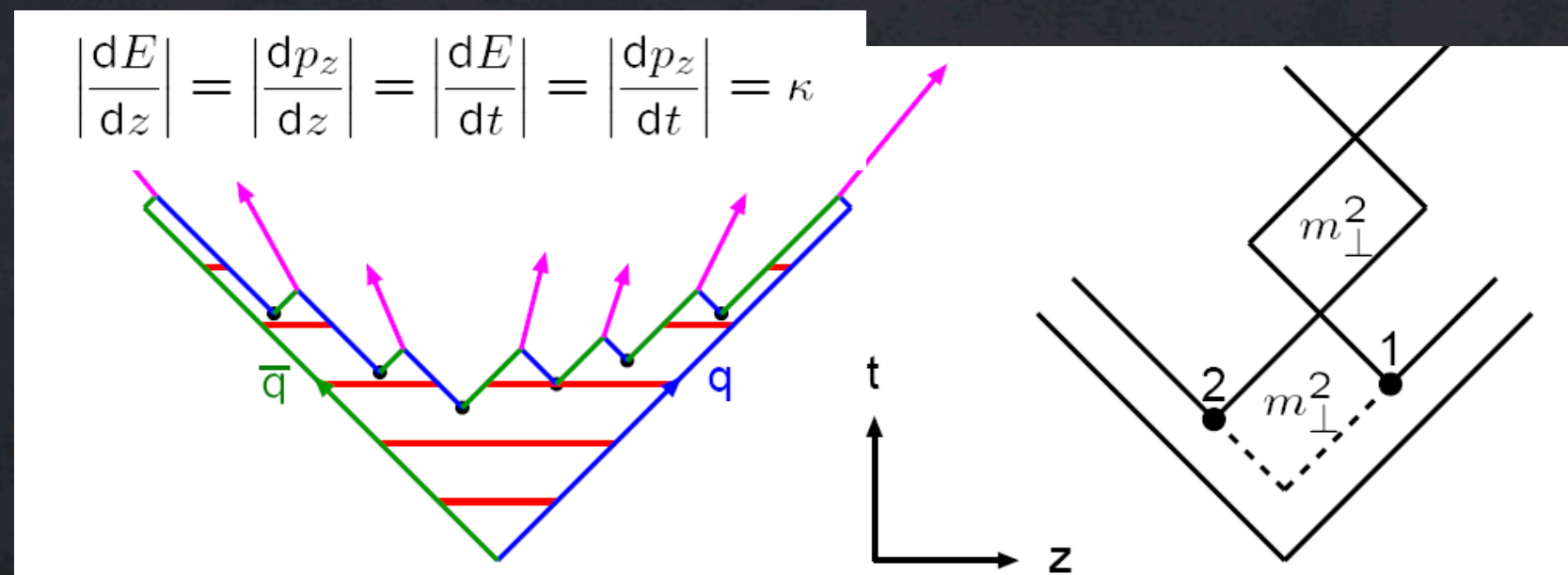


$\langle n_{\text{ch}} \rangle \approx c_0 + c_1 \ln E_{\text{cm}}, \sim \text{Poissonian multiplicity distribution}$



# → Hadrons

Repeat for large system → Lund Model



Note: string breaks causally disconnected

→ can proceed in arbitrary order (left-right, right-left, in-out, ...)

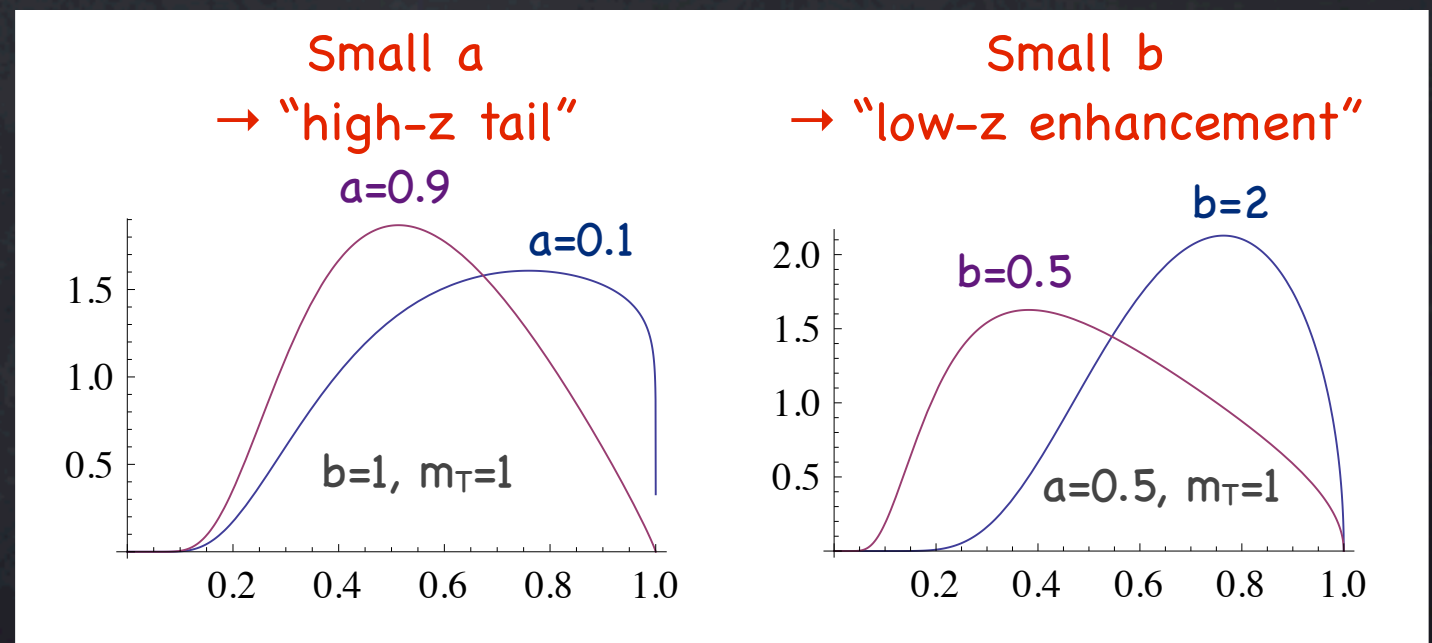
→ Justifies iterative ansatz (useful for MC implementation)

# Causality

Note: string breaks causally disconnected

- can proceed in arbitrary order (left-right, right-left, in-out, ...)
- Justifies iterative ansatz (useful for MC implementation)

Also constrains form  
of fragmentation  
function!  
(Left-Right Symmetry)



⇒ Lund symmetric fragmentation function  
$$f(z) \propto (1 - z)^a \exp(-bm_{\perp}^2/z)/z$$

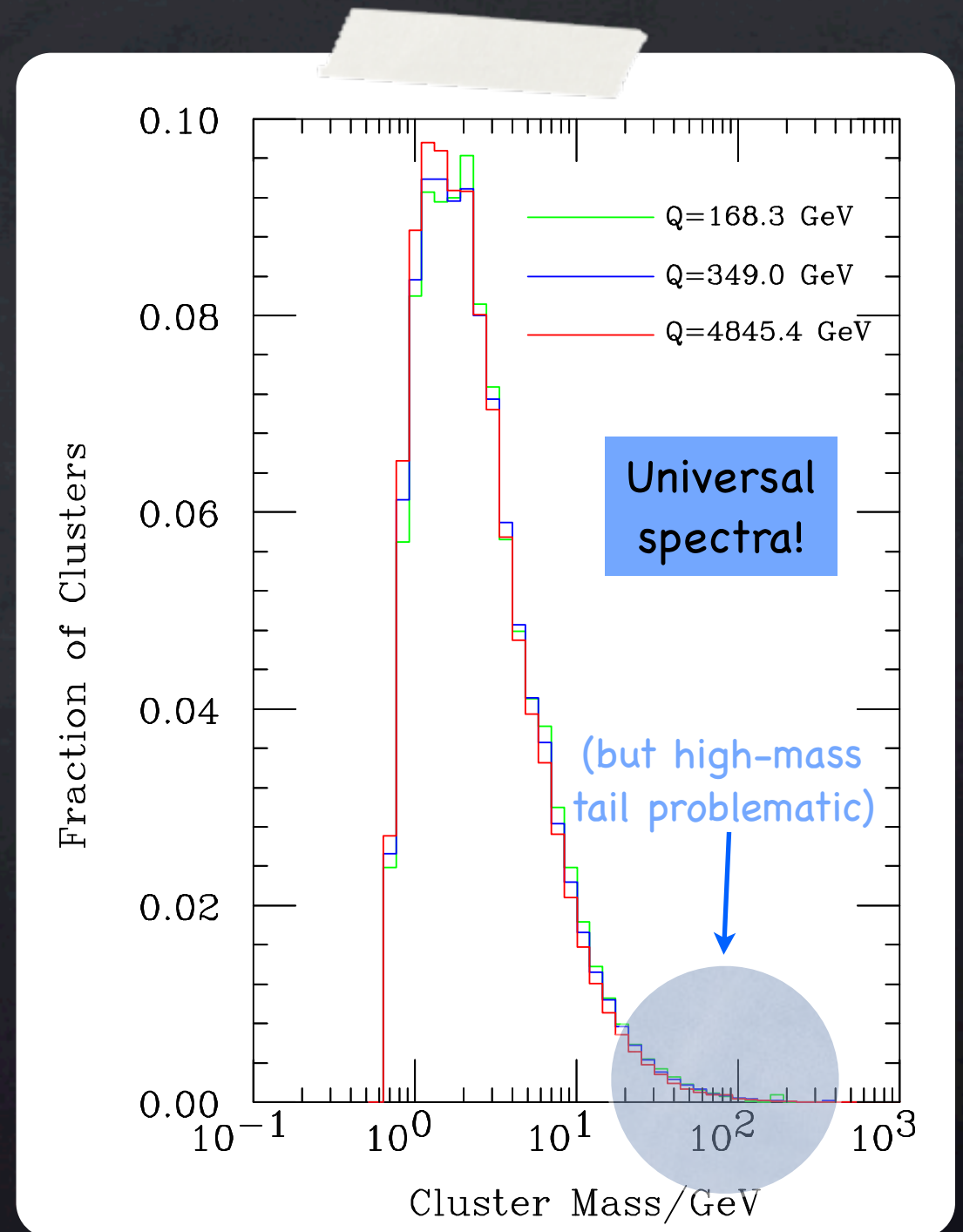
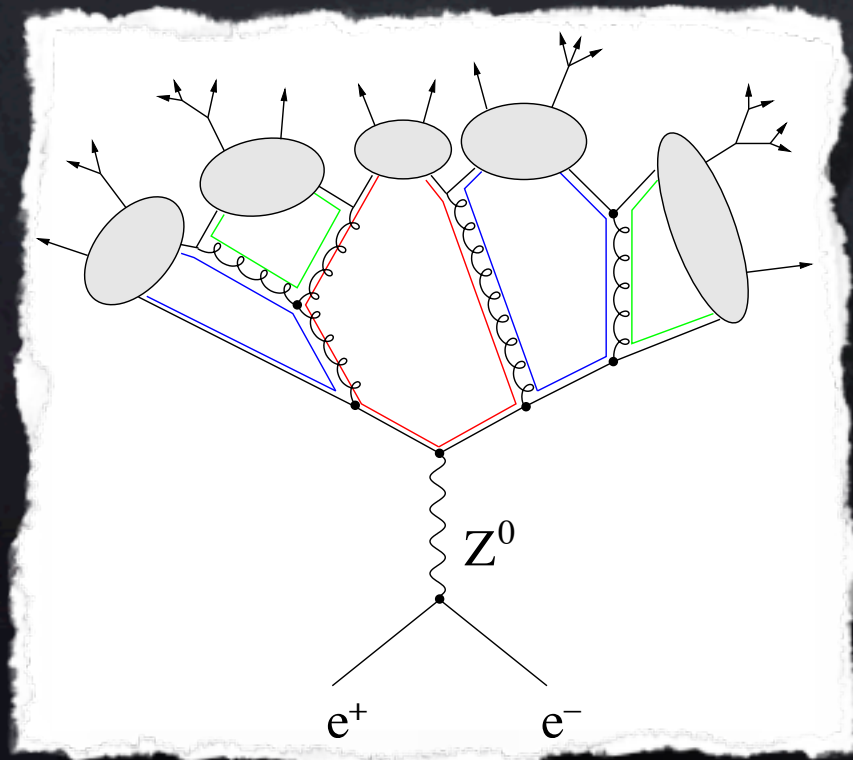


# Alternative: The Cluster Model

## "Preconfinement"

Force  $g \rightarrow qq$  splittings at  $Q_0$   
 $\rightarrow$  high-mass  $qq$  "clusters"

Isotropic 2-body decays to hadrons  
according to  $PS \approx (2s_1+1)(2s_2+1)(p^*/m)$



# Underlying Event

Nomenclature → what is what?

Perturbative? Or not?

(What) can we learn about it from Minimum-Bias?

What about diffraction?



# Additional Sources of Particle Production

## ► Starting point: Matrix Elements + Parton Showers

$n = \text{a handful}$   
+ resonance  
decays

$2 \rightarrow n$  hard parton scattering at (N)LO  
+ Bremsstrahlung  $\rightarrow 2 \rightarrow \infty$  at (N)LL

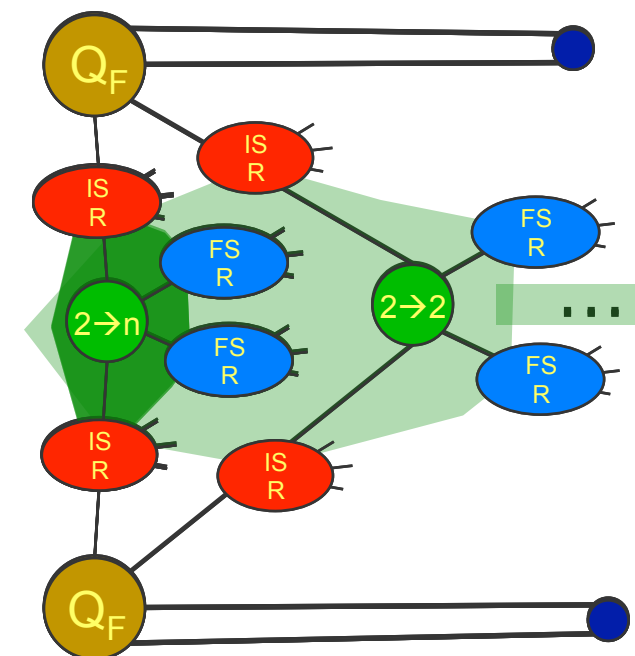
Hadrons are not elementary  
+ QCD diverges at low  $p_T$

$\rightarrow$  multiple perturbative parton-parton interactions

e.g.  $4 \rightarrow 4$ ,  $3 \rightarrow 3$ ,  $3 \rightarrow 2$

► No factorization theorem

$\rightarrow$  Herwig++, Pythia, Sherpa: MPI models

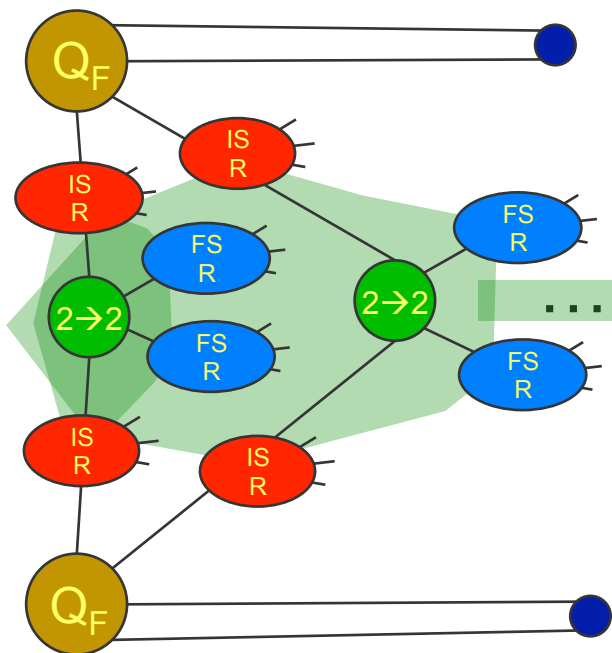


$$Q_F \gg \Lambda_{\text{QCD}}$$

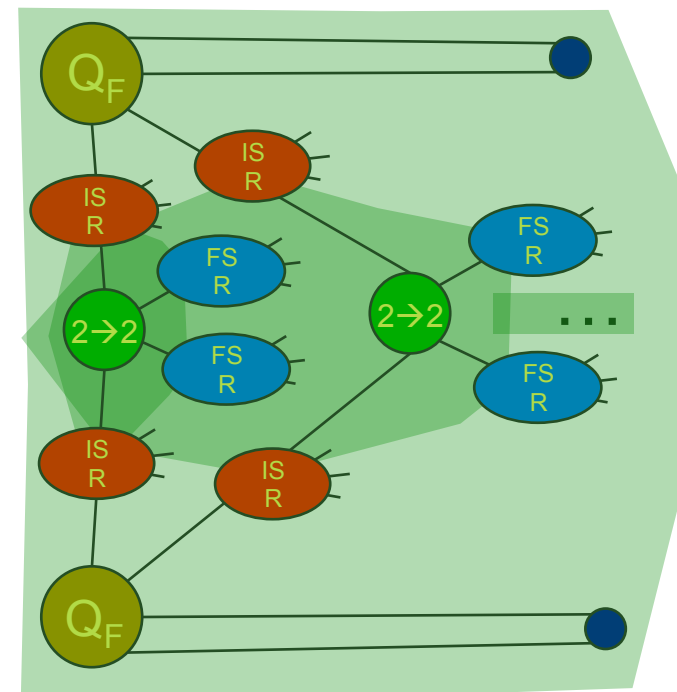
Underlying Event has  
perturbative part!

# Additional Sources of Particle Production

$Q_F \gg \Lambda_{\text{QCD}}$   
 ME+ISR/FSR  
 + perturbative MPI



+  
 Stuff at  
 $Q_F \sim \Lambda_{\text{QCD}}$



Need-to-know issues for IR sensitive quantities (e.g.,  $N_{\text{ch}}$ )

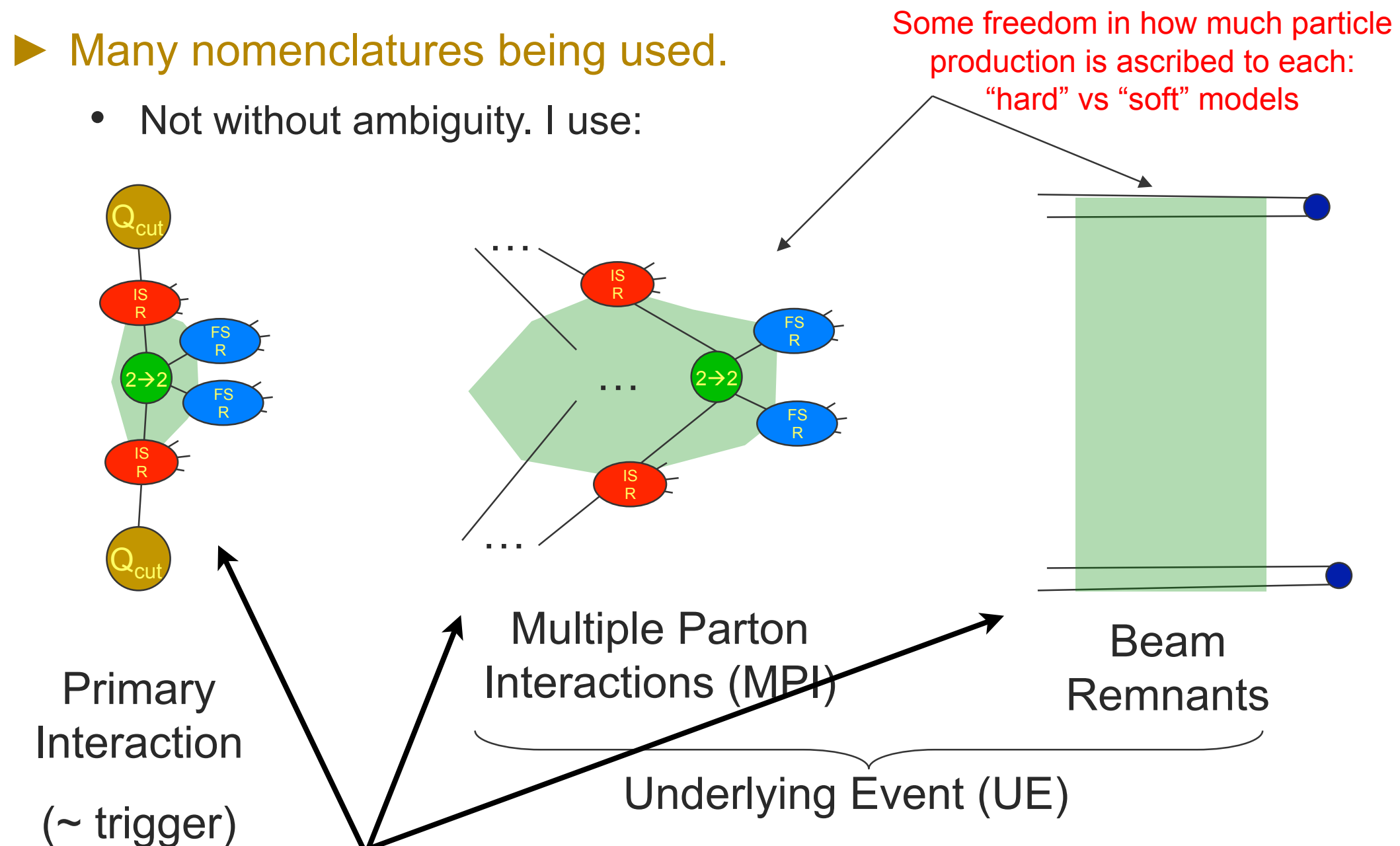


# What is What?

See also Tevatron-for-LHC Report of the QCD Working Group, hep-ph/0610012

► Many nomenclatures being used.

- Not without ambiguity. I use:



Note: each is colored → Not possible to separate cleanly at hadron level anyway

# What is Minimum-Bias?

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## The 'average' hadron-hadron collision

(TH) Reference laboratory for testing QCD models with almost unlimited statistics

(EXP) Benchmark for Luminosity Measurements

## The HARDEST physics process to study

Non-perturbative physics (no "hard trigger scale")

→ still don't have exact solutions

(PHENO) Important testing ground for new models

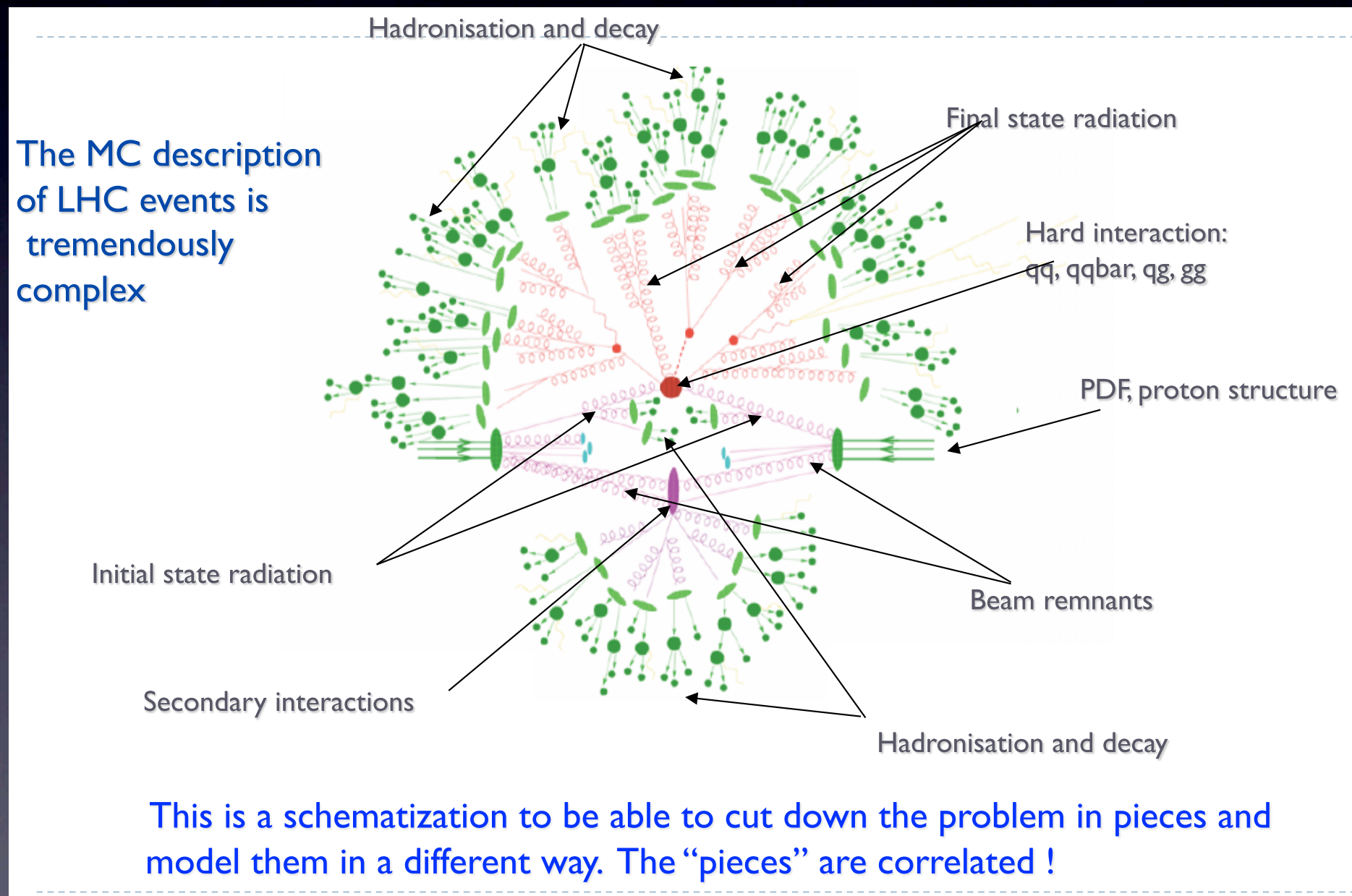
## → Constraints & Feedback to high- $p_T$ studies

Tails → Study evolution from soft to hard events



# Dissecting Minimum-Bias

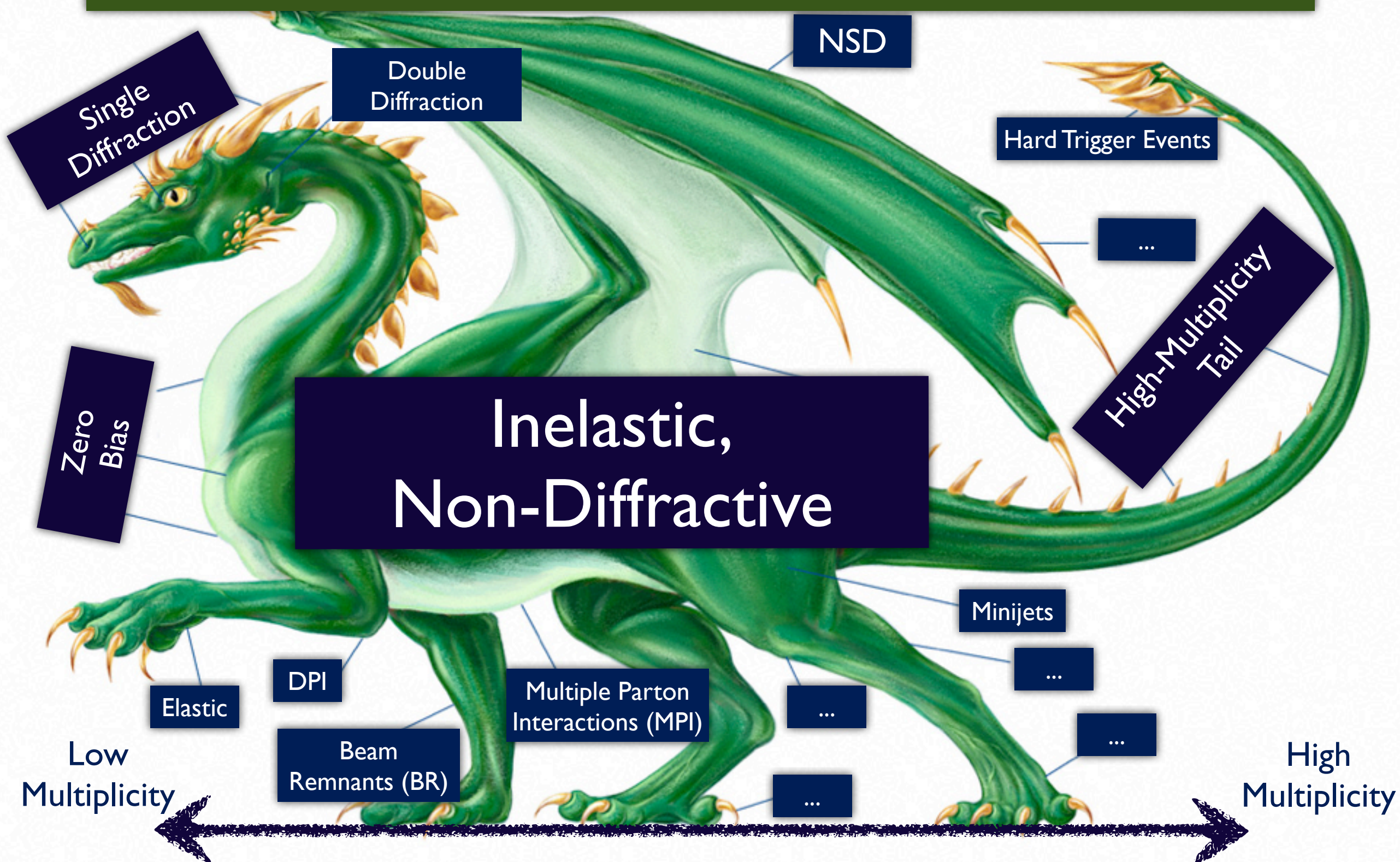
**A lab for** testing theory models and detector performance with high statistics



(slide from F. Cossutti (CMS), 7th MCnet Annual Meeting, January 2010)



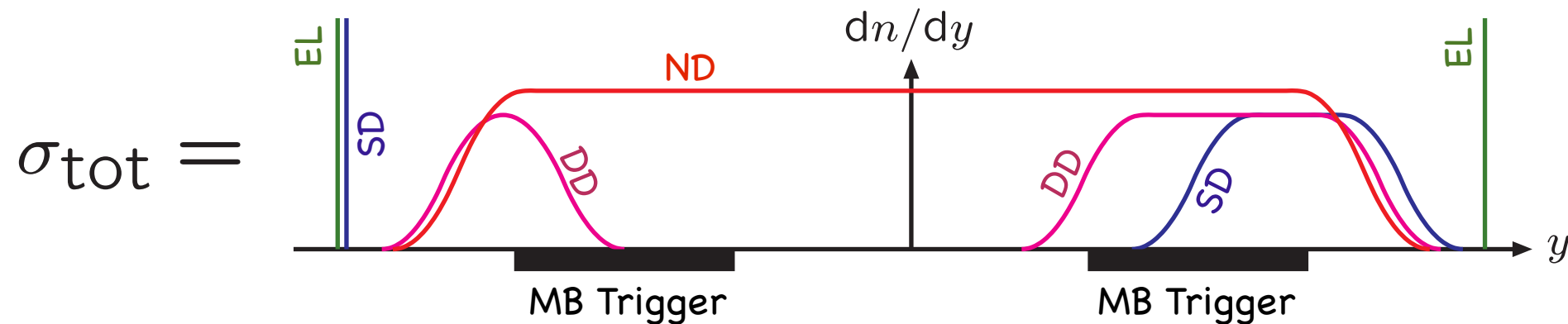
# Dissecting Minimum-Bias





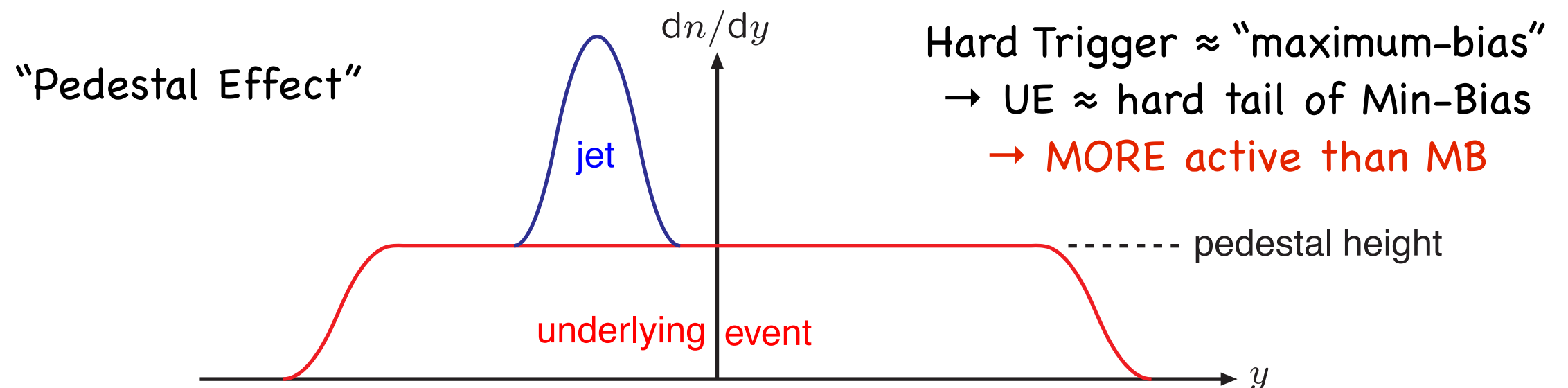
# What is Minimum-Bias?

$$= \left( \sigma_{\text{elastic}} + \sigma_{\text{single-diffractive}} + \sigma_{\text{double-diffractive}} + \dots + \sigma_{\text{non-diffractive}} \right) \times \epsilon_{\text{MB-trigger}}$$



reality:  $\sigma_{\text{min-bias}} \approx \sigma_{\text{non-diffractive}} + \sigma_{\text{double-diffractive}} \approx 2/3 \times \sigma_{\text{tot}}$

# What is Underlying Event?



# Multiple Parton Interactions?

## (M.P.I.)

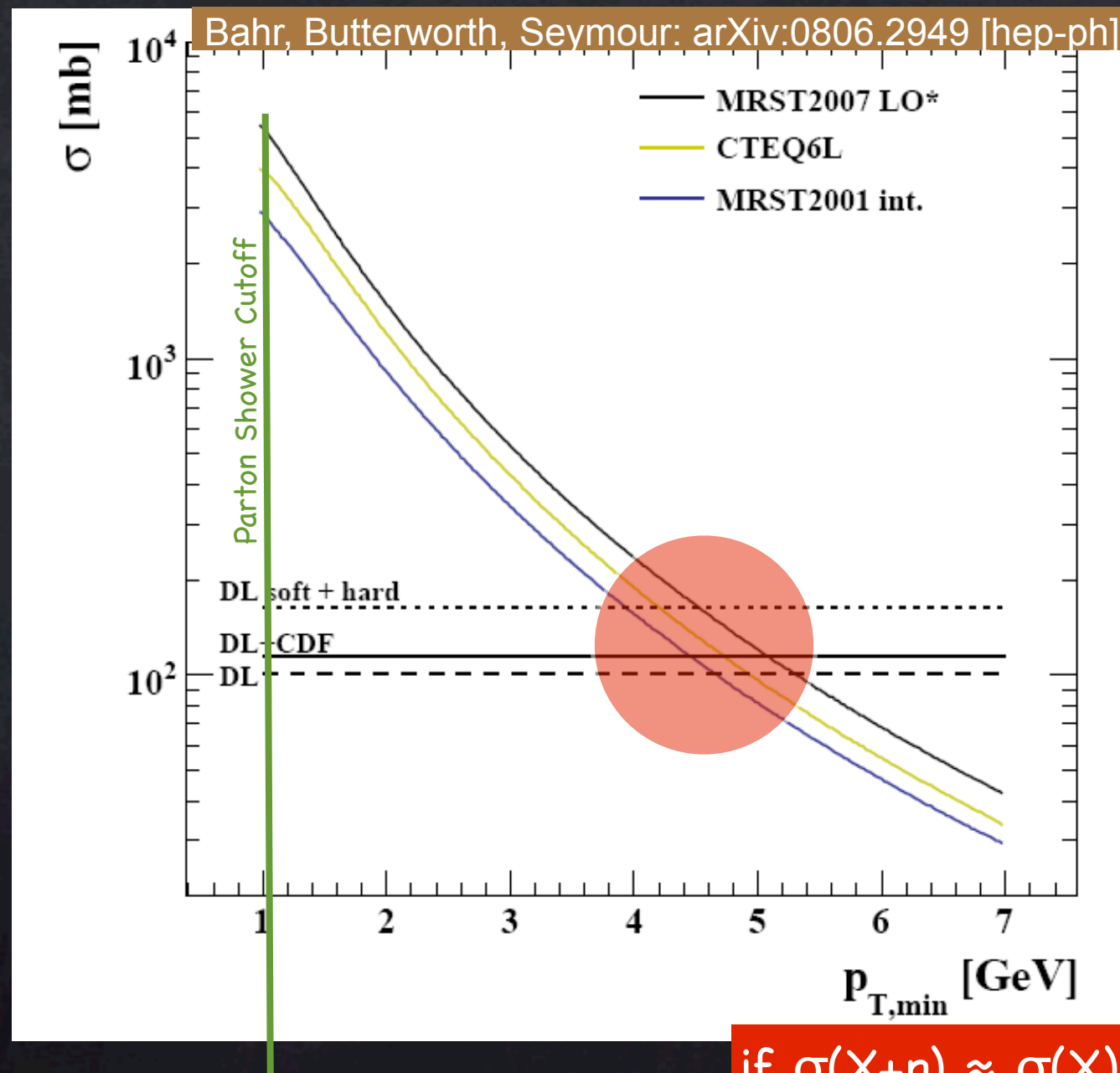
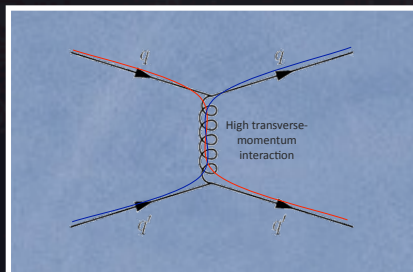
QCD

2 → 2

= Sum of

$qq' \rightarrow qq'$   
 $q\bar{q} \rightarrow q'\bar{q}'$   
 $q\bar{q} \rightarrow gg$   
 $qg \rightarrow qg$   
 $gg \rightarrow gg$   
 $gg \rightarrow q\bar{q}$

≈ Rutherford  
(t-channel gluon)



Becomes larger  
than total pp  
cross section?

At  $p_{\perp} \approx 5$  GeV

if  $\sigma(X+n) \approx \sigma(X)$  you got a problem  
fixed-order truncation not reliable



$$\sigma_{\text{parton-parton}} > \sigma_{\text{proton-proton}}$$

What does  $\sigma_{\text{parton-parton}}$  count?

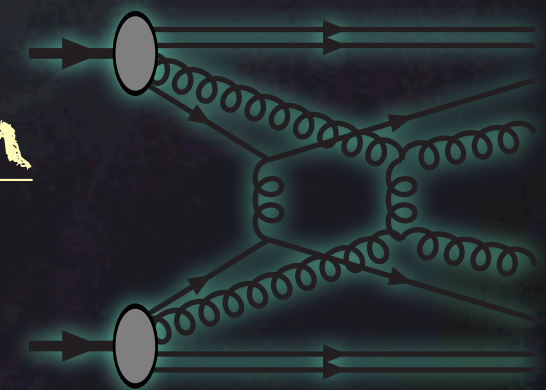
$$\frac{d\sigma_{2j}}{dp_{\perp}^2} = \sum_{i,j,k} \int dx_1 \int dx_2 \int d\hat{t} f_i(x_1, \mu_F^2) f_j(x_2, \mu_F^2) \frac{d\hat{\sigma}_{ij \rightarrow kl}}{d\hat{t}} \delta\left(p_{\perp}^2 - \frac{\hat{t}\hat{u}}{\hat{s}}\right) \propto \frac{1}{p_{\perp \min}^2} \quad (\text{neglecting pdf dependence and } \alpha_s \text{ running})$$

Inclusive number of PARTON-PARTON interactions

What does  $\sigma_{\text{proton-proton}}$  count?

Inclusive number of PROTON-PROTON interactions

→ Each proton-proton collision  
has many parton-parton interactions  
→ underlying event



# How many?

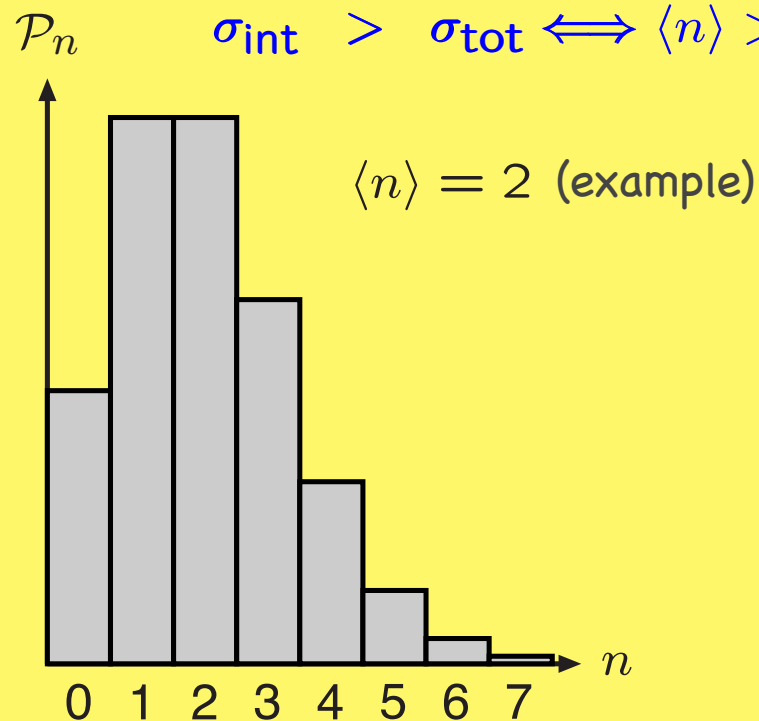
Naively  $\langle n_{2 \rightarrow 2}(p_{\perp \min}) \rangle = \frac{\sigma_{2 \rightarrow 2}(p_{\perp \min})}{\sigma_{\text{tot}}}$

Interactions independent (naive factorization)  $\rightarrow$  Poisson

$$\sigma_{\text{tot}} = \sum_{n=0}^{\infty} \sigma_n$$

$$\sigma_{\text{int}} = \sum_{n=0}^{\infty} n \sigma_n$$

$$\sigma_{\text{int}} > \sigma_{\text{tot}} \iff \langle n \rangle > 1$$



$$\mathcal{P}_n = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

## Real Life

Momentum conservation  
suppresses high- $n$  tail  
+ physical correlations  
 $\rightarrow$  not simple product



# Naive Factorization

Often used for simplicity

(i.e., assuming corrections are small / suppressed)

*CDF Collaboration, Phys. Rev. Lett. 79 (1997) 584*

## Measurement of Double Parton Scattering in $\bar{p}p$ Collisions at $\sqrt{s} = 1.8$ TeV

The double parton scattering (DP) process [1], in which two parton-parton hard scatterings take place within one  $\bar{p}p$  collision, can provide information on both the distribution of partons within the proton and on possible parton-parton correlations, topics difficult to address within the framework of perturbative QCD. The cross section for DP comprised of scatterings  $A$  and  $B$  is written

$$\sigma_{\text{DP}} \equiv \frac{\sigma_A \sigma_B}{\sigma_{\text{eff}}}, \quad (1)$$

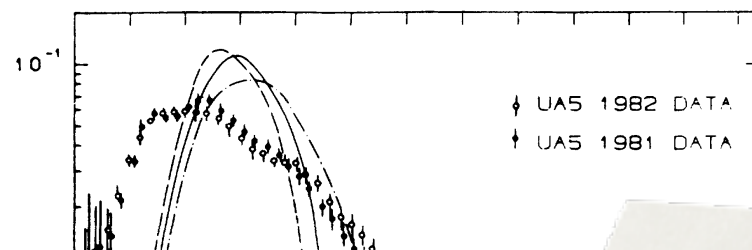
with a process-independent parameter  $\sigma_{\text{eff}}$  [2–5]. This expression assumes that the number of parton-parton interactions per collision is distributed according to Poisson statistics [6], and that the two scatterings are distinguishable [7]. Previous DP measurements have come

$\sigma_{\text{eff}} \approx$  “first moment” of  
multiple parton  
interaction distributions  
First rough  
characterization of MPI

But careful,  $\sigma_{\text{eff}}$  not  
valid / meaningful  
beyond factorized  
approximation!

Always report  
physical observables  
together with  
extracted quantities

# MPI and Min-Bias



without multiple interactions

Do not be scared of the failure of physical models  
Usually points to more interesting physics

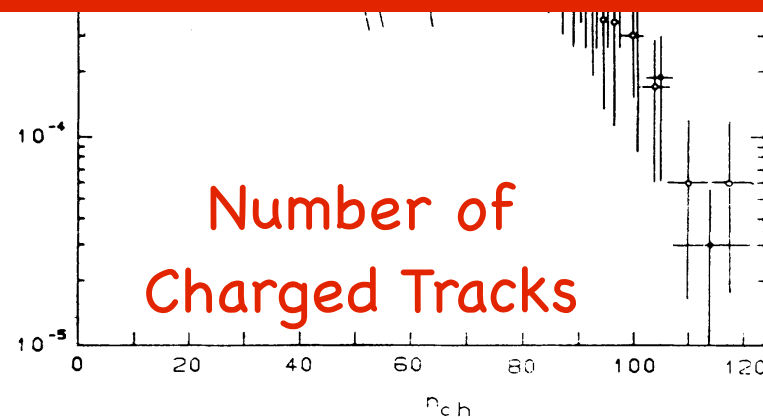


FIG. 3. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs simple models: dashed low  $p_T$  only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.

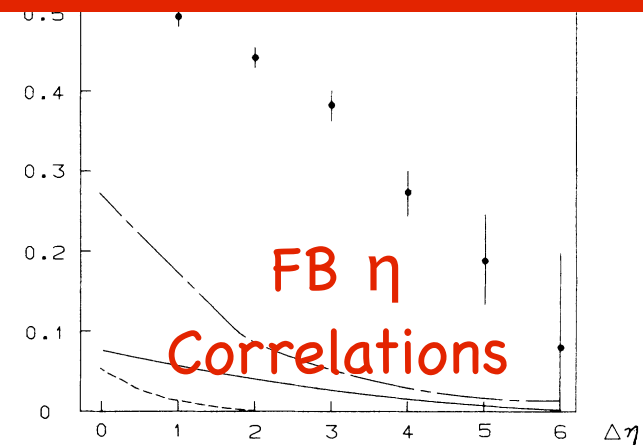


FIG. 4. Forward-backward multiplicity correlation at 540 GeV, UA5 results (Ref. 33) vs simple models; the latter models with notation as in Fig. 3.

Sjöstrand & v. Zijl, Phys.Rev.D36(1987)2019



# MPI and Min-Bias

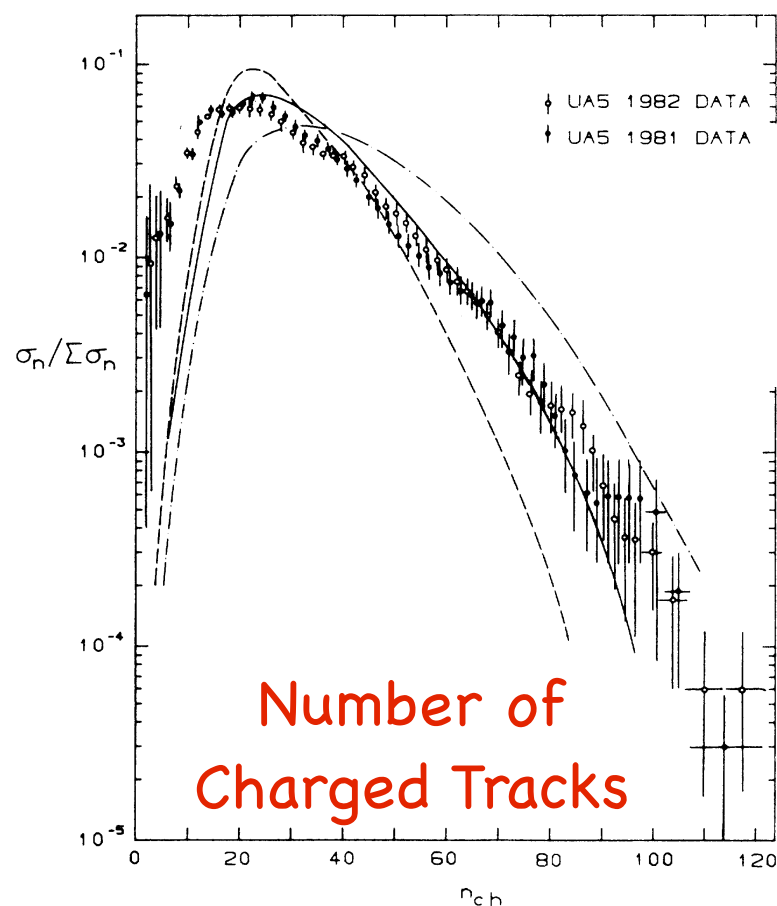


FIG. 5. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs impact-parameter-independent multiple-interaction model: dashed line,  $p_{Tmin}=2.0$  GeV; solid line,  $p_{Tmin}=1.6$  GeV; dashed-dotted line,  $p_{Tmin}=1.2$  GeV.

with multiple interactions

MPI also generate a  
“UE” in Min-Bias itself!

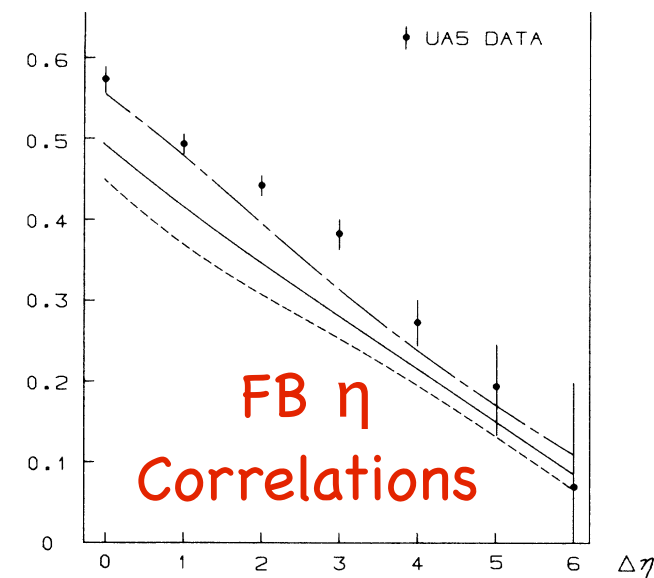
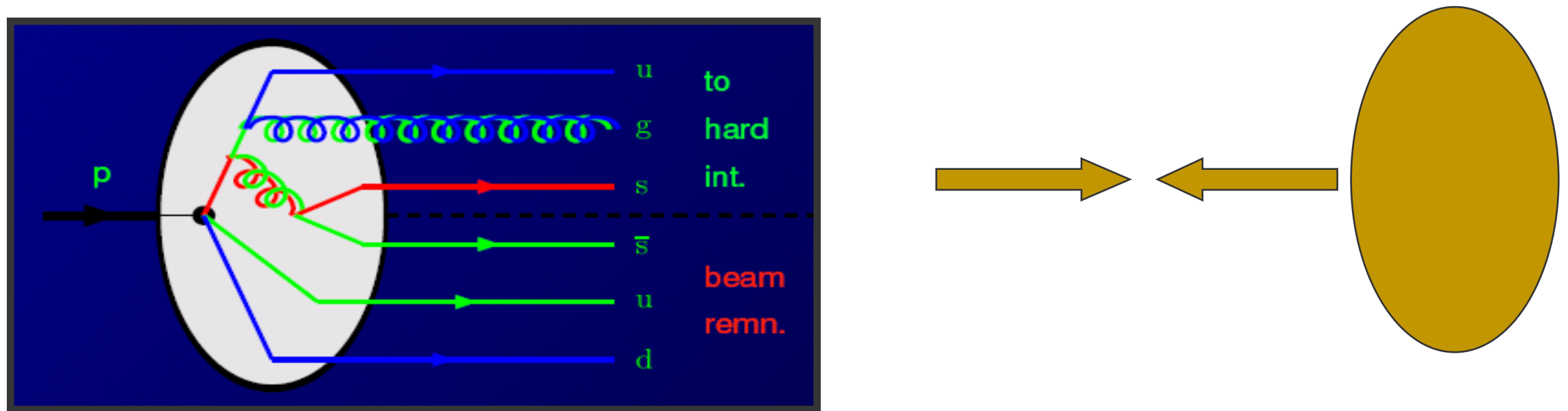


FIG. 6. Forward-backward multiplicity correlation at 540 GeV, UA5 results (Ref. 33) vs impact-parameter-independent multiple-interaction model; the latter with notation as in Fig. 5.

# Multi-Parton PDFs



How are the initiators and remnant partons correlated?

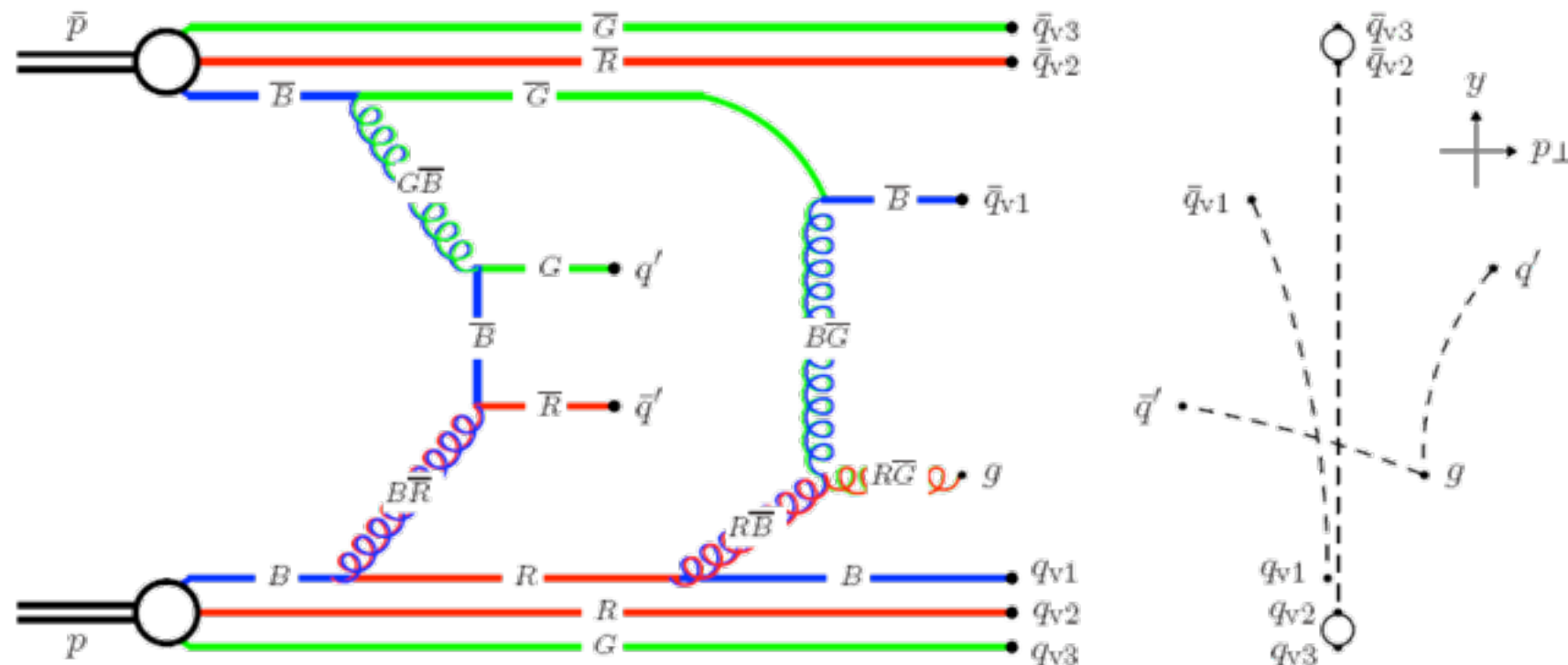


- in impact parameter?
- in flavour?
- in  $x$  (longitudinal momentum)?
- in  $k_T$  (transverse momentum)?
- in colour ( $\rightarrow$  string topologies!)
- What does the beam remnant look like?
- (How) are the showers correlated / intertwined?



# Colour and the UE

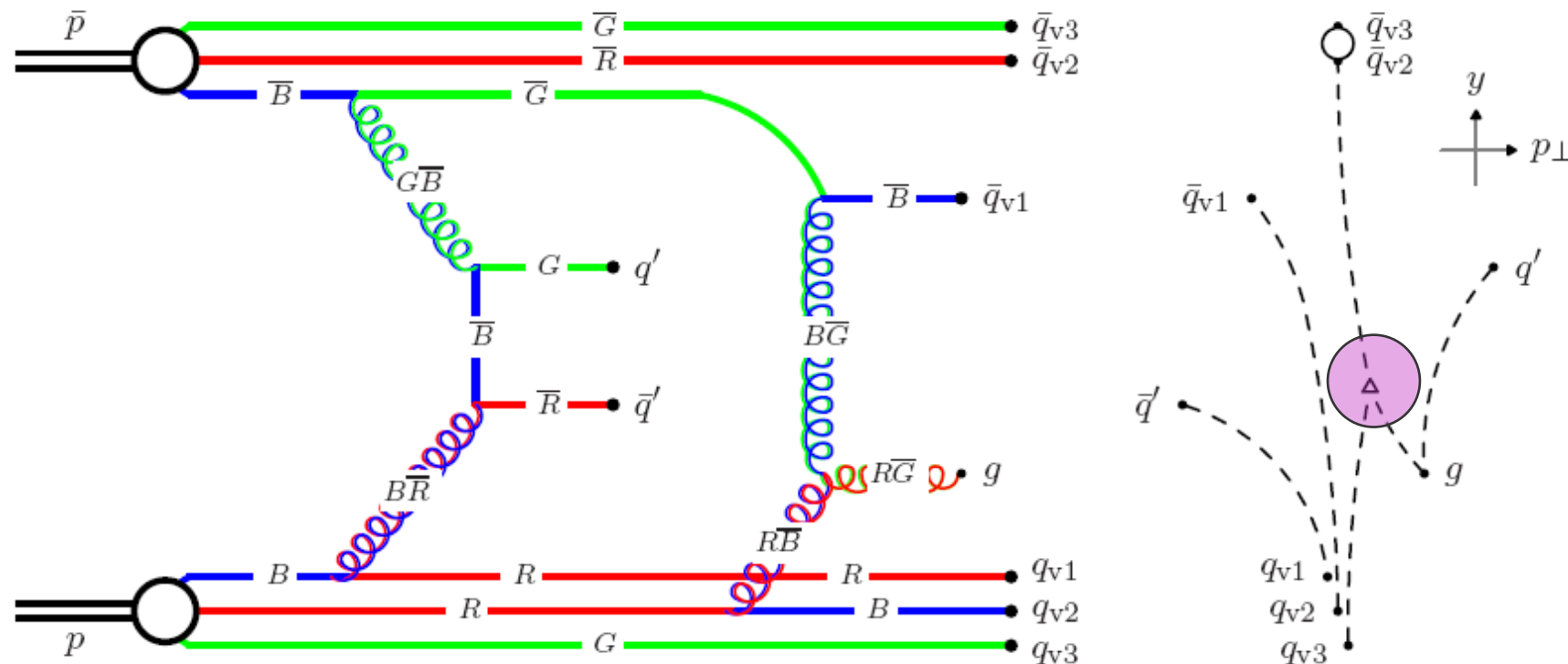
- The colour flow determines the hadronizing string topology
  - Each MPI, even when soft, is a color spark
  - Final distributions crucially depend on color space



# Colour and the UE

► The colour flow determines the hadronizing string topology

- Each MPI, even when soft, is a color spark
- Final distributions crucially depend on color space



Note: this just color **connections**, then there may be color **reconnections** too

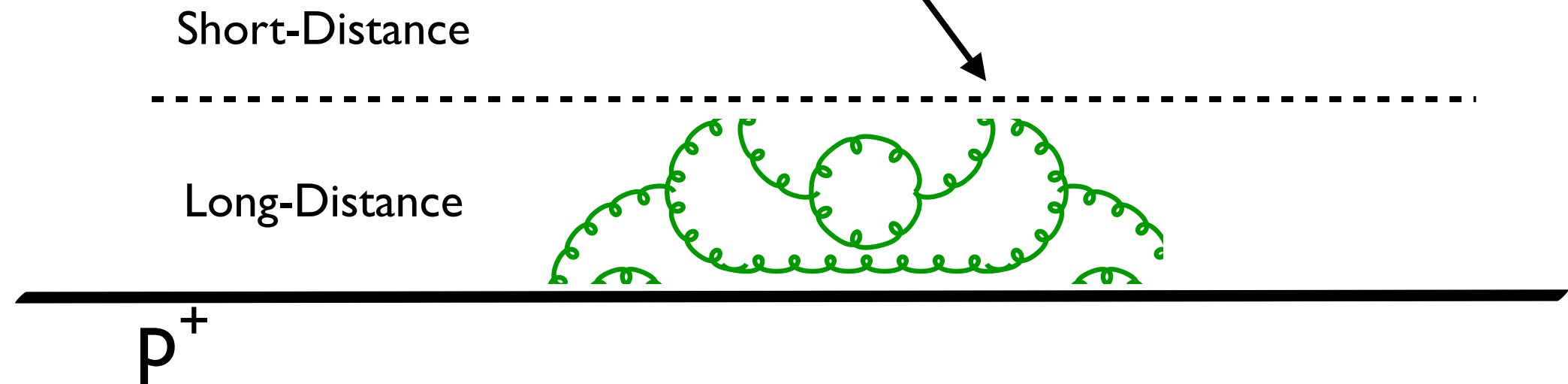


# (+ Diffraction)

“Intuitive picture”

Compare with  
normal PDFs

Hard Probe

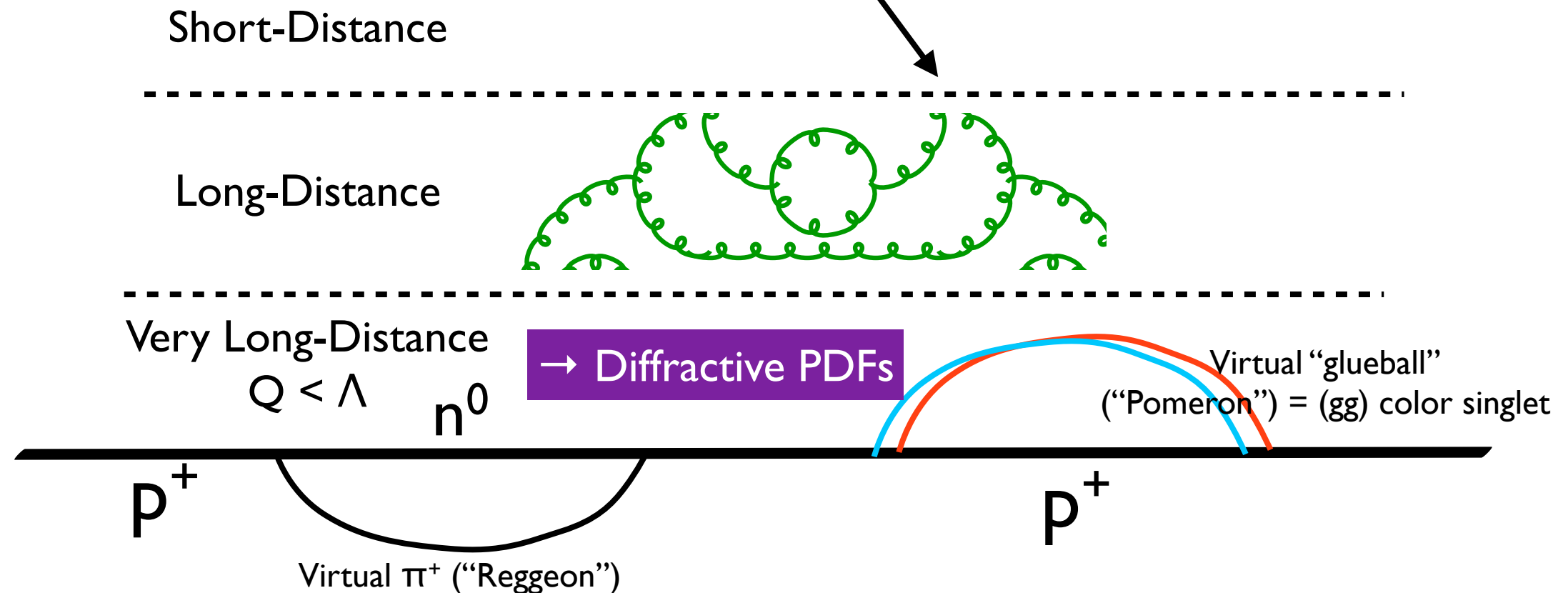


# (+ Diffraction)

“Intuitive picture”

Compare with  
normal PDFs

Hard Probe



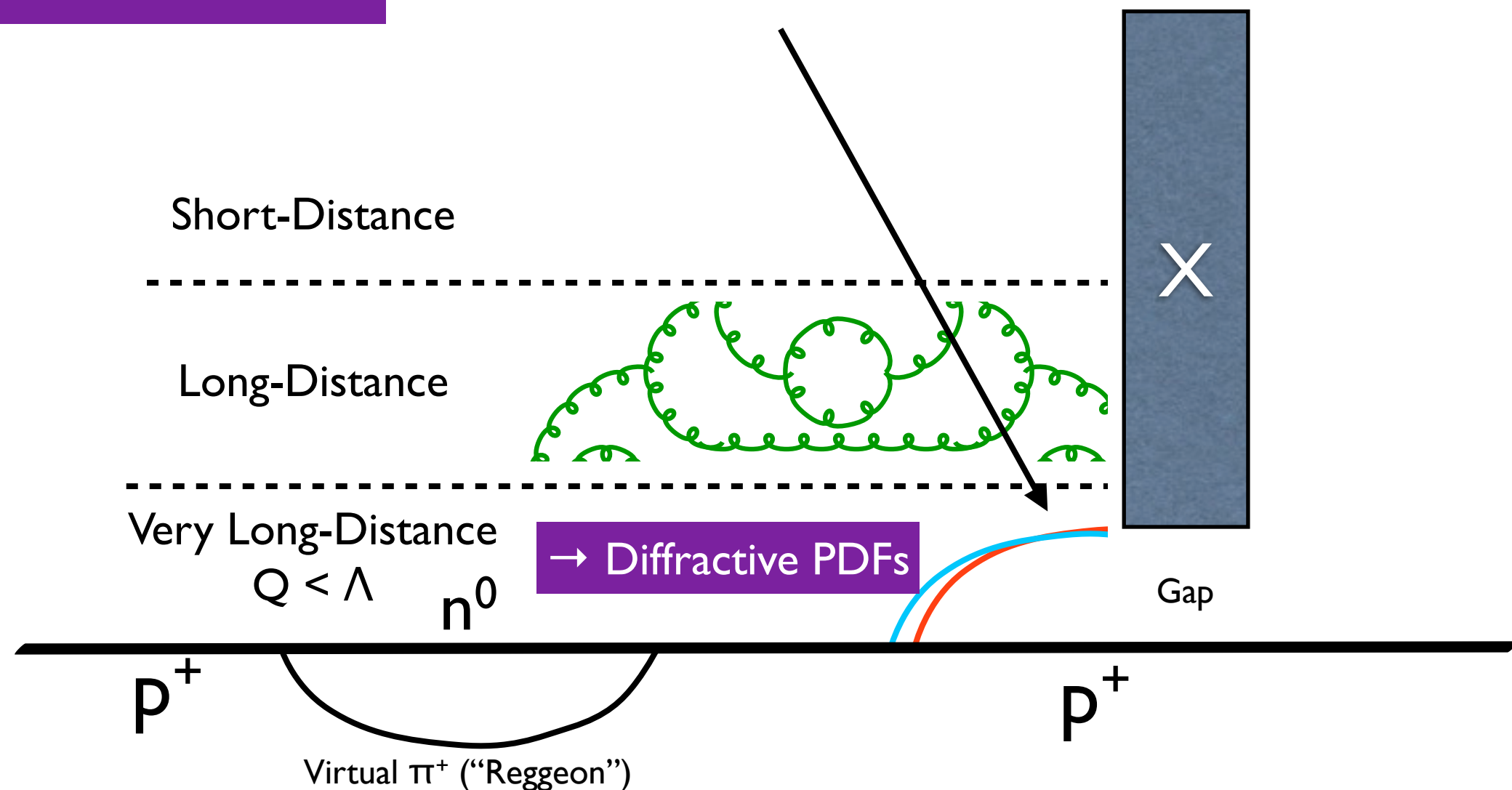


# (+ Diffraction)

“Intuitive picture”

Compare with  
normal PDFs

Hard Probe



# The Hard Tail

More about MC  
models in  
Lecture 4

Alright, but ...

If it's really **multi-PARTON** interactions – as opposed to just some additional soft gook – we should be able to see a **tail of HARD partonic scattering!**

Multiple (mini)jets

Already observed  
(E.g., AFS, CDF, D0)  $\rightarrow \sigma_{\text{eff}}$

Even Double Drell-Yan?

Will be searched for at LHC

# Infrared Summary

## Parton Densities

= Our beams!

Well constrained central fits at NLO and NNLO

Learning about precision issues: uncertainties, parametrization dependence, scheme dependence, mutually inconsistent data sets, ...

Learning about 'tuning'/optimization of LO sets

"Arbitrariness" from vice to virtue?

LO\*: allow (small) violations of momentum sum rule?

PDFs optimized for use with MC generators

Use approximate generator 'scheme' for evolution → formalize?

Still a developing field → developments yet to come!



# Infrared Summary

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## Fragmentation

Still an unsolved puzzle

→ Emergent degrees of freedom

Phenomenological models build on fundamental symmetries, perturbative limits, and lattice inputs

Much more sophisticated than simple fits

Still, probably unreasonable to ask for better than 10% precision on main IR quantities (e.g., number of tracks, proton/pion ratio, ...), and worse in tails.

LHC → important checks in situ (+ it's fun!)

# Underlying Event

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## Minimum-Bias

High-Statistics reference laboratory ('the LEP of hadron colliders')

Ideal for studies of non-pQCD properties

Including Fragmentation, diffraction, beam remnant blowup, ...

Again, 10% precision is probably the best we can do

Model power = simultaneous description of many observables

## Underlying Event

Pedestal effect: more active than minimum-bias

Dominating model: multiple parton interactions

Beware large fluctuations

+ Phenomenology → Theory?