



東京大学  
THE UNIVERSITY OF TOKYO

TODIAS  
東京大学国際高等研究所  
TODAI INSTITUTES FOR ADVANCED STUDY

KAVALI  
IPMU  
INSTITUTE FOR THE PHYSICS AND  
MATHEMATICS OF THE UNIVERSE

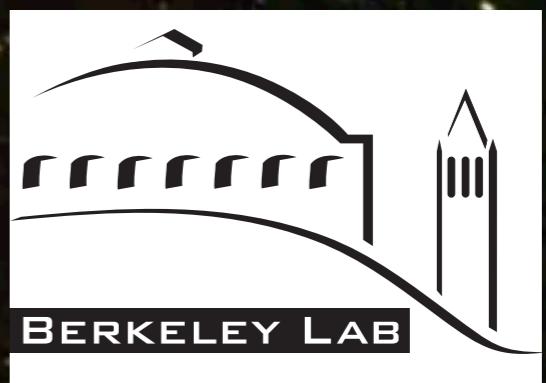
# Cosmology Dark Matter

AESHEP 2012年10月25日

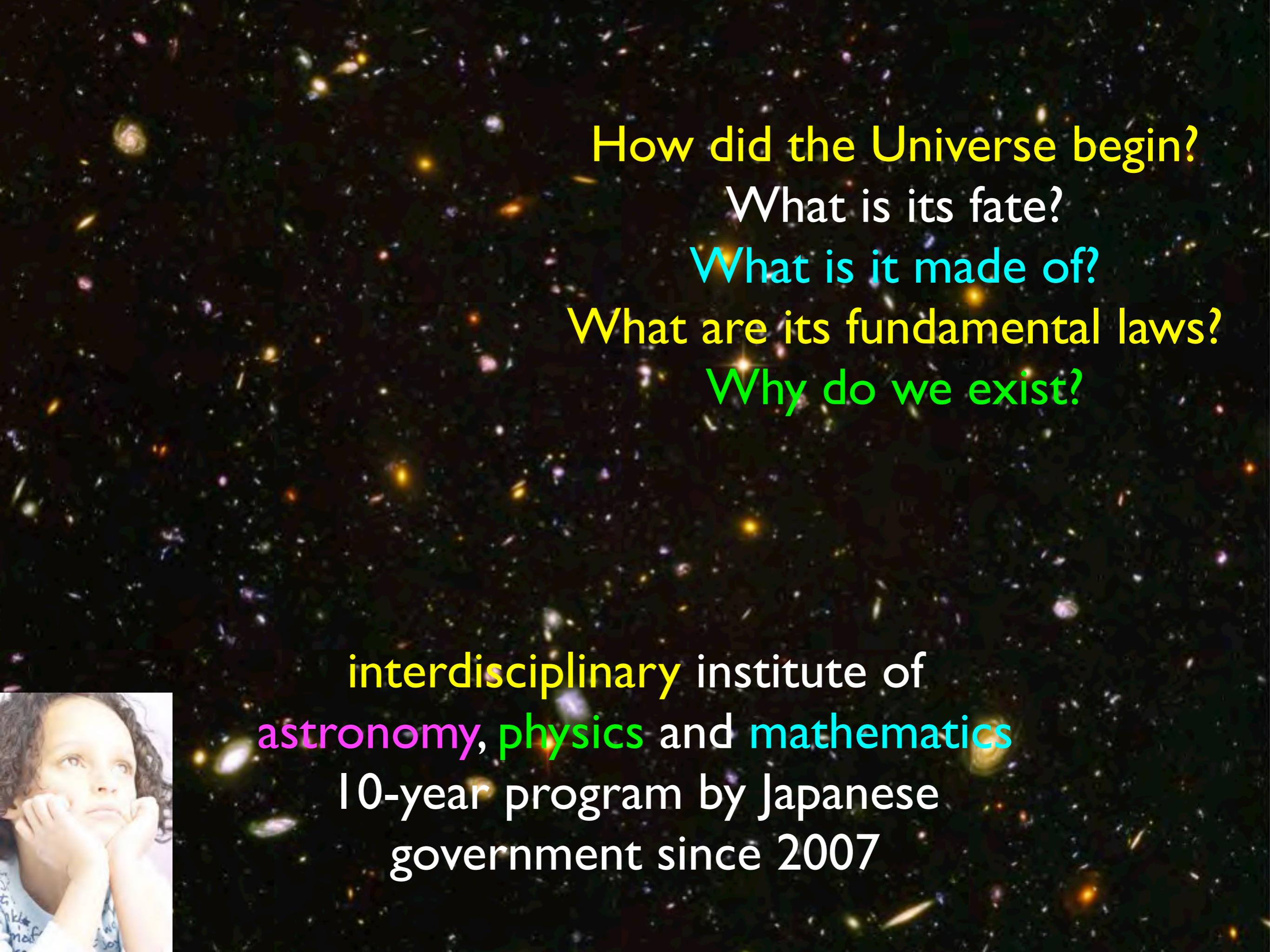
Kavli IPMU, University of Tokyo  
UC Berkeley, Lawrence Berkeley Laboratory  
Hitoshi Murayama



BERKELEY CENTER FOR THEORETICAL PHYSICS



BERKELEY LAB



How did the Universe begin?  
What is its fate?  
What is it made of?  
What are its fundamental laws?  
Why do we exist?



interdisciplinary institute of  
**astronomy, physics** and **mathematics**  
10-year program by Japanese  
government since 2007



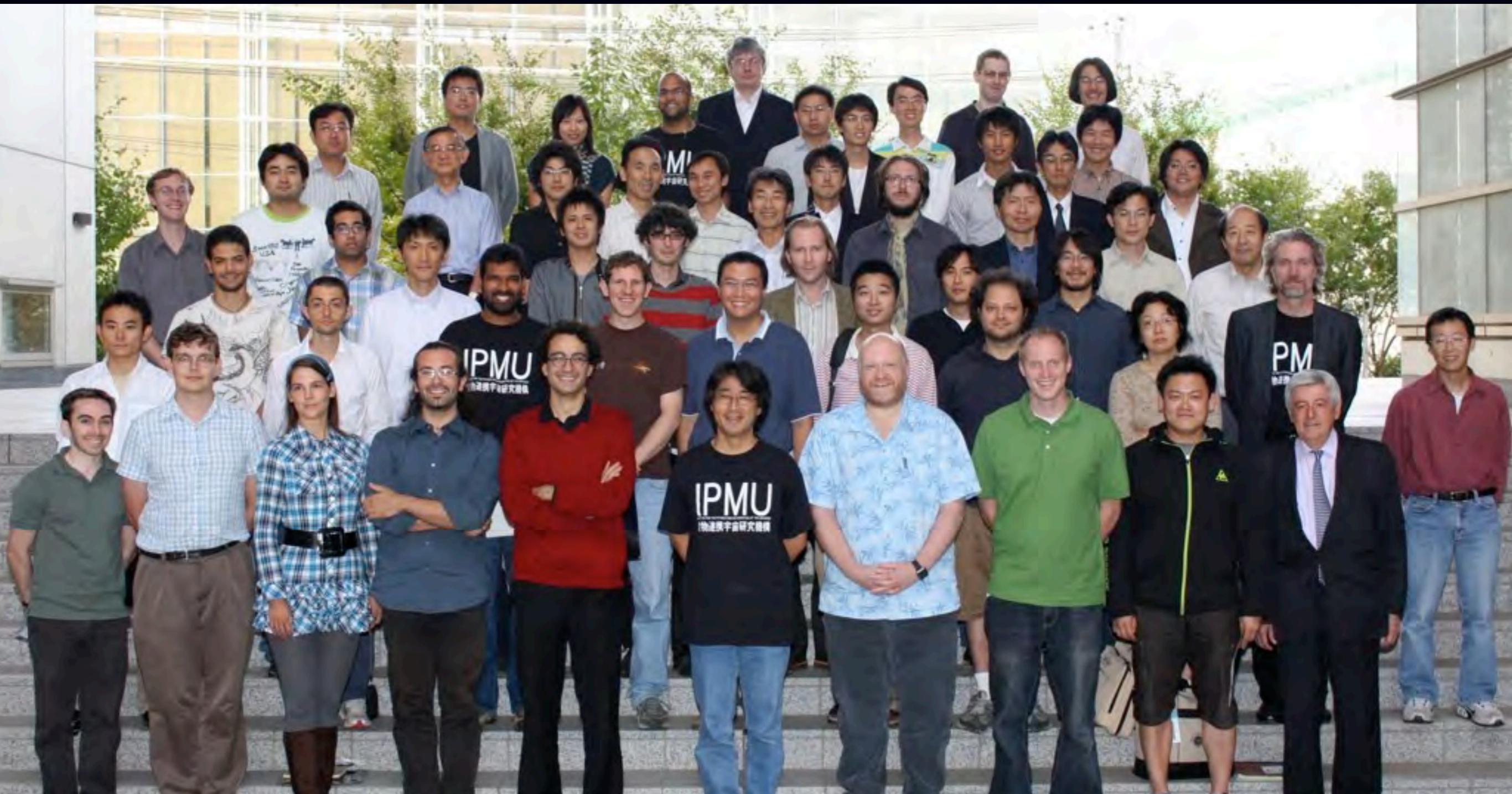
# Oct 2007

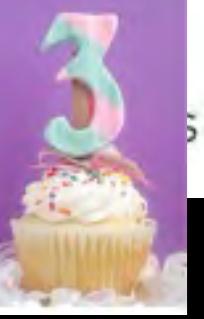
# Oct 2008



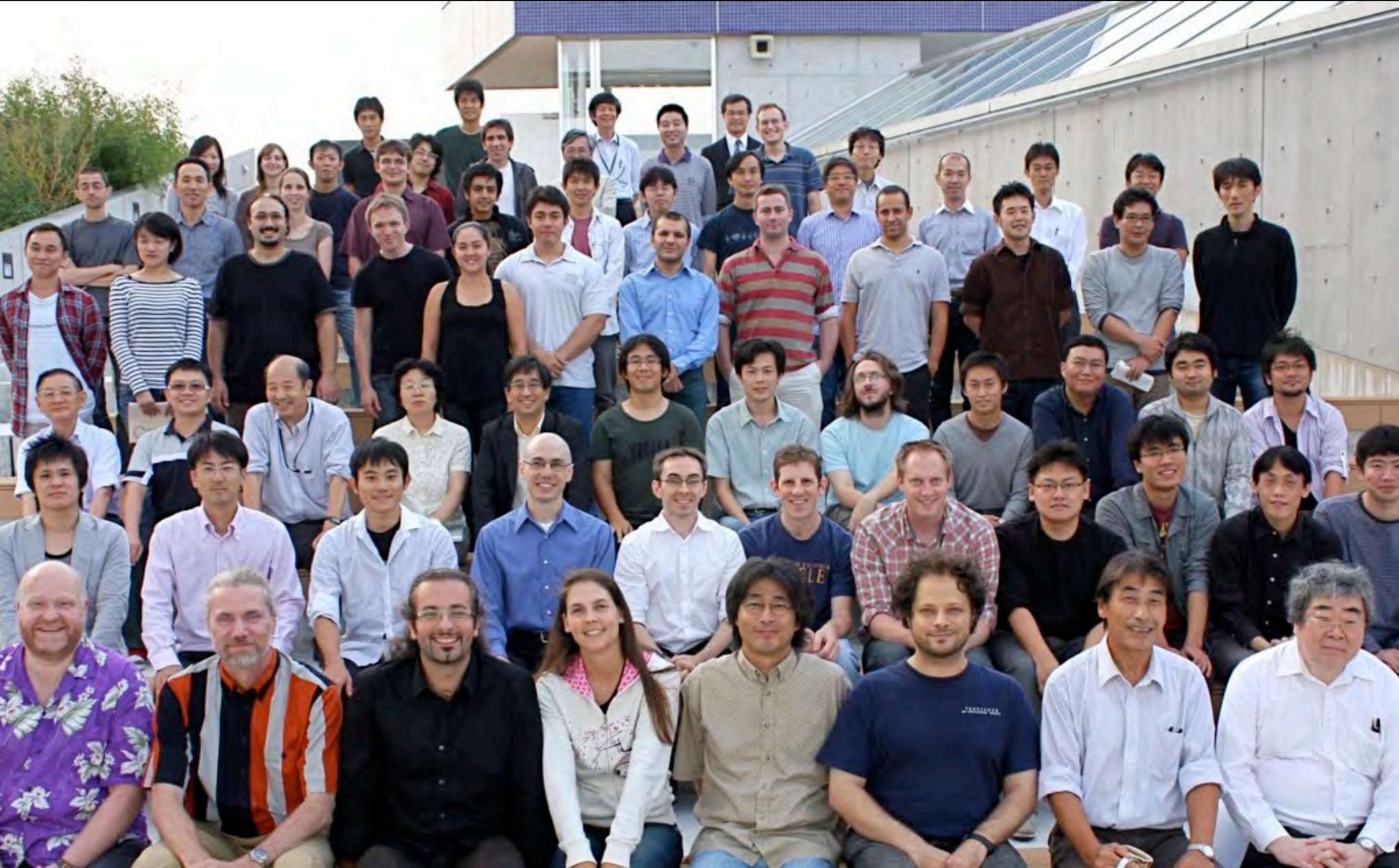


# Oct 2009





# Oct 2010



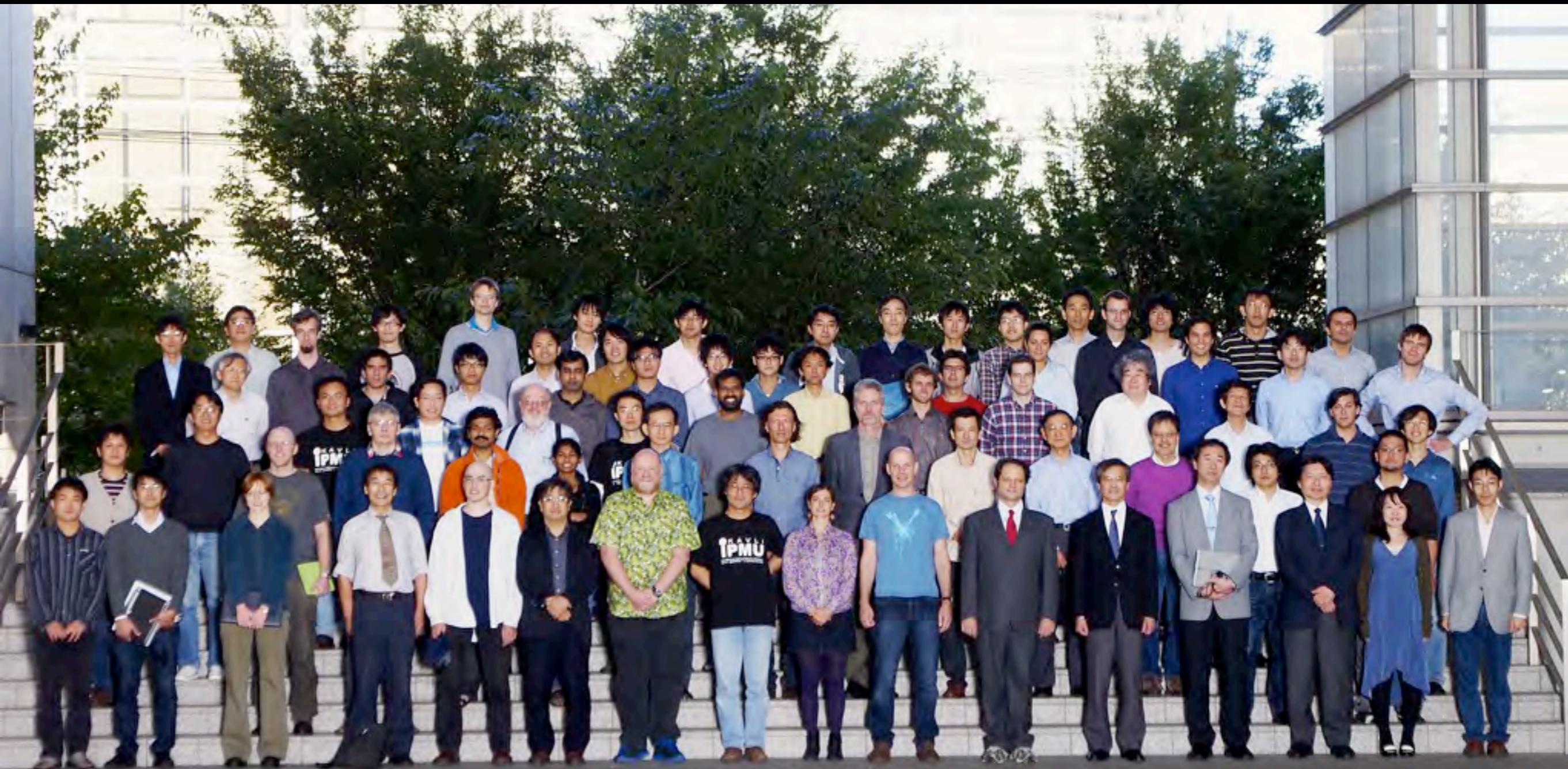


# Oct 2011

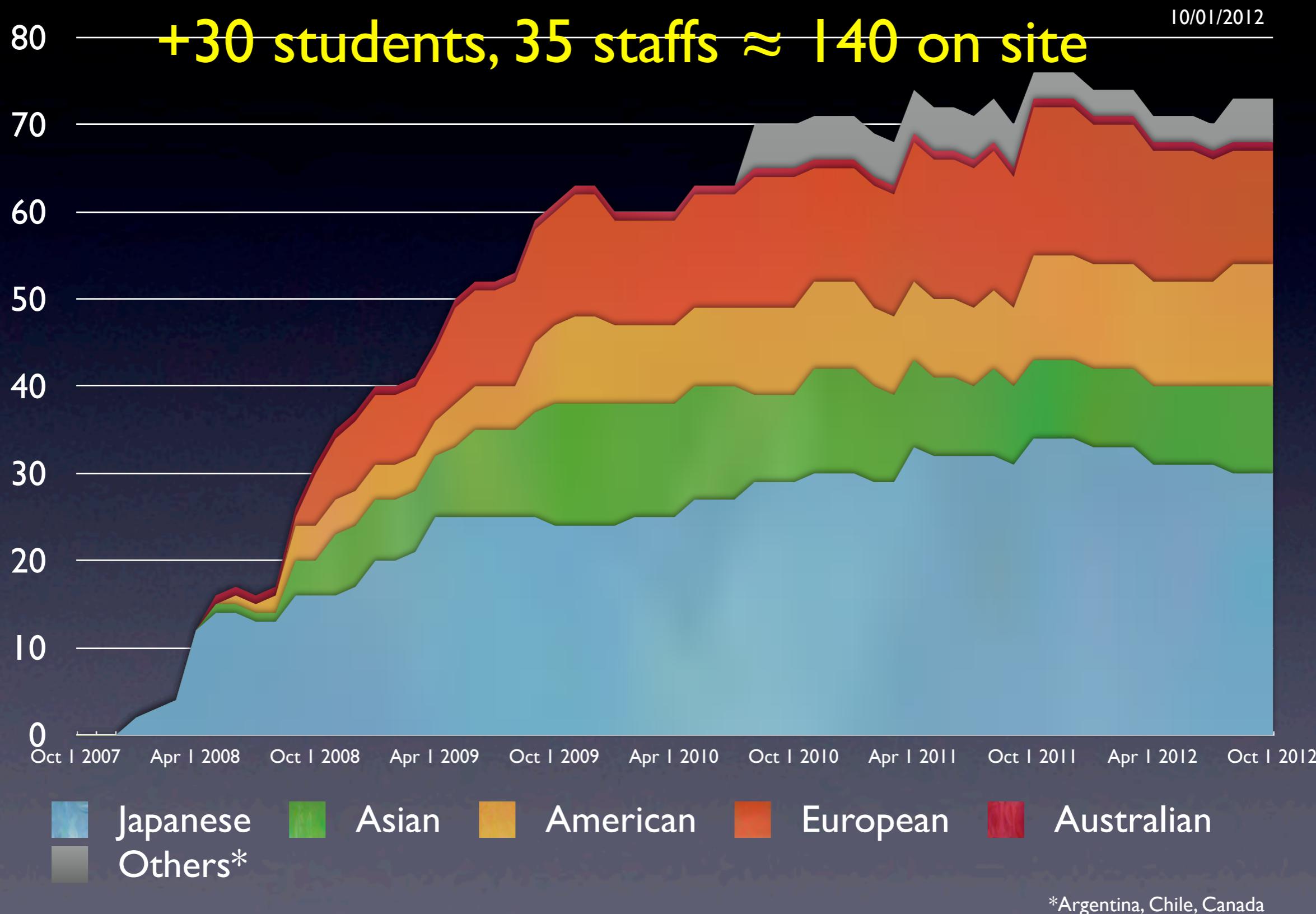




# Oct 2012



# Full-time Scientists paid by IPMU



brand-new building 2010



*“European town square”*

obelisk

*“L’Universo è scritto in  
lingua matematica”*

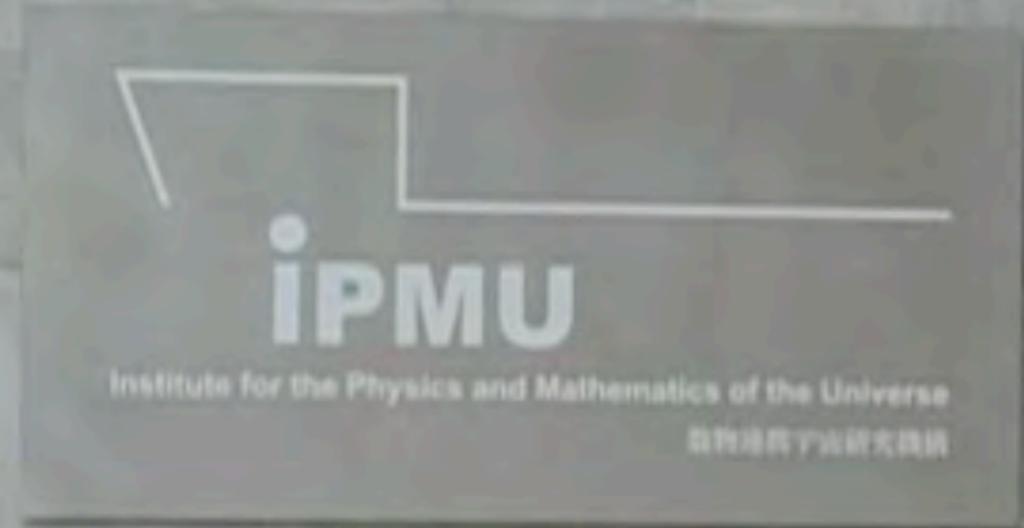


日本の頭脳

挑む天才たち

2007年設立  
東京大学国際高等研究所数物連携宇宙研究機構

Institute for the Physics and Mathematics of the Universe



Asahi TV

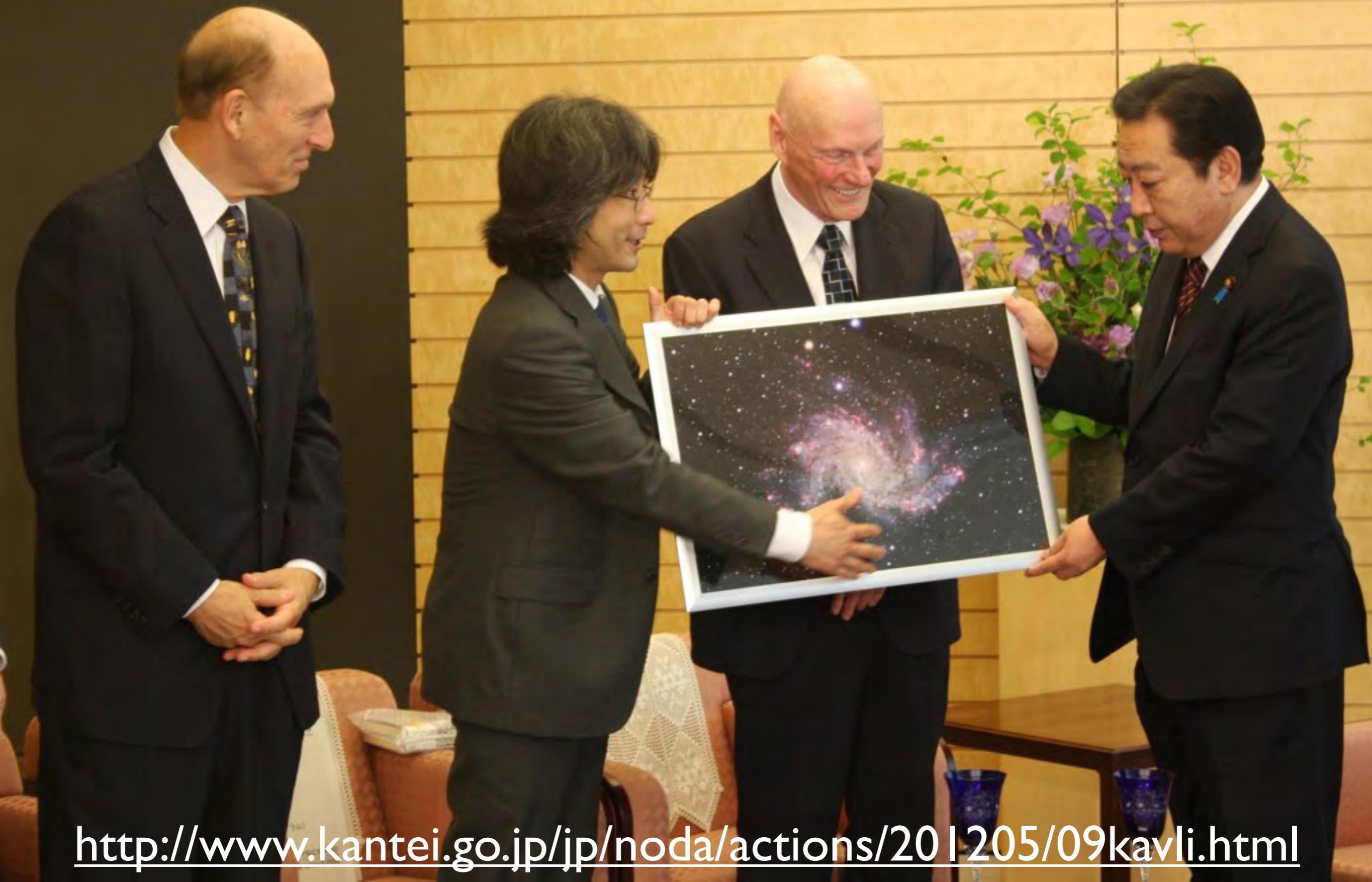


# THE KAVLI FOUNDATION



officially **Kavli IPMU** on April 1, 2012  
**First research institute in Japan** named  
after a donor, breaking new grounds

May 9, meeting with Prime Minister Noda



- *Basic research is very important, because it is a common resource shared by the whole humanity.*





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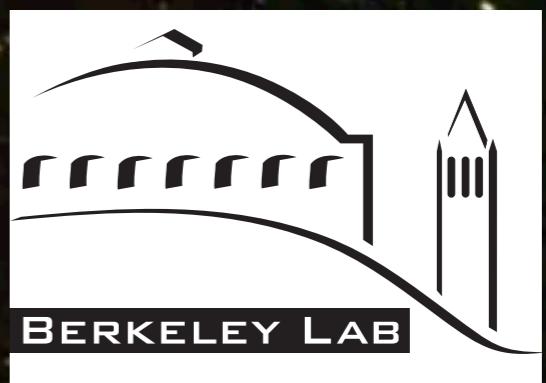
# Cosmology Dark Matter

AESHEP 2012年10月25日

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BERKELEY CENTER FOR THEORETICAL PHYSICS



BERKELEY LAB

*The whole  
Universe was  
smaller than  
an atom*

13.7Gyr

Big Bang

dark ages

particle soup

star

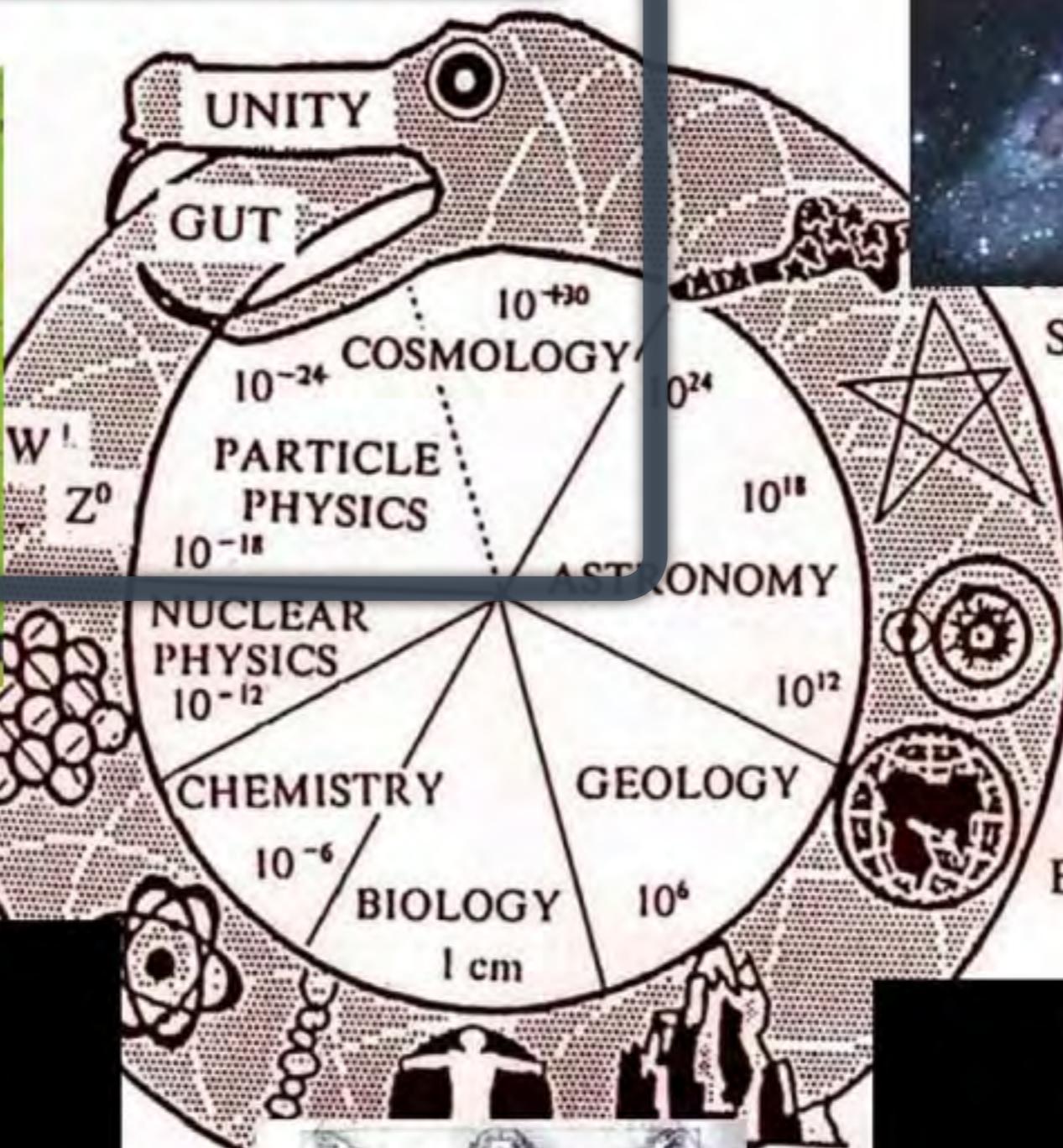
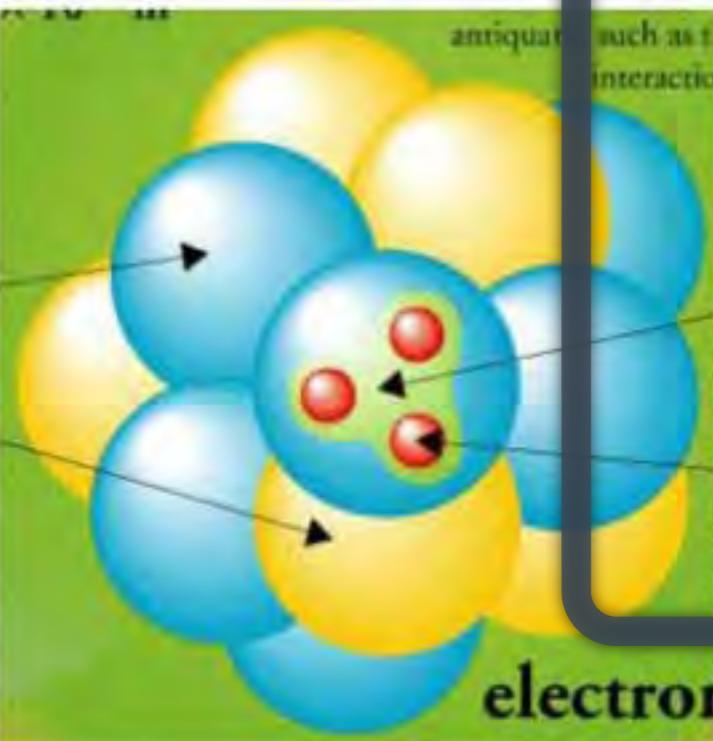
Earth



# ものの大きさ



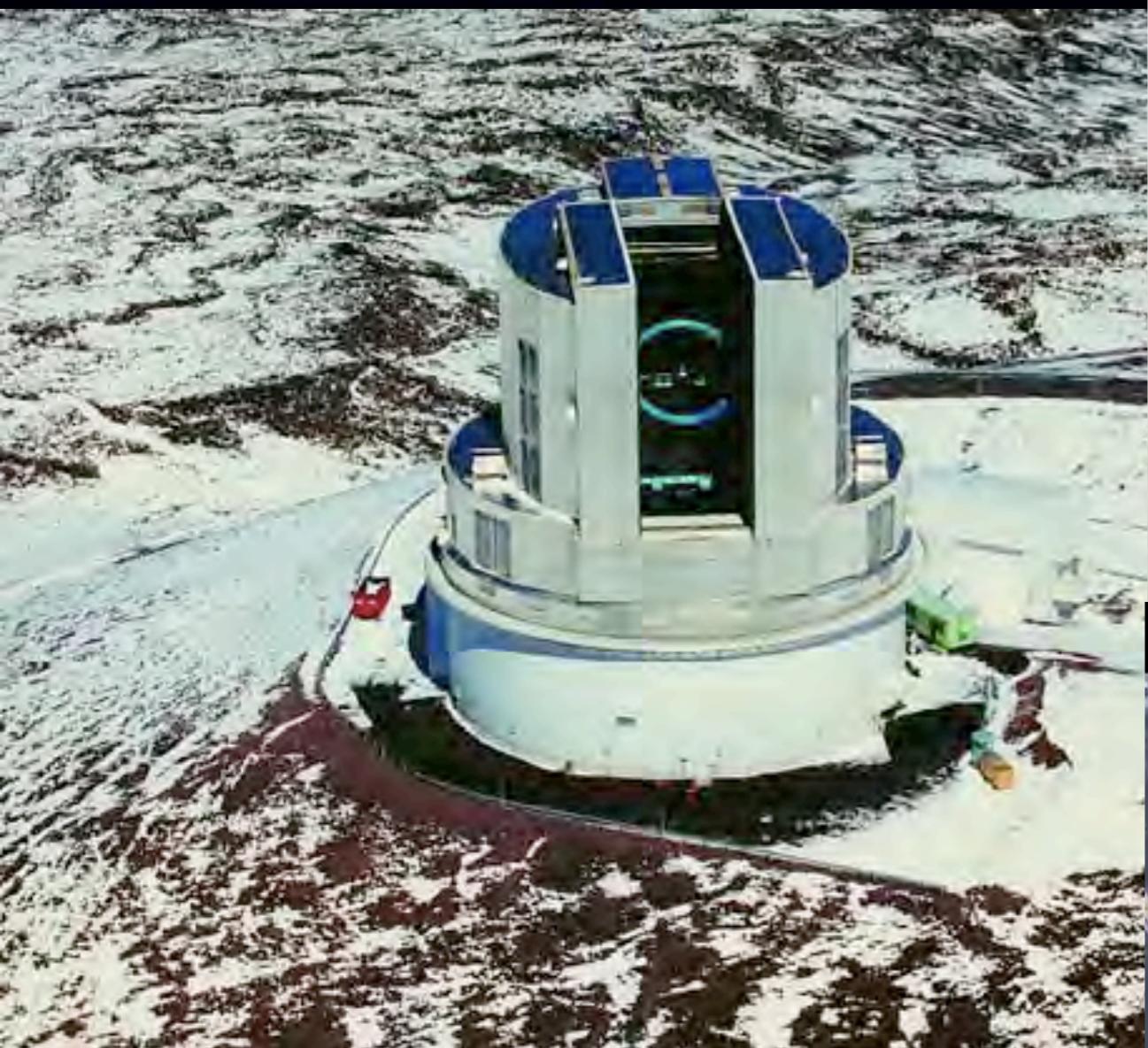
# ウロボロスの蛇



# タイムマシン



# タイムマシン



The background of the image is a dark, textured surface representing the void of space. It is densely populated with numerous small, glowing points of light of varying colors, primarily white, yellow, and orange, which represent distant stars and galaxies.

Solar System





2010年4月7日

375 km 上空

ももの皮

Sep 30, 2008

Kaguya

380,000km  
=1.3 light seconds

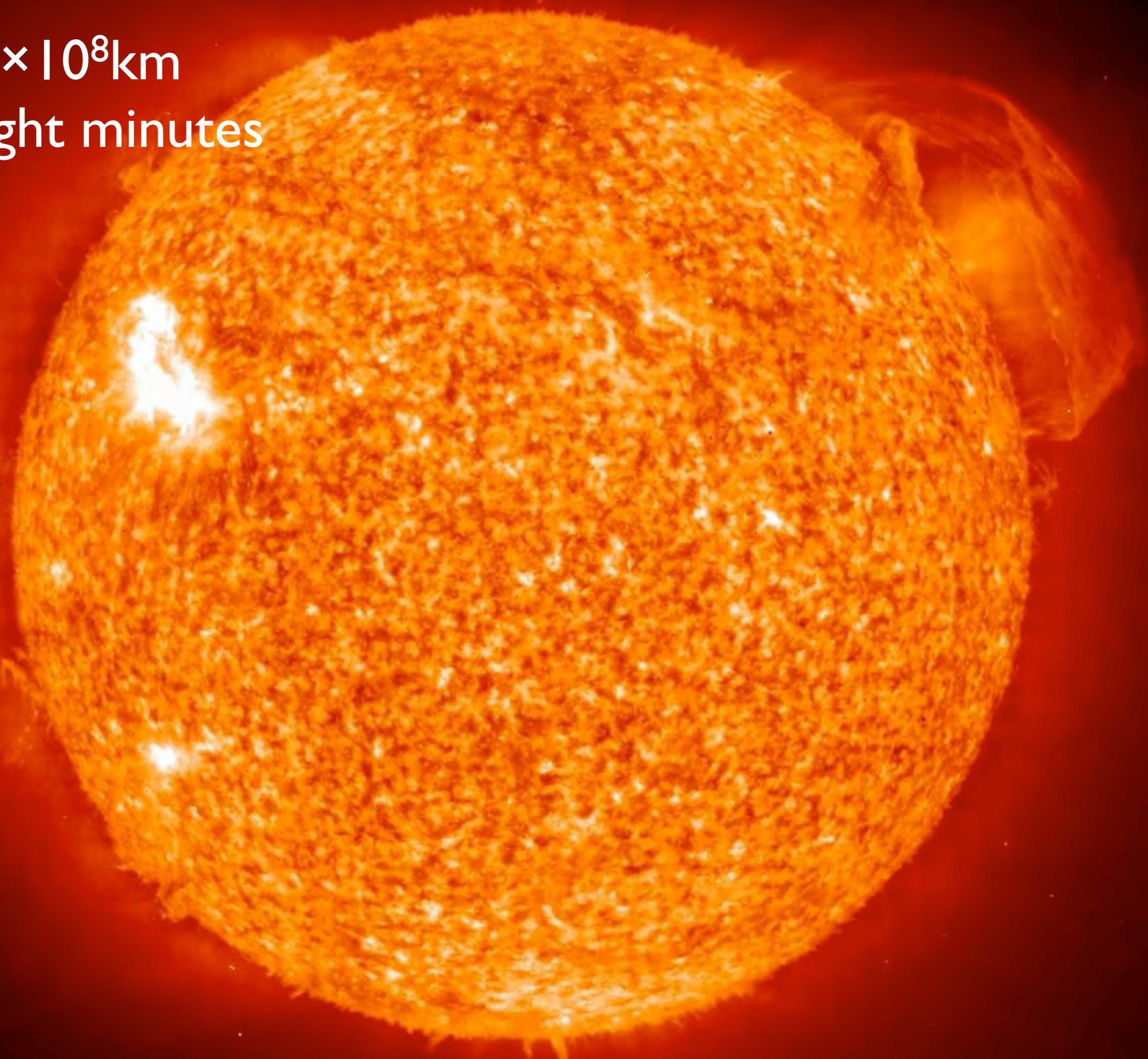
380,000km  
=1.3 light seconds



©JAXA/NHI

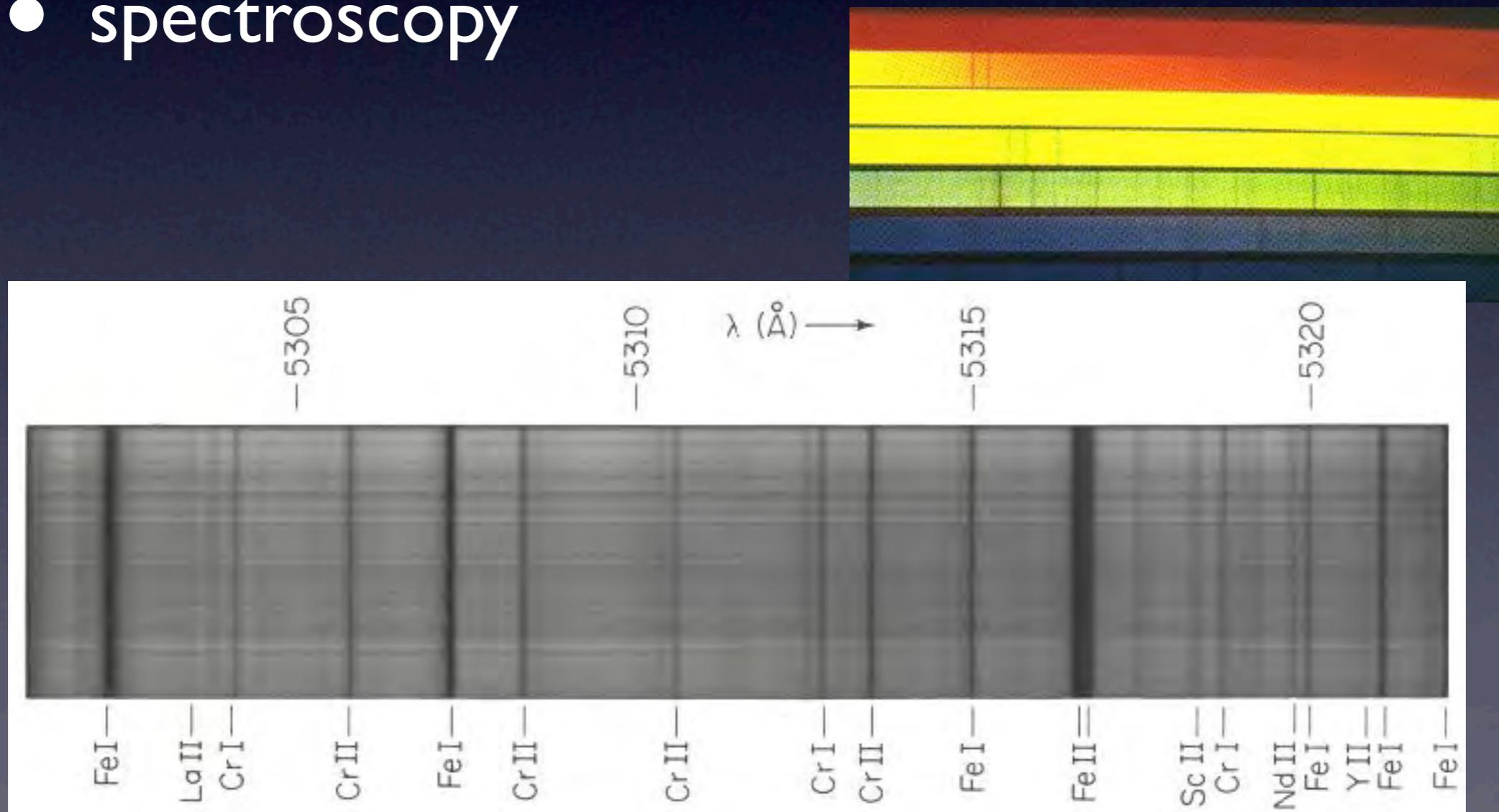
$1.5 \times 10^8 \text{ km}$

=8.3 light minutes



# Made of atoms

- everything around us is made of atoms
- stars are made of atoms, too
- spectroscopy



# PERIODIC TABLE OF THE ELEMENTS

<http://www.ktf-split.hr/periodni/cn/>

18 **VIIIA**

2 4.0026

**He**

HELIUM

GROUP

1 **IA**

1 1.0079

**H**

HYDROGEN

2 **IIA**

3 6.941

**Li**

LITHIUM

4 9.0122

**Be**

BERYLLIUM

RELATIVE ATOMIC MASS (1)

GROUP IUPAC

GROUP CAS

13

III A

ATOMIC NUMBER

5

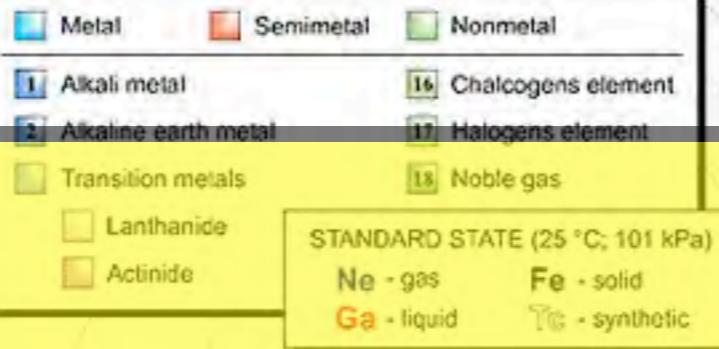
10.811

SYMBOL

**B**

BORON

ELEMENT NAME



13 **III A** 14 **IVA** 15 **VA** 16 **VIA** 17 **VIIA**

5 10.811 6 12.011 7 14.007 8 15.999 9 18.998 10 20.180

**B** **C** **N** **O** **F** **Ne**

BORON CARBON NITROGEN OXYGEN FLUORINE NEON

13 26.982 14 28.086 15 30.974 16 32.065 17 35.453 18 39.948

**Al** **Si** **P** **S** **Cl** **Ar**

ALUMINUM SILICON PHOSPHORUS SULPHUR CHLORINE ARGON

19 39.098 20 40.078 21 44.956 22 47.867 23 50.942 24 51.996 25 54.938 26 55.845 27 58.933 28 58.693 29 63.546 30 65.39 31 69.723 32 72.64 33 74.922 34 78.96 35 79.904 36 83.80

**K** **Ca** **Sc** **Ti** **V** **Cr** **Mn** **Fe** **Co** **Ni** **Cu** **Zn** **Ga** **Ge** **As** **Se** **Br** **Kr**

POTASSIUM CALCIUM SCANDIUM TITANIUM VANADIUM CHROMIUM MANGANESE IRON COBALT NICKEL COPPER ZINC GALLIUM GERMANIUM ARSENIC SELENIUM BROMINE KRYPTON

37 85.468 38 87.62 39 88.906 40 91.224 41 92.906 42 95.94 43 (96) 44 101.07 45 102.91 46 106.42 47 107.87 48 112.41 49 114.82 50 118.71 51 121.76 52 127.60 53 126.90 54 131.29

**Rb** **Sr** **Y** **Zr** **Nb** **Mo** **Tc** **Ru** **Rh** **Pd** **Ag** **Cd** **In** **Sn** **Sb** **Te** **I** **Xe**

RUBIDIUM STRONTIUM YTTRIUM ZIRCONIUM NIOBIUM MOLOBOLEMUM TECHNETIUM RUTHENIUM RHODIUM PALLADIUM SILVER CADMIUM INDIUM TIN ANTIMONY TELLURIUM IODINE XENON

55 132.91 56 137.33 57-71 La-Lu Lanthanide Hf Ta W Re Os Ir Pt Au Hg Tl Pb Bi Po At Rn

CAESIUM BARIUM Lanthanide HAFNIUM TANTALUM TUNGSTEN RHENIUM OSMIUM IRIDIUM PLATINUM GOLD MERCURY THALLIUM LEAD BISMUTH POLONIUM ASTATINE RADON

87 (223) 88 (226) 89-103 Ac-Lr Actinide Rf Db Sg Bh Hs Mt Uum Uuu Uub Uuq UNUNQUADIUM

114 (289) UNUNQUADIUM

metals

## LANTHANIDE

Copyright © 1998-2003 EniG ([eni@ktf-split.hr](mailto:eni@ktf-split.hr))

57 138.91 58 140.12 59 140.91 60 144.24 61 (145) 62 150.36 63 151.96 64 157.25 65 158.93 66 162.50 67 164.93 68 167.26 69 168.93 70 173.04 71 174.97

**La** **Ce** **Pr** **Nd** **Pm** **Sm** **Eu** **Gd** **Tb** **Dy** **Ho** **Er** **Tm** **Yb** **Lu**

LANTHANUM CERIUM PRASEOYMIUM NEODYMIUM PROMETHIUM SAMARIUM EUROPIUM GADOLINIUM TERBIUM DYSPROSIUM HOLMIUM ERBIUM THULIUM YTTERBIUM LUTETIUM

## ACTINIDE

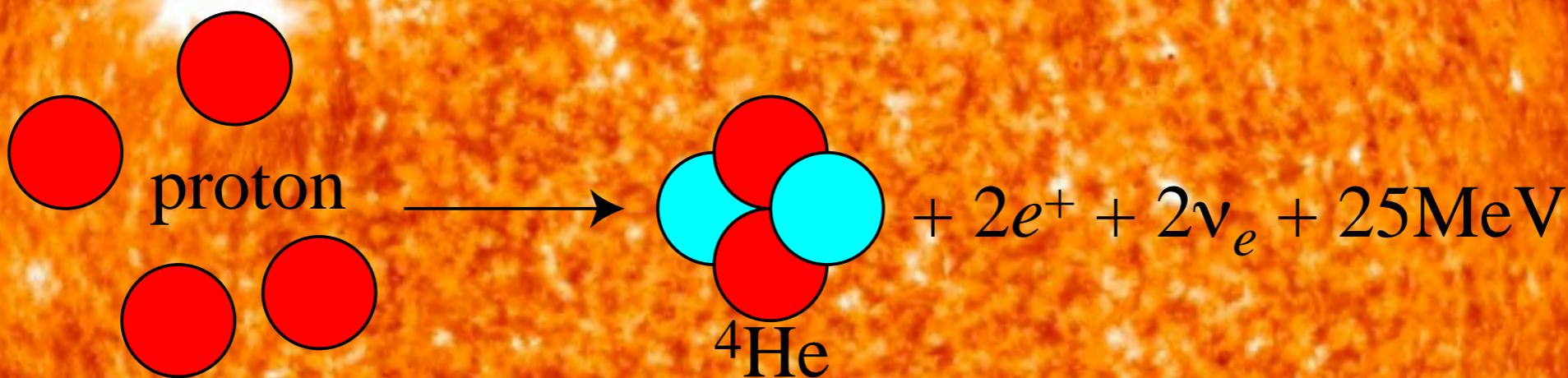
89 (227) 90 232.04 91 231.04 92 238.03 93 (237) 94 (244) 95 (243) 96 (247) 97 (247) 98 (251) 99 (252) 100 (257) 101 (258) 102 (259) 103 (262)

**Ac** **Th** **Pa** **U** **Np** **Pu** **Am** **Cm** **Bk** **Cf** **Es** **Fm** **Md** **No** **Lr**

ACTINIUM THORIUM PROTACTINIUM URANIUM NEPTUNIUM PLUTONIUM AMERICIUM CURIUM BERKELIUM CALIFORNIUM EINSTEINIUM FERMIIUM MENDELEVIUM NOBELIUM LAWRENCIUM

$1.5 \times 10^8 \text{ km}$

=8.3 light minutes

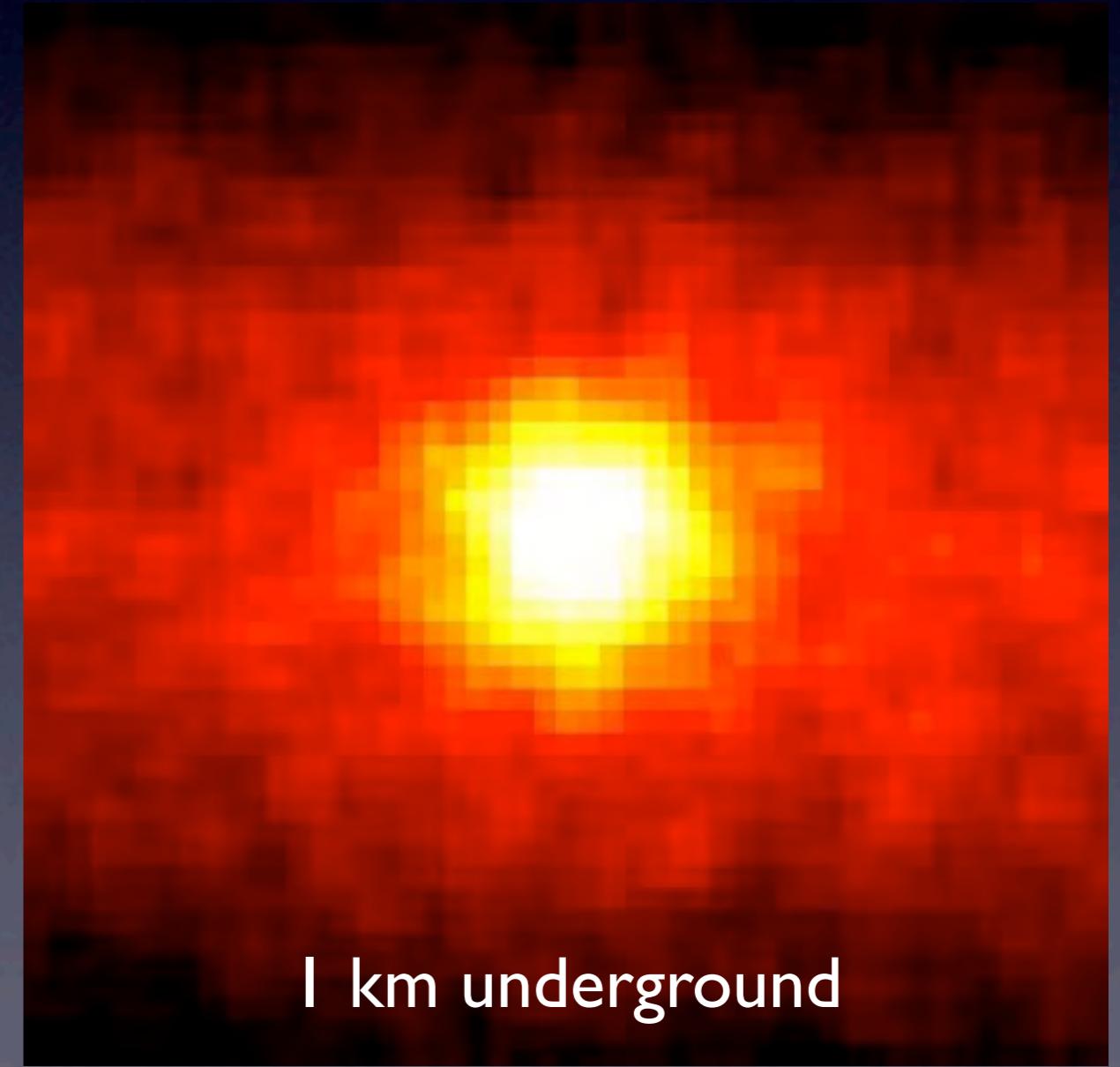
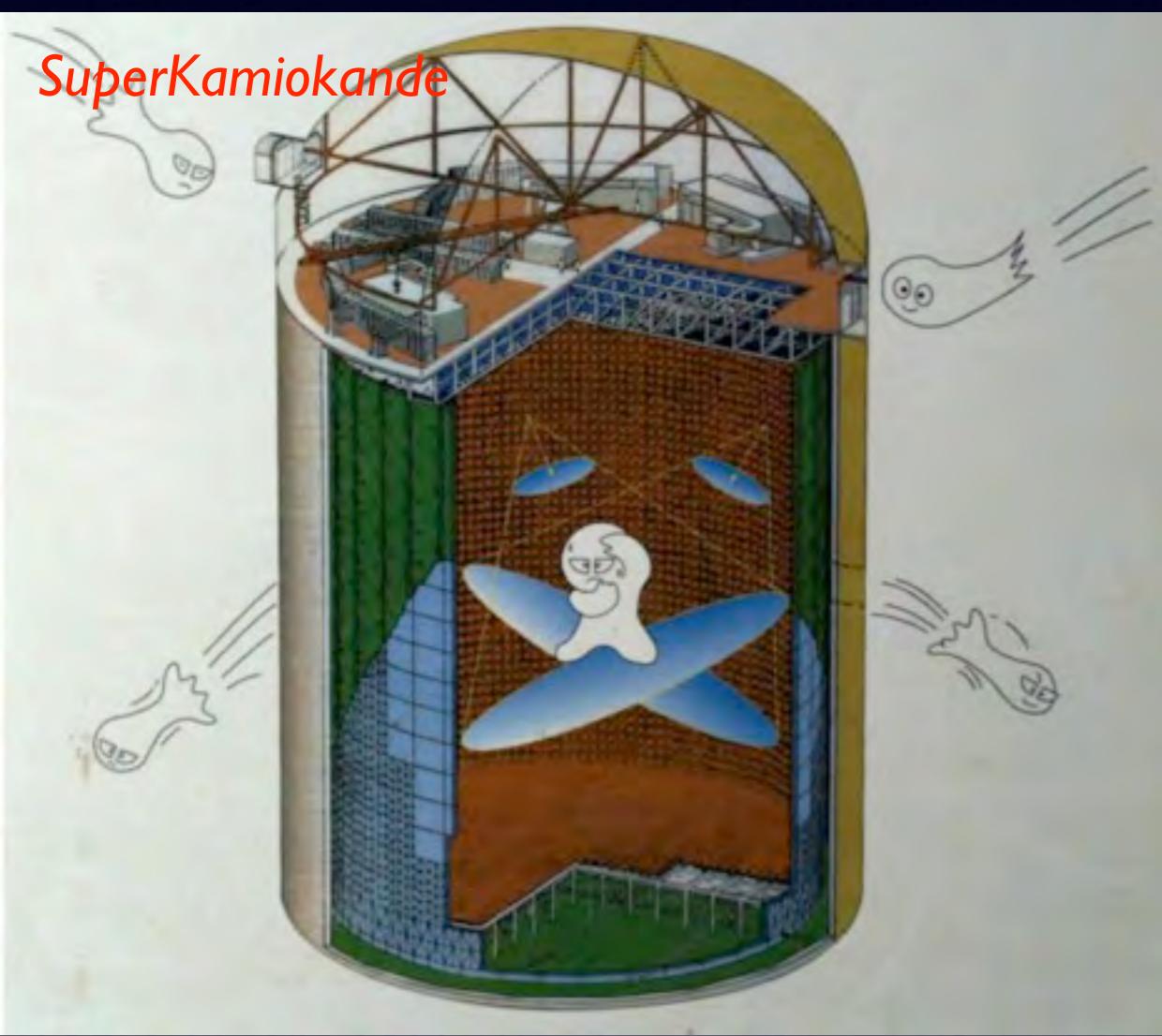
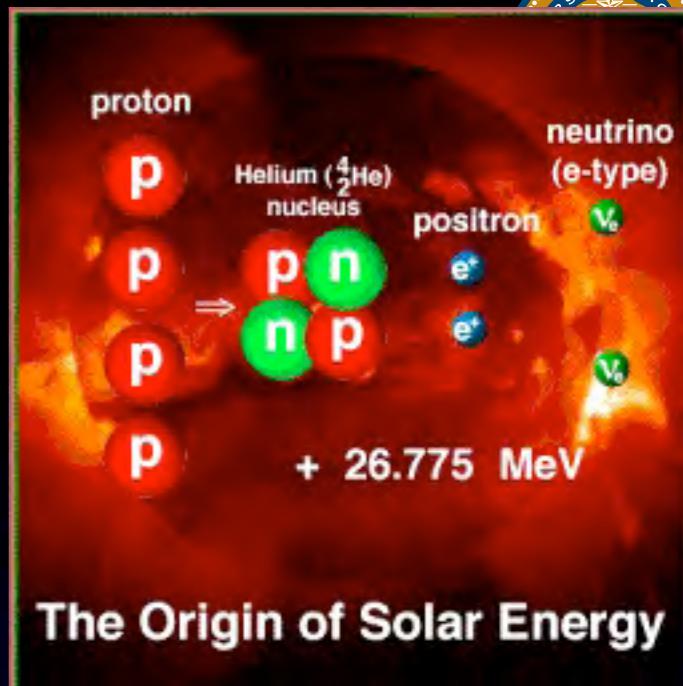


$$E=mc^2$$

5 Mt lighter every second  
turning mass to energy  
burning hydrogen

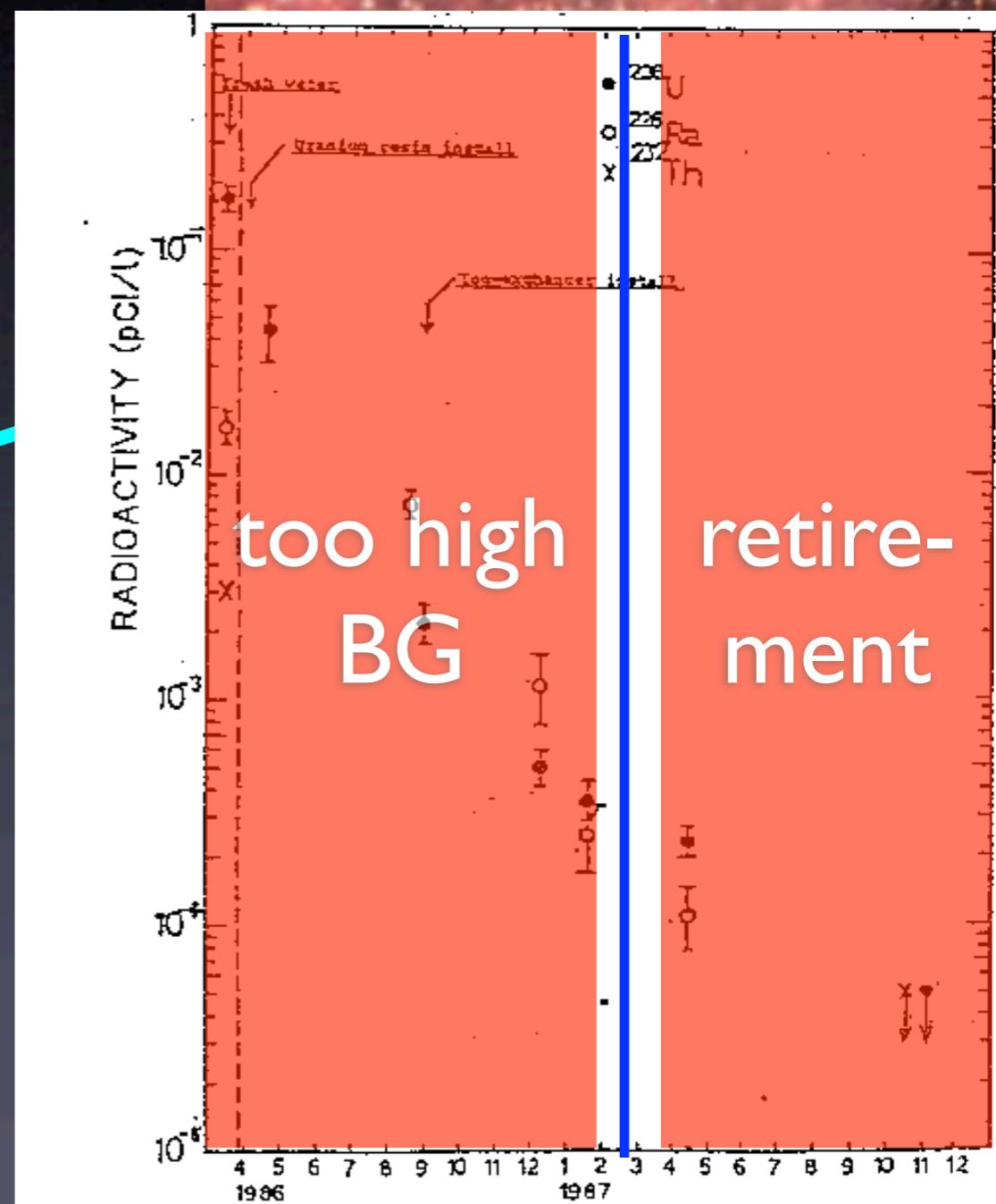
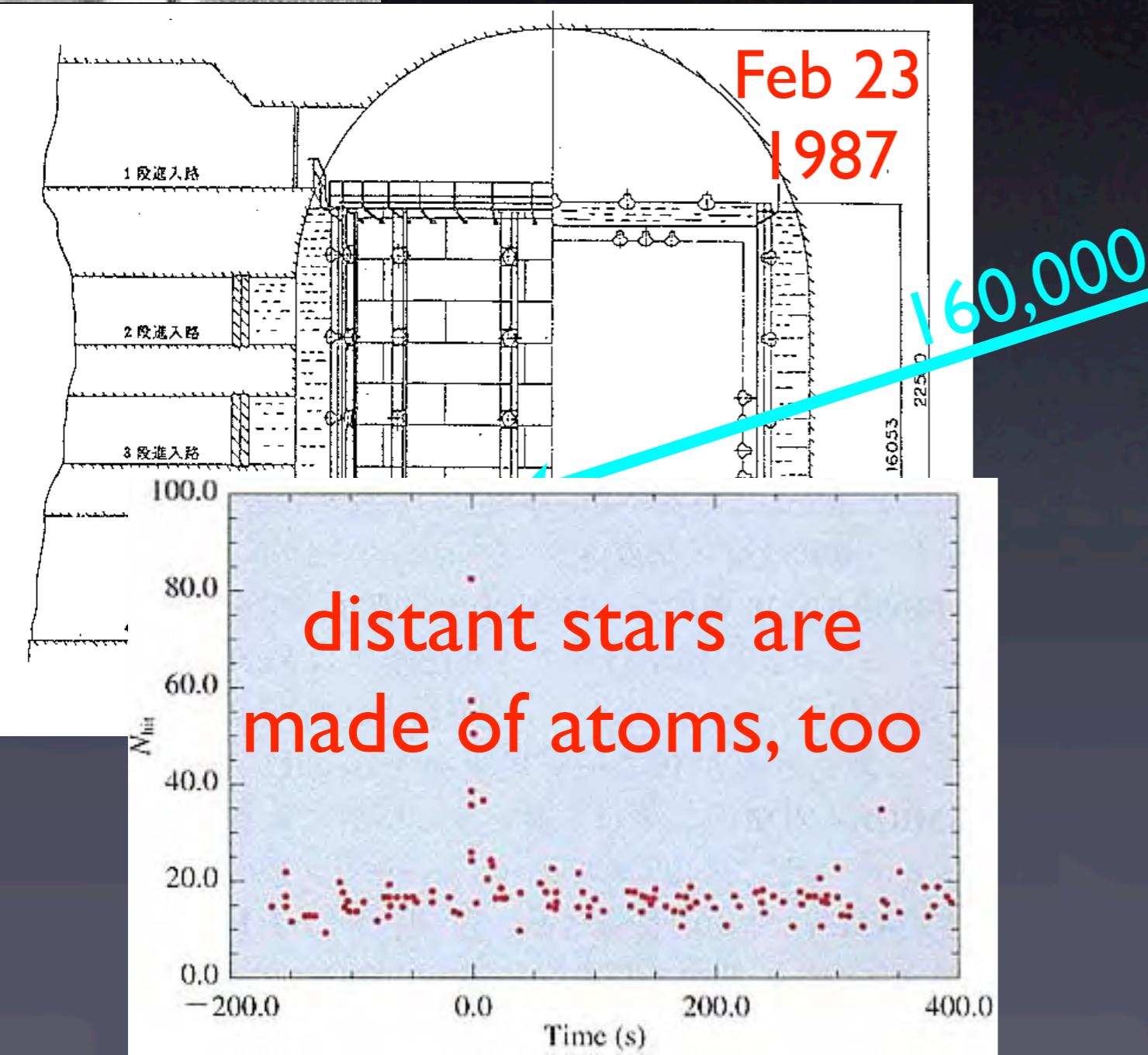
# proof

*nuclear fusion also produces neutrinos  
tens of trillions of neutrinos going through our body  
every second*





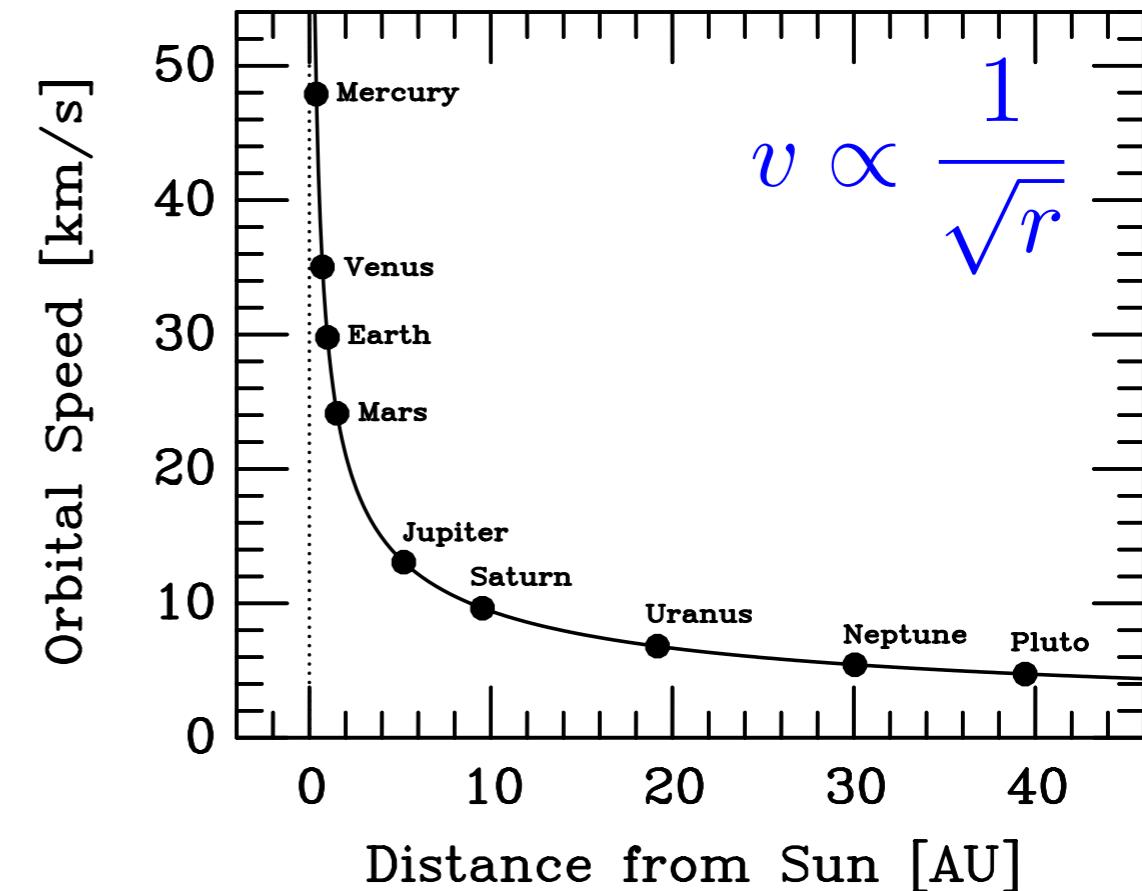
luck



Neptune  
4 light hours  
**Solar**

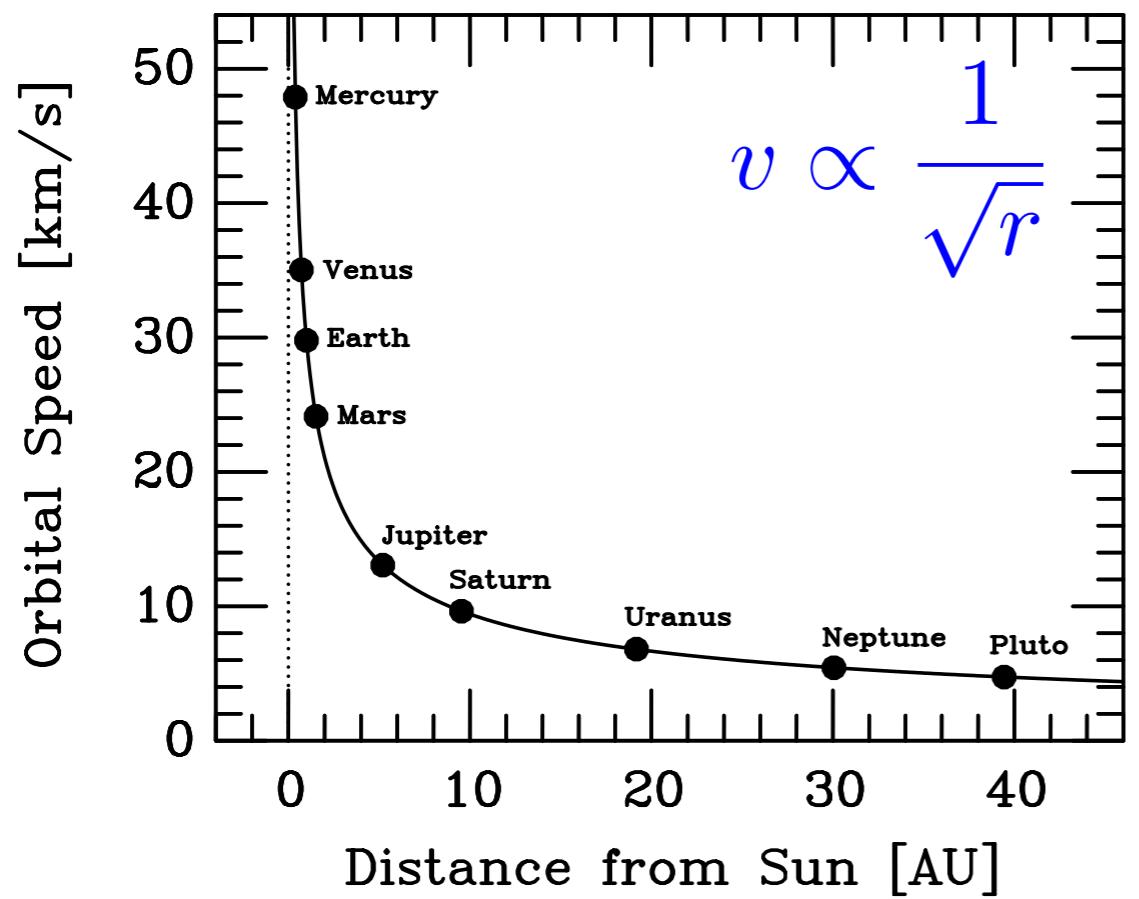


20 light minutes  
**Itokawa**



Earth revolves around the Sun with 30km/sec

# High School physics



$$F = \frac{GM_{\odot}m}{r^2}$$

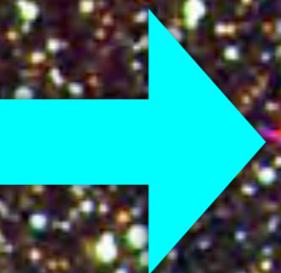
$$ma = m \frac{v^2}{r}$$

$$\frac{GM_{\odot}m}{r^2} = m \frac{v^2}{r}$$

$$v = \sqrt{\frac{GM_{\odot}}{r}} \propto \frac{1}{\sqrt{r}}$$

A dense field of stars of various colors and brightness against a dark background.

closest star



Proxima Centauri  
4.2 light years

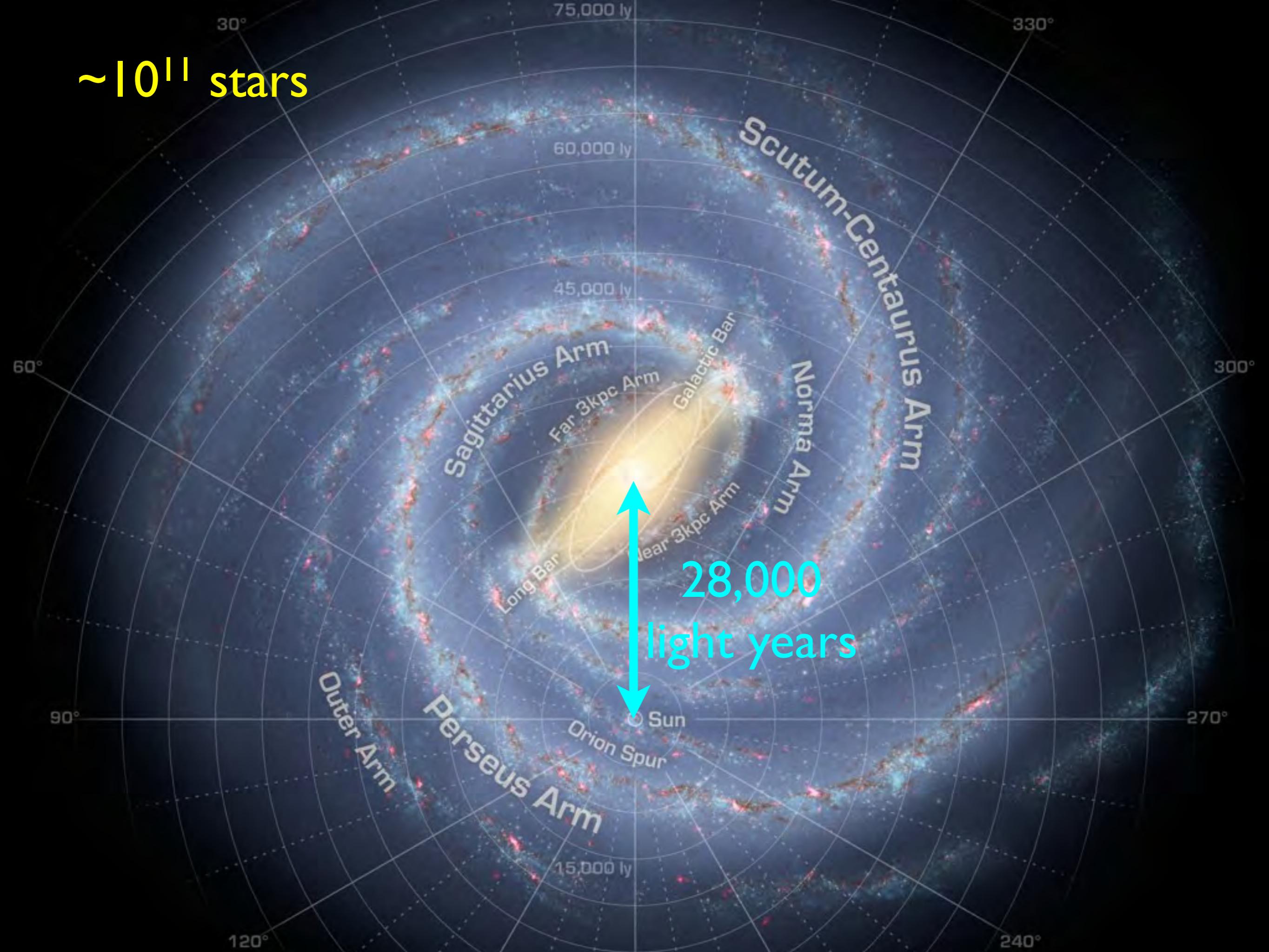


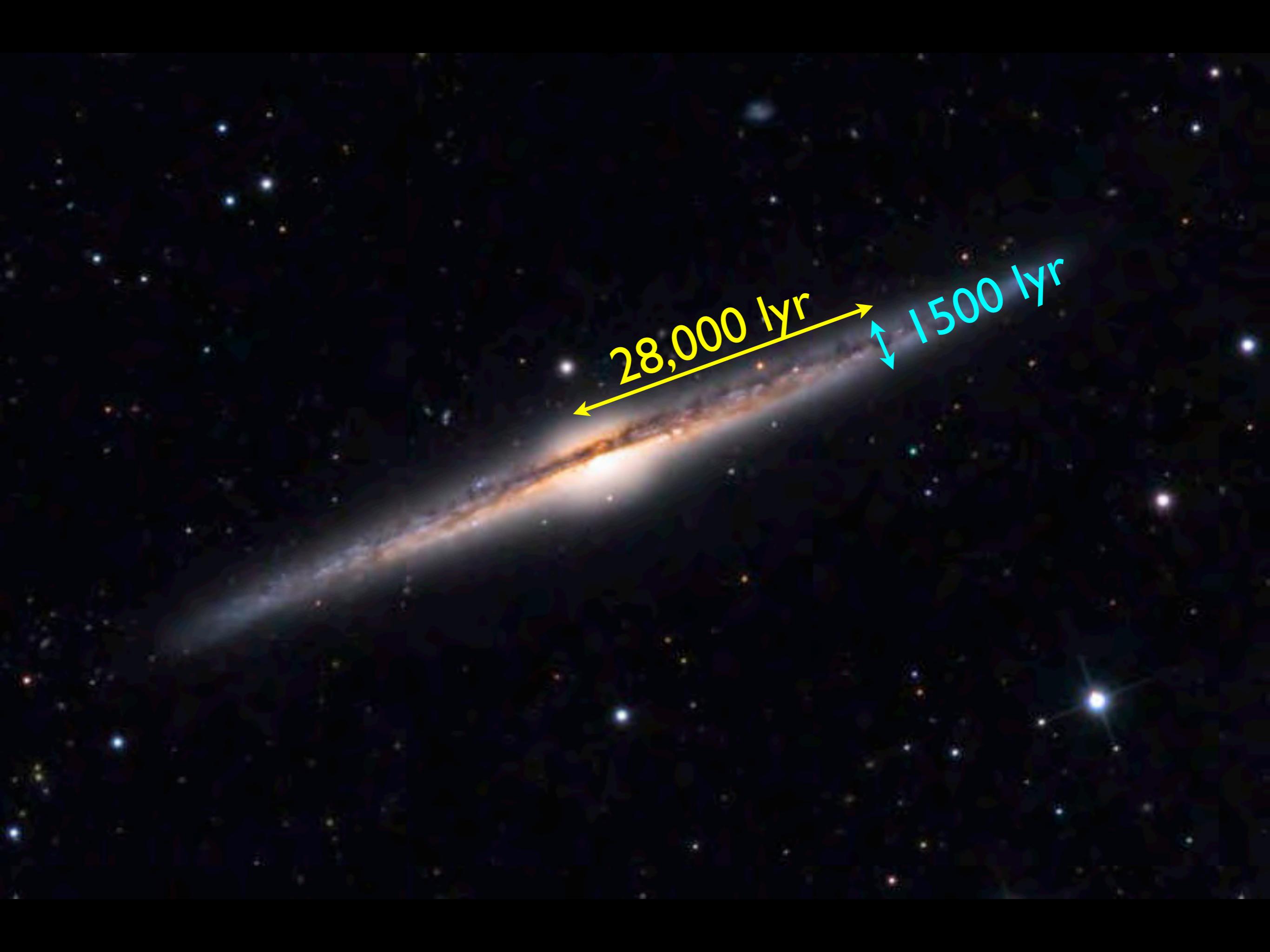
Milky Way and Galaxies





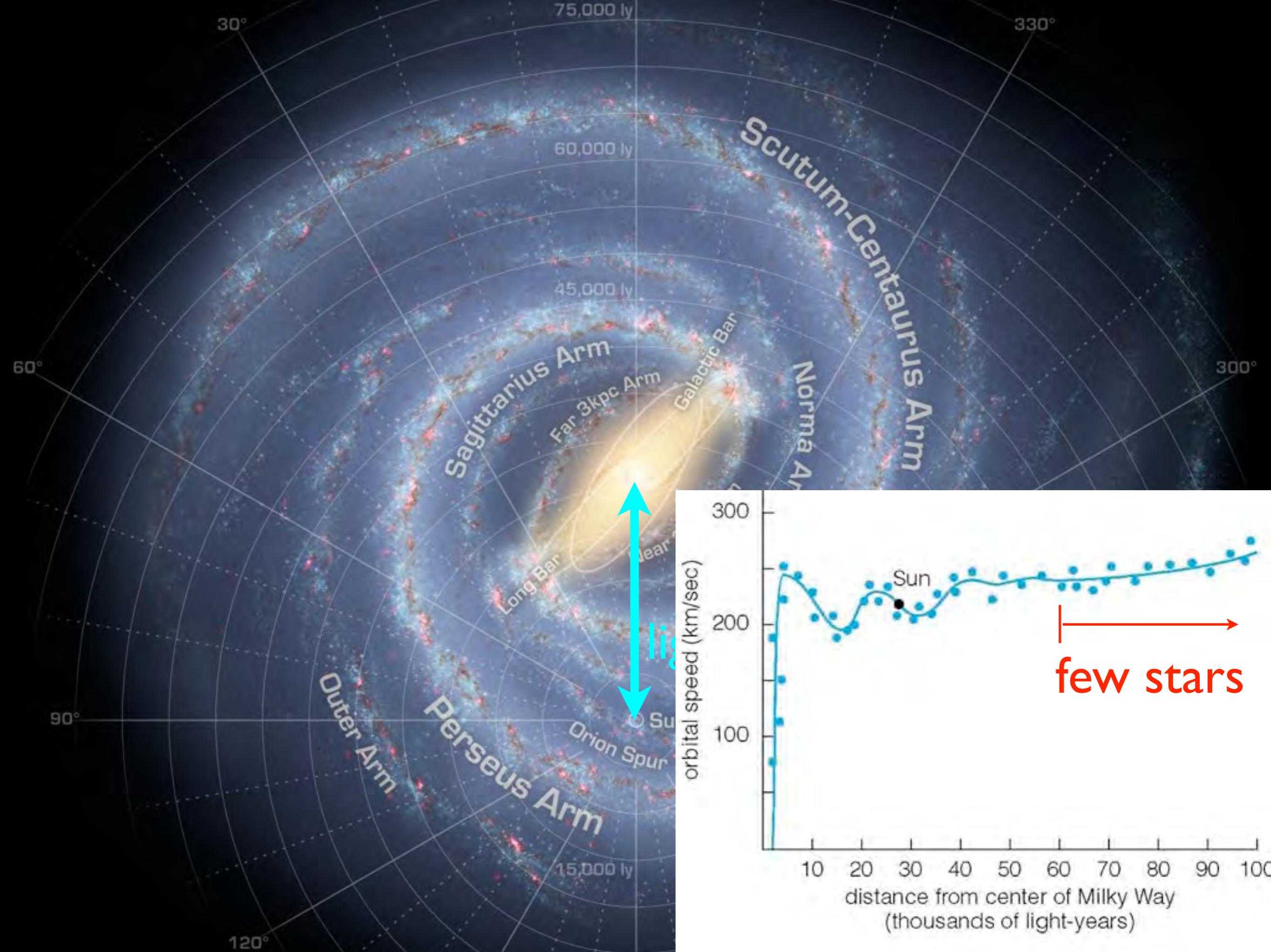
$\sim 10^{11}$  stars

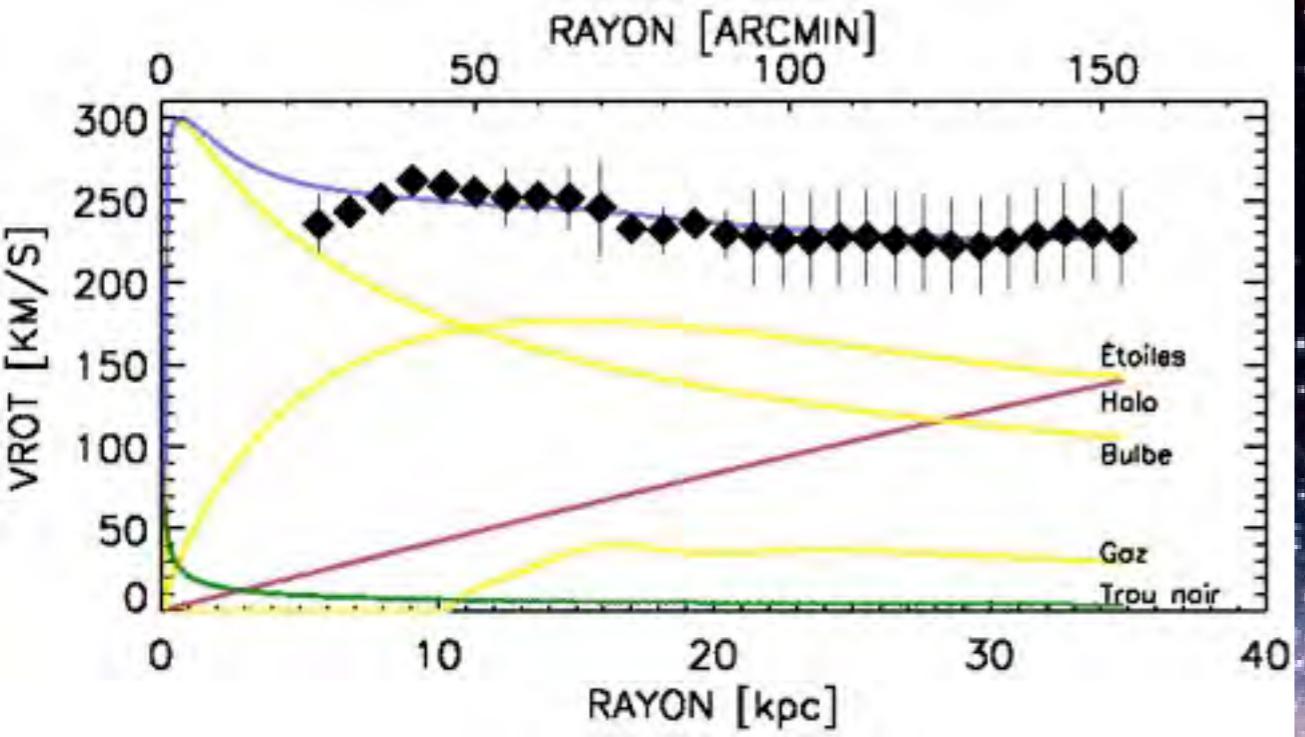




28,000 lyr

~1500 lyr





will collide with us  
in 4.5 billion years

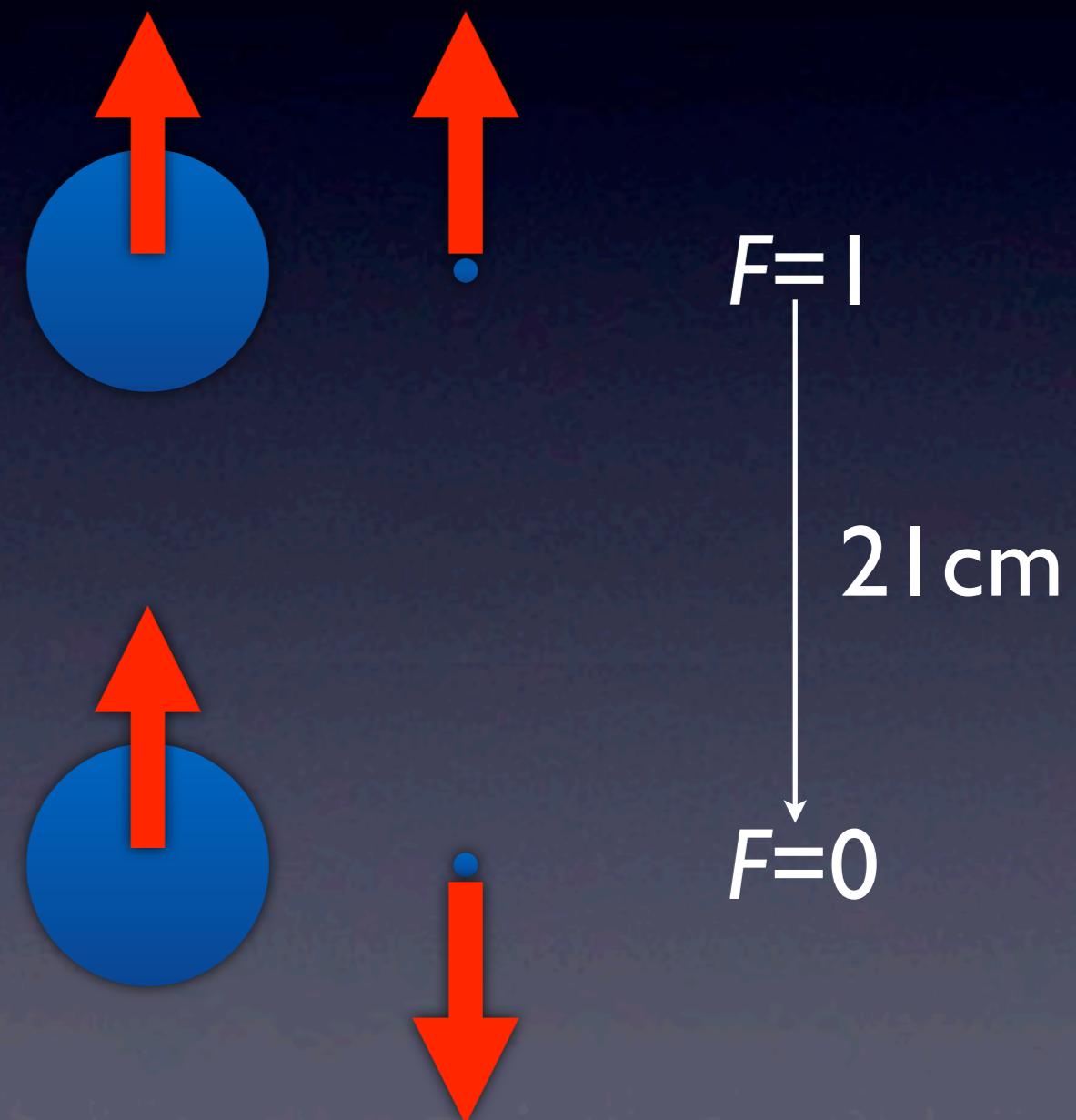


# How do we measure the rotation curve?

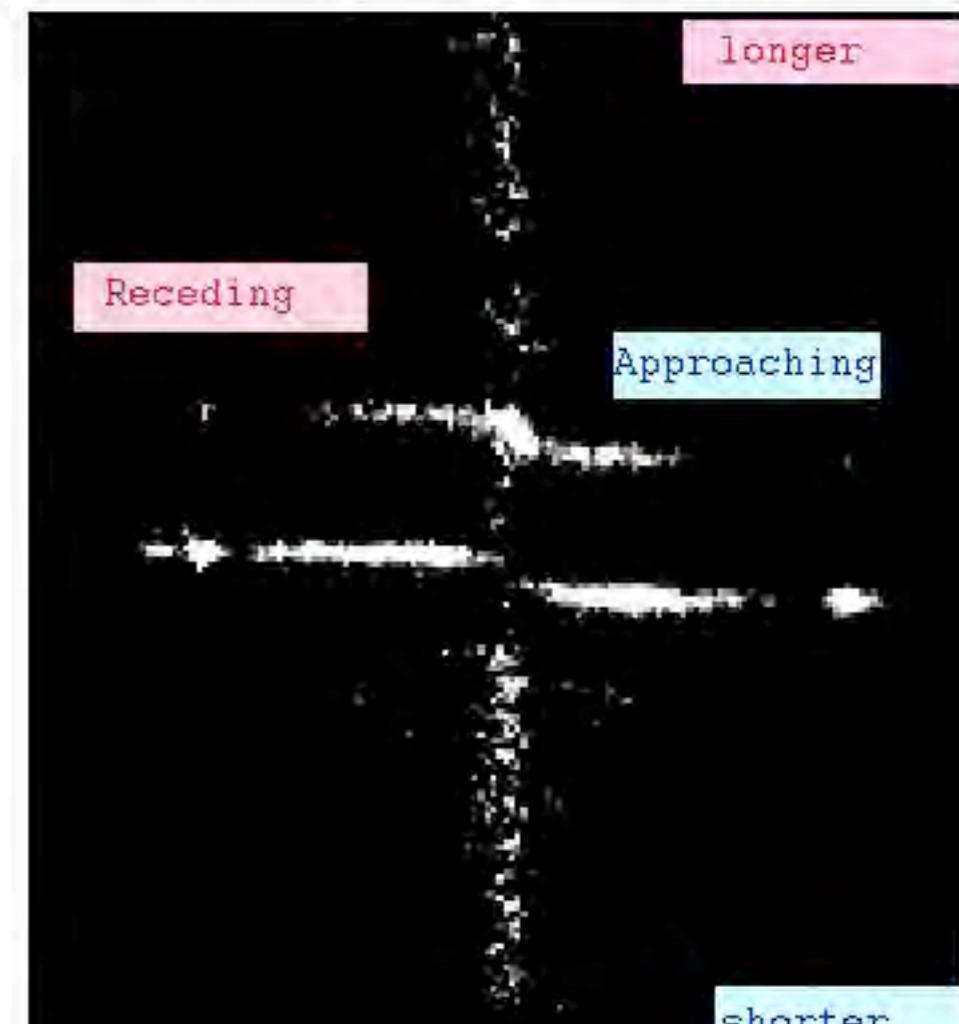
- Special relativity

$$f' = \sqrt{\frac{c \mp v}{c \pm v}} f \approx \left(1 \mp \frac{v}{c}\right) f$$

- hyperfine splitting in neutral hydrogen
- 21cm line can be excited by the cosmic microwave background
- $kT_0=0.23 \text{ meV}$
- $h c/21\text{cm} = 0.94 \mu\text{eV}$



## Galaxy rotation curve

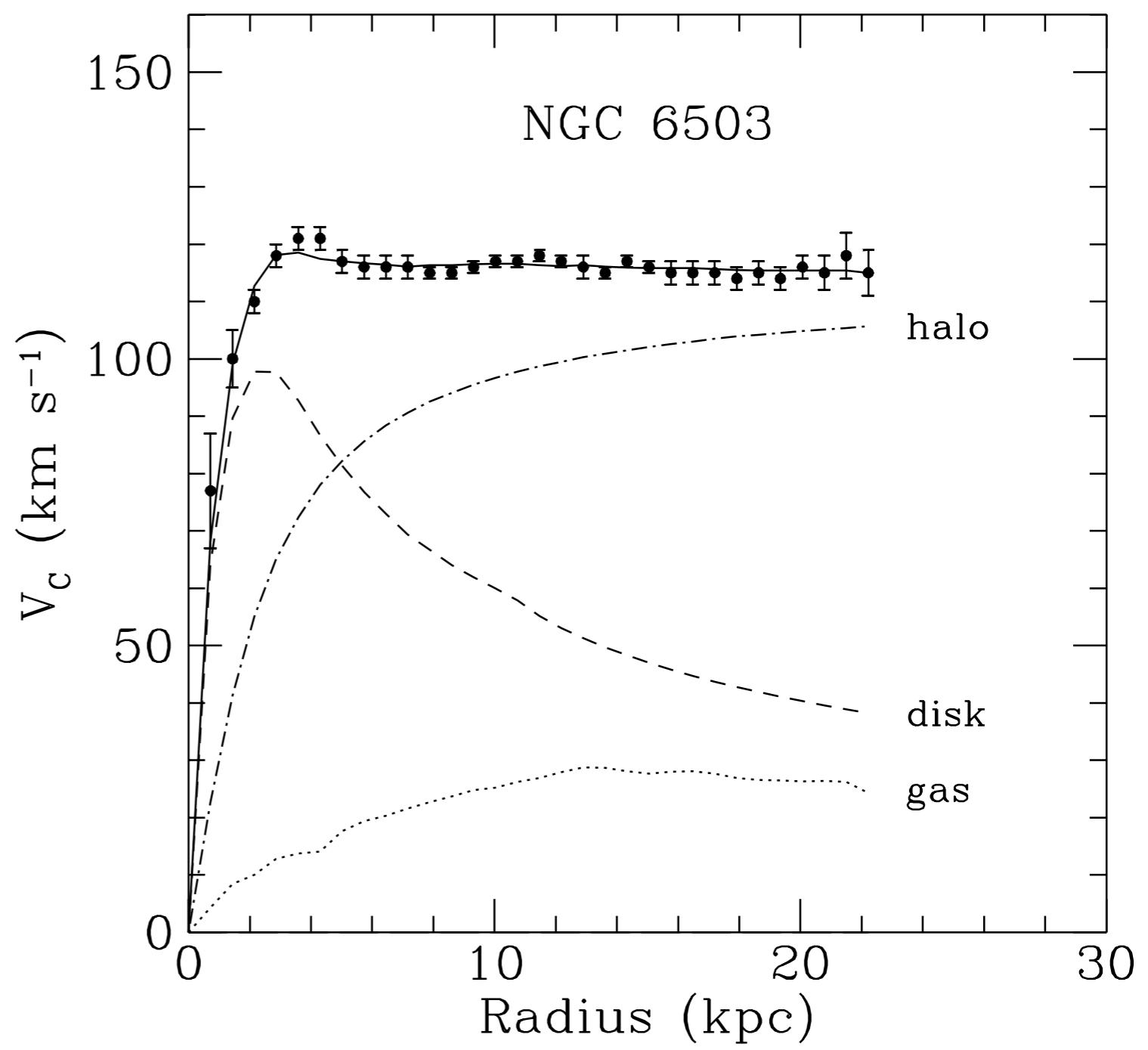
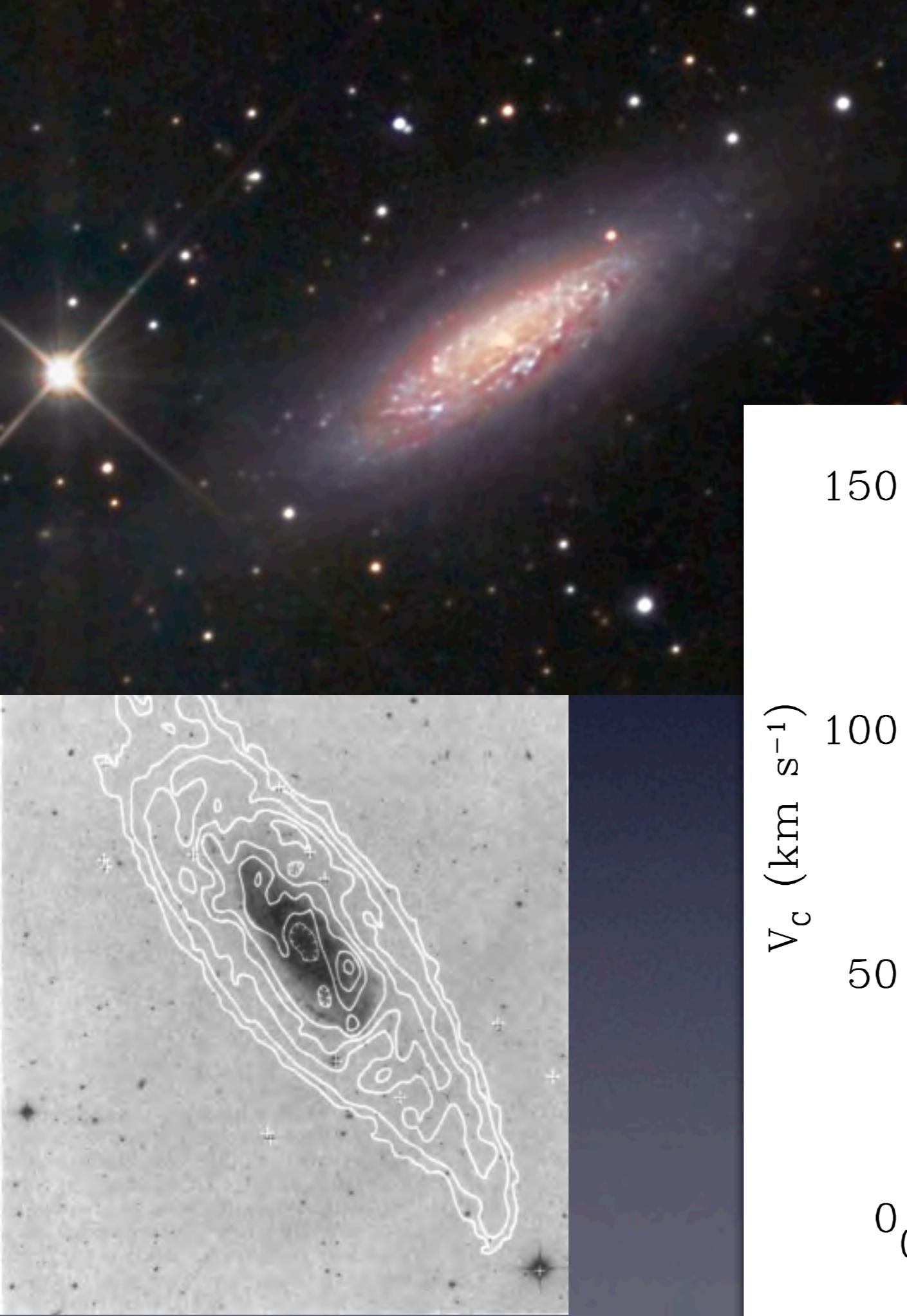


W A V E L E N G T H

Distance along galaxy major axis →

Vera  
Rubin  
1960s





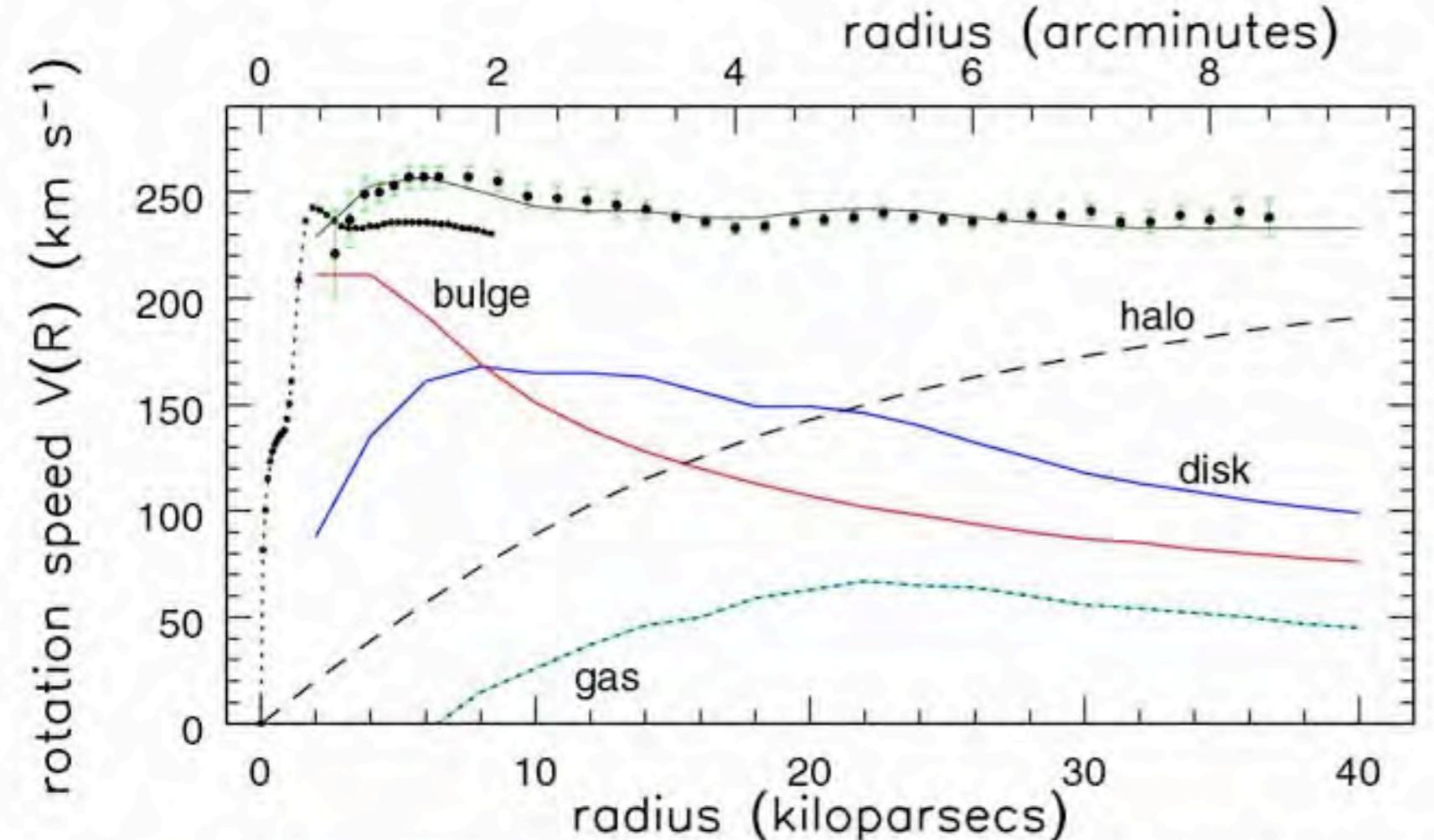
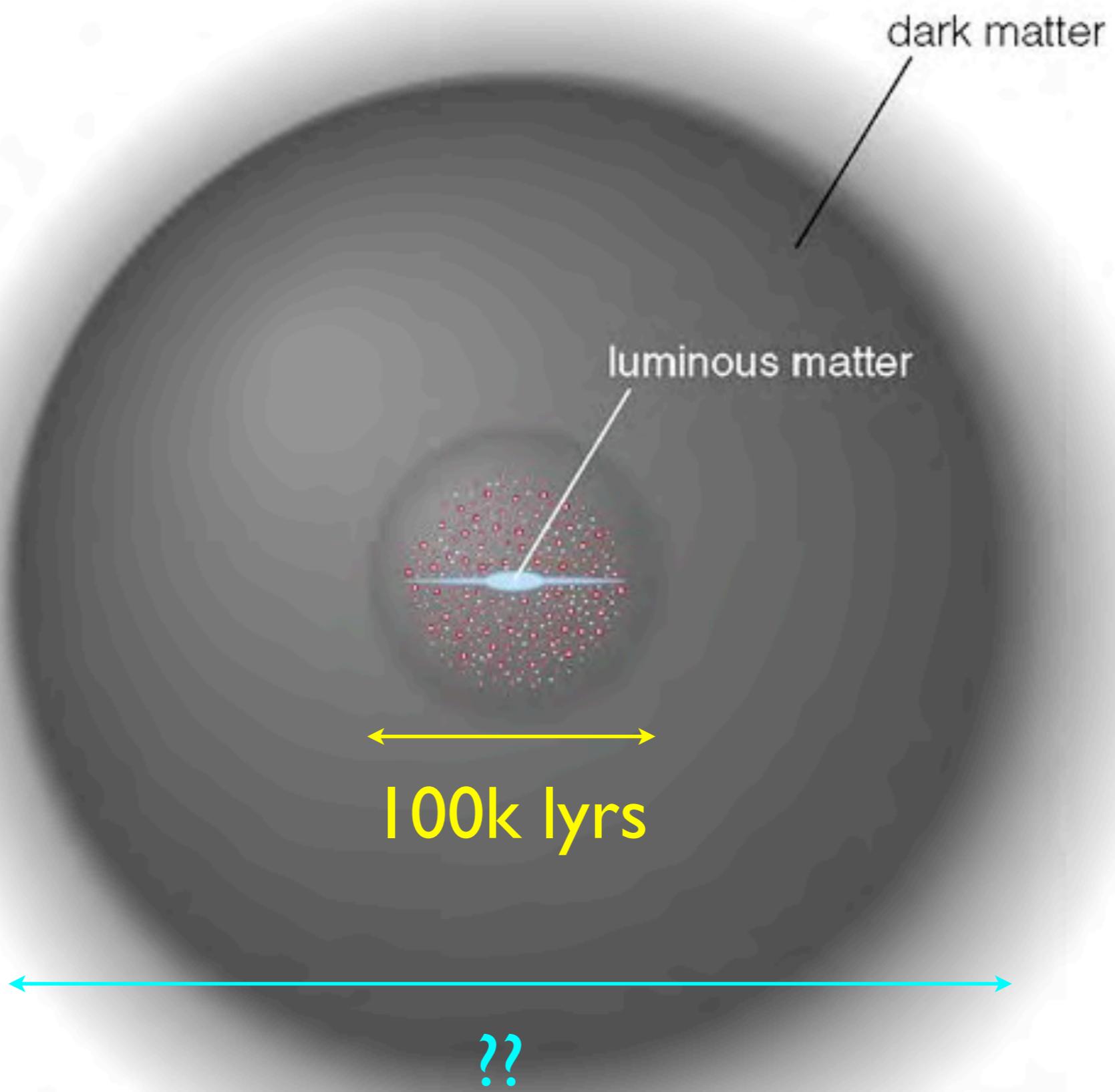


Fig 5.20 (Begeman, Sofue) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007





# connects galaxies

stacking 85k quasars near 20M galaxies

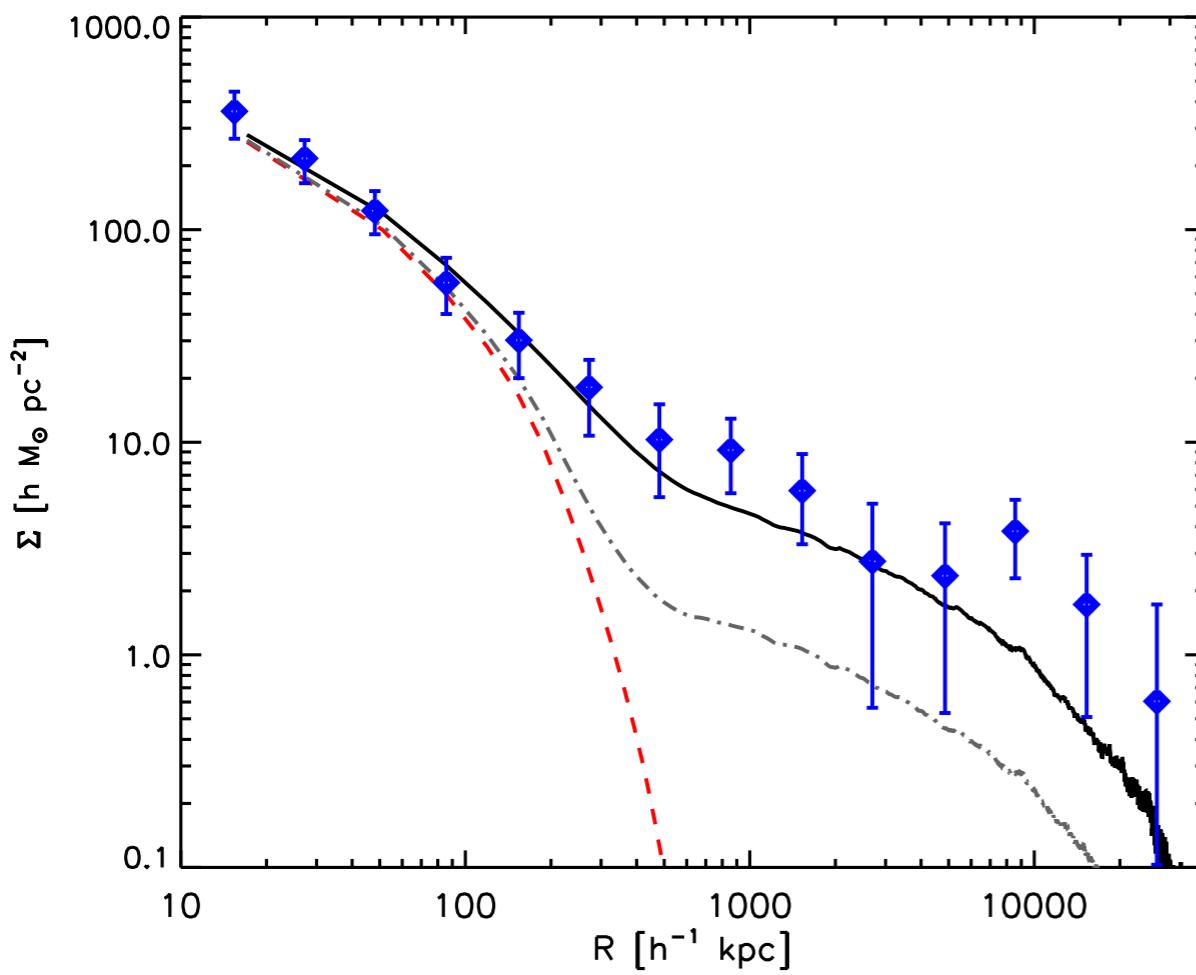
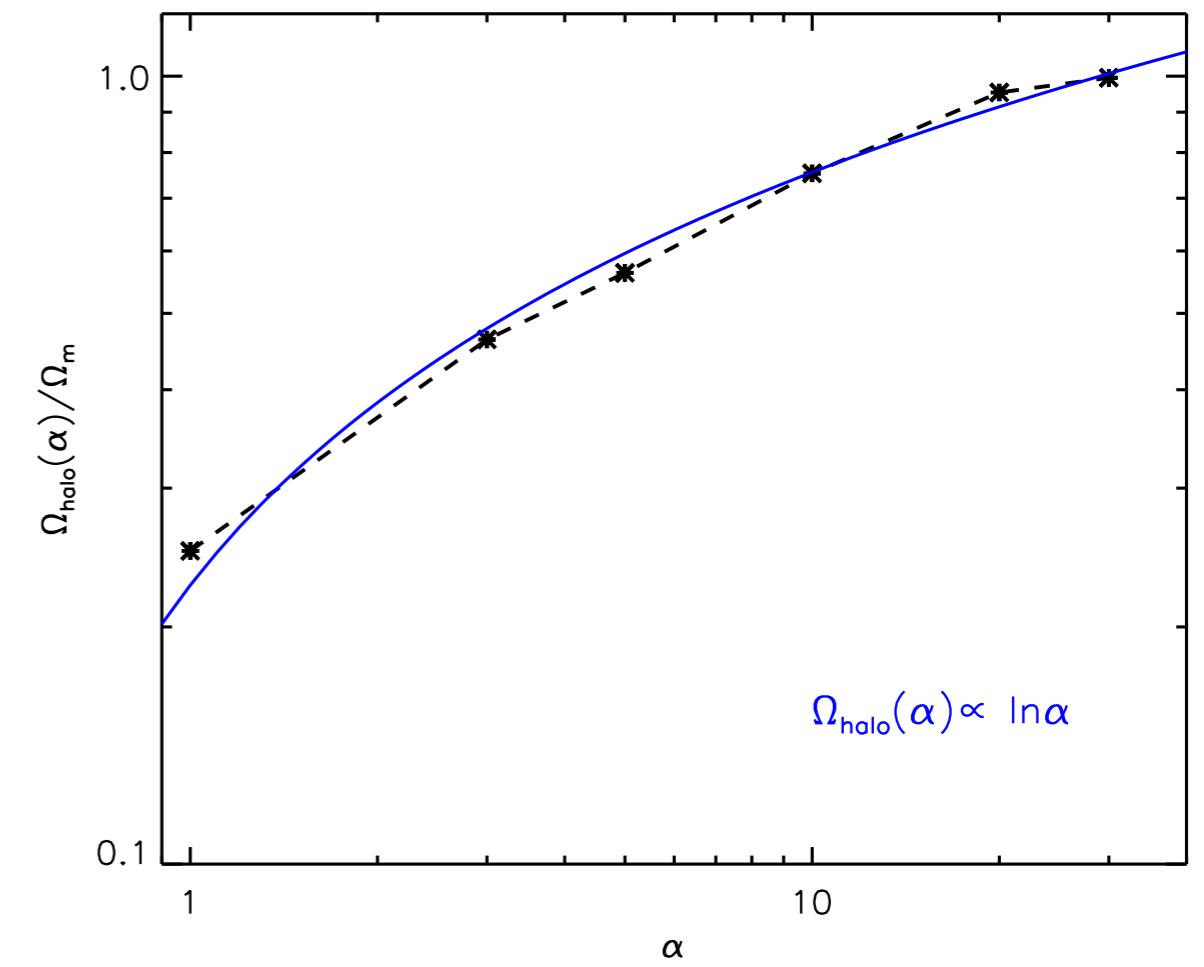
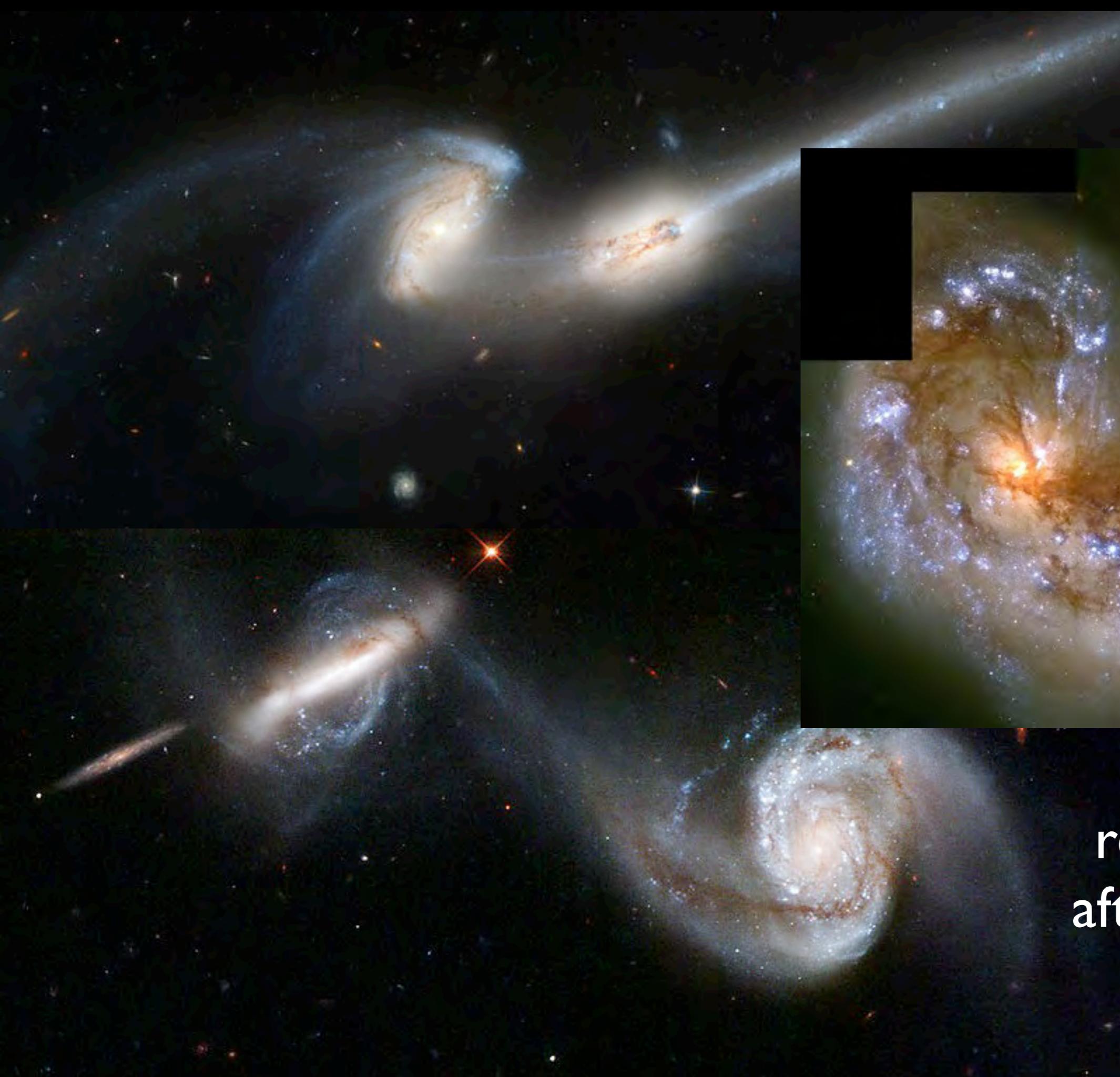


Fig. 2.— The mean surface mass density profile as a function of the distance from the centre of galaxies. The thick solid curve is the mean of all haloes above the mass threshold. The dash-dotted curve represents the contribution from particles bound to haloes, i.e., particles that reside within the virial radius of all haloes. The data with error bars are the observational estimate by MSFR given in the text.



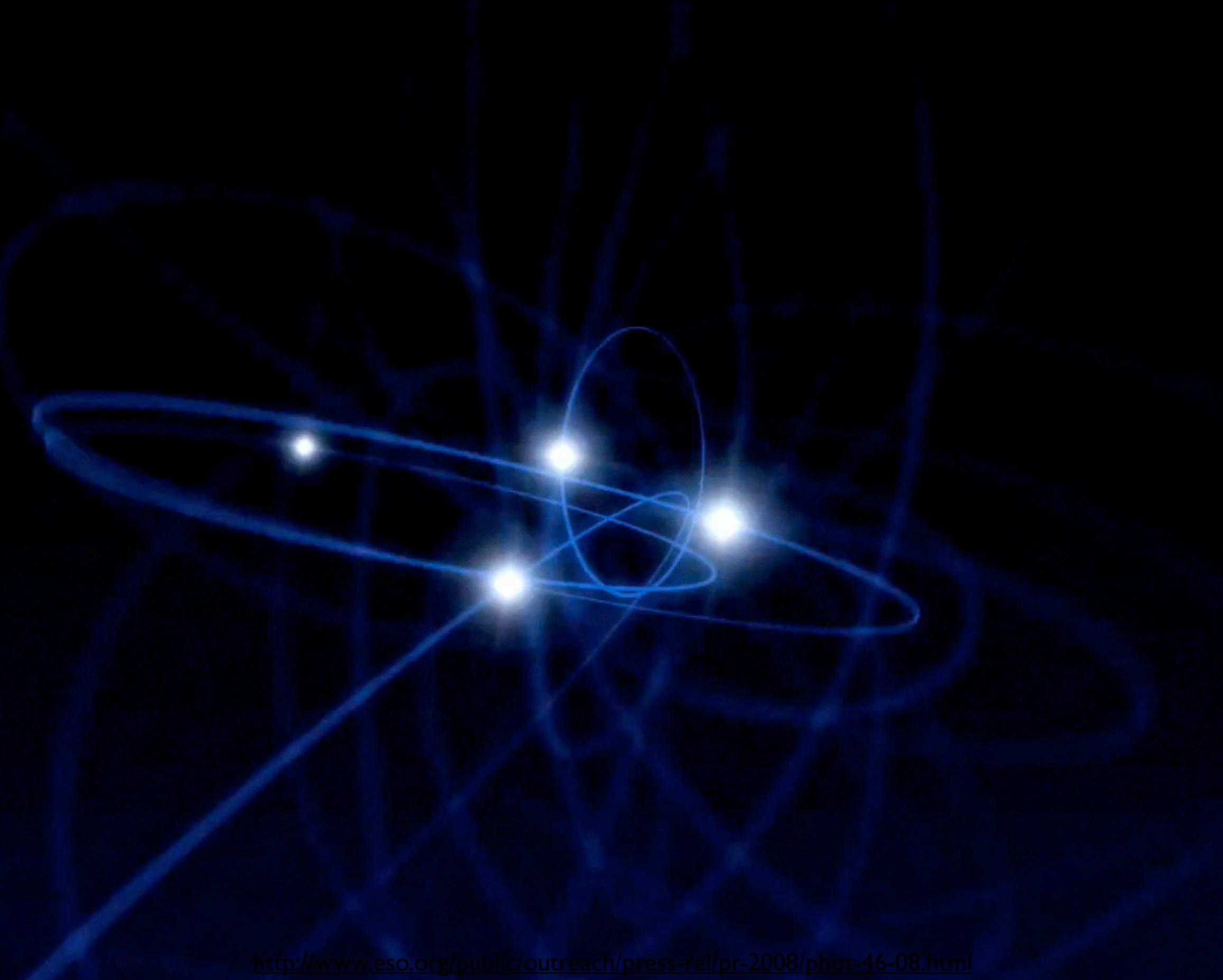
$$\Omega_{\text{halo}}(\alpha) \propto \ln \alpha$$



galaxies  
rejuvenated  
after mergers

# Center of Milky Way

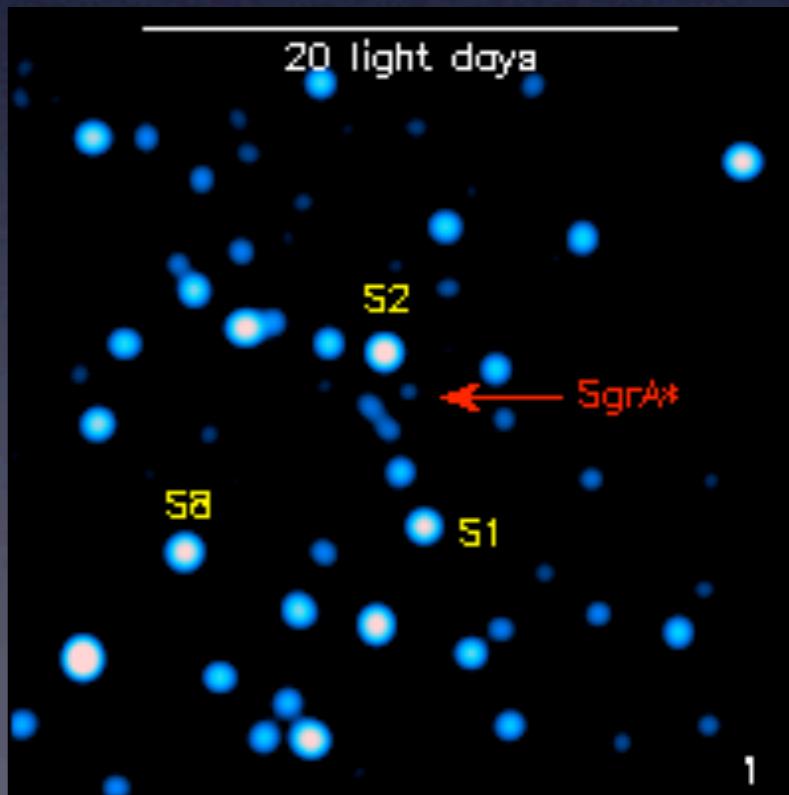




<http://www.eso.org/public/outreach/press-rel/pr-2008/phot-46-08.html>

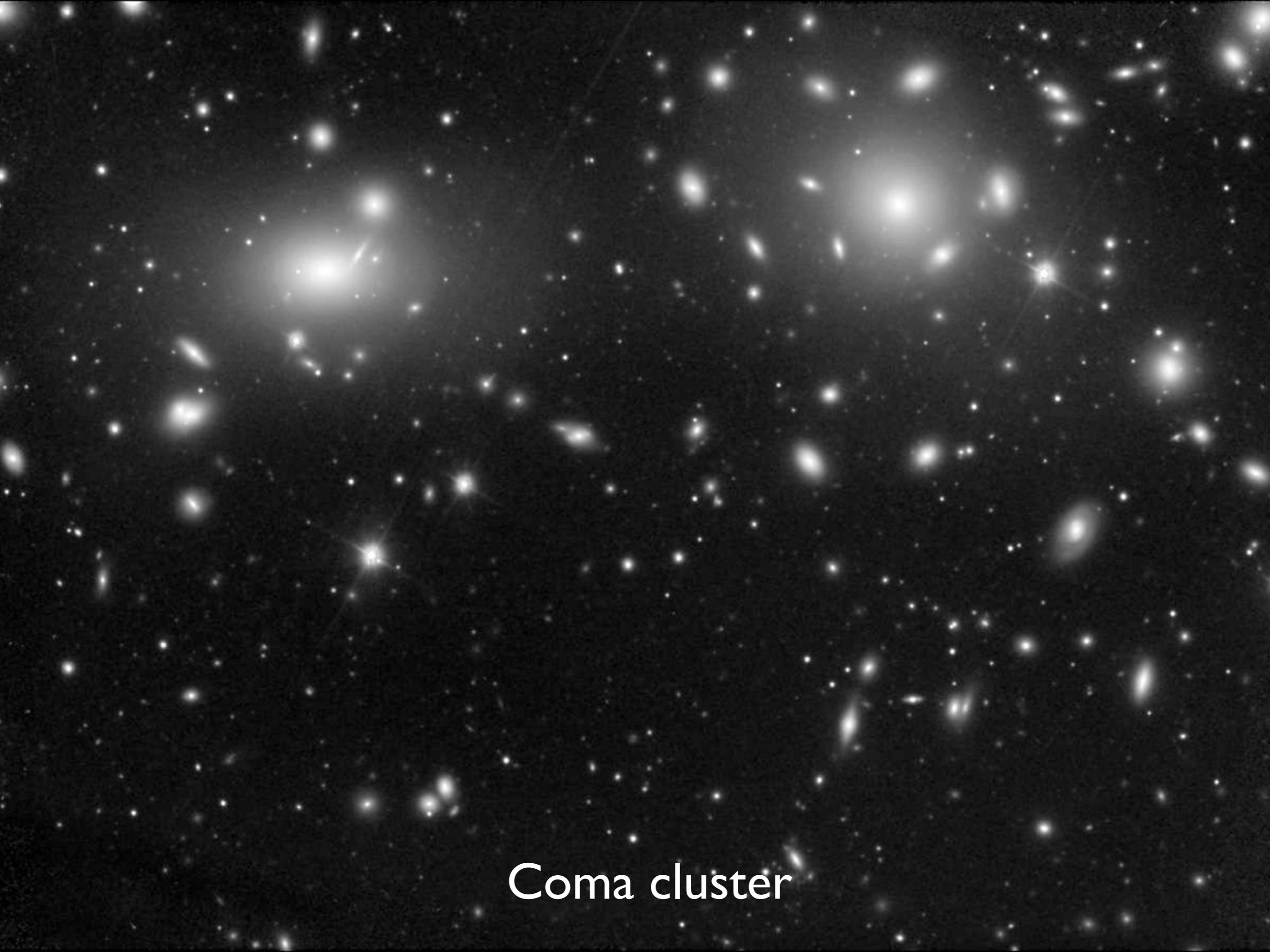
# Supermassive Black Hole

- supermassive blackhole of mass ~ $4M_{\text{sun}}$  at the center of Milky Way
- swallows gas around it
- “death cry” for about 30 min
- but can’t be dark matter, far less than 100billion stars



A dense cluster of galaxies against a dark background.

Cluster of Galaxies



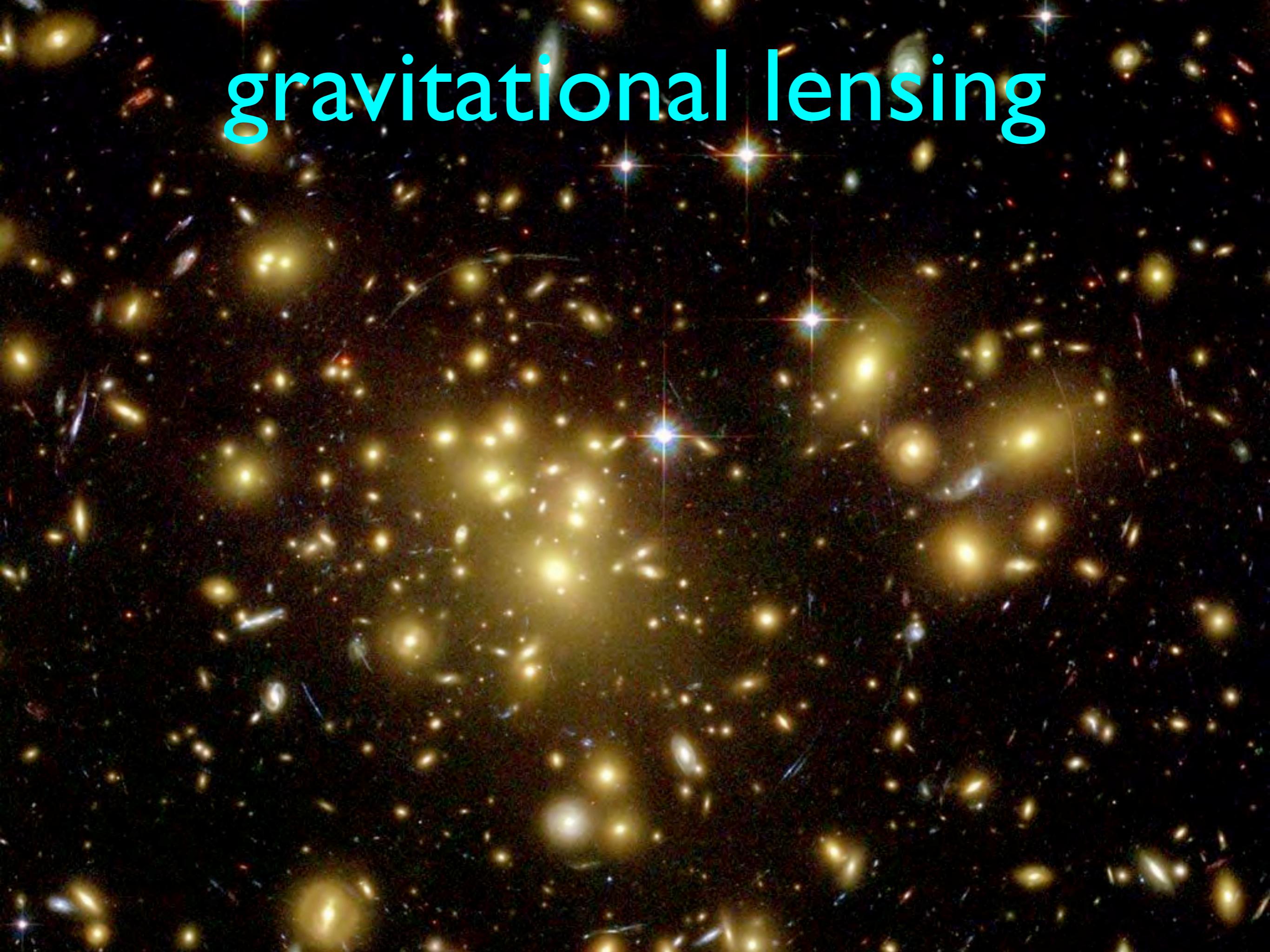
Coma cluster

# motion of galaxies

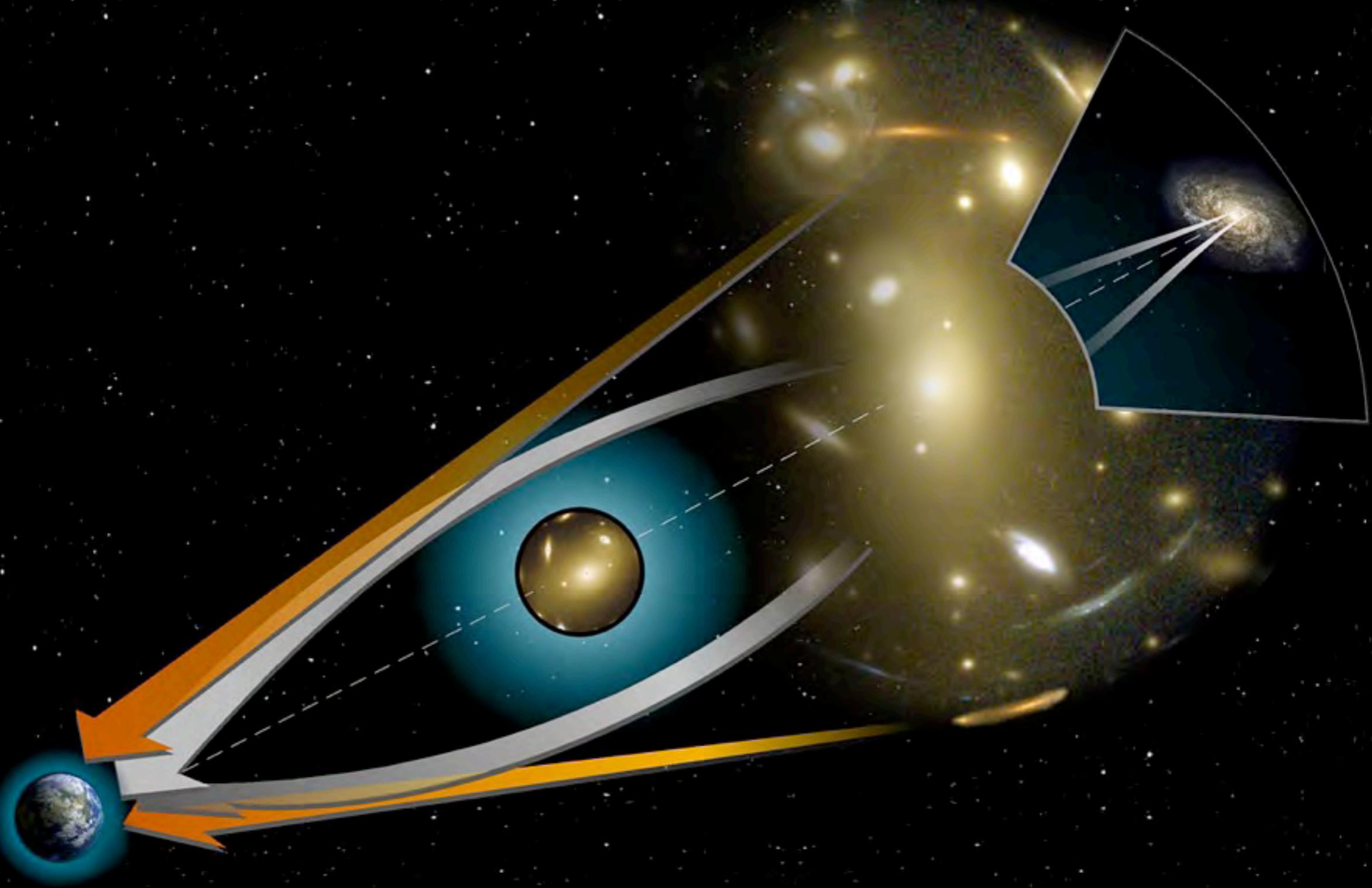
- galaxies are moving in the mutual gravitational potential
- assume virialized motion  $\langle v^2 \rangle = G_N M/r$
- but they are too fast, too
- first proposal of “dark matter”



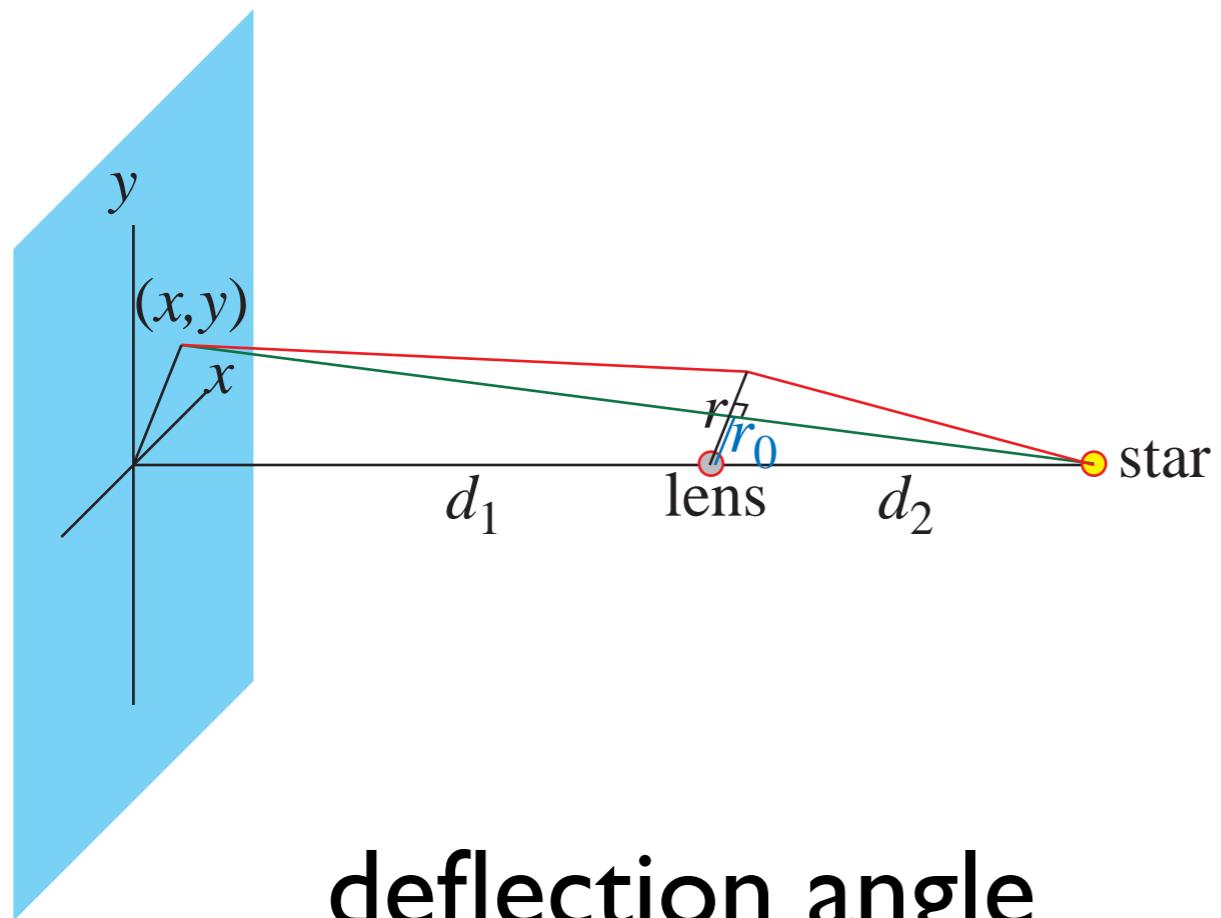
Fred Zwicky



gravitational lensing

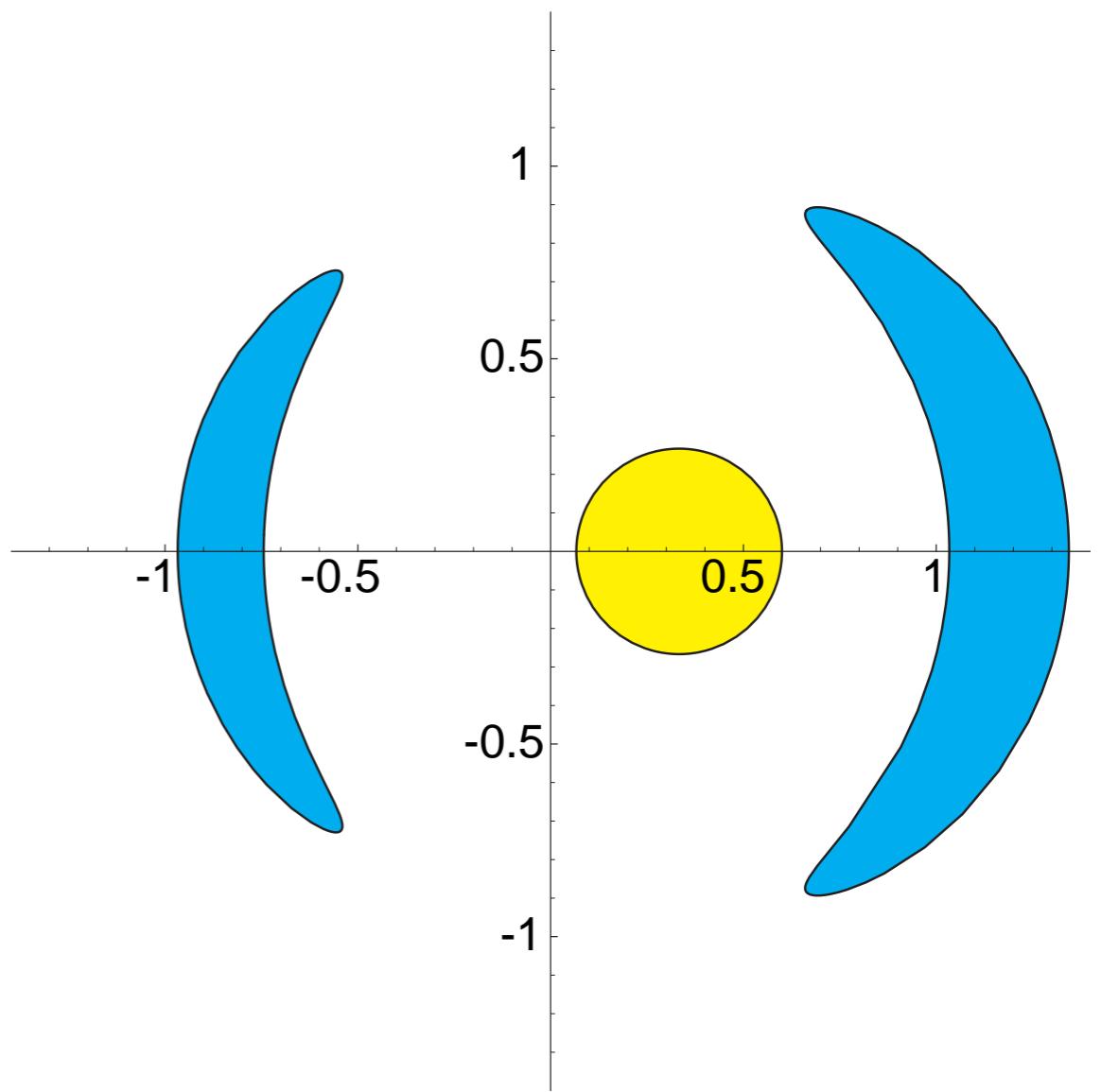


# gravitational lensing



deflection angle

$$\theta = 2 \frac{r_S}{r} = 4 \frac{G_N M}{c^2 r}$$



# gravitational lensing

$$ds^2 = \left(1 - \frac{r_S}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - r_S/r} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

$$g^{\mu\nu} \partial_\mu S \partial_\nu S = mc^2 = 0$$

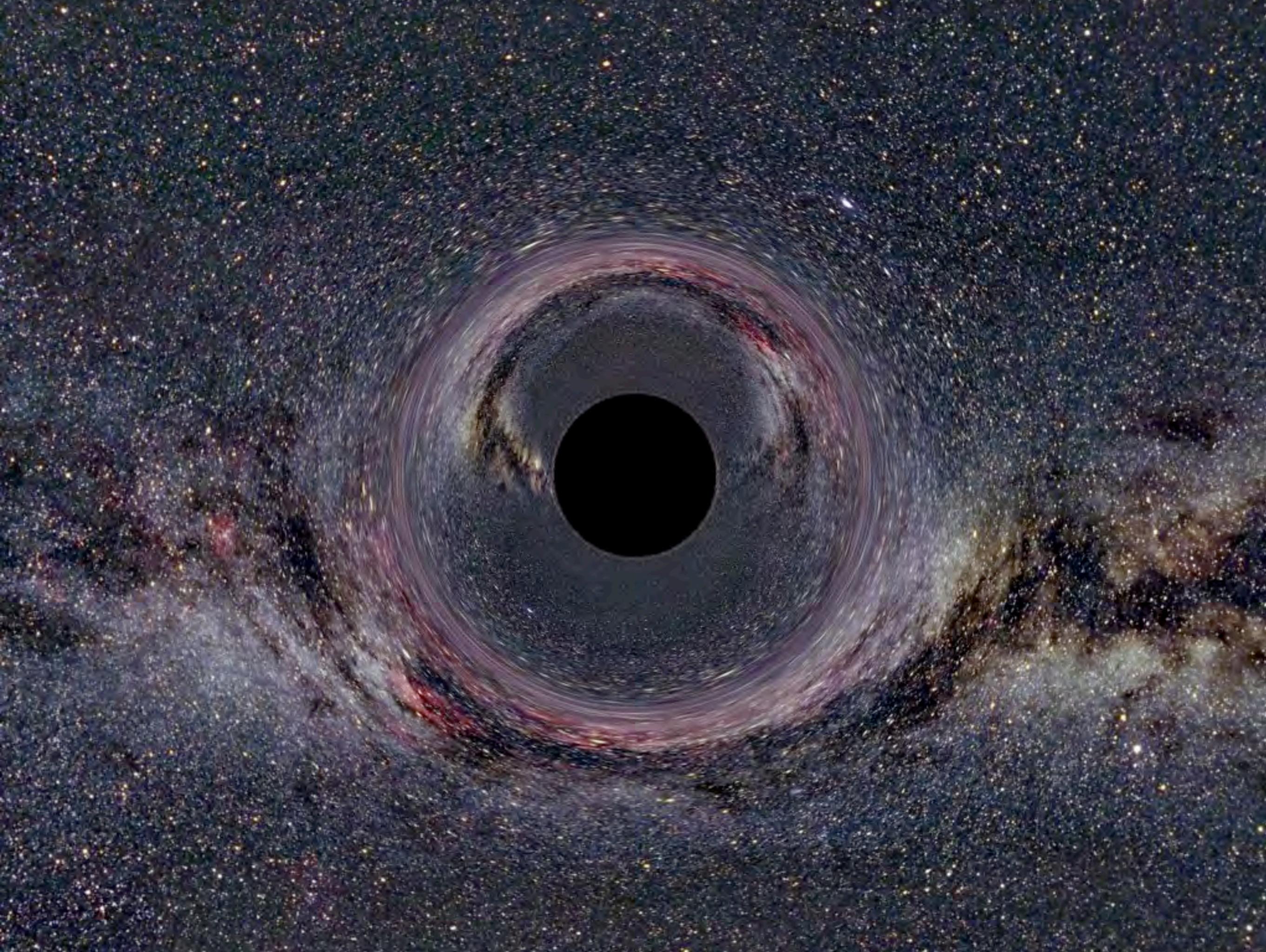
$$S(t, r, \theta) = Et - L\theta + W(E, r, L)$$

$$(\partial_r W)^2 = \left(1 - \frac{r_S}{r}\right)^{-2} \frac{E^2}{c^2} - \left(1 - \frac{r_S}{r}\right)^{-1} \frac{L^2}{r^2}$$

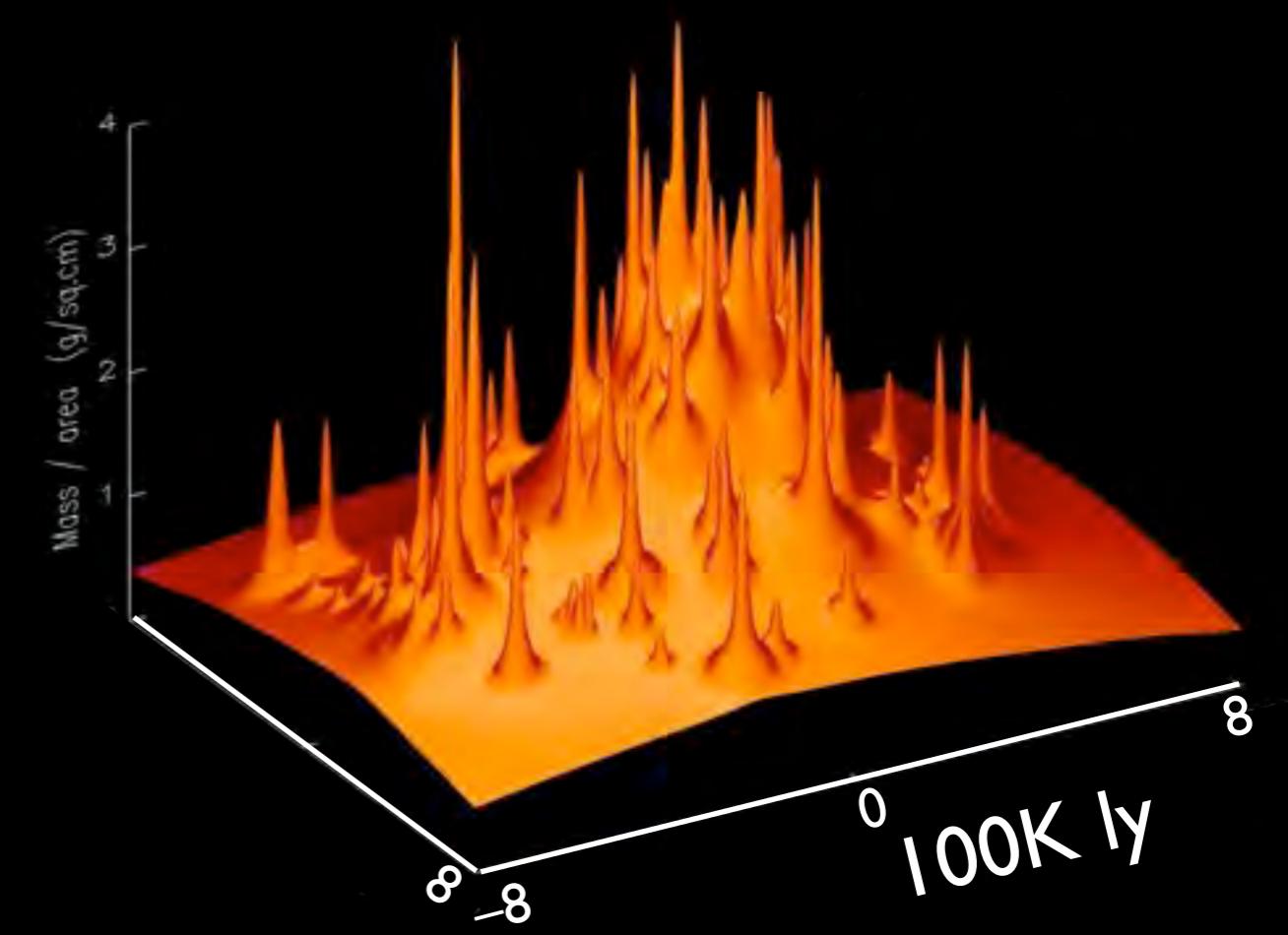
The closest approach  $r_c$  is where  $\partial_r W=0$

$$E^2 = c^2 \left(1 - \frac{r_S}{r_c}\right) \frac{L^2}{r_c^2}$$

$$\pi + \Delta\theta = \partial_L W = 2 \int_{r_c}^{\infty} \frac{dr}{r \sqrt{(1 - \frac{r_S}{r_c}) \frac{r^2}{r_c^2} - (1 - \frac{r_S}{r})}} = \pi + \frac{2r_S}{r_c} + O(r_S^2)$$

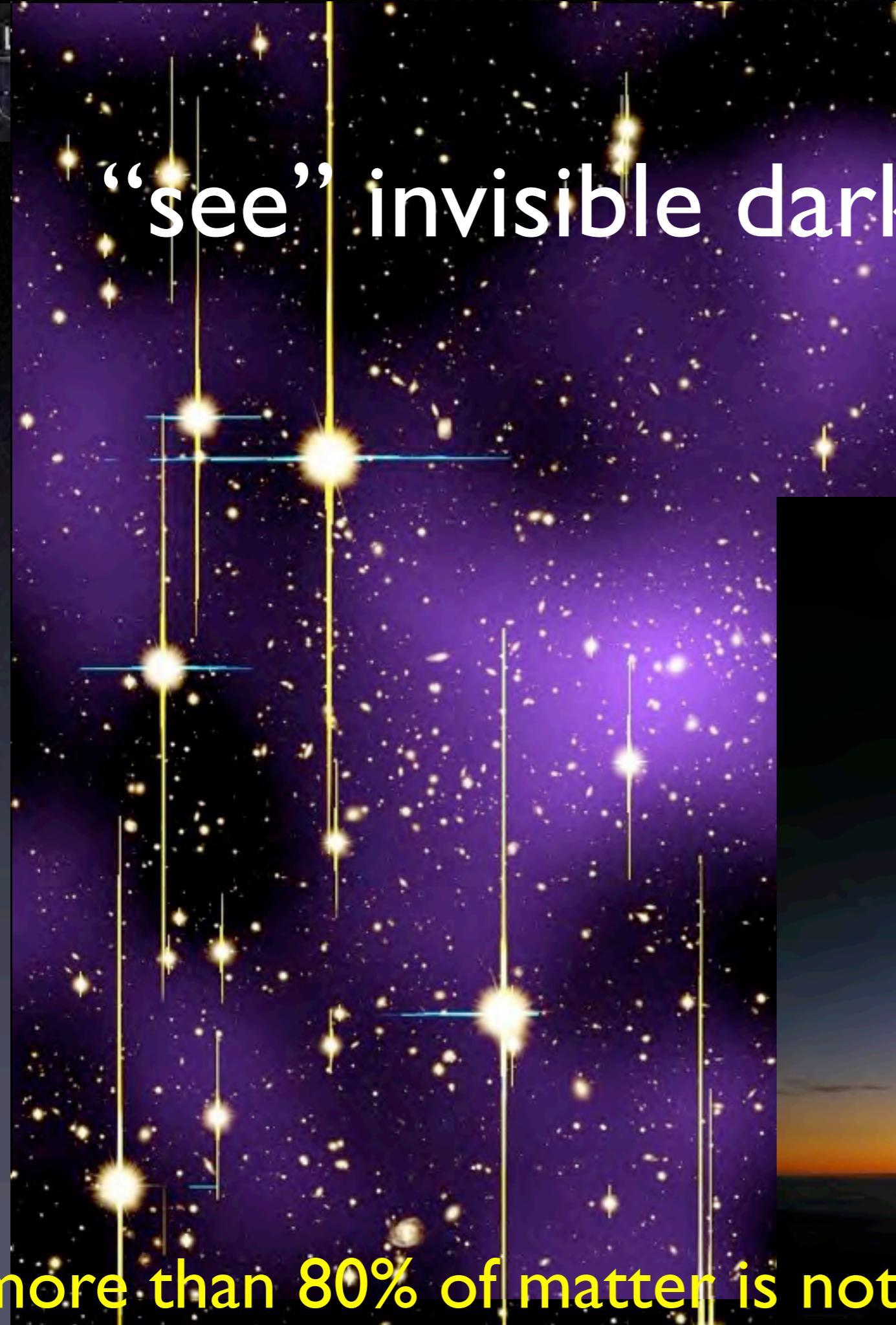


# Clusters of galaxies



distortion in **images** of BG galaxies → 2D map of dark matter

“see” invisible dark matter



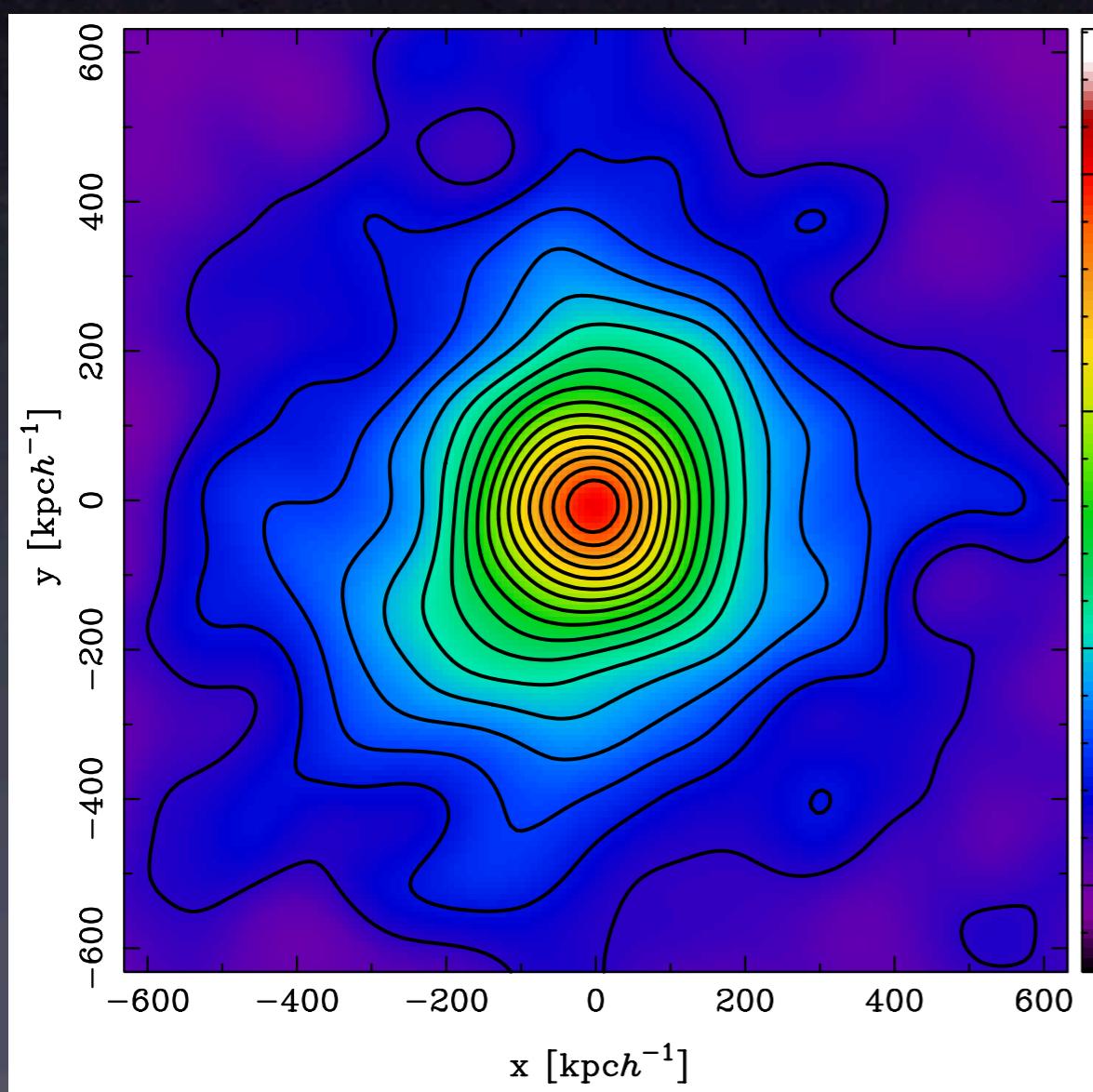
Subaru telescope



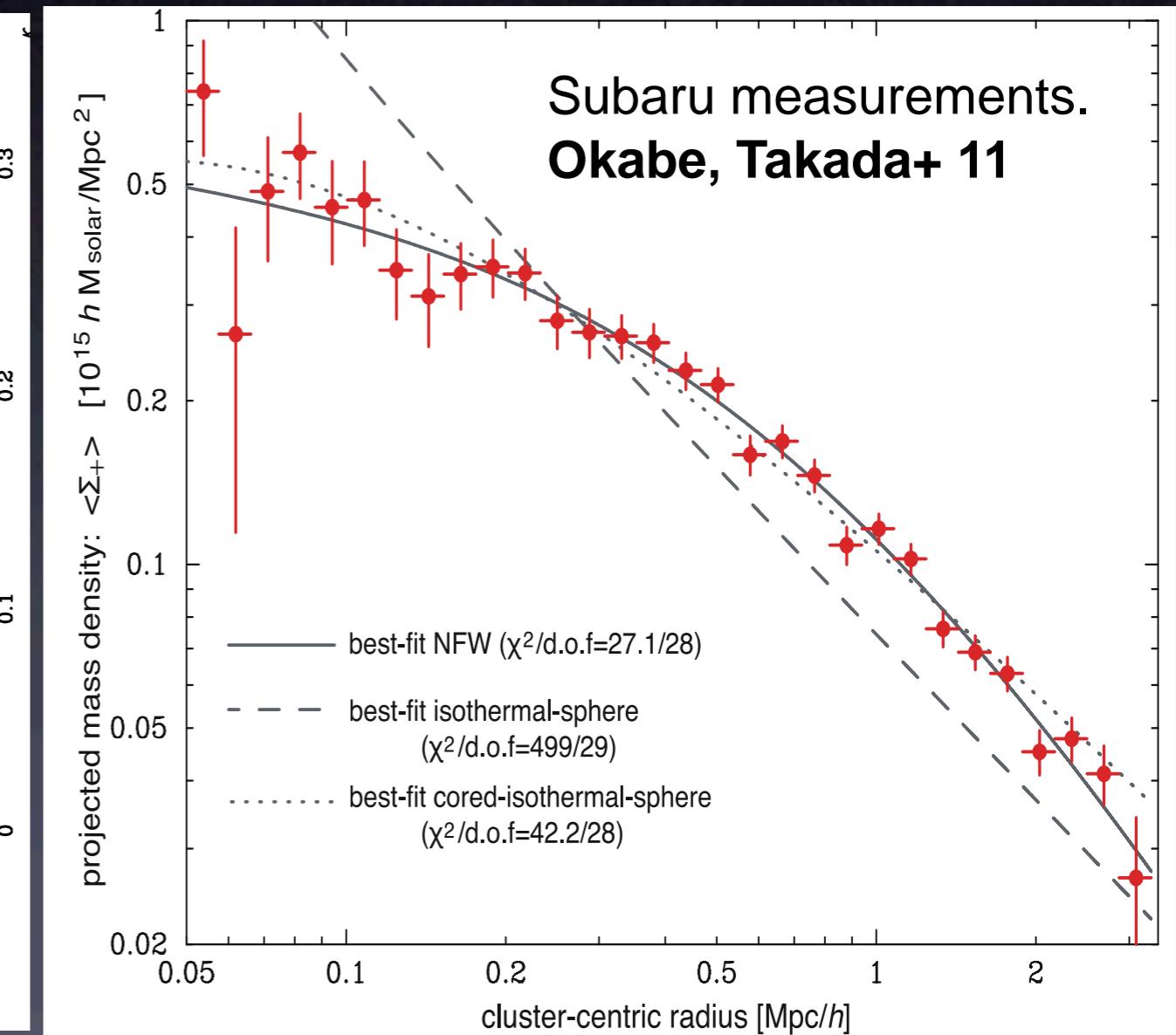
more than 80% of matter is not atoms!



# as expected by theory

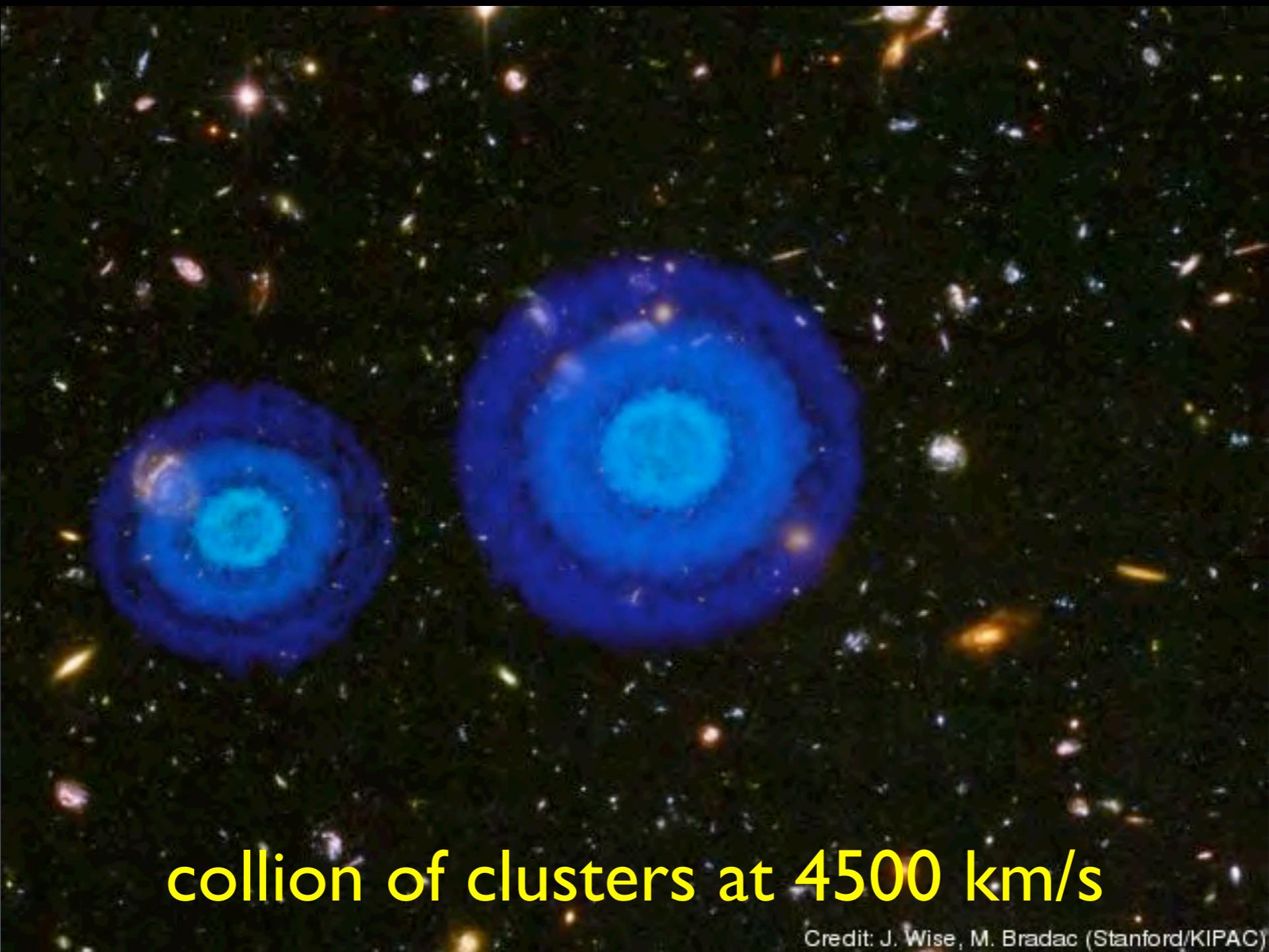


45 clusters stacked



consistent with NFW profile

# lucky we are not here



4 billion lyr away

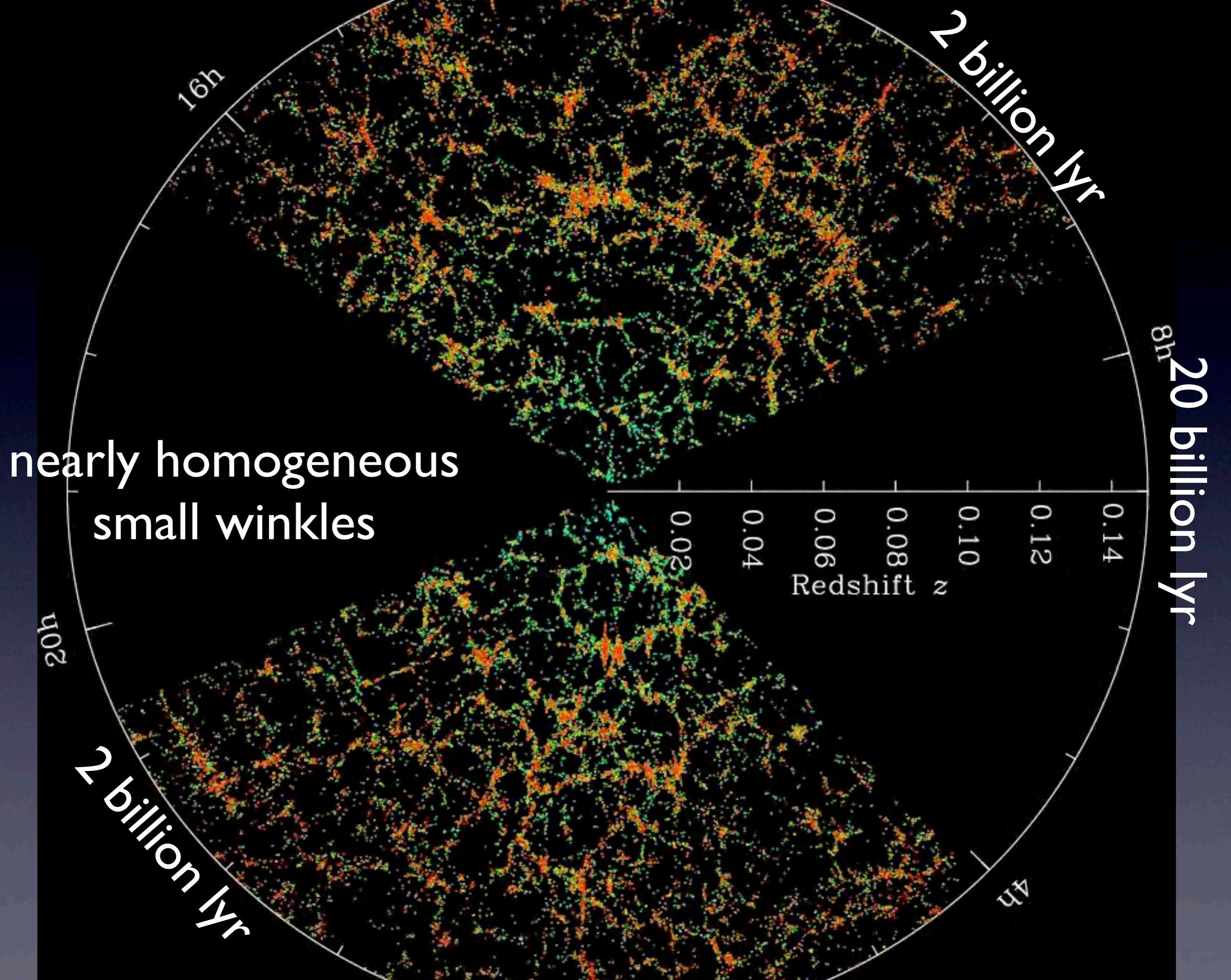
# Dark Matter



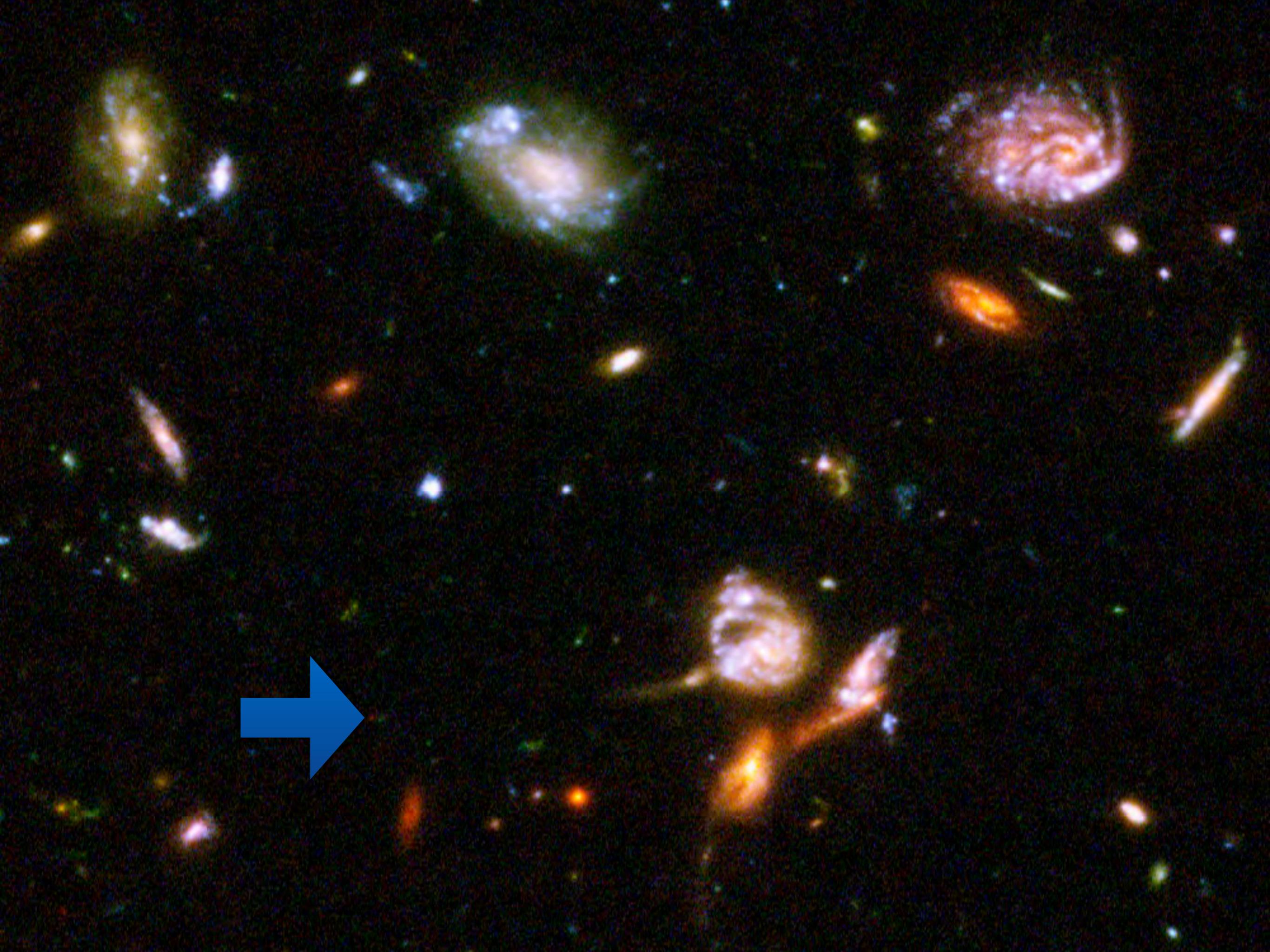


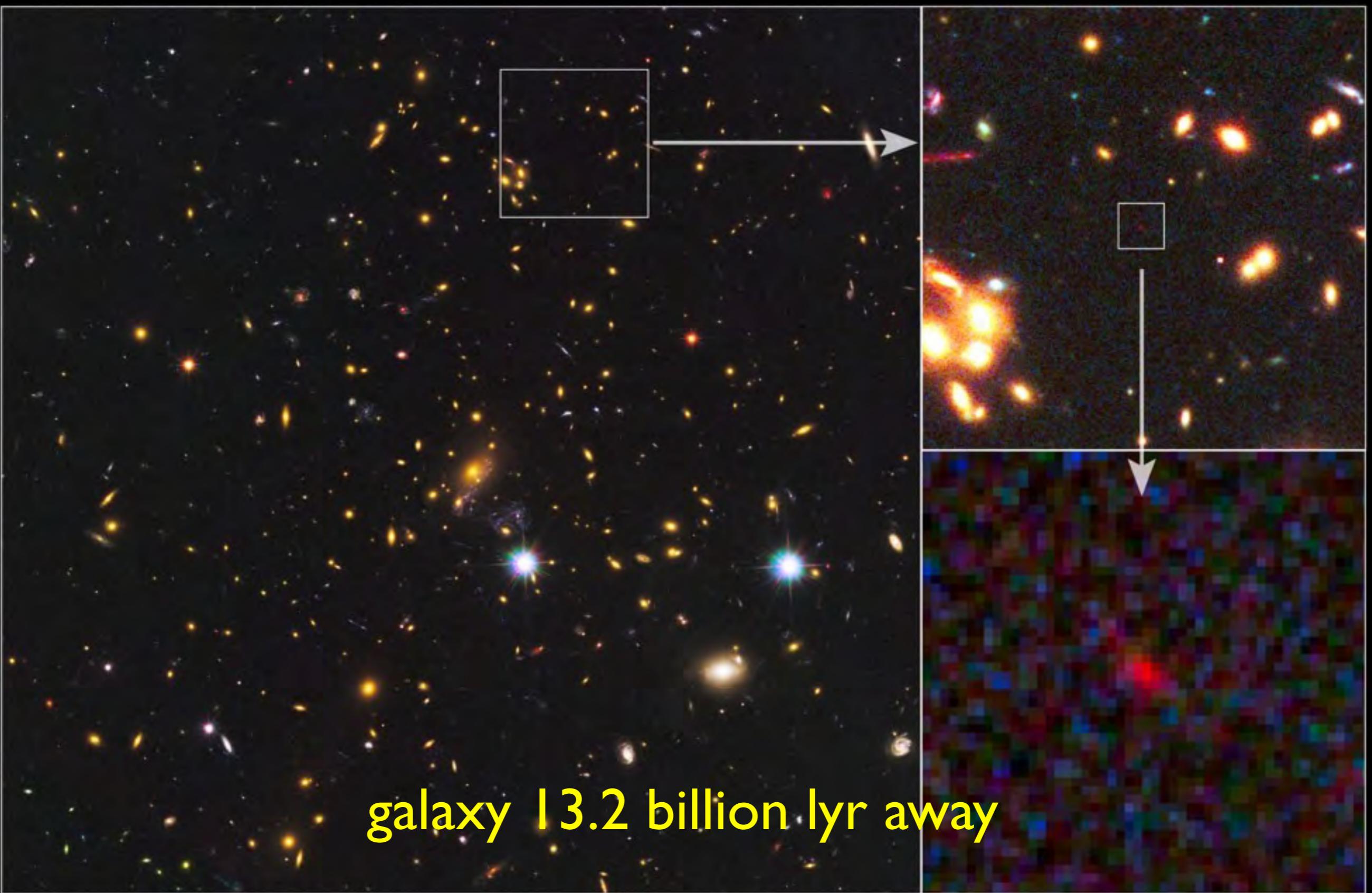
fly-through based on SDSS-III data











Galaxy Cluster MACS J1149+2223

A Distant Gravitationally-Lensed Galaxy at Redshift = 9.6  
NASA / ESA / STScI / J. Hora (Harvard-Smithsonian CfA)

High-Redshift Galaxy MACS1149-JD

Hubble Space Telescope • ACS • WFC3  
ssc2012-12a



# Expanding space

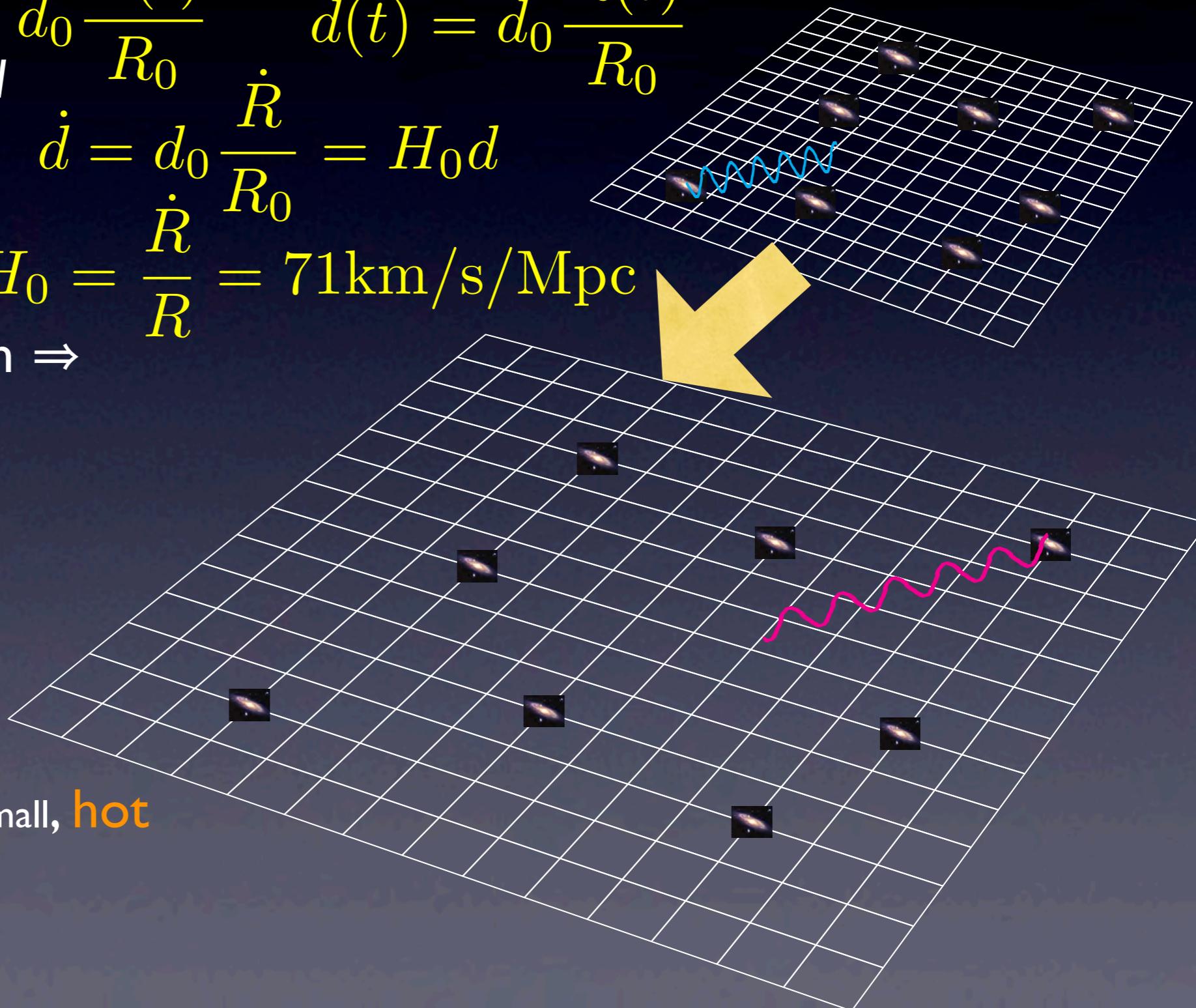
$$d = d_0 \frac{R(t)}{R_0} \quad \dot{d}(t) = d_0 \frac{\dot{R}(t)}{R_0}$$

- Hubble law:  $v=H_0 d$
- $\lambda=\lambda_0(1+z)$
- $z$ : redshift =  $R_0/R$   $H_0 = \frac{\dot{R}}{R} = 71 \text{ km/s/Mpc}$
- adiabatic expansion  $\Rightarrow$

$$T \propto R^{-1}$$

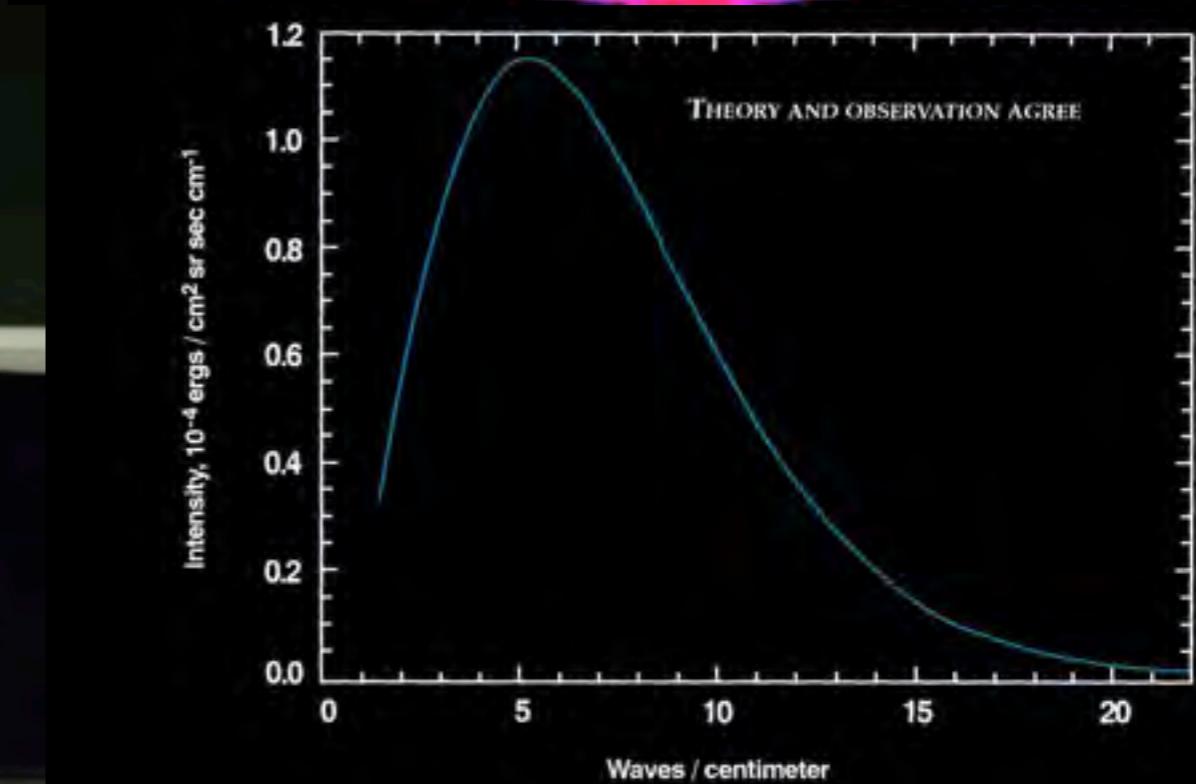
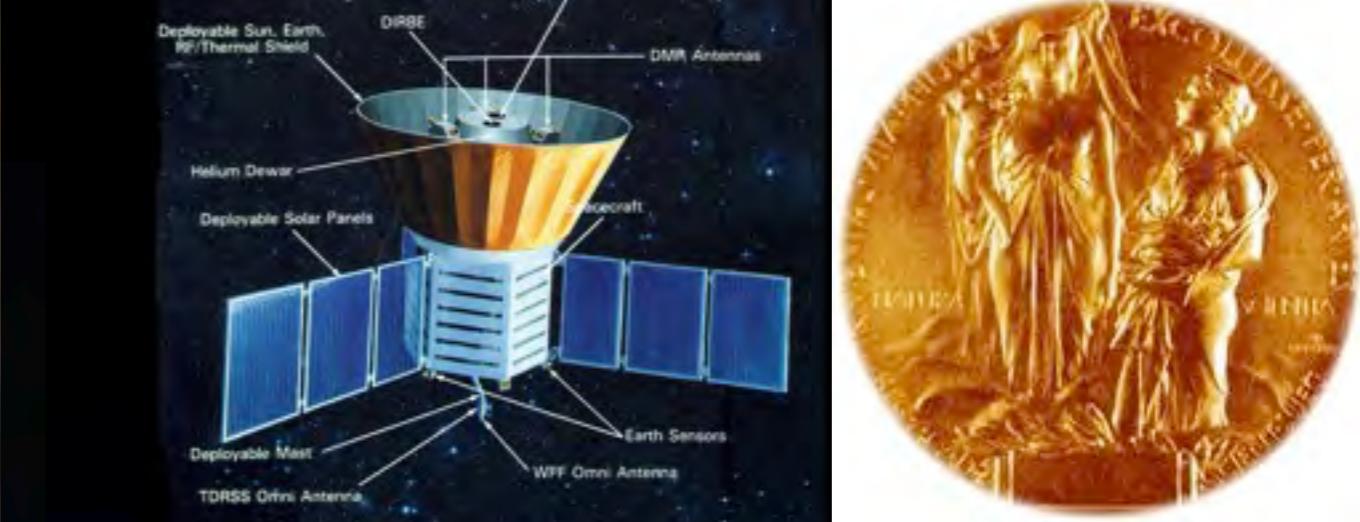
- $T=T_0(1+z)$
- bigger and **colder**
- Universe started small, **hot**

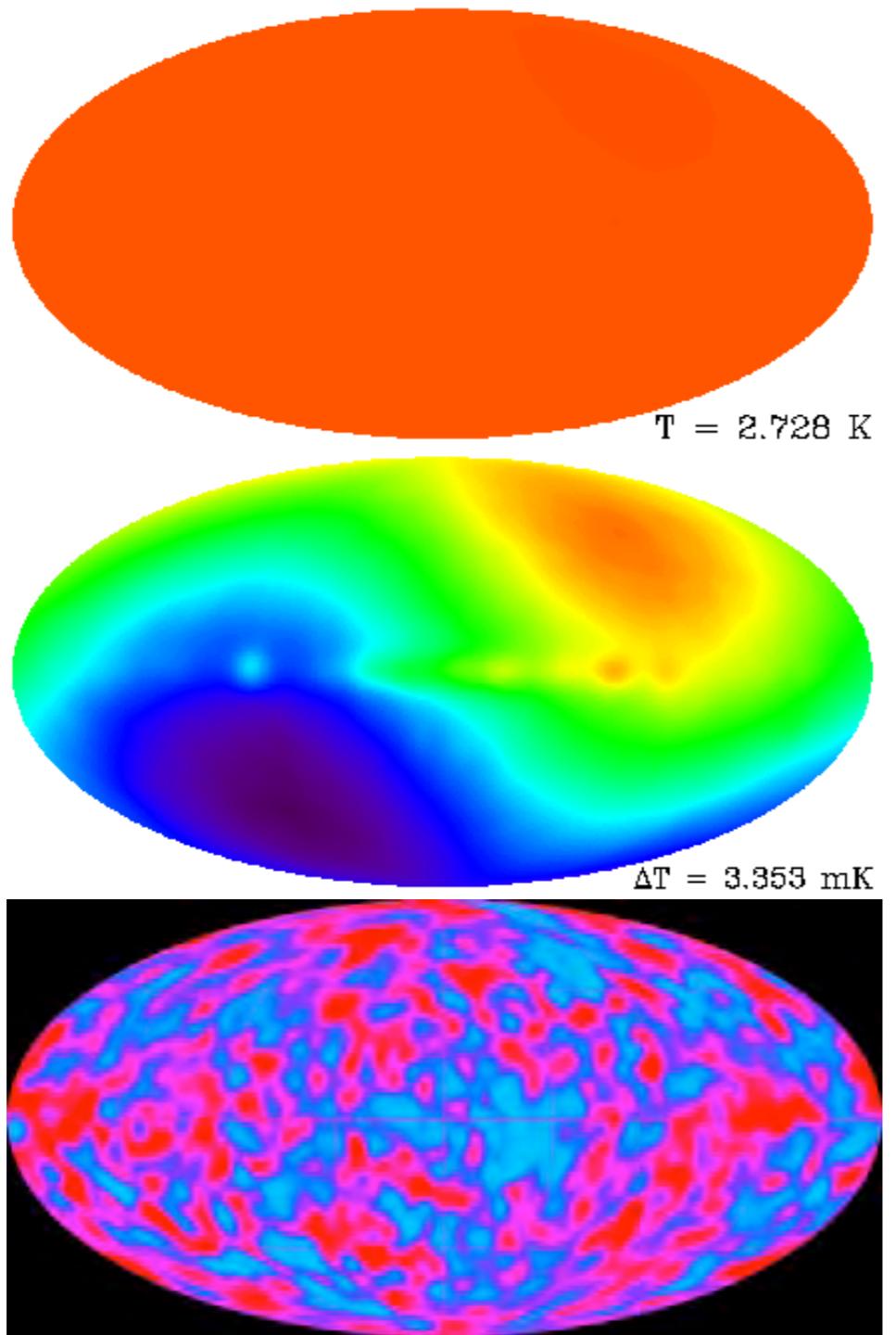
**Big Bang!**





13.7 G lyr away  
= 13.7 Gyr ago



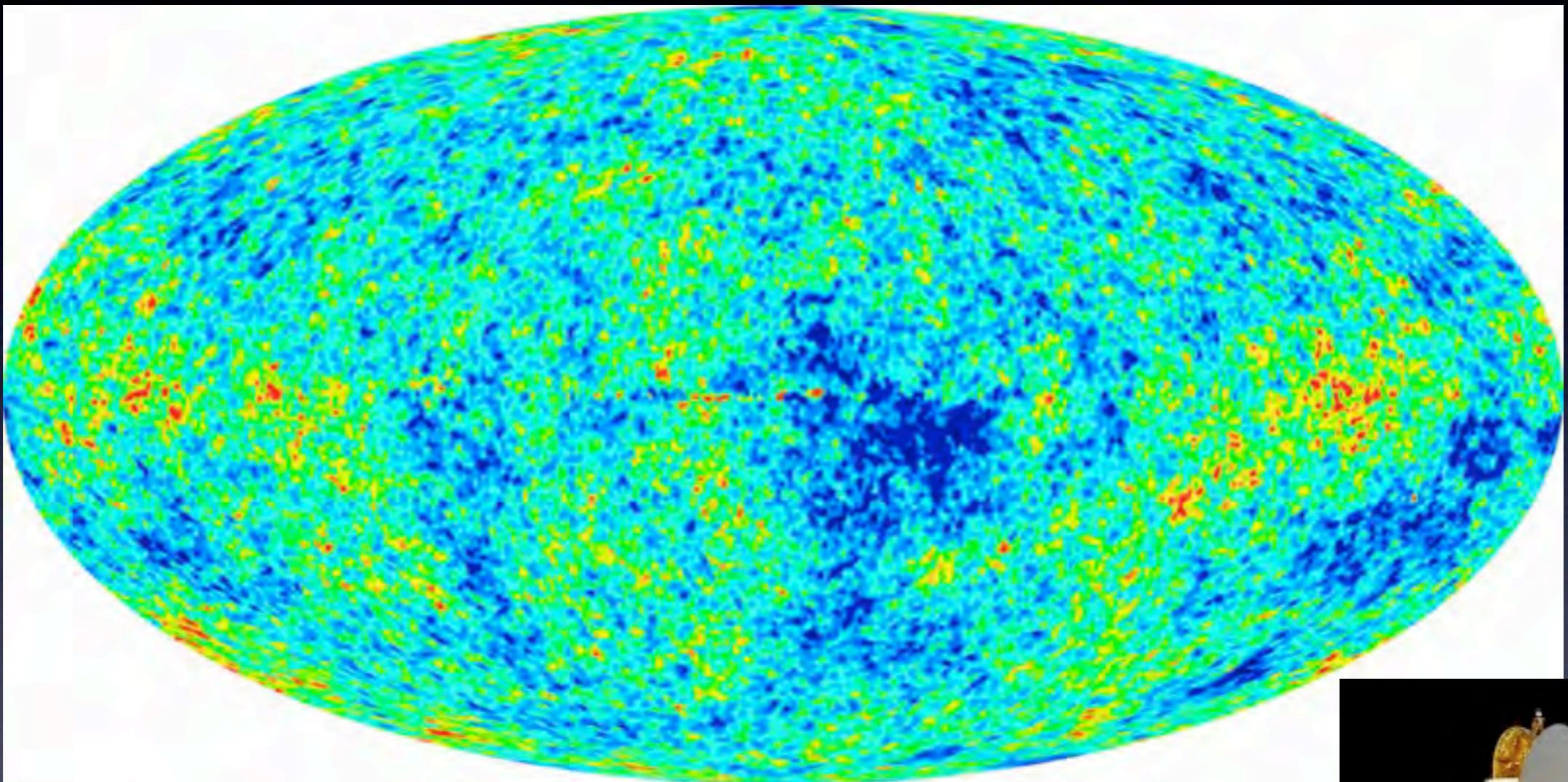


CMB temperature

CMB dipole  
we are moving at  
 $\sim 1\%$  of  $c$  relative to CMB

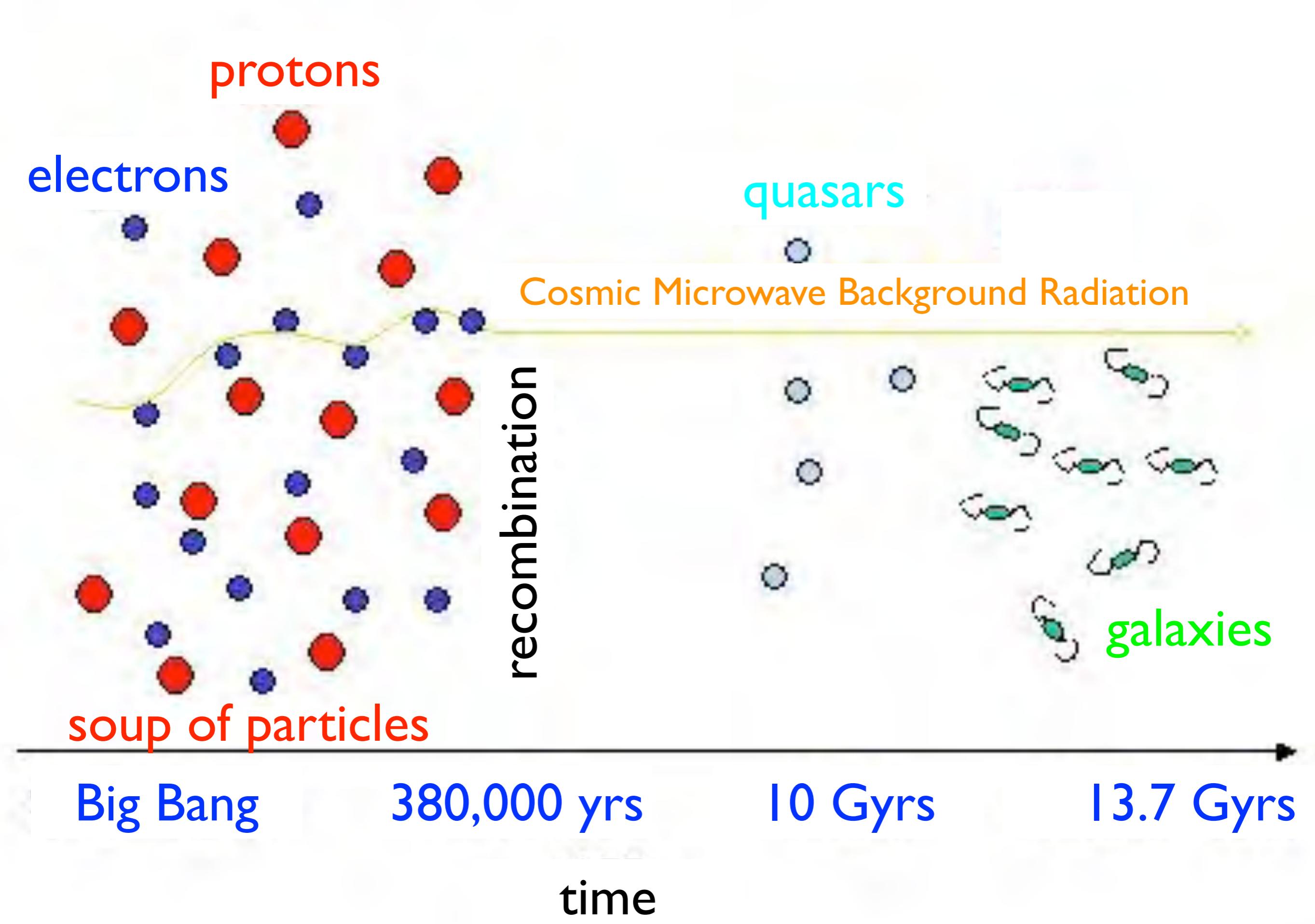
CMB anisotropy  
at  $\sim 10^{-5}$   
1mm ripple on 100m sea

# wall @ 13.7Glyr away

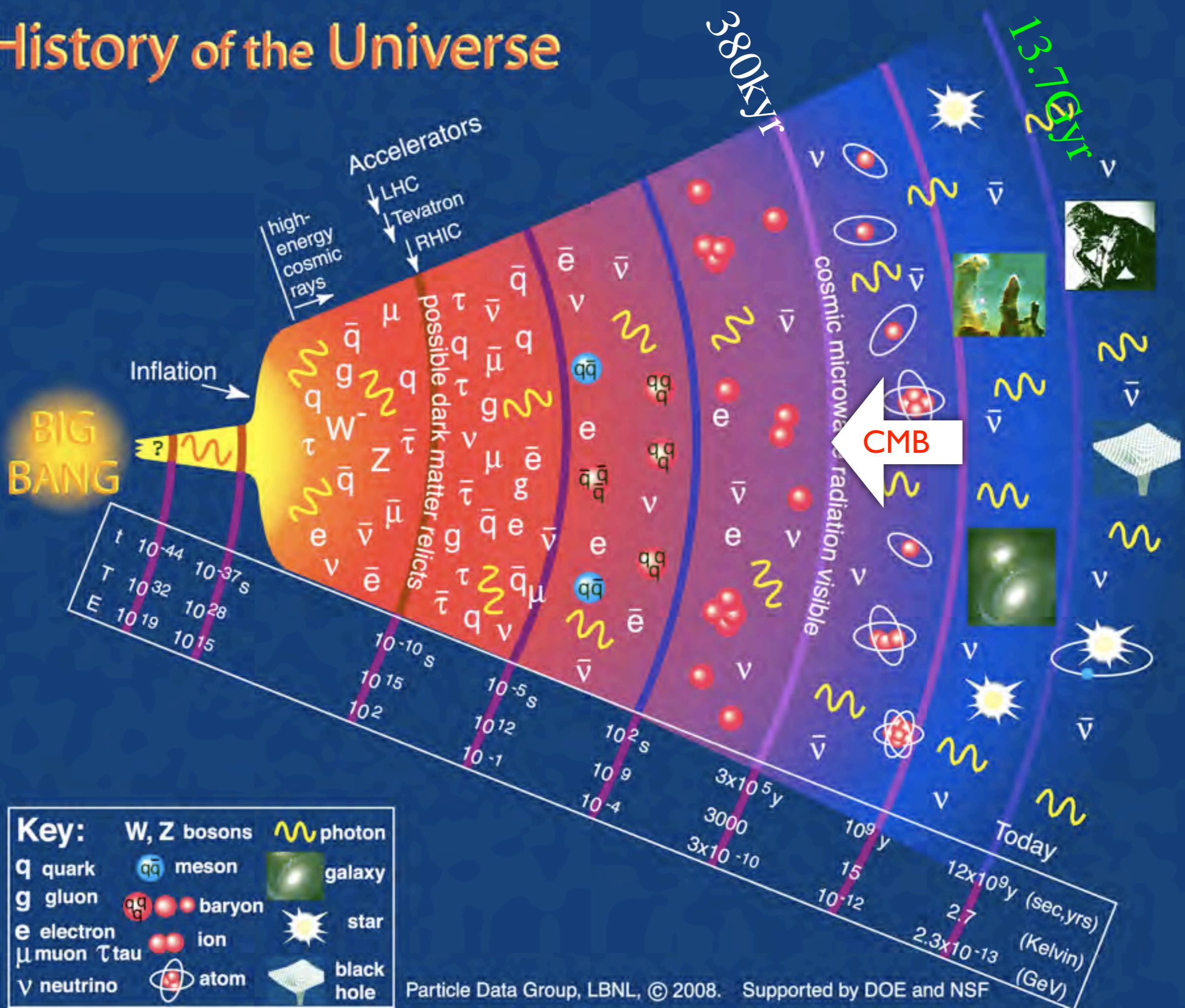


You can never see beyond this wall  
using light





# History of the Universe





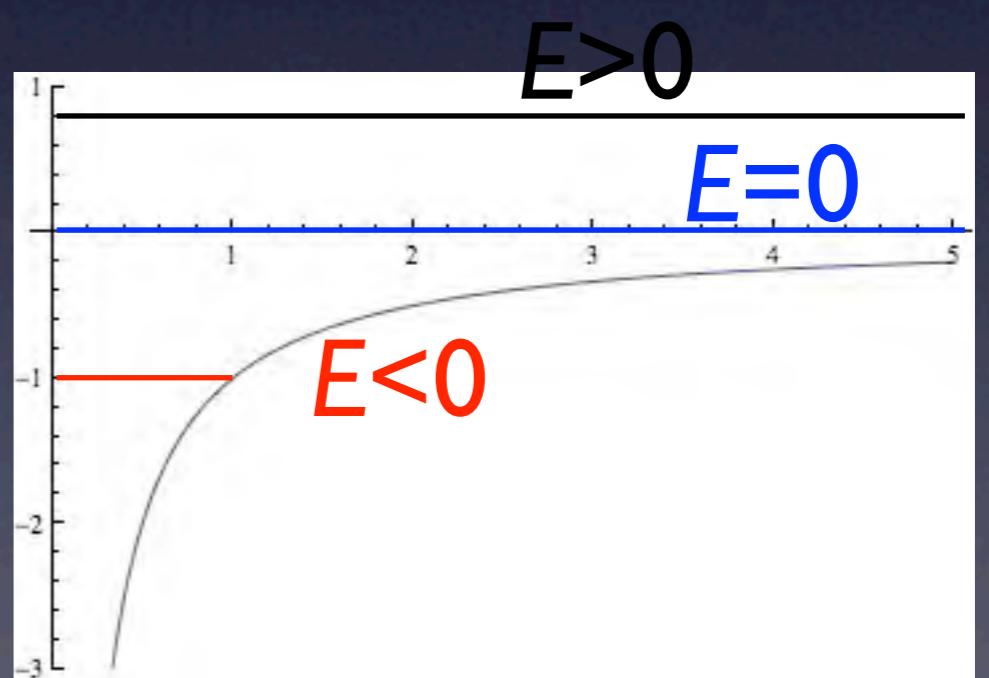
Friedmann Universe

# Friedmann Equation

$$E = \frac{m}{2} \dot{R}^2 - G_N \left( \frac{4\pi}{3} \rho R^3 \right) \frac{m}{R}$$

- “non-relativistic derivation”
- assume spherical distribution of mass density  $\rho$
- *same as full relativistic equation with a curvature term*

$$\left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi}{3} G_N \rho - \text{const}$$

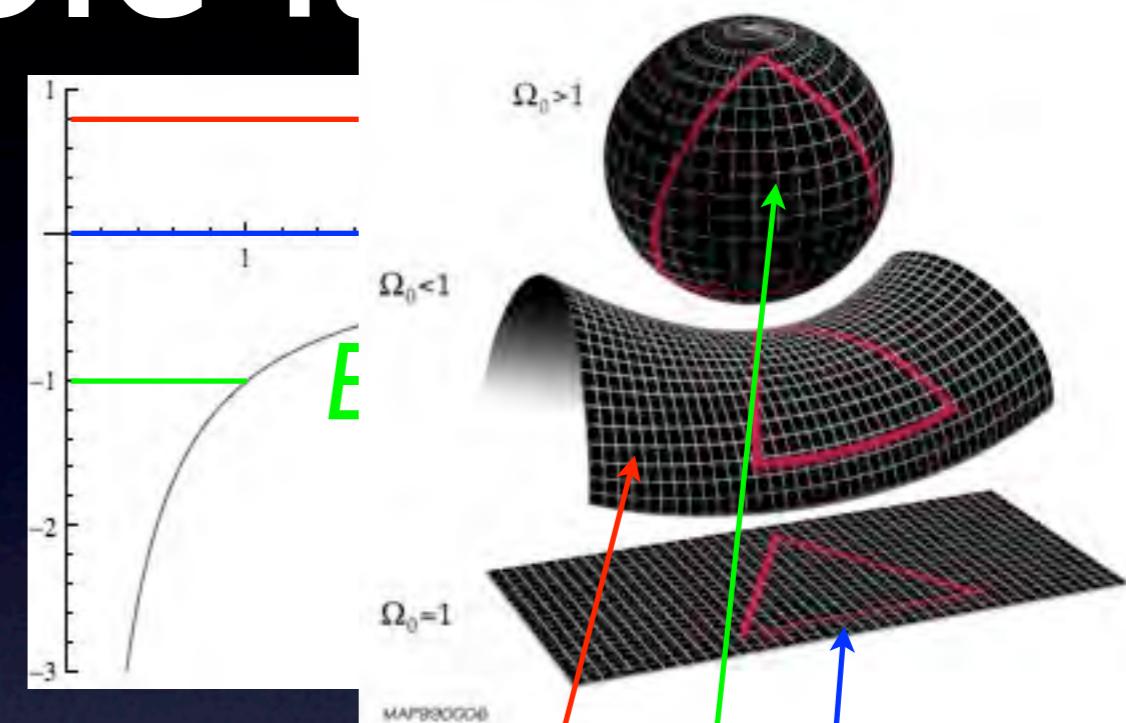
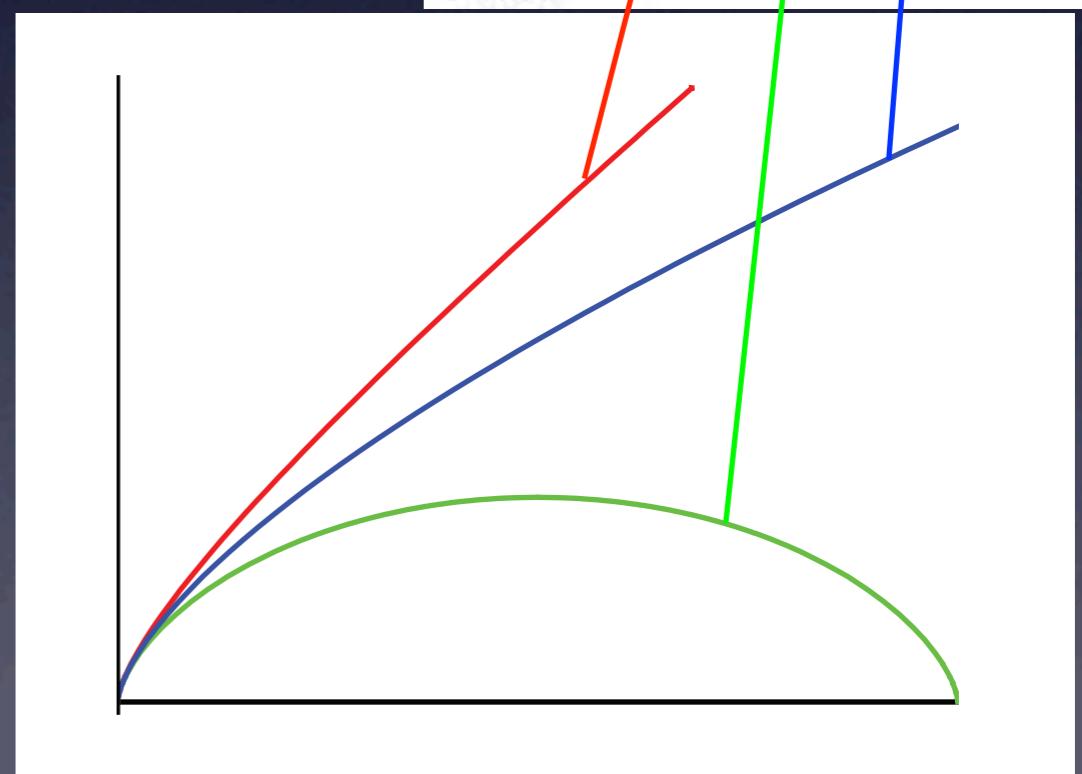


$$\frac{4\pi}{3} \rho R^3 = M$$

# three possible fates

- depending on the “energy”, there are three possible fates of the Universe
- adiabatic expansion  $\Rightarrow T \propto R^{-1}$
- past universe was smaller and hotter
- “energy” corresponds to the *curvature* of space
- actually, it is *flat*

size of the Universe



# Current Universe

- knowing the l.h.s. tells us the current energy density
- l.h.s. =  $H_0^2$  from Hubble law  $v=H_0 d$
- r.h.s. defines the *critical density*  $\rho_c$

$$\rho_c = \frac{3}{8\pi} G + N^{-1} H_0^2 = 5.3 \times 10^{-6} \text{GeVcm}^{-3}$$

- define  $\Omega_i = \rho_i / \rho_c$
- $\sum_i \Omega_i = 1$

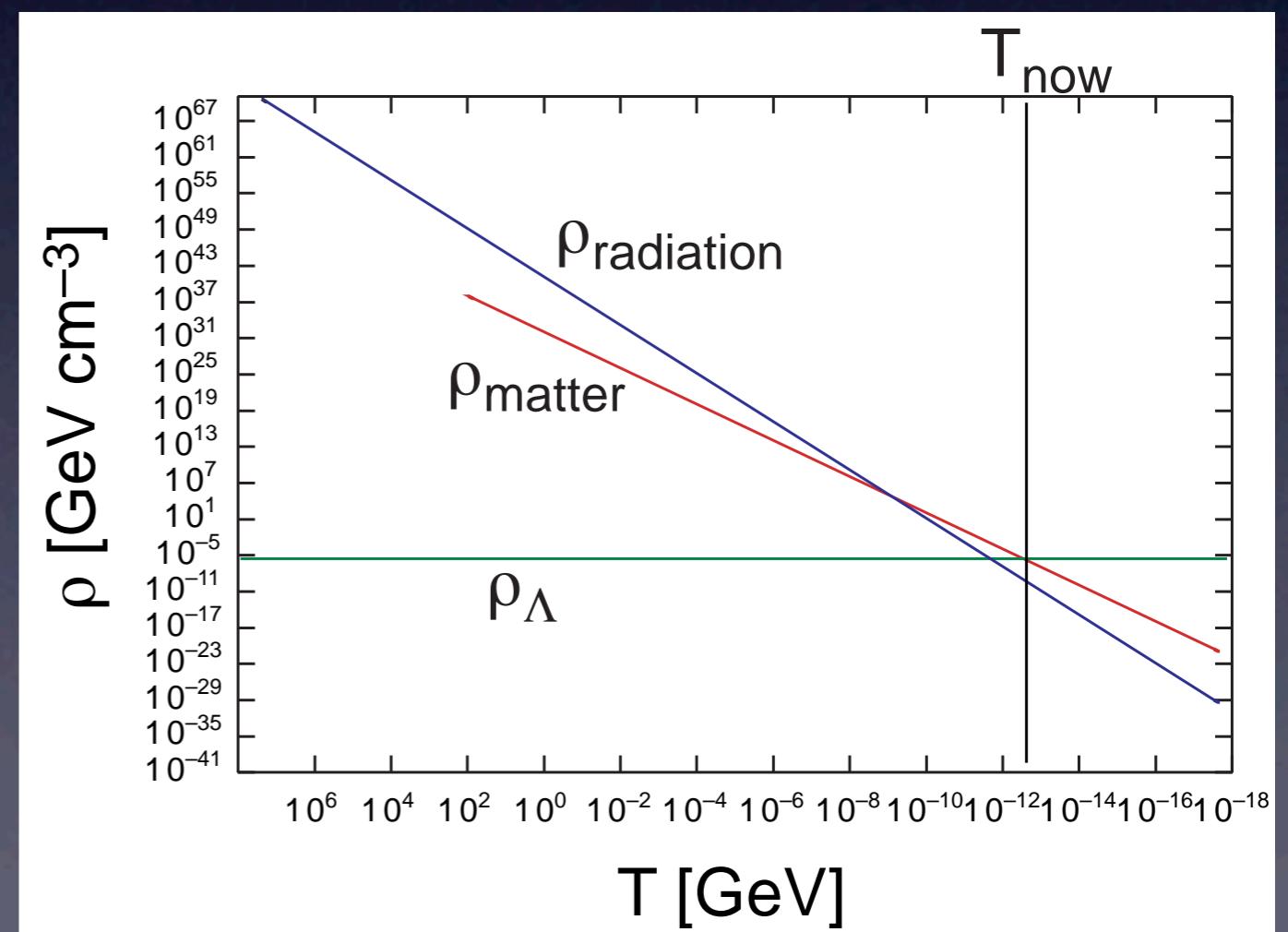
$$\left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi}{3} G_N \rho - \text{const}$$

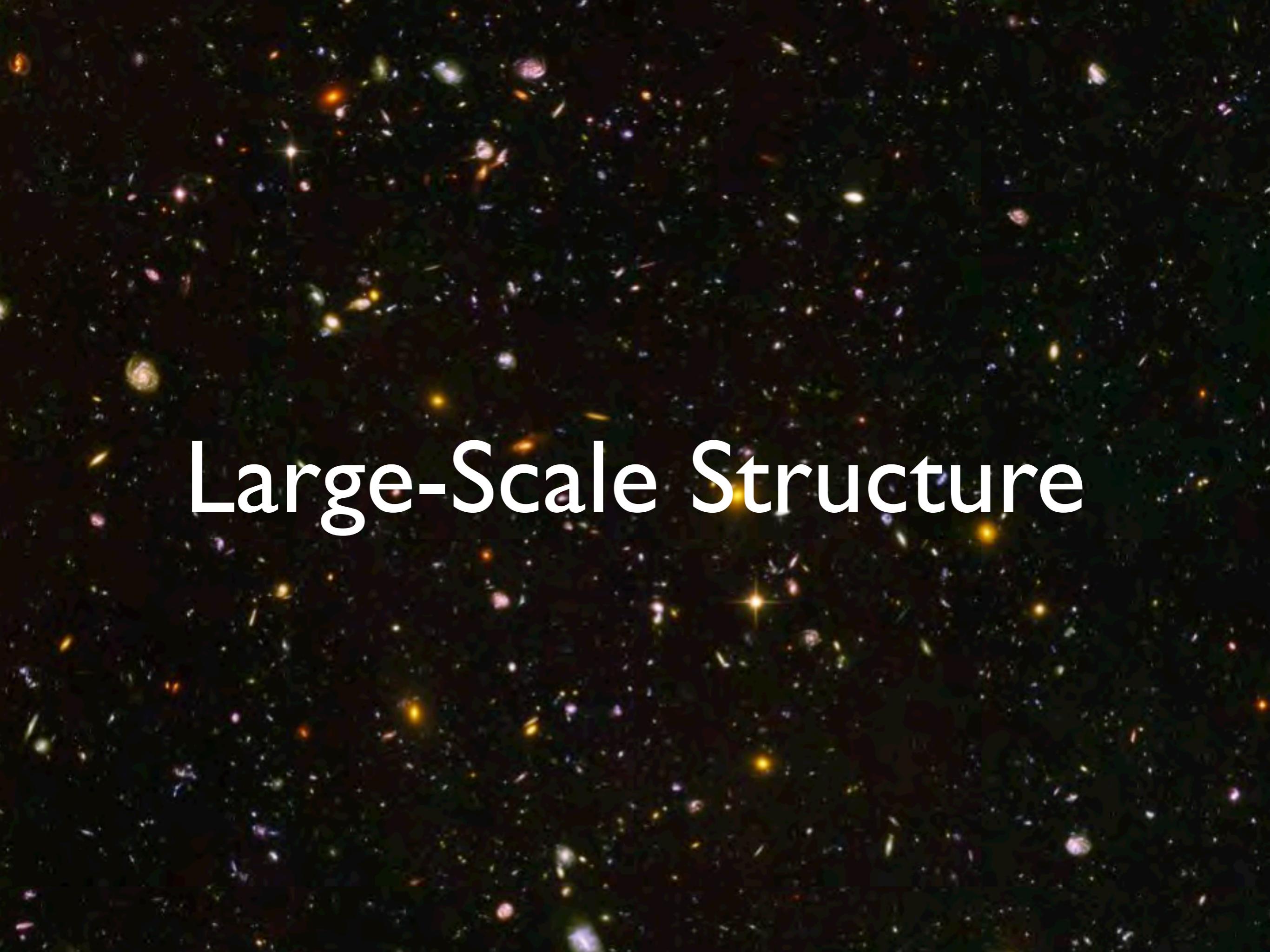
$$H_0 = \frac{\dot{R}}{R} = 71 \text{km/s/Mpc}$$

# Early Universe

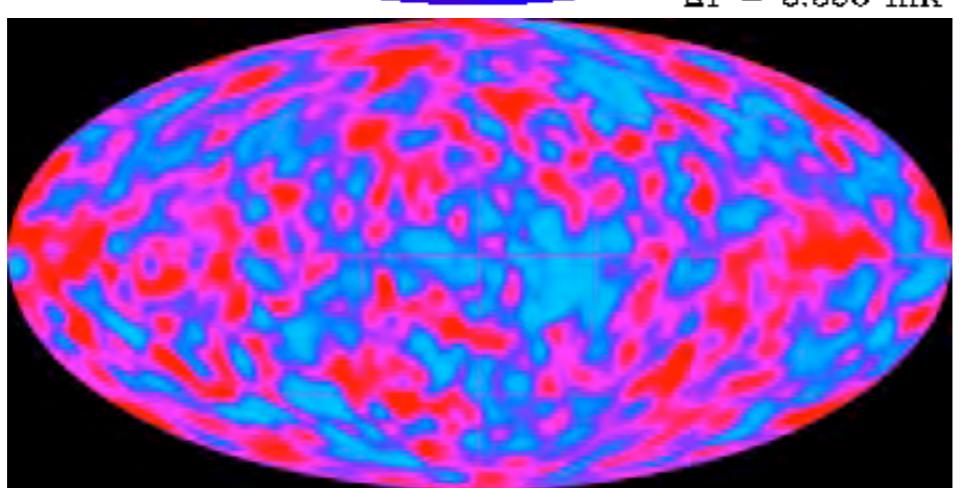
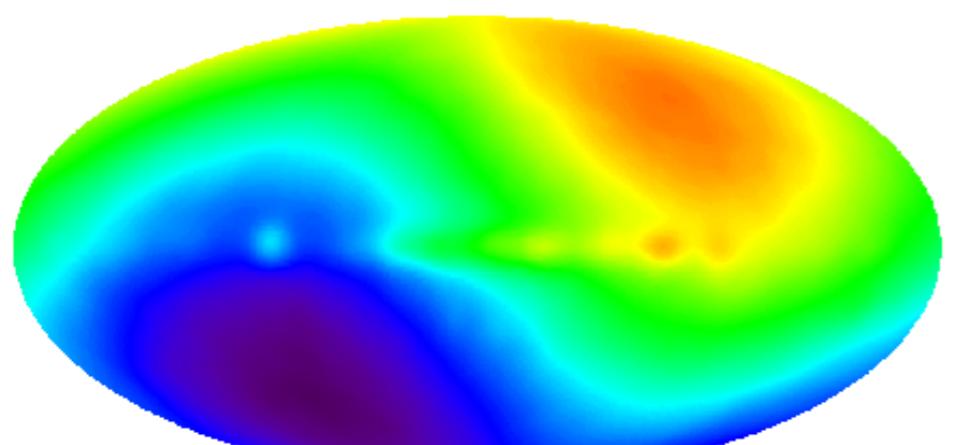
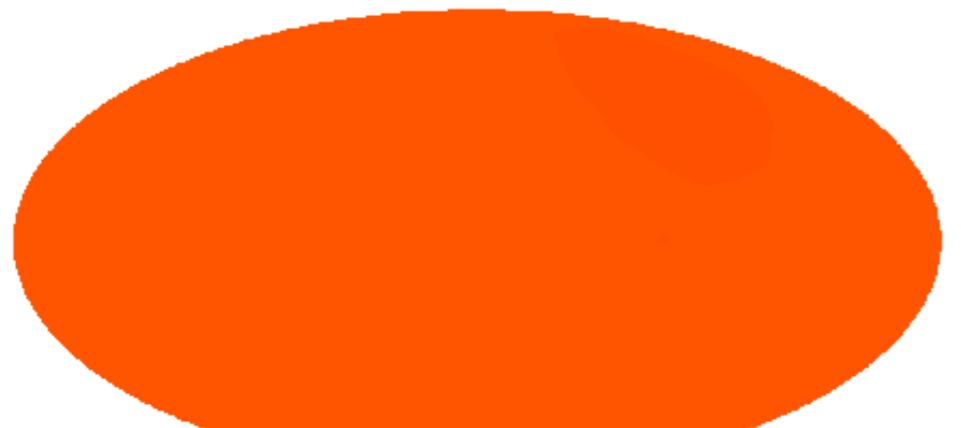
- adiabatic expansion  $\Rightarrow T \propto R^{-1}$
- $T = T_0(1+z)$
- $z$ : redshift  $= R_0/R$
- $\lambda = \lambda_0(1+z)$
- matter:  $\rho \propto R^{-3}$
- radiation (massless particles):  $\rho \propto R^{-4}$
- matter-radiation equality:  $z \approx 3300$
- recombination:  $z \approx 1300$

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3} G_N \rho - \text{const}$$



A dense field of galaxies of various sizes and colors, primarily yellow and white, set against a dark black background.

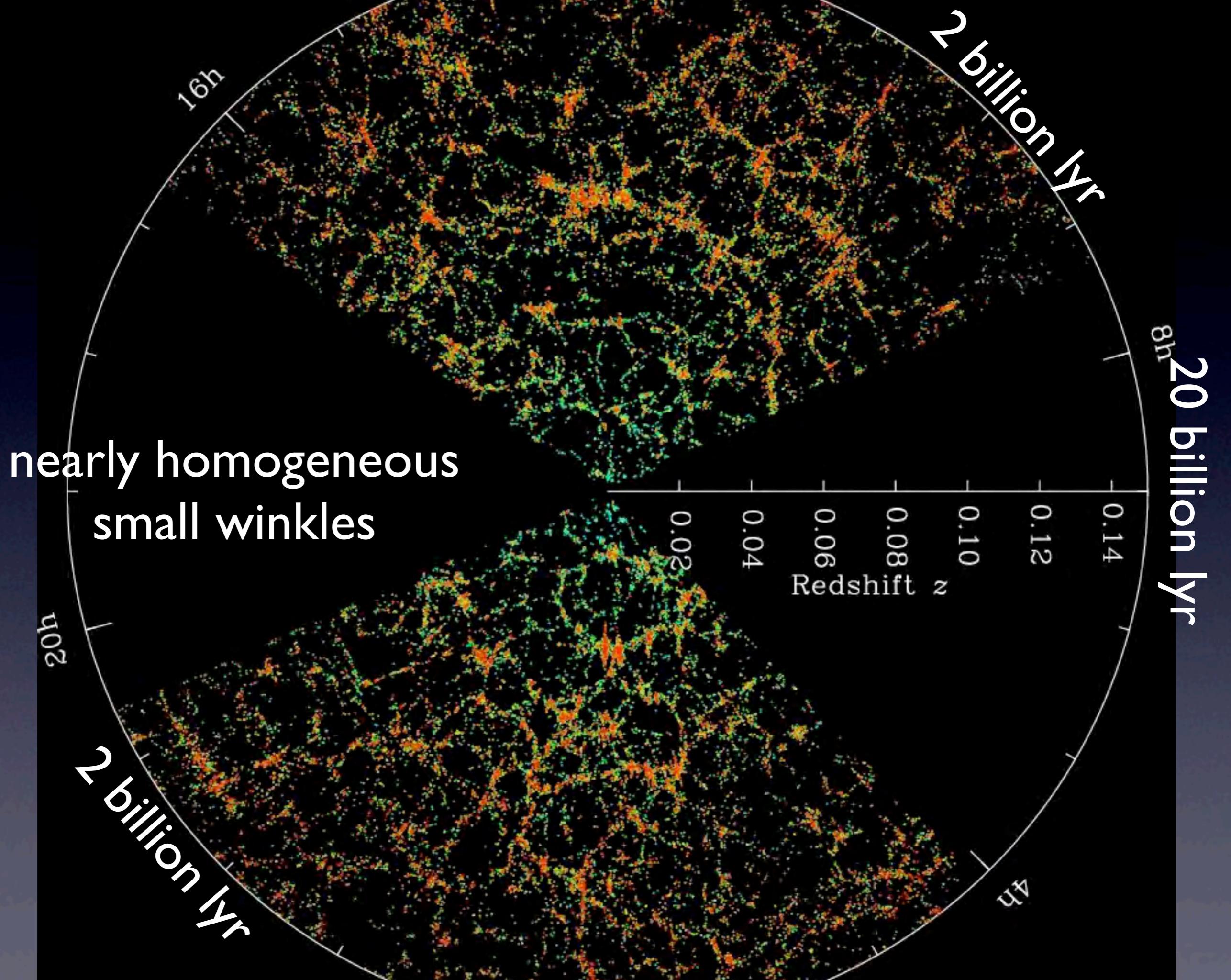
# Large-Scale Structure



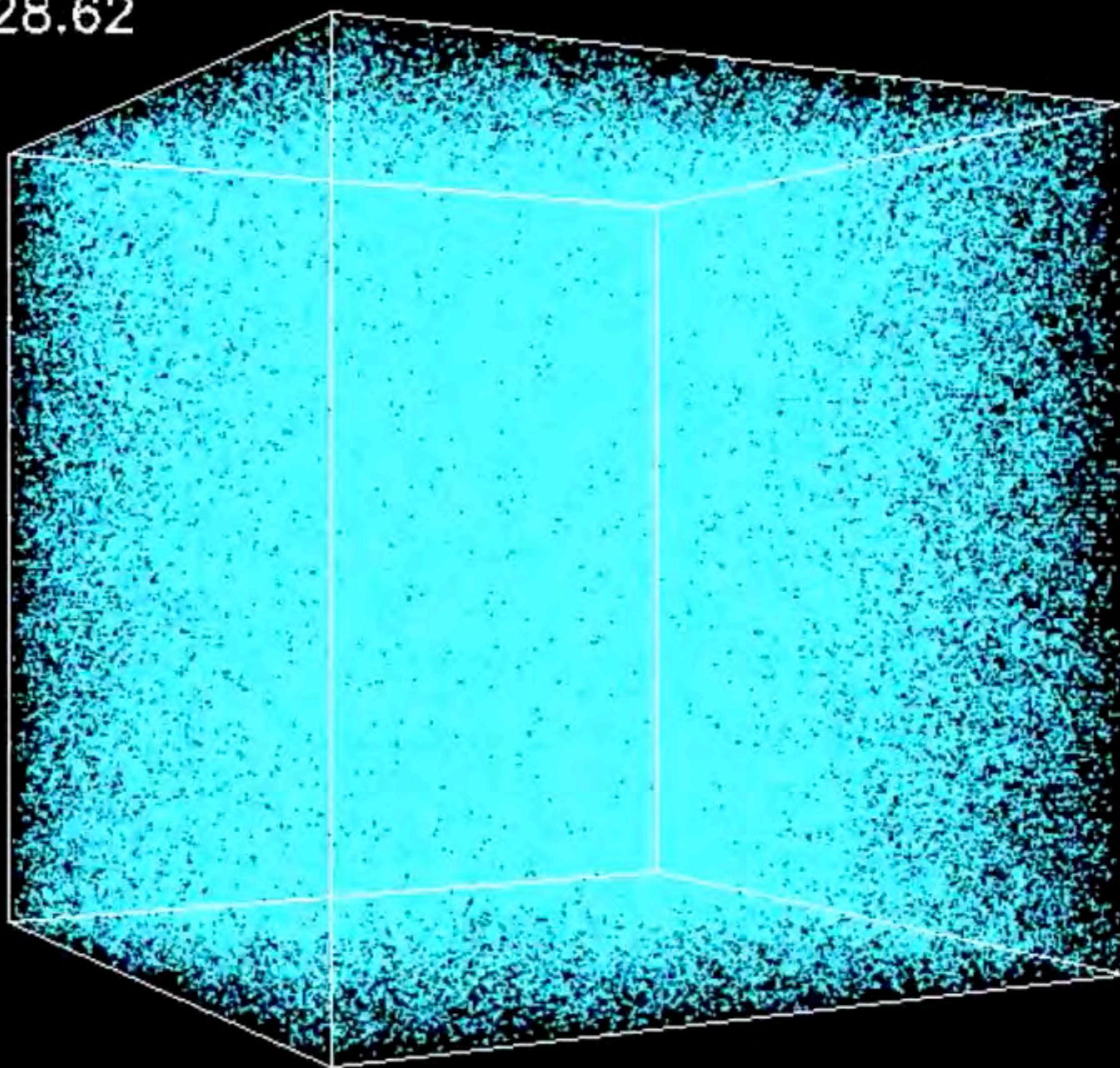
CMB temperature

CMB dipole  
we are moving at  
 $\sim 1\%$  of  $c$  relative to CMB

CMB anisotropy  
at  $\sim 10^{-5}$   
1mm ripple on 100m sea



$Z=28.62$



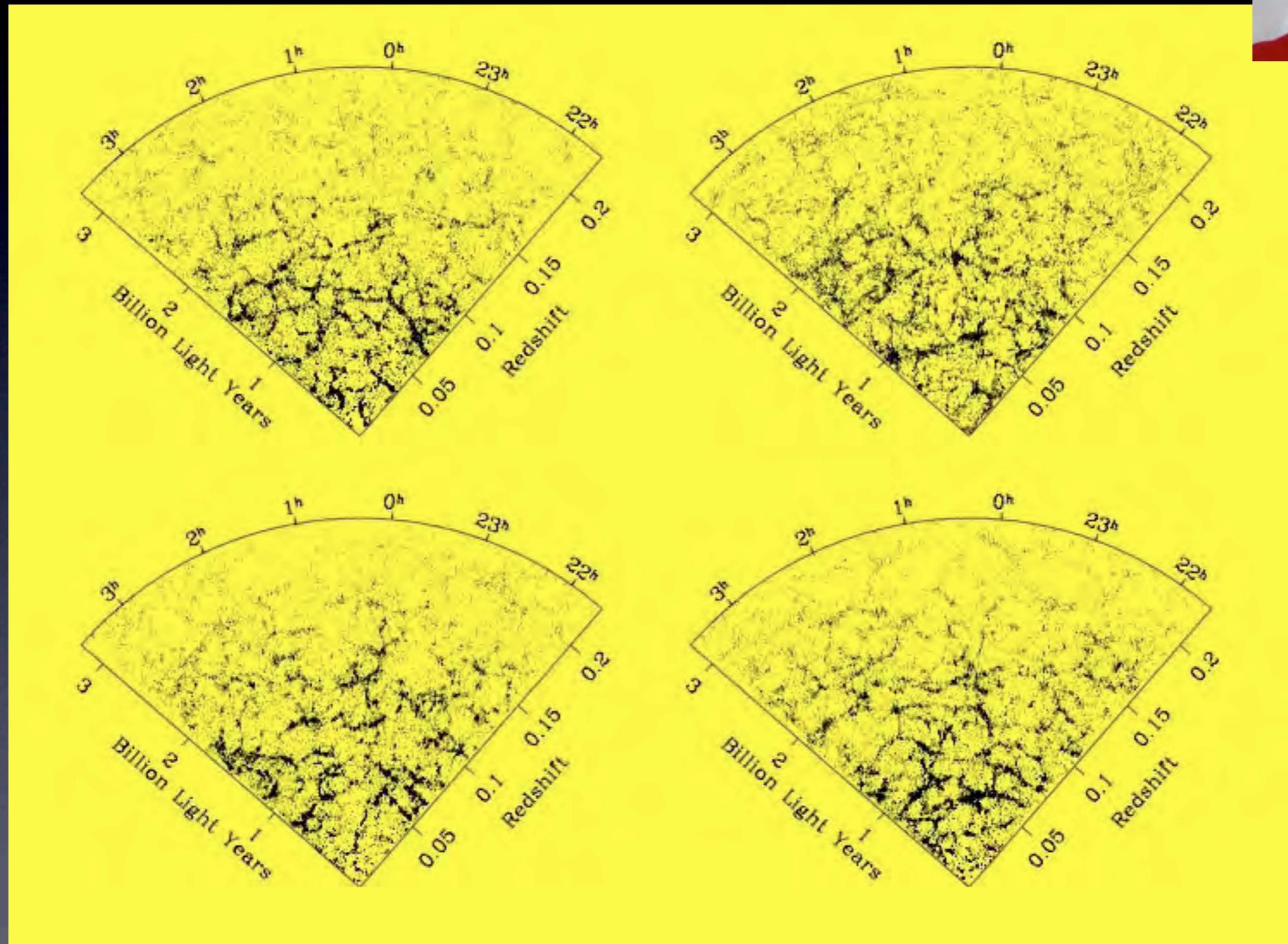
# Dark Matter is our birth mother



no dark matter

with dark matter

# one is real





# reenacting Big Ban with Cal Band





# Known Facts about Dark Matter

# Cold and Neutral

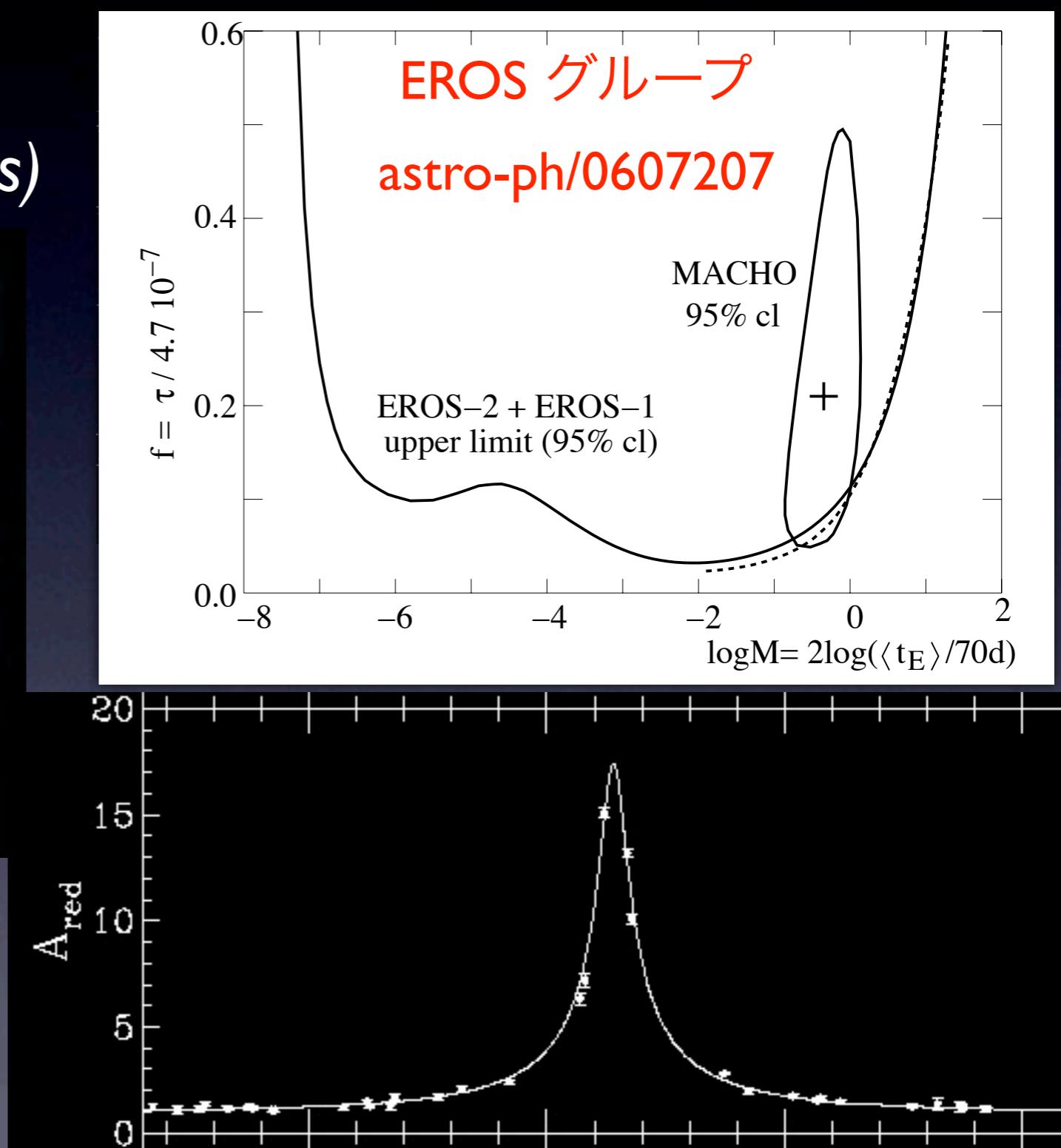
- By the time of matter-radiation equality and until now, dark matter must be non-relativistic and clump together by gravitational attraction
- must be electrically neutral

# Dim Stars?

Search for *MACHOs*  
(Massive Compact Halo Objects)



*Not enough of them!*



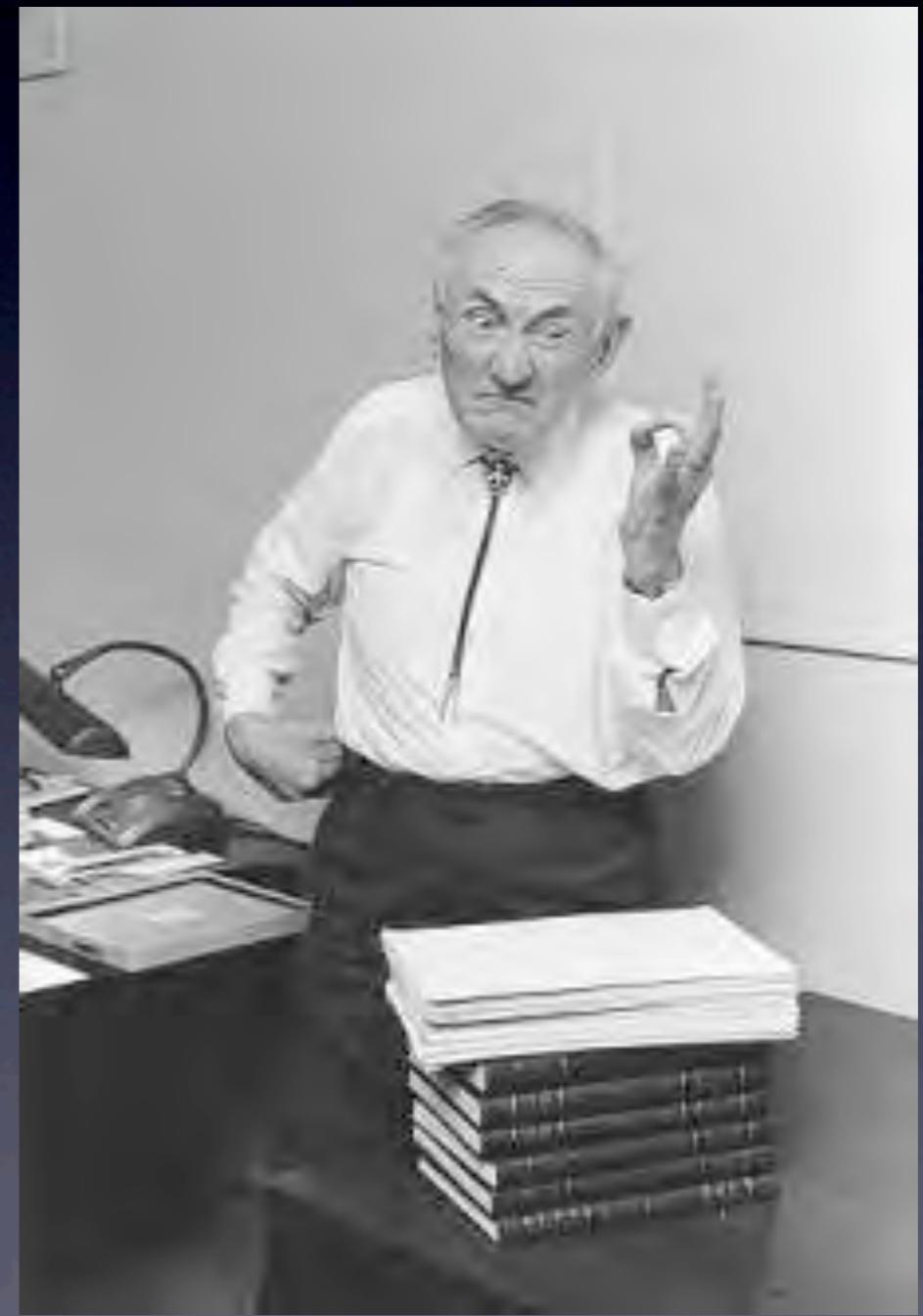
# Mass Limits

# “Uncertainty Principle”

- Clumps to form structure
- imagine  $V = G_N \frac{Mm}{r}$
- “Bohr radius”:  $r_B = \frac{\hbar^2}{G_N M m^2}$
- too small  $m \Rightarrow$  won’t “fit” in a galaxy!
- $m > 10^{-22} \text{ eV}$  “uncertainty principle” bound  
(modified from Hu, Barkana, Gruzinov, astro-ph/0003365)

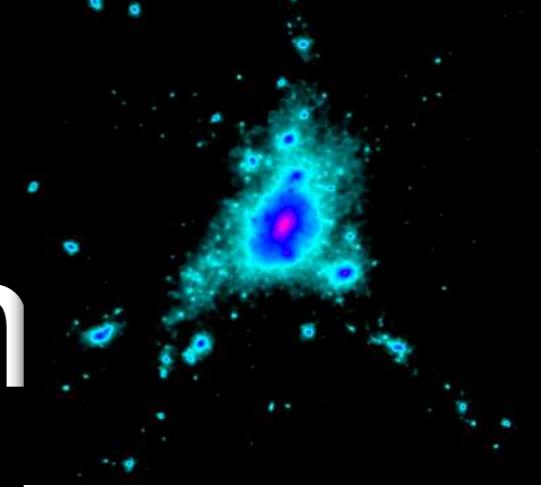
# Mass Limits

- $10^{-31}$  GeV to  $10^{50}$  GeV
- we narrowed it down to within 81 orders of magnitude
- a big progress in 70 years since Zwicky

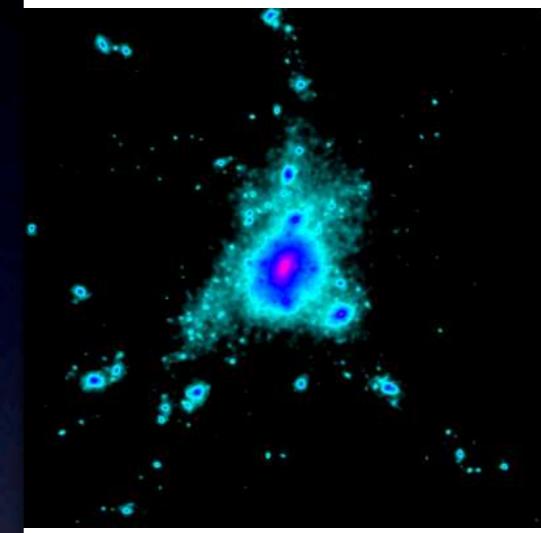
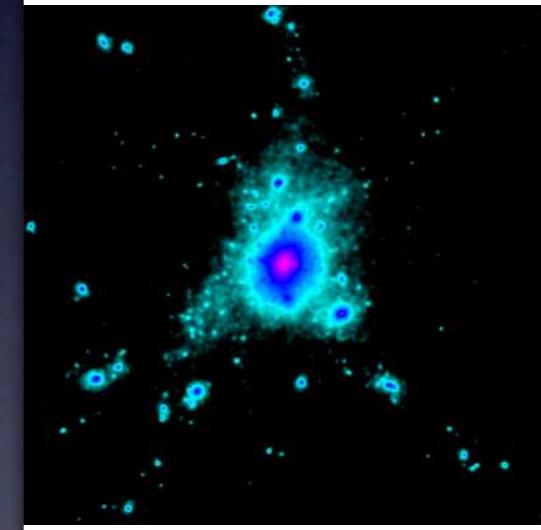
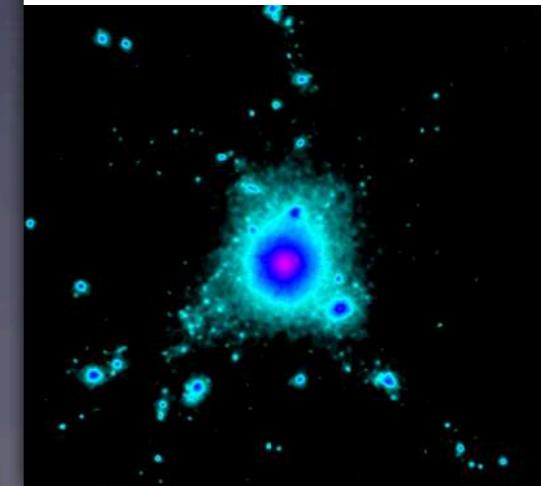


# Self-Coupling

- if self-coupling too big, will “smooth out” cuspy profile at the galactic center
- some people want it  
(Spergel and Steinhardt, astro-ph/9909386)
- need core  $< 35 \text{ kpc/h}$  from data  
 $\sigma < 1.7 \times 10^{-25} \text{ cm}^2 (\text{m/GeV})$   
(Yoshida, Springel, White, astro-ph/0006134)
- bullet cluster:  
 $\sigma < 1.7 \times 10^{-24} \text{ cm}^2 (\text{m/GeV})$   
(Markevitch et al, astro-ph/0309303)



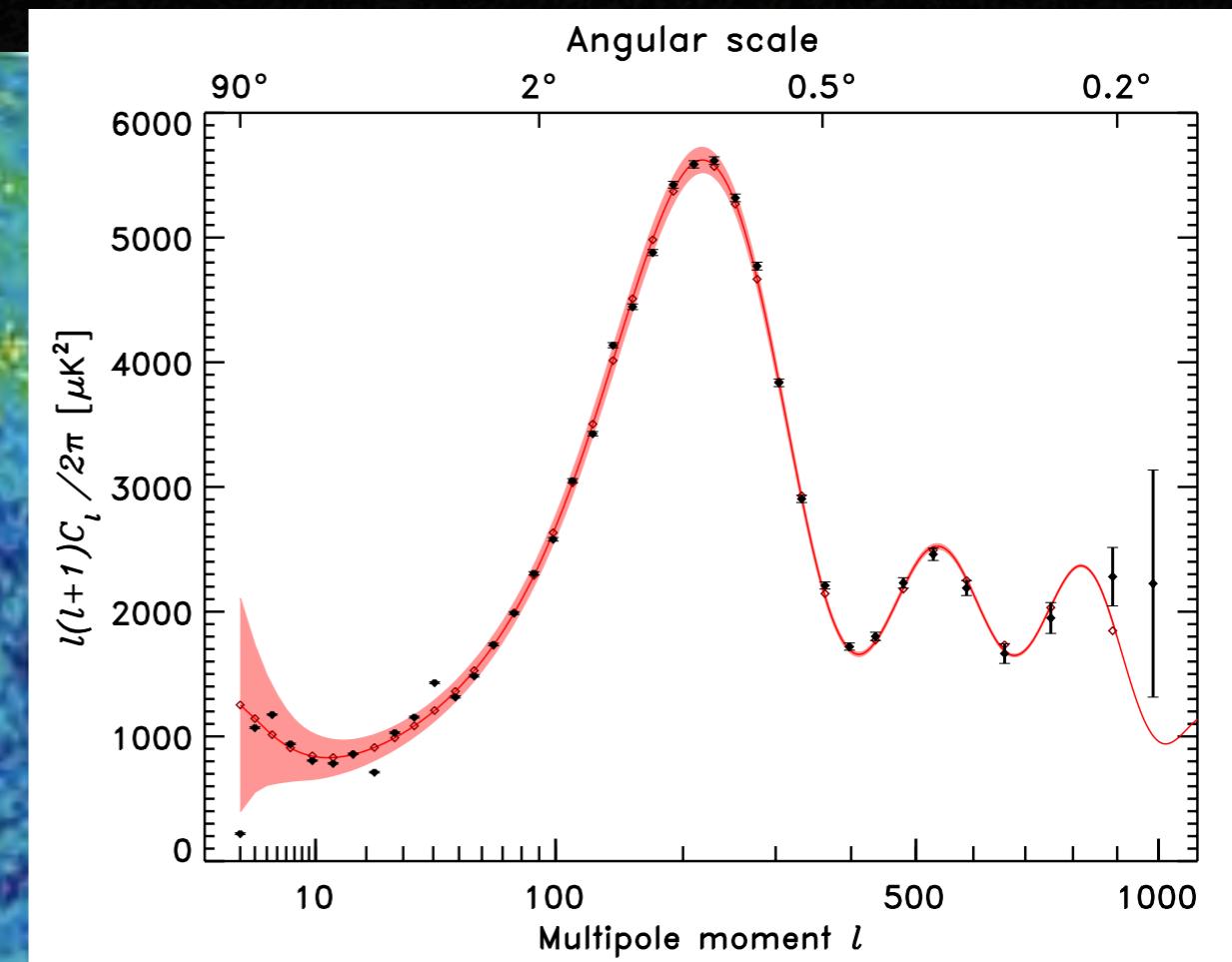
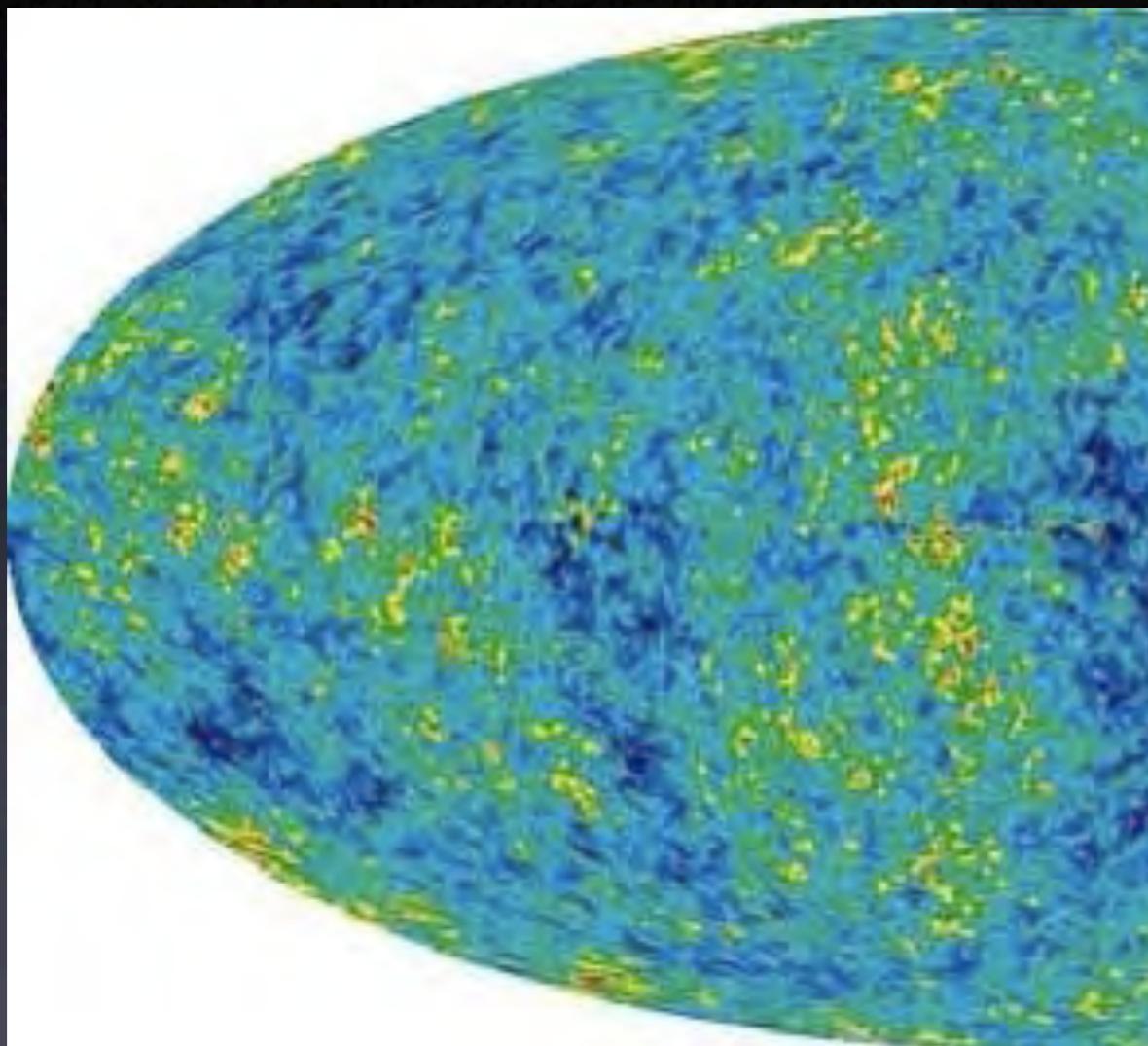
1 : 0.82 : 0.65

**S1Wa**  
 $\sigma^* = 0.1 \text{ cm}^2 \text{ g}^{-1}$   
 $r_c = 40 h^{-1} \text{kpc}$   
1 : 0.88 : 0.66**S1Wb**  
 $\sigma^* = 1.0 \text{ cm}^2 \text{ g}^{-1}$   
 $r_c = 100 h^{-1} \text{kpc}$   
1 : 0.91 : 0.72**S1Wc**  
 $\sigma^* = 10.0 \text{ cm}^2 \text{ g}^{-1}$   
 $r_c = 160 h^{-1} \text{kpc}$   
1 : 0.98 : 0.89

# Lifetime

- At least of the order of age of the universe  
14Gyr
- Beyond that, it depends on decay modes,  
branching fractions, all model-dependent

# Cosmological scales



$$\frac{\text{matter}}{\text{all atoms}} = 5.70^{+0.39}_{-0.61}$$

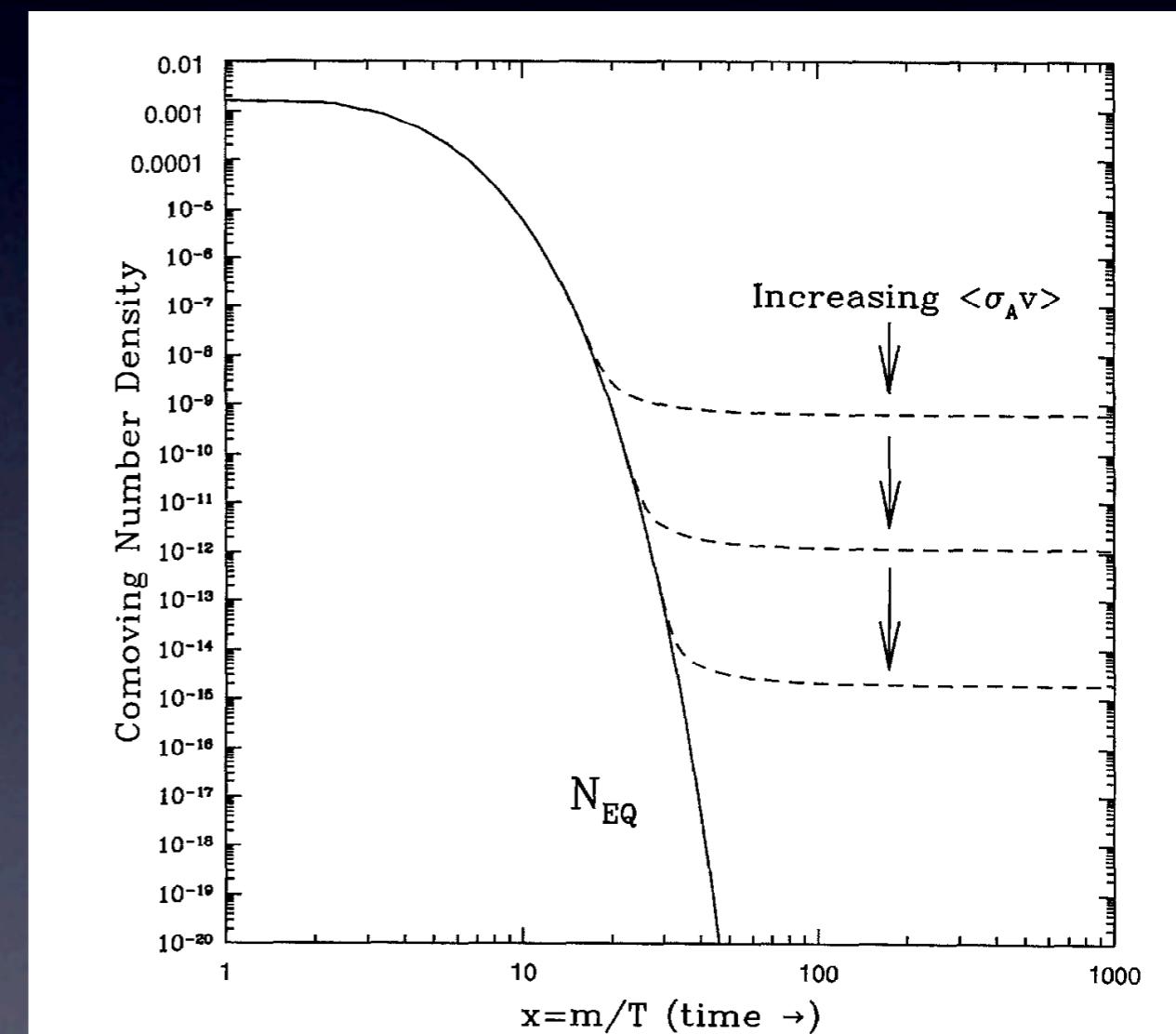




WIMP paradigm

# thermal relic

- thermal equilibrium when  $T > m_\chi$
- Once  $T < m_\chi$ , no more  $\chi$  created
- if stable, only way to lose them is annihilation
- but universe expands and  $\chi$  get dilute
- at some point they can't find each other
- their number in comoving volume “frozen”



# Freeze-out

$$H \approx g_*^{1/2} \frac{T^2}{M_{Pl}}$$

- WIMP freezes out when the annihilation rate drops below the expansion rate
- Yield  $Y = n/s$  constant under expansion
- stronger annihilation  $\Rightarrow$  less abundance

$$\Gamma_{\text{ann}} \approx \langle \sigma_{\text{ann}} v \rangle n$$

$$H(T_f) = \Gamma_{\text{ann}}$$

$$n \approx g_*^{1/2} \frac{T_f^2}{M_{Pl} \langle \sigma_{\text{ann}} v \rangle}$$

$$s \approx g_* T^3$$

$$Y = \frac{n}{s} \approx g_*^{-1/2} \frac{1}{M_{Pl} T_f \langle \sigma_{\text{ann}} v \rangle}$$

$$\Omega_\chi = \frac{m_\chi Y s_0}{\rho_c}$$

$$\approx g_*^{-1/2} \frac{x_f}{M_{Pl}^3 \langle \sigma_{\text{ann}} v \rangle} \frac{s_0}{H_0^2}$$

# Order of magnitude

- “Known”  $\Omega_\chi=0.23$  determines the WIMP annihilation cross section
- simple estimate of the annihilation cross section
- weak-scale mass!!!

$$\Omega_\chi \approx g_*^{-1/2} \frac{x_f}{M_{Pl}^3 \langle \sigma_{\text{ann}} v \rangle} \frac{s_0}{H_0^2}$$
$$\langle \sigma_{\text{ann}} v \rangle \approx \frac{1.12 \times 10^{-10} \text{GeV}^{-2} x_f}{g_*^{1/2} \Omega_\chi h^2}$$
$$\sim 10^{-9} \text{GeV}^{-2}$$
$$\langle \sigma_{\text{ann}} v \rangle \approx \frac{\pi \alpha^2}{m_\chi^2}$$
$$m_\chi \approx 300 \text{ GeV}$$

# therma

- Solve the Boltzmann eq

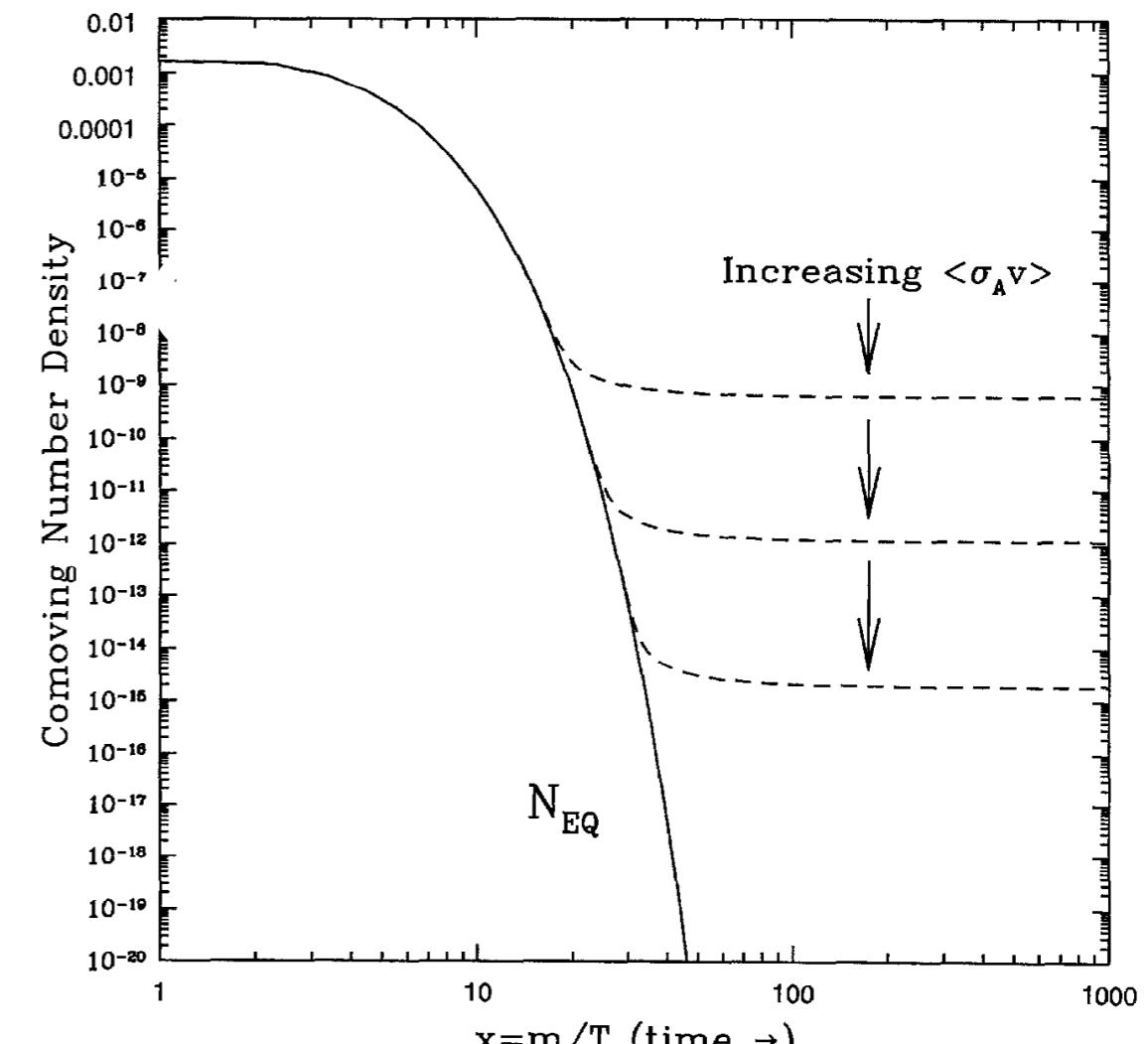
$$\frac{dn_1}{dt} + 3Hn_1 = - \int \prod_{i=1}^4 \frac{d^3 p_i}{(2\pi)^3 2E_i} |\mathcal{M}(p)| [f_1 f_2 (1 \pm f_3) (1 \pm f_4) -$$

- assume Maxwell distribution,  $T = z = \chi$ ,  $E_1 = E_2 = m_\chi$

$$\frac{dn}{dt} + 3Hn = -\langle \sigma_{\text{ann}} v \rangle (n^2 - n_{eq}^2)$$

- Note momentum dependence may be important close to thresholds, resonances
- reproduce the estimate with

$$x_f \approx 24 + \ln \frac{m_\chi}{100 \text{GeV}} + \ln \frac{\langle \sigma_{\text{ann}} v \rangle}{10^{-9} \text{GeV}^{-2}} - \frac{1}{2} \ln \frac{g_*}{100}$$



# WIMP

- A stable particle at the weak scale with “EM-strength” coupling naturally gives the correct abundance
- This is where we expect new particles because of the hierarchy problem!
- Many candidates of this type: SUSY, little Higgs with T-parity, Universal Extra Dimensions, etc
- If so, we may even create dark matter at accelerators

# Minimal Model

- Dark Matter clearly a new degree of freedom
- The smallest degree of freedom you can add to the QFT is a real Klein-Gordon field  $S$ : **dof=1**
- assign odd  $\mathbb{Z}_2$  parity to  $S$ , everything else even
- Most general renormalizable coupling

$$L_S = \frac{1}{2} \partial_\mu S \partial^\mu S - \frac{1}{2} m_S^2 S^2 - \frac{k}{2} |H|^2 S^2 - \frac{h}{4!} S^4.$$

# Consistency check

- correct Dark Matter abundance
- evades direct detection limits
- satisfies triviality/instability limits from RGE
- consistent with precision electroweak data

