Universität Zürich UZH

LHCb

HEP 2013
Stockholm
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(info@eps-hep2013.eu)

Studies of electroweak penguin transitions of $b \to s\mu\mu$

Nicola Serra on behalf of the LHCb collaboration

Outlook

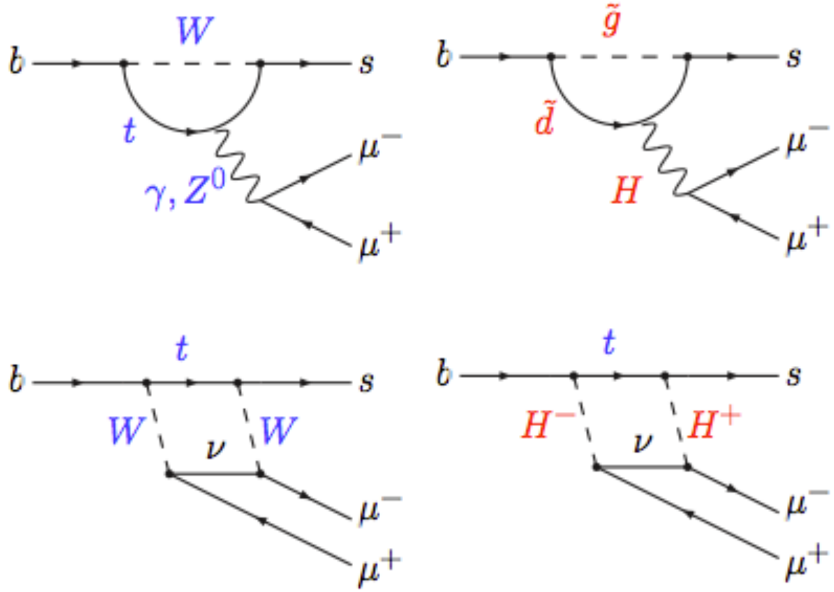
- $B^0 \to K^* \mu\mu$:
  - New observables shown here for the first time
- $B^+ \to K^+ \mu\mu$:
  - Time-integrated CP violation (new)
  - New resonance structure at low recoil (new)

    → see talk by Giovanni Carboni
- $B_s \to \phi \mu\mu$
- $\Lambda_b \to \Lambda \mu\mu$

→ see talk by Flavio Archilli for $B_s \to \mu\mu$

Introduction

Rare processes where new physics can enter to modify SM amplitudes



In the SM contribution from

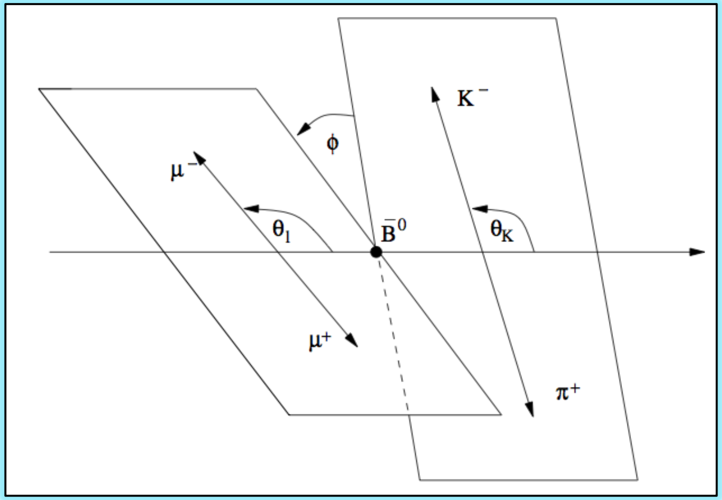
$$\mathcal{O}_7 \sim m_b(\bar{s}_L \sigma_{\mu\nu} b_R) F_{\mu\nu}$$

$$\mathcal{O}_9 \sim (\bar{s}b)_{V-A}(\bar{\ell}\ell)_V$$

$$\mathcal{O}_{10} \sim (\bar{s}b)_{V-A}(\bar{\ell}\ell)_A$$

$$H = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum (C_i^{SM} + C_i^{NP}) O_i^{SM} + \sum \frac{c}{\Lambda_{NP}} \; O_{NP}$$

$B^0 \to K^*(\to K^+\pi^-)\mu\mu$



The observables F_L and S_i are function of Wilson coefficients and form-factors

Well known observable is the forward-backward asymmetry of the dimuon system: $S_6^s = \frac{3}{4} A_{FB}$

$$\frac{1}{\Gamma}\frac{d^3(\Gamma+\bar{\Gamma})}{d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = \frac{9}{32\pi}\Big[\frac{3}{4}(1-F_L)\sin^2\theta_K + F_L\cos^2\theta_K + \frac{1}{4}(1-F_L)\sin^2\theta_K\cos 2\theta_\ell$$
$$- F_L\cos^2\theta_K\cos 2\theta_\ell +$$
$$S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + S_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi +$$
$$S_5\sin 2\theta_K\sin\theta_\ell\cos\phi + S_6^s\sin^2\theta_K\cos\theta_\ell +$$
$$S_7\sin 2\theta_K\sin\theta_\ell\sin\phi +$$
$$S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Big]$$

Altmannshofer et al. (2008)

$B^0 \to K^*(\to K^+\pi^-)\mu\mu$

Observables with limited dependence on form-factor uncertainty have been proposed by several authors:

Kruger-Matias (2005), Matias et al. (2012), Egede-Matias-Hurth-Ramon-Reece (2008), Bobeth-Hiller-Van Dyk (2010-11), Beciveric-Schneider (2012)

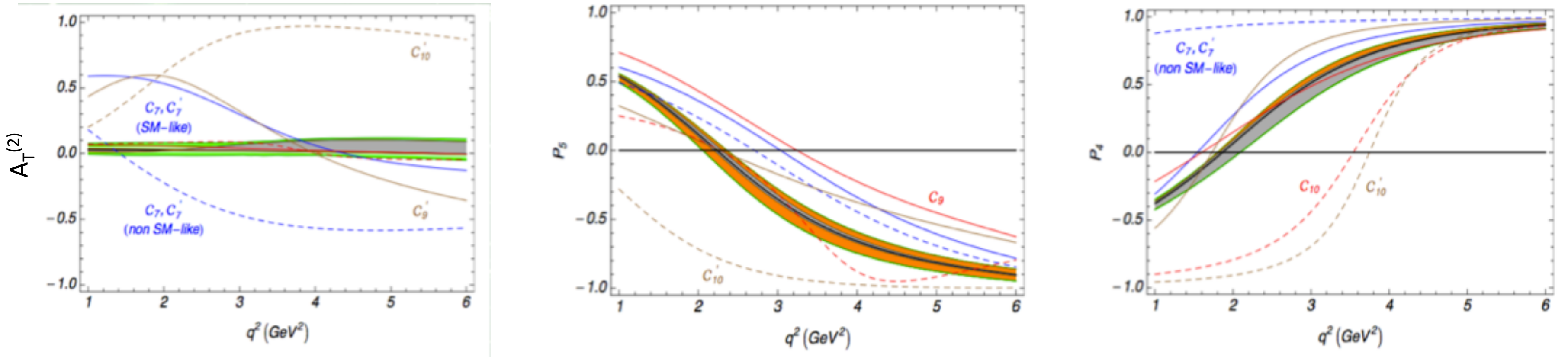
N.D.: There are other observables which are combination of those presented here

$$A_T^{(2)} = \frac{2S_3}{(1-F_L)}$$
$$A_T^{Re} = \frac{S_6}{(1-F_L)}$$
$$P'_4 = \frac{S_4}{\sqrt{(1-F_L)F_L}}$$
$$P'_5 = \frac{S_5}{\sqrt{(1-F_L)F_L}}$$
$$P'_6 = \frac{S_7}{\sqrt{(1-F_L)F_L}}$$
$$P'_8 = \frac{S_8}{\sqrt{(1-F_L)F_L}}$$

$$\frac{1}{\Gamma}\frac{d^3(\Gamma+\bar{\Gamma})}{d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = \frac{9}{32\pi}\Big[\frac{3}{4}(1-F_L)\sin^2\theta_K + F_L\cos^2\theta_K + \frac{1}{4}(1-F_L)\sin^2\theta_K\cos 2\theta_\ell$$
$$- F_L\cos^2\theta_K\cos 2\theta_\ell + \frac{1}{2}(1-F_L)A_T^{(2)}\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi +$$
$$\sqrt{F_L(1-F_L)}P'_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi + \sqrt{F_L(1-F_L)}P'_5\sin 2\theta_K\sin\theta_\ell\cos\phi +$$
$$(1-F_L)A_{Re}^T\sin^2\theta_K\cos\theta_\ell + \sqrt{F_L(1-F_L)}P'_6\sin 2\theta_K\sin\theta_\ell\sin\phi +$$
$$\sqrt{F_L(1-F_L)}P'_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + (S/A)_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Big]$$

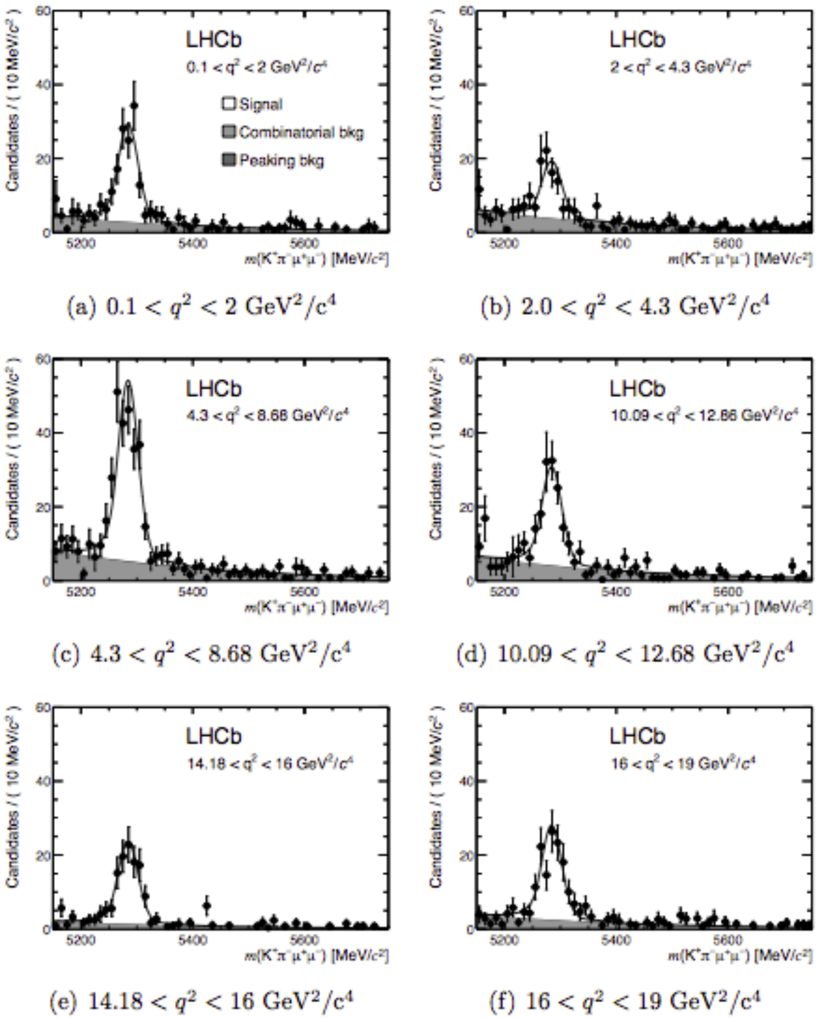
Complementarity

Reproduced from arXiv:1207.2753



- Important to measure all observables
- Example of complementarity: solid red curve and dashed red curve
- For discussion on the complementarity of S_i see Altmannshofer et al. arXiv:0811.1214
- For discussion of P_i complementarity see J. Matias et al. arXiv:1207.2753

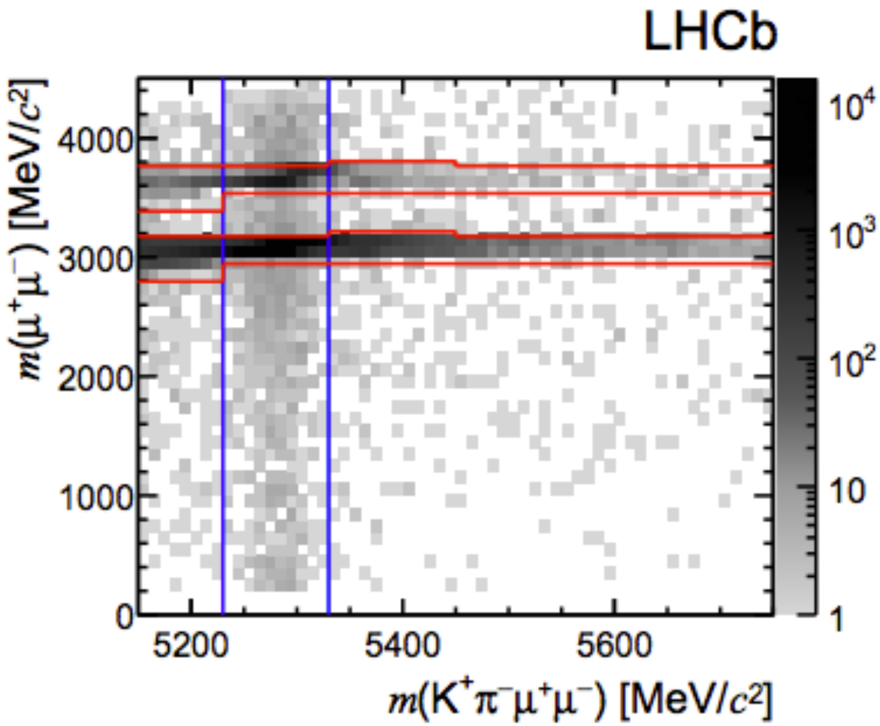
Analysis strategy



(a) $0.1 < q^2 < 2$ GeV^2/c^4
(b) $2.0 < q^2 < 4.3$ GeV^2/c^4
(c) $4.3 < q^2 < 8.68$ GeV^2/c^4
(d) $10.09 < q^2 < 12.68$ GeV^2/c^4
(e) $14.18 < q^2 < 16$ GeV^2/c^4
(f) $16 < q^2 < 19$ GeV^2/c^4

- Signal selected with a BDT (studied to keep the angular acceptance as flat as possible)
- Acceptance corrected in an event-by-event basis with the MC
- Data/MC agreement checked with control channels, e.g. $B^0 \rightarrow J/\psi K^*$
- Analysis in six bins of q^2 + the region $1<q^2<6\,GeV^2$ (theory preferred at large recoil)

Analysis strategy II



Vetoed regions of dimuon mass (q) consistent with charmonium resonances

Several other peaking backgrounds examined and reduced to a negligible level

“Folding technique”

By using the transformation $\phi \rightarrow \phi+\pi$ if $\phi<0$ (arXiv:1304.6325)

$$\frac{1}{\Gamma}\frac{\mathrm{d}^3(\Gamma+\bar{\Gamma})}{\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_K\,\mathrm{d}\phi} = \frac{9}{16\pi}\Big[\frac{3}{4}(1-F_L)\sin^2\theta_K + F_L\cos^2\theta_K + \frac{1}{4}(1-F_L)\sin^2\theta_K\cos 2\theta_\ell - F_L\cos^2\theta_K\cos 2\theta_\ell + \frac{1}{2}(1-F_L)A_T^{(2)}\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + \frac{1}{2}(1-F_L)A_T^{Re}\sin^2\theta_K\cos\theta_\ell + (S/A)_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Big]$$

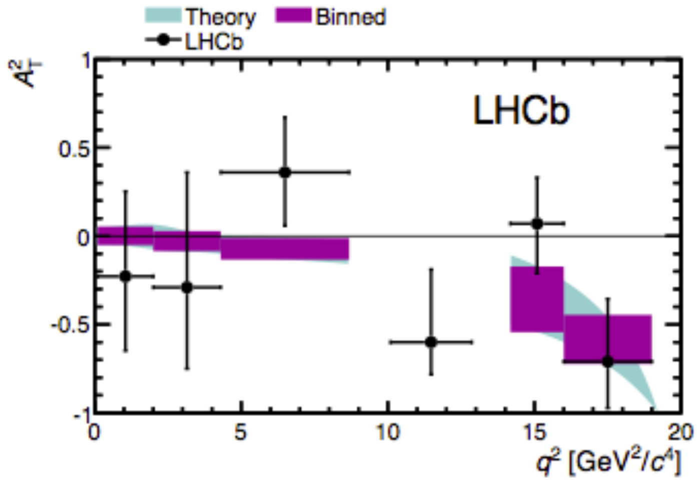
Measurement of the other observables with other folding techniques
For P_5' (or equivalently S_5) $\phi \rightarrow -\phi$ (if $\phi<0$) and $\vartheta_l \rightarrow \pi-\vartheta_l$ (if $\vartheta_l<\pi/2$)

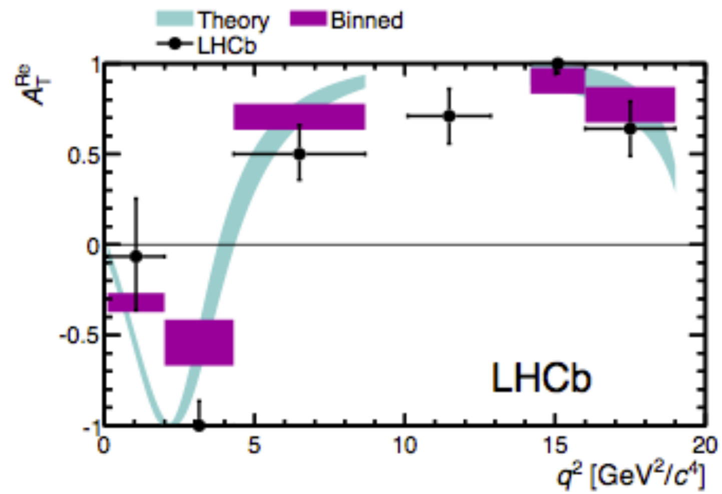
$$\frac{1}{\Gamma}\frac{\mathrm{d}^3(\Gamma+\bar{\Gamma})}{\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_K\,\mathrm{d}\phi} = \frac{9}{8\pi}\Big[\frac{3}{4}(1-F_L)\sin^2\theta_K + F_L\cos^2\theta_K + \frac{1}{4}(1-F_L)\sin^2\theta_K\cos 2\theta_\ell - F_L\cos^2\theta_K\cos 2\theta_\ell + \frac{1}{2}(1-F_L)A_T^{(2)}\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + \sqrt{F_L(1-F_L)}P_5'\sin 2\theta_K\sin\theta_\ell\cos\phi\Big]$$

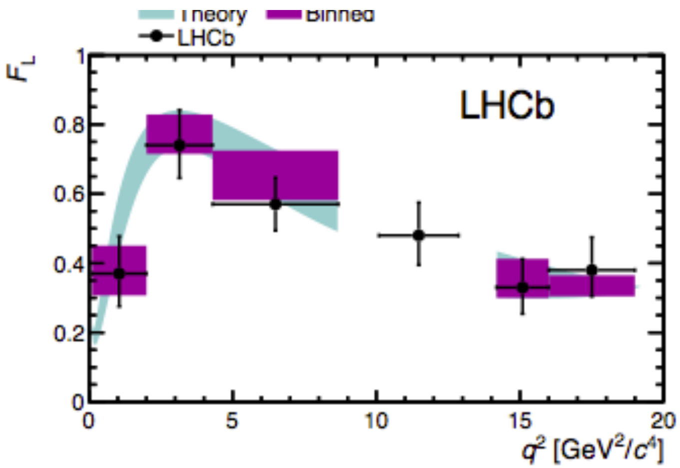
The other transformations for $P_{4,6,8}'$ (or $S_{4,7,8}$) are in the backup slides

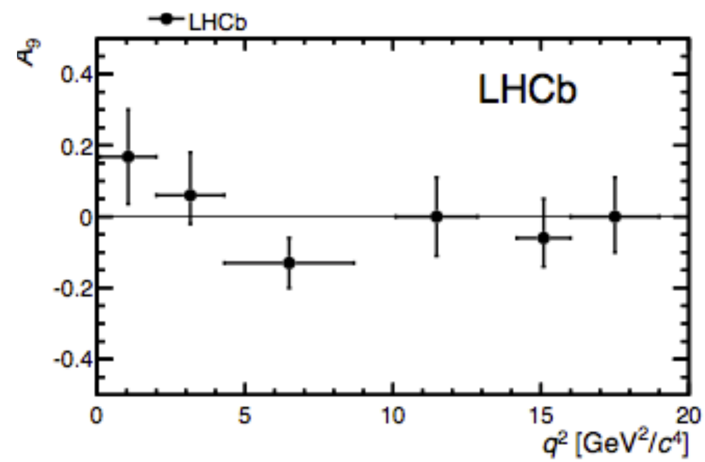
LHCb collaboration (1fb^{-1})
arXiv:1304.6325

Results





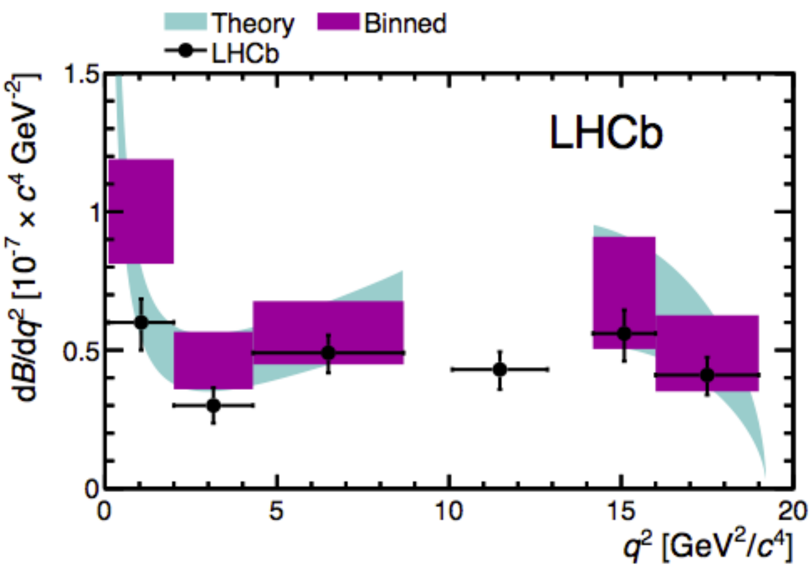


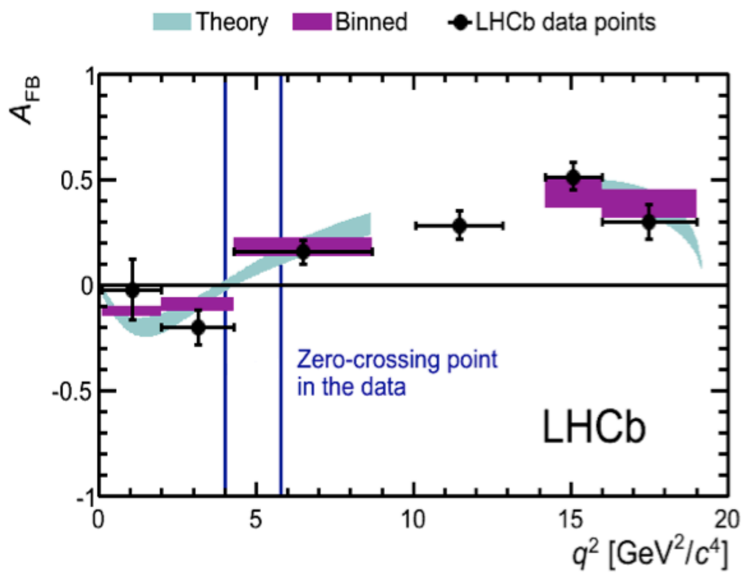


Good agreement with SM predictions
Bobeth-Hiller-Van Dyk (2011); Form-factor from Ball-Zwiky (2005);
Consistent with Matias et al.(2013)

Results

LHCb collaboration (1fb^{-1}), arXiv:1304.6325





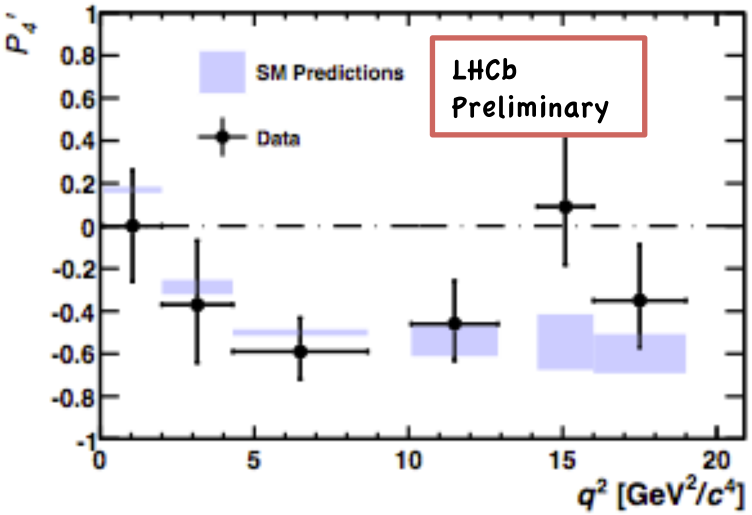
Good agreement with SM predictions

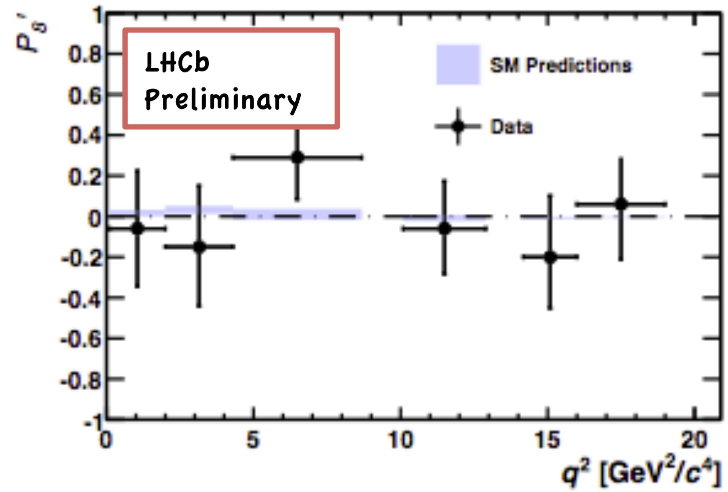
First measurement of the zero-crossing point: q_0^2=4.9 ± 0.9 GeV^2/c^4

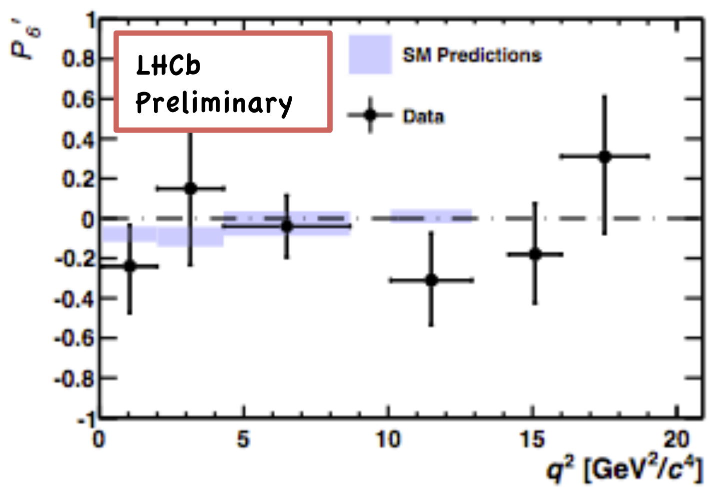
Results for new observables

NEW

LHCb collaboration ($1fb^{-1}$), LHCb-PAPER-2013-037





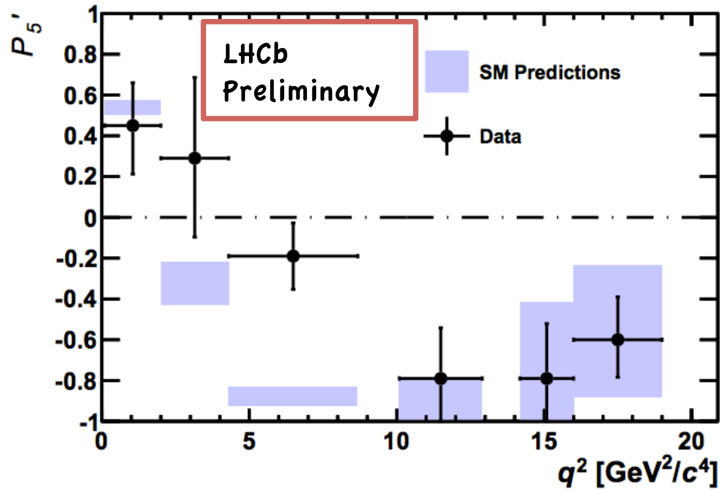


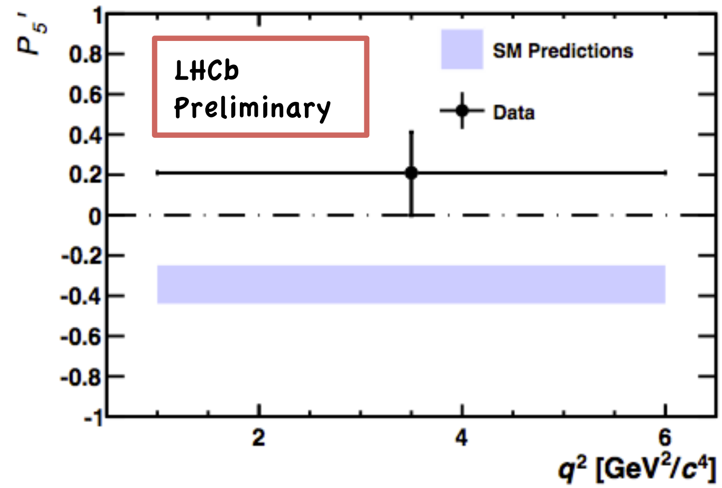
Good agreement with SM predictions (from J.Matias et al. arXiv:1303.5794)

Results for new observables

NEW

LHCb collaboration ($1 fb^{-1}$), LHCb-PAPER-2013-037





- Discrepancy with respect to SM predictions (arXiv:1303.5794) at low q^2
- 3.7 sigma discrepancy in the region $4.3<q^2<8.68$ GeV^2/c^4
- 0.5% probability (2.8 sigma) to observe such a deviation considering 24 independent measurements)
- 2.5 sigma discrepancy in the region $1.0<q^2<6.0$ GeV^2/c^4

N.B.: Jaeger-Camelich (arXiv:1212.2263) have predictions in the region $1.0<q^2<6.0$ GeV^2/c^4 with much larger theoretical error and small shift in the central value (QCD factorization breaking + ccbar loop)

Results for new observables

LHCb Preliminary

q^2 [GeV$^2/c^4$]	P'_4	P'_5	P'_6	P'_8
0.10 – 2.00	$0.00^{+0.26}_{-0.26} \pm 0.03$	$0.45^{+0.19}_{-0.22} \pm 0.09$	$-0.24^{+0.19}_{-0.22} \pm 0.05$	$-0.06^{+0.28}_{-0.28} \pm 0.02$
2.00 – 4.30	$-0.37^{+0.29}_{-0.26} \pm 0.08$	$0.29^{+0.39}_{-0.38} \pm 0.07$	$0.15^{+0.36}_{-0.38} \pm 0.05$	$-0.15^{+0.29}_{-0.28} \pm 0.07$
4.30 – 8.68	$-0.59^{+0.15}_{-0.12} \pm 0.05$	$-0.19^{+0.16}_{-0.16} \pm 0.03$	$-0.04^{+0.15}_{-0.15} \pm 0.05$	$0.29^{+0.17}_{-0.19} \pm 0.03$
10.09 – 12.90	$-0.46^{+0.20}_{-0.17} \pm 0.03$	$-0.79^{+0.16}_{-0.19} \pm 0.19$	$-0.31^{+0.23}_{-0.22} \pm 0.05$	$-0.06^{+0.23}_{-0.22} \pm 0.02$
14.18 – 16.00	$0.09^{+0.35}_{-0.27} \pm 0.04$	$-0.79^{+0.20}_{-0.13} \pm 0.18$	$-0.18^{+0.25}_{-0.24} \pm 0.03$	$-0.20^{+0.30}_{-0.25} \pm 0.03$
16.00 – 19.00	$-0.35^{+0.26}_{-0.22} \pm 0.03$	$-0.60^{+0.19}_{-0.16} \pm 0.09$	$0.31^{+0.38}_{-0.37} \pm 0.10$	$0.06^{+0.26}_{-0.27} \pm 0.03$
1.00 – 6.00	$-0.29^{+0.18}_{-0.16} \pm 0.03$	$0.21^{+0.20}_{-0.21} \pm 0.03$	$-0.18^{+0.21}_{-0.21} \pm 0.03$	$0.23^{+0.18}_{-0.19} \pm 0.02$

q^2 [GeV$^2/c^4$]	S_4	S_5	S_7	S_8
0.10 – 2.00	$-0.01^{+0.12}_{-0.12} \pm 0.03$	$0.22^{+0.09}_{-0.10} \pm 0.04$	$-0.12^{+0.11}_{-0.11} \pm 0.03$	$-0.04^{+0.13}_{-0.12} \pm 0.01$
2.00 – 4.30	$-0.14^{+0.13}_{-0.12} \pm 0.03$	$0.11^{+0.14}_{-0.13} \pm 0.03$	$0.06^{+0.15}_{-0.15} \pm 0.02$	$-0.05^{+0.12}_{-0.12} \pm 0.02$
4.30 – 8.68	$-0.29^{+0.06}_{-0.06} \pm 0.02$	$-0.09^{+0.08}_{-0.08} \pm 0.01$	$-0.03^{+0.07}_{-0.08} \pm 0.04$	$0.13^{+0.08}_{-0.08} \pm 0.01$
10.09 – 12.90	$-0.22^{+0.09}_{-0.08} \pm 0.02$	$-0.40^{+0.08}_{-0.10} \pm 0.10$	$-0.17^{+0.11}_{-0.12} \pm 0.03$	$-0.03^{+0.10}_{-0.10} \pm 0.01$
14.18 – 16.00	$0.05^{+0.14}_{-0.08} \pm 0.01$	$-0.38^{+0.10}_{-0.09} \pm 0.09$	$-0.08^{+0.13}_{-0.14} \pm 0.01$	$-0.10^{+0.13}_{-0.12} \pm 0.02$
16.00 – 19.00	$-0.16^{+0.11}_{-0.09} \pm 0.01$	$-0.29^{+0.09}_{-0.08} \pm 0.04$	$0.15^{+0.16}_{-0.15} \pm 0.03$	$0.03^{+0.12}_{-0.12} \pm 0.02$

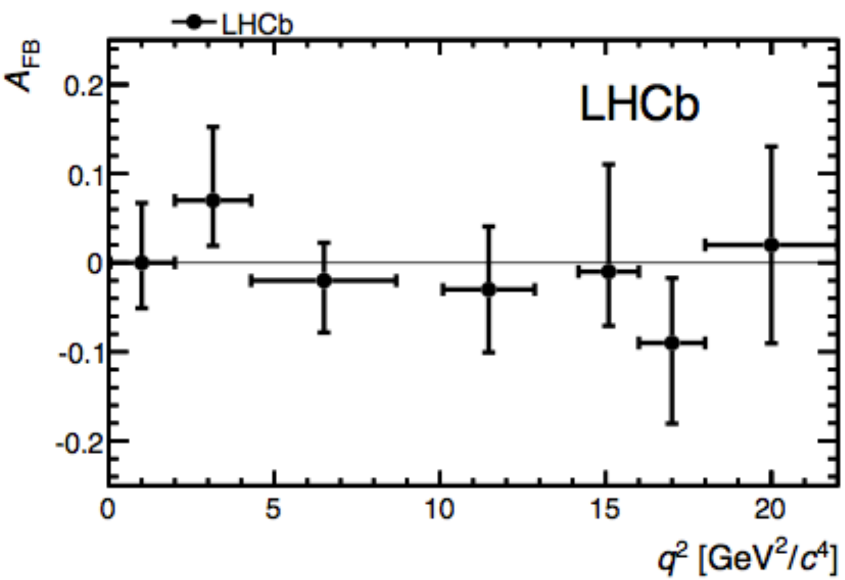
$B^+ \to K^+ \mu\mu$

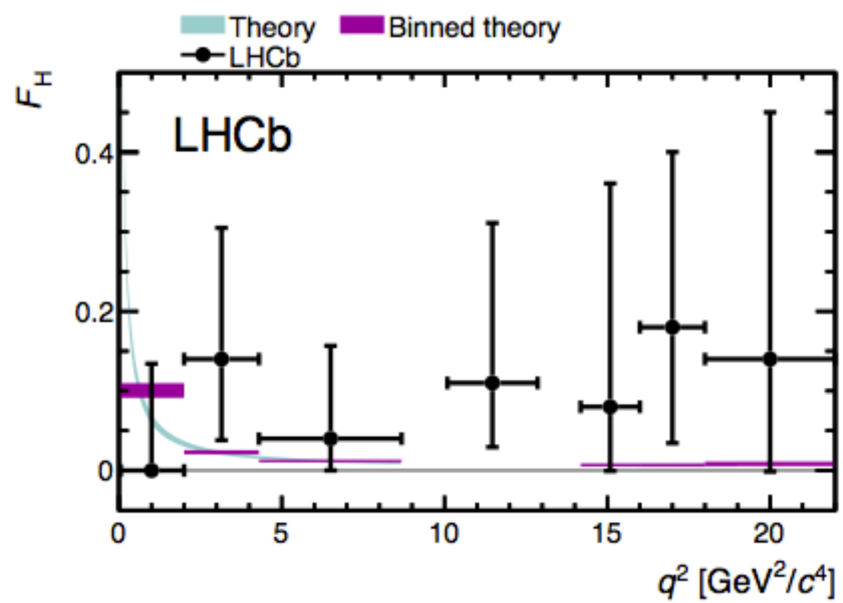
LHCb collaboration ($1 fb^{-1}$)
JHEP 02 (2013) 105

Decay described by a single angle ϑ_l

$$\frac{1}{\Gamma}\frac{d\Gamma[B^+ \to K^+\mu^+\mu^-]}{d\cos\theta_l} = \frac{3}{4}(1-F_H)(1-\cos^2\theta_l) + \frac{1}{2}F_H + A_{FB}\cos\theta_l$$

F_H and A_{FB} sensitive to O_P and O_S
In the SM both observables are about zero

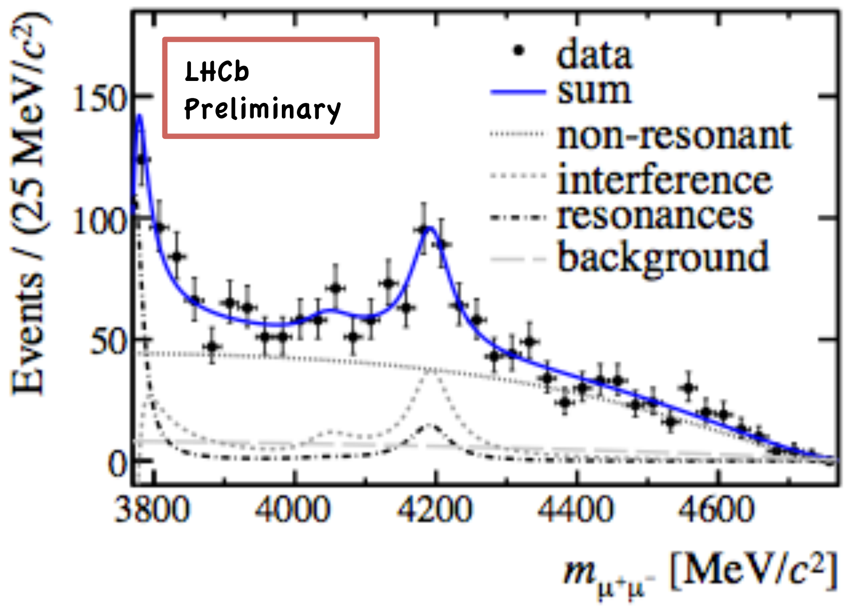


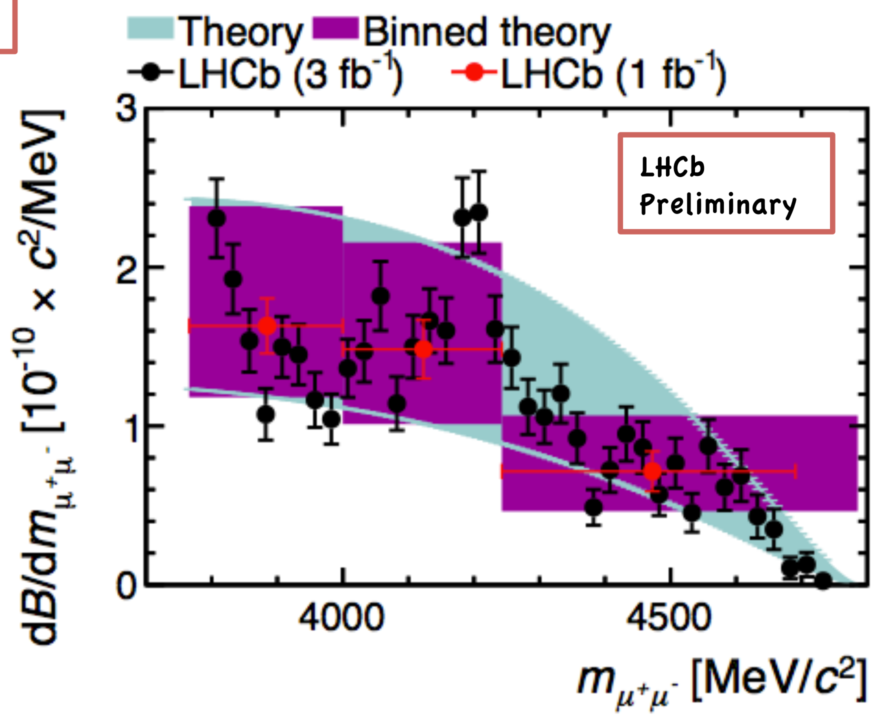


New resonance structure

NEW

LHCb collaboration ($3fb^{-1}$)
LHCb-PAPER-2013-039





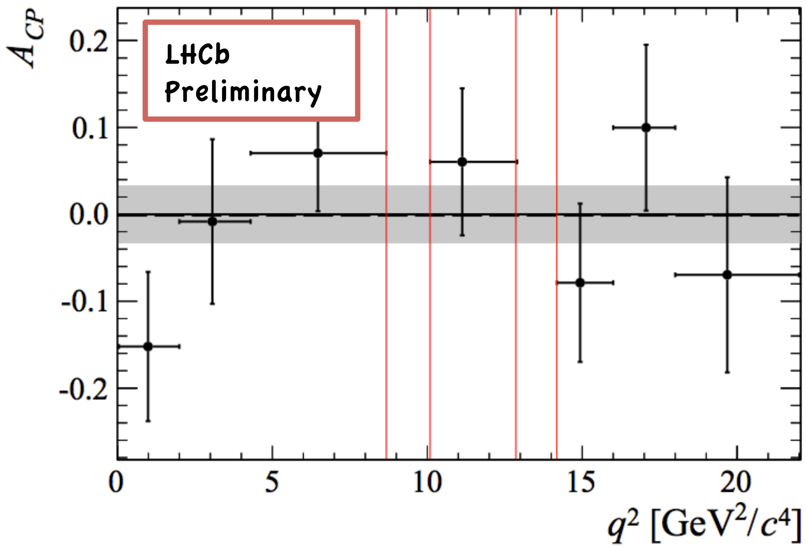
Observation of a resonant structure (ψ(4160)) at high q^2
First observation of $B^+ \to K^+\psi(4160)$ and of $\psi(4160) \to \mu\mu$
Unexpected result → large influence on EWP at low recoil

See talk by Giovanni Carboni

Time-integrated CP Violation

NEW

LHCb collaboration ($1fb^{-1}$)
LHCb-PAPER-2013-043



Production and detection asymmetry taken into account by using the $B^+ \to J/\psi K^+$ decay

$$A_{CP}(K^+\mu\mu) = A_{RAW}(K^+\mu\mu) - A_{RAW}(J/\psi K^+)$$

Left-right detector asymmetry removed averaging magnet polarities

LHCb Preliminary:
$A_{CP}(K^+\mu\mu) = 0.000 \pm 0.033(stat) \pm 0.005(syst) \pm 0.007\ (norm)$

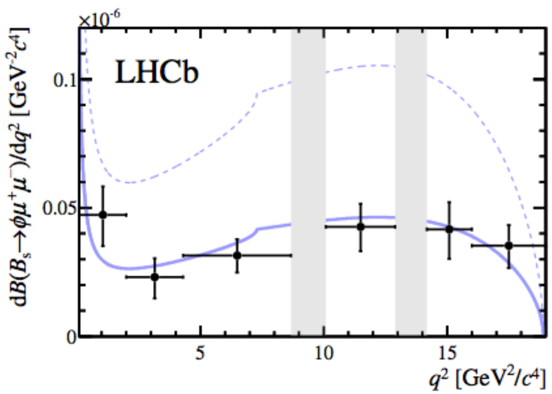
$B_s^0 \to \phi \mu\mu$

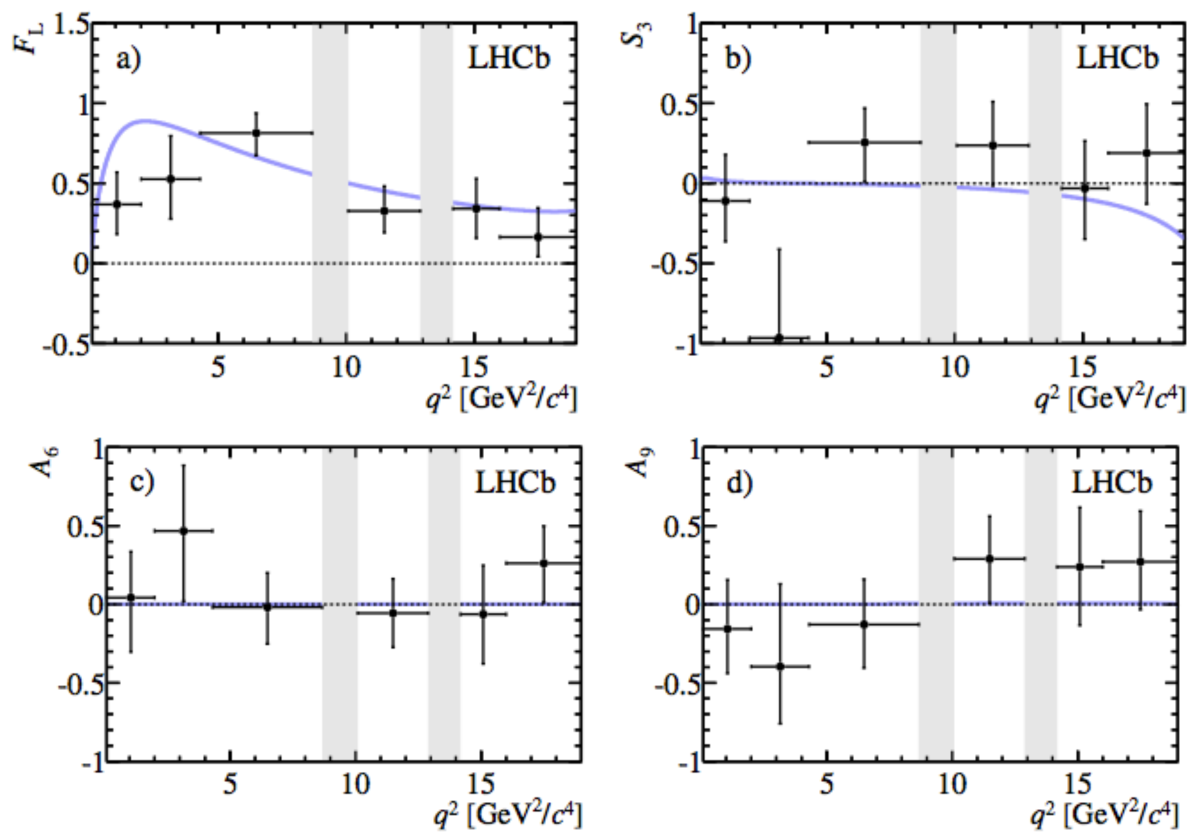
LHCb collaboration ($1fb^{-1}$)
arXiv:1305.2168

Similar to $B^0 \to K^* \mu\mu$, but not self tagging.

for certain angular terms only A are accessible!

See Altmannshofer et al. (2008) and Bobeth-Hiller-Piranishvili (2008)



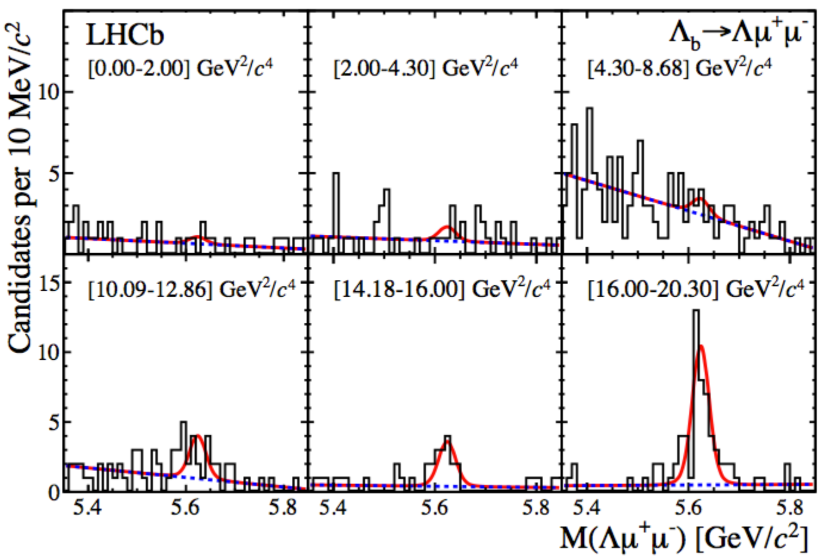


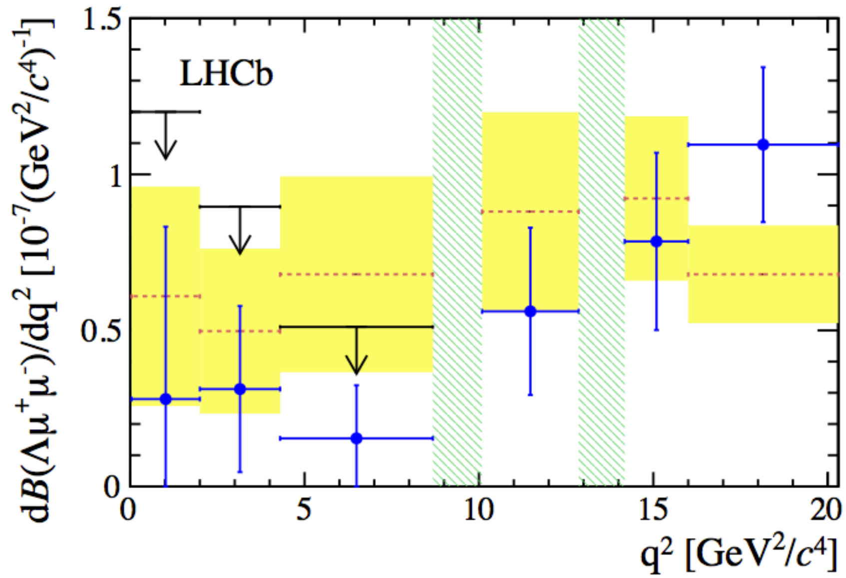
- Total BR lower than predictions
- Good agreement for the angular observables

$\Lambda_b \to \Lambda \mu\mu$

LHCb collaboration ($1 fb^{-1}$), arXiv:1306.2577

- Additional complexity/observables wrt mesons (Hiller et al. 2007 and ref therein)
- First step measuring the yield
- Decay $\Lambda_b \to \Lambda J/\psi$ used as a control channel





Predictions from Detmold et al. (2012)

Observed about 80 events in the whole q^2 region
$BR(\Lambda_b \to \Lambda \mu\mu) = (0.96 \pm 0.16(stat) \pm 0.13(syst) \pm 0.21(norm)) \times 10^{-6}$

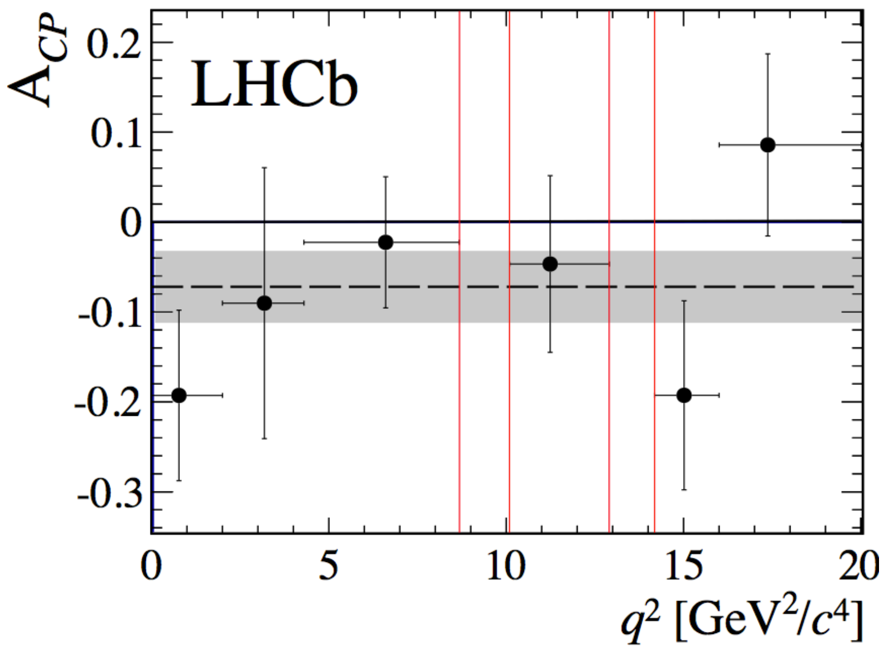
Conclusions

- LHCb has new results on rare electroweak penguins
- Generally good agreement with the SM, apart for a local discrepancy in the low q^2 region in the observable P_5' in the decay $B^0 \rightarrow K^* \mu\mu$
- Most of the results are obtained with $1fb^{-1}$… stay tuned for the $3fb^{-1}$ analyses

Backup slides

Time-integrated CP Violation

Phys. Rev. Lett. 110 (2013) 031801



$A_{CP}(K^*\mu\mu)$ = -0.072 ± 0.040 ± 0.005

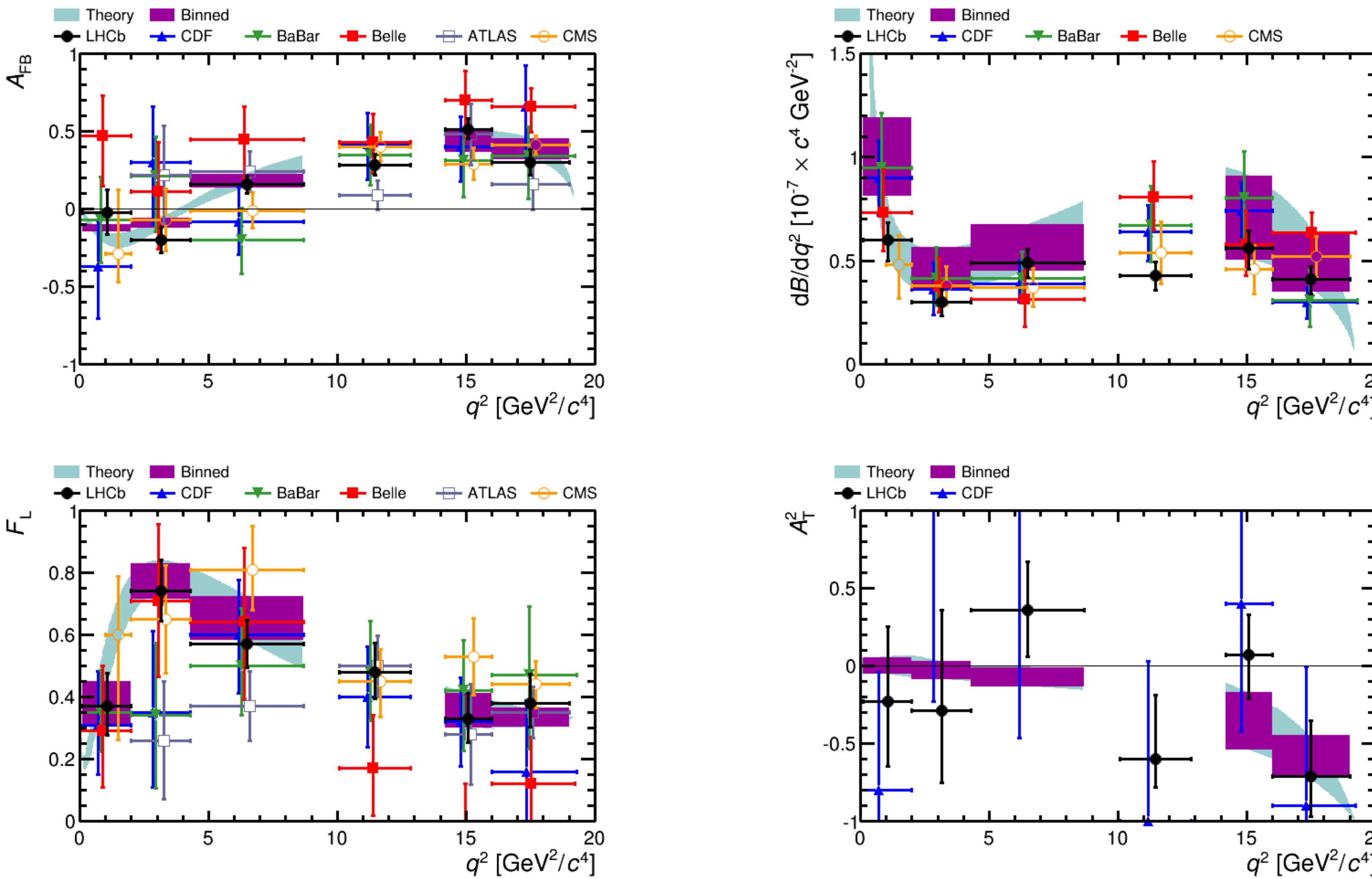
Production and detection asymmetry taken into account by using the $B^0 \rightarrow J/\psi K^*$ decay

$A_{CP}(K^*\mu\mu) \approx A_{RAW}(K^*\mu\mu) - A_{RAW}(J/\psi K^*)$

Kinematic difference taken into account by reweighting

Left-right detector asymmetry removed averaging magnet polarities

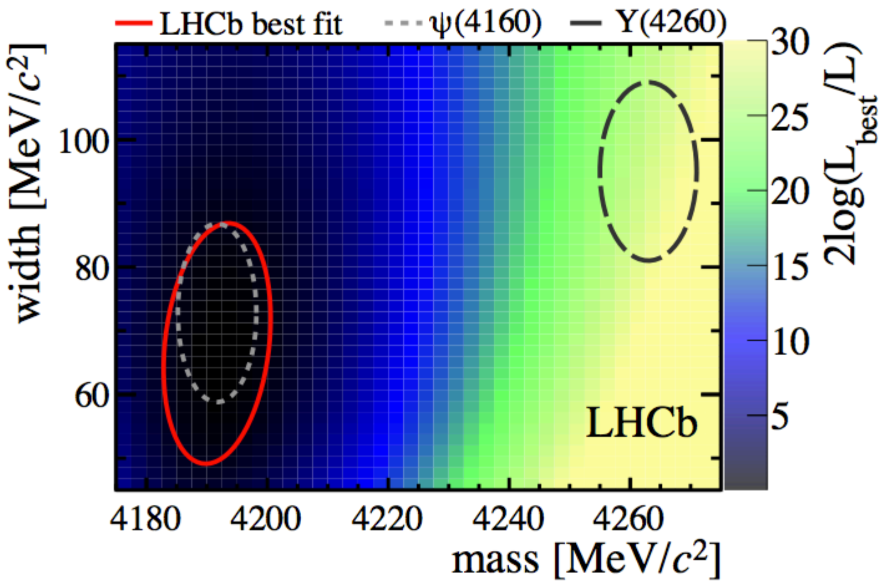
Comparison with other experiments



New resonance structure

$$\mathcal{P}_{\text{sig}} \propto P(m_{\mu^+\mu^-})\,|\mathcal{A}|^2\, f^2(m^2_{\mu^+\mu^-})\,,$$
$$|\mathcal{A}|^2 = |A^{\text{V}}_{\text{nr}} + \sum_k e^{i\delta_k} A^k_{\text{r}}|^2 + |A^{\text{AV}}_{\text{nr}}|^2\,,$$

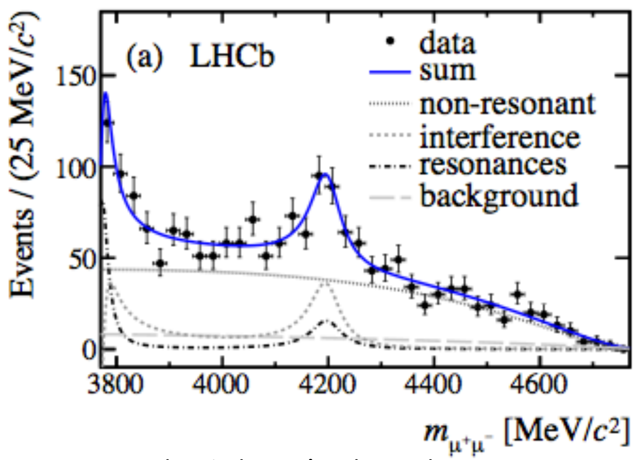
	Unconstrained	$\psi(4160)$
$\mathcal{B}[\times 10^{-9}]$	$3.9\,^{+0.7}_{-0.6}$	$3.5\,^{+0.9}_{-0.8}$
Mass [MeV/c^2]	$4191\,^{+9}_{-8}$	4190 ± 5
Width [MeV/c^2]	$65\,^{+22}_{-16}$	66 ± 12
Phase [rad]	-1.7 ± 0.3	-1.8 ± 0.3



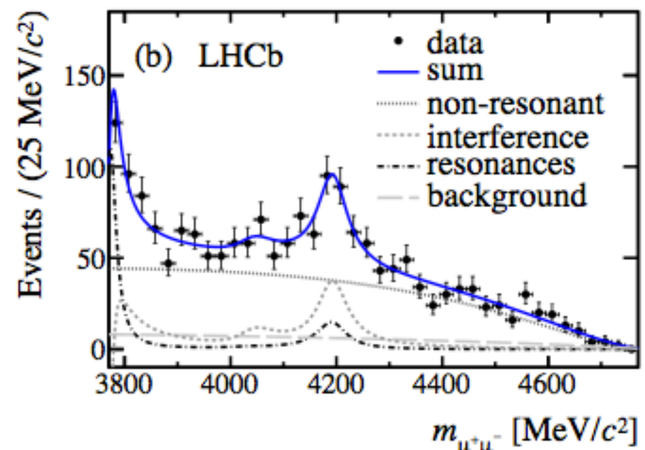
- Significance of the new structure exceed 6 sigmas
- Compatible with hypothesis of ψ(4160)
- Hypothesis of Y(4260) disfavored at 4 sigmas

Fit with various hypotheses

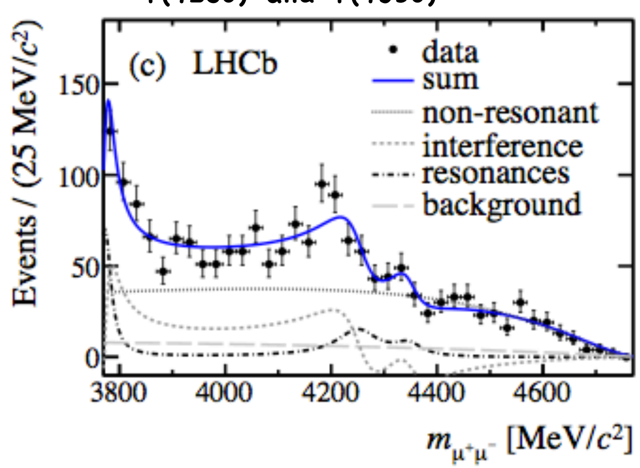
Free extra resonance



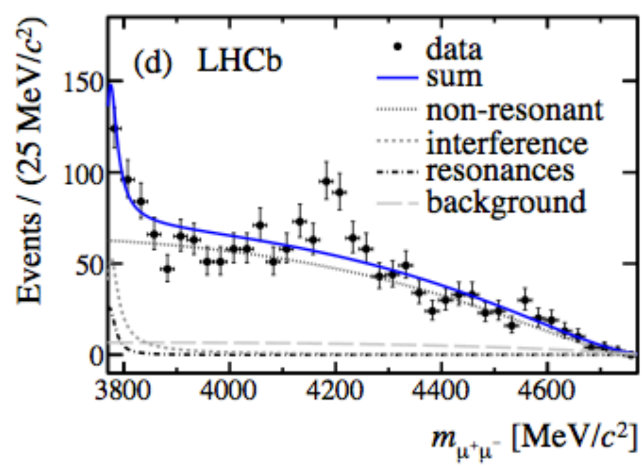
ψ(4040) and ψ(4160)



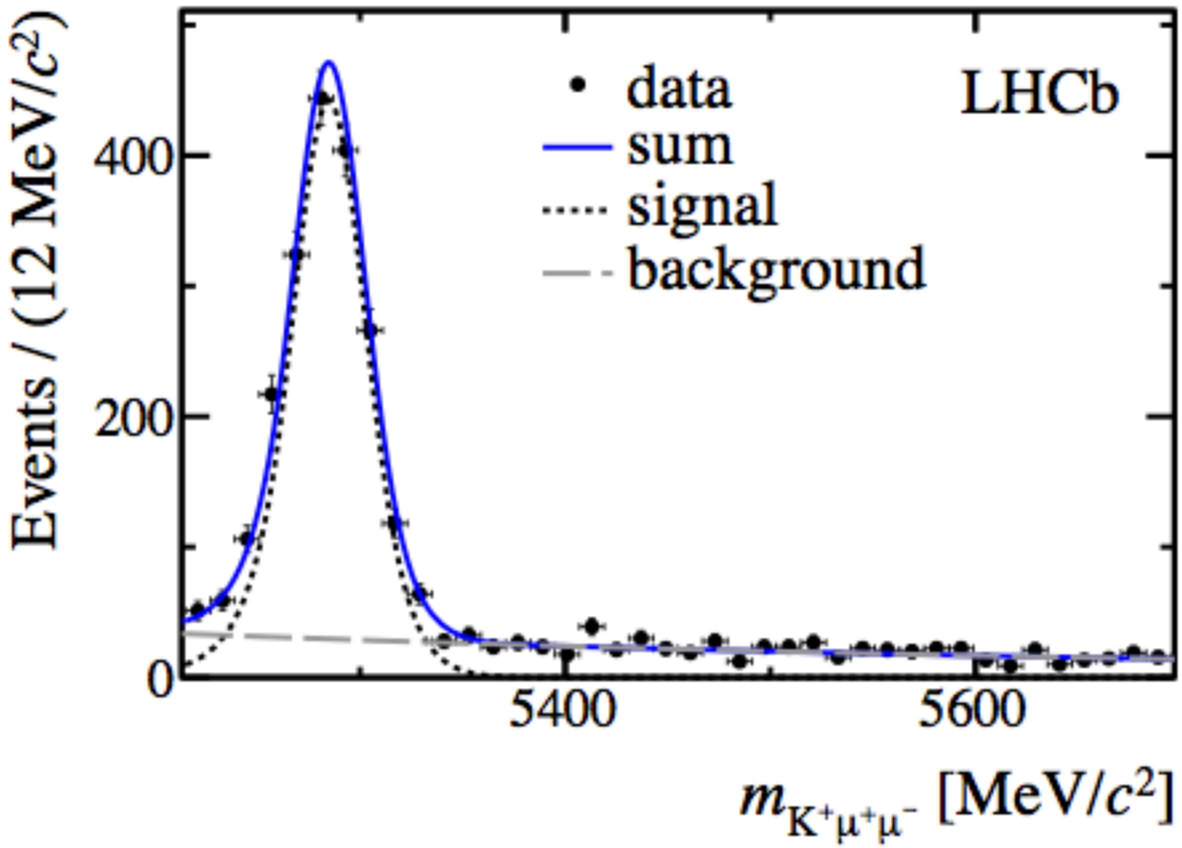
Y(4260) and Y(4350)



Non resonance only

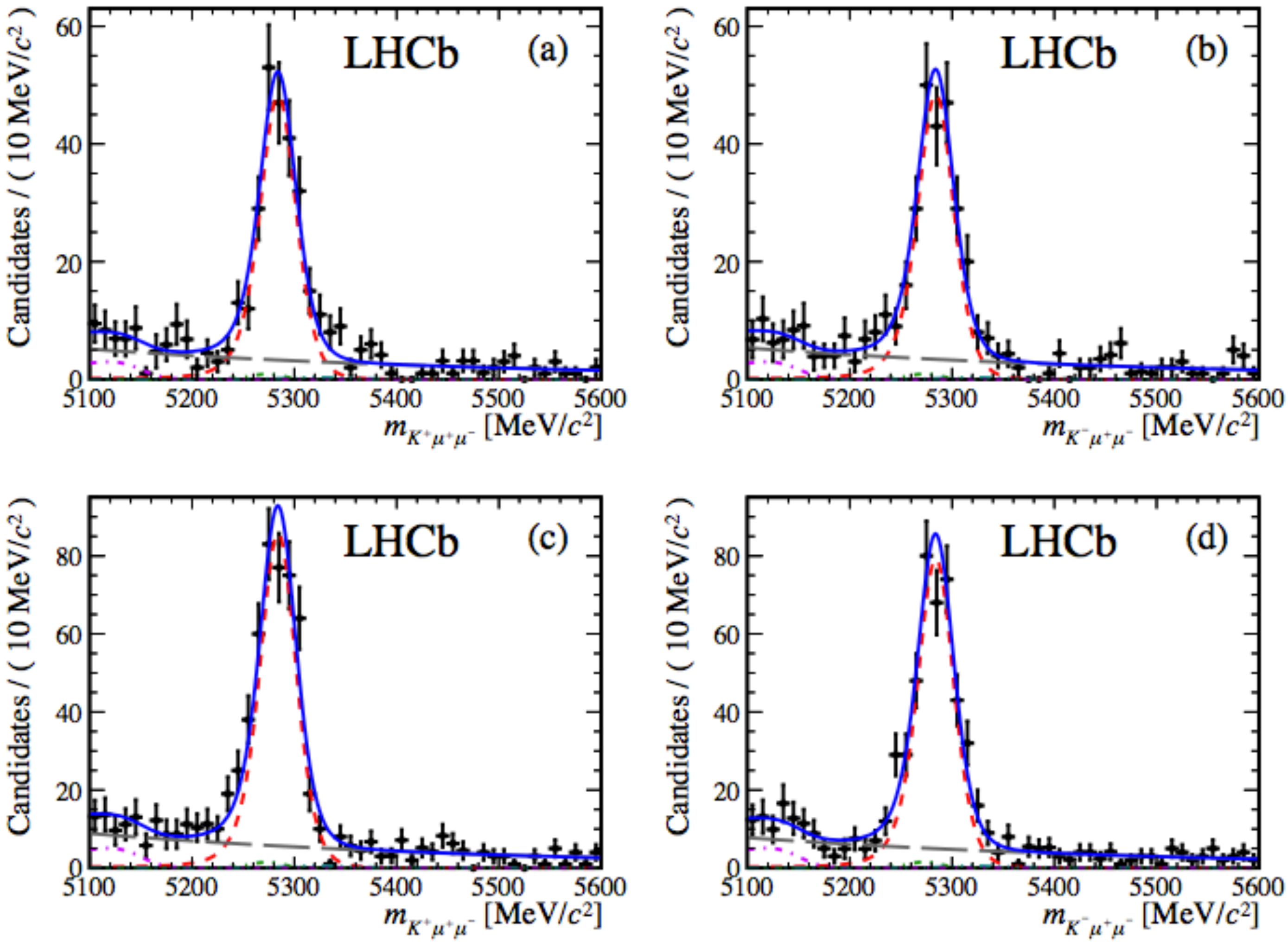


B-mass fit

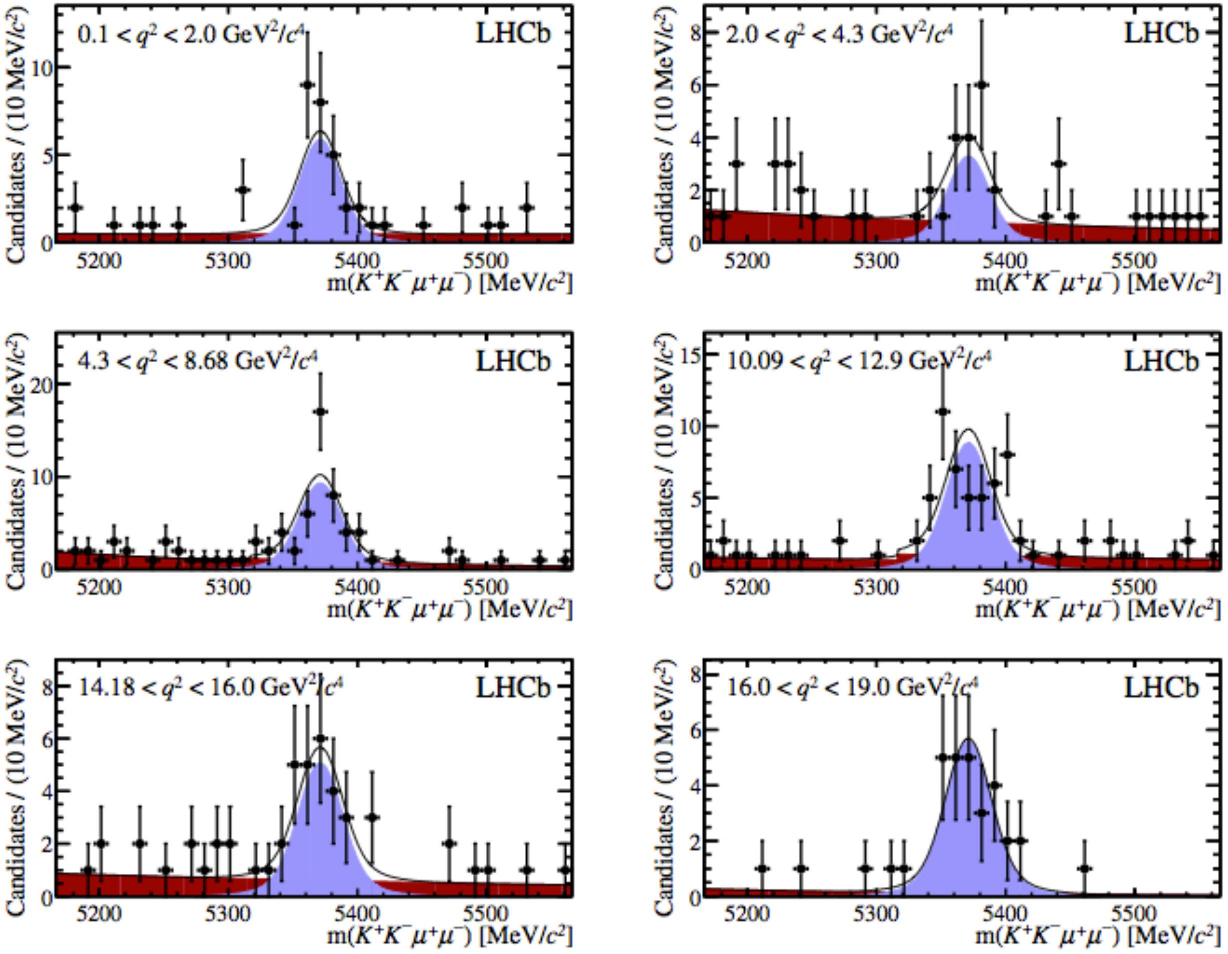


Events / (12 MeV/c^2)
400
200
0
data
sum
signal
background
LHCb
5400
5600
$m_{K^+\mu^+\mu^-}$ [MeV/c^2]

A_{CP} $(B^+ \rightarrow K^+ \mu\mu)$



$B_s^0 \to \phi\mu\mu$



$B_s^0 \to \phi\mu\mu$

$$\frac{1}{\mathrm{d}\Gamma/\mathrm{d}q^2}\frac{\mathrm{d}^4\Gamma}{\mathrm{d}q^2\mathrm{d}\cos\theta_\ell\mathrm{d}\cos\theta_K\mathrm{d}\Phi} = \frac{9}{32\pi}\Big[S_1^s\sin^2\theta_K + S_1^c\cos^2\theta_K + S_2^s\sin^2\theta_K\cos 2\theta_\ell + S_2^c\cos^2\theta_K\cos 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\Phi + S_4\sin 2\theta_K\sin 2\theta_\ell\cos\Phi + A_5\sin 2\theta_K\sin\theta_\ell\cos\Phi + A_6\sin^2\theta_K\cos\theta_\ell + S_7\sin 2\theta_K\sin\theta_\ell\sin\Phi + A_8\sin 2\theta_K\sin 2\theta_\ell\sin\Phi + A_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\Phi\Big]$$

Projecting over the three angles separately:

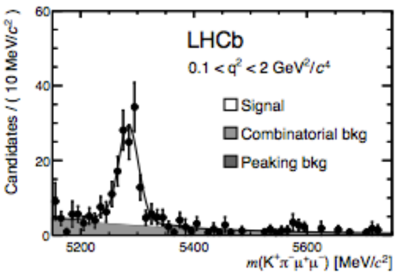
$$\frac{1}{\mathrm{d}\Gamma/\mathrm{d}q^2}\frac{\mathrm{d}^2\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_K} = \frac{3}{4}(1-F_\mathrm{L})(1-\cos^2\theta_K) + \frac{3}{2}F_\mathrm{L}\cos^2\theta_K,$$

$$\frac{1}{\mathrm{d}\Gamma/\mathrm{d}q^2}\frac{\mathrm{d}^2\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_\ell} = \frac{3}{8}(1-F_\mathrm{L})(1+\cos^2\theta_\ell) + \frac{3}{4}F_\mathrm{L}(1-\cos^2\theta_\ell) + \frac{3}{4}A_6\cos\theta_\ell,$$

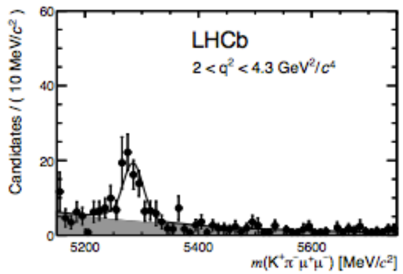
$$\frac{1}{\mathrm{d}\Gamma/\mathrm{d}q^2}\frac{\mathrm{d}^2\Gamma}{\mathrm{d}q^2\,\mathrm{d}\Phi} = \frac{1}{2\pi} + \frac{1}{2\pi}S_3\cos 2\Phi + \frac{1}{2\pi}A_9\sin 2\Phi,$$

New observables $B^0 \rightarrow K^*\mu\mu$

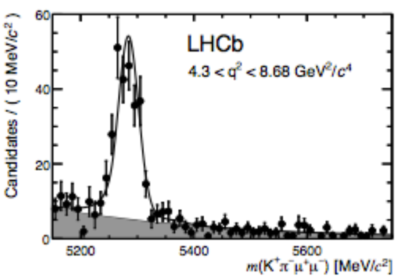
B-mass plots



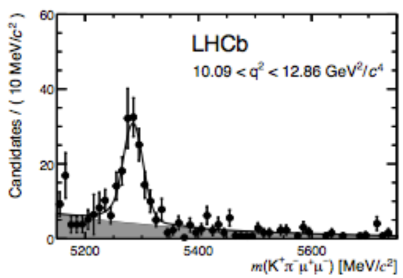
(a) $0.1 < q^2 < 2\ \mathrm{GeV}^2/c^4$



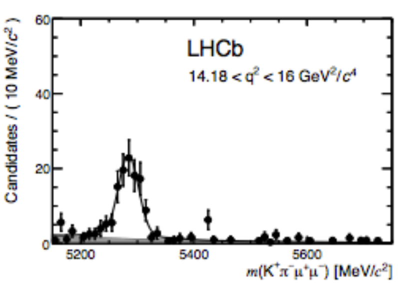
(b) $2.0 < q^2 < 4.3\ \mathrm{GeV}^2/c^4$



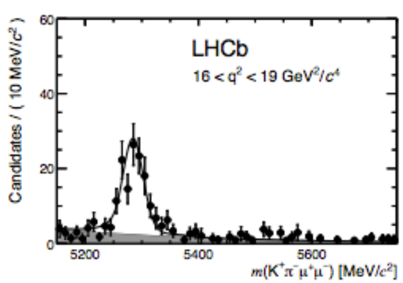
(c) $4.3 < q^2 < 8.68\ \mathrm{GeV}^2/c^4$



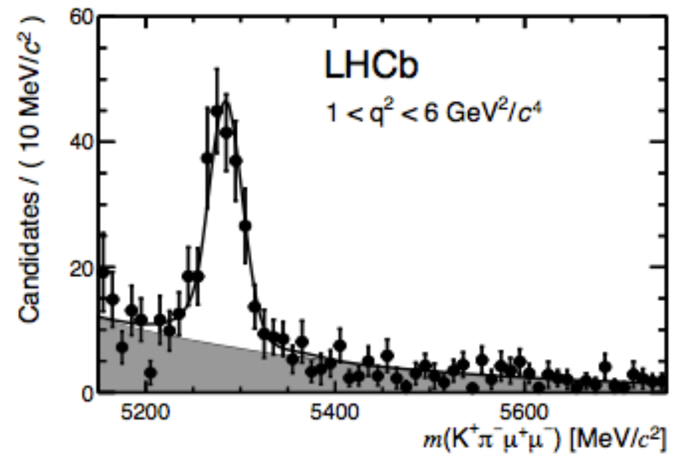
(d) $10.09 < q^2 < 12.68\ \mathrm{GeV}^2/c^4$



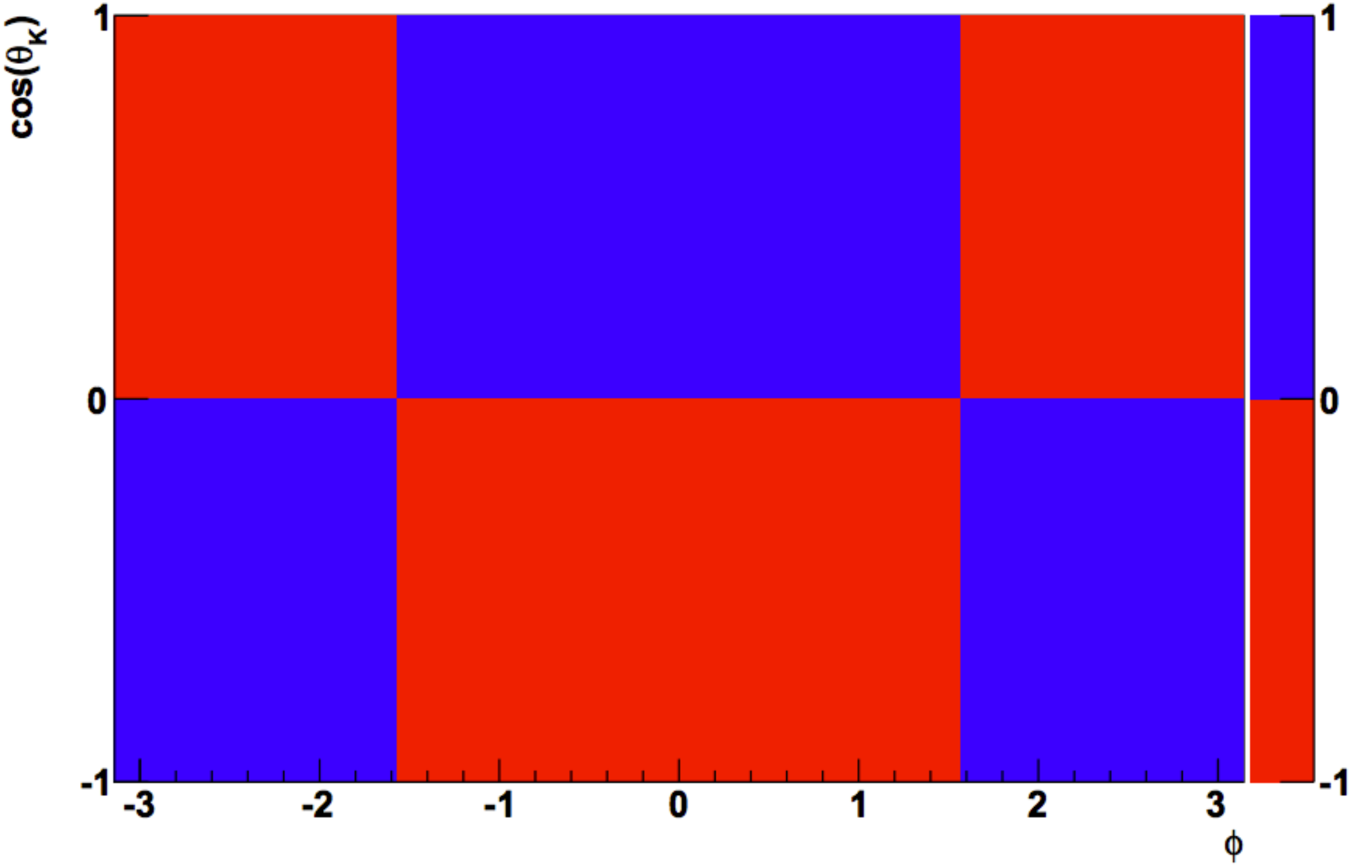
(e) $14.18 < q^2 < 16\ \mathrm{GeV}^2/c^4$



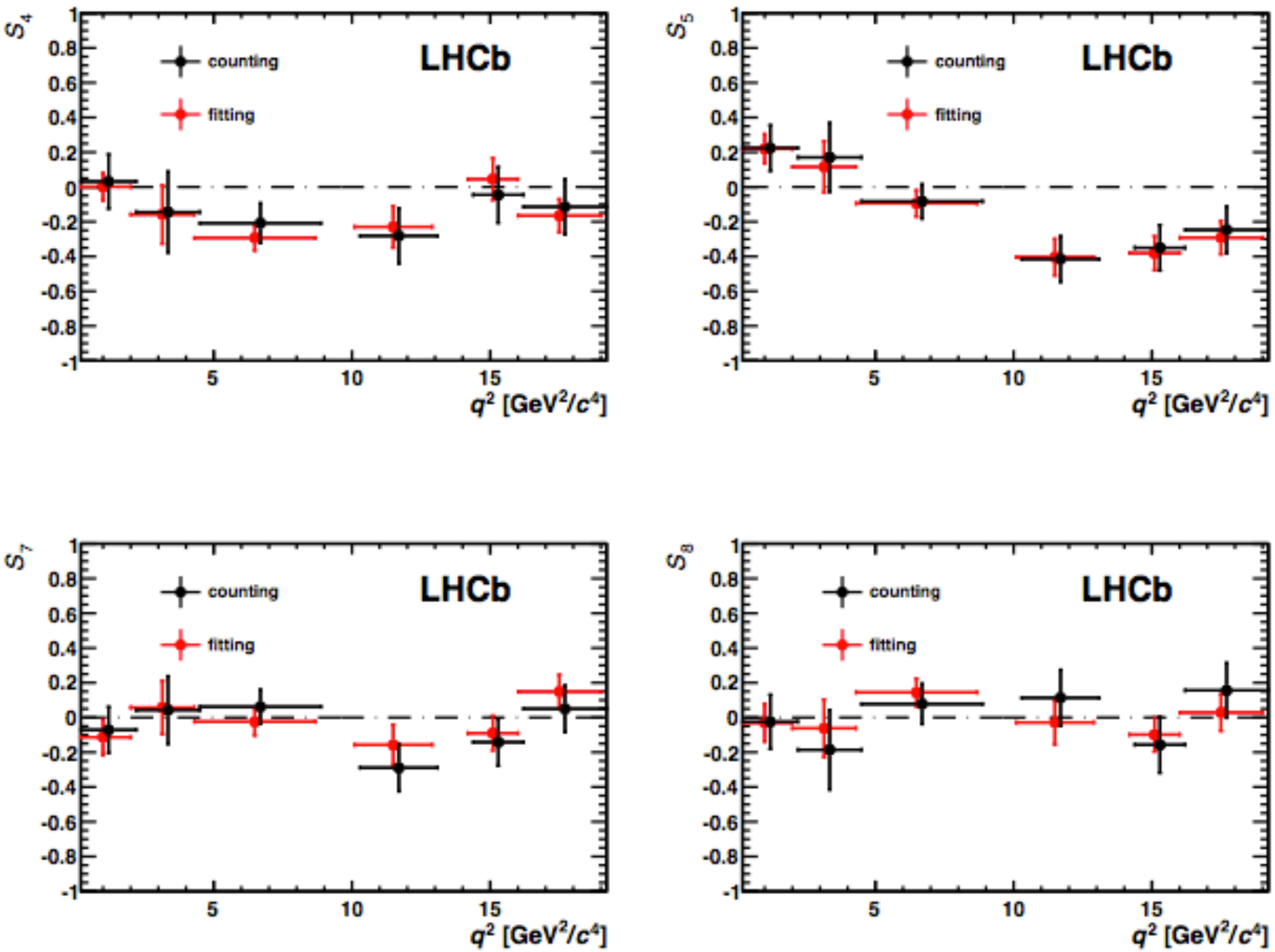
(f) $16 < q^2 < 19\ \mathrm{GeV}^2/c^4$



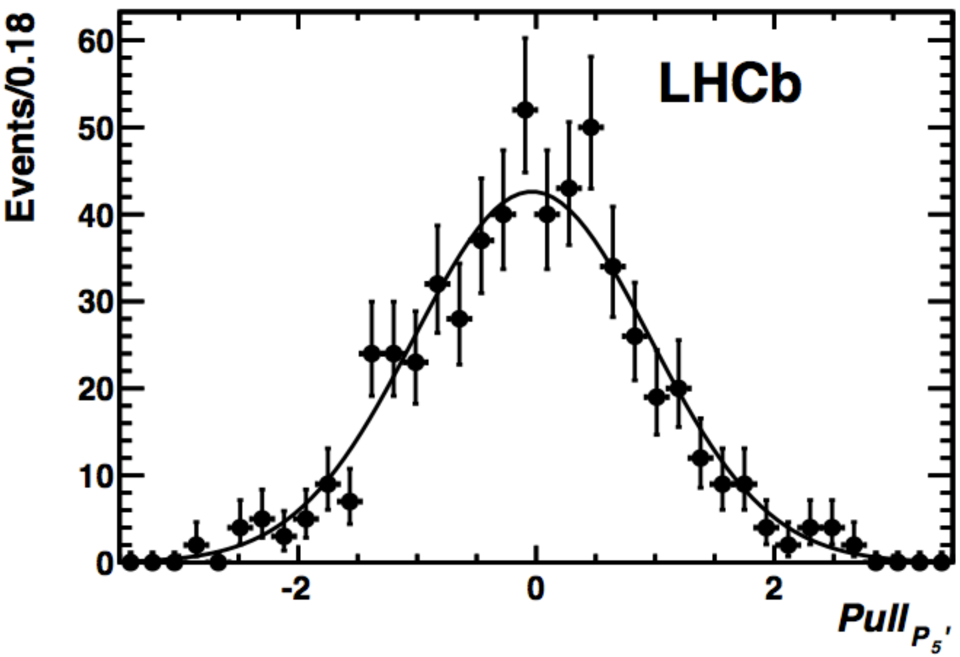
S_5 counting



Counting VS fitting



X-checks with $B^0 \to J/\psi K^*$



- Procedure repeated with the full sample of $B^0 \to J/\psi K^*$
- Procedure repeated chopping off the full sample in about 300 sub-samples

Foldings

$$P'_4,\ S_4:\ \begin{cases} \phi \to -\phi & \text{for } \phi < 0 \\ \phi \to \pi - \phi & \text{for } \theta_\ell > \pi/2 \\ \theta_\ell \to \pi - \theta_\ell & \text{for } \theta_\ell > \pi/2, \end{cases}$$

$$P'_5,\ S_5:\ \begin{cases} \phi \to -\phi & \text{for } \phi < 0 \\ \theta_\ell \to \pi - \theta_\ell & \text{for } \theta_\ell > \pi/2, \end{cases}$$

$$P'_6,\ S_7:\ \begin{cases} \phi \to \pi - \phi & \text{for } \phi > \pi/2 \\ \phi \to -\pi - \phi & \text{for } \phi < -\pi/2 \\ \theta_\ell \to \pi - \theta_\ell & \text{for } \theta_\ell > \pi/2, \end{cases}$$

$$P'_8,\ S_8:\ \begin{cases} \phi \to \pi - \phi & \text{for } \phi > \pi/2 \\ \phi \to -\pi - \phi & \text{for } \phi < -\pi/2 \\ \theta_K \to \pi - \theta_K & \text{for } \theta_\ell > \pi/2 \\ \theta_\ell \to \pi - \theta_\ell & \text{for } \theta_\ell > \pi/2. \end{cases}$$

Pdfs to fit

$$Pdf = \frac{9}{8\pi}\left[\frac{3}{4}(1-F_L)\sin^2\theta_K + F_L\cos^2\theta_K + \frac{1}{4}(1-F_L)\sin^2\theta_K\cos 2\theta_\ell - F_L\cos^2\theta_K\cos 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + \sqrt{F_L(1-F_L)}P_5'\sin 2\theta_K\sin\theta_\ell\cos\phi\right]$$

$$Pdf = \frac{9}{8\pi}\left[\frac{3}{4}(1-F_L)\sin^2\theta_K + F_L\cos^2\theta_K + \frac{1}{4}(1-F_L)\sin^2\theta_K\cos 2\theta_\ell - F_L\cos^2\theta_K\cos 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + \sqrt{F_L(1-F_L)}P_4'\sin 2\theta_K\sin 2\theta_\ell\cos\phi\right]$$

$$Pdf = \frac{9}{8\pi}\left[\frac{3}{4}(1-F_L)\sin^2\theta_K + F_L\cos^2\theta_K + \frac{1}{4}(1-F_L)\sin^2\theta_K\cos 2\theta_\ell - F_L\cos^2\theta_K\cos 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + \sqrt{F_L(1-F_L)}P_6'\sin 2\theta_K\sin\theta_\ell\sin\phi\right]$$

$$Pdf = \frac{9}{8\pi}\left[\frac{3}{4}(1-F_L)\sin^2\theta_K + F_L\cos^2\theta_K + \frac{1}{4}(1-F_L)\sin^2\theta_K\cos 2\theta_\ell - F_L\cos^2\theta_K\cos 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + \sqrt{F_L(1-F_L)}P_8'\sin 2\theta_K\sin 2\theta_\ell\sin\phi\right]$$

S-wave

Differential decay rate

$$(1 - F_{\rm S})W_{\rm P} + \frac{9}{32\pi}\left(W_{\rm S} + W_{\rm SP}\right)$$

S-wave

$$W_{\rm S} = \frac{2}{3}F_{\rm S}\sin^2\theta_\ell$$

Interference terms

$$\frac{4}{3}A_{\rm S}\sin^2\theta_\ell\cos\theta_K + A_{\rm S}^{(4)}\sin\theta_K\sin 2\theta_\ell\cos\phi +$$
$$A_{\rm S}^{(5)}\sin\theta_K\sin\theta_\ell\cos\phi + A_{\rm S}^{(7)}\sin\theta_K\sin\theta_\ell\sin\phi$$
$$+A_{\rm S}^{(8)}\sin\theta_K\sin 2\theta_\ell\sin\phi\ . \qquad (9)$$